Clarifying the interior dynamics of charged black holes

Holographic applications:
from Quantum Realms to the Big Bang

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Towards classifying the interior dynamics of charged black holes with scalar hair

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Clarifying Kasner dynamics inside anisotropic black hole with vector hair

JHEP04(2025)179

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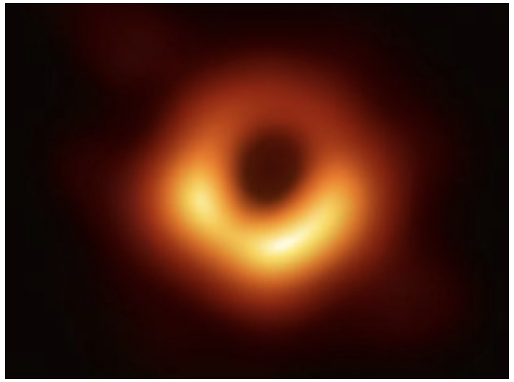
Background

1. The nature of black hole



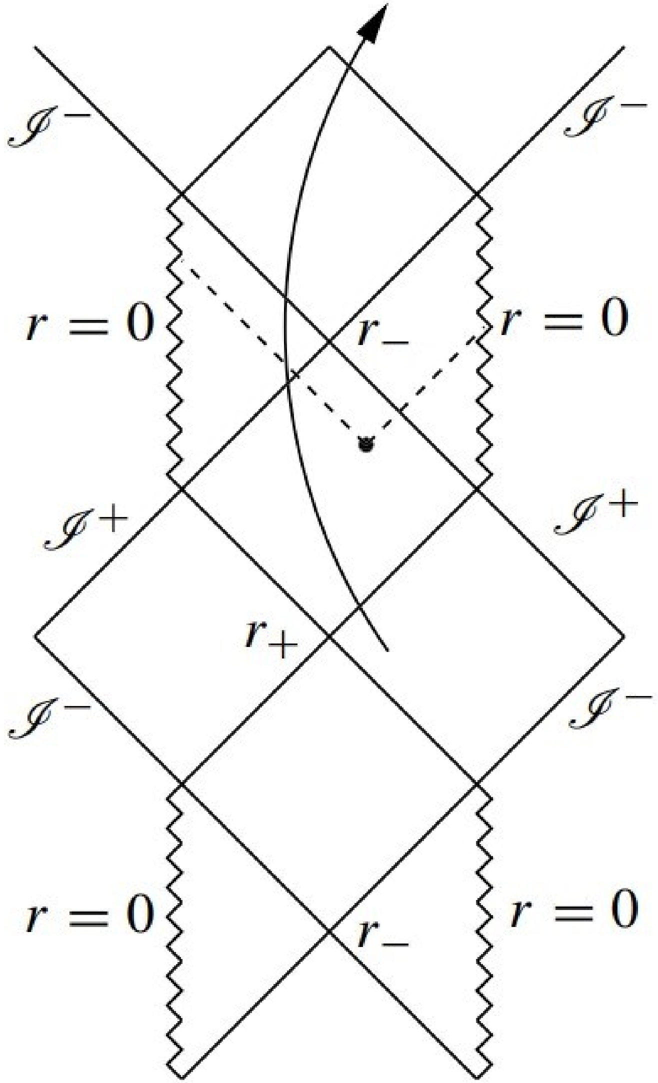
Singularity

2. Singularity

(1) Penrose Singularity Theorem

(2) Belinskii, Khalatnikov and Lifshitz (BKL) hypothesis

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + C_t \tau^{2p_t} \mathrm{d}t^2 + C_{x_i} \tau^{2p_{x_i}} \mathrm{d}\Sigma^2_{d-1,0}\,, \quad \sum_{i=1}^{D-1} p_i = \sum_{i=1}^{D-1} p_i^2 = 1\,.$$



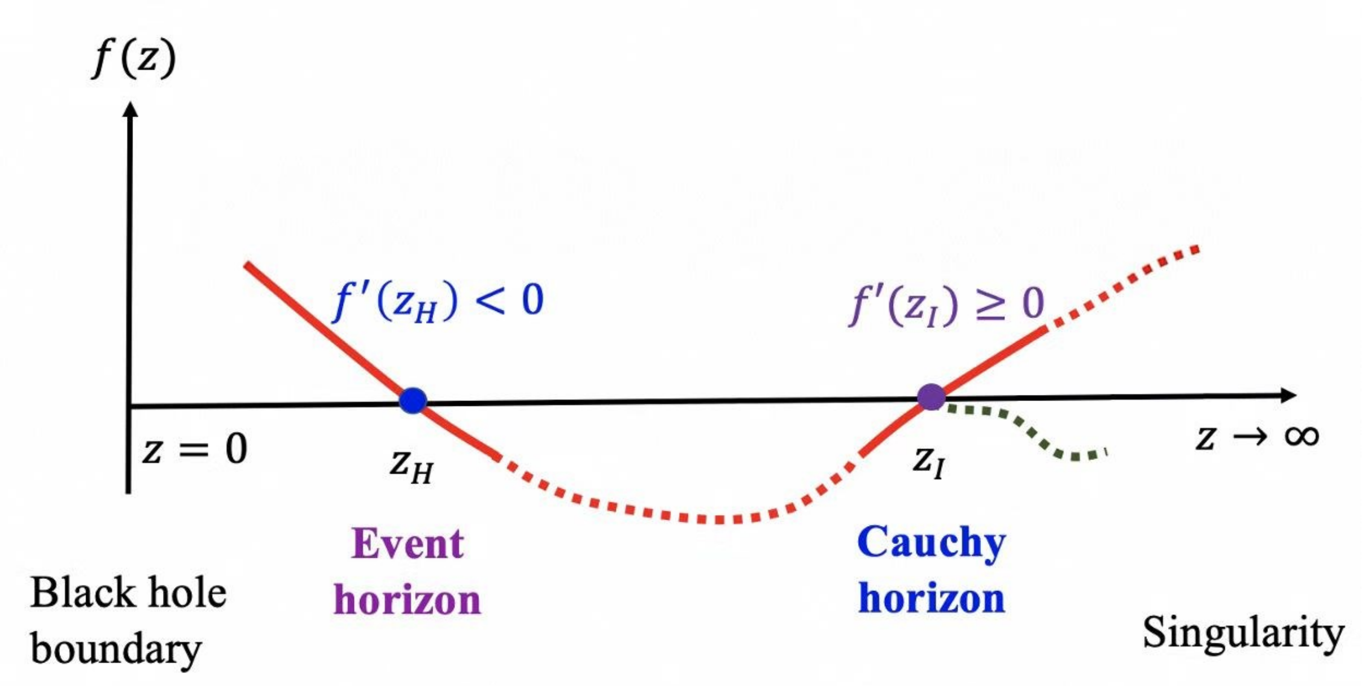
RN Spacetime

Without Cauchy Horizon

$$ds^2 = \frac{1}{z^2}\left[-f(z)e^{-\chi(z)}dt^2 + \frac{dz^2}{f(z)} + d\Sigma^2_{d-1,k}\right]$$

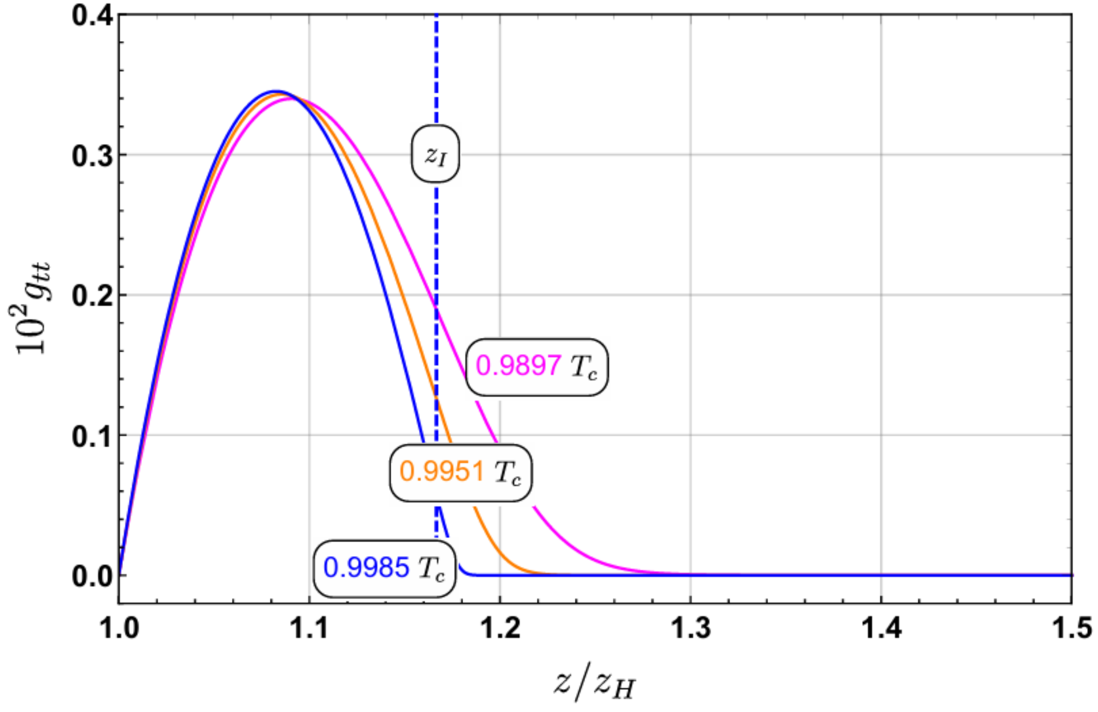
Roots of the metric function f(z):

Scaling symmetry, EoMs, energy condition……

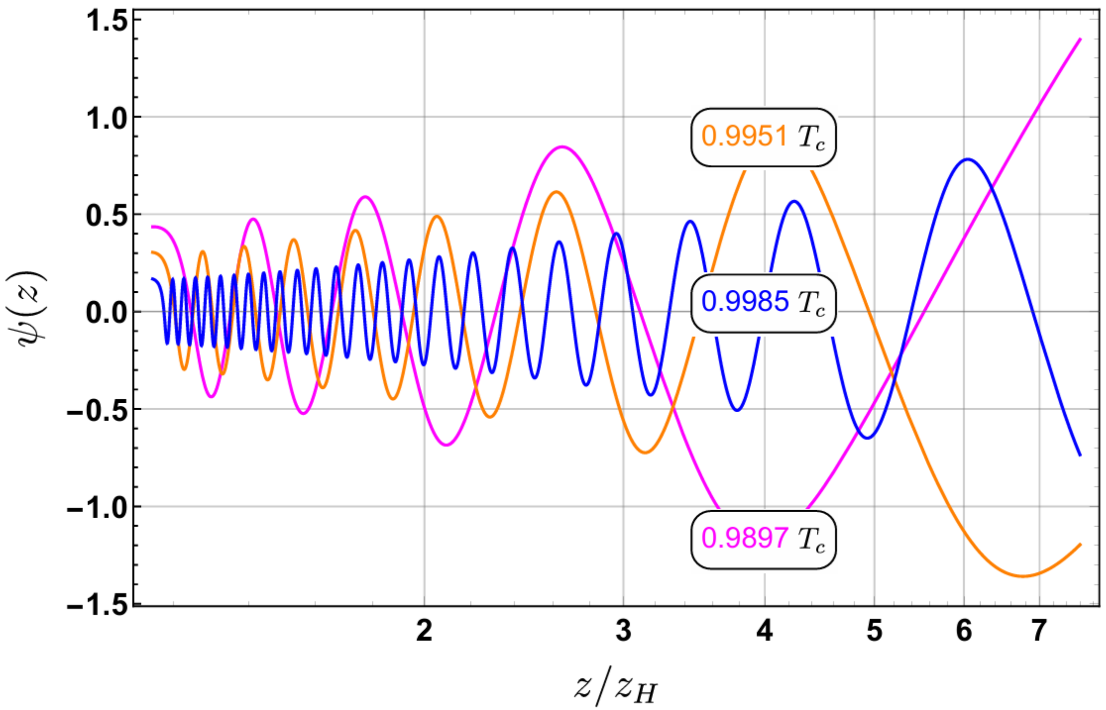


Without Cauchy Horizon

Einstein-Rosen Bridge collapse



Josephson oscillation



→ Kasner epoch

[1] Y.-S. An, L. Li, F.-G. Yang and R.-Q. Yang, Interior structure and complexity growth rate of holographic superconductor from M-theory, JHEP 08 (2022) 133 [2205.02442].

From Kasner behavior to Billiards

1. Belinskii-Khalatnikov-Lifshitz Singularity(BKL limit)

   (1) Close to a space-like singularity

   (2) The spatial points dynamically decouple (a space-like singularity is local). ➡ (ODE with respect to time).

2. Kasner solution

   Power law evolution of scale factors:

$$ds^2 = -\mathrm{d}\tau^2 + C_t \tau^{2p_t} \mathrm{d}t^2 + C_x \tau^{2p_x} \mathrm{d}\Sigma^2_{d-1,0} \,,$$

3. Kasner-like behavior

   Mimick the Kasner solution at each spatial point.

Cosmological Billiards

A billiard motion in a region of Lobachevskii space
(An hyperboloid in the space of logarithmic scale factors)

$$S[G_{MN}, \phi, A^{(p)}] = \int d^D x \sqrt{-G} \left[R - \partial_M \phi \partial^M \phi - \frac{1}{2} \sum_p \frac{1}{(p+1)!} e^{\lambda_p \phi} F^{(p)}_{M_1 \cdots M_{p+1}} F^{(p)\, M_1 \cdots M_{p+1}} \right] + \dots$$

Beautiful physical picture!

BKL limit

$$H_\infty(\lambda, \pi_\lambda, \gamma, \pi_\gamma, Q, P) = \frac{1}{4} \left[-\pi_\lambda^2 + \pi_\gamma^2 \right] + \sum_{A'} \Theta(-2 w_{A'}(\gamma)),$$

Kinetic term

Wall: define the billiard table

Beyond Billiard
Complicated potential?
Modified Gravity?

[2] T. Damour, M. Henneaux and H. Nicolai, Cosmological billiards, Class. Quant. Grav. 20(2003) R145 [hep-th/0212256].

From the EoMs

Challenges and Attempts

(1) Nonlinear effect is important → The details of the model is sensitive

(a) Neutral scalar hair, arXiv:2004.01192&2006.10056
(b) Charged scalar hair, arXiv:2008.12786
……

→ More general?

(2) Vector field → Anisotropic geometry → More complicated behavior

(a) Some rules were summarized, arXiv: 2112.04206

(b) Kasner Oscillation, arXiv: 2210.01046

(3) EoMs are differential equations → Hard to get analytical description

(a) Analytical transformation rules(scalar hair), arXiv: 2205.02442

Method

Method 1:

Kasner hypothesis → Action → Possible Kasner behavior

→ Simple, general

Method 2:

Action → EoMs → (the order of terms; The limit of near singularity($z \to \infty$)) → Possible Kasner behavior / Conditions for transformations

→ More detail

Example 1(JHEP02(2024)169)

General scalar couplings and local U(1) symmetry

The action and metric:

$$S = \frac{1}{2\kappa_N^2} \int d^{d+1}x\sqrt{-g}\,[R+\mathcal{L}]\ ,$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\psi)^2 - \mathcal{F}(\psi)(\partial_\mu\theta - qA_\mu)^2 - V(\psi) - \frac{Z(\psi)}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{e}^{-\chi(z)}\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}\Sigma^2_{d-1,k}\right], \quad \psi = \psi(z), \quad A = A_t(z)\mathrm{d}t\,,$$

Result: Kasner Structure

With approximations, in the far interior $z \longrightarrow \infty$ (e.g. $\mathcal{F}(\psi) \backsim \exp(\kappa\psi)$):

$$\psi = \alpha \ln z + C_\psi \,, \qquad \chi = \frac{\alpha^2}{d-1} \ln z + C_\chi \,,$$

$$A_t' = C_{A_t} z^{d-3} \mathrm{e}^{-\chi/2} \,, \qquad h' = C_h z^{d-3} \mathrm{e}^{-\chi/2} \,,$$

$$\Big\downarrow \quad \tau \sim z^{-\left(\frac{d}{2} + \frac{\alpha^2}{4(d-1)}\right)}$$

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + c_t \tau^{2p_t} \mathrm{d}t^2 + c_s \tau^{2p_s} \mathrm{d}\Sigma_{d-1,k}^2, \quad \psi \simeq -\sqrt{2} p_\psi \ln \tau \,,$$

$$p_t = \frac{\alpha^2 - 2(d-1)(d-2)}{\alpha^2 + d(d-1)}, \quad p_s = \frac{4(d-1)}{\alpha^2 + d(d-1)}, \quad p_\psi = \frac{2\sqrt{2}(d-1)\alpha}{\alpha^2 + d(d-1)} \,,$$

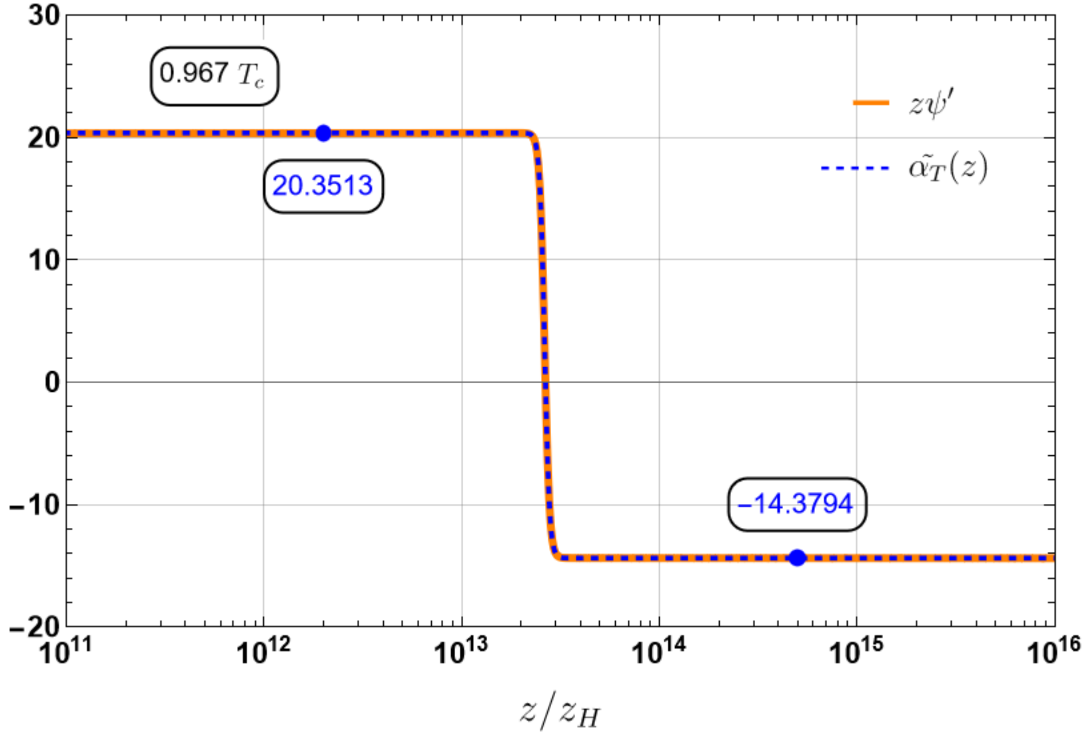
$$p_t + (d-1)p_s = 1, \quad p_t^2 + (d-1)p_s^2 + p_\psi^2 = 1 \,,$$

Other terms come into play ➡ Transformations are triggered

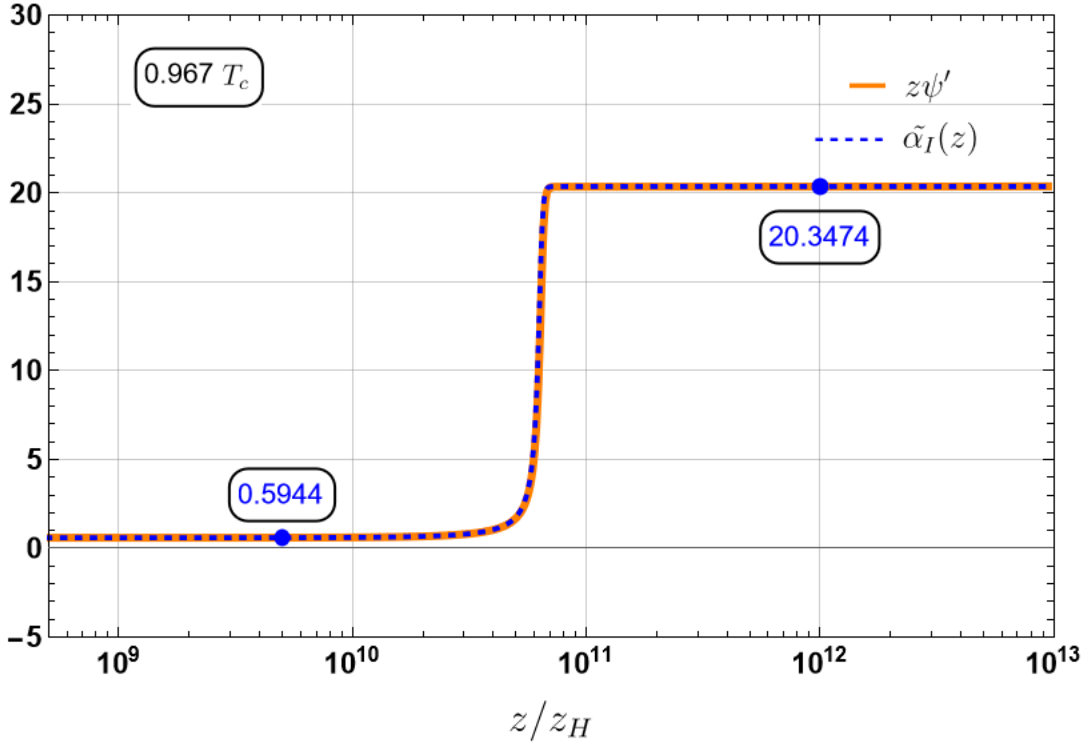
Result: Kasner Alternation

The laws of Kasner Transformation(e.g. $\mathcal{F}(\psi) \backsim \exp(\kappa\psi)$):

$$\begin{cases} Kasner\ Inversion, & \alpha\alpha_I = 2(d-1)(d-2), & |\alpha| < \sqrt{2(d-1)(d-2)} \\ Kasner\ Transition, & \alpha + \alpha_T = \frac{4}{\kappa}(d-1), & \kappa\alpha > 2(d-1) \end{cases}$$

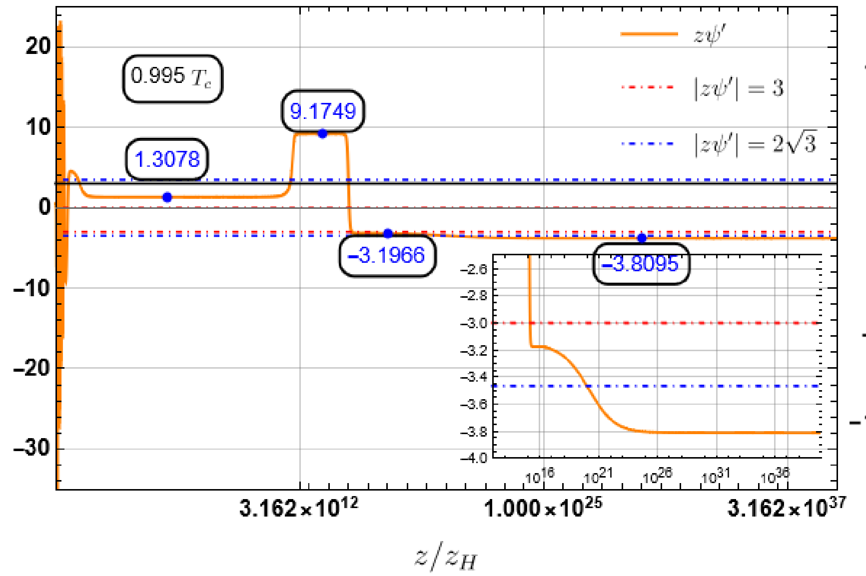


Transition(d=4, κ=2)



Inversion(d=4 , κ=2)

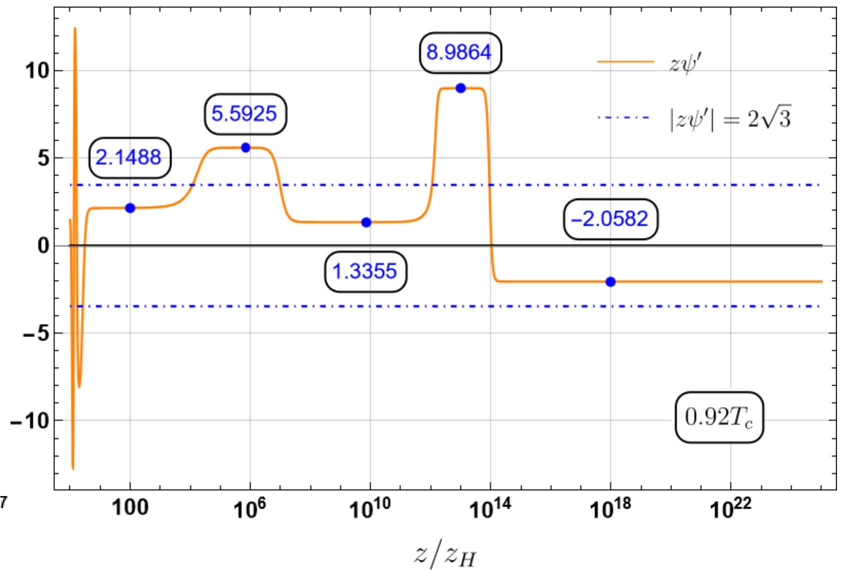
Result: Classification



Competition

$\sqrt{2(d-1)(d-2)} > 2(d-1)/|\kappa|$

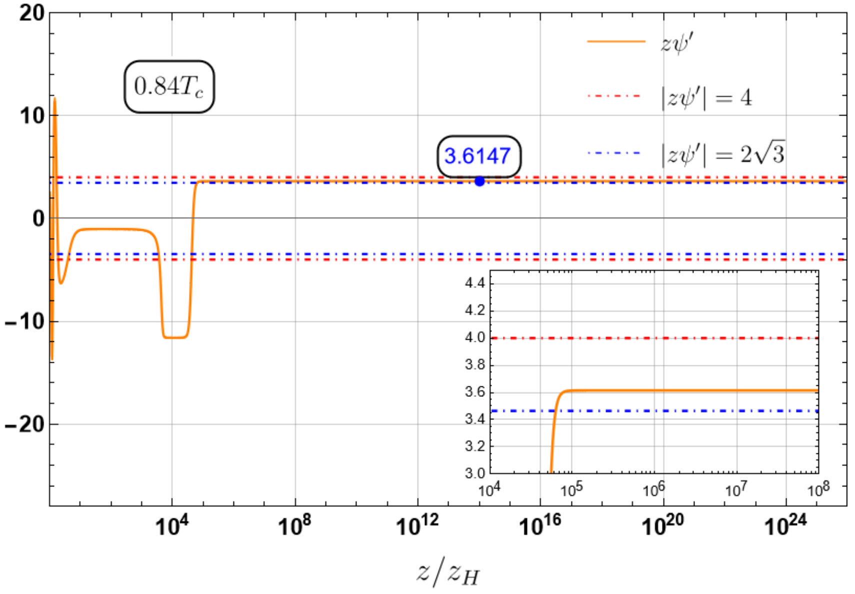
(d=4, κ=2)



Infinite Alternation

$\sqrt{2(d-1)(d-2)} = 2(d-1)/|\kappa|$

(d=4, κ=$\sqrt{3}$)



Stable Kasner

$\sqrt{2(d-1)(d-2)} < 2(d-1)/|\kappa|$

(d=4, κ=3/2)

Example 2(JHEP04(2025)179)

Complex vector field ρ_μ charged under a U(1) gauge field A_μ:

Action and metric:

$$S = \frac{1}{\kappa^2_{(d+1)}} \int \mathrm{d}^{d+1}x \sqrt{-g}\,(R - 2\Lambda + \mathcal{L}_m)\ , \quad \mathcal{L}_m = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\rho^\dagger_{\mu\nu}\rho^{\mu\nu}\ ,$$

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(-f(z)e^{-\chi(z)}\mathrm{d}t^2 + \frac{1}{f(z)}\mathrm{d}z^2 + e^{2(d-2)\zeta(z)}\mathrm{d}x^2 + \frac{1}{e^{2\zeta(z)}}\mathrm{d}\Sigma^2_{d-2}\right)\ .$$

$$A_\nu \mathrm{d}x^\nu = \phi(z)\mathrm{d}t\ , \quad \rho_\mu \mathrm{d}x^\mu = \psi(z)\mathrm{d}x.$$

Result: Kasner Structure

With approximations, in the far interior $z \longrightarrow \infty$:

$$\zeta = \beta \ln z + C_\zeta, \quad z\rho_x' = C_{z\rho_x'} z^{2(d-2)\beta-2}, \quad A_t' = C_{A_t'} z^{d-3} \mathrm{e}^{-\chi/2},$$
$$\chi = 2(d-2)\beta^2 \ln z + C_\chi, \quad h' = C_{h'} z^{d-3} \mathrm{e}^{-\chi/2},$$

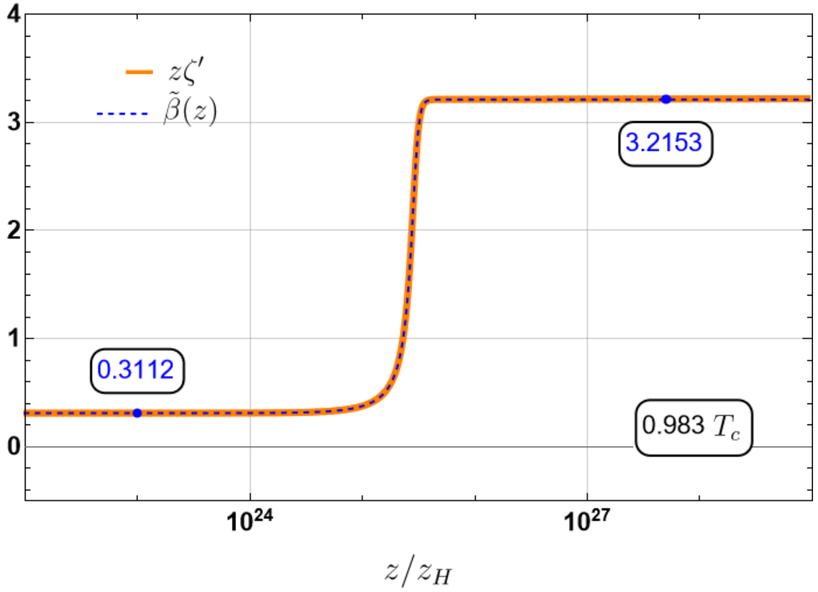
$$\Big\downarrow \quad \tau \sim z^{-\frac{d}{2}-\frac{(d-2)\beta^2}{2}}$$

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + c_t \tau^{2p_t} \mathrm{d}t^2 + c_x \tau^{2p_x} \mathrm{d}x^2 + c_\sigma \tau^{2p_\sigma} \mathrm{d}\Sigma_{d-2}^2\,,$$

$$p_t + p_x + (d-2)p_\sigma = 1, \quad p_t^2 + p_x^2 + (d-2)p_\sigma^2 = 1.$$

Result: Kasner Alternation

where $\beta_c = -1 + \sqrt{2(d-1)/(d-2)}$.

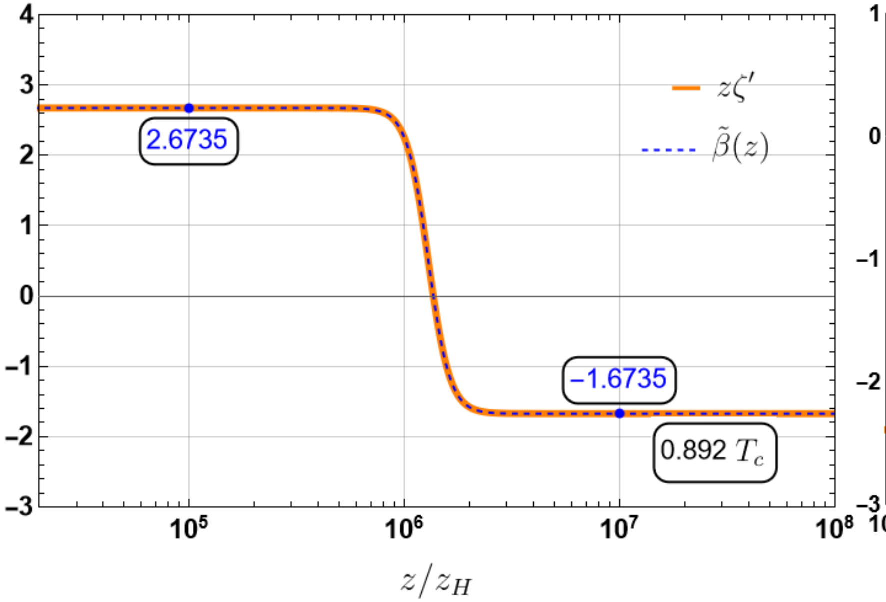


Inversion($1 < \beta < \beta_c$)

$d = 5,\ T = 0.983T_c$

The relationship of β

$$\beta\beta_I = 1\,,$$

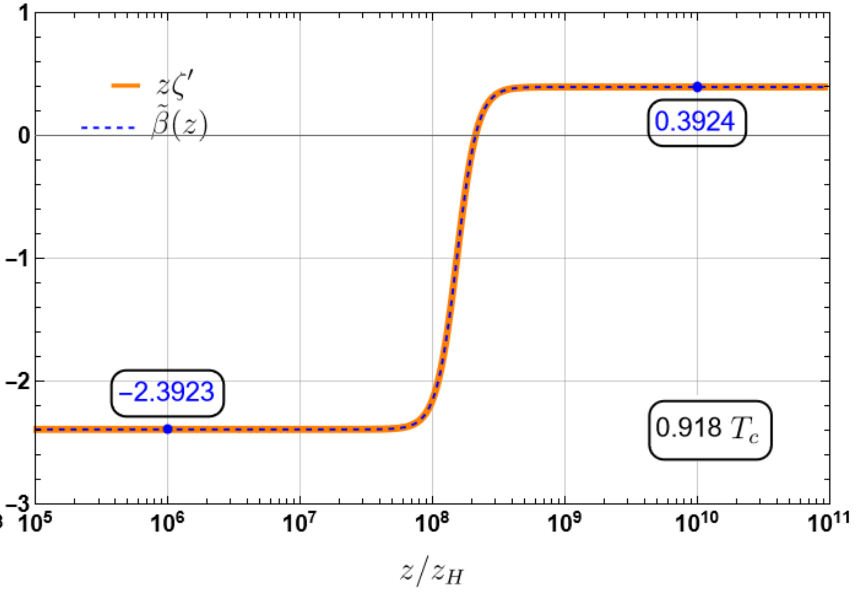


Transition($\beta > \beta_c$)

$d = 5,\ T = 0.892T_c$

The relationship of β

$$\beta + \beta_T = \frac{2}{d-2}\,.$$



Reflection($\beta < -1$)

$d = 5,\ T = 0.918T_c$

The relationship of β

$$\beta + \beta_R = -2\,,$$

Result: Classification

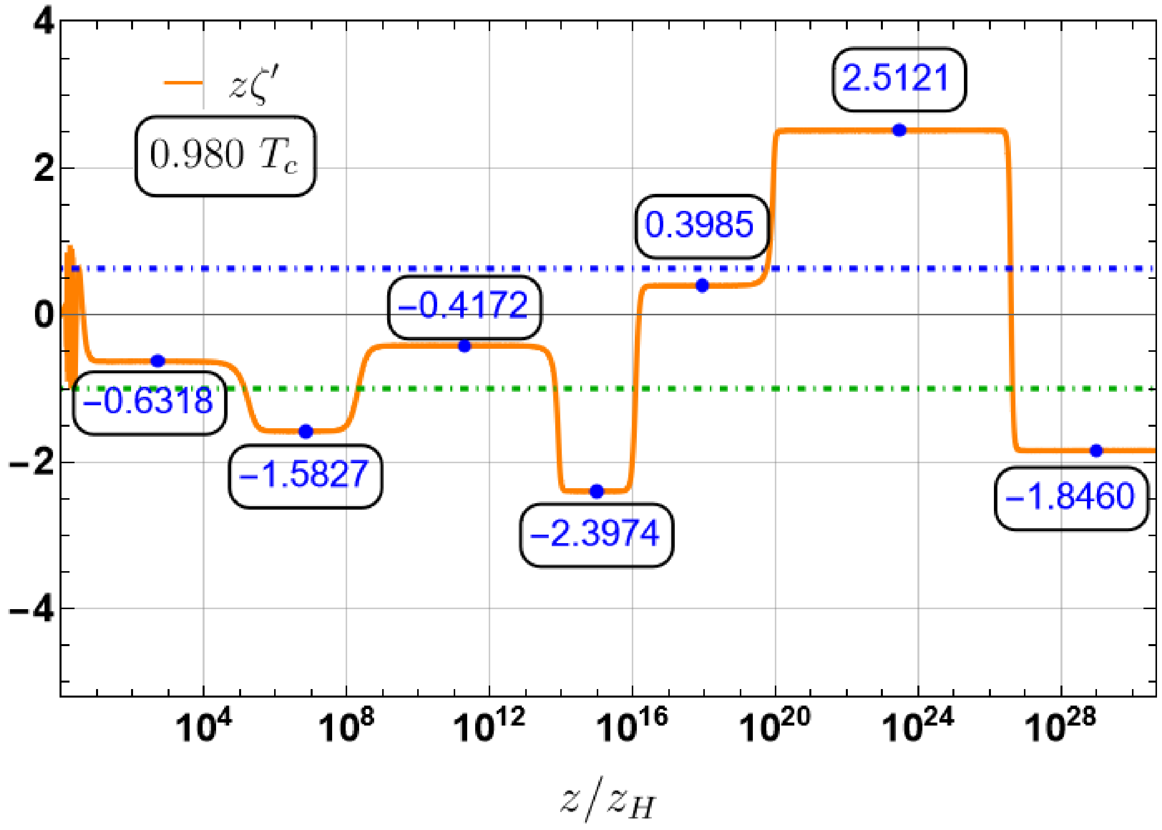
The laws of Kasner transformation:

$$\begin{cases} \text{Kasner transition}: \beta + \beta_T = \frac{2}{d-2}, \quad \beta > \beta_c\,, \\ \text{Kasner inversion}: \quad \beta\,\beta_I = 1, \quad -1 < \beta < \beta_c\,, \\ \text{Kasner reflection}: \beta + \beta_R = -2, \quad \beta < -1\,, \end{cases}$$

where $\beta_c = -1 + \sqrt{2(d-1)/(d-2)}$.

Blue dashed curve:
the boundary
between inversion and transition

Green dashed curve:
the boundary
between inversion and reflection



$d = 5,\ T = 0.980T_c$

Summary:

1. Rich interior dynamics of black holes
   Cauchy horizon, ER collapse, Josephson oscillation, Kasner epoch…

2. Different Method approaching space-like singularities
   Cosmological Billiards, BKL limit, EoMs…

3. The transformation between Kasner epochs, and their analyzed transformation laws

4. Possible extensions in other models
   Modified gravity, model with low symmetry…

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Thanks！

谢谢！