Chiral phase transition: Effective field theory and holography

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Based on arXiv:2412.08882, Yanyan Bu, Zexin Yang(PRD)

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contents

Introduction

- Goal: Develop a SK effective action for QCD matter near the chiral phase transition

- Background: QCD matter with 2 flavor quarks at finite temperature but with vanishing baryon density, slightly above phase transition point.

- Focus: We basically focus on 2 flavors quarks in chiral limit! We mainly stand the perspective of $SU(2)_L \times SU(2)_R$ flavor symmetry!

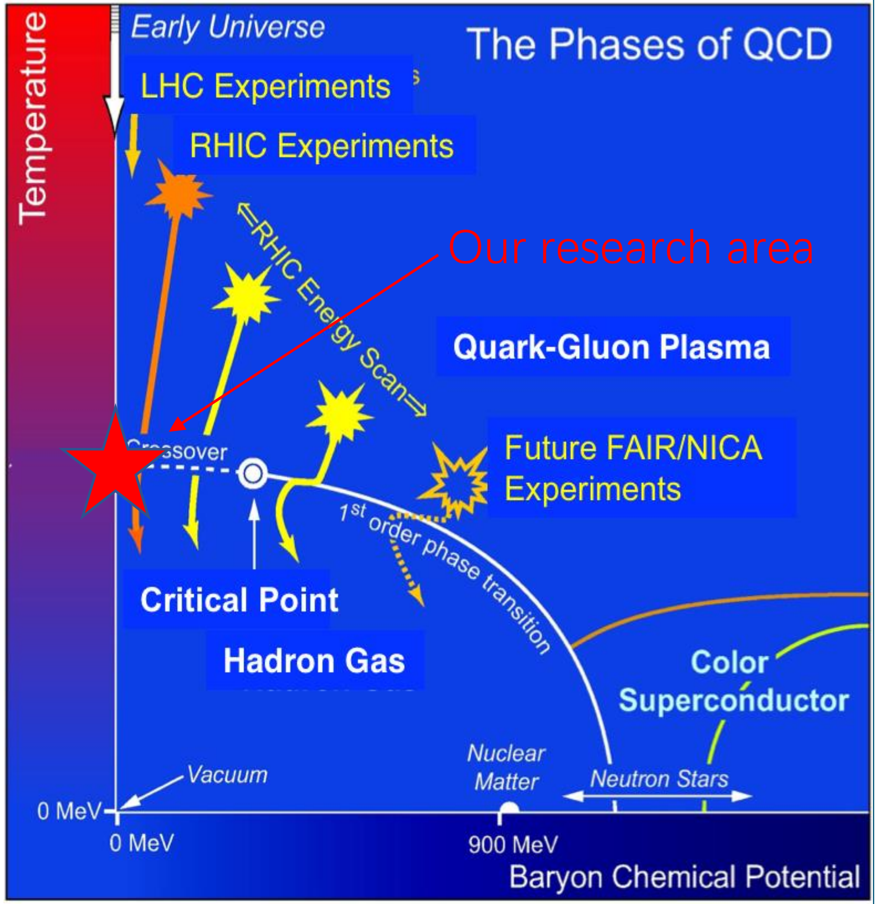

Figure 1 QCD phase diagram

Nayak T K. Journal of Physics: Conference Series, 2020, 1602(1) : 012003.

Introduction

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[arXiv:2411.08016, M. Hongo, N. Sogabe, M.A. Stephanov, and H.-U. Yee.]

[arXiv:2005.02885, E. Grossi, A. Soloviev, D. Teaney, F. Yan.]

We both consider flavor currents and order parameter!

[arXiv:2304.06008, A. Donos, P. Kailidis]

Nearly critical U(1) superfluid: Relation and difference?

- **Facts:** In the low-energy regime of QCD in vacuum, hadrons are basic degree of freedom.

Methods: EFT construction and holographic calculation.

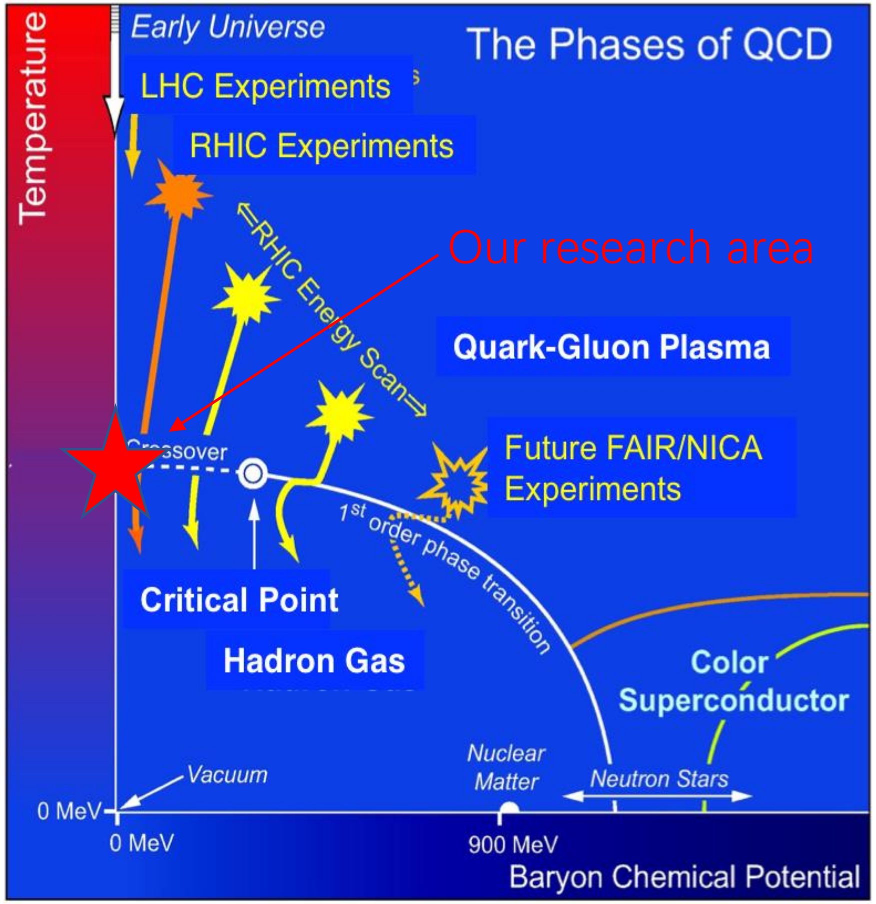

Figure 1 QCD phase diagram

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SK EFT Side

[arXiv:1805.09331, P. Glorioso, H. Liu]

[arXiv:2007.13753, P. Glorioso, L. V. Delacrétaz, X. Chen, R. M. Nandkishore, A. Lucas]

[arXiv:2205.00195, Y. Bu, X. Sun, B. Zhang]

- Degrees of freedom: order parameter $\mathcal{O}$(u,d quarks pairing condensation) and chiral conserved charge ρ_L and ρ_R
- Schwinger-Keldysh formalism: $O \to O_1,\ O_2 \Leftrightarrow O_a,\ O_r$
- Variables Definition:

$$B_\mu \equiv \mathcal{U}(\varphi)\,(\mathcal{A}_\mu + \mathrm{i}\partial_\mu)\mathcal{U}^\dagger(\varphi), \quad C_\mu \equiv \mathcal{U}(\phi)\,(\mathcal{V}_\mu + \mathrm{i}\partial_\mu)\mathcal{U}^\dagger(\phi),$$

$$\Sigma \equiv \mathcal{U}(\varphi)\mathcal{O}\mathcal{U}^\dagger(\phi).$$

Where $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$ are background gauge fields, $\mathcal{U}(\varphi) = e^{\mathrm{i}\varphi^a(x)t^a}$, $\mathcal{U}(\phi) = e^{\mathrm{i}\phi^a(x)t^a}$.

- Chemical potentials and conserved charges: $B_0 \sim \rho_L,\ C_0 \sim \rho_R$

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- Principles

◆ Symmetries:

The constraints implied by the unitarity of time evolution

Global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauge symmetry,

Chemical shift symmetry,

$$\varphi_r^{\mathrm{a}} \to \varphi_r^{\mathrm{a}} + \sigma_L^{\mathrm{a}}(\vec{x}),\ \phi_r^{\mathrm{a}} \to \phi_r^{\mathrm{a}} + \sigma_R^{\mathrm{a}}(\vec{x}), \quad \text{others unchanged},$$

$$\mathcal{D}_{Li} \equiv \partial_i - \mathrm{i}[B_{ri},\cdot], \quad \mathcal{D}_{Ri} \equiv \partial_i - \mathrm{i}[C_{ri},\cdot],\ \mathcal{D}_i = \partial_i - \mathrm{i}B_{ri}\cdot + \mathrm{i}\cdot C_{ri}.$$

Dynamic KMS symmetry, Onsager relation……

◆ Building Blocks:

$$B_{a\mu},\ C_{a\mu},\ \mathcal{D}_i B_{a\mu},\ \mathcal{D}_i C_{a\mu},\ B_{rv},\ C_{rv},\ \mathcal{D}_i B_{rv},\ \mathcal{D}_i C_{rv}, \partial_v B_{ri}, \partial_v C_{ri}$$

$$\Sigma_a,\ \Sigma_r,\ \tilde{\mathcal{D}}_i\Sigma_a,\ \tilde{\mathcal{D}}_i\Sigma_r,\ \mathcal{F}_{rij} \equiv \partial_i B_{rj} - \partial_j B_{ri} - i[B_{ri}, B_{rj}]$$

◆ Organized by number of fields and derivatives, by order of space and time derivatives

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$$\mathcal{L}_{diff}=\frac{\mathrm{i}}{2}u_0B_{av}B_{av}+\frac{\mathrm{i}}{2}u_1B_{ai}B_{ai}+\mathrm{i}u_2B_{av}(\mathcal{D}_i\partial_vB_{ai})+a_0B_{av}B_{rv}+a_1B_{av}\partial_vB_{rv}+a_2B_{ai}\partial_vB_{ri}+a_3B_{av}(\mathcal{D}_i\partial_vB_{ri})+a_4B_{ai}(\mathcal{D}_i\partial_vB_{rv})$$
$$+a_5B_{av}\mathcal{D}_i(\mathcal{D}_iB_{rv})+a_6B_{ai}\mathcal{D}_i(\mathcal{D}_i(\mathcal{D}_iB_{rv}))+\frac{\mathrm{i}}{2}u_3C_{av}C_{av}+\frac{\mathrm{i}}{2}u_4C_{ai}C_{ai}+\mathrm{i}u_5C_{av}(\mathcal{D}_i\partial_vC_{ai})+a_7C_{av}C_{rv}+a_8C_{av}\partial_vC_{rv}$$
$$+a_9C_{ai}\partial_vC_{ri}+a_{10}C_{av}(\mathcal{D}_i\partial_vC_{ri})+a_{11}C_{ai}(\mathcal{D}_i\partial_vC_{rv})+a_{12}C_{av}\mathcal{D}_i(\mathcal{D}_iC_{rv})+a_{13}C_{ai}\mathcal{D}_i(\mathcal{D}_i(\mathcal{D}_iC_{rv}))$$

$$\mathcal{L}_{\Sigma}=\mathrm{i}v_0\Sigma_a^\dagger\Sigma_a+v_1\Sigma_a^\dagger\partial_v\Sigma_a+\mathrm{i}v_2\left(\tilde{\mathcal{D}}_i\Sigma_a^\dagger\right)\left(\tilde{\mathcal{D}}_i\Sigma_a\right)+b_0\Sigma_r^\dagger\Sigma_a+b_0^*\Sigma_a^\dagger\Sigma_r$$
$$+b_1\Sigma_a\partial_v\Sigma_r^\dagger+b_1^*\Sigma_a^\dagger\partial_v\Sigma_r+b_2\left(\tilde{\mathcal{D}}_i\Sigma_r\right)^\dagger\left(\tilde{\mathcal{D}}_i\Sigma_a\right)+b_2^*\left(\tilde{\mathcal{D}}_i\Sigma_a\right)^\dagger\left(\tilde{\mathcal{D}}_i\Sigma_r\right)$$

$$\mathcal{L}_3=c_0\Sigma_r\Sigma_r^\dagger B_{av}+d_0\Sigma_r^\dagger\Sigma_rC_{av}+c_1\Sigma_r\Sigma_a^\dagger B_{rv}+d_1\Sigma_a^\dagger\Sigma_rC_{rv}+c_1^*\Sigma_a\Sigma_r^\dagger B_{rv}+d_1^*\Sigma_r^\dagger\Sigma_aC_{rv}+c_2\partial_v\Sigma_r\cdot\Sigma_a^\dagger B_{rv}+d_2\Sigma_a^\dagger\partial_v\Sigma_r\cdot C_{rv}$$
$$+c_2^*\Sigma_a\cdot\partial_v\Sigma_r^\dagger B_{rv}+d_2^*\partial_v\Sigma_r^\dagger\cdot\Sigma_aC_{rv}+c_3\partial_v\Sigma_r\cdot\Sigma_r^\dagger B_{av}+d_3\Sigma_r^\dagger\partial_v\Sigma_r\cdot C_{av}+\mathrm{i}c_4\left(\tilde{\mathcal{D}}_i\Sigma_r\right)\Sigma_r^\dagger B_{ai}+\mathrm{i}d_4\Sigma_r^\dagger\left(\tilde{\mathcal{D}}_i\Sigma_r\right)C_{ai}$$
$$-\mathrm{i}c_4^*\Sigma_r\left(\tilde{\mathcal{D}}_i\Sigma_r\right)^\dagger B_{ai}-\mathrm{i}d_4^*\left(\tilde{\mathcal{D}}_i\Sigma_r\right)^\dagger\Sigma_rC_{ai}+\varpi_1B_{rv}B_{ai}(\mathcal{D}_iB_{rv})+\varpi_2B_{rv}B_{ai}\partial_vB_{ri}+\varpi_3C_{rv}C_{ai}(\mathcal{D}_iC_{rv})+\varpi_4C_{rv}C_{ai}\partial_vC_{ri}$$

$$\mathcal{L}_4=\chi_1\Sigma_a^\dagger\Sigma_r\Sigma_r^\dagger\Sigma_r+\chi_1^*\Sigma_r^\dagger\Sigma_a\Sigma_r^\dagger\Sigma_r+c_5\Sigma_r\Sigma_r^\dagger B_{rv}B_{av}+d_5\Sigma_r^\dagger\Sigma_rC_{rv}C_{av}+c_6\Sigma_r\Sigma_a^\dagger B_{rv}B_{rv}+d_6\Sigma_a^\dagger\Sigma_rC_{rv}C_{rv}$$
$$+c_6^*\Sigma_a\Sigma_r^\dagger B_{rv}B_{rv}+d_6^*\Sigma_r^\dagger\Sigma_aC_{rv}C_{rv}+\chi_2\Sigma_r^\dagger B_{rv}\Sigma_rC_{av}+\chi_3\Sigma_r^\dagger B_{av}\Sigma_rC_{rv}+\chi_4\Sigma_r^\dagger B_{rv}\Sigma_aC_{rv}+\chi_4^*\Sigma_a^\dagger B_{rv}\Sigma_rC_{rv}$$

SK EFT Side

- Definition of currents:

$$J_L^\mu \equiv \frac{\delta S_{eff}}{\delta \mathcal{A}_{a\mu}}, \quad J_R^\mu \equiv \frac{\delta S_{eff}}{\delta \mathcal{V}_{a\mu}}$$

- Here, $\mathcal{A}_\mu, \mathcal{V}_\mu$ are external gauge fields, and $\varphi_\mu, \ \phi_\mu$ are diffusion fields.
- Variation principles:

$$\frac{\delta S_{\text{eff}}}{\delta \varphi_a} = 0 \Rightarrow \partial_\mu J_L^\mu = 0, \quad \frac{\delta S_{\text{eff}}}{\delta \phi_a} = 0 \Rightarrow \partial_\mu J_R^\mu = 0, \quad \frac{\delta S_{eff}}{\delta \Sigma_a^\dagger} = 0 \Rightarrow \frac{J_{\mathcal{O}}}{b_1^*} = \zeta$$

- Redefinition of order parameter and its covariant derivative:

$$\Sigma_r \to \tilde{\Sigma}_r = \mathcal{U}_L^\dagger \Sigma_r \mathcal{U}_R, \quad \tilde{\mathcal{D}}_i \Sigma_r \to \hat{\mathcal{D}}_i \tilde{\Sigma}_r = \partial_i \tilde{\Sigma}_r - \mathrm{i} A_{L;ri} \tilde{\Sigma}_r + \mathrm{i} \tilde{\Sigma}_r A_{R;ri}$$

- Stochastic equations :

$$\partial_0 \tilde{\Sigma}_r = \frac{b_0}{b_1^*} \tilde{\Sigma}_r - \frac{b_2}{b_1^*} \hat{\mathcal{D}}_i^\dagger \left(\hat{\mathcal{D}}_i \tilde{\Sigma}_r \right) + \frac{c_0}{b_1^* a_0} \rho_L \tilde{\Sigma}_r + \frac{d_0}{b_1^* a_7} \tilde{\Sigma}_r \rho_R + \frac{1}{b_1^*} \left(\chi_1 + \frac{c_0^2}{a_0} + \frac{d_0^2}{a_7} \right) \tilde{\Sigma}_r \tilde{\Sigma}_r^\dagger \tilde{\Sigma}_r$$

$$+ \frac{c_6}{b_1^* a_0^2} \tilde{\Sigma}_r \rho_L \rho_L + \frac{d_6}{b_1^* a_7^2} \tilde{\Sigma}_r \rho_R \rho_R + \frac{\chi_4}{\varpi_4^* w_3 w_{15}} \tilde{\Sigma}_r \rho_L \rho_R + \theta$$

$$\partial_0 \rho_L = \frac{\beta u_1}{2 a_0} \nabla^2 \rho_L - \frac{\varpi_1 + \varpi_2}{2 a_0^2} \nabla^2 \rho_L^2 + \frac{\beta u_1 c_1}{2 a_0} \nabla^2 \left(\tilde{\Sigma}_r \tilde{\Sigma}_r^\dagger \right) - \mathrm{i} c_4 \left(\nabla^2 \tilde{\Sigma}_r \cdot \tilde{\Sigma}_r^\dagger - \nabla^2 \tilde{\Sigma}_r^\dagger \cdot \tilde{\Sigma}_r \right)$$

$$\partial_0 \rho_R = \frac{\beta u_4}{2 a_7} \nabla^2 \rho_R - \frac{\varpi_3 + \varpi_4}{2 a_7^2} \nabla^2 \rho_R^2 + \frac{\beta u_4 d_1}{2 a_7} \nabla^2 \left(\tilde{\Sigma}_r \tilde{\Sigma}_r^\dagger \right) - \mathrm{i} d_4 \left(\nabla^2 \tilde{\Sigma}_r \cdot \tilde{\Sigma}_r^\dagger - \nabla^2 \tilde{\Sigma}_r^\dagger \cdot \tilde{\Sigma}_r \right)$$

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- HH's model F:

$$\frac{\partial \psi}{\partial t} = -2\Gamma_0 \frac{\delta F}{\delta \psi^*} - i g_0 \psi \frac{\delta F}{\delta m} + \theta,$$

$$\frac{\partial M}{\partial t} = \lambda_0^m \nabla^2 \frac{\partial F}{\partial m} + 2 g_0 \,\mathrm{Im}\left(\psi^* \frac{\delta F}{\delta \psi^*}\right) + \zeta,$$

$$F[\psi, m] = F_0 - \int d^d x \{h_m(\mathbf{x}, t) m + \mathrm{Re}[h(\mathbf{x}, t)\psi^*]\},$$

$$F_0 = \int d^d x \left\{\frac{1}{2}\tilde{r}_0 |\psi|^2 + \frac{1}{2}|\nabla\psi|^2 + \tilde{u}_0 |\psi|^4 + \frac{1}{2} C_0^{-1} m^2 + \gamma_0 m |\psi|^2\right\},$$

- Rewrite the stochastic equations:

$$\partial_v \tilde{\Sigma}_r = -2\Gamma_0 \frac{\delta F}{\delta \tilde{\Sigma}_r^\dagger} - i g_{0L} \tilde{\Sigma}_r \frac{\delta F}{\delta \rho_L} - i g_{0R} \tilde{\Sigma}_r \frac{\delta F}{\delta \rho_R} + \theta$$

$$\partial_0 \rho_L = d_0^{\rho_L} \nabla^2 \frac{\delta F}{\delta \rho_L} + 2 g_{0L} \,\mathrm{Im}\left(\tilde{\Sigma}_r^\dagger \frac{\delta F}{\delta \tilde{\Sigma}_r^\dagger}\right) + \xi_1$$

$$\partial_0 \rho_R = d_0^{\rho_R} \nabla^2 \frac{\delta F}{\delta \rho_R} + 2 g_{0R} \,\mathrm{Im}\left(\tilde{\Sigma}_r^\dagger \frac{\delta F}{\delta \tilde{\Sigma}_r^\dagger}\right) + \xi_2$$

Model	Designation	System
Relaxational	A	Kinetic Ising anisotropic magnets
	B	Kinetic Ising uniaxial ferromagnet
	C	Anisotropic magnets structural transition
Fluid	H	Gas–liquid binary fluid
Symmetric planar magnet	E	Easy-plane magnet, $h_z = 0$
Asymmetric planar magnet	F	Easy-plane magnet, $h_z \neq 0$ superfluid helium
Isotropic antiferromagnet	G	Heisenberg antiferromagnet
Isotropic ferromagnet	J	Heisenberg ferromagnet

Figure 2 Hohenburg's classification of some dynamical models after treatment with the renormalization group method

[Hohenberg P C, Halperin B I. Reviews of Modern Physics, 1977, 49(3) : 435.]

Holographic Side

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- AdS/QCD is an excellent method for studying strong coupling QCD systems in the low energy region

[arXiv:hep-ph/0501128, J. Erlich, E. Katz, D. T. Son, M. A. Stephanov]

[arXiv:hep-ph/0602229, A. Karch, E. Katz, D. T. Son, M. A. Stephanov]

[arXiv:1810.07019, Jianwei Chen, Song He, Mei Huang, Danning Li]

[arXiv:2210.09088, X. Cao, M. Baggioli, H. Liu, D. Li]

[arXiv:1511.02721, K. Chelabi, Z. Fang, M. Huang, D. Li, Y. Wu]

- Consider a modified AdS/QCD model:

$$S_0 = \int d^5x\sqrt{-g}\,\mathrm{Tr}\left\{-|DX|^2 + \left(m_0^2 - \frac{\mu_c^2}{r^2}\right)|X|^2 + a|X|^4 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right\}$$

- Modify the effective mass, to adjust the system to the critical point:

$$m_0^2 \to m_c^2 = m_0^2 - \frac{\mu_c^2}{r^2}$$

Holographic Side

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- X is 5d dual field of $\langle q\bar{q}\rangle$, A_L, A_R are chiral gauge fields. Their boundary behaviors are:

$$A_{L\mu}(r=\infty_s,\ x^\alpha)=B_{s\mu}+\cdots,\ A_{R\mu}(r=\infty_s,\ x^\alpha)=C_{s\mu}+\ldots,$$
$$X(r=\infty_s,\ x^\alpha)=\frac{m_s}{r}+\frac{\Sigma_s}{r^3}+\cdots,\ X^\dagger(r=\infty_s,\ x^\alpha)=\frac{m_s^\dagger}{r}+\frac{\Sigma_s^\dagger}{r^3}+\cdots,$$

- The partition function in bulk

$$Z_{AdS}=\int[DA_{LM}][DA_{RM}][DX][DX^\dagger]\mathrm{e}^{\mathrm{i}S_0[A_{LM},A_{RM},X,X^\dagger]+\mathrm{i}S_{bdy}}$$

- The partition function on boundary

$$Z_{bdy}=\int[DB_\mu][DC_\mu][D\Sigma][D\Sigma^\dagger]\mathrm{e}^{\mathrm{i}S_0|_{\mathrm{p.o.s}}[B_\mu,C_\mu,\Sigma,\Sigma^\dagger]+\mathrm{i}S_{\mathrm{bdy}}}$$

- The partition function on EFT side

$$Z_{eff}=\int[D\varphi_r][D\varphi_a][D\phi_r][D\phi_a][D\Sigma_r][D\Sigma_a]e^{\mathrm{i}S_{\mathrm{eff}}[B_{r\mu},C_{r\mu},\Sigma_r;B_{a\mu},C_{a\mu},\Sigma_a]},$$

Holographic Side

- Read the effective action on boundary:

$$S_{eff} = S_0|_{\text{p.o.s}}[B_\mu, C_\mu, \Sigma, \Sigma^\dagger] + S_{\text{bdy}} = S_{\text{eff}}[B_{r\mu}, C_{r\mu}, \Sigma_r; B_{a\mu}, C_{a\mu}, \Sigma_a]$$

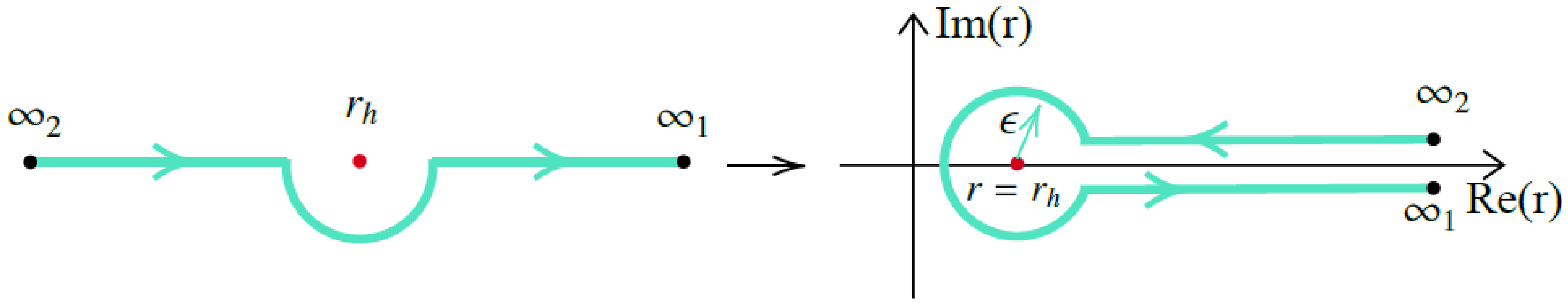

Figure 3 Left: complex double AdS contour； Right: holographic SK contour

[Glorioso P, Crossley M, Liu H. arXiv preprint arXiv:1812.08785, 2018.]

- Next, solve the EOM of bulk fields X, A_L, A_R on holographic SK contour:

$$D^M D_M X + \left(m_0^2 - \frac{\mu_c^2}{r^2}\right) X + 2a(X^\dagger X)X = 0, \quad (D^M D_M X)^\dagger + \left(m_0^2 - \frac{\mu_c^2}{r^2}\right) X^\dagger + 2a(XX^\dagger)X^\dagger = 0$$

$$\frac{1}{g_5^2}[-\nabla^N (F_L)_{NM} + i[A_L^N, (F_L)_{NM}]] + J_{LM} = 0, \quad \frac{1}{g_5^2}[-\nabla^N (F_R)_{NM} + i[A_R^N, (F_R)_{NM}]] + J_{RM} = 0$$

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- The numerical integration method is used for numerical solution for scalar field X
- Eventually, we can obtain the boundary effective action and the coefficients.

$$u_0=u_3=0,\ u_1=u_4=-\frac{2}{\pi},\ a_0=a_7=-2,\ v_0=0.220,\ v_1=0,\ v_2=0.0579\mathrm{i},$$

$$b_0=b_0^*=0,\ b_1=b_1^*=-0.348,\ b_2=b_2^*=-0.121,\ c_0=d_0=0,\ c_4=0.121,\ d_4=-0.121,$$

$$\varpi_1=\varpi_3=\log(2),\ \varpi_2=\varpi_4=\log(2),$$

$$\chi_1=(-0.00730-0.0113\mathrm{i})a, c_5=c_5^*=0.0672,\ c_6=c_6^*=0.0336,\ d_6=d_6^*=0.0336,$$

$$\chi_2=-0.1340,\ \chi_3=-0.1340,\ \chi_4=\chi_4^*=-0.0672$$

Summary and Outlook

- We have constructed a Wilsonian EFT of QCD matter near the chiral phase transition.
- The set of stochastic equations resemble the model F of Hohenberg-Halperin classification
- We have confirmed the EFT construction by deriving the boundary effective action for a modified AdS/QCD model, which naturally incorporates spontaneous chiral symmetry breaking.

Several future directions:

- ➢ physical consequences of higher-order terms in stochastic equations
- ➢ explicit breaking of chiral symmetry
- ➢ Color superconductivity（EFT of electromagnetic superconductivity has been constructed. Based on this, we will extend this method to color superconductivity）

Thank you for listening

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