Black Hole Superradiance

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Based on PRD 106 064016 (2022), PRD 107 064037(2023), PRD 107 075009 (2023), PRD 110 083029 (2024), arXiv: 2501.09280

In collaboration with Shoushan Bao, Leidong Cheng, Yinda Guo, Nayun Jia, Tianjun Li, Qixuan Xu, Xin Zhang

04/12/2025, MEPA 2025, Nanjing

Outline

- Motivation
- BH Superradiance
- Observables
- Summary

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- **Motivation**
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Axions

- The QCD axion is proposed to solve the strong CP problem.

  Peccei & Quinn (1977) Weinberg (1978), Wilczek (1978)

  m_a: 10^{-6} eV to 10^{-2} eV

- The axion-like particle (ALP) is a hypothetical **ultralight scalar** to solve the dark matter problem.

  e.g. Hui et.al. PRD (2017)

  m_a: 10^{-22} eV to keV ⟹ λ: nano-meter to light-year

  Both QCD axion and ALP are called **axion** in this talk

- The axion is popular dark matter candidate

  axions are wavelike, different from WIMP

- The axion may or may not couple to SM sector

  How to detect axions if they are not coupled to SM?

Model-indep. Probe of Axions

- BH superradiance provides a model-independent probe of axions

  only assume axion has a small mass

- Non-interacting axions in Kerr metric

$$(\nabla^{\nu}\nabla_{\nu} + \mu^{2})\Phi = 0 \qquad \mu\text{: axion mass}$$

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$$(\nabla^{\nu}\nabla_{\nu} + \mu^2)\Phi = 0$$

μ: axion mass

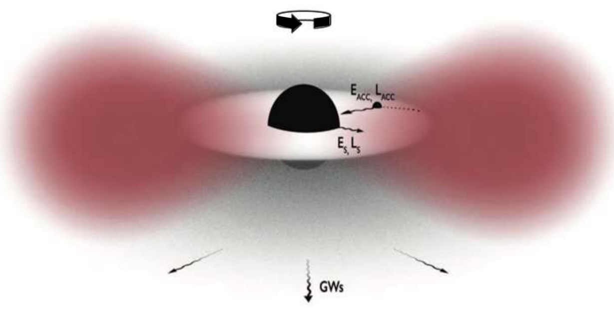
- The bound state eigen-energy is **complex** $\omega_{n\ell m} = \omega_{n\ell m}^{(R)} + i\omega_{n\ell m}^{(I)}$

  Three indices: $(n \geq 0, l, m)$, similar to hydrogen atom

- The condensate mass and angular momentum grow exponentially

$$M_s \propto \exp(2\omega_{nlm}^{(I)} t)$$

If $\omega_{nlm}^{(I)} > 0$, called superradiance rate



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Superradiance Rate

$$M_s \propto \exp(2\omega_{nlm}^{(I)}\, t)$$

M: BH mass
μ: axion mass
$G_N = 1$

- Analytic approximation at LO of $\alpha = M\mu$

KLEIN-GORDON EQUATION AND ROTATING BLACK HOLES #3

Steven L. Detweiler (Yale U.) (1980)

Published in: *Phys.Rev.D* 22 (1980) 2323-2326

DOI cite claim reference search 617 citations

- Complicated numerical solution

Instability of the massive Klein-Gordon field on the Kerr spacetime #4

Sam R. Dolan (University Coll., Dublin) (May, 2007)

Published in: *Phys.Rev.D* 76 (2007) 084001 • e-Print: 0705.2880 [gr-qc]

pdf DOI cite claim reference search 487 citations

NLO Calculation

- We confirm a mistake in the LO calculation of $\omega_{nlm}^{(I)}$, and calculate the NLO contribution for the first time.

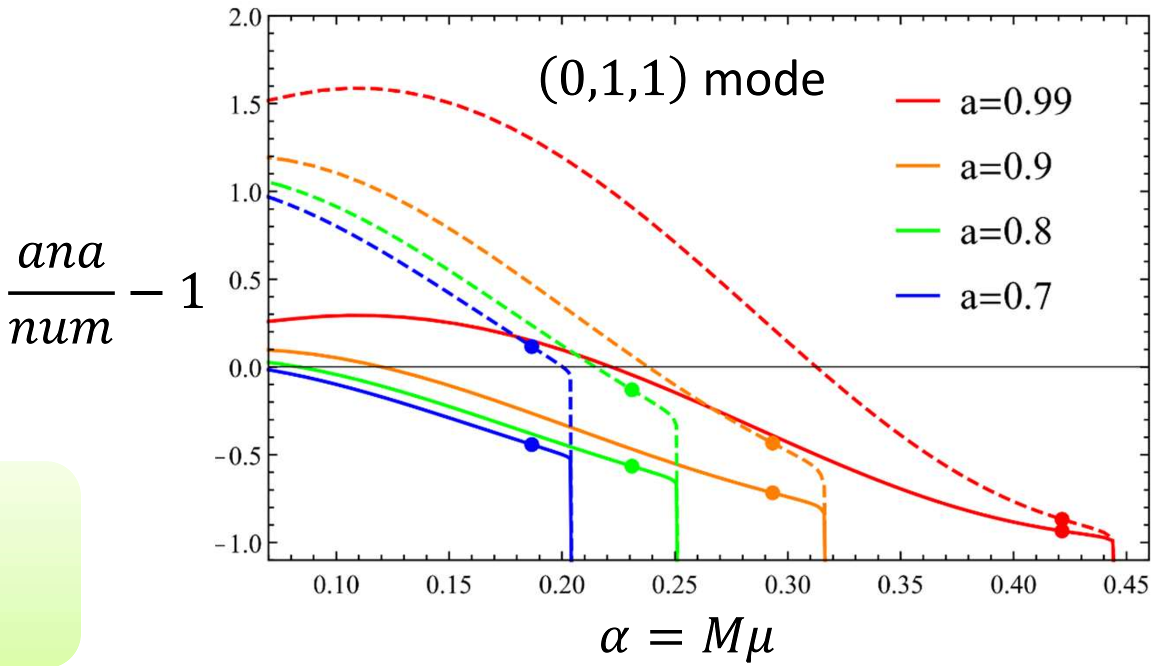
- Leading order

  At small α with $a = 0.99$,

  Err. of original result $\sim$ 150%

  Err. of corrected result $\sim$ 30%

Analytic and numerical results do not converge at $\alpha \to 0$!



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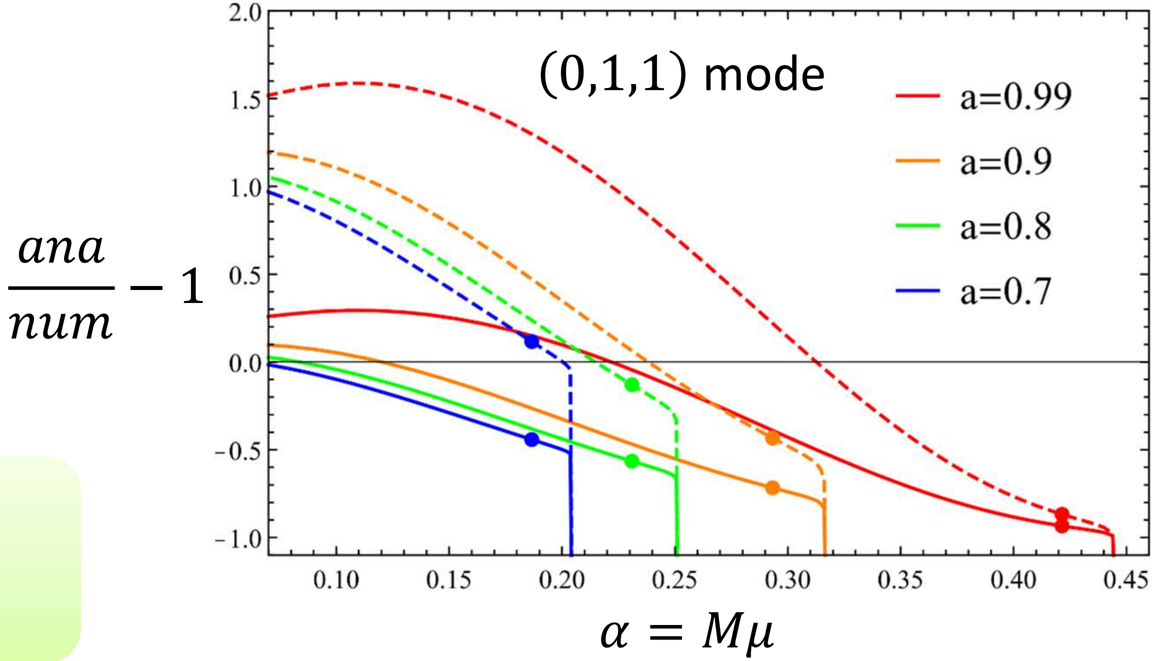
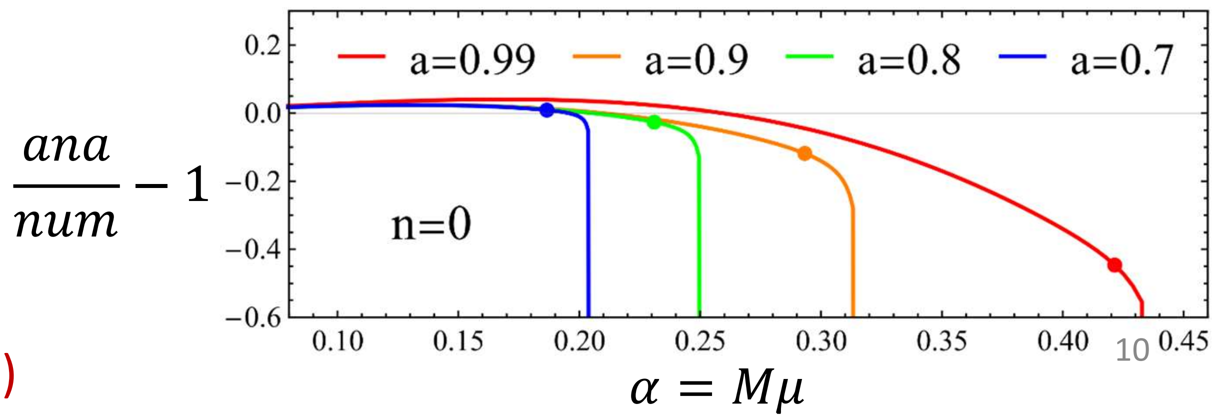


- Next-to-leading order

  At small α, **err.** $< 5\%$

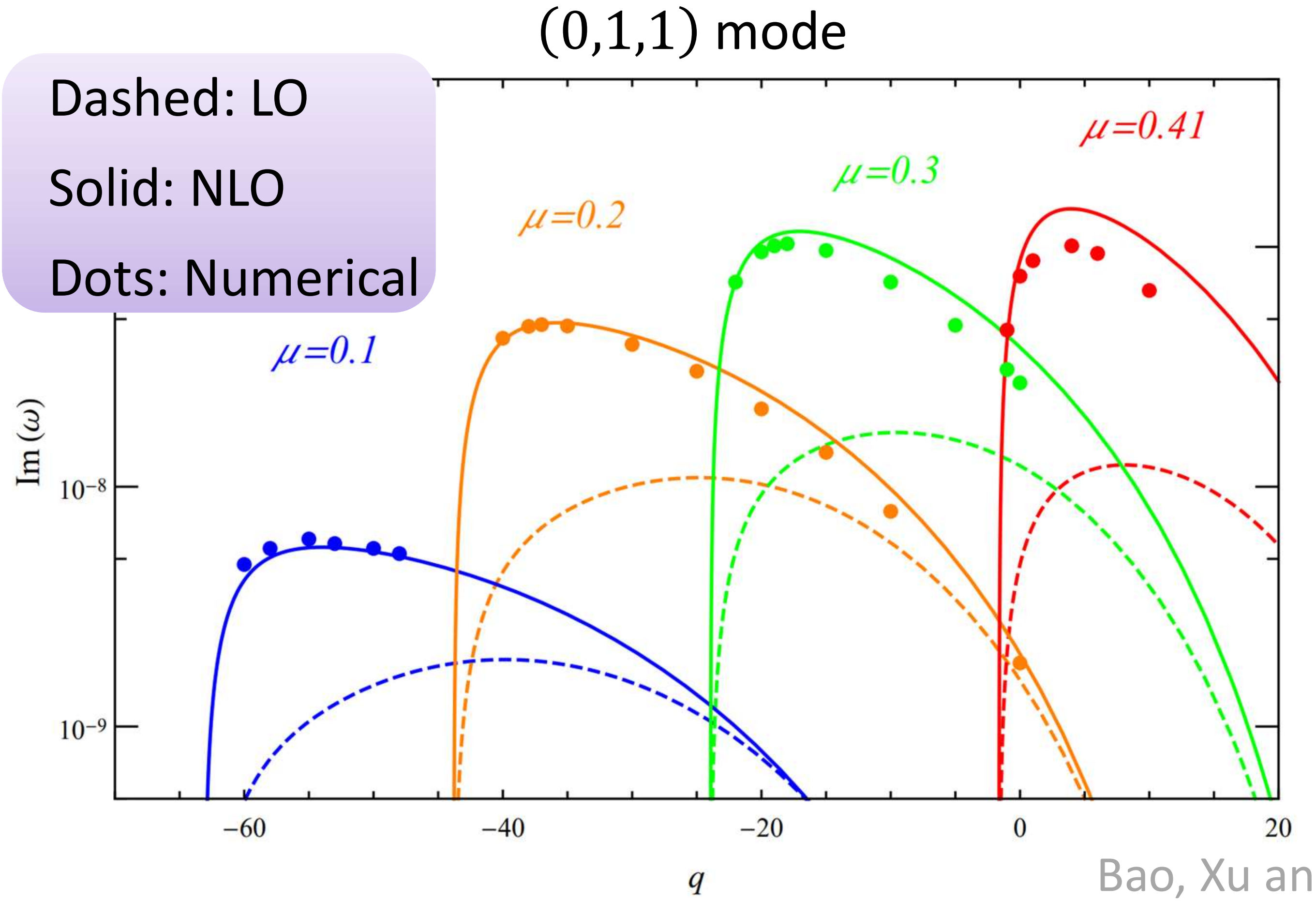


Bao, Xu and **HZ,** PRD (2022)

NLO Sol. of KNBH

- NLO solution greatly improves the precision

BH mass is normalized to 1, BH charge $Q = 0.02$

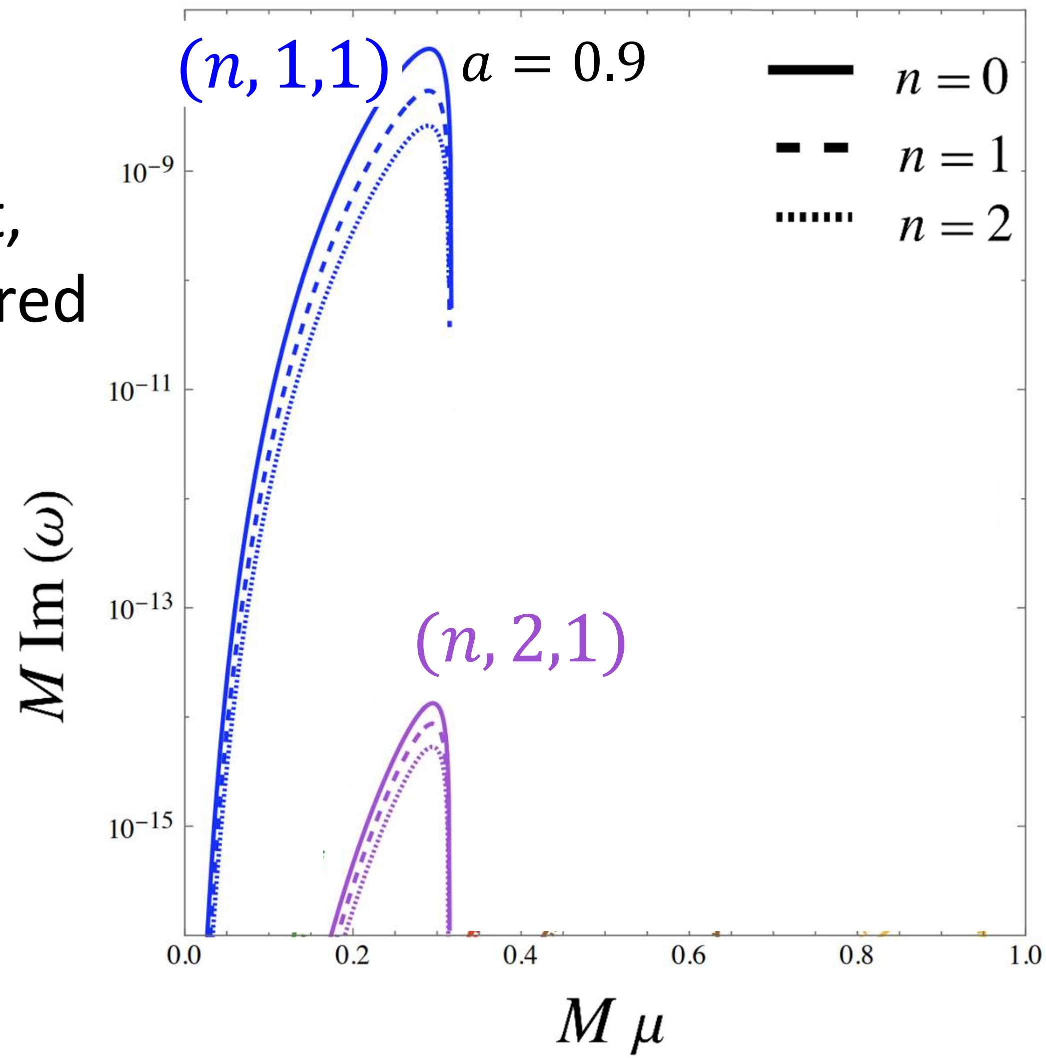


Bao, Xu and **HZ,** PRD (2023)

- In the rest of this part, I focus only on Kerr BH.

Important Modes

- Three indices $(n \geq 0, l, m)$
- (n, l, l) are the most important, modes with $m \neq l$ can be ignored

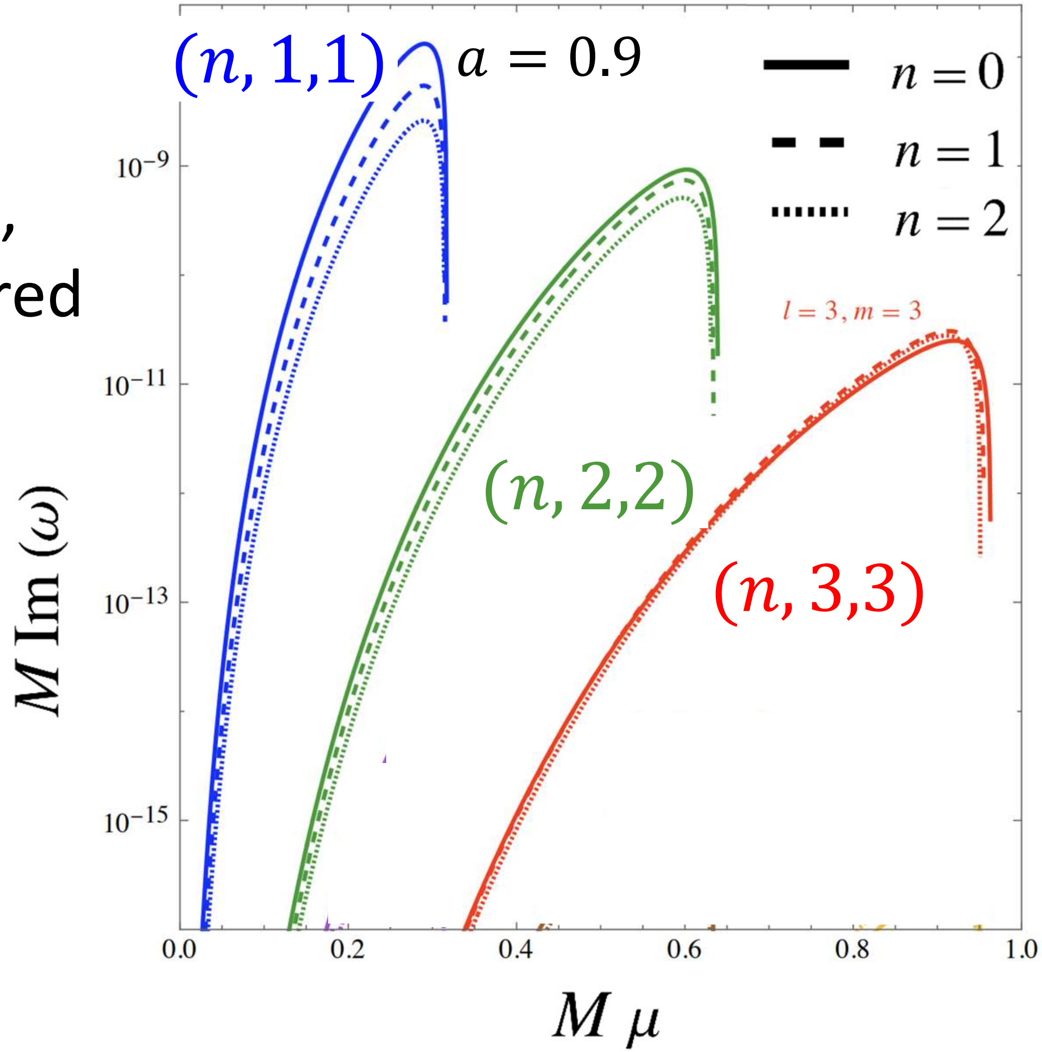


Important Modes

- Three indices $(n \geq 0, l, m)$
- (n, l, l) are the most important, modes with $m \neq l$ can be ignored
- (n, l, l) modes with different l have very different rates

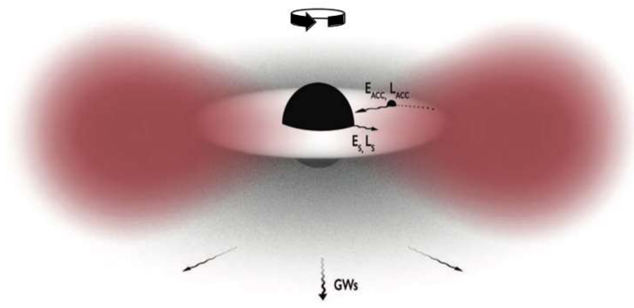
  $\Rightarrow$ Very different time scales

  Can consider a single l in each time range
- In each l-stage, $(0, l, l)$ is the fastest, followed by $(1, l, l)$



Isolated BH-condensate Evolution

- Consider **non-interacting real scalar field** in a Kerr metric



- Evolution equations of the BH-condensate

M: BH mass

J: BH angular mom.

$M_s^{(nlm)}$: mass of mode (nlm)

$J_s^{(nlm)}$: angular mom. of mode (nlm)

$$\dot{M} = -\sum_{nlm} 2M_s^{(nlm)}\omega_I^{(nlm)},$$

$$\dot{J} = -\sum_{nlm} 2mM_s^{(nlm)}\omega_I^{(nlm)}/\omega_R^{(nlm)}$$

$$\dot{M}_s^{(nlm)} = 2M_s^{(nlm)}\omega_I^{(nlm)} - \boxed{\dot{E}_{\mathrm{GW}}^{(nlm)}}$$

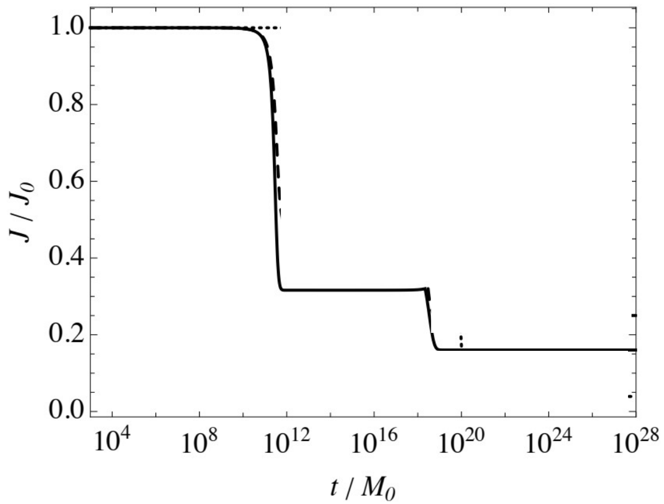
$$\dot{J}_s^{(nlm)} = 2mM_s^{(nlm)}\omega_I^{(nlm)}/\omega_R^{(nlm)} - \boxed{m\dot{E}_{\mathrm{GW}}^{(nlm)}/\omega_R^{(nlm)}}$$

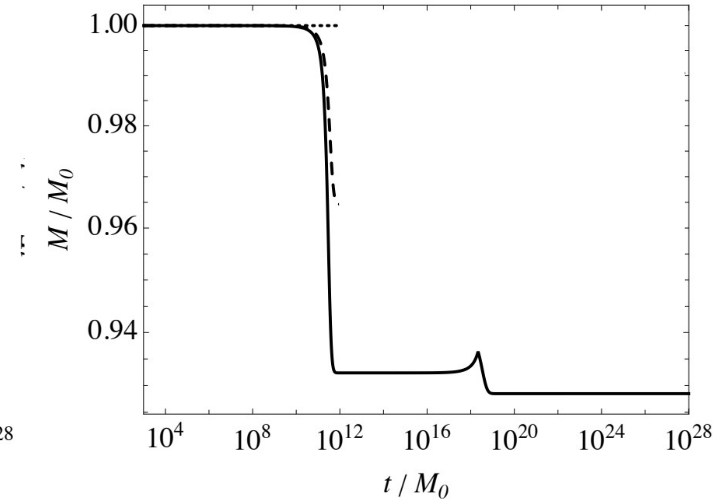
Energy and angular mom. radiated by GW

Isolated BH-condensate Evolution

- Keep four $(n, l, m = l)$ modes: $(0,1,1), (1,1,1), (0,2,2), (1,2,2)$
- The superradiant cloud **changes BH spin** and **emits GW**.

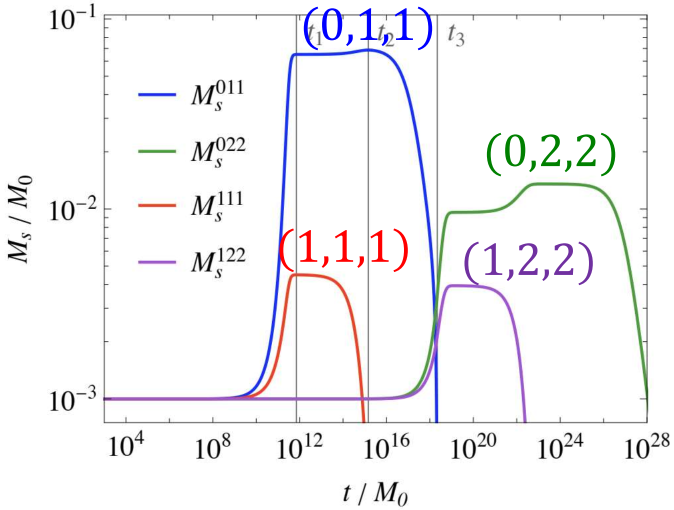
BH angular momentum





BH mass

Condensate mass



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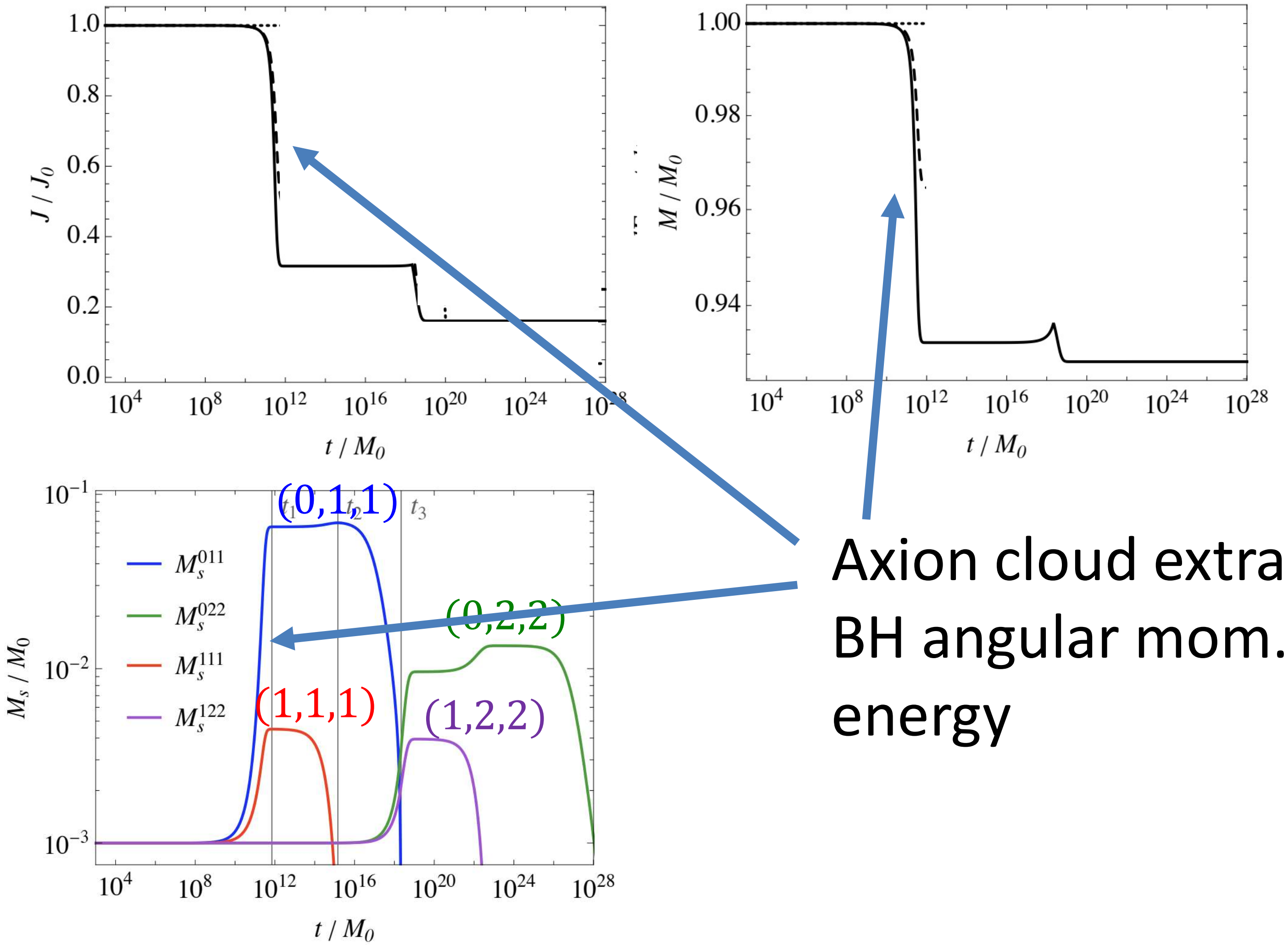

Axion cloud extract BH angular mom. and energy

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Isolated BH-condensate Evolution

- Keep four $(n, l, m = l)$ modes: $(0,1,1), (1,1,1), (0,2,2), (1,2,2)$
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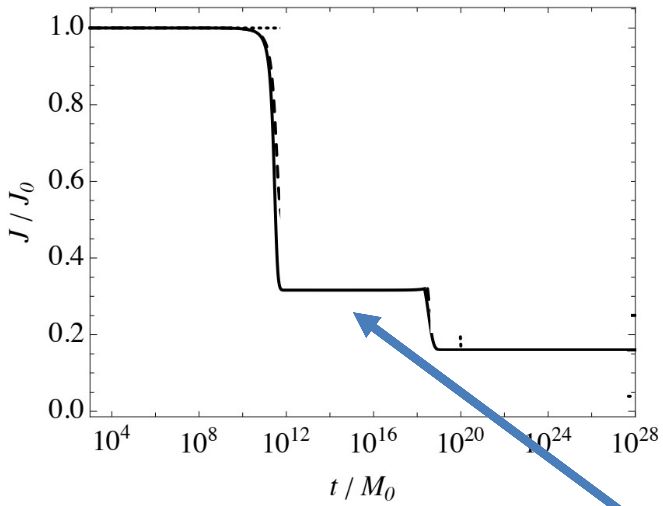
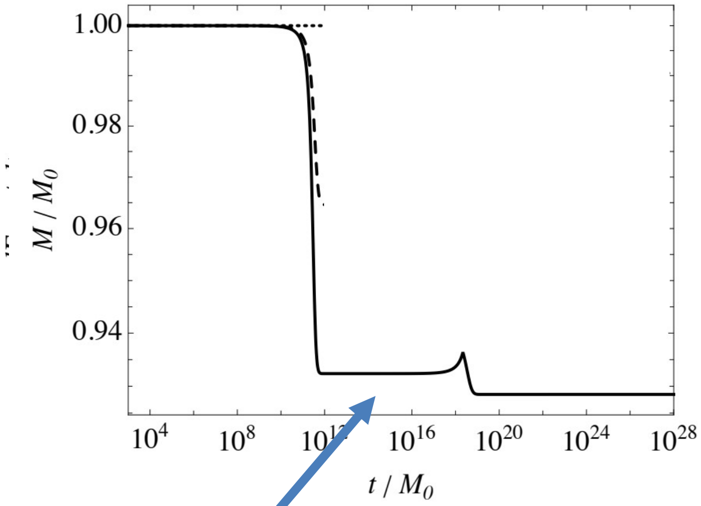
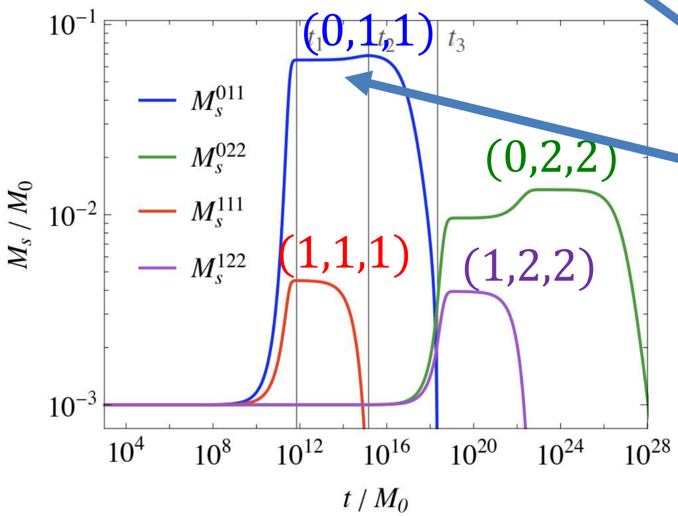
BH angular momentum





BH mass

Condensate mass



In the presence of the cloud, the BH angular mom. and mass are nearly constants

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Isolated BH-condensate Evolution

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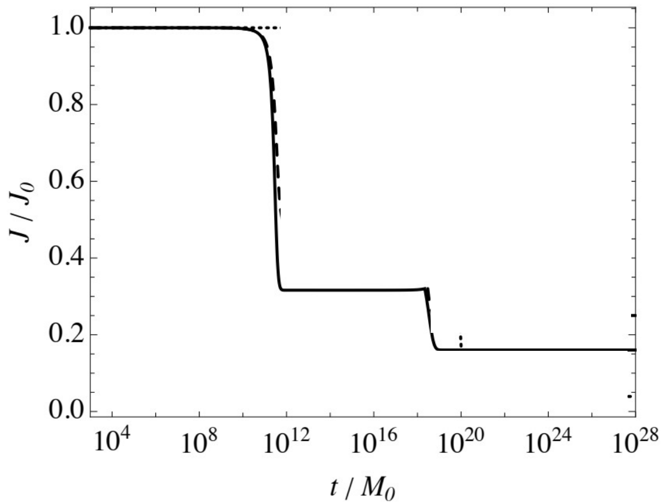
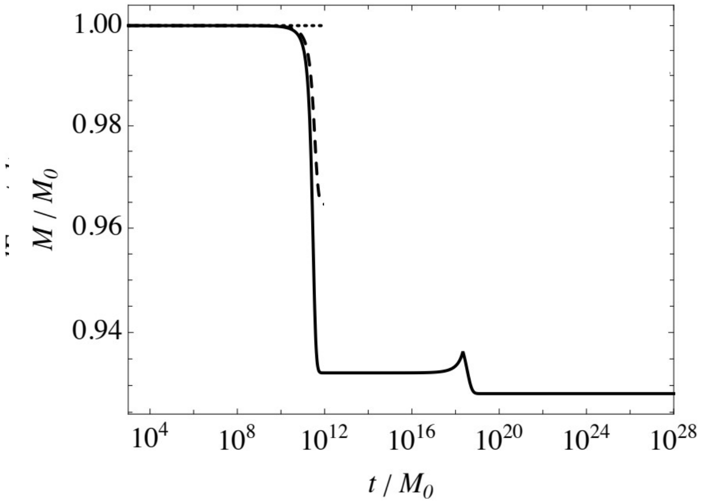
BH angular momentum





BH mass

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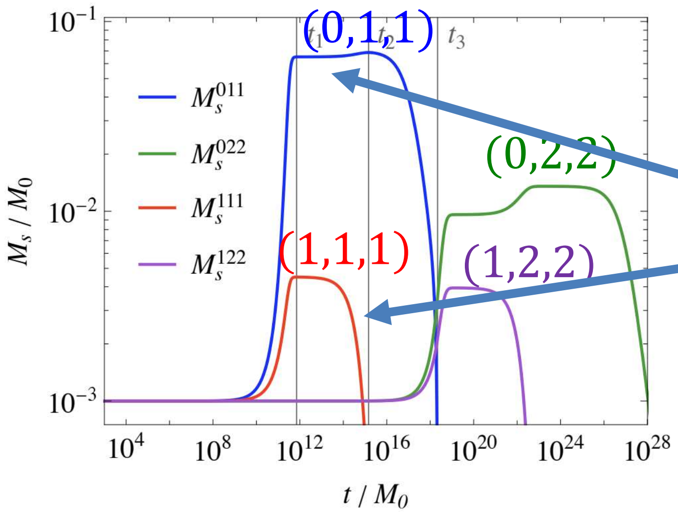

$(0, l, l)$ and $(1, l, l)$ coexist for a long time.

$N_{111} \sim \frac{1}{10} \mathrm{N}_{011}$

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  - BH distribution
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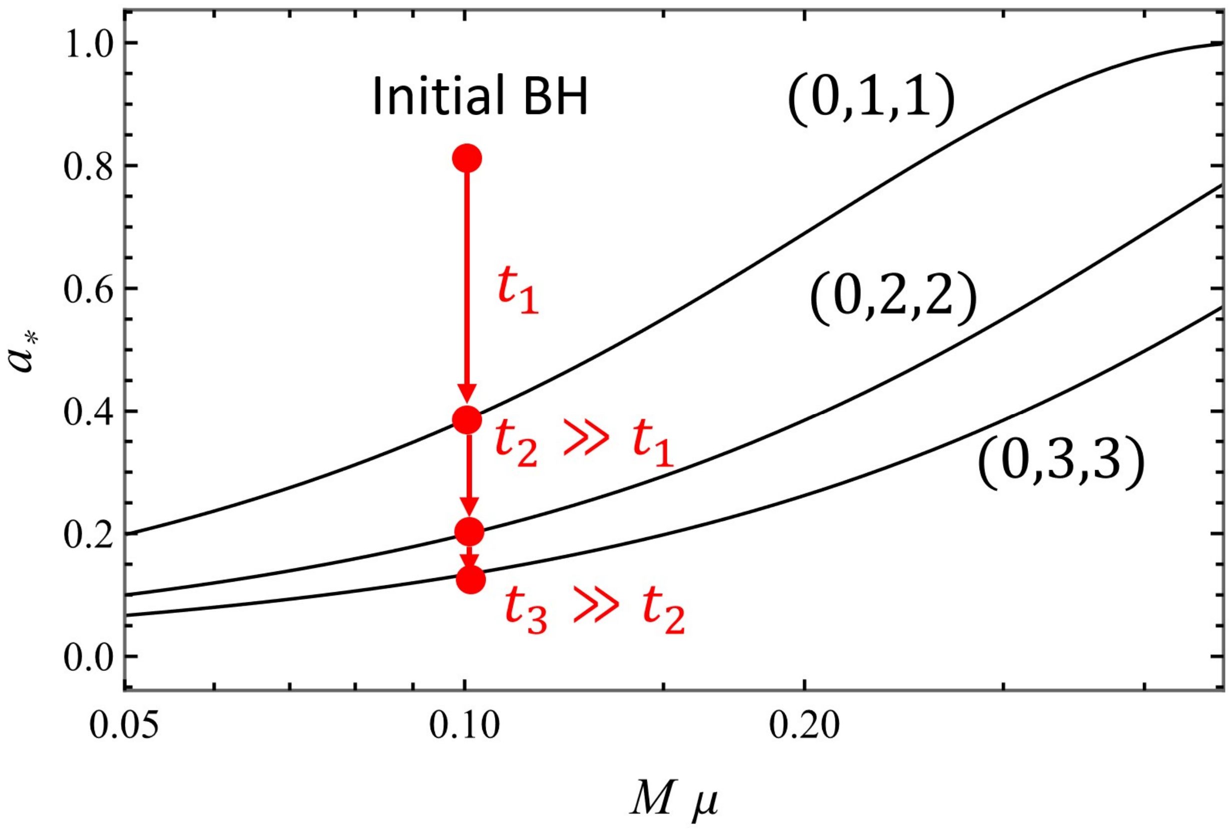
Evolution on the Regge Plot

- With superradiance, the **BH spin drops a lot (~50%)** and the BH mass decreases slightly (a few percent)
- On Regge plot, the BH steps down on each Regge trajectory, which is determined by the superradiance condition



$$a_*^{(nlm)} \approx \frac{4m\,M\mu}{m^2 + 4(M\mu)^2}$$

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High-spin BHs are Rare

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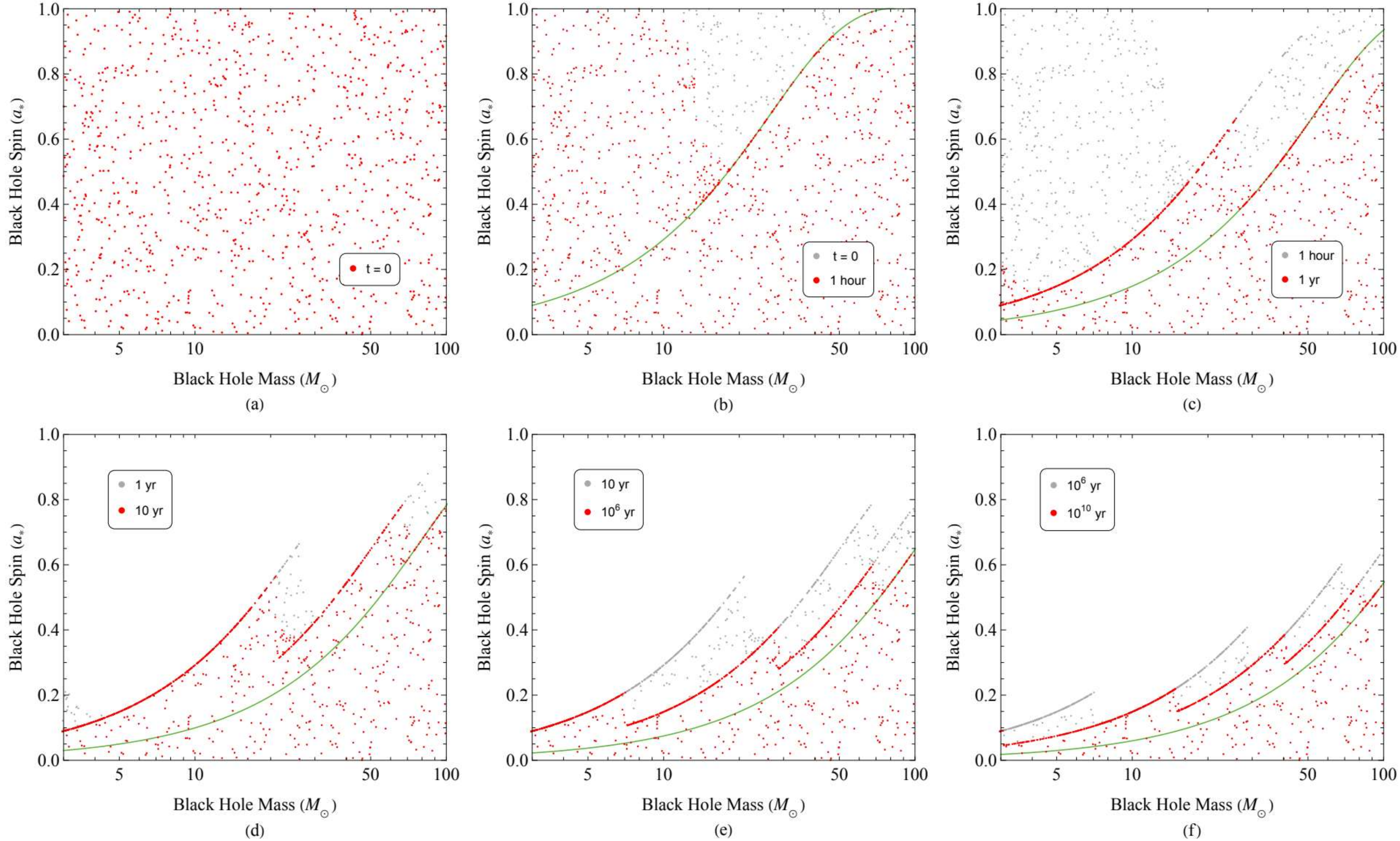
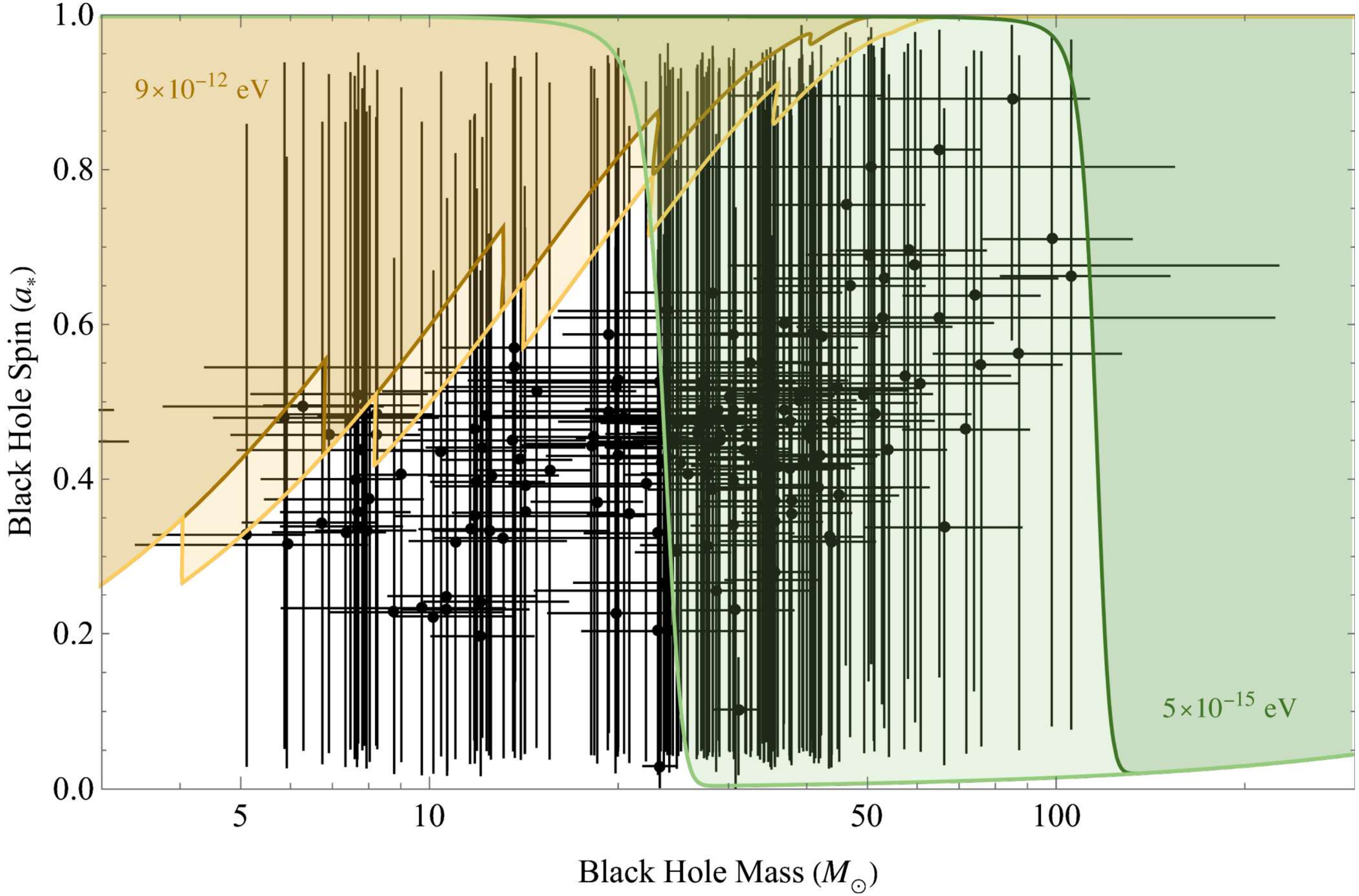


- Angular momentum transfers from BH to the cloud.
- BHs prefer to reside on Regge trajectories.

BH Spin Measurement

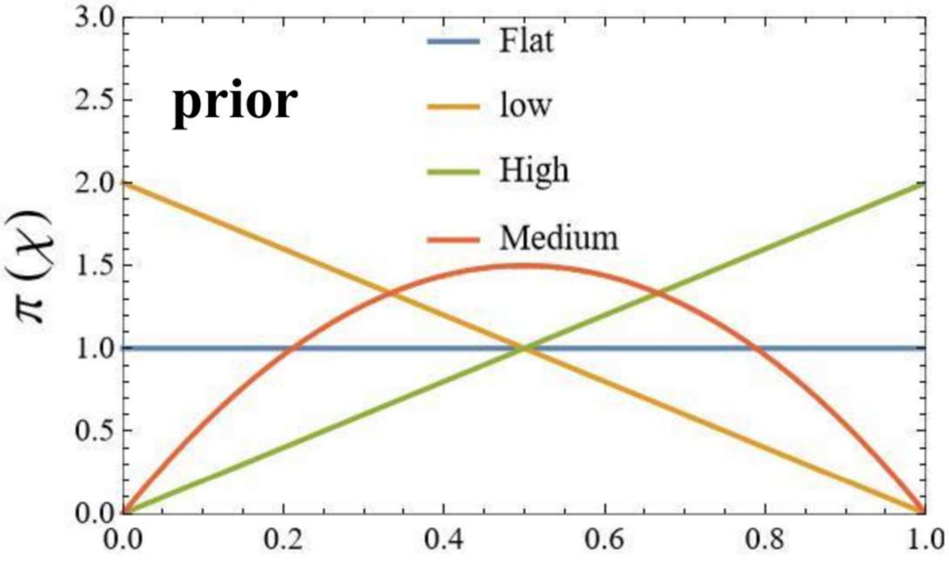
- LVK can measure the individual BH spin, with huge error.
- The error can be reduced to ~30% in the future

LIGO, Virgo, PRX 2016

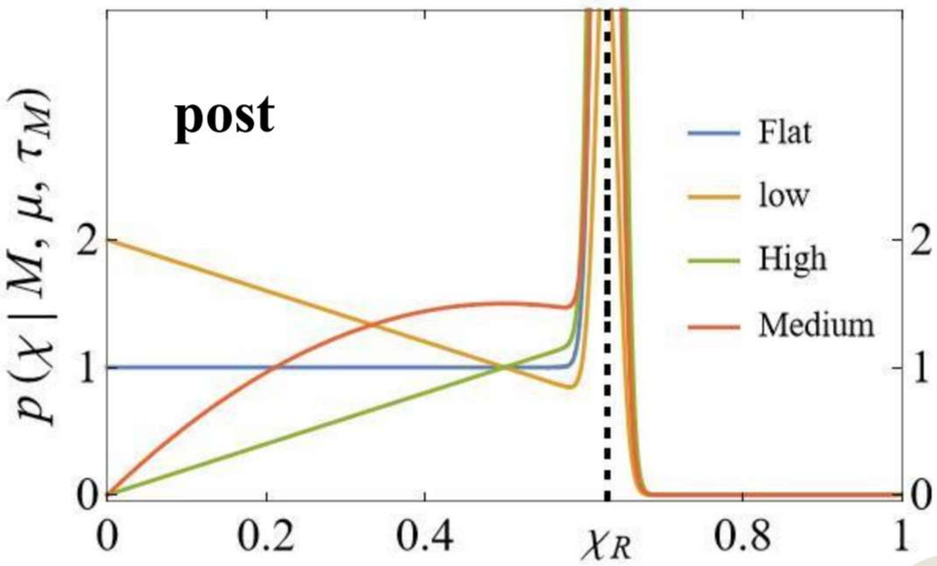


Survive with the Current Data

- Consider 3 scenarios: **high**, **flat**, **low** to estimate the effect of the initial BH spin.



BH spin a



BH spin a

- Assume the Lifetime of BHs distributes log-uniformly between 10^6 to 10^{10} years

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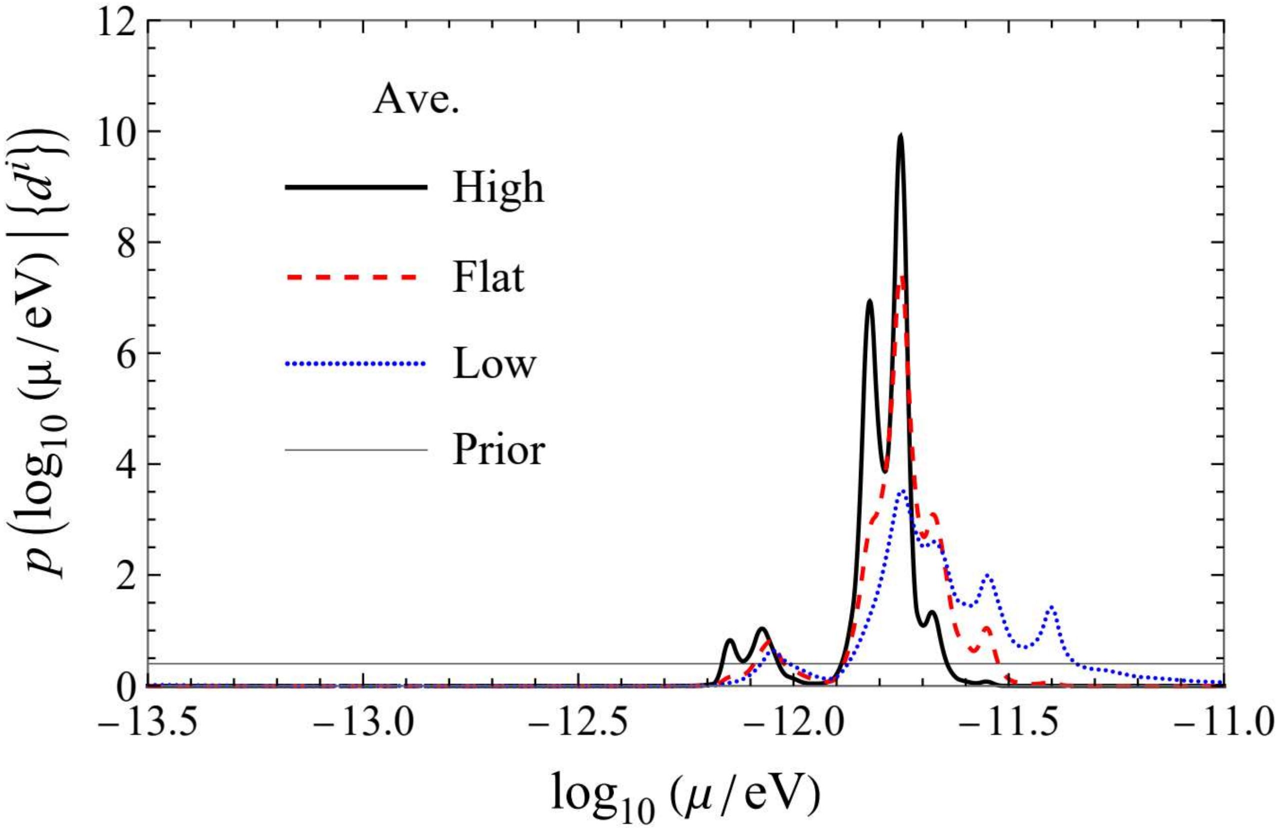
Constrain Scalar Mass

- **Include all BBHs in three phases of GTWC data** reported by LVK collaboration, only excluding the events with neutron.
- **scalar mass prior** is log-uniform between $10^{-13.5}$ to 10^{-11} eV.



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- Two slightly favored ranges are identified, but evidence is weak.

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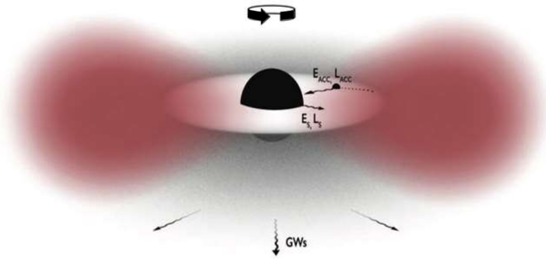
GW Emission from Real Fields

- Previous calculation only consider the $(n = 0, l = 1, m = 1)$ mode



Monochromatic, constant energy flux,
Cannot distinguish from neutron stars

- Different modes have slightly different angular speeds

$$\phi(t, \vec{r}) = \sum_{l,m} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{lm}(r) S_{lm}(\theta) + \text{c.c.} \right]$$

$$\omega_R^{nlm} \approx \mu \left[1 - \frac{\alpha^2}{2(n+l+1)^2} \right] + O(\alpha^4)$$

e.g.

$$\cos[(\omega + \Delta\omega)t] + \cos[(\omega - \Delta\omega)t] = 2\cos(\Delta\omega\, t)\cos(\omega t)$$

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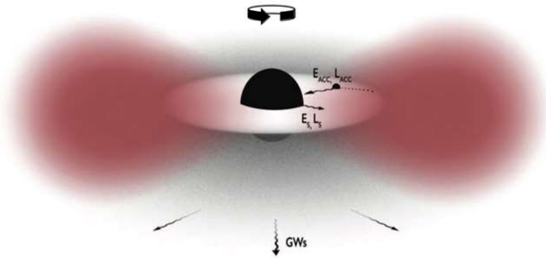

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Modulation of amp. and energy flux. **Beat!**

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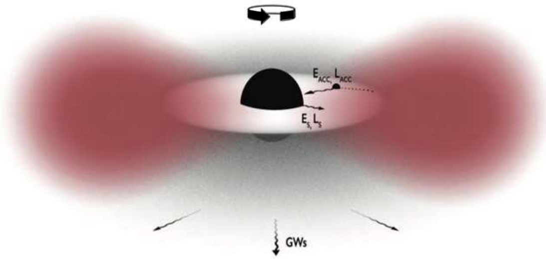

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e.g $\cos[(\omega+\Delta\omega)t] + \cos[(\omega-\Delta\omega)t] = 2\cos(\Delta\omega\, t)\cos(\omega t)$

- Strength of the beat signal.

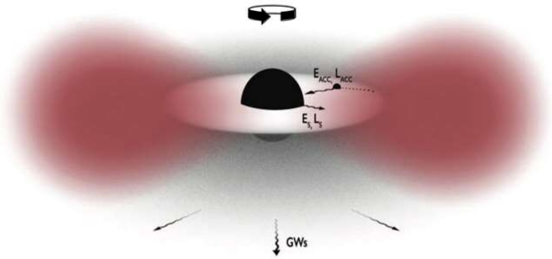
Two (0,1,1) axions ⟹ graviton: Amp.$\propto N_{011}$, freq. $= 2\omega^{011}$

(0,1,1) + (1,1,1) ⟹ graviton: Amp.$\propto \sqrt{N_{011}N_{111}}$, freq. $= \omega^{011} + \omega^{111}$

Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}$

GW Emission from Real Fields

- Previous calculation only consic



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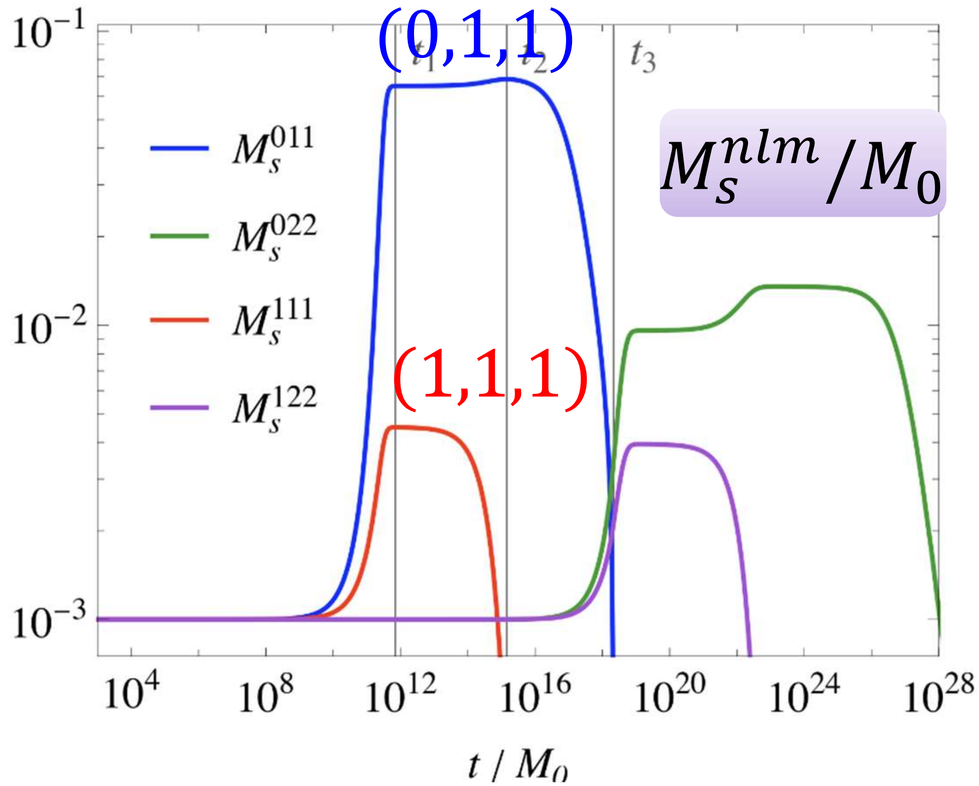

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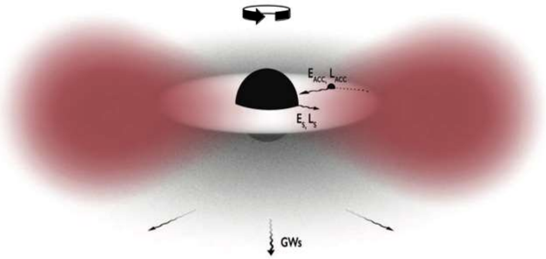

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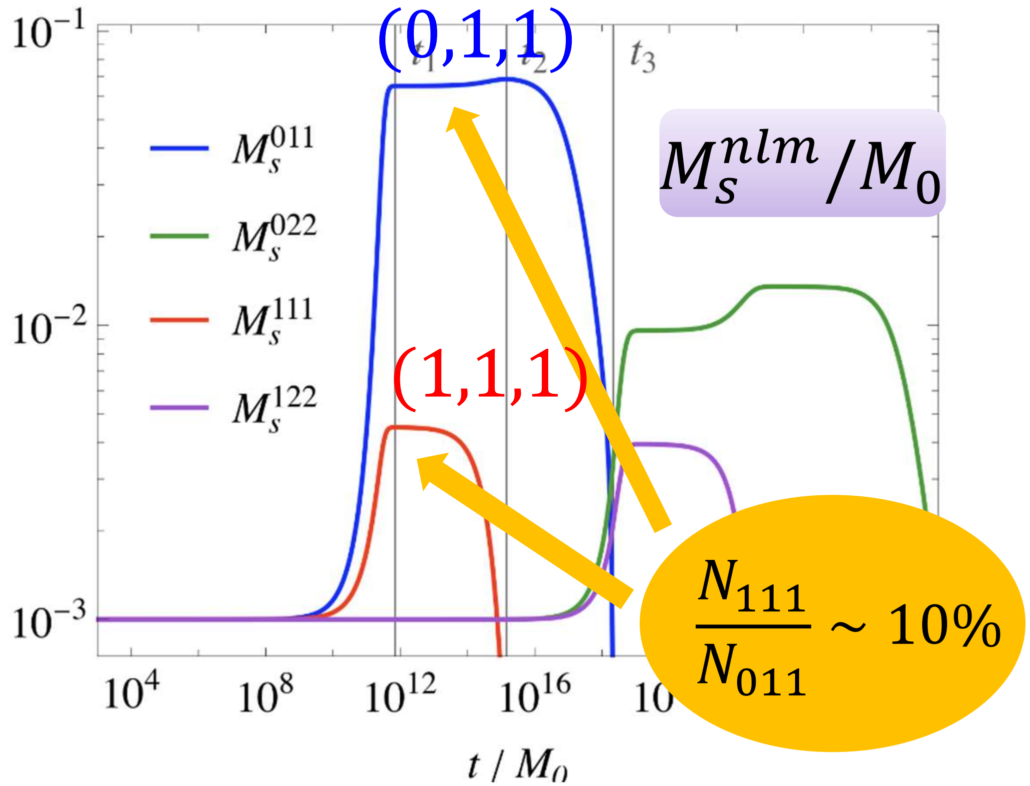

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Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}$

GW Emission

- Use Teukolsky formalism to calculate the beat signal

LO

NLO

$\sqrt{N_{111}/N_{011}}$

suppressed

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\Bigg\{ \boxed{\frac{N_{011}^2}{\omega^{(011)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_1)}\right|^2}{\tilde{\omega}_1^2}} + \frac{N_{111}^2}{\omega^{(111)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_2)}\right|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde{\omega}_3}\right|^2}{\tilde{\omega}_3^2}$$

$$\boxed{+4\sqrt{\frac{N_{011}^3 N_{111}}{\omega^{(011)^3}\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_3)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right]}$$

$$+2\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|}{\tilde{\omega}_1\tilde{\omega}_2}\cdot\cos\left[2\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_2)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right]$$

$$+4\sqrt{\frac{N_{011}N_{111}^3}{\omega^{(011)}\omega^{(111)^3}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_2\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_2)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\right]\Bigg\}.$$

$$\tilde{\omega}_1 \equiv 2\omega^{(011)},\ \tilde{\omega}_2 \equiv 2\omega^{(111)},$$
$$\tilde{\omega}_3 \equiv \omega^{(011)}+\omega^{(111)},\ \tilde{\omega}_4 \equiv \omega^{(111)}-\omega^{(011)}$$

Guo, Bao and **HZ,** PRD (2023)

GW Emission

- Use Teukolsky formalism to calculate the beat signal

LO

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\left\{ \boxed{\frac{N_{011}^2}{\omega^{(011)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_1)}\right|^2}{\tilde{\omega}_1^2}} + \frac{N_{111}^2}{\omega^{(111)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_2)}\right|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde{\omega}_3}\right|^2}{\tilde{\omega}_3^2}\right.$$

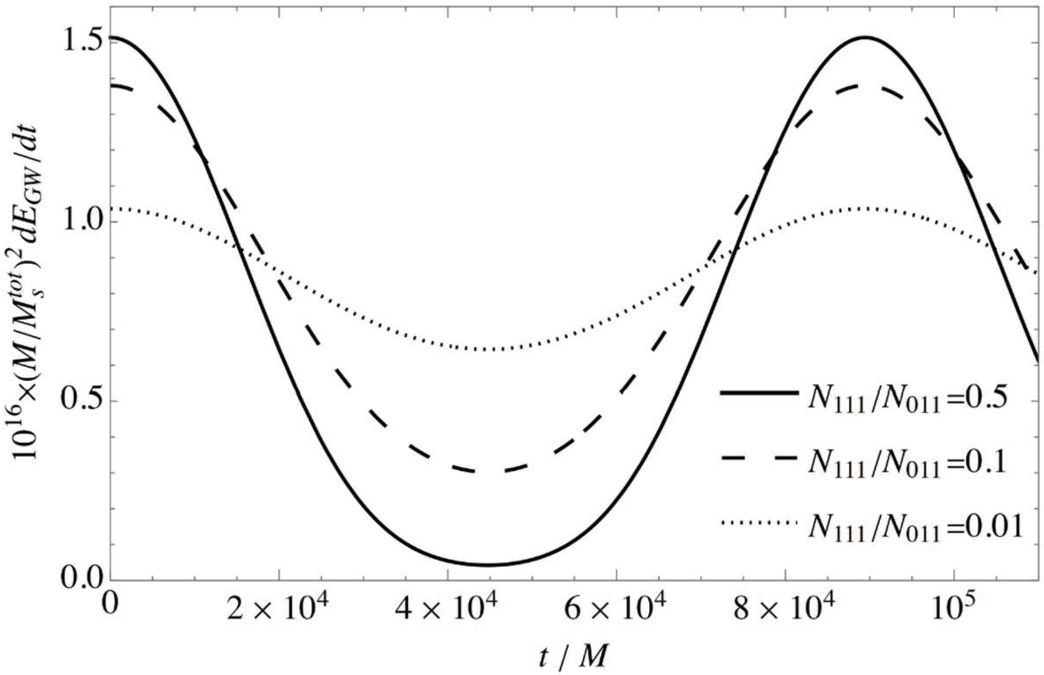
NLO

$\sqrt{N_{111}/N_{011}}$ suppressed

$$\boxed{+4\sqrt{\frac{N_{011}^3 N_{111}}{\omega^{(011)^3}\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_3)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right]}$$

$+2 \ \ldots \ \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\Big]$

$+4 \ \ldots \ + \phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\Big]\Big\}.$

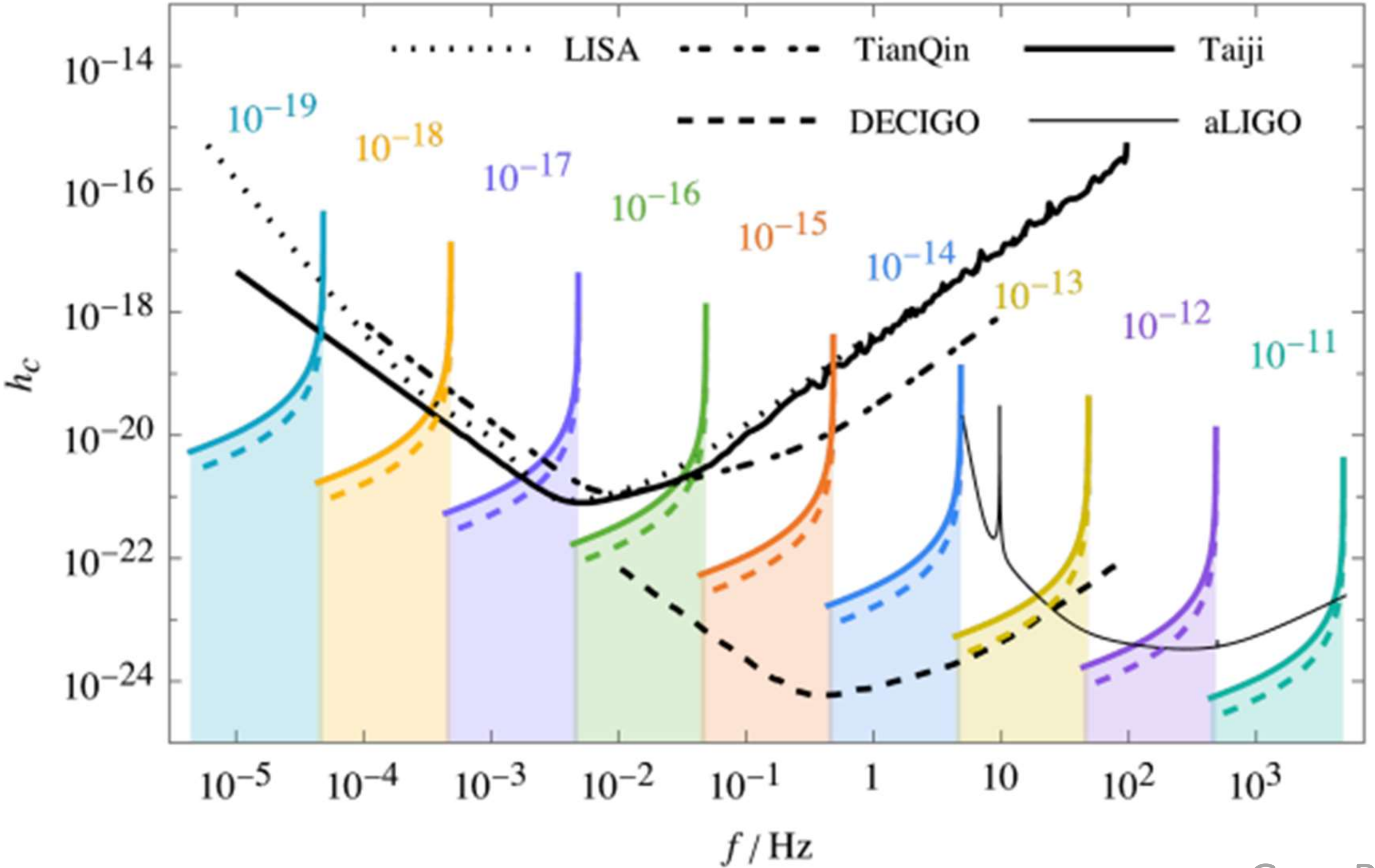


)11)

'RD (2023)

GW Beat: Observation

- The current and future GW telescope can cover a large range of scalar mass.



Parameters:
$M\mu = 0.17$
$M_s/M = 0.1,$
$N_{111}/N_{011} = 0.1$

z changes from 10^{-3} to 10

Guo, Bao and **HZ,** PRD (2023)

Summary

- If axion (ALP) de Brogile wave length $\sim$ BH horizon size, the axion field could grow exponentially around Kerr BHs.
- We improve the widely-used analytic expression for superradiance rate, reducing error from 150% to $\leq 5\%$.
- BH Superradiance provides a model-independent approach to search for axions.
- The axion condensate greatly modifies the BH spin distribution, which could be used to constrain axion mass.
- The GW emitted by superradiant axion condensate has a unique beat signature.

Summary

- If axion (ALP) de Brogile wave length ~ BH horizon size, the axion field could grow exponentially around Kerr BHs.
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Thank you!

5月9-12日

山东青岛

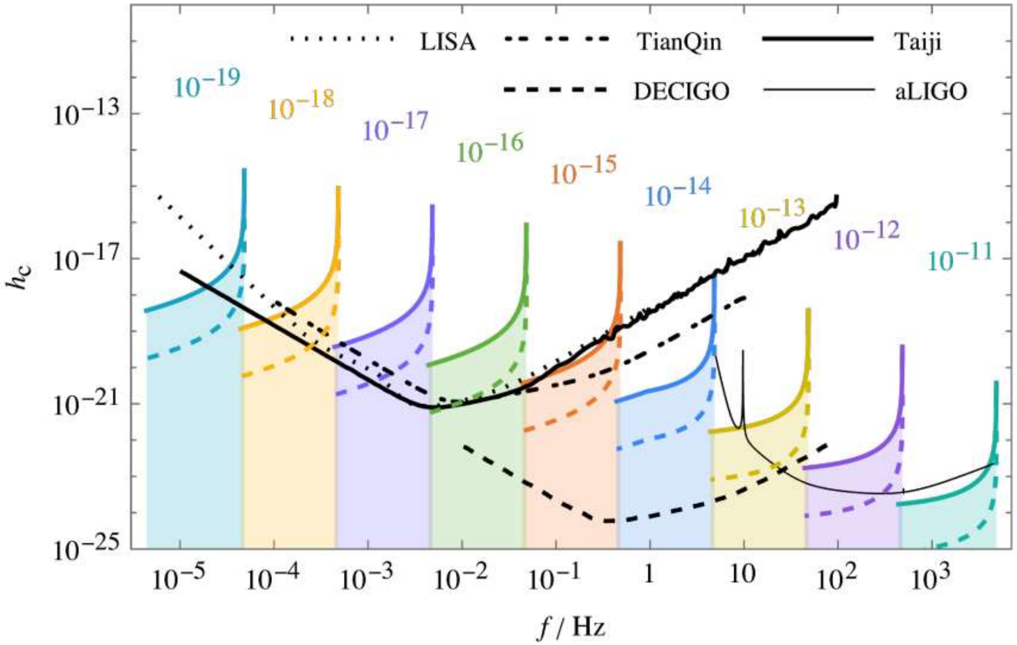
诚邀各位专家学者
前来参会！

报名截止时间：
4月25日

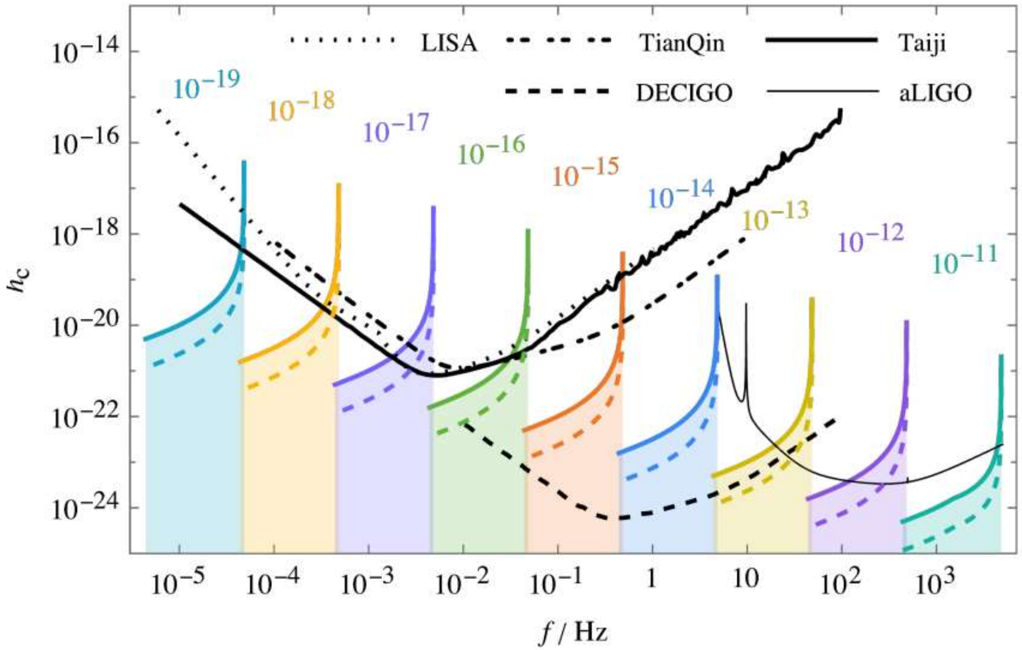
Fields with Other Spins

- Real vector

J=m=1 stage



J=m=2 stage



Guo, Jia, Bao, **HZ,** Zhang, PRD (2024)

- Real tensor field: unknown
- Complex field does not radiate GW. Then BHs reside on the first Regge trajectory for very long time.