Holographic Kibble-Zurek Mechanism with Discrete Symmetry Breaking

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Holographic applications: from quantum realms to the big bang

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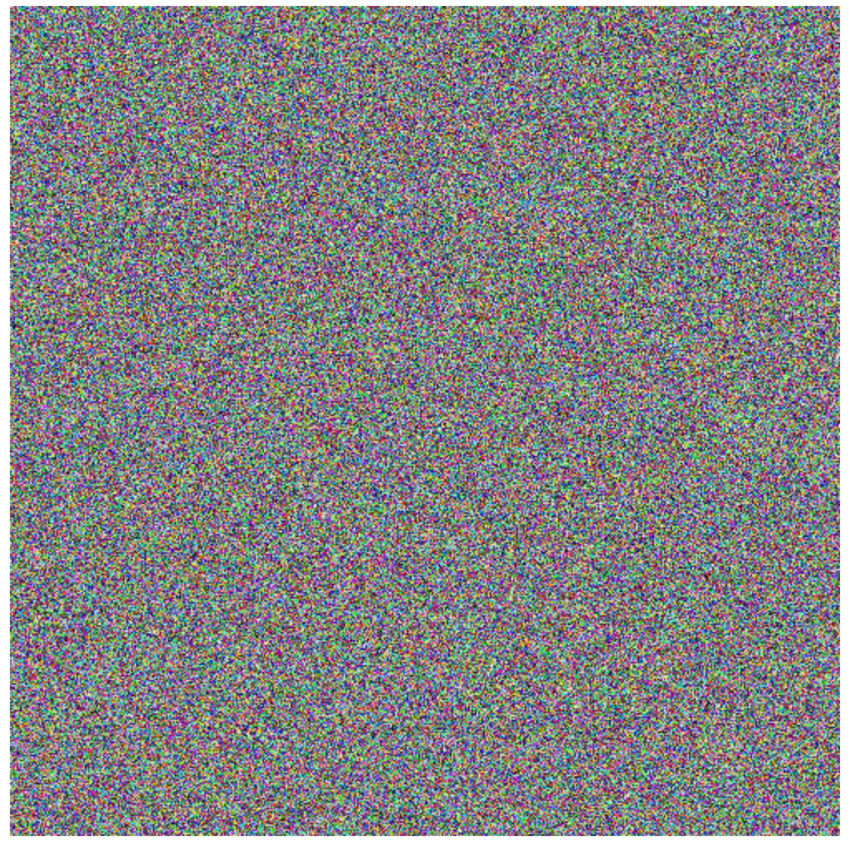
- **Summary**

Kibble-Zurek mechanism (KZM)

Tom W.B. Kibble

Wojciech H. Zurek

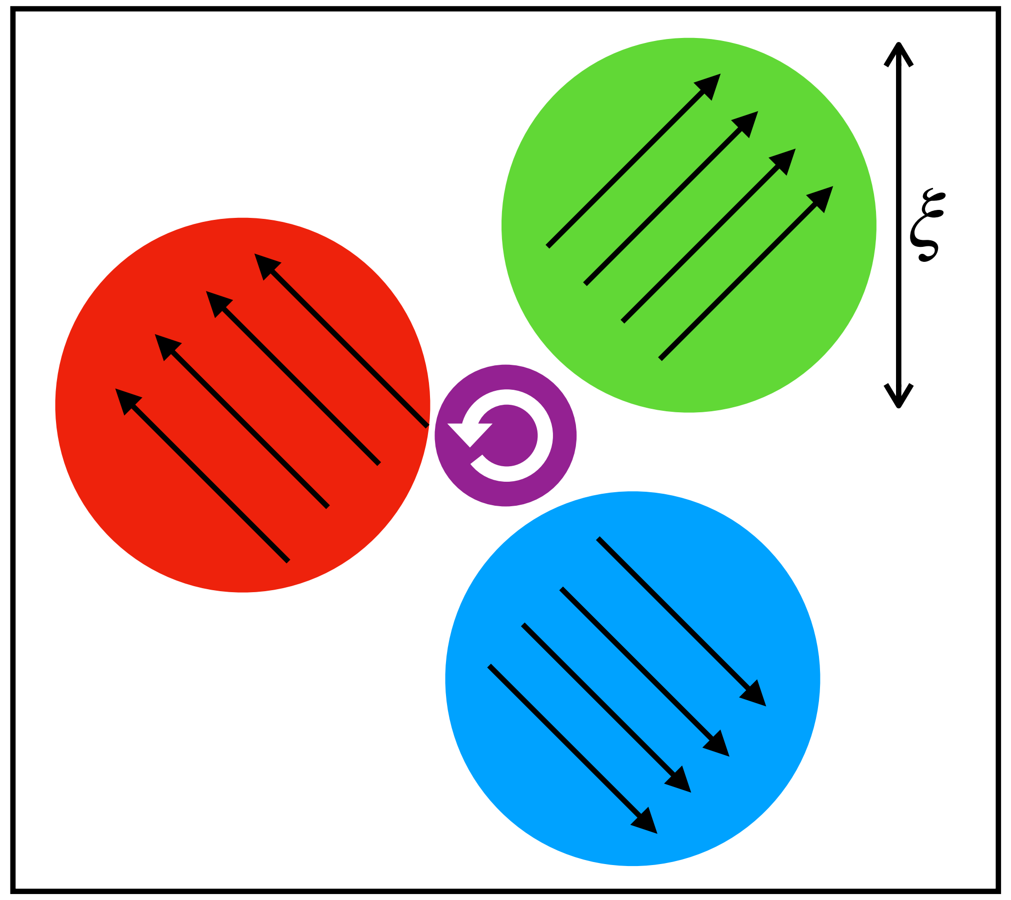
- **KZM: topological defects number vs quench rate**



linear quench

→

across critical point



disordered

ordered

- Near critical point of continuous phase transition

$$\xi \propto |\epsilon|^{-\nu}, \quad \tau \propto |\epsilon|^{-z\nu}. \qquad \epsilon = 1 - T/T_c = t/\tau_Q$$

coherence length   relaxation time

- KZM predicts a universal power law relation between the *number density of topological defects* and the *quench rate* τ_Q

$$n \propto \left(\tau_Q\right)^{\frac{-(D-d)\nu}{1+z\nu}}$$

D: dimension of space
d: dimension of defects

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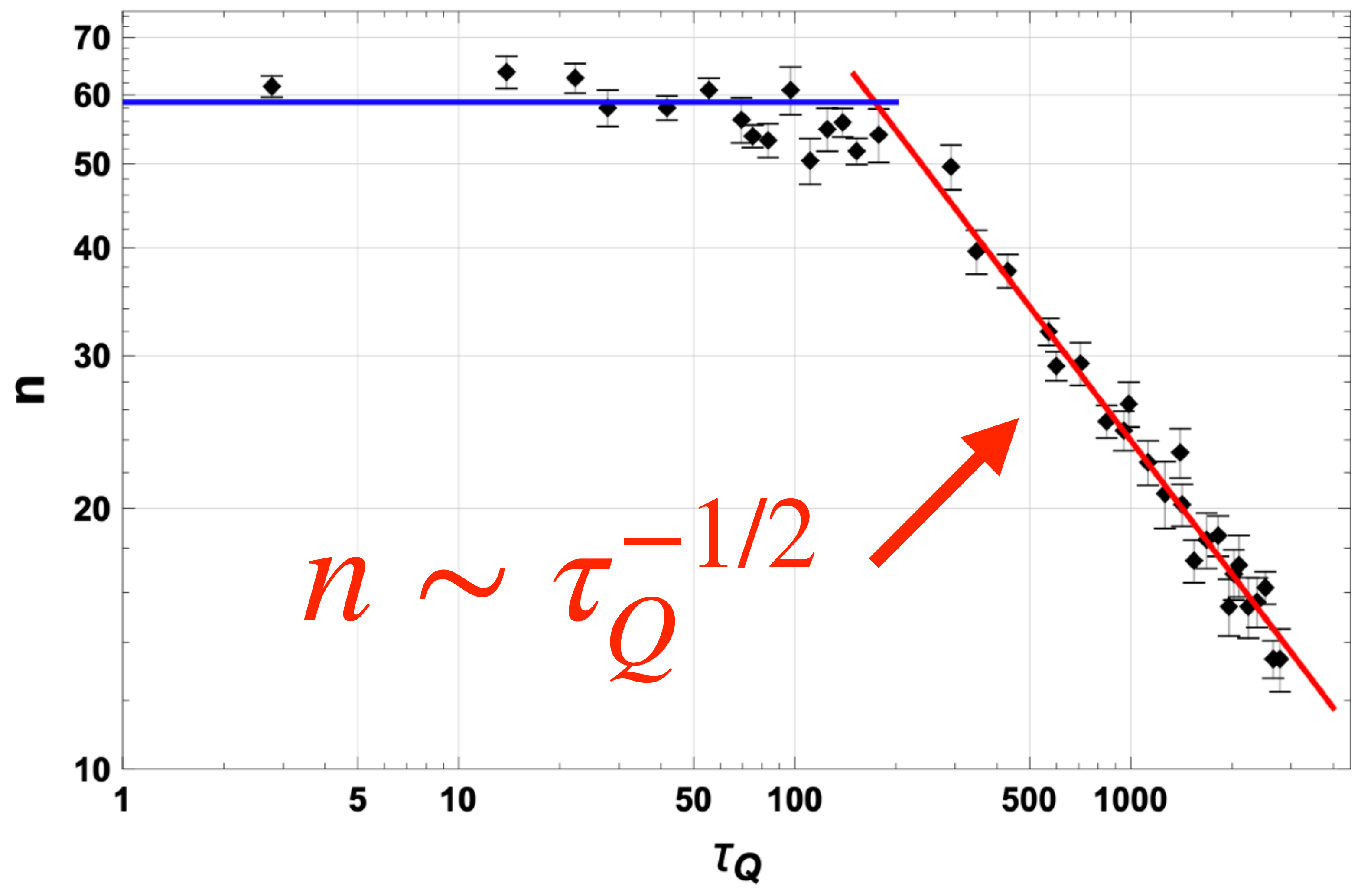


Confirmed by various experiments in condensed matter

- Liquid crystals: *Chuang, et.al., Science 251 (1991) 1336; Bowick, et.al.,Science 263 (1994) 943; Digal, et.al., PRL 83 (1999) 5030*

- He-3 superfluids: *Baeuerle,et.al.,Nature 382 (1996) 332; Ruutu et al. , Nature 382 (1996) 334*

- Thin-film superconductors: *Maniv,et.al., PRL 91 (2003) 197001; PRL 104, 247002 (2010).*

- Quantum optics: *Xu, et.al., PRL,112, 035701(2014)*

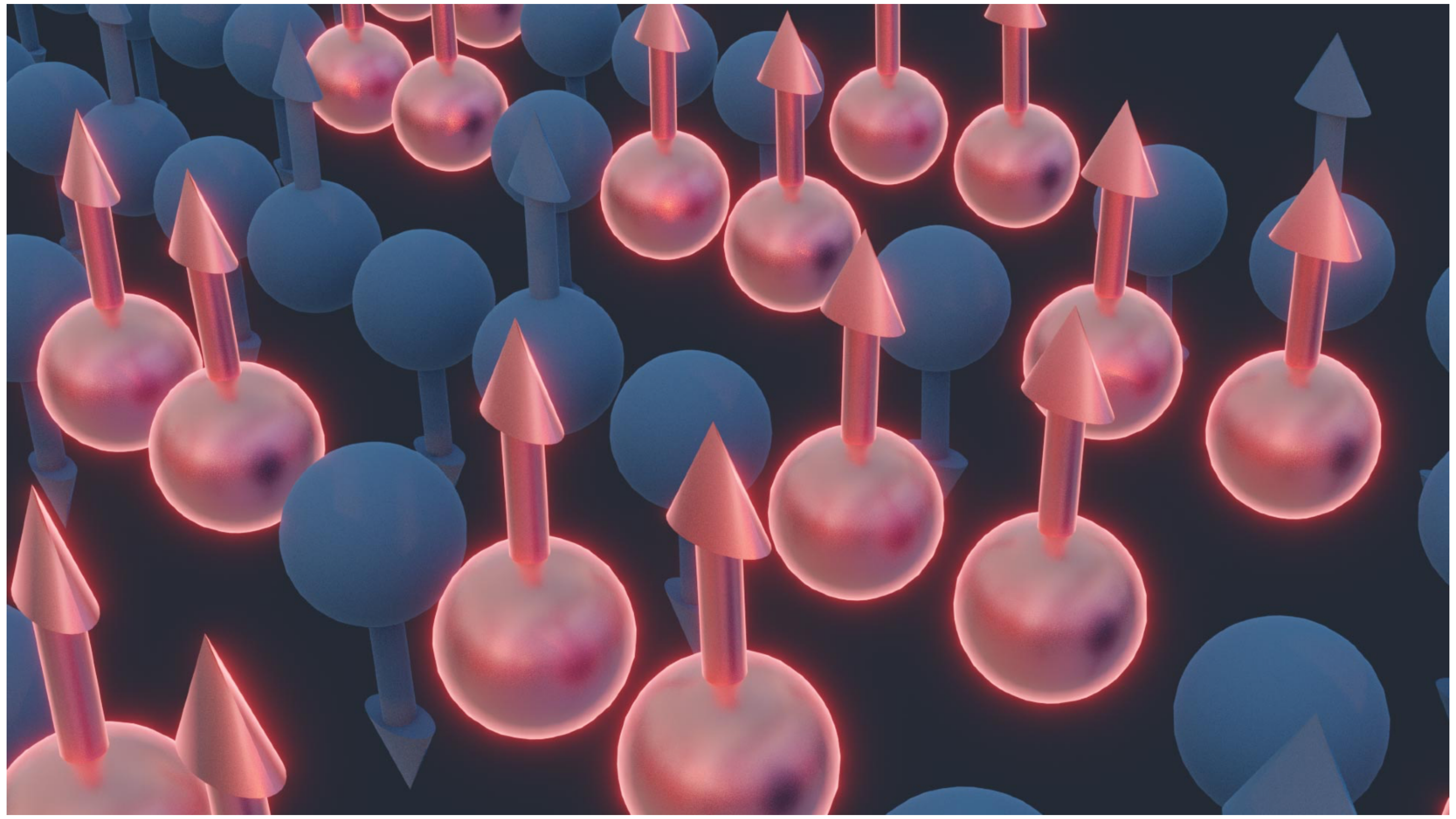
…

Holographic KZM with U(1) symmetry breaking

- **Winding numbers in 1+1 dim holographic superfluid:** ***Sonner, del Campo and Zurek, 1406.2329***
- **Vortices in 2+1 dim holographic superfluid:** ***Chesler, Garcia-Garcia and Liu, 1407.1862***
- **Magnetic vortices in 2+1 dim holographic superconductors:** ***Zeng, Xia, HQZ, 1912.08332***

  …

How to realize discrete symmetry breaking in holography? 🤔



Simulate the kinks and domain walls in spin chain with strong couplings

Holographic kinks

Z-H Li, H-Q Shi, HQZ [2207.10995]

- Complex scalar fields with U(1) gauge fields

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\tilde{\Psi}|^2 - m^2|\tilde{\Psi}|^2$$

$$D_\mu = \nabla_\mu - iA_\mu$$

- Gauge transformation

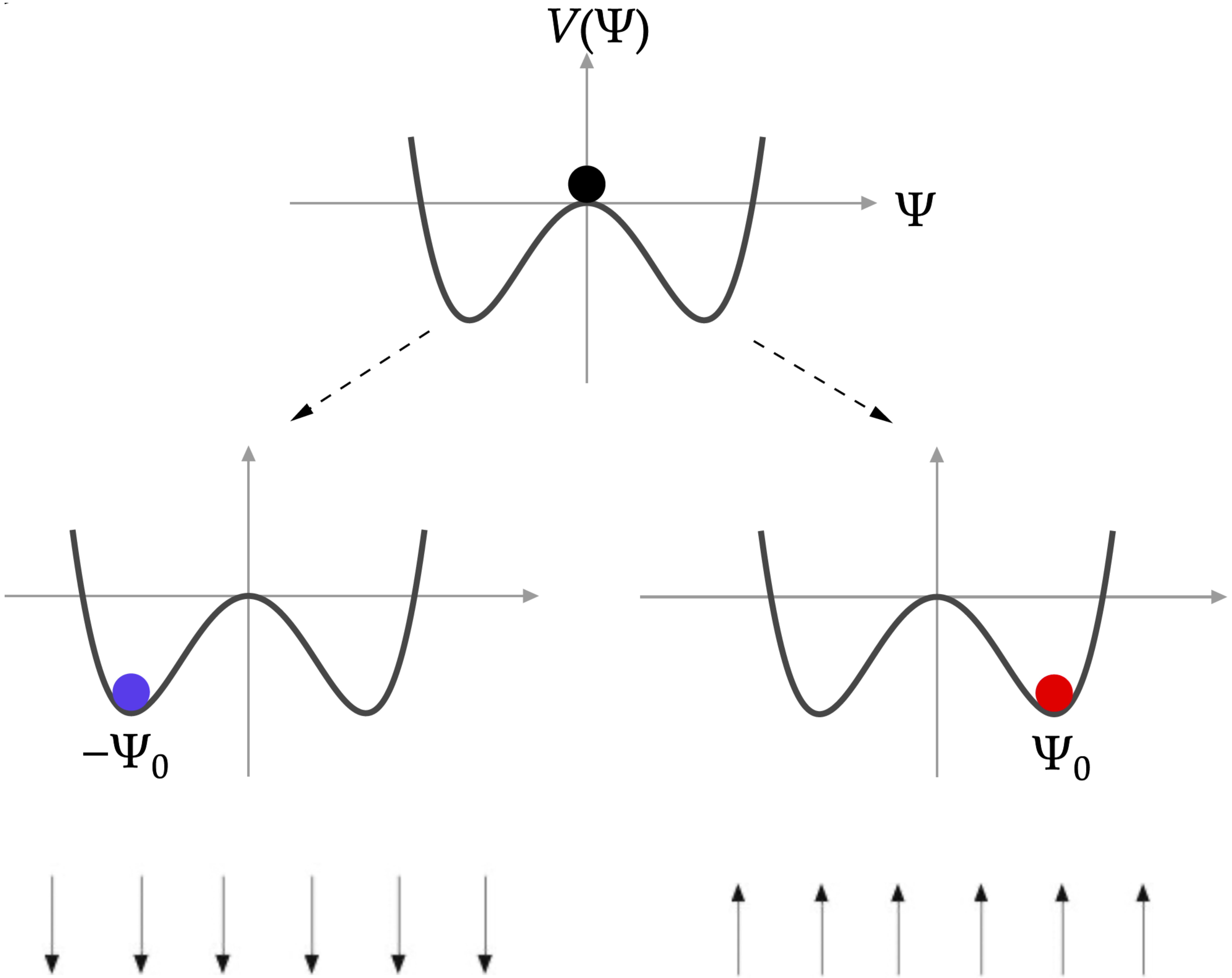
$$\tilde{\Psi} = \Psi e^{i\lambda}, \qquad A_\mu = M_\mu + \partial_\mu\lambda,$$

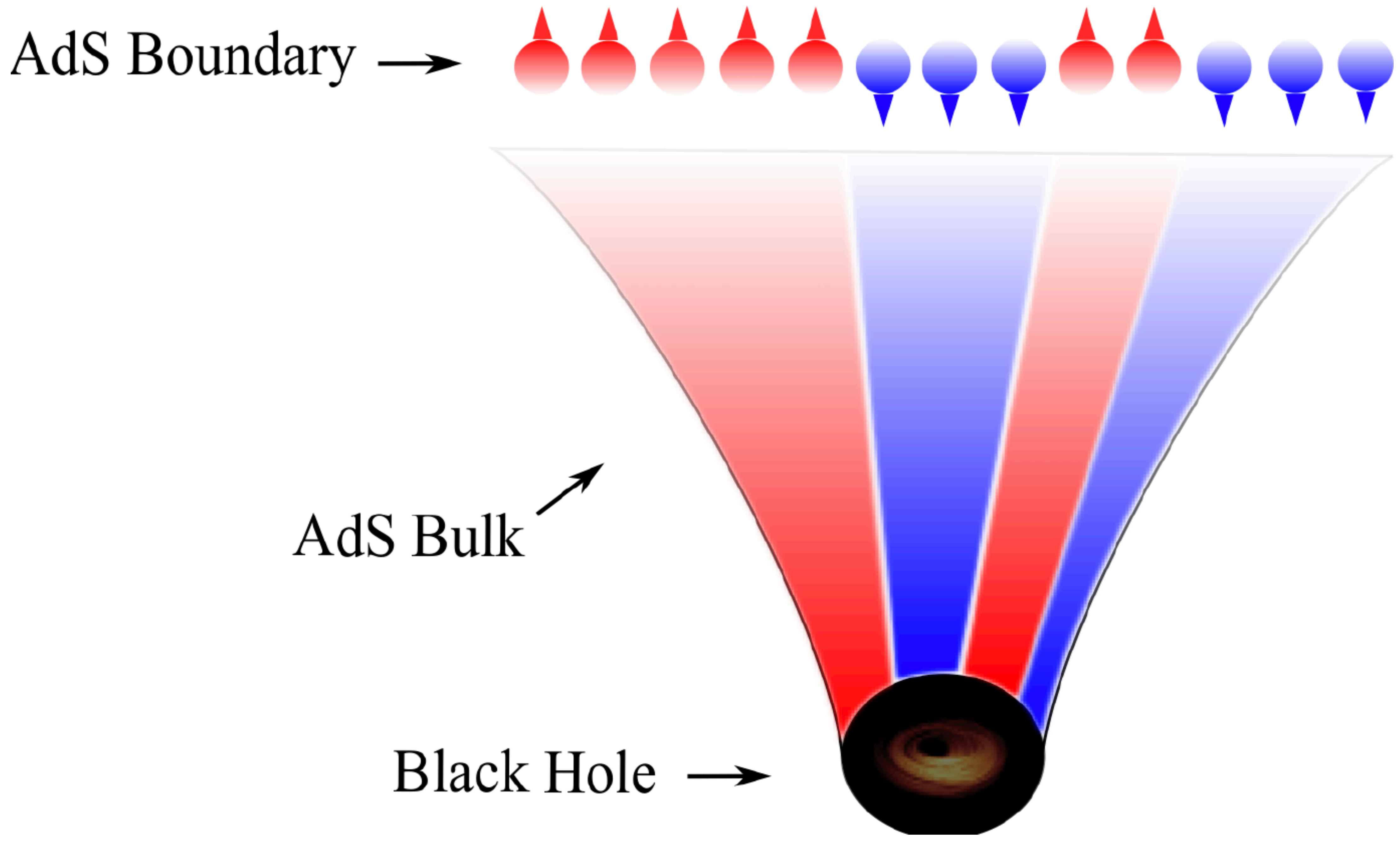
- EoM of real fields

$$(\nabla_\mu - iM_\mu)(\nabla^\mu - iM^\mu)\Psi - m^2\Psi = 0, \qquad \nabla_\mu F^{\mu\nu} = 2M^\nu\Psi^2.$$

Z_2 symmetry: $+\Psi \leftrightarrow -\Psi$

•Simulate a holographic spin chain





•Eddington-Finkelstein coordinates

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 - 2dtdz + dx^2 + dy^2\right] \qquad f(z) = 1 - (z/z_h)^3$$

•Ansatz of fields

turn on all the fields, and all fields depend on (t, z, x)

• Note: must include M_z, 4 independent equations to solve 4 fields

$$\begin{aligned}
\nabla_\mu \nabla^\mu \Psi - M_\mu M^\mu \Psi - m^2 \Psi &= 0, \\
(\nabla_\mu M^\mu)\,\Psi + 2M^\mu \nabla_\mu \Psi &= 0, \\
\nabla_\mu F^{\mu\nu} &= 2M^\nu \Psi^2.
\end{aligned}$$

$$\Rightarrow \quad 0 \equiv \nabla_\nu\left(\nabla_\mu F^{\mu\nu}\right) \Rightarrow \nabla_\nu\left(2M^\nu \Psi^2\right) = 0$$

- **Initial condition**

Static, spatial independent: EoMs of gauge fields becomes

$$0 = -\frac{2\Psi^2 M_t}{z^2} + f\partial_z^2 M_t,$$

$$0 = -\frac{2\Psi^2 M_z}{z^2} + \partial_z^2 M_t,$$

$\Rightarrow M_z = \frac{M_t}{f}$

$$0 = -\frac{2\Psi^2 M_x}{z^2} + f'\partial_z M_x + f\partial_z^2 M_x.$$

$\Rightarrow M_x = 0$

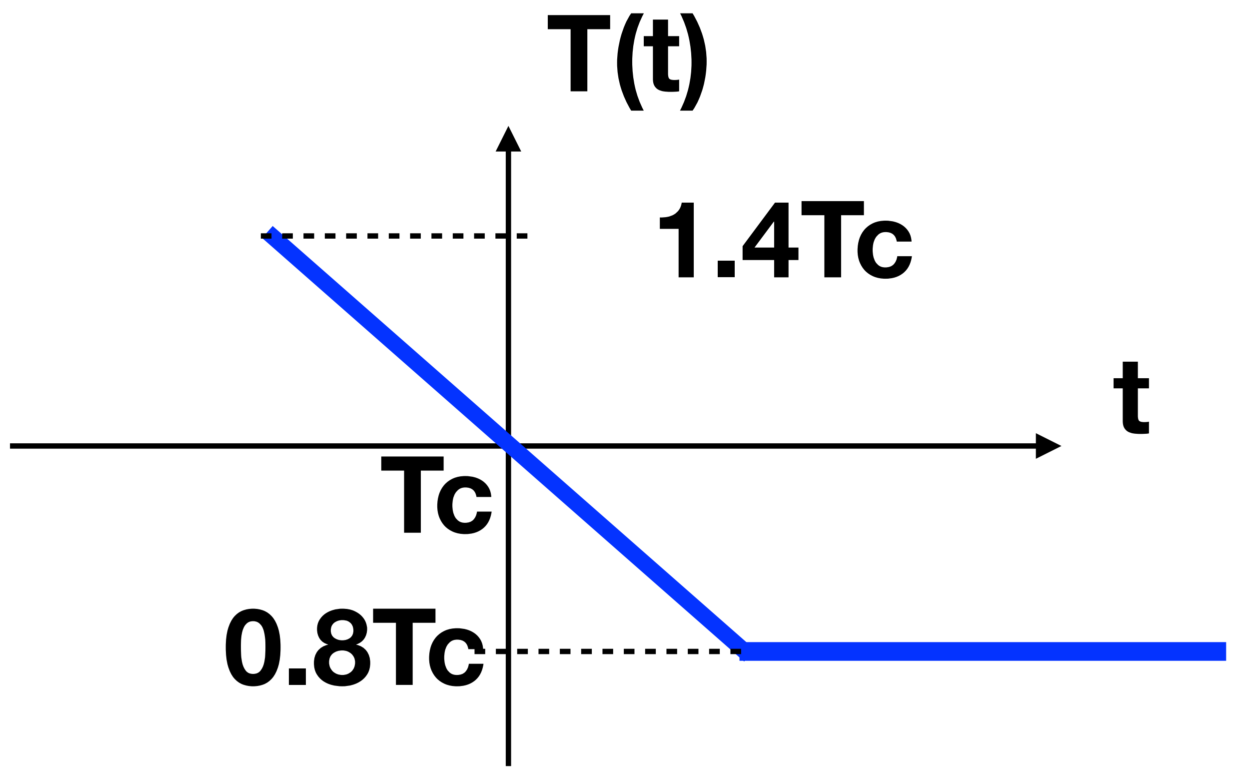
In normal state $\Psi = 0$, $M_t = \mu - \mu z$, $M_z = (\mu - \mu z)/f$

- **Quench chemical potential = quench temperature**

$$T(t)/T_c = 1 - t/\tau_Q$$

$$\downarrow$$

$$\mu(t) = \mu_c/(1 - t/\tau_Q)$$



$\mu_c \approx 4.06$ **is the critical chemical potential in static case**

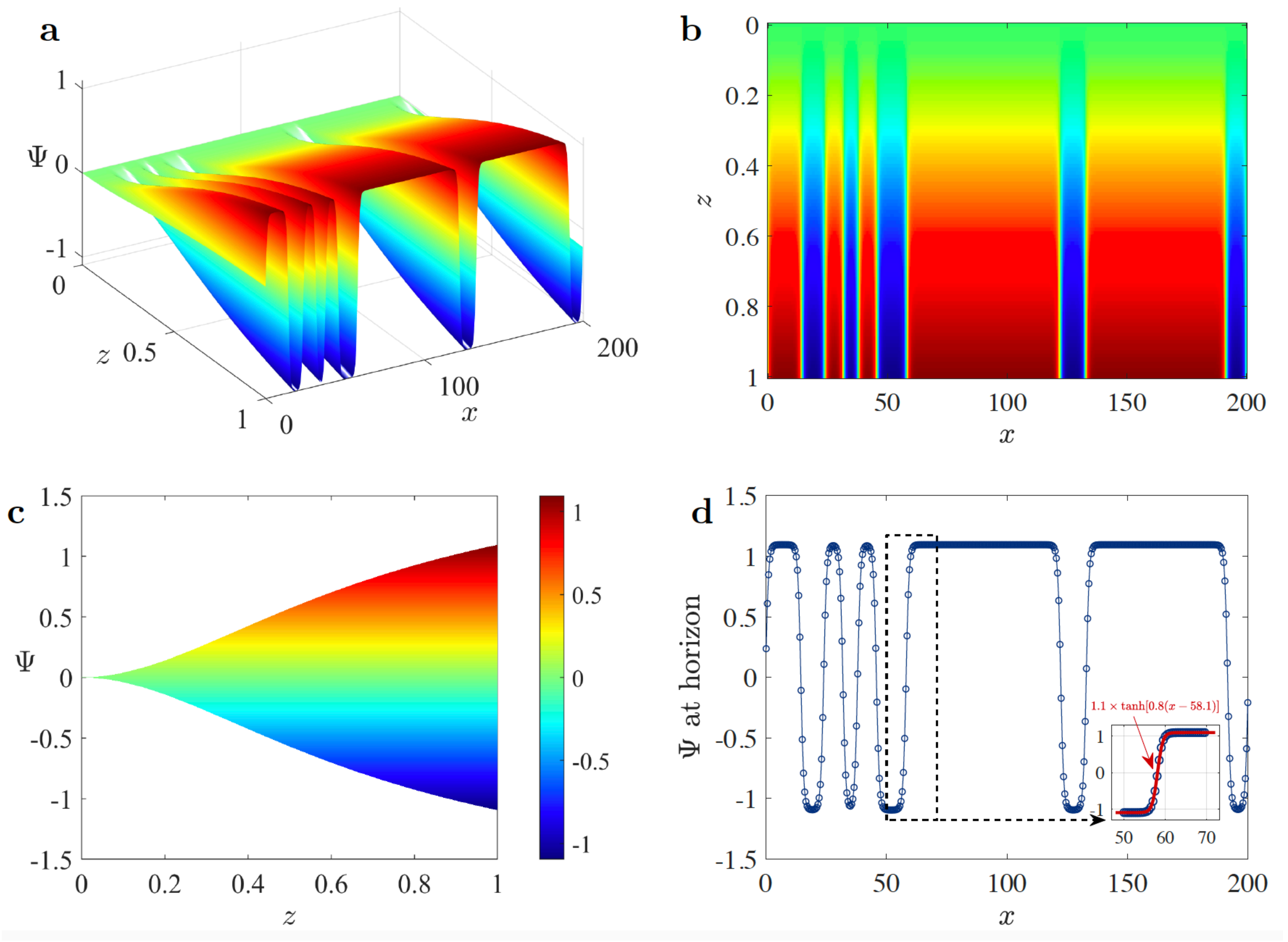
- **Small fluctuations of scalar field at initial time**

Gaussian white noise $\zeta(x_i, t)$: $\langle \zeta(x_i, t) \rangle = 0$

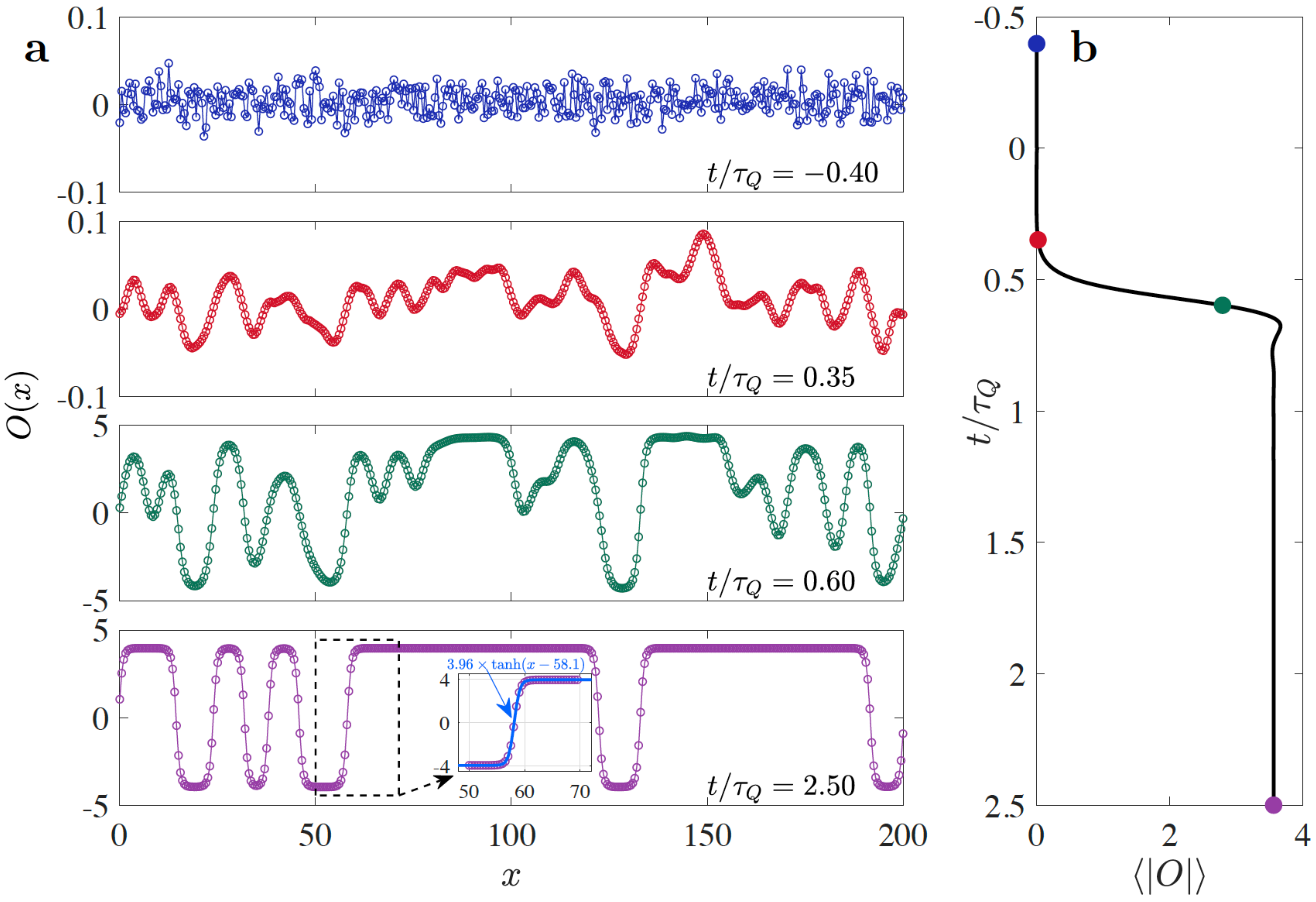
$$\langle \zeta(x_i, t)\zeta(x_j, t') \rangle = h\delta(t - t')\delta(x_i - x_j)$$

$$h = 0.001$$

•Kink hairs in the bulk



• **Time evolution of kinks on boundary**



•Beyond KZ scaling relation

del Campo, 1806.10646

Kinks in one dimensional quantum spin chain, the first three cumulants $\kappa_1, \kappa_2, \kappa_3$ have the following universal relations

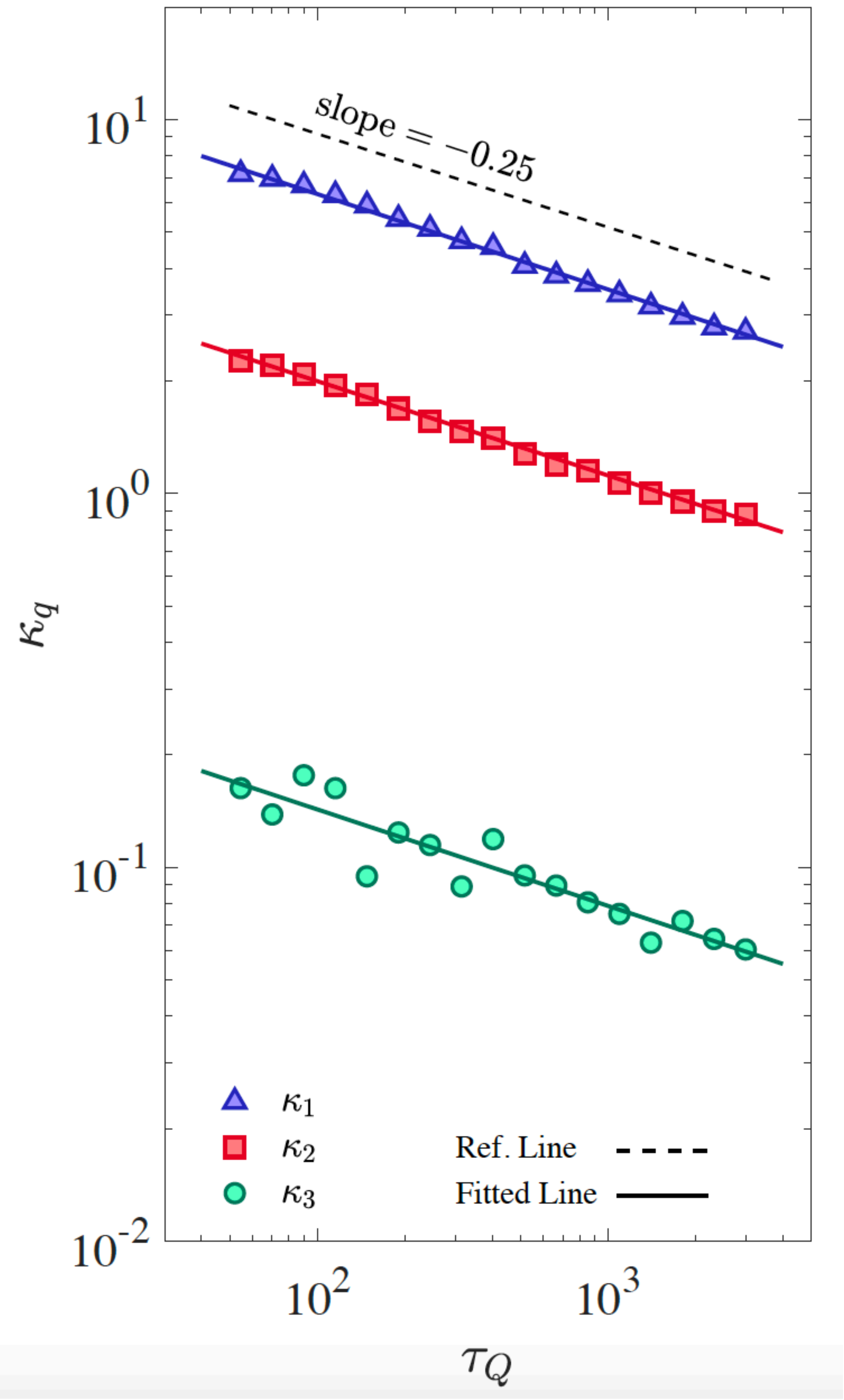
$$\kappa_1 = \langle n \rangle \propto \tau_Q^{-(D-d)\nu/(1+z\nu)}$$

$$\kappa_2 = \langle n^2 \rangle - \langle n \rangle^2 = \frac{2-\sqrt{2}}{2}\kappa_1 \approx 0.29\kappa_1$$

$$\kappa_3 = \langle (n - \langle n \rangle)^3 \rangle = (1 - 3\sqrt{2} + 2/\sqrt{3})\kappa_1 \approx 0.033\kappa_1$$

•Cumulants vs. quench rate in holographic kinks

$(D = 1, d = 0, \nu = 1/2, z = 2)$



$\langle n \rangle = \kappa_1 \propto \tau_Q^{-1/4}$

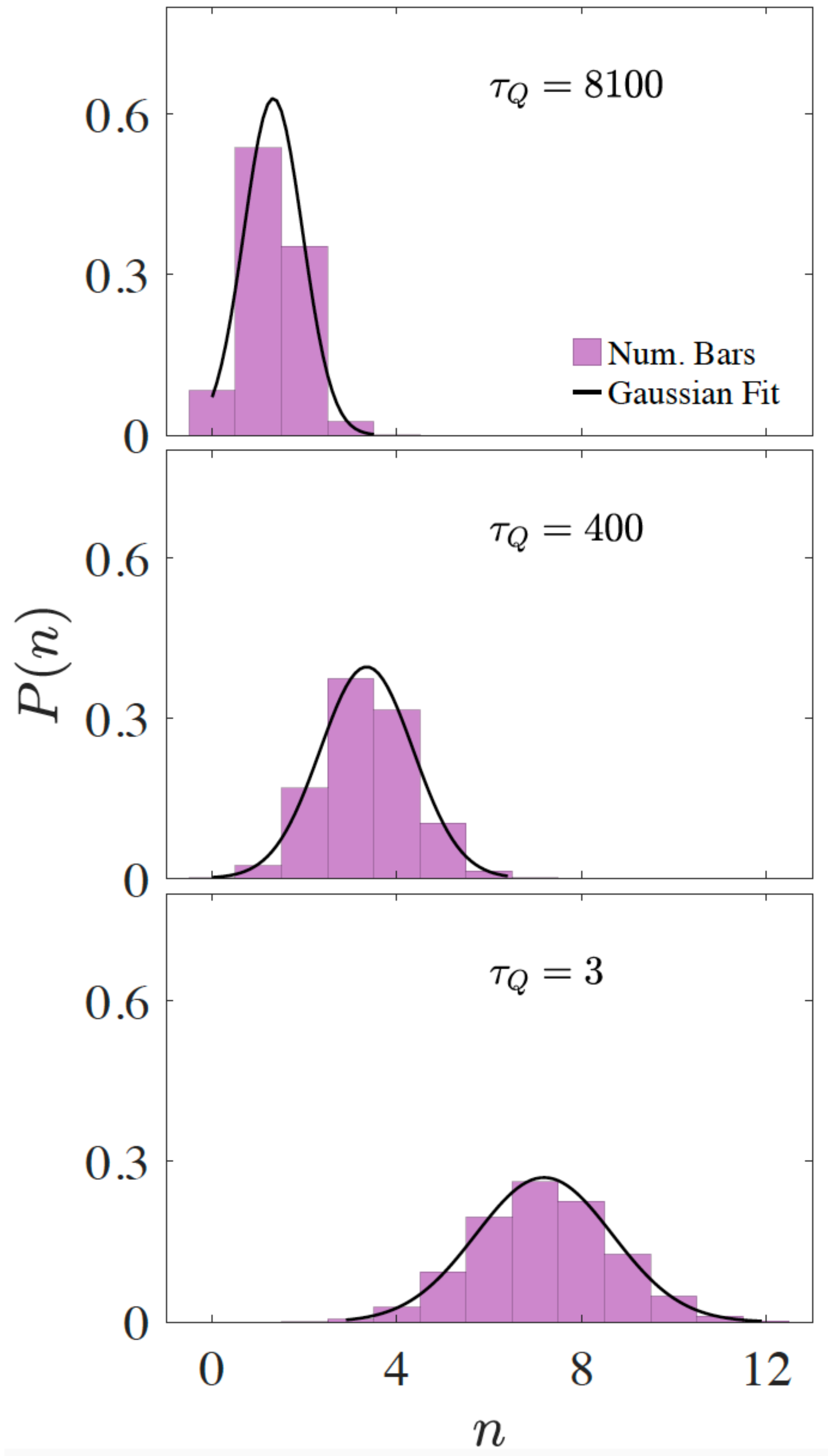
$\kappa_2/\kappa_1 \approx 0.312$

$\kappa_3/\kappa_1 \approx 0.023$

•Gaussian distribution in large trial number

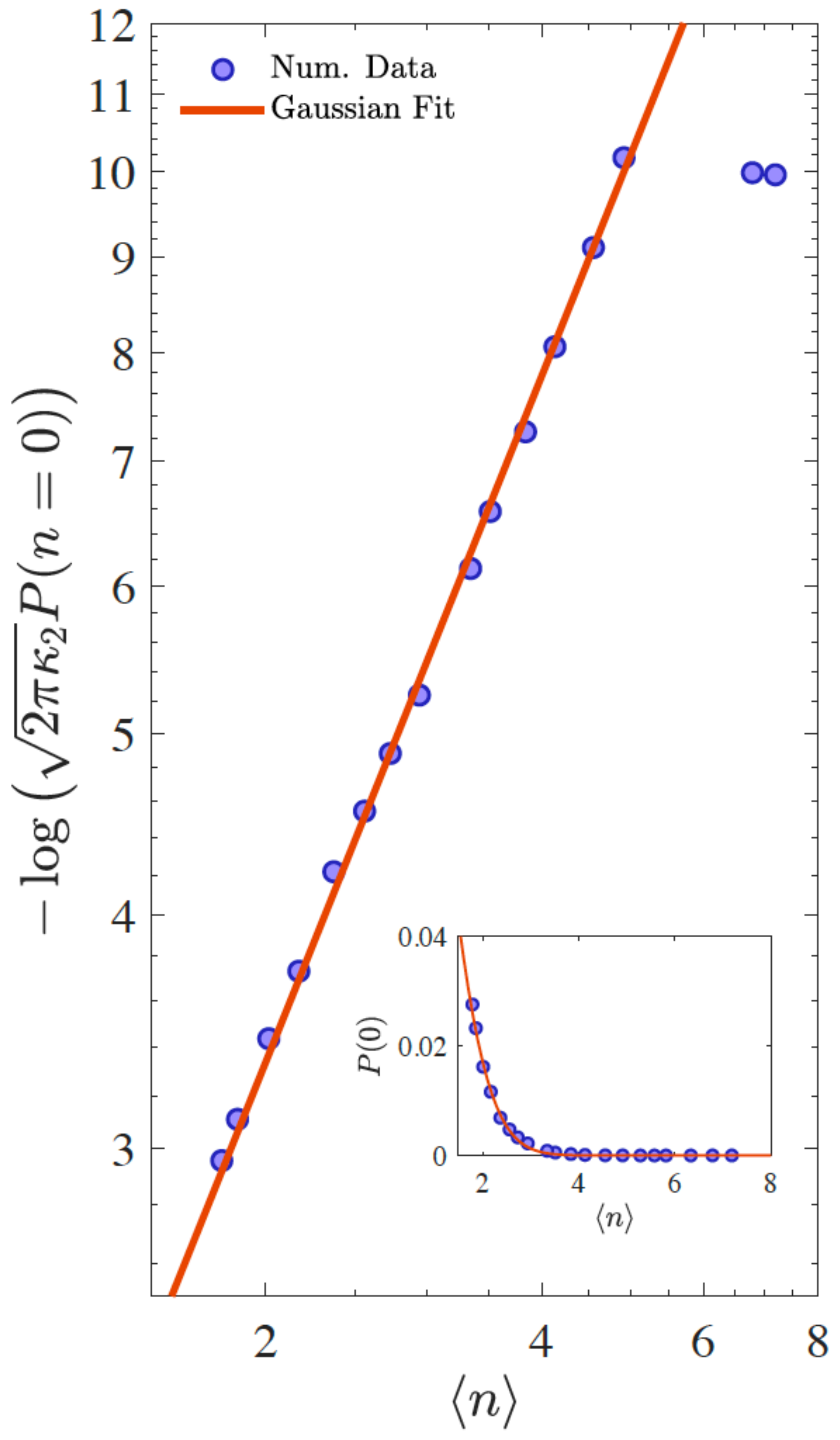
In the limit of large trial number with fixed average probability, distribution becomes Gaussian (Central limit theorem)

$$P(n) \approx \frac{1}{\sqrt{2\pi\kappa_2}} \exp\left[-\frac{(n-\langle n\rangle)^2}{2\kappa_2}\right]$$



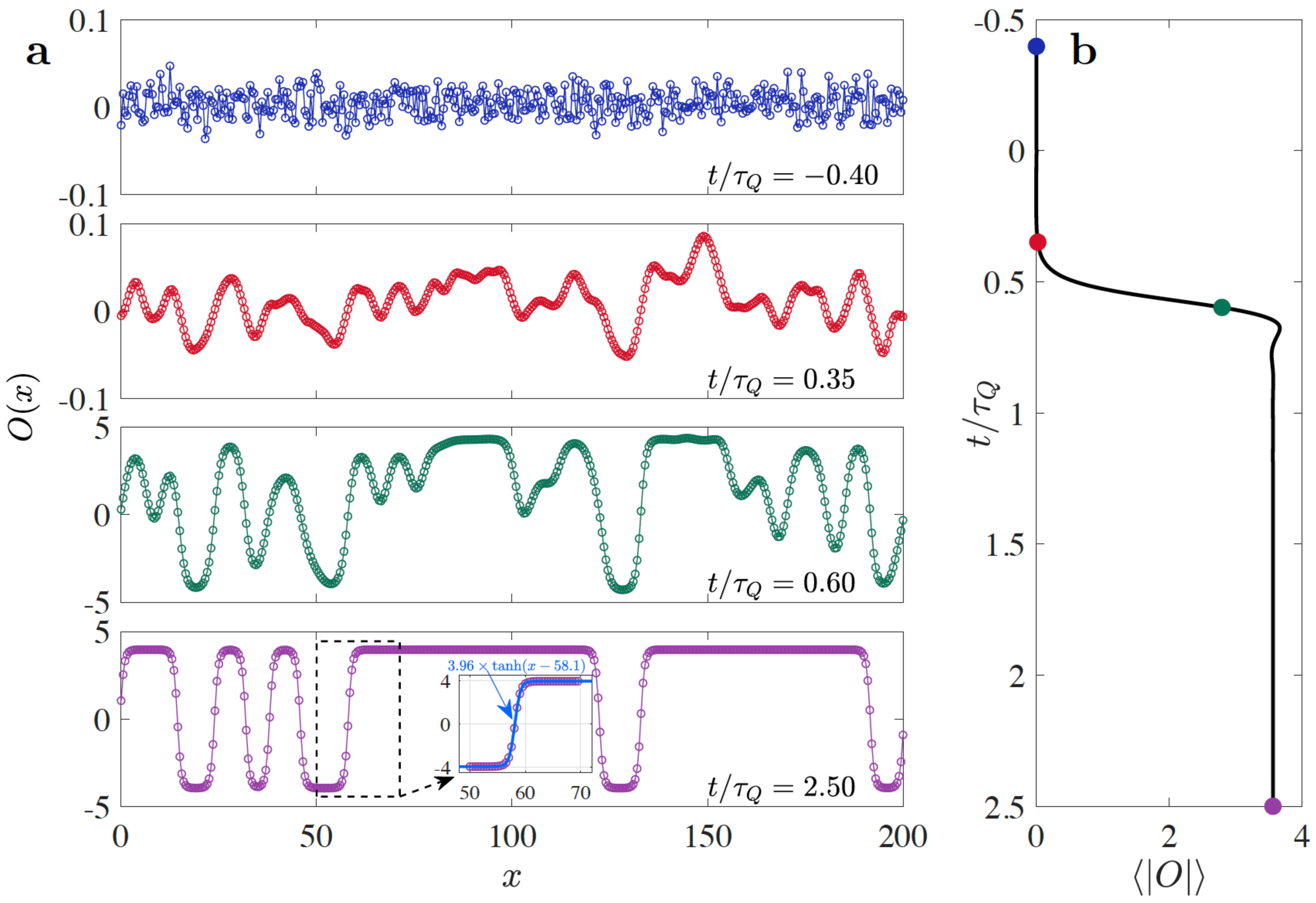
- **Adiabaticity limit: P(n=0)**

$$P(n=0) \approx \frac{1}{\sqrt{2\pi\kappa_2}} \exp^{-\frac{\langle n\rangle^2}{2\kappa_2}}$$



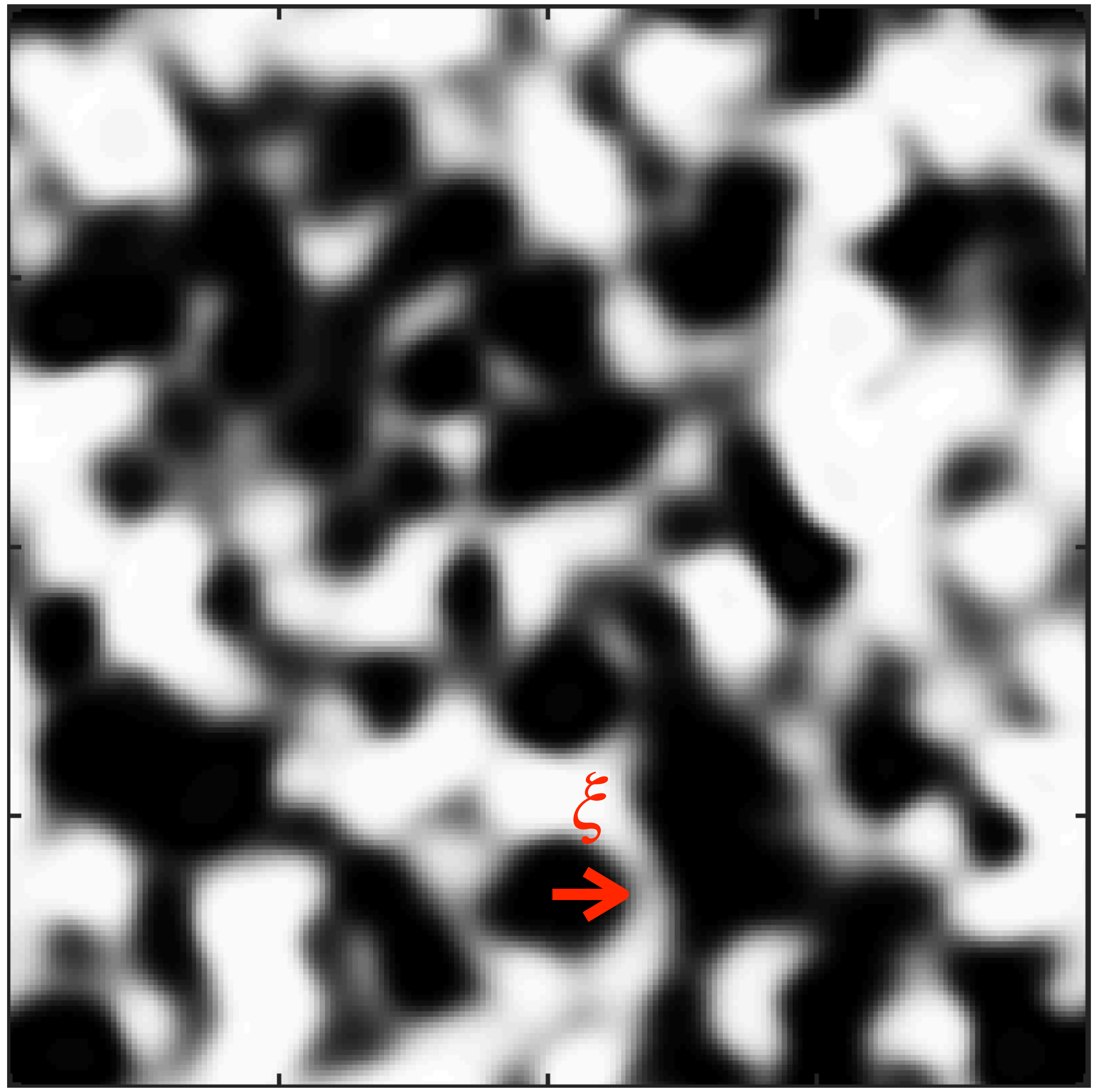
- **KZM is valid near the critical point**

- **Kinks satisfy the KZM away from critical point, because they are stable at late time.**



Holographic domain walls

🤔**How the coarsening dynamics governs the domain wall length far-away from critical point?**



Area A

- **Domain wall length vs. time**

A.J. Bray (1994), Adv. in Phys.

the length scale $\xi(t) \sim t^{1/2}$

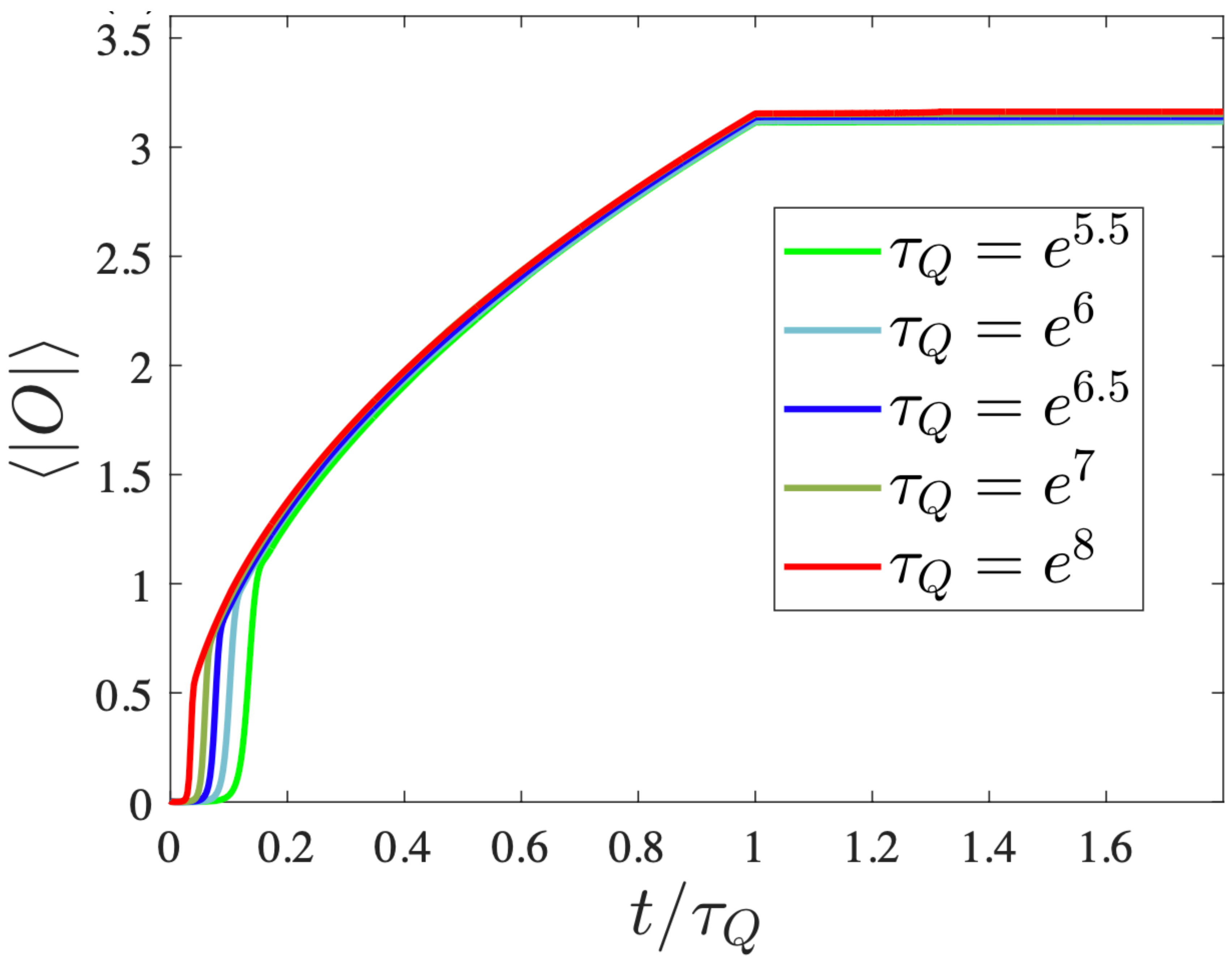
Number of domains: $n = A/\pi\xi^2$

Length of domain walls: $L \approx n \cdot 2\pi\xi = 2A/\xi$

$$L \propto t^{-1/2}$$

Tian-Chi Ma, Han-Qing Shi, HQZ, Adolfo del Campo, 2406.05167

- **Condensate at large t and large τ_Q**



Tian-Chi Ma, Han-Qing Shi, HQZ, Adolfo del Campo, 2406.05167

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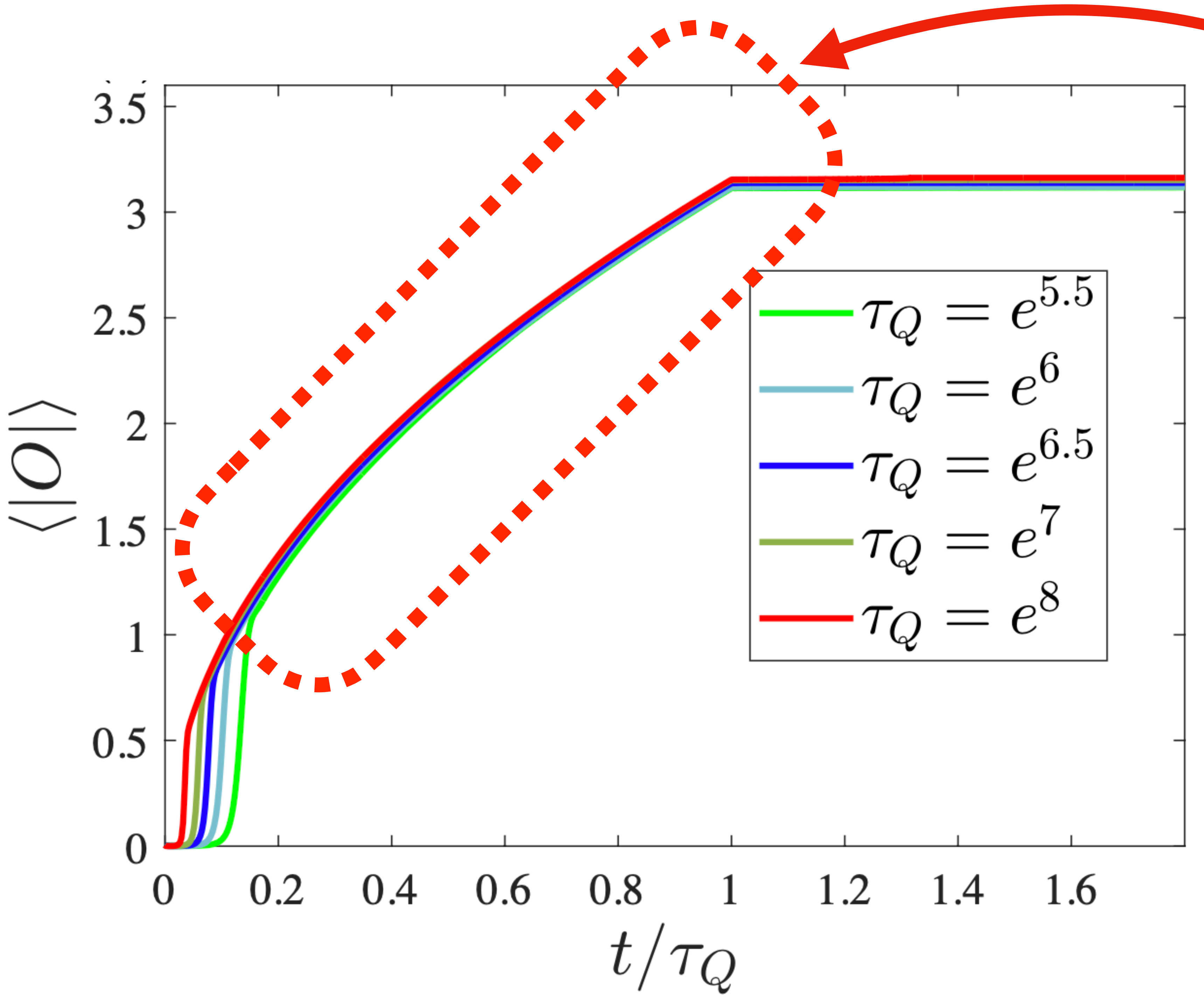
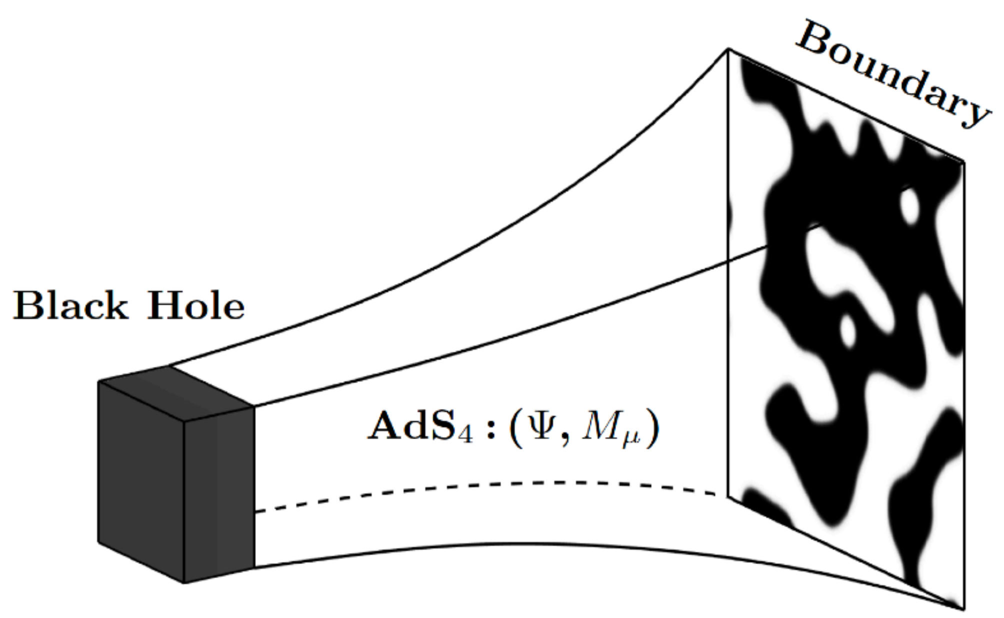
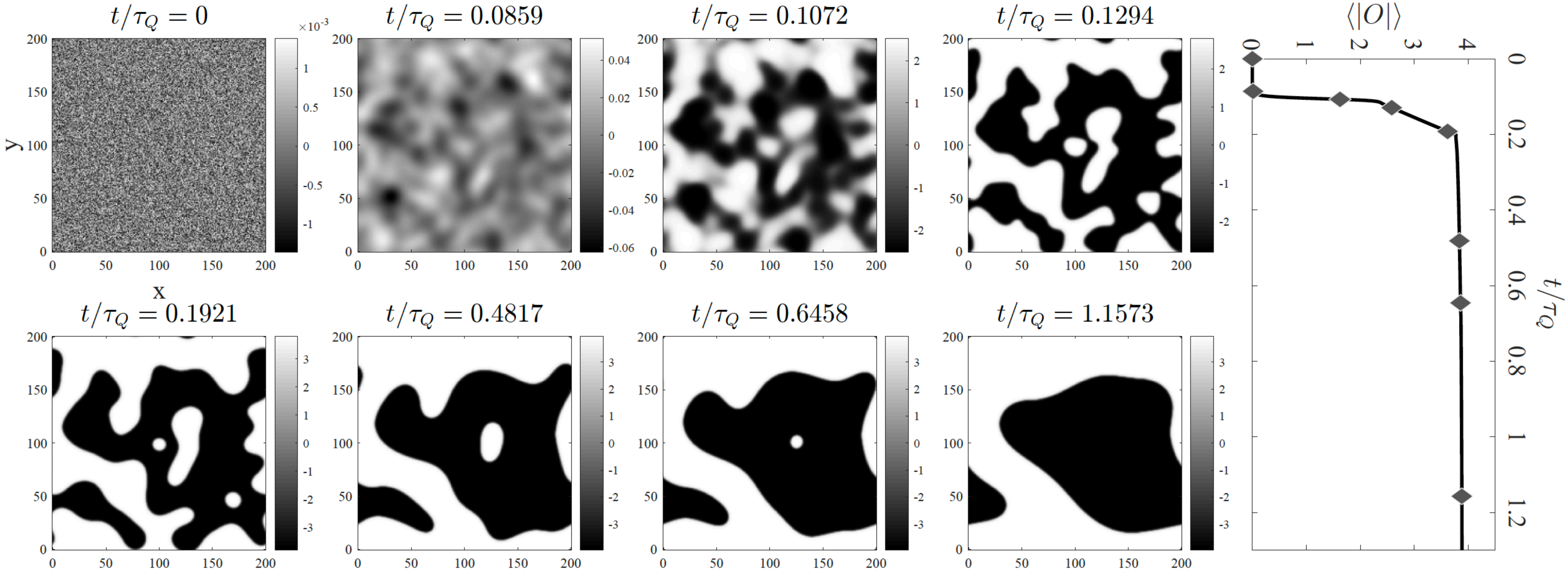


Adiabatic evolution at large time $t \propto \tau_Q$

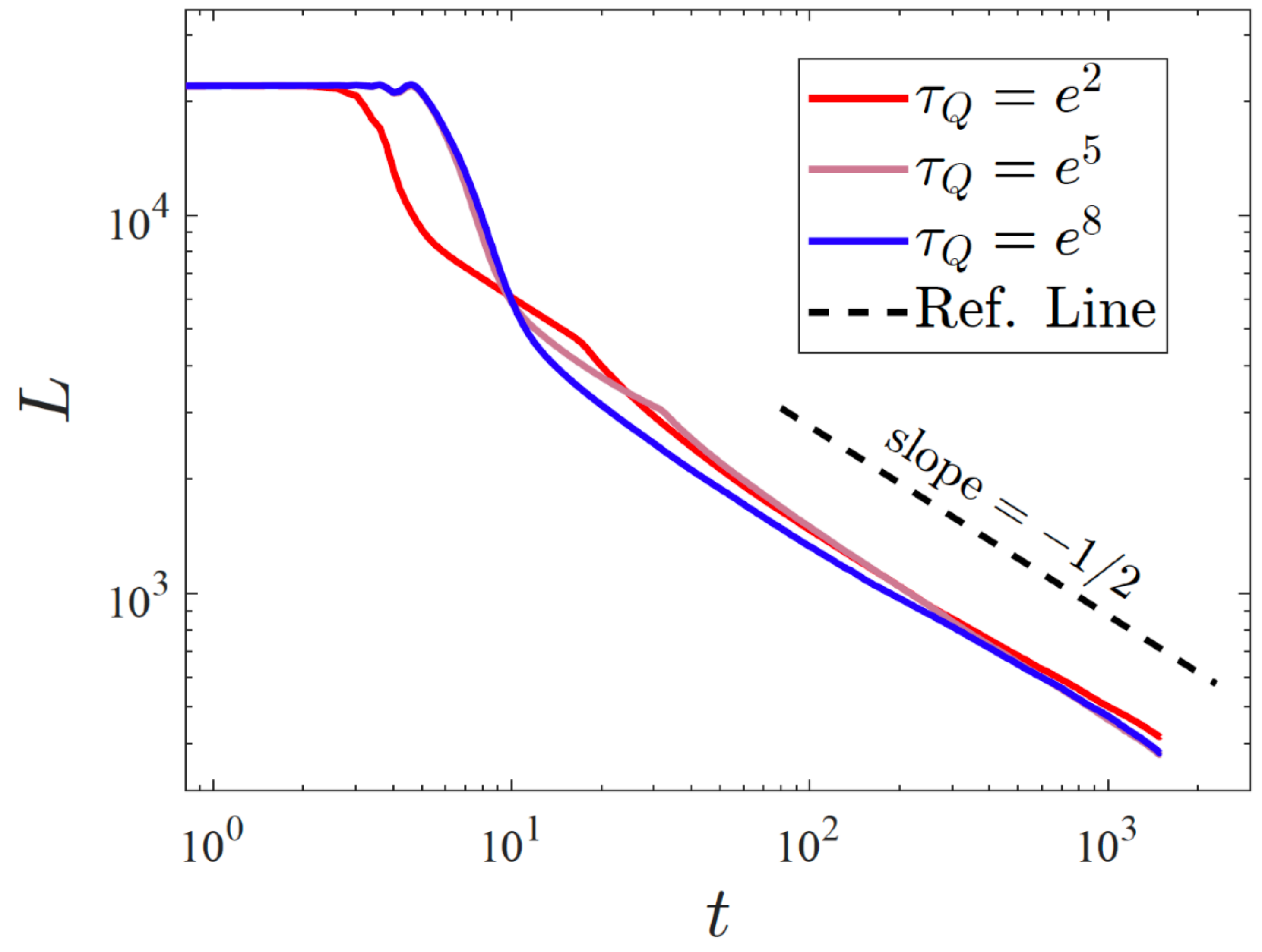
$$L \propto t^{-1/2} \propto \tau_Q^{-1/2}$$



•Time evolution of domain walls

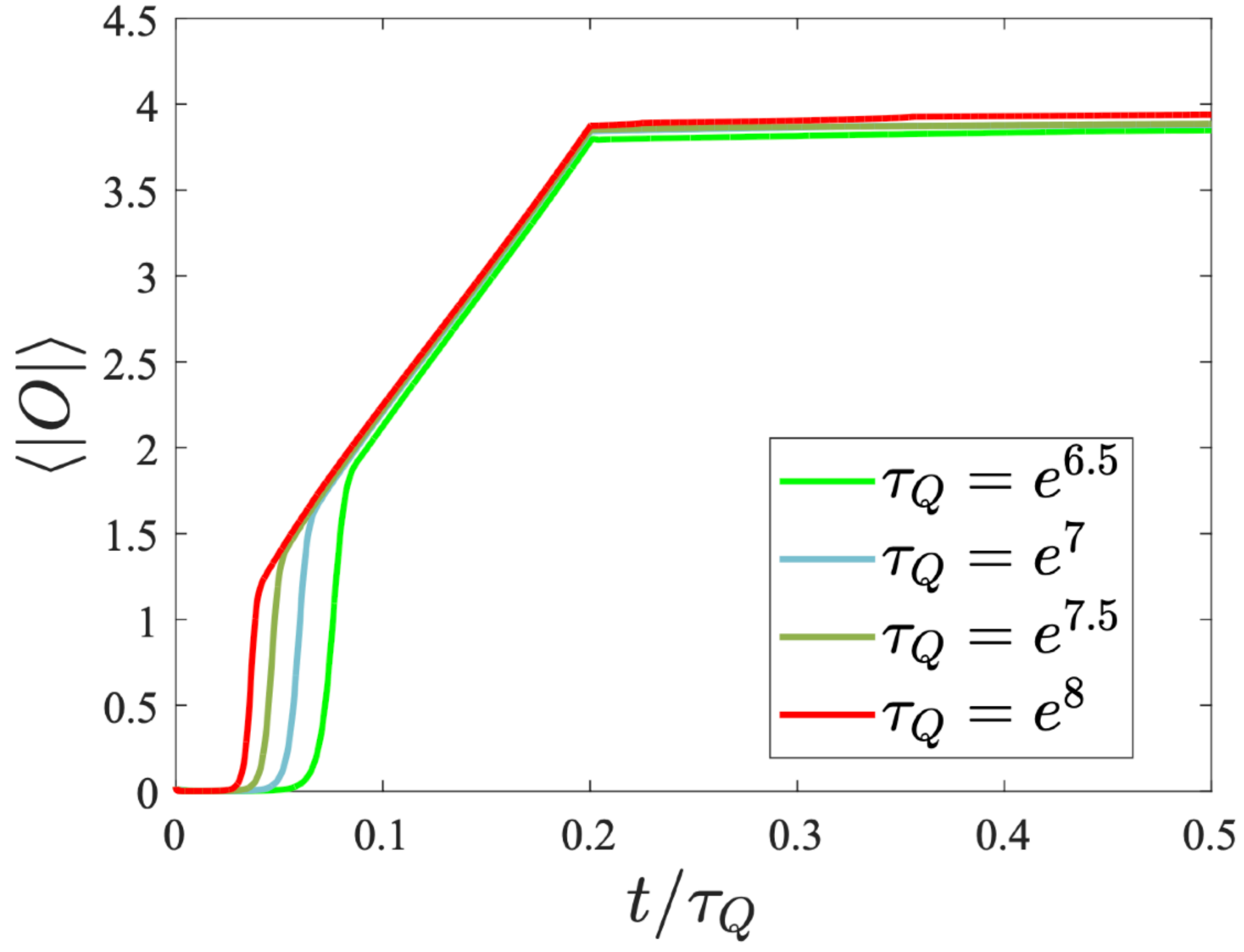


- **Domain wall length vs. time**



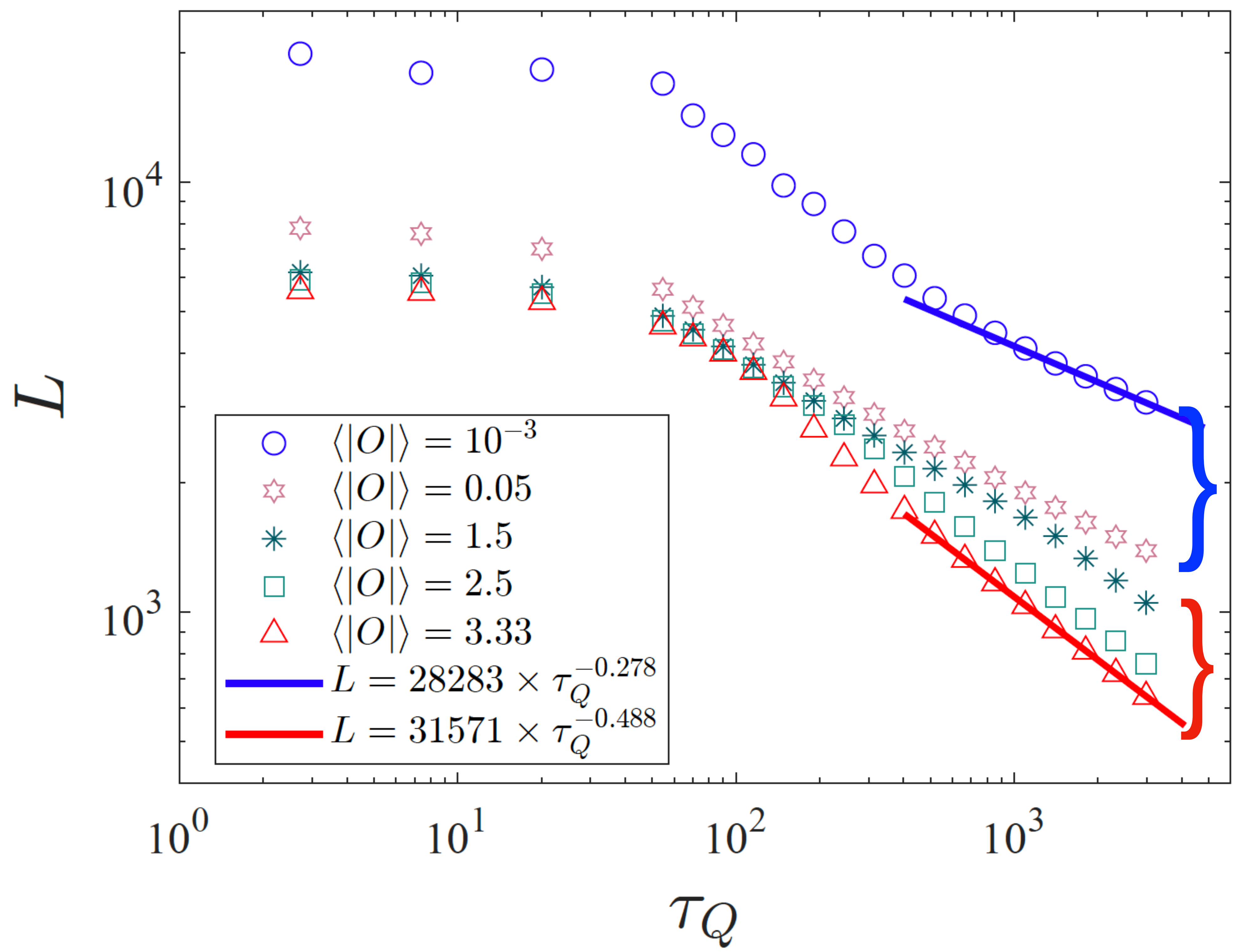
$L \propto t^{-1/2}$

- **Condensate at large t and large τ_Q**



Adiabatic evolution $t \propto \tau_Q$

- **Domain wall length vs. quench rate**



KZM

$L \propto \tau_Q^{-1/4}$

Beyond KZM

$L \propto \tau_Q^{-1/2}$

Near critical point

$L \propto \tau_Q^{-(D-d)\nu/(1+z\nu)}$,

$(D = 2, d = 1, \nu = 1/2, z = 2)$

Far-away from critical point

$L \propto t^{-1/2} \propto \tau_Q^{-1/2}$

Summary

- We have realized the kinks and domain wall structures holographically
- The distribution of kink numbers satisfies the KZM
- However, due to the coarsening dynamics, the KZ scalings for domain walls are only satisfied nearby the critical point
- Away from the critical point, this relation would be destroyed, and satisfy another power-law for domain wall

Thank you for listening!