# Wess-Zumino-Witten interactions of axions

Jia Liu
Peking University

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*Collaborated with: Yang Bai, Ting-Kuo Chen, Jia Liu, Xiaolin Ma*

Workshop on Multi-front Exotic phenomena in Particle and Astrophysics (MEPA 2025)
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# Outlines

- Axion models and general searches
- Problems in axion-meson interactions
  - Consistency of the interaction model
  - Wess-Zumino-Witten interactions of QCD
- Wess-Zumino-Witten interactions of Axions
- Summary

# The QCD axion and the Strong CP problem

$$\mathscr{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - \left(\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \mathrm{h.c.}\right)$$

A total derivative, but is allowed by non-trivial QCD vacuum

$$G^a_{\mu\nu}\tilde{G}^{a\mu\nu} = \partial_\mu K^\mu,$$

$$K^\mu = \epsilon^{\mu\nu\alpha\beta}\left(G^a_\nu \partial_\alpha G^a_\beta + \frac{g}{3} f^{abc} G^a_\nu G^b_\alpha G^c_\beta\right)$$

- The CKM matrix from $M_{u,d}$
  - CP violating phase $\theta_{\mathrm{CP}}$ ~ 1.2 radian
- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right]$$

- $\bar{\theta}$ is invariant under quark chiral rotation
- According to neutron EDM experiment

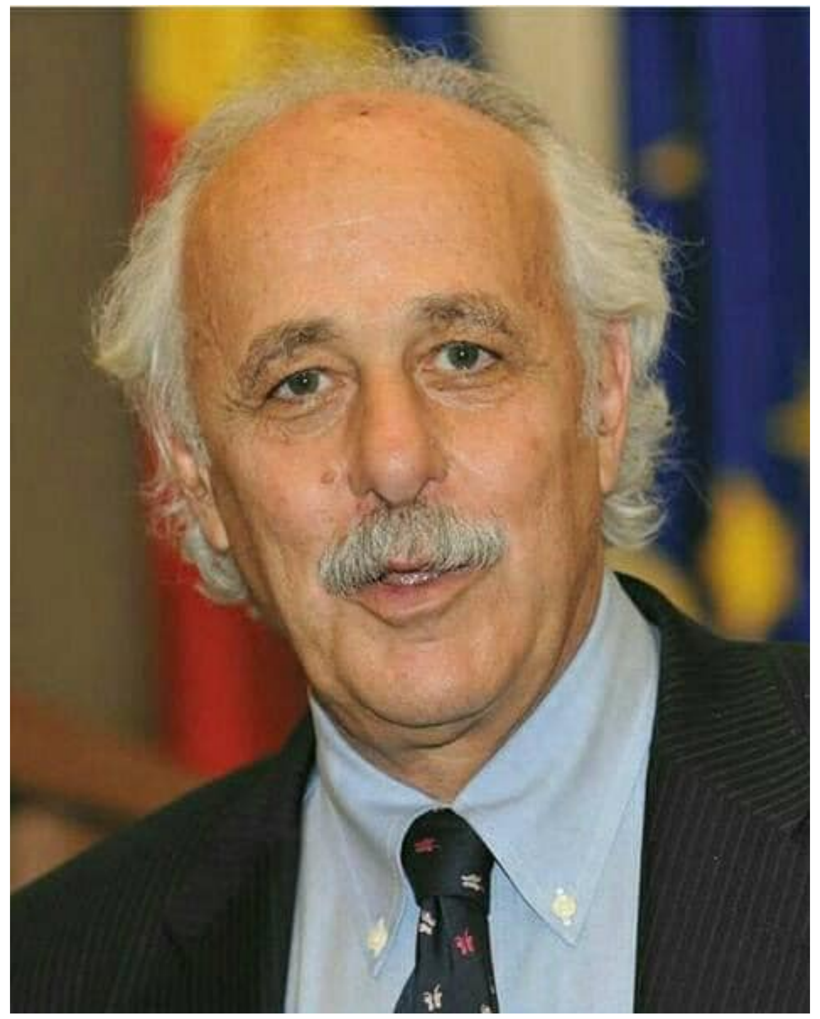
$$d^n_{\mathrm{EDM}} \sim \bar{\theta} \times 10^{-16}\ \mathrm{e\ cm}$$

$$d^n_{\mathrm{exp}} < 10^{-26}\ \mathrm{e\ cm}$$

$$\bar{\theta} \lesssim 1.3 \times 10^{-10}\ \mathrm{radian}$$

Why did not see strong CP?

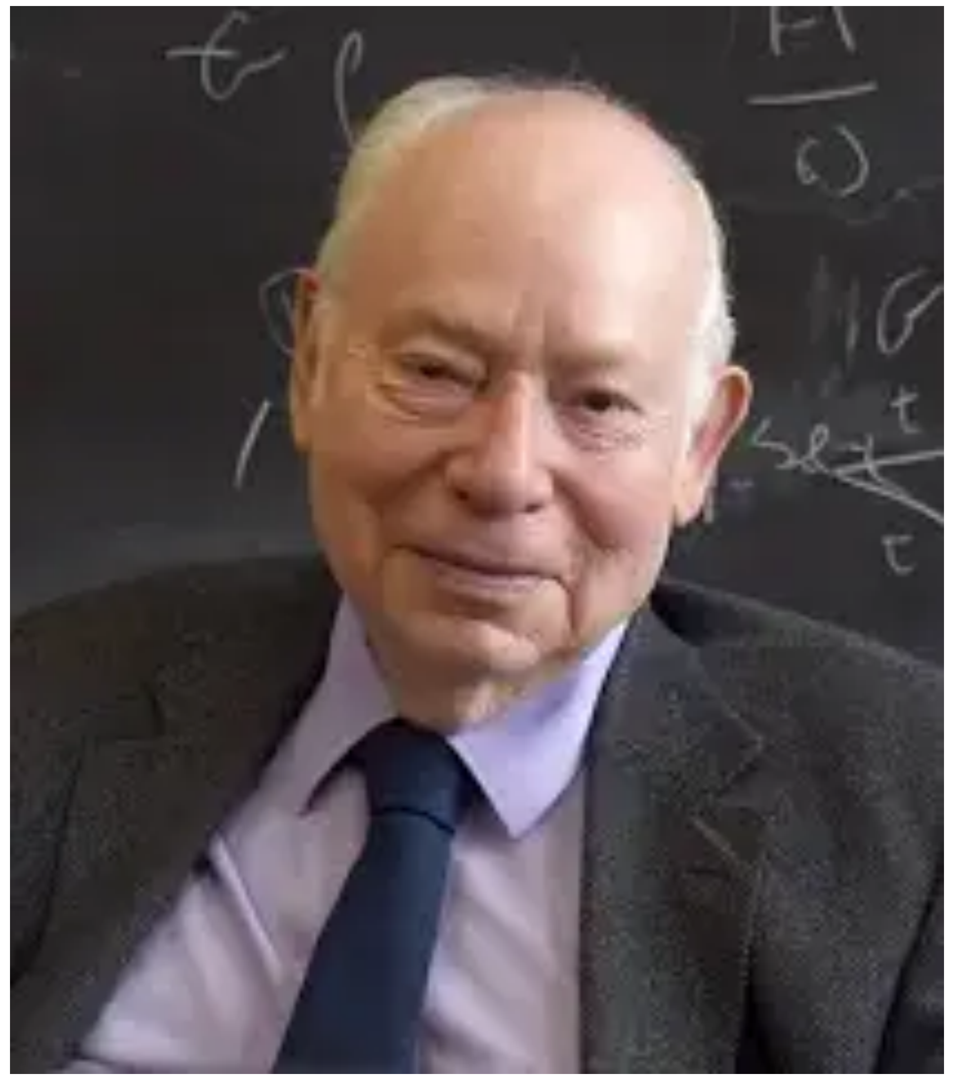
# The Peccei-Quinn solution to Strong CP problem

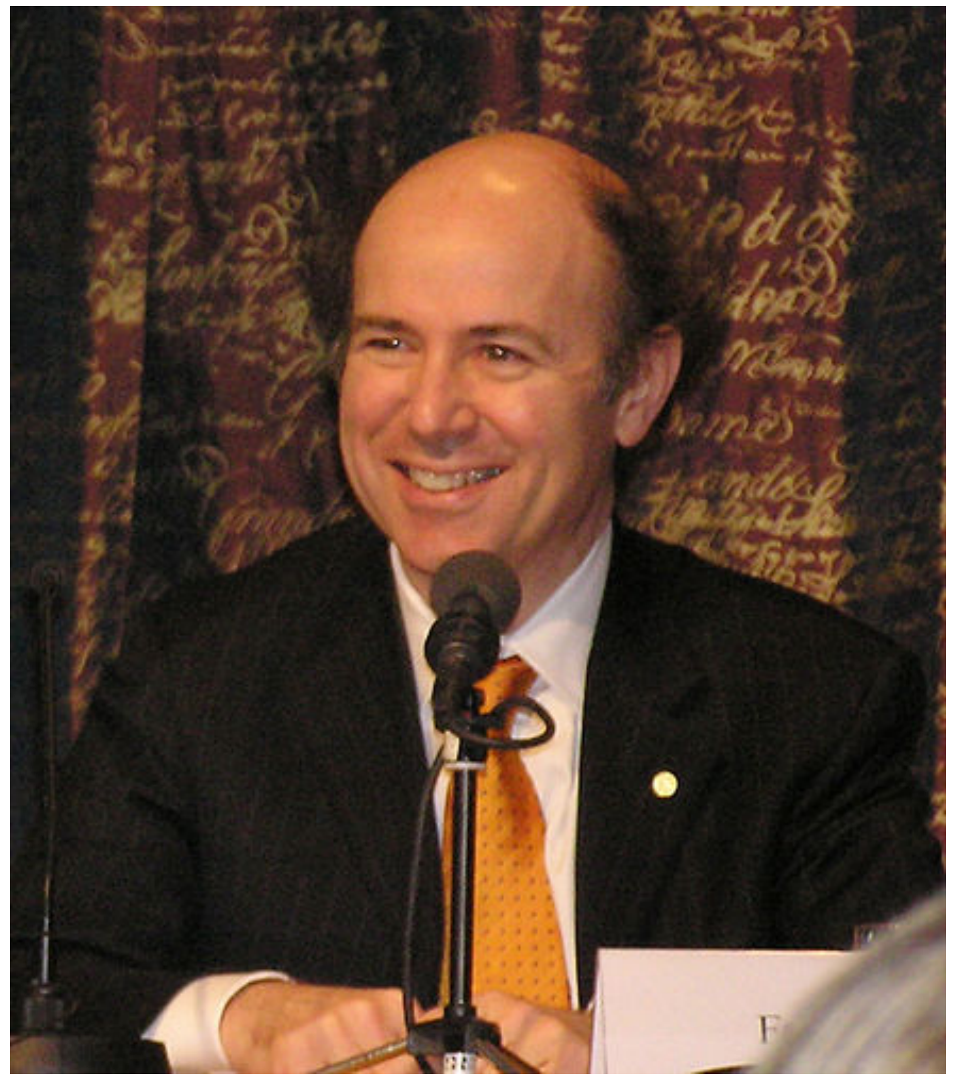


- Experiment requires $\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right] \lesssim 10^{-1}\mathrm{rad}$
- PQ: promote the constant $\bar{\theta}$ to a dynamical field, a(x)
- Introduce a *global* PQ-symmetry $U(1)_{\mathrm{PQ}}$, *anomalous* under the QCD
  - $a \to a + \kappa f_a \ \Rightarrow \mathcal{S} \to \mathcal{S} + \frac{\kappa}{32\pi^2}\int d^4x G\tilde{G}$, cancels $\bar{\theta}$
- Vafa-Witten theorem: vector-like theory (QCD) has ground state $\langle\bar{\theta}\rangle = 0$

# PQWW Axion

- In 1978, Weinberg and Wilzeck realize there is an light particle





**Axion!!!**

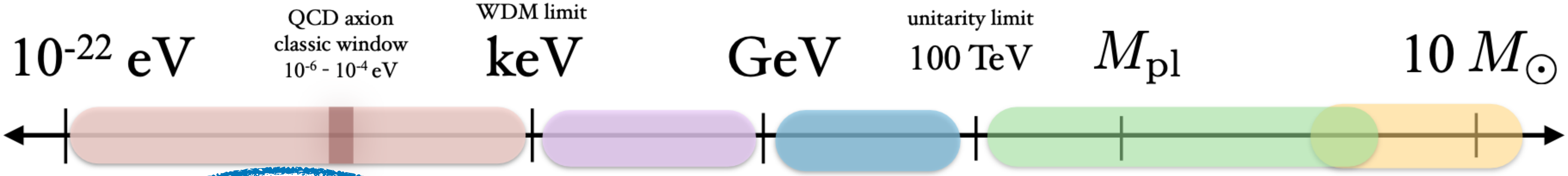
- It can wash out the unwanted strong CP phase
- QCD axion : $m_a^2 f_a^2 \approx \Lambda_{\mathrm{QCD}}^4$; Neutrino Seesaw: $m_\nu m_{N_R} \approx (y v_h)^2$
- Light particles can probe high scale physics!

# PQWW Axion

- PQWW axion assumes breaking scale $f_a \sim v_{\rm EW}$
- Axion mass from $100\ \rm keV \sim 1\ MeV$, and the coupling strength is large $1/f_a$
- PQWW axion is quickly ruled out by
  - Lab constraints: $K^\pm \to \pi^\pm + a$, $J/\Psi \to \gamma + a$, and $\Upsilon \to \gamma + a$
  - Astrophysical constraints: Red giant and Supernovae
- *(Invisible)* QCD axion: the leading axion models are KSVZ/DFSZ model with $f_a \gg v_{\rm EW}$

# The dark matter candidate models

1904.07915，TASI lecture



``Ultralight” DM

non-thermal
bosonic fields

``Light” DM

dark sectors
sterile ν
can be thermal

WIMP

Composite DM
(Q-balls, nuggets, etc)

Primordial
black holes

**Axion and ALP dark matter**

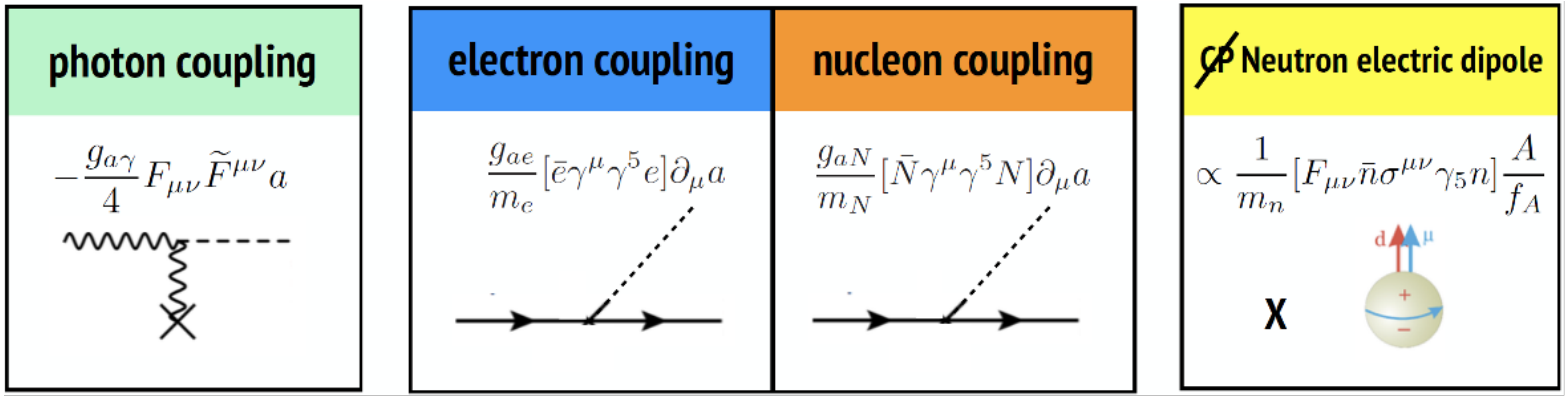
**HEP at a cross-road: explore all directions!**

# The axion effective Lagrangian at quark-level

- Axion can couple to SM gauge bosons and fermions

$$\mathscr{L}_{\rm ALP} = g_{ag}\frac{a}{f_a}G\tilde{G} + g_{a\gamma}\frac{a}{f_a}F\tilde{F} + g_{af}\frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f$$

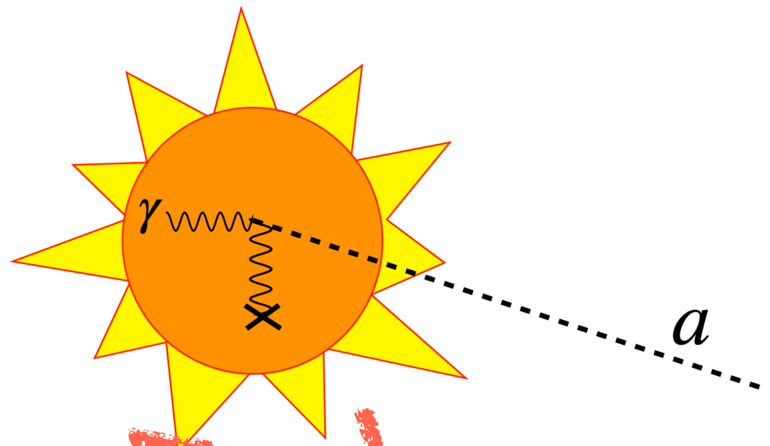
- Detection of axion through various couplings

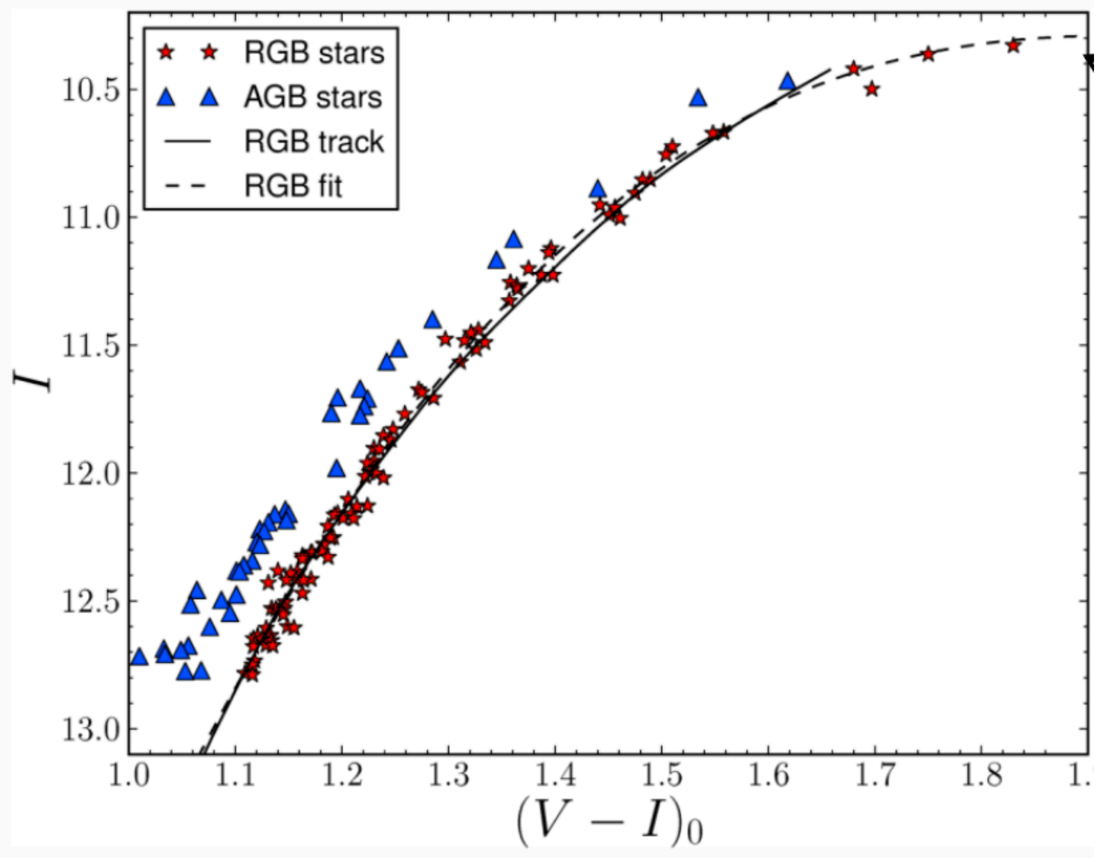


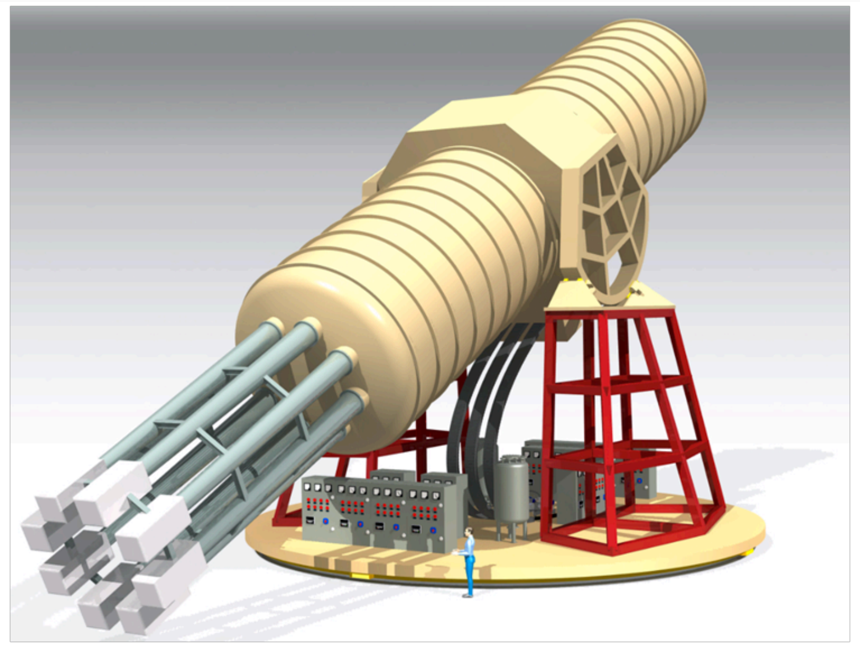
# Experimental searches for axions

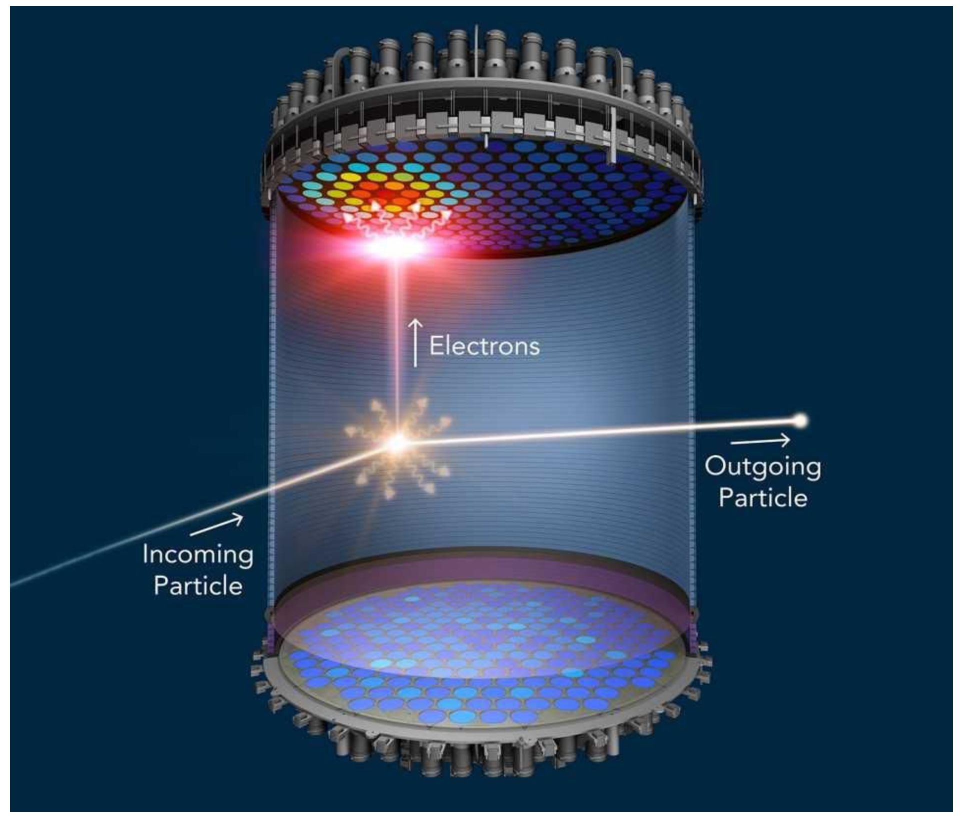
**Methodology:**

- Dark Matter Axion: haloscopes ..
- Non-DM searches:
  - Rare meson decays
  - Stellar cooling
  - Supernova
  - Helioscopes: solar axion (CAST, IAXO, or DM direct detection searches)
  - Light shining through walls
  - Polarization
  - Fifth force
  - Radio wave detection









# The resonant searches for ALP via photon coupling

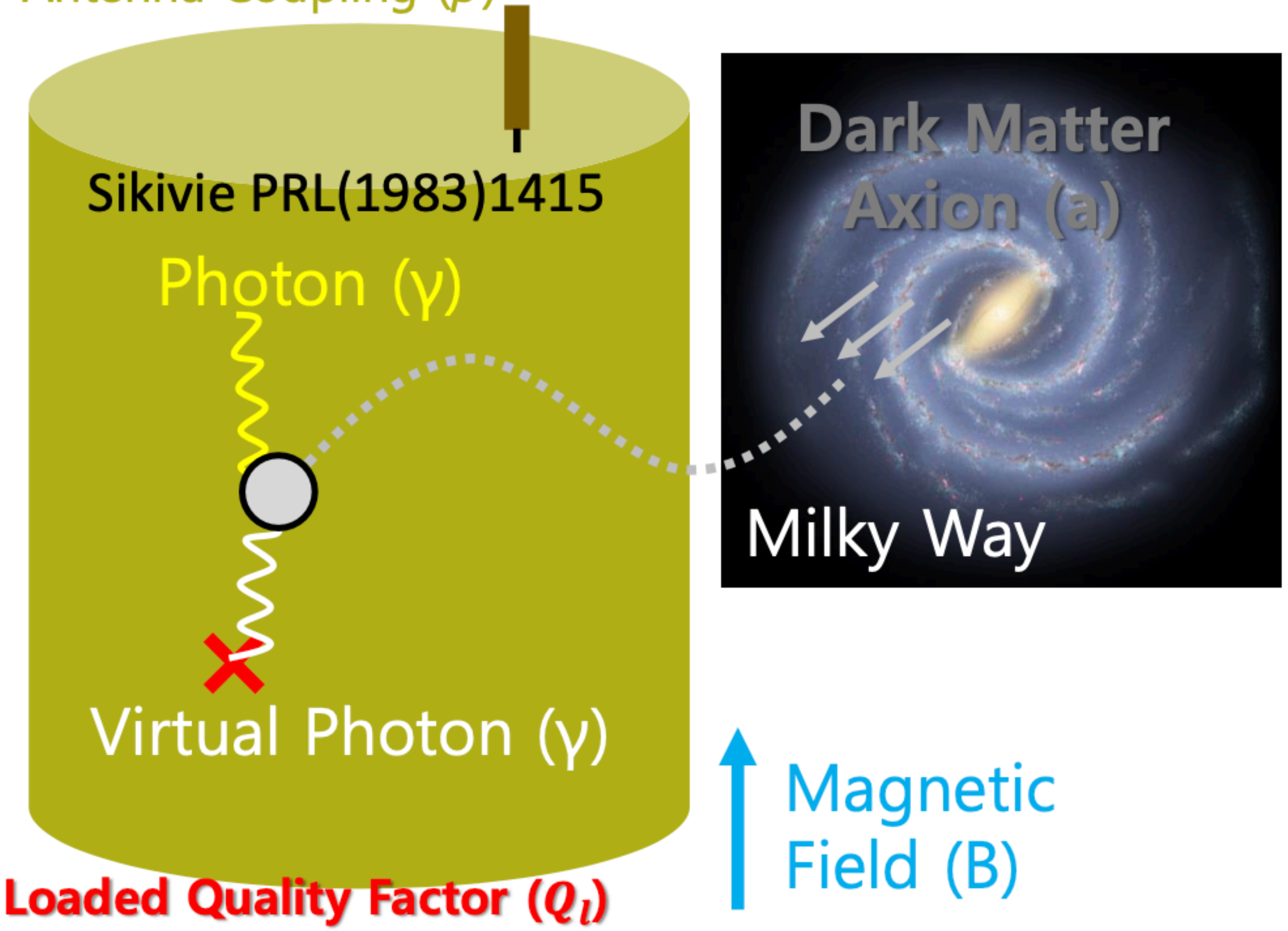
https://cajohare.github.io/AxionLimits/

# The resonant searches for ALP via photon coupling

- Tuning cavity resonant frequency to match axion mass

$$g_{a\gamma\gamma} a F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \sim g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

From Danho Ahn@Patras2023



**Loaded Quality Factor ($Q_l$)**
Volume (V)
Resonant Angular Frequency ($\omega_0$)
Form Factor (C)
System Temperature ($T_{sys}$)

Coupling Constant
Dark Matter Axion Density

Signal Power

$$P_{sig} = \frac{\beta}{1+\beta} g_{a\gamma\gamma}^2 \frac{\rho_a}{m_a^2} \boldsymbol{B}^2 V \omega_0 C \frac{Q_a \boldsymbol{Q_l}}{Q_a + \boldsymbol{Q_l}}$$

Axion Mass
Axion Quality Factor

Kim *et al.* JCAP03(2020)066

Scan Rate

$$\frac{df}{dt} \propto \frac{\boldsymbol{B}^4 V^2 C^2}{k_B^2 T_{sys}^2} \boldsymbol{Q_l} Q_a$$

$$\boldsymbol{Q_l} \gg Q_a \sim 10^6$$

System Noise Temperature ~ 200 mK

Refer to Session 02, Thu, Dr. Jinsu Kim

# The resonant searches of nucleon couplings

- The ALP DM field

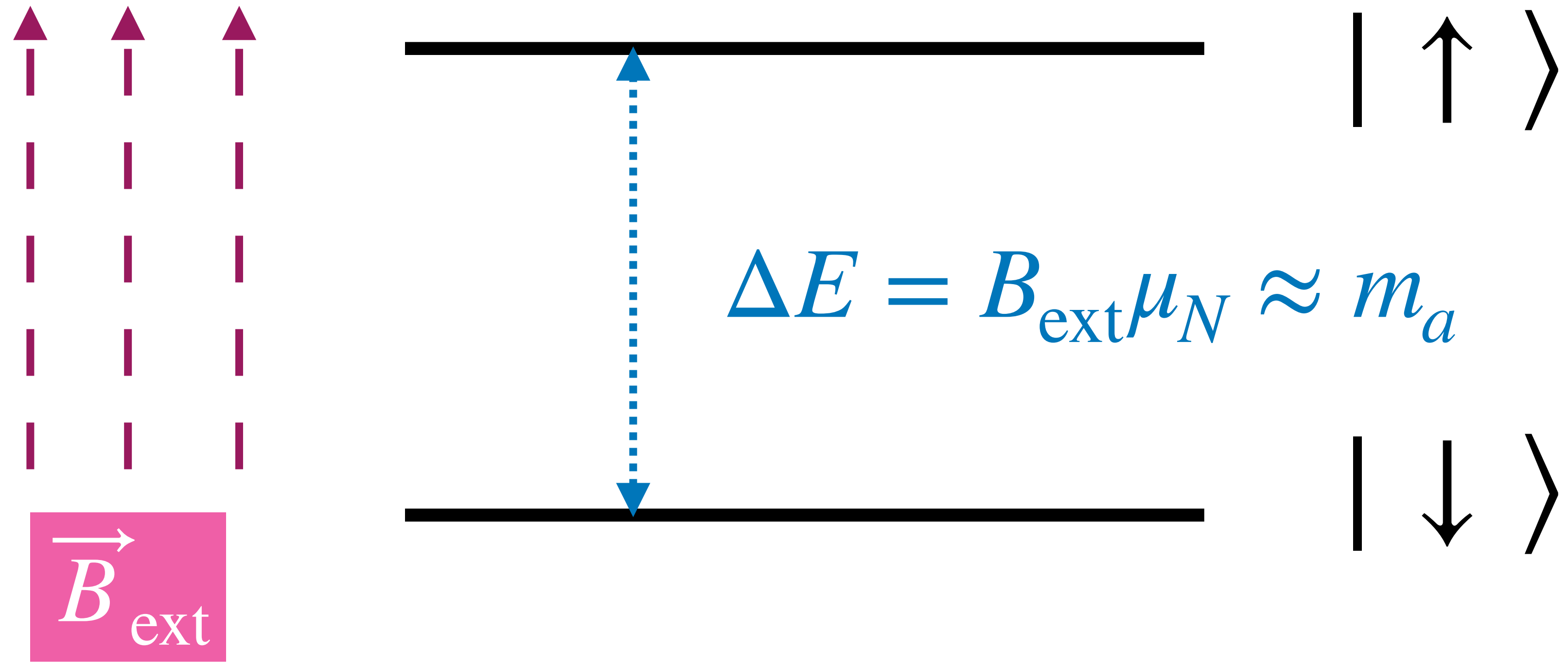
$$a(x,t) \approx a_0 \cos(\omega t - \vec{p} \cdot \vec{x} + \theta_0)$$

- The axion-wind Hamiltonian

$$g_{aNN}\frac{\partial_\mu a}{2f_a}\bar{N}\gamma^\mu\gamma_5 N \to H = g_{aNN}\vec{\nabla} a \cdot \vec{\sigma}_N$$

$$\approx g_{aNN}\vec{v}_a \cdot \vec{\sigma}_N \times \sqrt{2\rho_a}\,\sin(p \cdot x)$$

- A Zeeman split in B field

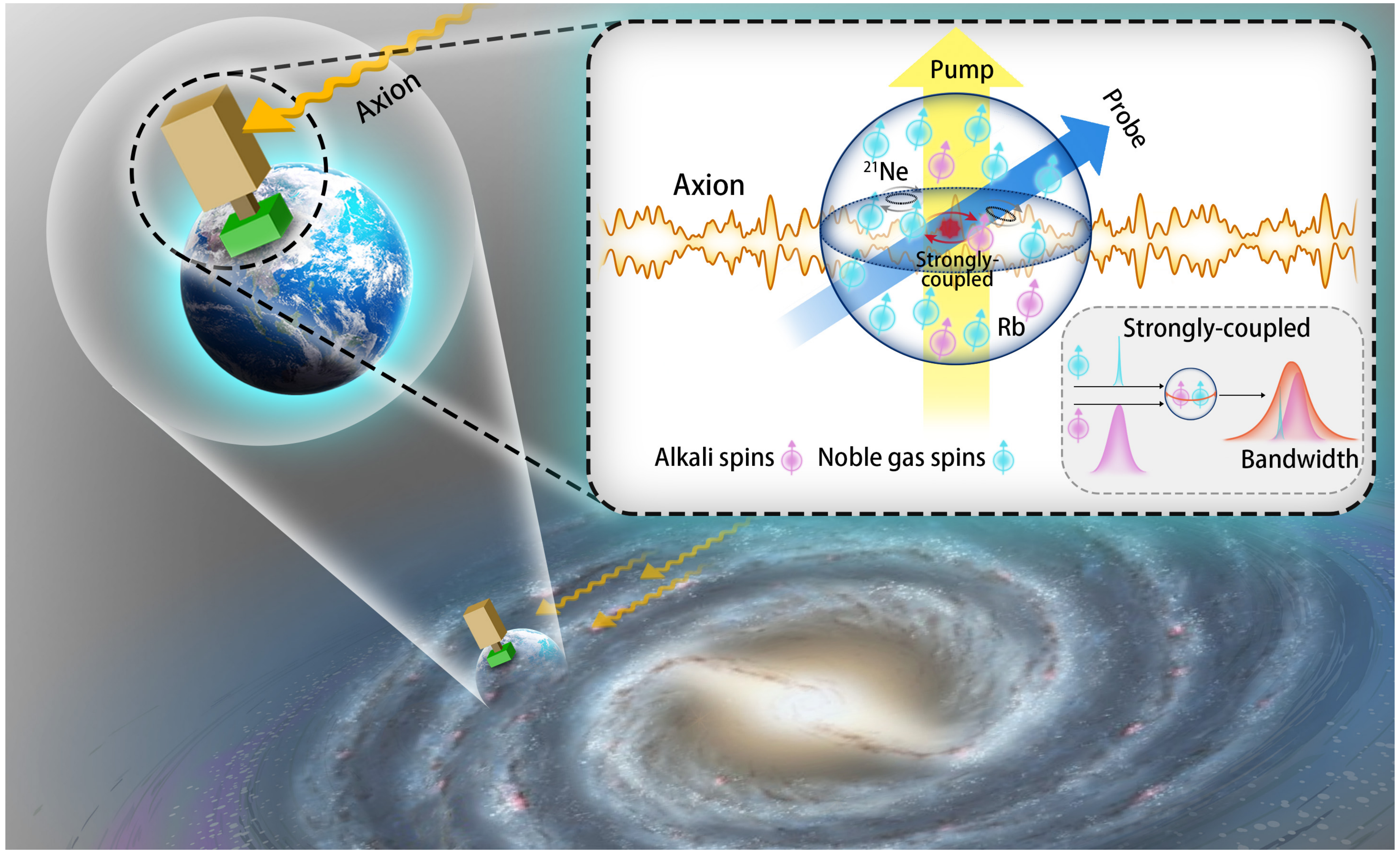


$|\uparrow\rangle$

$\Delta E = B_{\text{ext}}\mu_N \approx m_a$

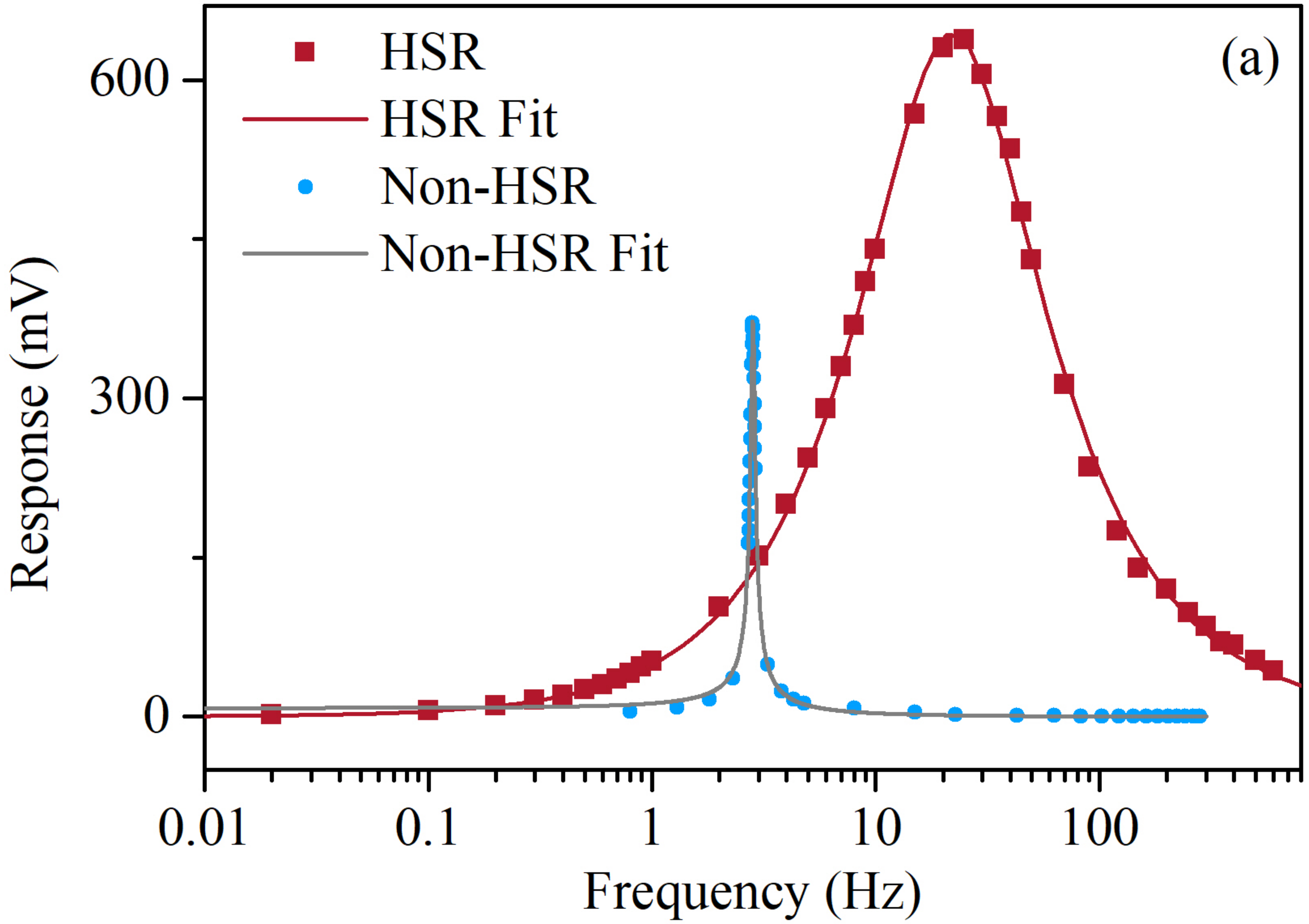
$|\downarrow\rangle$

$\vec{B}_{\text{ext}}$



# Comagnetometer in Hybrid Spin Resonance: New Method

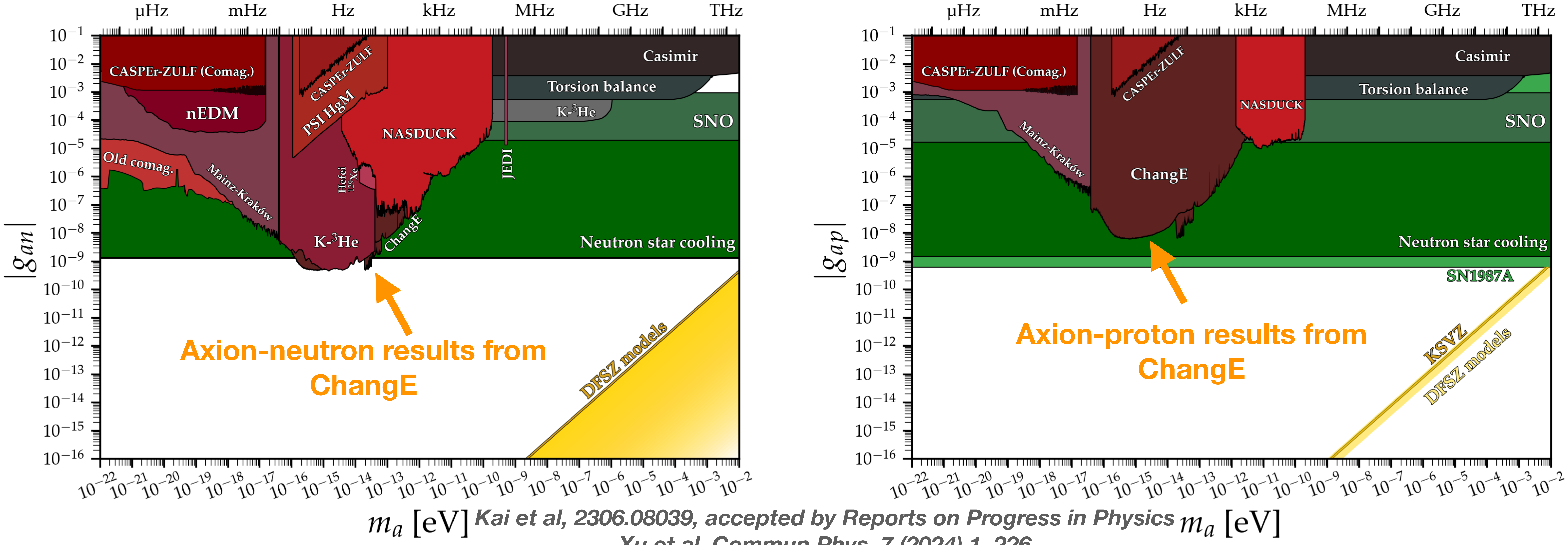
- Method: tune external B field to make Larmor frequency equal
  - HSR: $\omega_K \approx \omega_{\rm Ne}$
  - Width $\Gamma_n$ is about 100 Hz now
  - NMR: $\omega_{\rm Ne} \approx m_a c^2$
- Smaller amplification but with much wider resonance
  - Do not need to scan (e.g. 35 days)
  - Long-time measurement at single point to compensate amplification lost



ChangE experiment: Kai Wei, .. **JL** .. et al, 2306.08039

# ChangE results

- ChangE experiments set competitive limits on ALP-nucleon couplings (AxionLimits version)
- Improving ALP-proton coupling limits by $10^5 - 10^6$
- Provideing best limits on ALP-neutron couplings at ~[0.02, 0.2] Hz and [10, 200] Hz



**Axion-neutron results from ChangE**

**Axion-proton results from ChangE**

*Kai et al, 2306.08039, accepted by Reports on Progress in Physics*
*Xu et al, Commun.Phys. 7 (2024) 1, 226*

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# The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real

- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

# The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

Bauer et al, PRL 127 (2021), 081803

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right]q(x) \qquad \mathrm{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

# The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\boldsymbol{\theta}_L \equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} \qquad U_L \equiv \exp\left[-i\boldsymbol{\theta}_L a/f_a\right]$$

$$\boldsymbol{\theta}_R \equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \qquad U_R \equiv \exp\left[-i\boldsymbol{\theta}_R a/f_a\right]$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \rightarrow \begin{pmatrix} m_{u,0} e^{-2i\kappa_{u,0} c_{gg} \frac{a}{f_a}} & 0 \\ 0 & m_{d,0} e^{-2i\kappa_{d,0} c_{gg} \frac{a}{f_a}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger [\mathbf{k}_{L,0} + c_{gg} \boldsymbol{\theta}_{L,0}] U_L \rightarrow \mathbf{k}_{L,0} + c_{gg} \boldsymbol{\theta}_{L,0}$$

$$\mathbf{k}_R(a) = U_R^\dagger [\mathbf{k}_{R,0} + c_{gg} \boldsymbol{\theta}_{R,0}] U_R \rightarrow \mathbf{k}_{R,0} + c_{gg} \boldsymbol{\theta}_{R,0}$$

**Anomalous axion contribution**

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg} \mathrm{Tr}\left[\mathbf{Q}^2 \boldsymbol{\kappa}_{q,0}\right]$$

# The consistent ChPT axion Lagrangian at meson level

$$\mathscr{L}_{\mathrm{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a\boldsymbol{\tau}^a]$$

$$\mathscr{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\mathrm{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$$

# The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x)$$

- The most important channel BR($K \to \pi a$) is off by a factor of 37 for 35 years

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \to \pi a$ and $\pi^- \to e^- \bar{\nu}_e a$ have been obtained for all axion couplings, only in 2021

Bauer et al, PRL 127 (2021), 081803

# Axion couplings to other mesons/baryons/EFT

- Axion couplings to other mesons , e.g. $\eta$, $\eta$’ etc

  Gao, Guo, Oller, Zhou JHEP04(2023)022; Wang, Guo, Zhou PRD 109(2024)075030; Wang, Guo Lu, Zhou 2403.16064; Cao, Guo, 2408.15825

- Axion couplings to baryons

  Vonk, Guo, Meissner JHEP03(2020)138, Lu, Du, Guo, Meissner, Vonk JHEP05(2020)001, Vonk Guo, Meissner JHEP08(2021)024 …

- Axion coupling to DM

  Cheng, Bian, Zhou,  Phys.Rev.D 104 (2021) 6, 063010;
  Yang, Feng, Wu, J. Phys. G: Nucl. Part. Phys. 51 065201(2024);

- Axion EFT

  Hu, Jiang, Li, Xiao, Yu, PRD 103(2021)095025; Song, Sun, Yu JHEP01(2024)161;

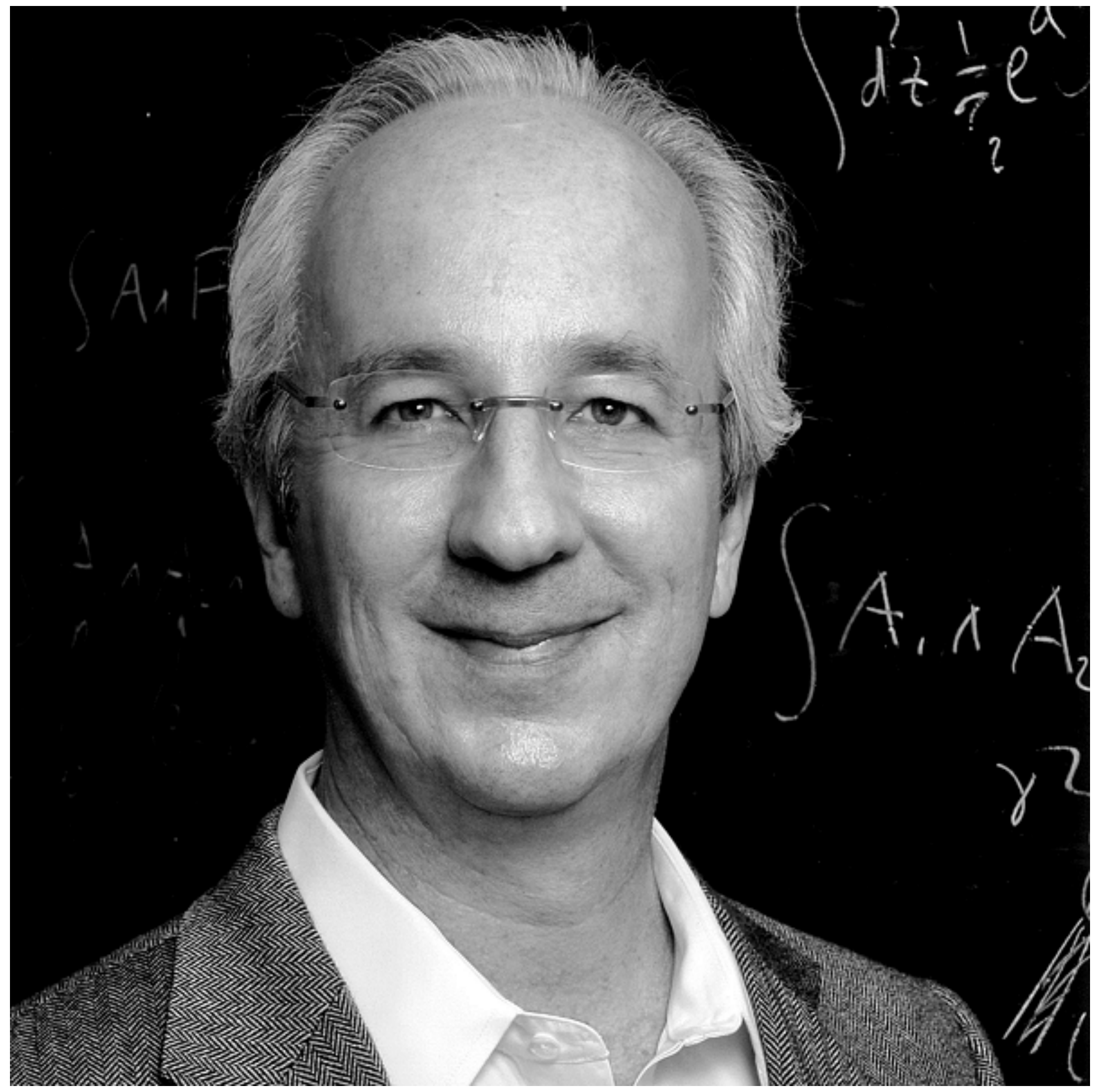
- Etc …

# Wess-Zumino-Witten Interactions in QCD

- WZW terms can describe anomalies in QCD, ensuring gauge invariance and completing chiral L

- Low-energy dynamics of mesons:
  e.g. multiple mesons and photons interactions
  $\pi_0/\eta/\eta' \to \gamma\gamma, \eta' \to 4\pi, \gamma^* \to 3\pi, 5\pi$

- Axion should be involved in WZW interactions systematically, not only in $a - \gamma - \gamma$ interactions
  - Proposed by Harvey Hill Hill in [PRL 99 (2007) 261601], but not solved previously



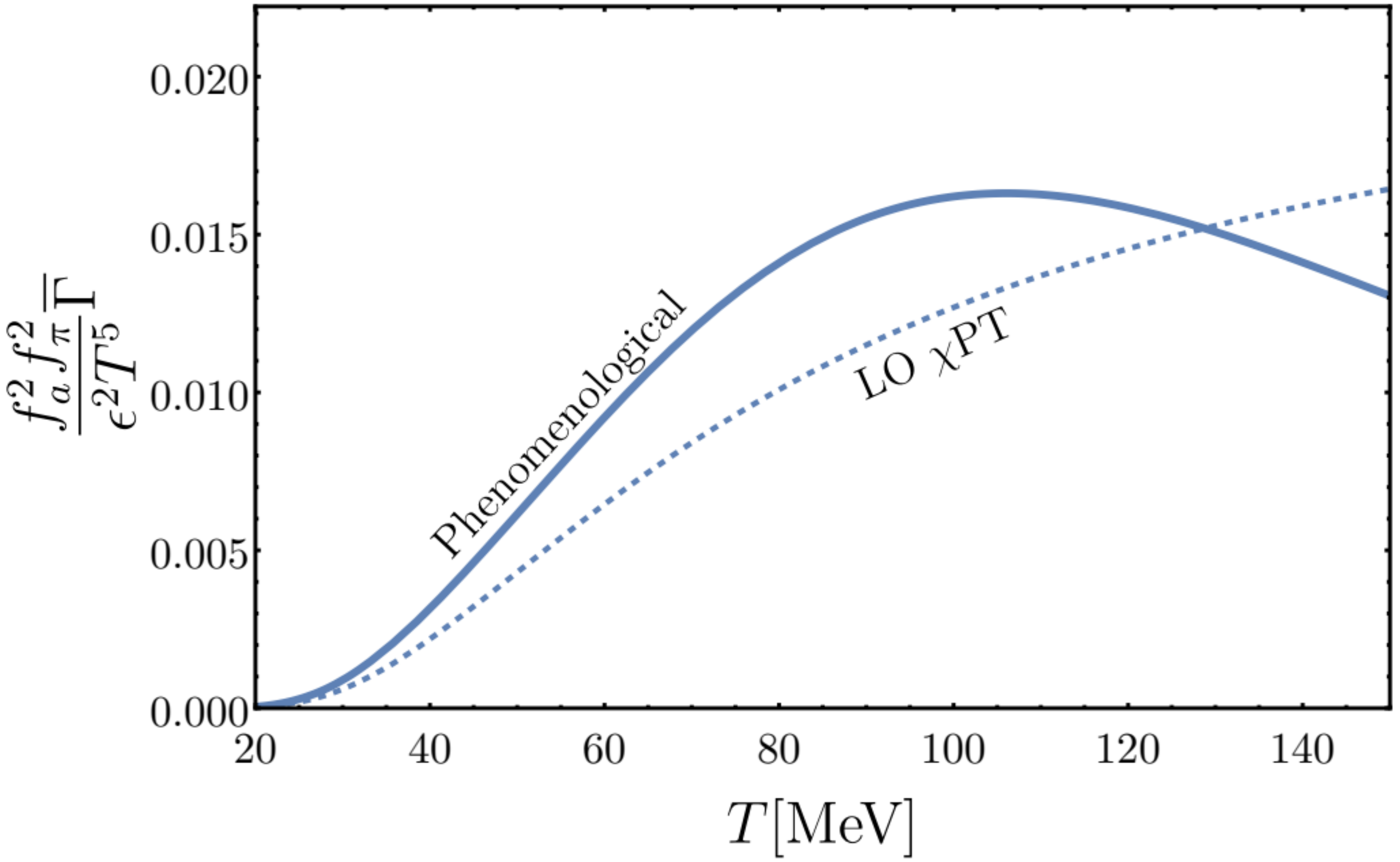
**Jeffrey Harvey,**
**String theorists from UChicago**
**Dirac Medal in 2023**

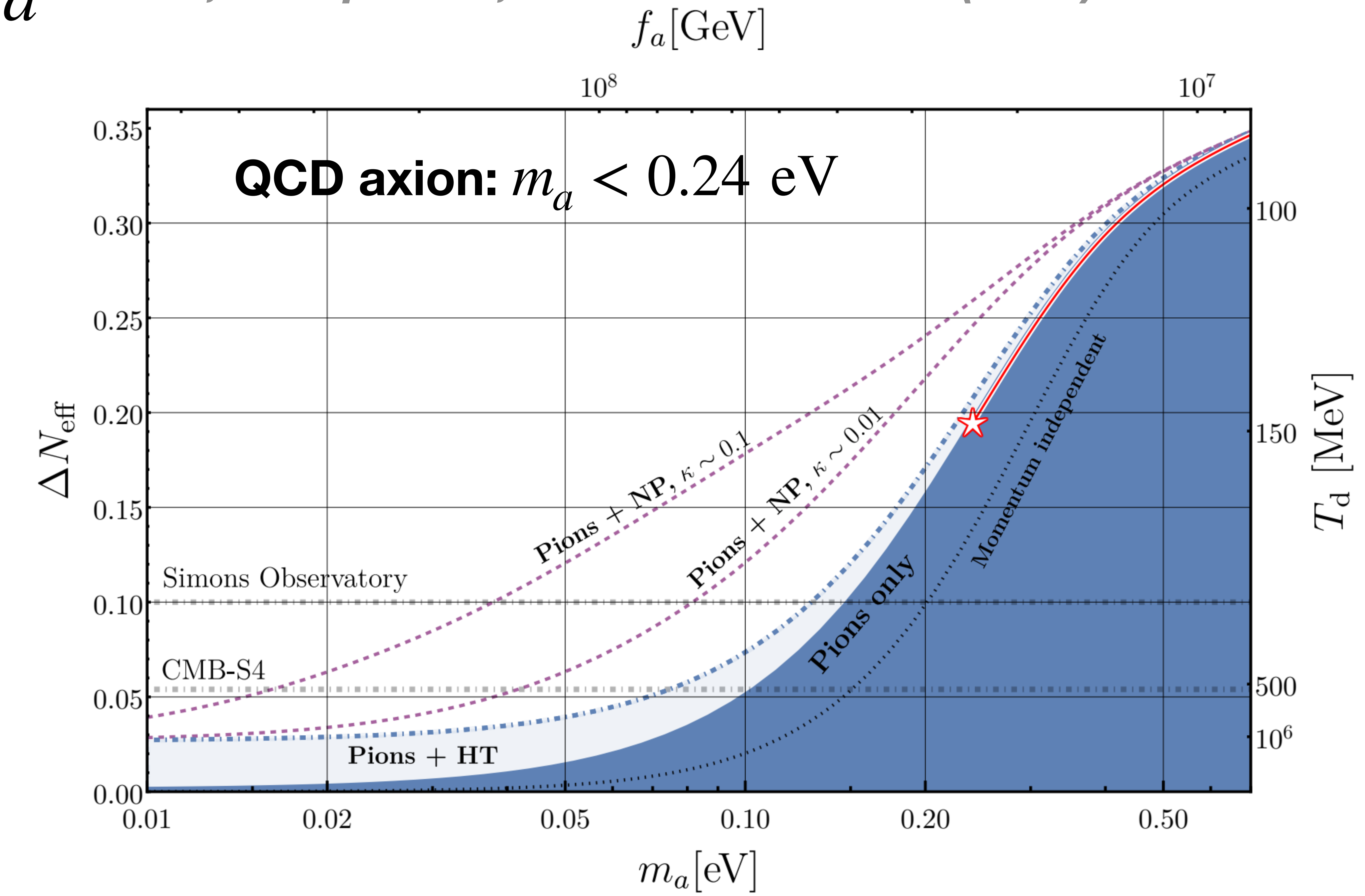
# Challenges in axion-meson interactions

- 1. The issue of chiral basis dependence: The choice of different quark chiral bases alters the form of the axion-meson interaction terms, but physical observables must remain independent of the chiral basis selection.
- 2. WZW interaction involving axions: The low-energy theory of axion-meson interactions must incorporate the Wess-Zumino-Witten (WZW) term while ensuring that physical results are unaffected by the choice of chiral basis.
- 3. The problem of mixed quantum anomalies: Mixed quantum anomalies among the axion's Peccei-Quinn (PQ) symmetry, vector meson symmetry, and the gauge symmetries of the Standard Model may violate gauge invariance.

# Why accurate interactions are important?

- 1. Prediction for thermal axion and its near future test by CMB observation
  - Thermal axion production (high T): $q\bar{q} \to ga, qg \to qa$ *Ferreira, Notari PRL 120 (2018)191301*
  - QCD phase transition: *D'Eramo, Hajkarim, Yun PRL 128 (2022)152001*
  - Improved axion-pion scattering production: $\pi\pi \leftrightarrow \pi a$ *Notari, Rompineve, Villladoro PRL 131 (2023)011004*

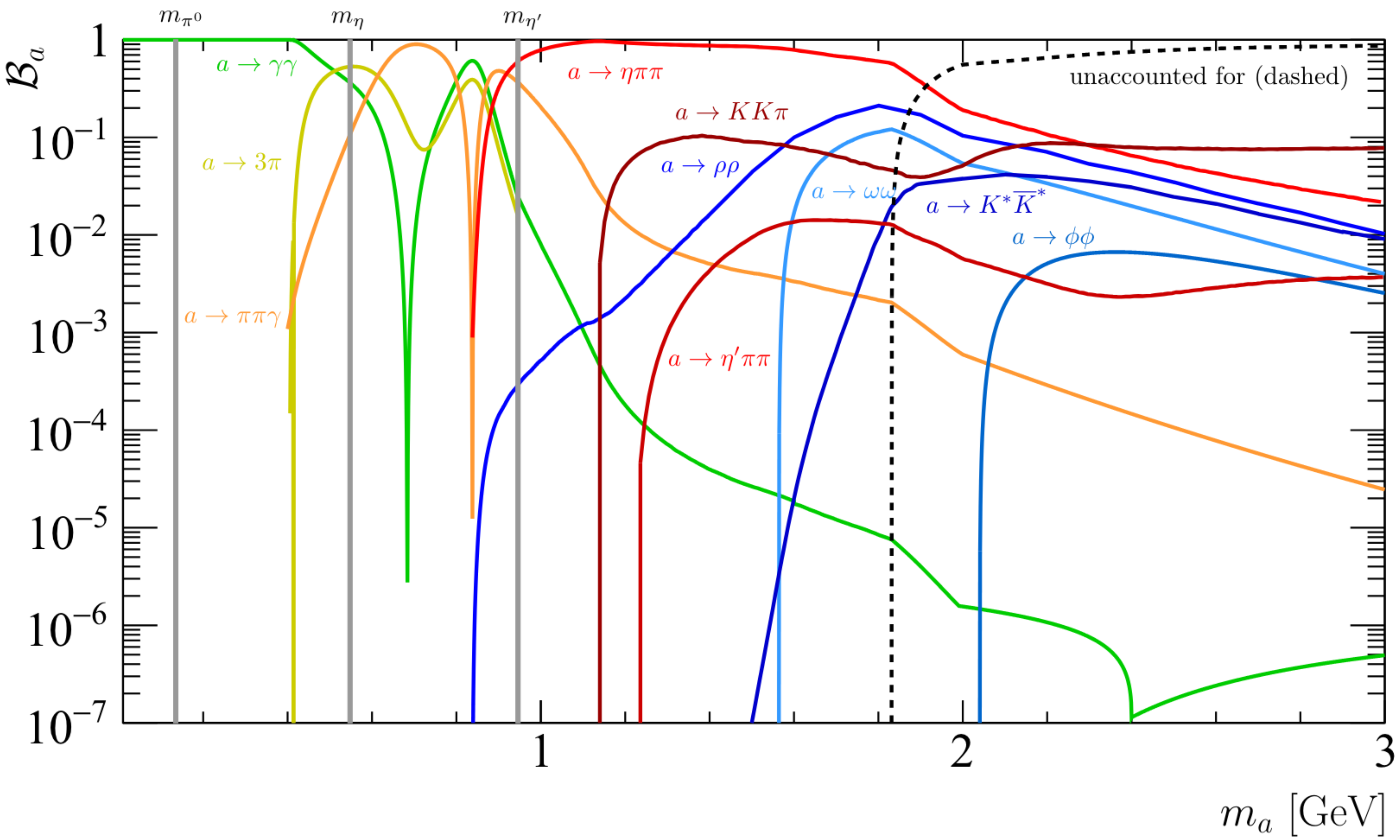


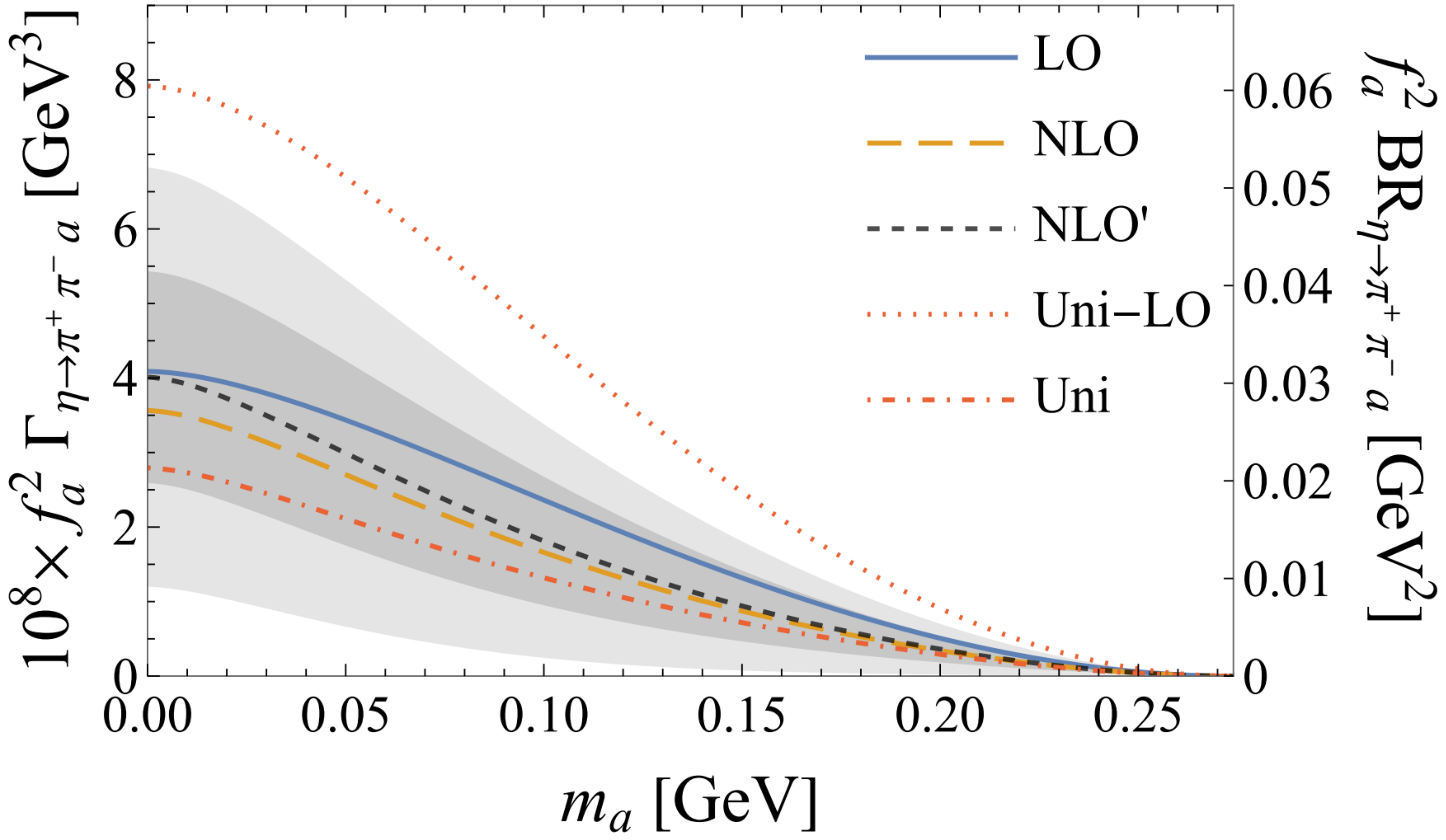


# Why accurate interactions are important?

- 2. Axion related exotic decay width and BR
  - Axion decay BR: *Aloni, Soreq, Williams PRL 123 (2019) 031803*
  - Other meson decays to Axion: $K \to \pi a, \eta \to \pi\pi a$ etc…

*Bauer et al, PRL 127 (2021), 081803*
*Wang, Guo Lu, Zhou 2403.16064*





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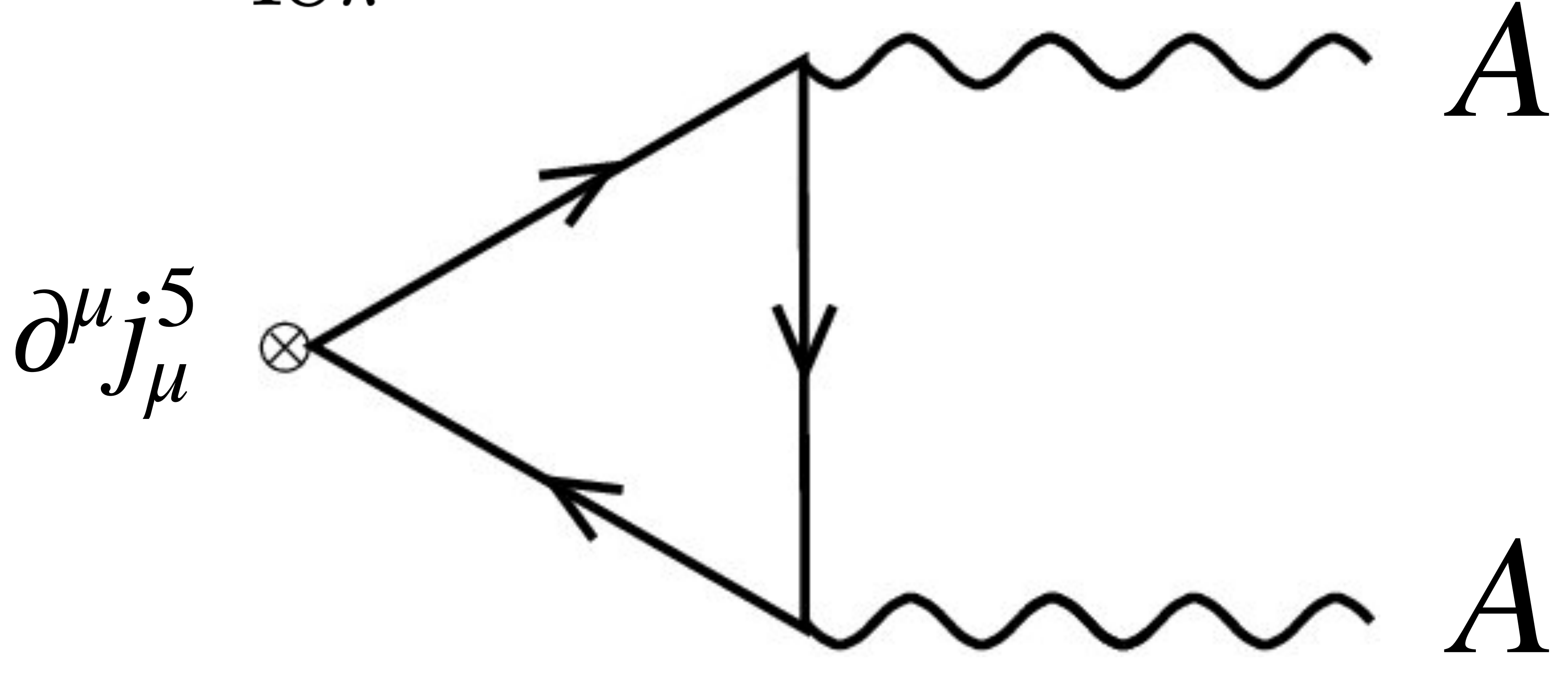
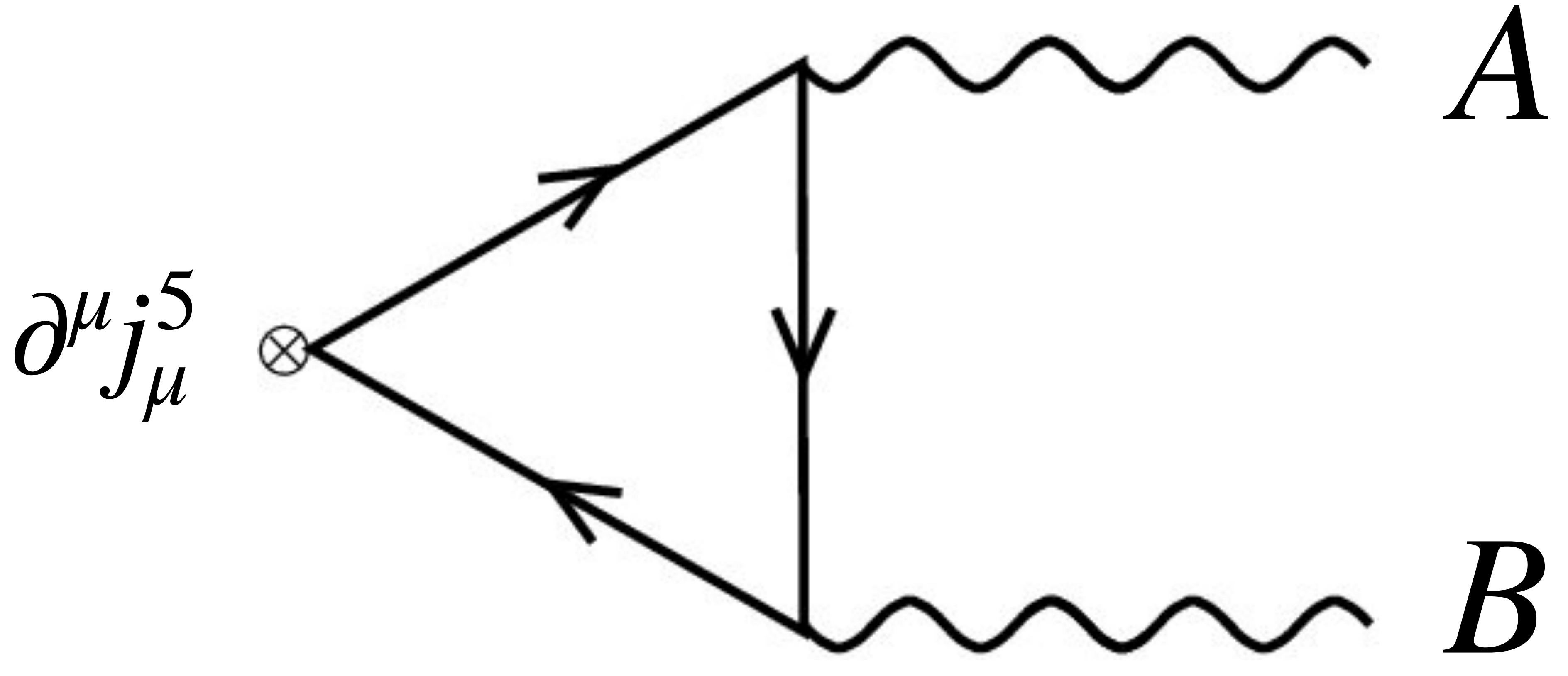
# Global currents and background vector fields

- Background fields can couple to currents of $\mathscr{L}_{\chi\mathrm{PT}}$
    - Baryon currents $U(1)_B$ in neutron star, $\omega$ meson
    - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j_\mu^5 = \frac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



$$\delta\partial^\mu j_\mu^5 \propto \epsilon_{\mu\nu\rho\sigma}A^{\mu\nu}B^{\rho\sigma}$$



# WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
  - Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
    Similar to Hidden Local Symmetry

$$\mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathscr{C} \int Tr\Big[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L)\mathbb{B}_L + \frac{1}{2}\mathbb{A}_L(\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i \mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i \mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2} \mathbb{A}_L \mathbb{B}_L^3\Big] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

# Axion treatment as a fictitious background field

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

*Yang Bai, Ting-Kuo Chen, JL, Xiaolin Ma*
*Phys.Rev.Lett. 134, (2025) 081803*

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \to D_\mu + i\dfrac{\partial_\mu a}{f_a}\left(\mathbf{k}_L P_L + \mathbf{k}_R P_R\right)$
- Hints from ChPT L: $D^\mu U \to D^\mu U - i\dfrac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

# Axion treatment as a fictitious background field

- Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
  Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields:
  $$\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \frac{da}{f_a}$$

- 2-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{A}_L = \frac{e}{s_{\rm W}} W^a \frac{\boldsymbol{\tau}^a}{2} + \frac{e}{c_{\rm W}} W^0 \mathbf{Y}_Q, \ \mathbb{A}_R = \frac{e}{c_{\rm W}} W^0 \mathbf{Y}_q$$

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho_0 \end{pmatrix} + g' \begin{pmatrix} \omega & \\ & \omega \end{pmatrix} \boxed{+ (\mathbf{k}_{L,0} + \mathbf{k}_{R,0}) \frac{da}{f}}$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 & \sqrt{2}a^+ \\ \sqrt{2}a^- & -a_1 \end{pmatrix} + g' \begin{pmatrix} f_1 & \\ & f_1 \end{pmatrix} \boxed{+ (\mathbf{k}_{L,0} - \mathbf{k}_{R,0}) \frac{da}{f}}$$

# The consistent full axion Lagrangian at low-energy

- ChPT:

$$\mathscr{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\,\mathrm{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f}\sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1\mathscr{A}_2} F_{\mathscr{A}_1\,\mu\nu}\widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- Full WZW: $\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$

***Yang Bai, Ting-Kuo Chen, JL, Xiaolin Ma***
***Phys.Rev.Lett. 134, (2025) 081803***

- Full $\mathscr{L}$: $\mathscr{L}^{\mathrm{full}}_{\mathrm{axion}} \equiv \left[\mathscr{L}_{\chi\mathrm{PT}} + \mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$

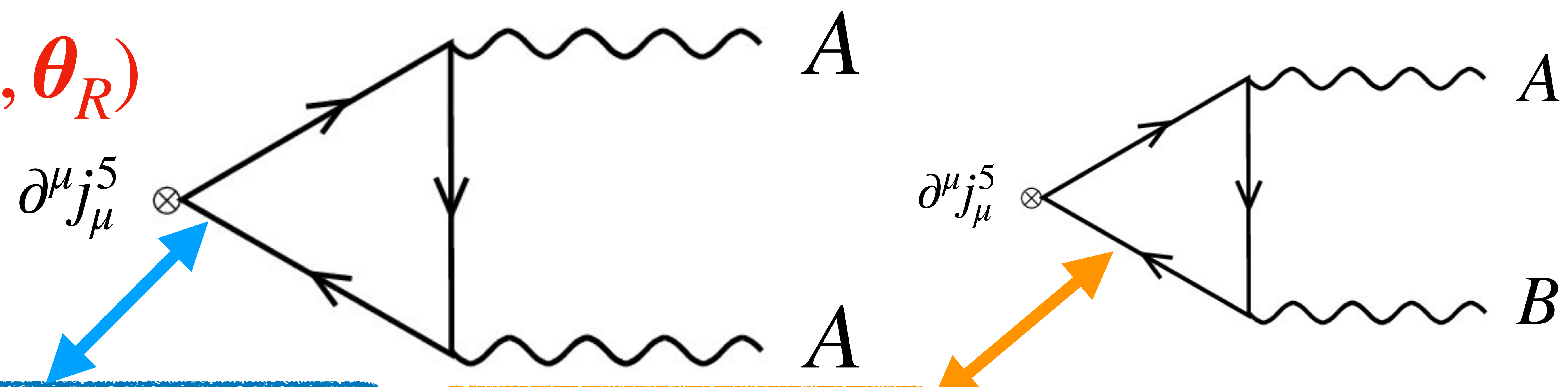
# Matching between $\mathscr{L}_{\text{eff}}$ and $\mathscr{L}^{\text{full}}_{\text{axion}}$

$$\begin{tikzcd}
\mathscr{L}_{\text{eff},\,0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0}) \arrow[d, "q_0 = \exp\left( -i\, c_{gg}\, \boldsymbol{\kappa}_{q,0}\, \gamma_5 \frac{a}{f} \right) q"] & \\
\mathscr{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \arrow[r, "q' = \exp\left[ i \left( \boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5 \right) \frac{a}{f} \right] q"'] \arrow[d, "\text{matching}"'] & \mathscr{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathscr{L}^{\text{ano}}_{a} \arrow[d, "\text{matching}"] \\
\begin{array}{l} \mathscr{L}^{\text{full}}_{\text{axion}} \equiv \mathscr{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \\ \quad + \mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \end{array} \arrow[r, "U' = U_L^{\dagger} U U_R"] & \begin{array}{l} \mathscr{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ + \mathscr{L}^{\text{full}}_{\text{WZW}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ + \delta \mathscr{L}^{\text{ano}}_{\text{WZW}} \end{array}
\end{tikzcd}$$

# The axion anomalous interactions

$$\delta \mathscr{L}_a^{\mathrm{ano}} = -\,\delta\left[\mathscr{L}_{\mathrm{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions

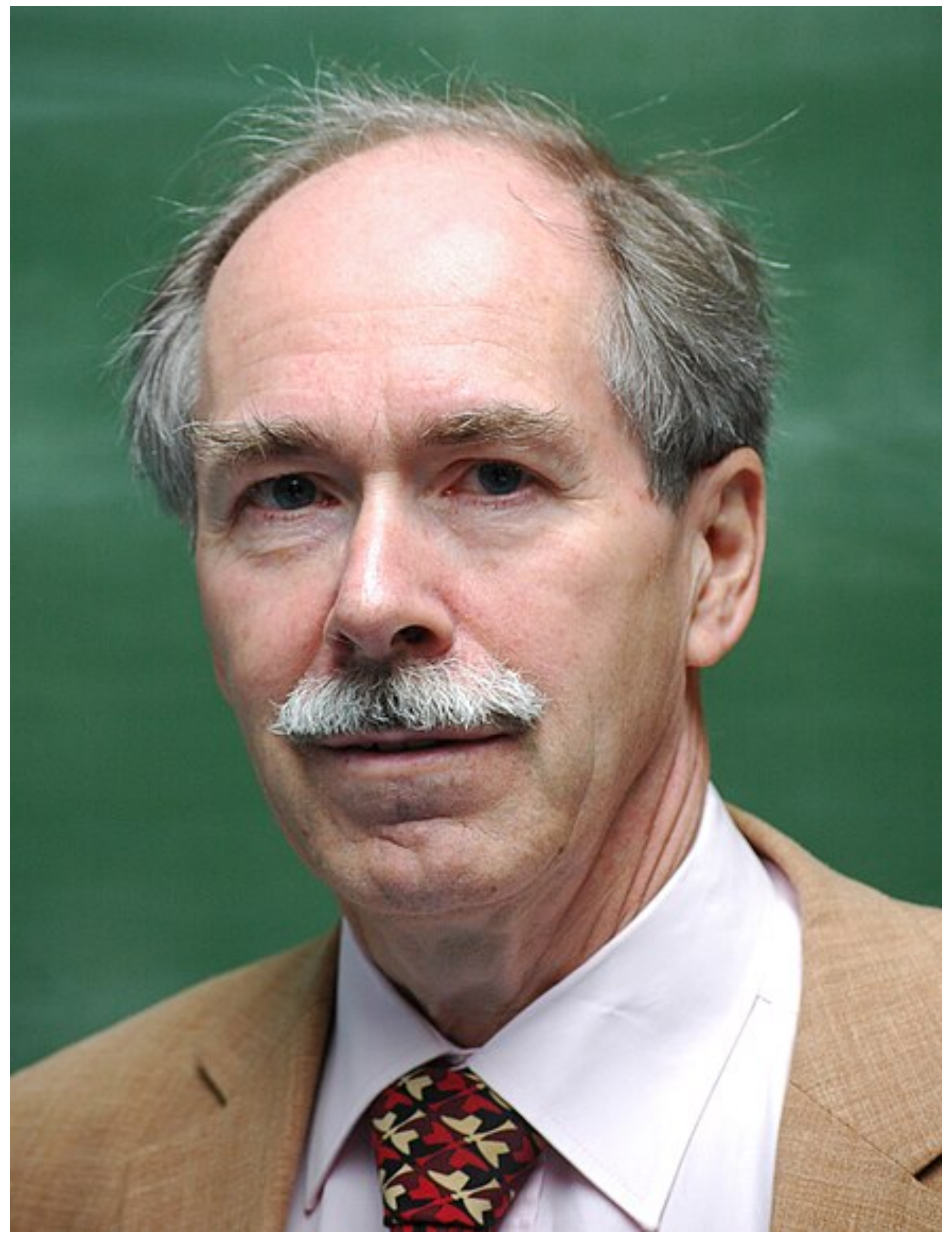


$$\delta\left[\Gamma_{\mathrm{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\boxed{\frac{a}{f}}\int \mathrm{Tr}\Big\{\boxed{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2}+\boxed{3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L)}+D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+\,i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

# Gerard 't Hooft UV-IR anomaly matching condition

- 't Hooft anomaly matching requires that the chiral anomaly of the flavor symmetry remain invariant under a change in the energy scale of the degrees of freedom.



**Gerard ’t Hooft**

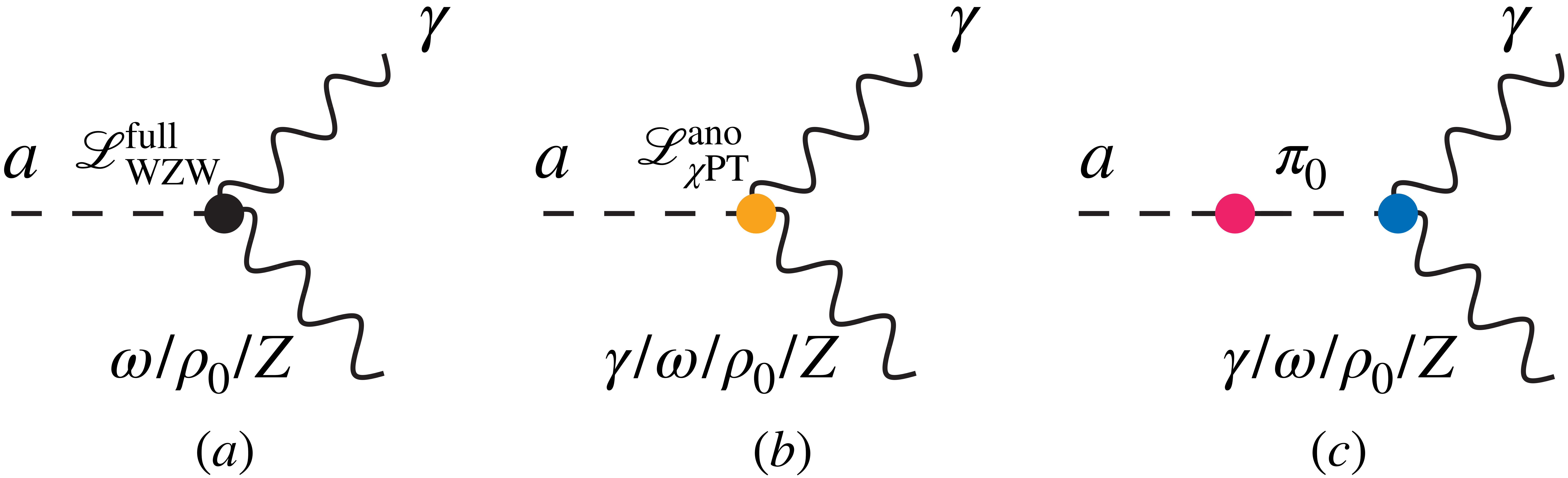
$$\delta \mathscr{L}_a^{\text{ano}} = -\,\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = \delta \mathscr{L}_{\text{WZW}}^{\text{ano}}$$

$$\begin{tikzcd}
\mathscr{L}_{\text{eff},\,0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0}) \arrow[d, "q_0 = \exp\left(-i\, c_{gg}\, \boldsymbol{\kappa}_{q,0}\, \gamma_5 \frac{a}{f}\right) q"] & \\
\mathscr{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \arrow[r, "q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right)\frac{a}{f}\right] q"'] \arrow[d, "\text{matching}"'] & \mathscr{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \boxed{\delta \mathscr{L}_a^{\text{ano}}} \arrow[d, "\text{matching}"] \arrow[d, leftrightarrow, bend left=60, "\text{Anomaly Matching}"] \\
\begin{array}{l} \mathscr{L}_{\text{axion}}^{\text{full}} \equiv \mathscr{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \\ \quad + \mathscr{L}_{\text{WZW}}^{\text{full}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \end{array} \arrow[r, "U' = U_L^\dagger U U_R"] & \begin{array}{l} \mathscr{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ + \mathscr{L}_{\text{WZW}}^{\text{full}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ \boxed{+\,\delta \mathscr{L}_{\text{WZW}}^{\text{ano}}} \end{array}
\end{tikzcd}$$

# Consistent physical amplitudes

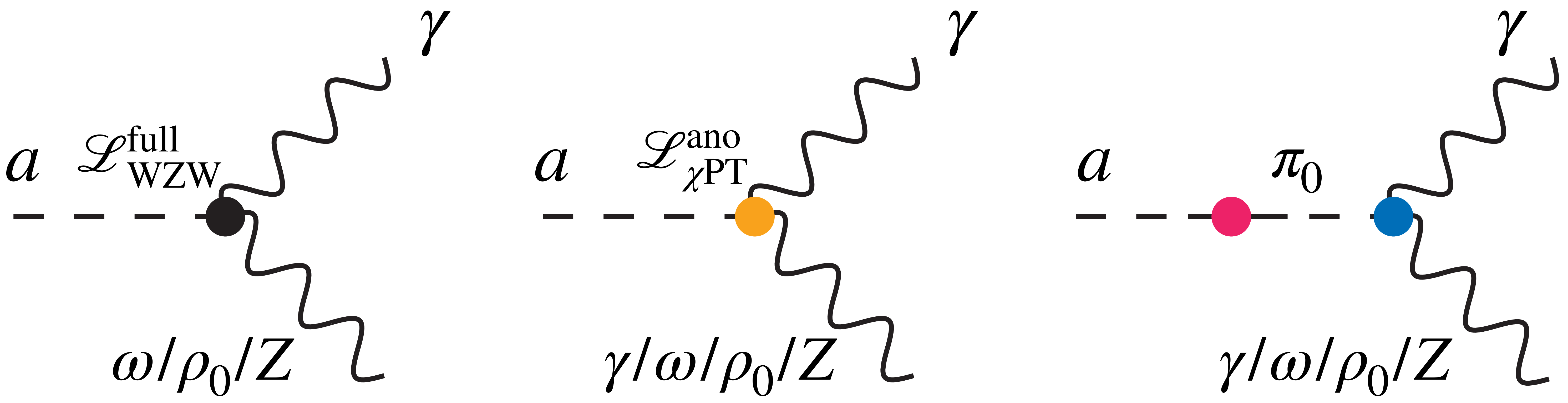
- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations

- Full WZW interactions are important for a-A-B amplitudes



# Consistent physical amplitudes for $a - \gamma - \gamma$

- Auxiliary rotations are cancelled



$(a)$ $(b)$ $(c)$

$$ad\gamma d\gamma:\quad c_{\rm WZW}\equiv 0 \qquad ad\gamma d\gamma:\quad c_{\rm ano}\equiv -\frac{e^2N_c}{48\pi^2 f}12(Q_u^2\kappa_u+Q_d^2\kappa_d) \qquad \pi_0 d\gamma d\gamma:\quad c_{\pi_0}\equiv \frac{e^2N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d^2-Q_u^2)$$

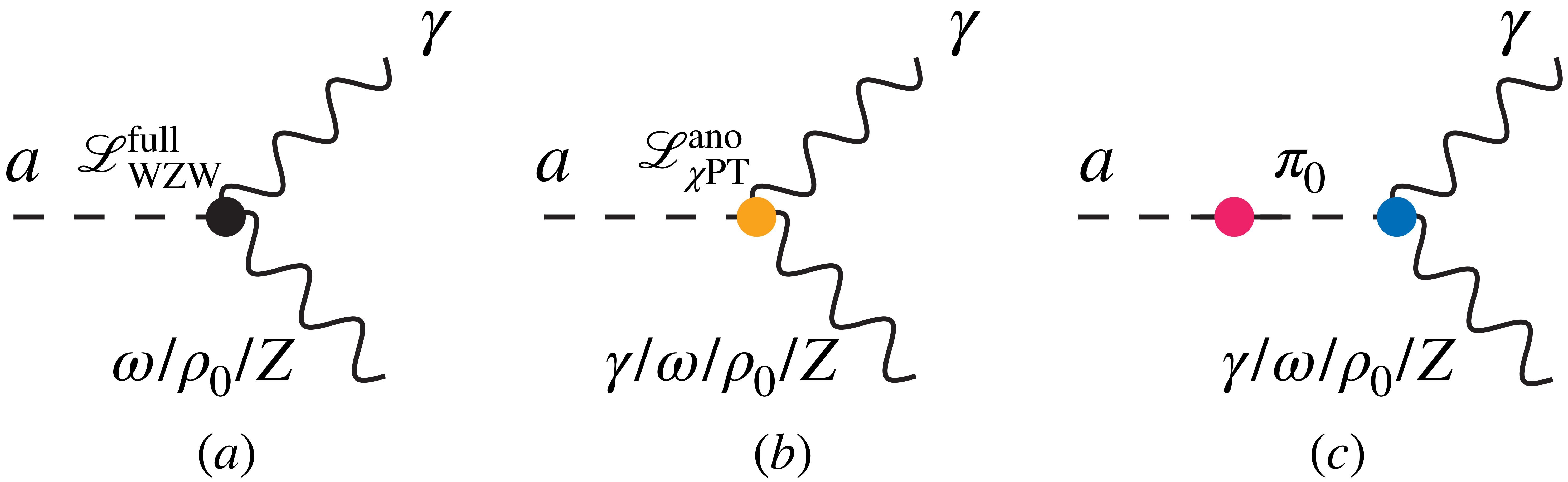
$$\mathscr{M}(a\to\gamma\gamma)\textbf{(auxiliary)} = CF\times\left(c_{\rm ano}+\theta'_{a-\pi_0}c_{\pi_0}+c_{\rm WZW}\right)$$

$$= CF\times e^2\left\{\frac{-N_c}{48\pi^2 f_a}12(Q_u^2\kappa_u+Q_d^2\kappa_d)+i\frac{f_\pi}{\sqrt{2}f}\left[(\kappa_u-\kappa_d)p_a^2-2\frac{m_u\kappa_u-m_d\kappa_d}{m_u+m_d}m_\pi^2\right]\frac{i}{p_a^2-m_\pi^2}\times\frac{N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d^2-Q_u^2)\right\}$$

$$\begin{matrix}\kappa_u+\kappa_d=0 \\ p_a^2=m_a^2\end{matrix}\quad \to 0$$

# Consistent physical amplitudes for $a - \gamma - \omega$

- Auxiliary rotations are cancelled



$(a)$ $(b)$ $(c)$

$a d\omega d\gamma:\ \ c_{\rm wzw} = \frac{-eg'N_c}{48\pi^2 f}6(Q_u\kappa_u + Q_d\kappa_d)$ $a d\omega d\gamma:\ \ c_{\rm ano} = \frac{-eg'N_c}{48\pi^2 f}6(Q_u\kappa_u + Q_d\kappa_d)$ $\pi_0 d\omega d\gamma:\ \ c_{\pi_0} = \frac{eg'N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d - Q_u)$

$\mathscr{M}(a \to \omega\gamma)$**(auxiliary)** $= CF \times (c_{\rm ano} + \theta'_{a-\pi_0}c_{\pi_0} + c_{\rm wzw})$

$$= CF \times eg'\left[\frac{-N_c}{48\pi^2 f}12(Q_u\kappa_u + Q_d\kappa_d) + i\frac{f_\pi}{\sqrt{2}f}((\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d}m_\pi^2)\frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d - Q_u)\right]$$

$\to 0$

# Consistent amplitudes for three point vertex

$$c^{\text{eff}}_{\gamma\gamma} = c^0_{\gamma\gamma} + \boxed{\frac{e^2 c_{gg}}{16\pi^2 f}}\left(-\frac{10}{3} - 2\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{e^2}{16\pi^2 f}\frac{m_a^2}{m_\pi^2 - m_a^2}(c_u - c_d)$$

$$c^{\text{eff}}_{\omega\gamma} = \boxed{eg'}\left\{\frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}(c_d + c_Q - 2c_u)\right\}$$

$$c^{\text{eff}}_{\rho\gamma} = eg\left\{\frac{-3c_{gg}}{8\pi^2 f} - \frac{1}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}\left(3c_Q - 2c_u - c_d\right)\right\}$$

$$c^{\text{eff}}_{\gamma Z} = c^0_{\gamma Z} + \frac{N_c c_{gg}}{48\pi^2 f}\frac{e^2}{s_w c_w}(-9 + 20 s_w^2) - c_{\pi_0}\frac{f_\pi}{\sqrt{2}f}\left(\frac{m_a^2}{m_\pi^2 - m_a^2}\frac{c_d - c_u}{2} - c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{N_c}{48\pi^2 f}\frac{2e^2}{s_{2w}}(c_d + 2c_u + 3c_Q)$$

$\mathbf{k}_{L,0} = \{c_Q, c_Q\}$ $\quad$ $\mathbf{k}_{R,0} = \{c_u, c_d\}$

# Summary

- A full chiral axion Lagrangian for axion-pseudo-vector meson
- $\mathscr{L}_{\text{axion}}^{\text{full}} \equiv \left[\mathscr{L}_{\chi\text{PT}} + \mathscr{L}_{\text{WZW}}^{\text{full}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$
  - 1. Wess-Zumino-Witten counter term is included for gauge invariance
  - 2. t'Hooft UV-IR anomaly matching is achieved
  - 3. Consistent physical amplitudes without auxiliary rotation parameters
- New search channels, e.g. $\omega - \gamma - a$ vertex, for BESIII & STCF
- Future plan: extending to three light quarks scheme
  Needs to deal with η'; vector meson mediated processes in astrophysics

*Thank you!*

# Backup: 3-flavor quarks

$$U = \exp\left[(\sqrt{2}i/f_\pi)\pi^a \boldsymbol{\tau}^a\right] \equiv \exp\left[(\sqrt{2}i/f_\pi)\mathbf{\Phi}\right]$$

$$\mathbf{\Phi} = \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g\begin{pmatrix} \rho_0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*+} \\ \sqrt{2}\rho^- & -\rho_0 + \omega & \sqrt{2}K^{*0} \\ \sqrt{2}K^{*-} & \sqrt{2}\bar{K}^{*0} & \sqrt{2}\phi \end{pmatrix}$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g\begin{pmatrix} a_1 + f_1 & \sqrt{2}a^+ & \sqrt{2}K_A^{*+} \\ \sqrt{2}a^- & -a_1 + f_1 & \sqrt{2}K_A^{*0} \\ \sqrt{2}K_A^{*-} & \sqrt{2}\bar{K}_A^{*0} & \sqrt{2}f_s \end{pmatrix}$$

# Prove of consistency for 3-flavor quarks

$$\begin{tikzcd}
\mathscr{L}^0_{\rm eff}(q, \mathbf{m}^0_q, \mathbf{k}^0_L, \mathbf{k}^0_R, c_{gg}) \arrow[r, "q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right)\frac{c_{gg}a}{f}\right]q"'] \arrow[d, "\text{matching}"'] & \mathscr{L}_{\rm eff}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R, c'_{gg}) + \delta\mathscr{L}^{\rm ano}_a \arrow[d, "\text{matching}"] \\
\begin{array}{l} \mathscr{L}^{\rm full}_{\rm axion} \equiv \mathscr{L}_{\chi{\rm PT}}(U, \mathbf{m}^0_q, \mathscr{A}_{L/R} + \mathbf{k}^0_{L/R}da) \\ \quad + \mathscr{L}^{\rm full}_{\rm WZW}(U, \mathbf{m}^0_q, \mathscr{A}_{L/R} + \mathbf{k}^0_{L/R}da) \\ \quad - \frac{\tau}{2}(-i\log[{\rm det}U] - 2c_{gg}a/f_a)^2 \end{array} \arrow[r, "U' = U_L^\dagger U U_R"] & \begin{array}{l} \mathscr{L}_{\chi{\rm PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R}da) \\ + \mathscr{L}^{\rm full}_{\rm WZW}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R}da) \\ + \delta\mathscr{L}^{\rm ano}_{\rm WZW} - \frac{\tau}{2}(-i\log[{\rm det}U] - 2c'_{gg}a/f_a)^2 \end{array}
\end{tikzcd}$$