Holographic multipartite entanglement from upper bound configurations of entropy combinations

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Motivation and summary

- **Quantum entanglement**: essential for building spacetime geometry and for governing the quantum properties in quantum gravity systems;
- **Holographic entanglement structure:** a very special one, strong, multi-partite entanglement required in quantum error correction;
- **Detecting the holographic entanglement structure:** various bipartite and multipartite entanglement measures in holography;
- An important measure: the **conditional mutual information (CMI)** between A and B with the condition E:

$$I(A:B|E) = S_{AE} + S_{BE} - S_E - S_{ABE}$$

- CMI quantifies the correlation between A and BE that is not just due to the correlation between A and E
- CMI≥MI: MI fails to capture how ρ_{AB} is embedded in the full system
- Strong subadditivity ensures the non-negativity of CMI

Motivation and summary

- **CMI as a probe of the holographic entanglement structure:**
  - ➢Reveals real-space entanglement behavior, short/long range entanglement;
  - ➢Captures multipartite entanglement contributions: the multipartite entanglement that two distant small subsystems participate would contribute to the CMI;
- A new proposal on the relation between quantum entanglement and geometry: **radial scale ↔ boundary real space scale entanglement** (Ju et.al, 2024 & Ji et.al, 2025) **IR geometry** --- long distance entanglement structure;

  UV geometry --- short distance entanglement structure;
- **The conditional region E** has been chosen to be the subregion between the two subsystems.

Motivation and summary

- ***Questions:***

➢ What is the role of this conditional region?

➢ Does this choice optimally reveal entanglement structure?

➢ Does it maximize CMI between subsystems?

- ***Answer: No***
- What if we tune this conditional region: the behavior of CMI under variation of E reflects the multipartite entanglement structure, especially the upper bound of CMI
- CMI could be rewritten as: $I(A:B|E) = -I_3(A:B:E) + I(A:B)$
- The upper bound of CMI ⟷ upper bound of $-I_3(A:B:E)$ with fixed A and B

Motivation and summary

- **Goal:**
  - ➢ Identify the conditional region E that maximizes CMI $I(A:B|E)$, and determine the upper bound value of CMI.
  - ➢ Generalizations: the upper bound of $(-1)^n I_n$ with general n and even more general combinations of entanglement entropy for n subregions
- **Key findings:**
  - ✦ The region E in the upper bound limit typically consists of infinitely many intervals;
  - ✦ The upper bound values of $(-1)^n I_n$ and more general entropy combinations reveal the multipartite entanglement of the holographic quantum many-body system: no real bipartite entanglement in holography! All few partite entanglement emerge from more partite entanglement
- Important for the understanding of the holographic multipartite entanglement structure as well as for strongly coupled quantum many-body systems

Outline

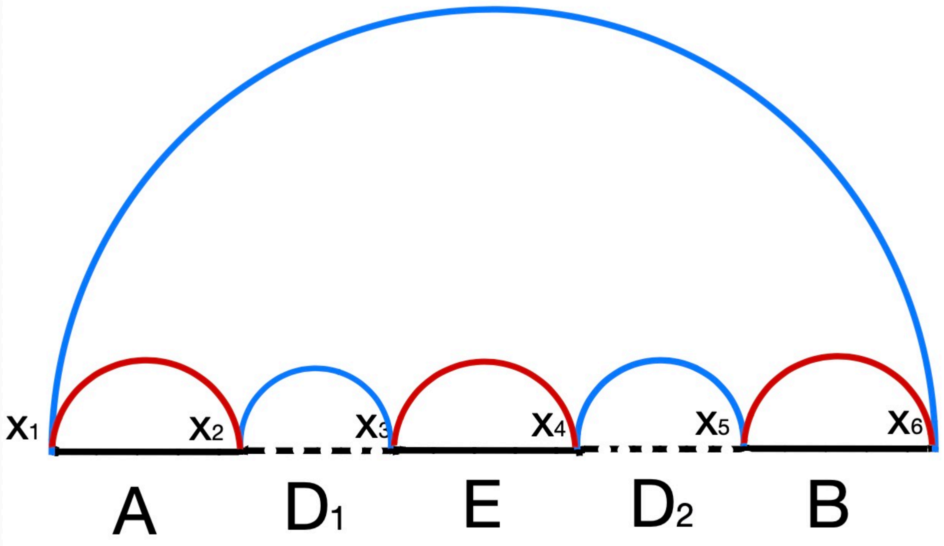
- **Upper bound of CMI (or $-I_3$) in holography:** fixing two subregions and tuning one
  - ➢A straightforward way of calculation
  - ➢A more universal method: (dis)connectivity conditions
  - ➢Fewer partite entanglement coming from more partite entanglement: more evidence
- **Upper bound of general $(-1)^n I_n$ in holography:** (dis)connectivity conditions
- **Upper bound of more general entropy combinations:** the lamp diagram method
- **Open questions:** the holographic exclusive multi-partite entanglement configuration

II. Upper bound of CMI (or $-I_3$) in holography

- **A straightforward search for the upper bound of CMI in AdS_3/CFT_2:**
- Based on the RT formula: $S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$
- For fixed A and B, vary E from the interval between A and B and search for the E that maximizes the CMI

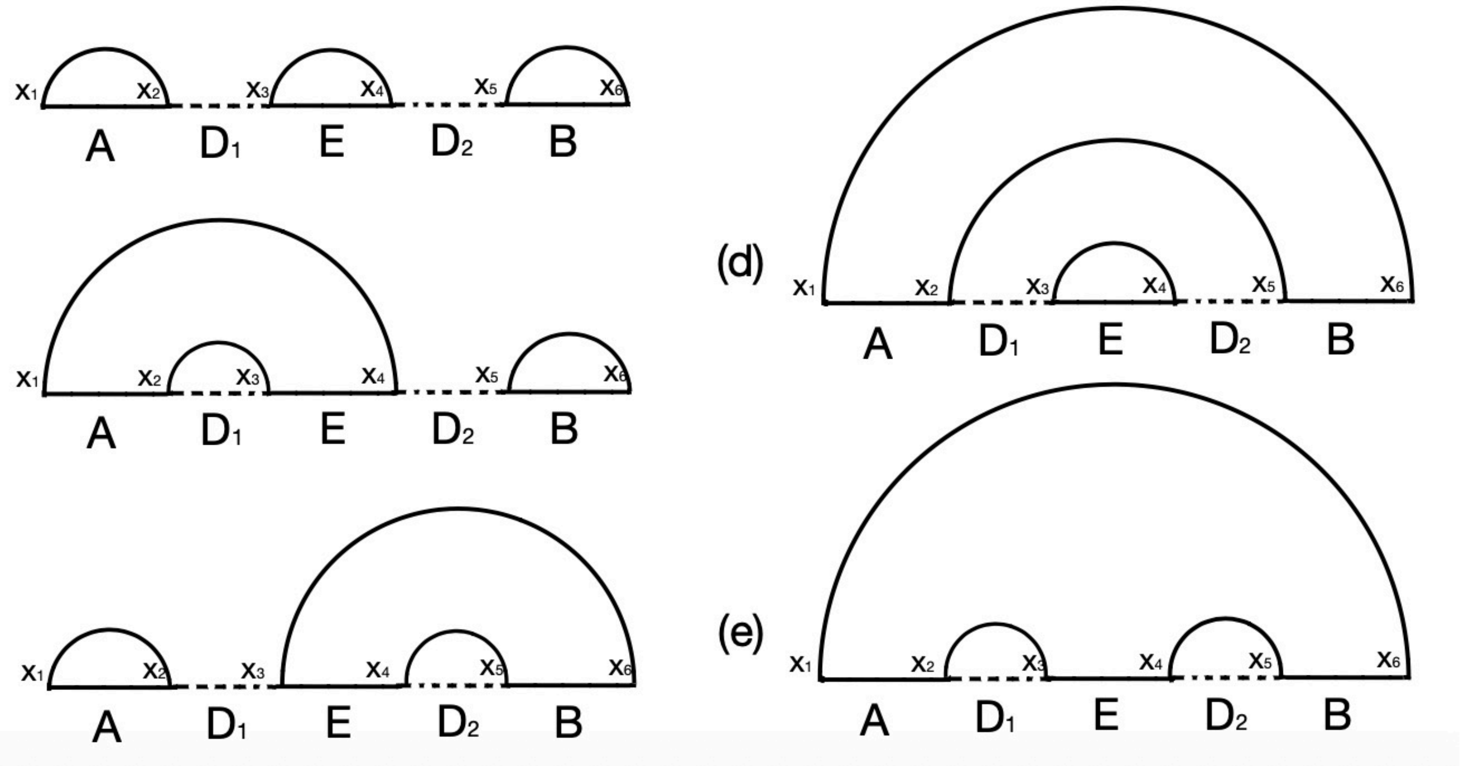
$$I(A:B|E) = S_{AE} + S_{BE} - S_E - S_{ABE}$$

- First assume E has only one connected interval; minimal surfaces chosen depending on the cross ratios



II. Upper bound of CMI (or $-I_3$) in holography

- Consider all possible RT surface configurations, choice of which depends on the cross ratios
- Example: all possible geodesic configurations for the calculation of $S(ABE)$



II. Upper bound of CMI (or $-I_3$) in holography

- The resulting value of CMI for E being only one connected interval, which depends on various cross ratios

$$I(A:B|E) = \frac{c}{3}\log\frac{\max\{1, X(A:E), X(B:E), X(A:B), X(A:B)/X(D_1:D_2)\}}{\max\{1, X(A:E)\}\max\{1, X(B:E)\}}$$

- Cross ratios defined by

$$X(A:E) = \frac{x_{12}x_{34}}{x_{23}x_{14}}, \quad X(E:B) = \frac{x_{34}x_{56}}{x_{45}x_{36}}, \quad X(A:B) = \frac{x_{12}x_{56}}{x_{25}x_{16}}, \quad X(D_1:D_2) = \frac{x_{23}x_{45}}{x_{34}x_{25}}$$

II. Upper bound of CMI (or $-I_3$) in holography

- We can find the maximum of the result tuning cross ratios related to E
- There is indeed an upper bound for I(A : B|E) when varying the length of E: at $X(A:E) = X(B:E) = 1$;
- The condition of a cross ratio being 1 is just the critical point for the entanglement phase transition between the connected and disconnected phases of the entanglement wedge;
- The interval E should be tuned to reach the critical points of both the entanglement phase transitions with A and B simultaneously: it is always possible;
- Example: when the length of A=B=1 separated at distance 1, we have E=0.464102 sitting at the middle at the CMI upper bound

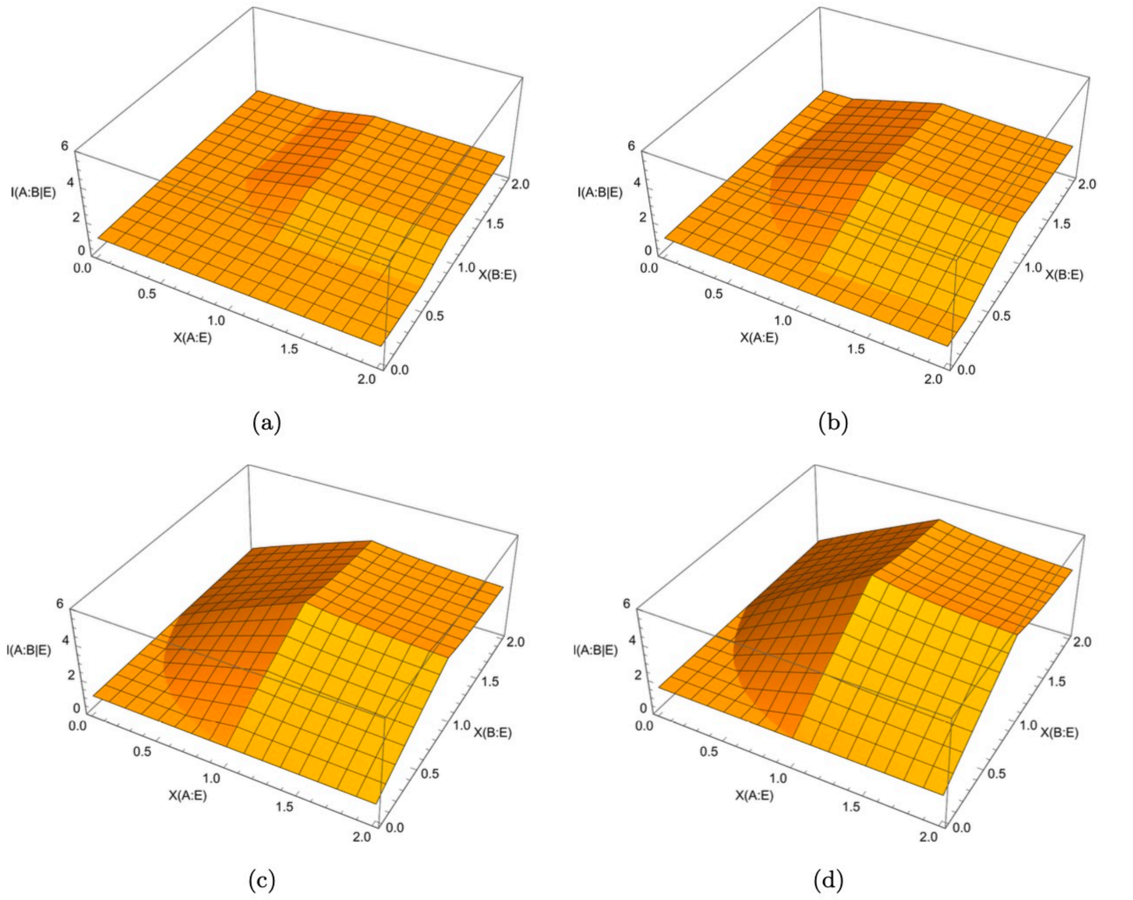
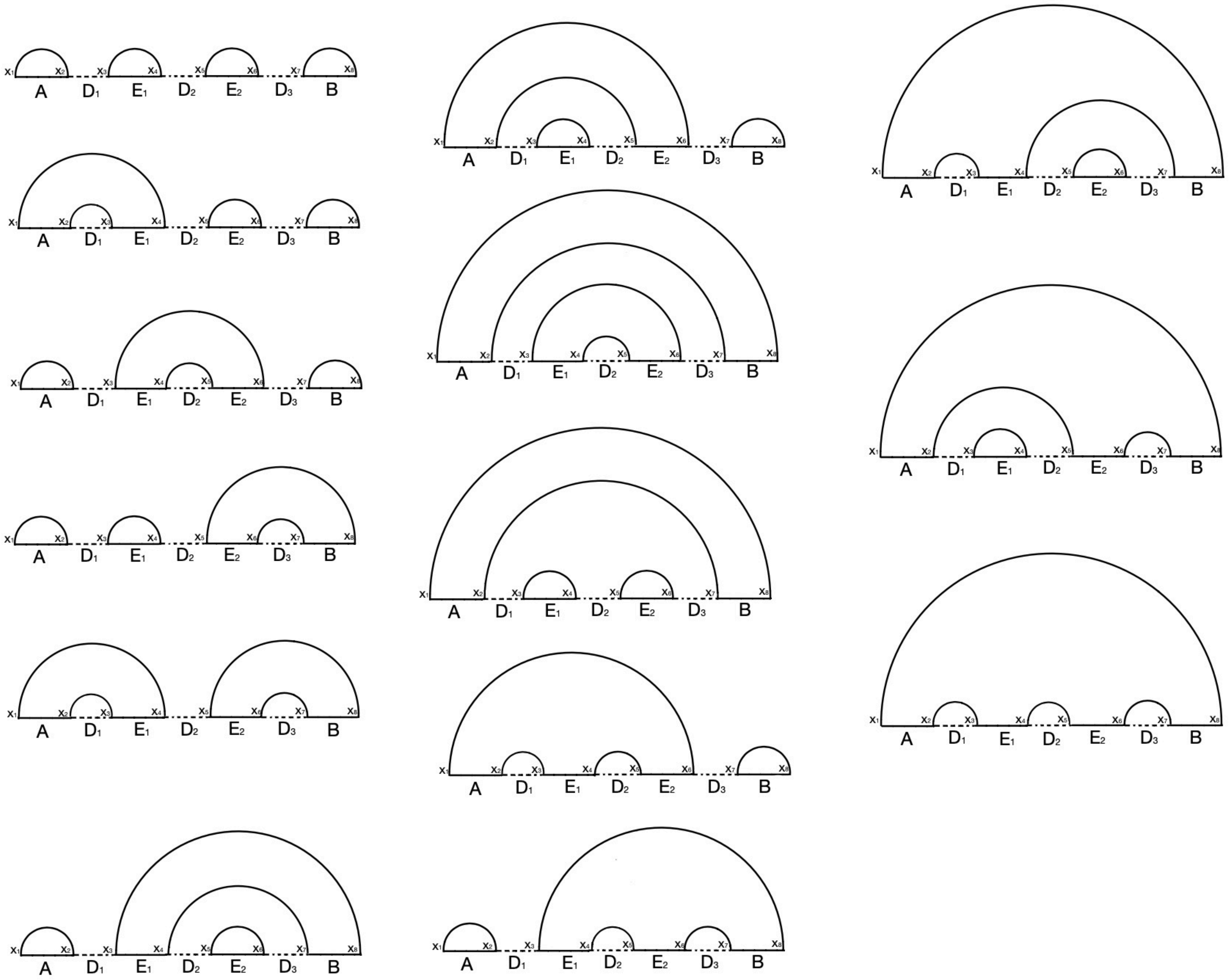


Figure 4. $I(A:B|E)$ as a function of $X(A:E)$ and $X(B:E)$ for different values of $X(A:B)$: (a) $X(A:B) = 0.2$, (b) $X(A:B) = 0.5$, (c) $X(A:B) = 1$, (d) $X(A:B) = 1.5$.

II. Upper bound of CMI (or $-I_3$) in holography

- Complexities arise when we know that E could be a set of disjoint intervals, while not just one interval
- Now let us assume E has **two intervals:** more complicated possibilities for RT surfaces
- Example: all geodesic possibilities for the calculation of $S(ABE)$



II. Upper bound of CMI (or $-I_3$) in holography

- Using similar calculations we can find the result of the CMI as a function of 11 cross ratios with 5 of them independent; as one cross ratio ($X(A{:}B)$) is fixed and an interchange symmetry between A and B, only 2 independent ones left;

$$I(A:B|E) = \frac{c}{3}\log\frac{\max\{1,X_2\}\max\{1,X_1,X_2,X_3,X_5,X_6,X_8,X_1X_5,X_2X_8,X_1X_{10},X_5X_{11},X_3/X_4,X_6/X_7,(X_2X_8)/X_9\}}{\max\{1,X_1,X_2,X_3,X_3/X_4\}\max\{1,X_2,X_5,X_6,X_6/X_7\}}$$

- The maximum of this function is found using numerics to be at a special value of the two cross ratios, which corresponds to: *E_1 and E_2 should reach the critical points of four sets of phase transitions between the connected and disconnected phases of (1) E_1 and A; (2) E_1 and E_2D_3B; (3) E_2 and B; (4) E_2 and AD_1E_1.*

II. Upper bound of CMI (or $-I_3$) in holography

- But when E has more intervals, it will become more complicated.
- From the calculations we have: when region E is one/two intervals inside the gap region between regions A and B, CMI reaches the upper bound when two/four entanglement phase transitions of RT surfaces occur simultaneously.
- This could be generalized to the case of E being m disjoint intervals:
- **Multi-entanglement phase transition rule:** for fixed A and B, the conditional mutual information $I(A:B|E)$ for region E being m disjoint boundary intervals reaches its upper bound at the critical point where 2m entanglement phase transitions happen simultaneously.
- These 2m phase transition conditions uniquely determine the position of the 2m endpoints of intervals in E, i.e. the configuration of E at maximum CMI.

II. Upper bound of CMI (or $-I_3$) in holography

- **The maximal CMI occurs precisely at entanglement phase transition critical points when varying conditional region E.**
- An intuitive sketch of why this is true from the iterative optimization approach, the hill-climbing algorithm:
  - Modify the endpoints of each interval of E_i in E while fixing other intervals;
  - Each modification maximizes the CMI locally;
  - Repeat this procedure for all intervals;
- As we move the end point of E_i in one direction, CMI increases or decreases monotonically (or remains constant) until an entanglement phase transition happens;
- As a result, CMI must reach its maximum at critical points of entanglement phase transitions

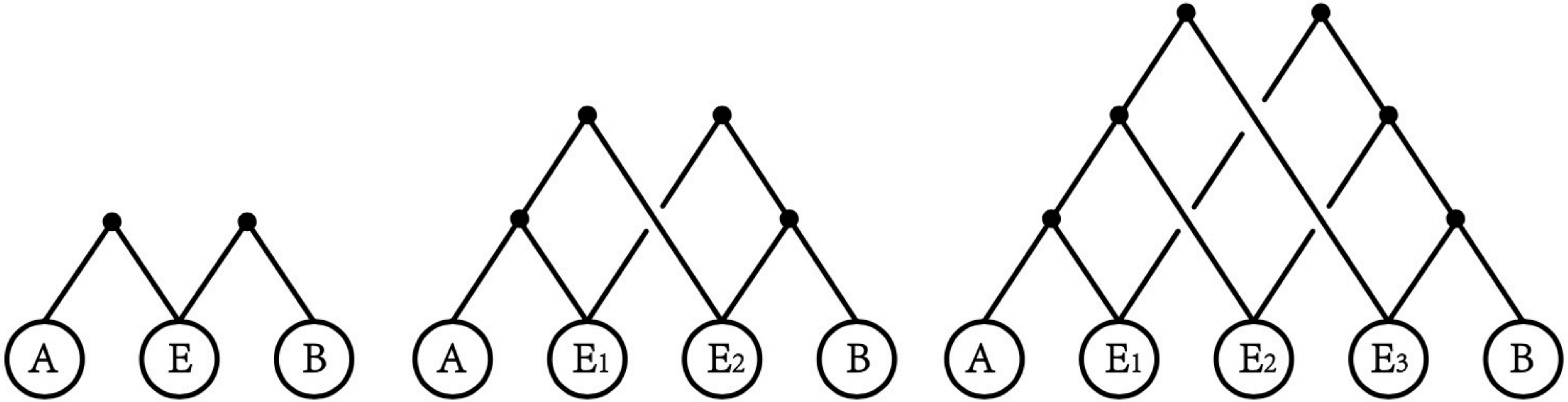
II. Upper bound of CMI (or $-I_3$) in holography

- For E being m intervals with m fixed:
  - Multiple but finite many configurations satisfy phase transition criteria (2m critical points)
  - Only one configuration achieves global CMI maximum
  - Requires exhaustive comparison of all critical configurations

- **Multi-entanglement phase transition (MPT) diagram:** which could depict the phase transition conditions in a concise way at the multi-entanglement phase transition critical point, to find the correct phase transition points

II. Upper bound of CMI (or $-I_3$) in holography

- Examples of the MPT diagram:



- Dots indicating entanglement phase transition critical points
- Several rules to plot the correct diagram
- Only one correct diagram when E stays between A and Bs

II. Upper bound of CMI (or $-I_3$) in holography

Generalized to the case of intervals of E living in different regions: MPT diagrams not unique

Example: four sets of distinct values when m=2

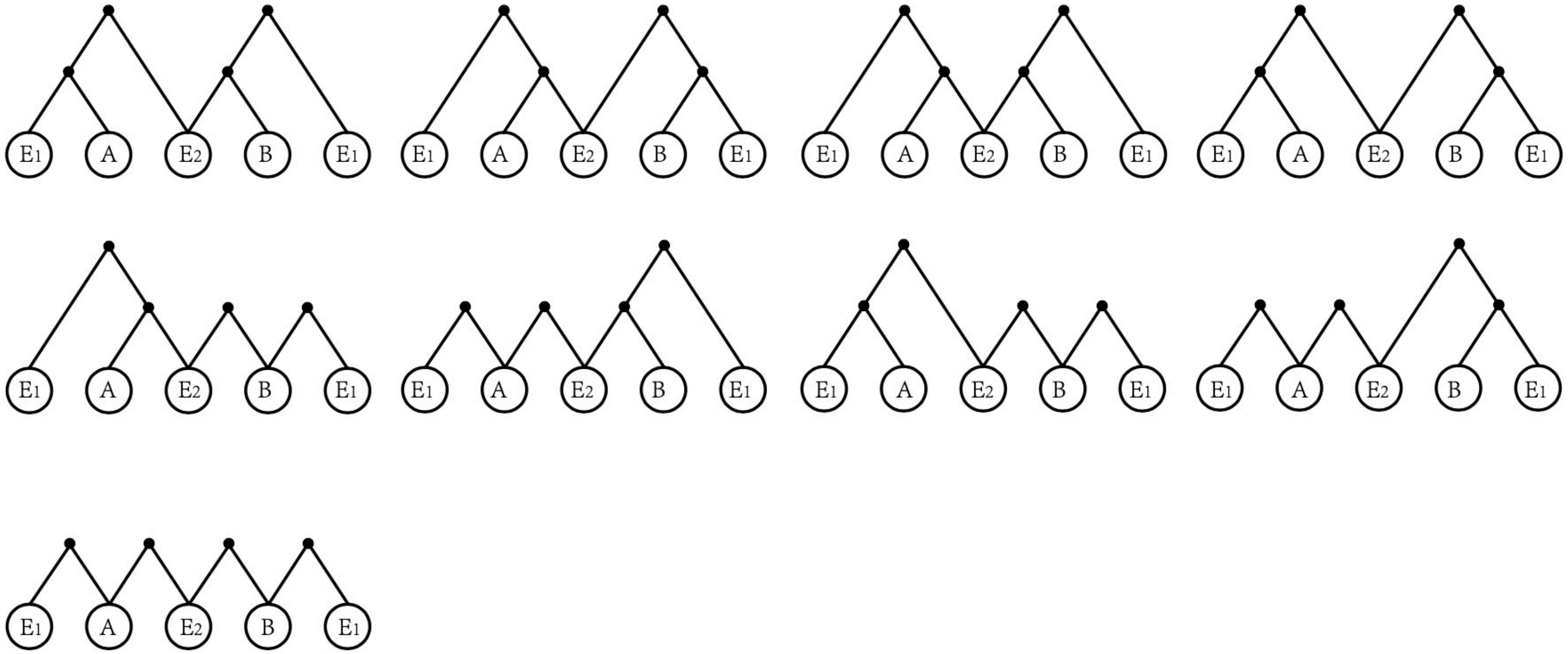
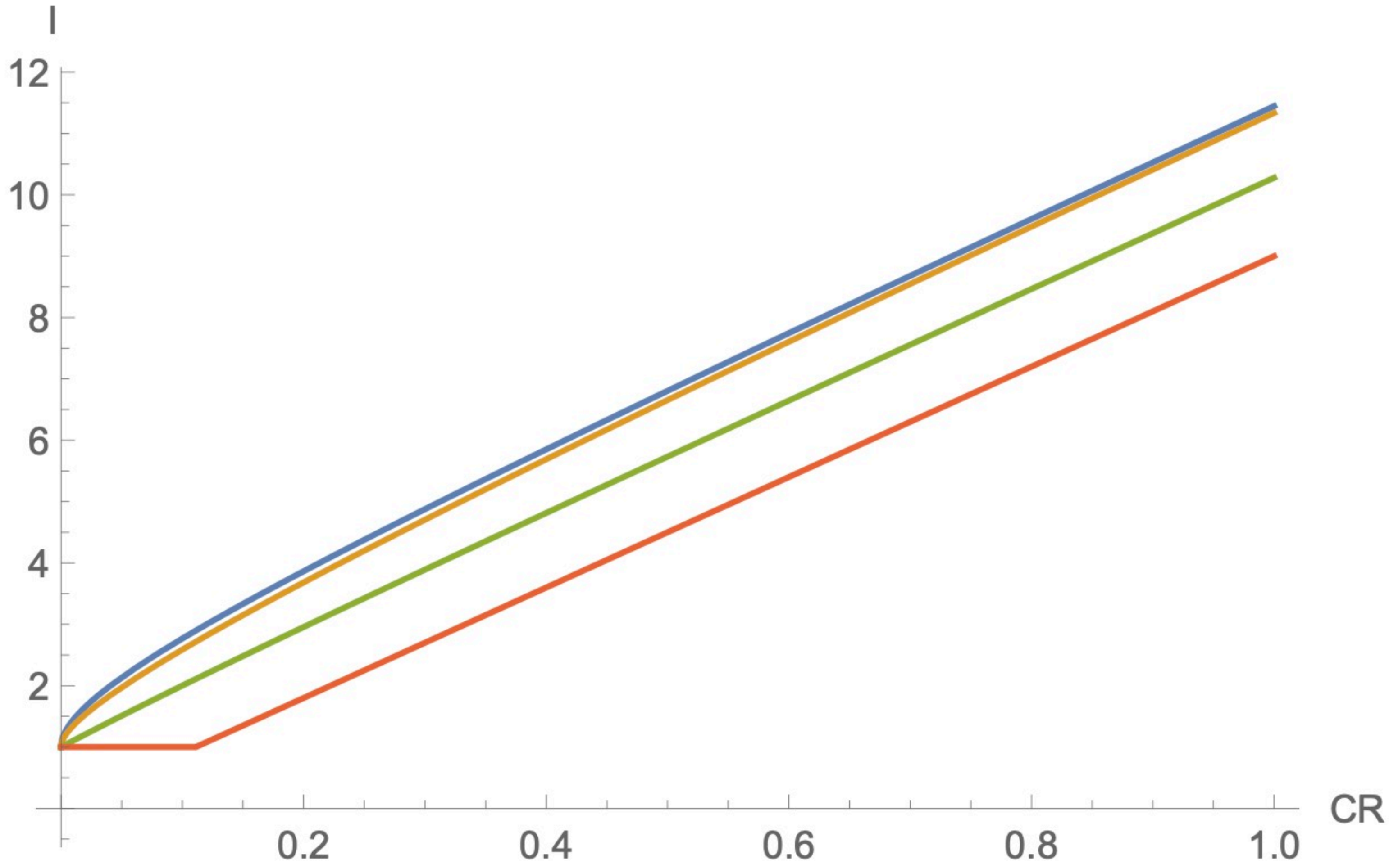


Figure 9. $m = 2$ MPT diagrams for the case of E living in both gap regions inside and outside of A and B.

II. Upper bound of CMI (or $-I_3$) in holography

Compare the four distinct values to get the maximum: the firs two MPT diagrams have the maximum value



II. Upper bound of CMI (or $-I_3$) in holography

- Challenge: though the total number of consistent MPT diagrams is finite, it is still very large, and drawing them one by one would be a formidable task.
- Solution: we have to find **stricter constraints for MPT diagrams**.
- **Constraints:** on the connectivity of entanglement wedges in the configuration that maximizes the CMI
  - Constraint I: the entanglement wedge of ABE must be totally connected.
  - Constraint II: the entanglement wedge of E inside a gap must be totally disconnected.
  - Constraint III, **disconnectivity condition**: the mutual information between region $A(B)$ and E vanishes.

$$I(A:E) = I(B:E) = 0. \tag{3.1}$$

II. Upper bound of CMI (or $-I_3$) in holography

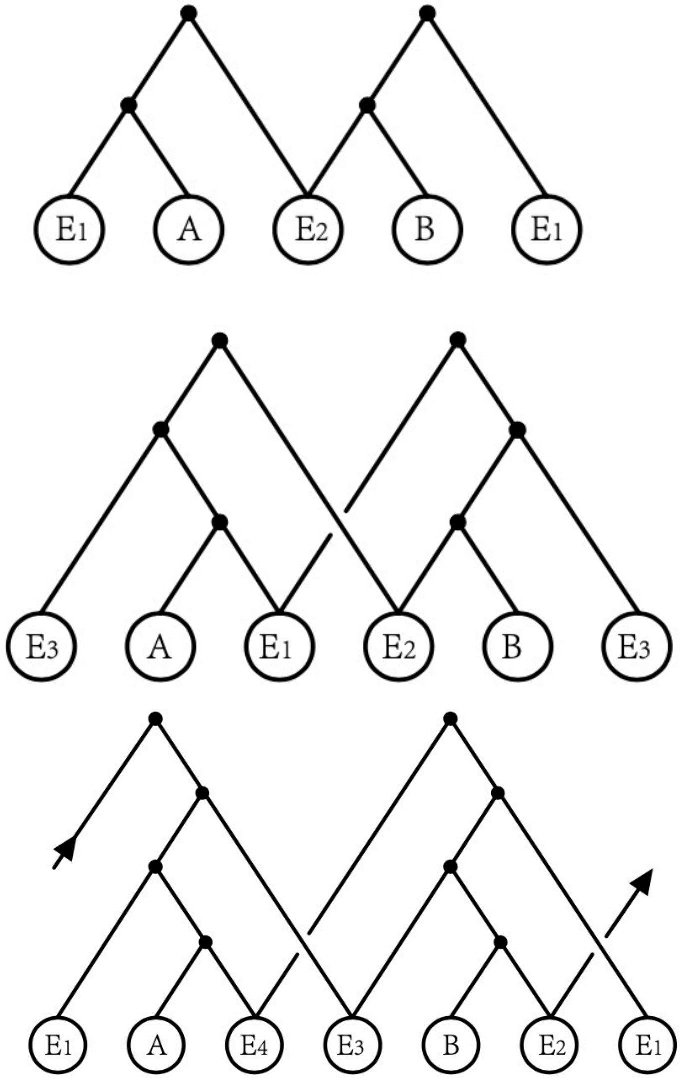
- These constraints are crucial in the following analysis and we will generalize them into a more universal method for more general entropy combinations

- Intuitive explanations for the three constraints:
  - For the first two constraints: for a given configuration, we could always find a configuration that satisfies these two constraints with the same value of CMI;
  - For the third constraint: from analysis of the contribution of each term in the CMI, we could see that the value of CMI will not be affected by the phase transition of AE_i , only when the whole entanglement wedge AE reaches its phase transition critical point could CMI be affected.

II. Upper bound of CMI (or $-I_3$) in holography

- Under these constraints, the number of MPT diagrams is greatly reduced.

- We can observe that the MPT diagram which reaches the maximum value has two features:
  - The dots are arranged in a "zigzag" pattern: with transitions involving left and right E_i regions interchangeably.
  - The order of the phase transition between E_i and A is the exact opposite of the order of the phase transition between E_i and B.



II. Upper bound of CMI (or $-I_3$) in holography

- Varying the number of intervals in E: divergence behavior of CMI
- Final calculation of the upper bound values of CMI:
- $\exp\left(\frac{I(A:B|E)}{2}\right)$ grows with m, and is divergent when m goes to infinity
- The divergence behavior

$$-I_3 \sim 2\log(2m+1) \propto 2\log\frac{1}{\epsilon}+O(1), \qquad \text{(one gap)},$$

$$-I_3 \sim 2\log(am^2+bm+c) \propto 4\log\frac{1}{\epsilon}+O(1), \quad \text{(two gaps)},$$

$$-I_3 \sim 2\log(am^4+\dots) \propto 8\log\frac{1}{\epsilon}+O(1), \qquad \text{(four gaps)}.$$

- Consistent with the quantum information upper bound $I(A:B|E) \leq 2\min(S_A, S_B)$.

which is saturated at

$$I(A:B) = I(A:E) = I\left(A:(ABE)^c\right) = 0, \quad I(A:BE) = 2S_A.$$

II. Upper bound of CMI (or $-I_3$) in holography

- Interpretation of the results:
- **Genuine tripartite entanglement** among subsystems A, B, and E: all bipartite mutual information vanishes ($I(A:B)=I(A:E)=I(B:E)=0$) while $I(A:BE)=2S_A$
- Significance of the measure of I_3:
  - While I_3 typically fails as a tripartite entanglement measure
  - In this specific configuration, I_3 effectively quantifies true tripartite entanglement
- Nearly all short range entanglement degrees of freedom in A and B (UV divergent terms) are participating in the tripartite entanglement with E.
- A stronger conclusion later

II. Upper bound of CMI (or $-I_3$) in holography

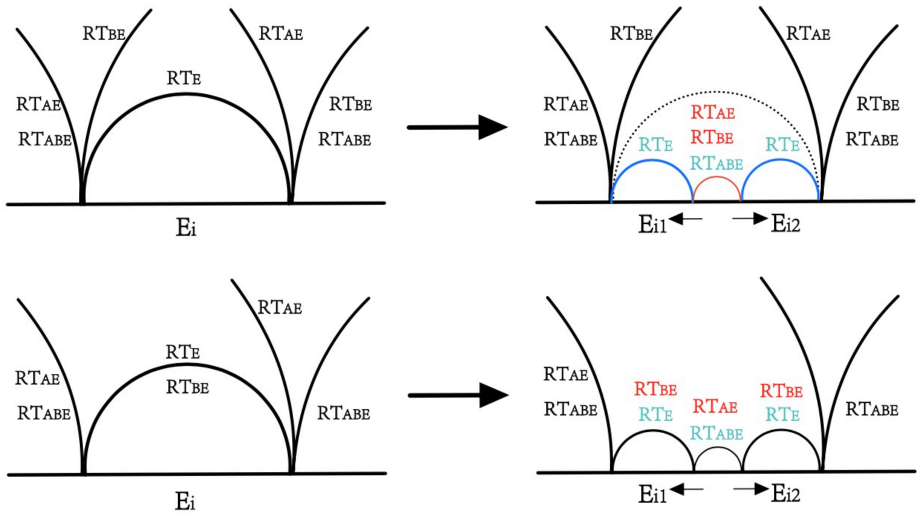
- The MPT method, only for the UV behavior of the entanglement, not useful in higher dimensions which have more degrees of freedom in determining E;
- **A more universal method: (dis)connectivity conditions**
- Inspired by the previous direct calculations

- In this new method, we focus on configurations of E that satisfies:
  - I, the entanglement wedge of ABE is fully connected;
  - II, the entanglement wedge of E is fully disconnected;
  - III, the disconnectivity condition $I(A:E) = I(B:E) = 0$

II. Upper bound of CMI (or $-I_3$) in holography

- The first two constraints for the same reason.
- The third constraint: a weaker form of disconnectivity condition
- Given a configuration E with a non-vanishing I(A: E) or I(B : E), there always exists a disconnected configuration of E with vanishing I(A: E) and I(B : E) whose CMI I(A: B|E) is not less than the former one.

Right figure: for a configuration whose intervals of E connect with both A and B inside the entanglement wedge of AE and BE respectively, one can always find another configuration whose intervals connect to at most one of A and B inside the entanglement wedge of AE and BE, with larger CMI.



II. Upper bound of CMI (or $-I_3$) in holography

- Explicit examples for this calculation of the upper bound value of CMI:
- *1, Asymptotic AdS_3*
- CMI simplified under the disconnectivity conditions

$$
\begin{aligned}
I(A:B|E) &= S_{AE} + S_{BE} - S_E - S_{ABE} \\
&= S_A + S_E + S_B + S_E - S_E - S_{ABE} \\
&= S_A + S_B + \sum_{i=1}^{m} S_{E_i} - \sum_{i=1}^{m+2} S_{\text{Gap}_i},
\end{aligned}
$$

- An upper bound on the last two terms coming from the constraint of the disconnectivity condition: the lengths of the connected geodesics must be larger than that of the disconnected geodesics as the latter are the real RT surfaces

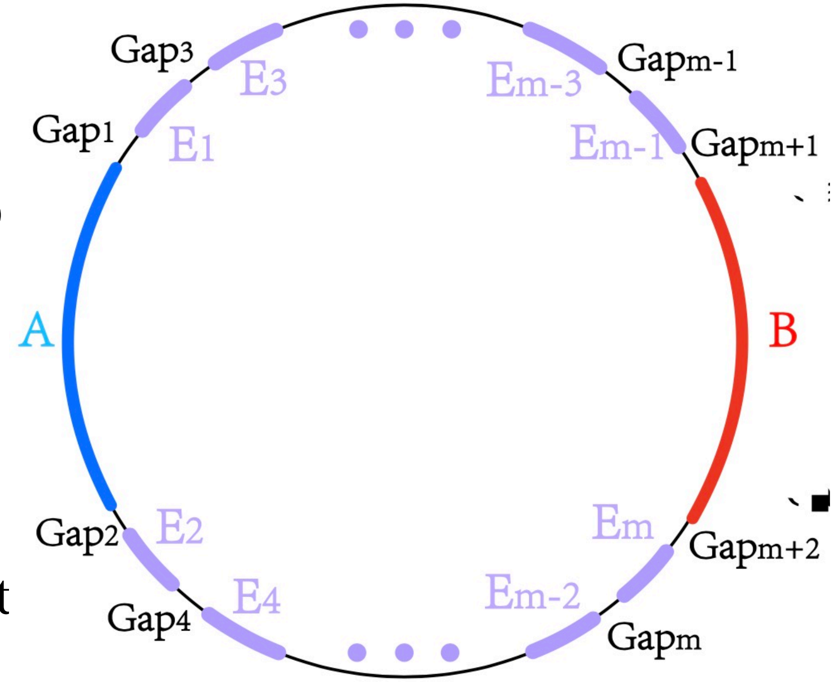
$$
I(A:B|E) \leq S_{\text{Gap}_{m+1}B\text{Gap}_{m+2}} + S_B - S_{\text{Gap}_{m+1}} - S_{\text{Gap}_{m+2}}
$$

II. Upper bound of CMI (or $-I_3$) in holography

- When m approaches infinity, it becomes $I(A:B|E) \leq 2\min(S_A, S_B)$

  which is the quantum information theoretic upper bound for any quantum system

- The saturation of this upper bound: when the system reaches the phase transition point of the RT surface of region AE (BE) if B(A) has smaller entropy, with m approaching infinity.



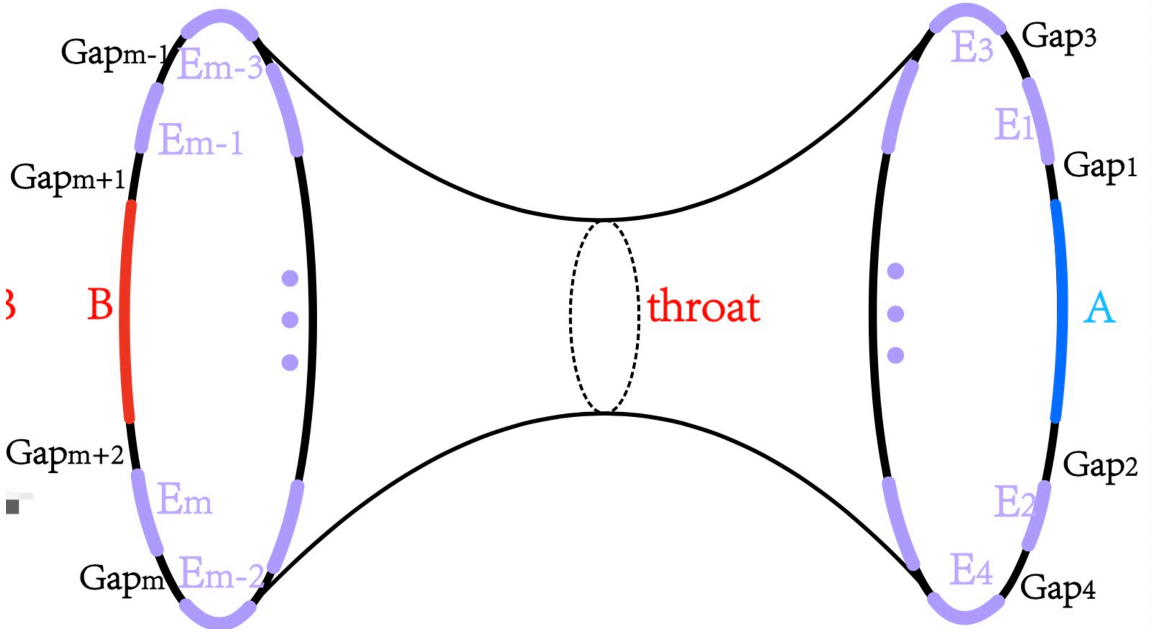
- At the saturation: I (A:B)=I (A:E)=I(B:E)=0,

- I (A:BE)=$2S_A$, all degrees of freedom in A, including the IR degrees of freedom, contribute to the tripartite entanglement

II. Upper bound of CMI (or $-I_3$) in holography

- Explicit examples for this calculation of the upper bound value of CMI:
- *2, Two sided black hole*
- The upper bound is

$$I(A:B|E) \leq 2S_{\text{throat}}$$



- The long-range nature in the tripartite global entanglement in ABE

II. Upper bound of CMI (or $-I_3$) in holography

- Interpretation of the results:
- As A and B are two small, distant, arbitrarily chosen subregions, this implies that in holographic systems, any two small distant subregions are highly tripartite entangled with a third region.
- All bipartite entanglement between A and BE emerges from the tripartite global entanglement among A, B and E.
- Another piece of evidence:

A constrained upper bound: given A, B with $I(A:B) \neq 0$, $a \subset A$ and $b \subset B$, constrain e to be within AB, $-I_3(a:b:e) \leq I(A:B)$.

when it saturates: tripartite entanglement for two sufficiently small subregions a ⊂A and b ⊂B to participate along with another region e ⊂AB

II. Upper bound of CMI (or $-I_3$) in holography

- Conclusion:
- No Bell pairs exist in holographic states; all bipartite entanglement emerges from tripartite entanglement; any two small distant subsystems are highly tripartite entangling with another system.
- Will be upgraded to a more partite version after the upper bound of I_n is obtained

III. Upper bound of $(-1)^n I_n$ in holography

- The upper bound of general $(-1)^n I_n$ when n-1 subregions are fixed while one subregion E varied
- Using the (dis)connectivity conditions
  - I. EW (ABCDFG….E) being totally connected.
  - II. EW (E) being totally disconnected.
  - III. Entanglement wedges of any k<n subregions containing E are disconnected

- **A dimensional difference:**
  - In AdS_3/CFT_2, upper bound of $(-1)^n I_n$ is finite if we fix n-1 subregions and tune one region; while in higher dimensions, the upper bound of $(-1)^n I_n$ is UV divergent, which is $2S_A$
  - Note that if we fix n-2 subregions and tune two regions, the upper bound of $(-1)^n I_n$ in AdS_3/CFT_2 could be divergent

III. Upper bound of $(-1)^n I_n$ in holography

- At the upper bound configuration:
  - The n subregions have global quantum entanglement with connected entanglement while any k<n subregions are disconnected without quantum entanglement
  - Therefore the upper bound of $(-1)^n I_n$ reflects genuine global multipartite entanglement
- The upgraded conclusion:
  These subregions fully participate in the n-partite global entanglement, where all m-partite entanglement among m-partitions of these n subregions arises from it
- There is no genuine few partite entanglement in holography, all fewer partite entanglement arises from more partite entanglement

IV. Upper bound of more general entropy combinations

- The upper bound of $(-1)^n I_n$ reveals the multipartite entanglement structure at the upper bound configuration
- Consider more general entropy combinations other than $(-1)^n I_n$
- Holographic Upper Bound $\leq$ Information Theoretical Upper Bound
- Upper bound for more general entropy combinations with fixed n subregions and one varying region E: reveals more multipartite entanglement structures that these subsystems participate
- We utilize the following systematic procedure to evaluate the upper bound.
  - 1. Determine the (dis)connectivity of each entanglement wedge in the upper bound configuration via a so-called lamp diagram.
  - 2. Calculating the exact upper bound for general configurations of n fixed regions using constraints from fake RT surfaces being larger than real RT surfaces.
- Note that we consider balanced (in E) entropy combinations

IV. Upper bound of more general entropy combinations

- Step 1: determining the (dis)connectivity of all entanglement wedges containing E, utilizing a lamp diagram
- all connectivity refers to the connectivity between region E and the rest in the corresponding entanglement wedges.
- To find the (dis)connectivity of all entanglement wedges containing E at the upper bound configuration, we must show that if a configuration of E yields entanglement wedges that do not satisfy the required (dis)connectivity conditions, then there always exists another configuration E' that does satisfy them, with the value of the entropy combination being no less than that of the original configuration E
- EW(E) being totally disconnected; EW(AB…E) being totally connected for the same reasons

IV. Upper bound of more general entropy combinations

- **The lamp diagram to determine the (dis)connectivity condition**
- One lamp diagram: a set of (dis)connectivity conditions of each entanglement wedge represented by each dot.
- Grey dots indicate connectivity of the corresponding entanglement wedge while colored dots indicate disconnectivity.
- The specific color is determined by the sign in front of the entropy term.
- Positive sign: blue; negative sign: red; if S(AE) does not appear in the combination, in black.

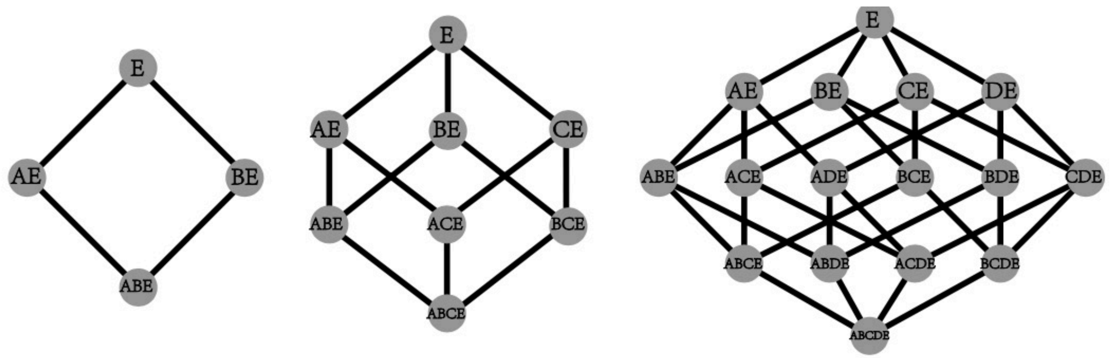


Figure 1. Examples of lamp diagrams for entropy combinations with fixed $n = 2, 3, 4$ subregions (left, middle, right). The grey (other colored) dots of the three diagrams, as labeled, represent the respective connectivity (disconnectivity) of the entanglement wedge of every term in I_{n+1} that contains E. We denote the change of the color from grey to other colors (red, blue or black) as "illuminating or lighting a lamp". Meanwhile, the lines display the relationship between these connectivity conditions: lighting a lamp requires the illumination of all lamps above it which connect with it by lines.

IV. Upper bound of more general entropy combinations

- Splitting process represented by a sequence of lamp diagrams
- A sequence of lamp diagrams indicates the splitting process: each splitting step would lead to the next lamp diagram in the sequence.
- Arrows: the direction in which the value of the entropy combination under study increases.
- Diagrams connected by arrows represent configurations that can be transformed by a splitting process.

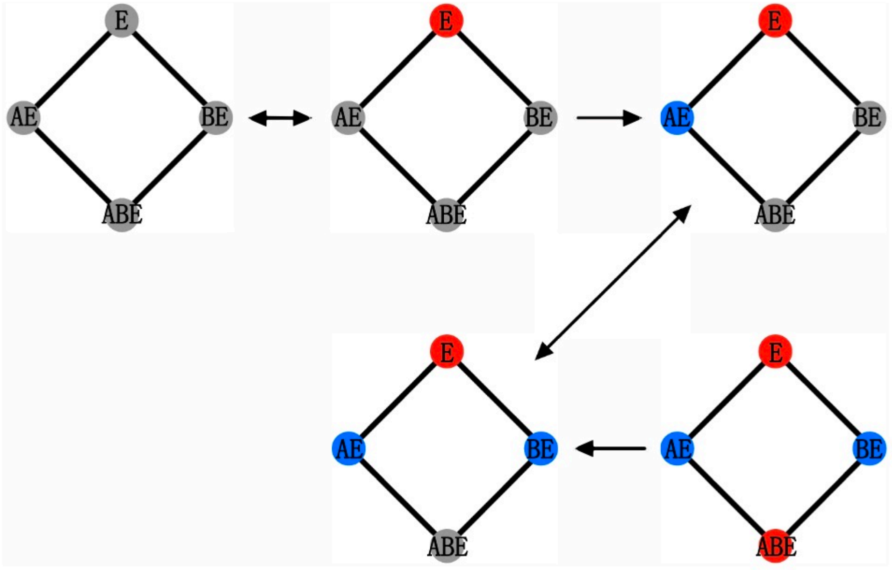


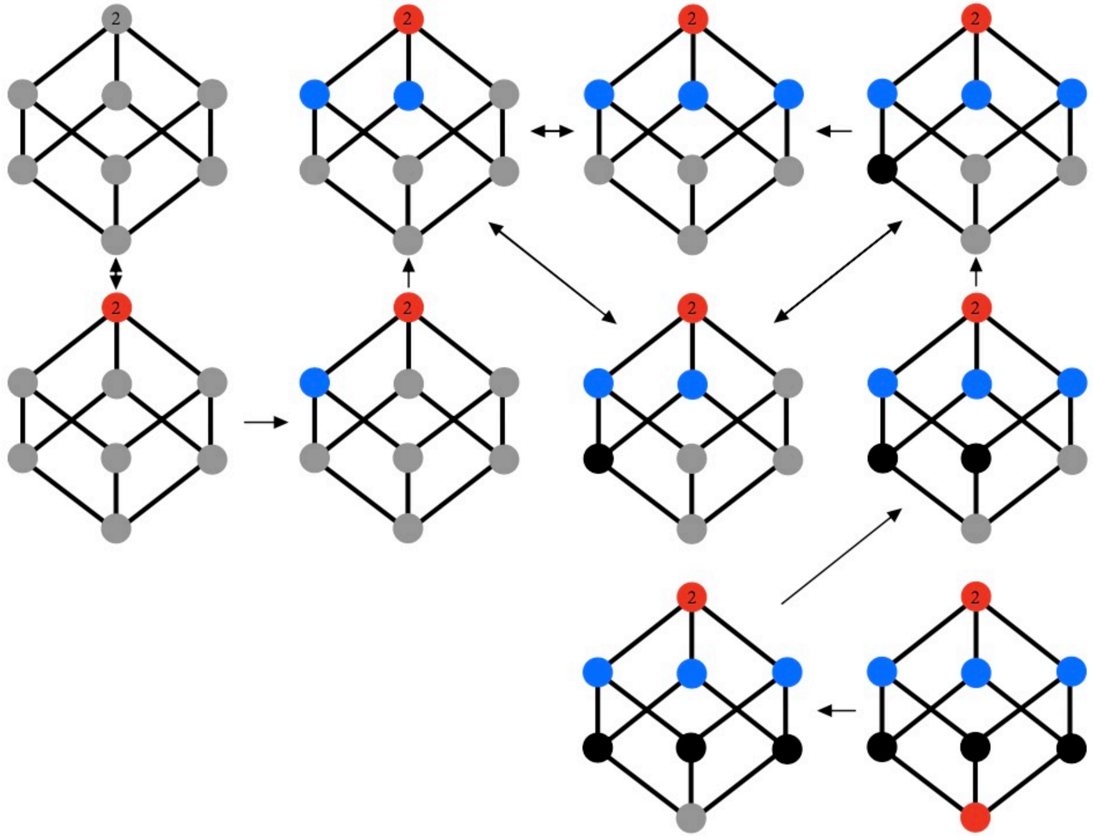
diagram to obtain the disconnectivity condition for the CMI $I(A:B|E)$,

IV. Upper bound of more general entropy combinations

- Two types of possible balanced entropy combinations could use this method to obtain the upper bound:
- The CMI type entropy combinations display only one local maximum configuration;
- While the I_4 type entropy combinations feature two local maxima of subregion configurations, one of which can be shown to be not the true maximum. Eventually, configurations corresponding to the global maximum in both the two cases can be verified to satisfy the required disconnectivity condition.

IV. Upper bound of more general entropy combinations

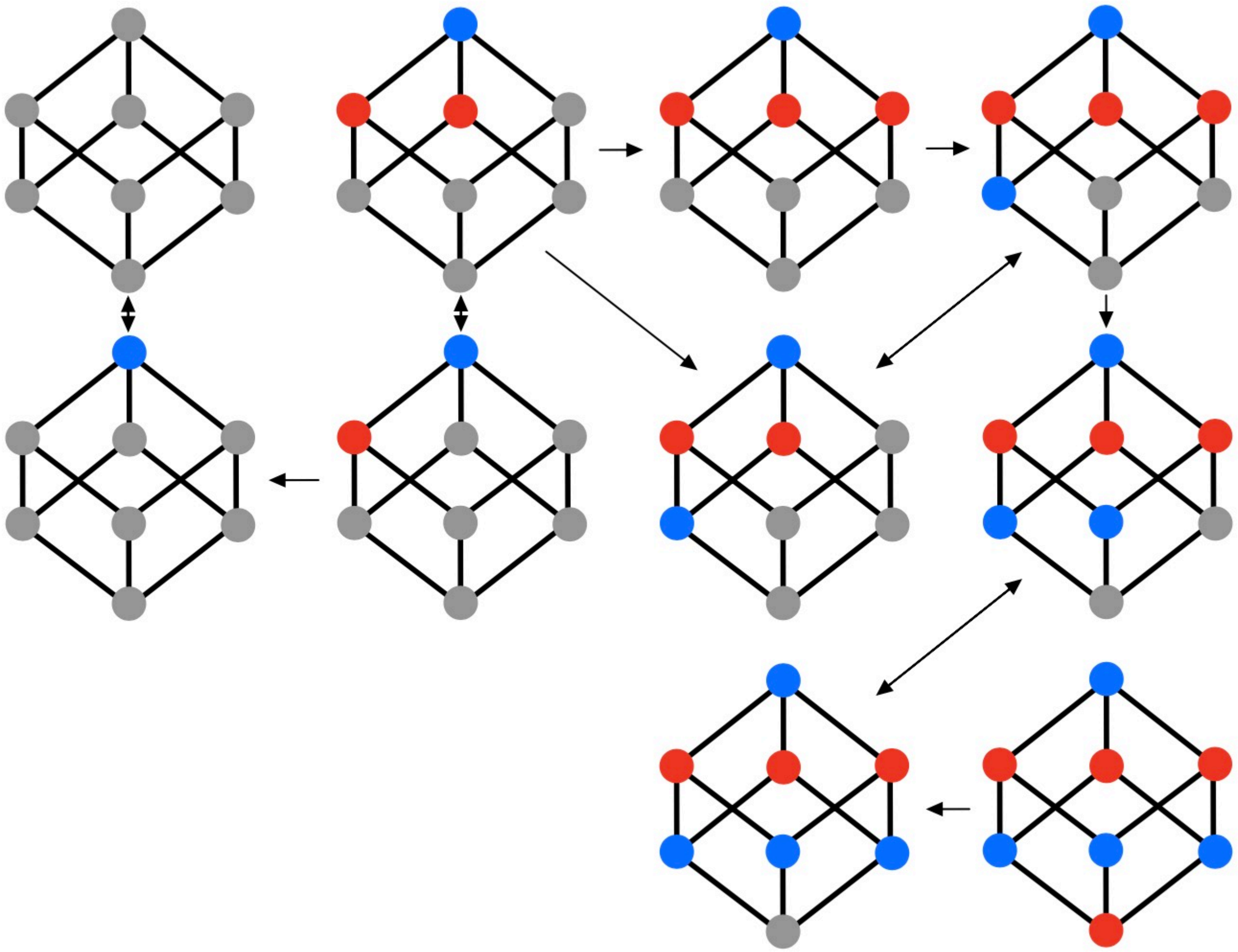
- A CMI type diagram:
- If the number of red lamps is greater than the number of blue lamps in the current lamp diagram, then the splitting process to the next step will increase the value of the entropy combination; if the number of blue lamps is greater than the number of red lamps in the current diagram, then the splitting process to the next step will decrease the value of the entropy combination; if the numbers are equal, the next step of the splitting process will not change the value of the entropy combination



diagrams that determine the disconnectivity conditions for 3-CMI.

IV. Upper bound of more general entropy combinations

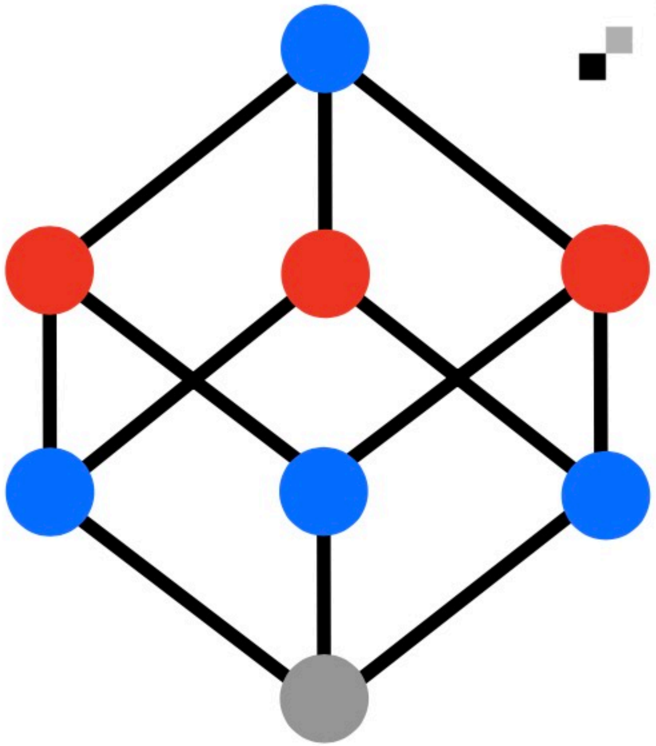
- An I_4 type diagram:



. Lamp diagram to prove the I_4 version disconnectivity condition. From now

IV. Upper bound of more general entropy combinations

- In both cases, the upper bound configurations
  - EW(E) disconnected
  - EW(ABCD…E) connected
  - All other entanglement wedges containing E disconnected: the same configuration for upper bound of I_n



IV. Upper bound of more general entropy combinations

- Step 2: Derive the upper bound: **constraints from fake RT surfaces**
  - Explicit inequalities that constrain the upper bound based on the principle that the minimal RT surface is always smaller than the non-minimal (or fake) ones.
  - Achieving this first requires a universal classification of all the gap regions according to their adjacent subregions, in order to derive the explicit constraints from fake RT surfaces.
  - Finally, with classified gap regions, we could derive the upper bound from fake RT surfaces and find the tightest upper bound using various analysis.

IV. Upper bound of more general entropy combinations

- Explicit upper bound values for examples of entropy combinations could be found respectively, as a function of the fixed n subregions and existing gap regions, which we do not show here;
- Upper bound values may not reach the quantum information theoretic upper bound;
- A dimensional difference due to the classification of gap regions;

Conclusion and open questions

- An IR term could diverge when the number of intervals of a region tends to infinity.
- There are no bell pairs existing in holography. All bipartite entanglement emerged from tripartite entanglement. Any distant A, B are highly tripartite entangling.
- The upper bound of $\mathrm{I}_4(\mathrm{A}:\mathrm{B}:\mathrm{C}:E)$ is finite in $\mathrm{AdS}_3/\mathrm{CFT}_2$ and infinite in higher dim holography, which reveals the fundamental difference in multipartite entanglement structure in different dimensions.
- The upper bound we find can be regarded as unbalanced holographic entropy inequalities. Our work provides a method to investigate them further in the future.

Open questions: the HEGMEC configuration

- Holographic exclusive genuine multipartite entanglement configuration: n subsystems with connected entanglement wedge while any n-1 of them have no quantum entanglement
- Only genuine n-multipartite entanglement exists in the n subsystems
- The upper bound configuration being a special one with $(-1)^n I_n$ reaching the maximum: it is the upper bound configuration of a lot of entropy combinations at given n, all these entropy combinations are candidates describing multipartite entanglement of this pattern
- Connection with holographic quantum error correction
- A special pattern of multipartite entanglement, needs more analysis: properties of this configuration helps us identify the holographic multipartite entanglement and the multipartite entanglement pattern of strongly coupled quantum systems.

- Open questions:
- Multipartite entanglement measures tested in the HEGMEC: EWCS, squashed multipartite entanglement, multi-entropy, genuine multi-entropy, Latent entropy, etc.
- Fixing m subregions and tune n-m subregions: already some preliminary results, more to be investigated;
- Further understanding of the holographic multi-partite entanglement beyond the upper bound analysis;
- Evolvement of the multi-partite entanglement structure in nonequlibrium
- Relation between the upper bound structure and geometry
- Connection with strongly coupled quantum many-body systems
- Connection with holographic entanglement inequalities

Thank you!