# 早期宇宙真空拓扑缺陷与暗物质

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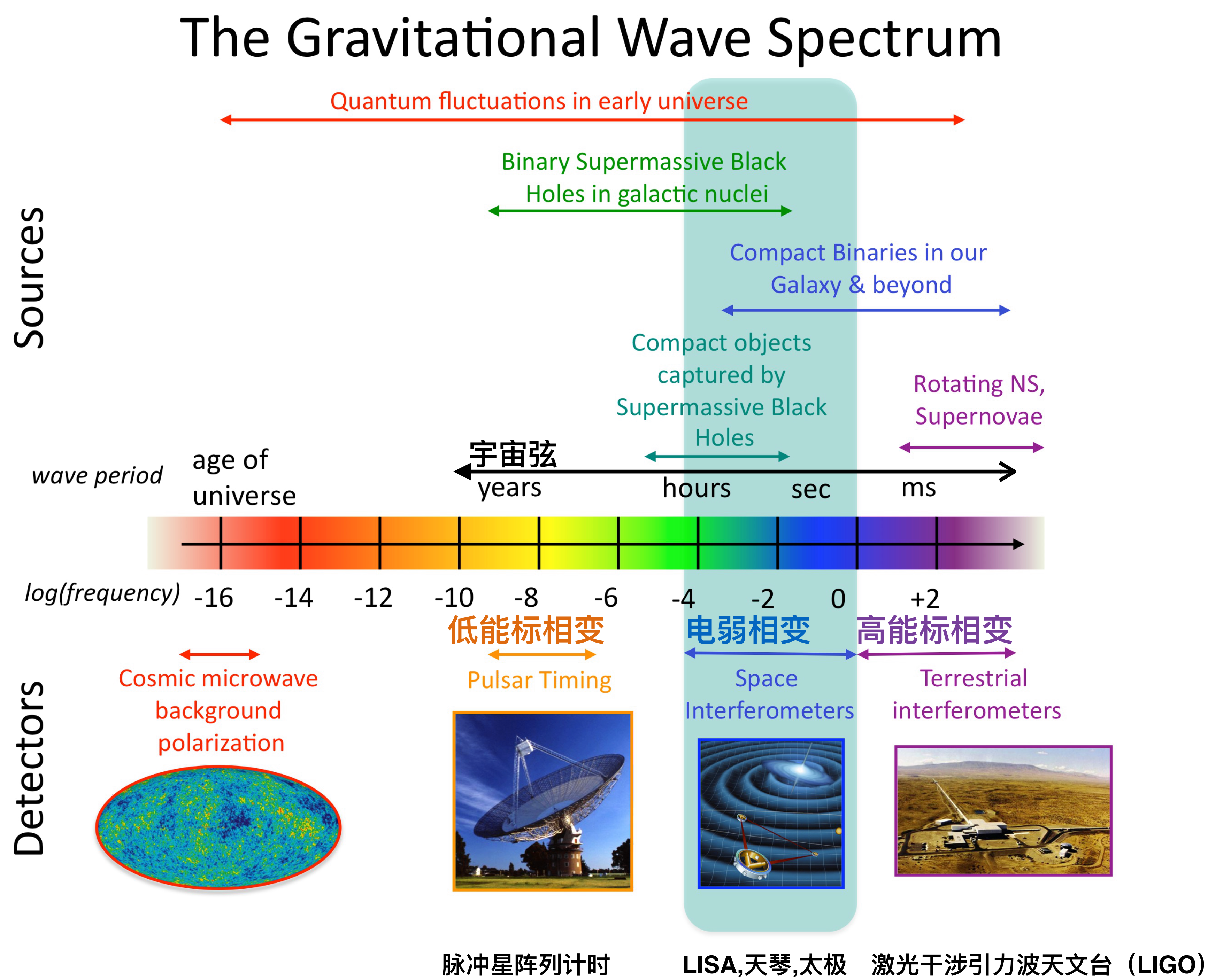
重庆大学

MEPA2025
2025/4/12@南京

# 报告内容

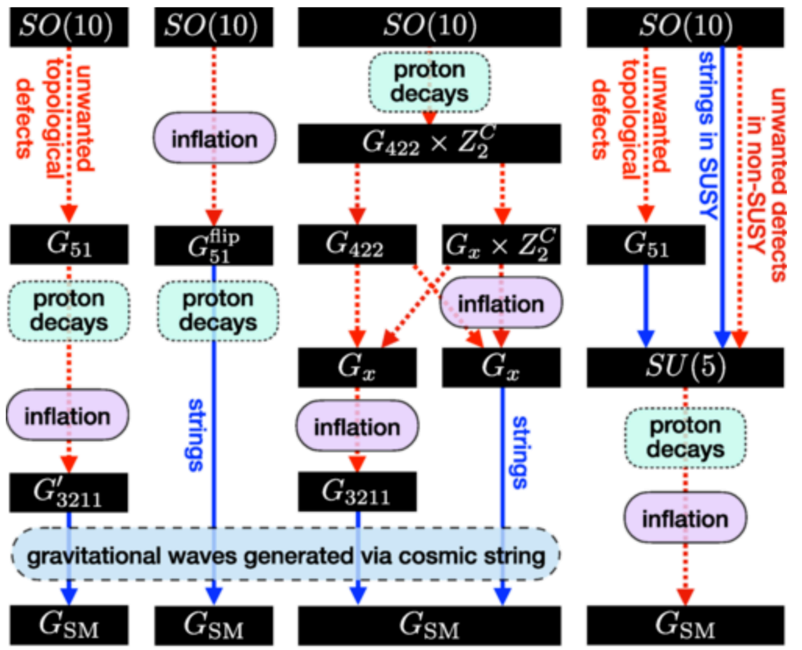
## 研究背景　随机引力波与基础物理

**随机引力波探测开启了探索早期宇宙背后基础物理的一个新的窗口**



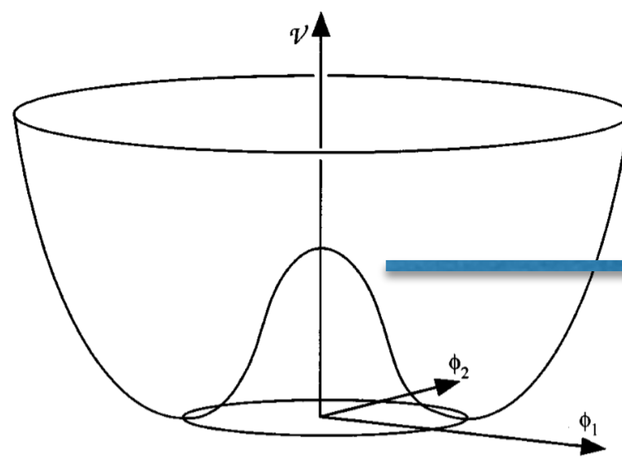
# ▸ 宇宙弦

**通常形成于GUTs**

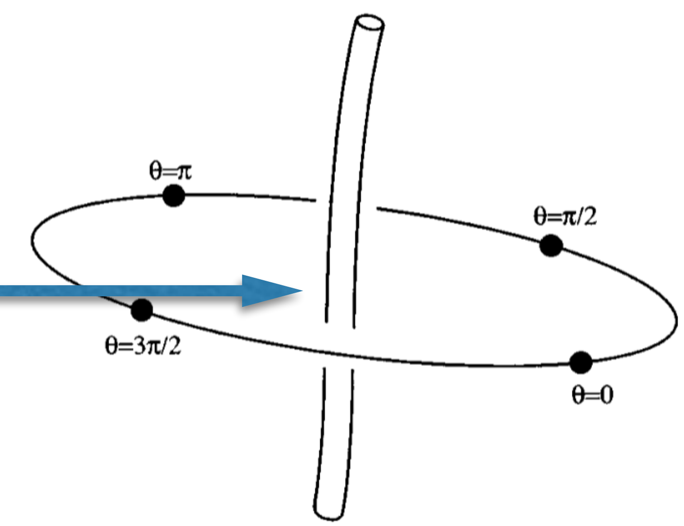



Stephen F. King, Silvia Pascoli, Jessica Turner, Ye-Ling Zhou, Phys. Rev. Lett. **126**, 021802

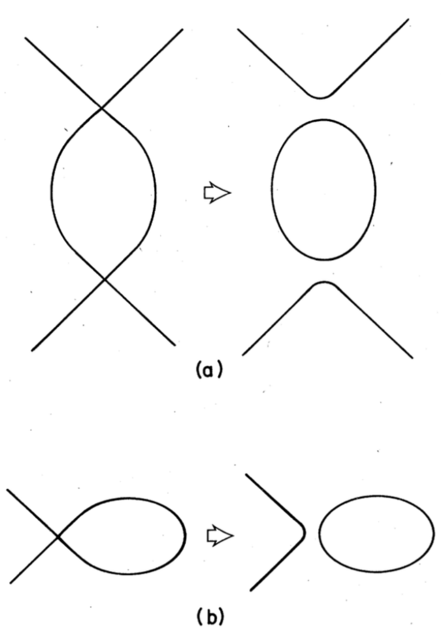
**相变后U(1) 对称性自发破缺**




**宇宙弦**



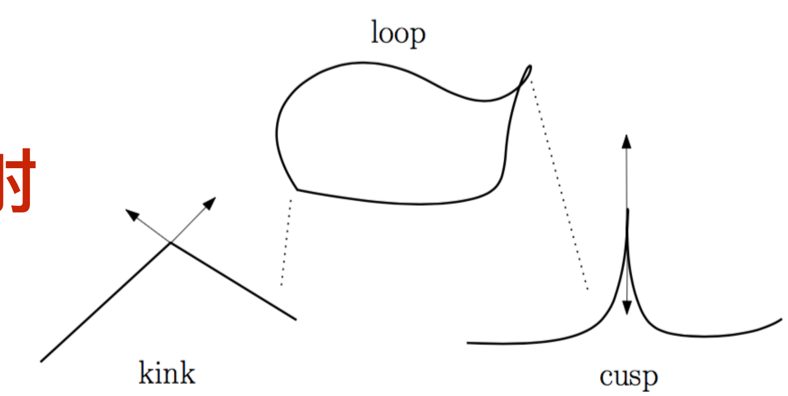

T. W. B. Kibble




Phys.Rev.D 30 (1984) 2036

**宇宙弦成圈**

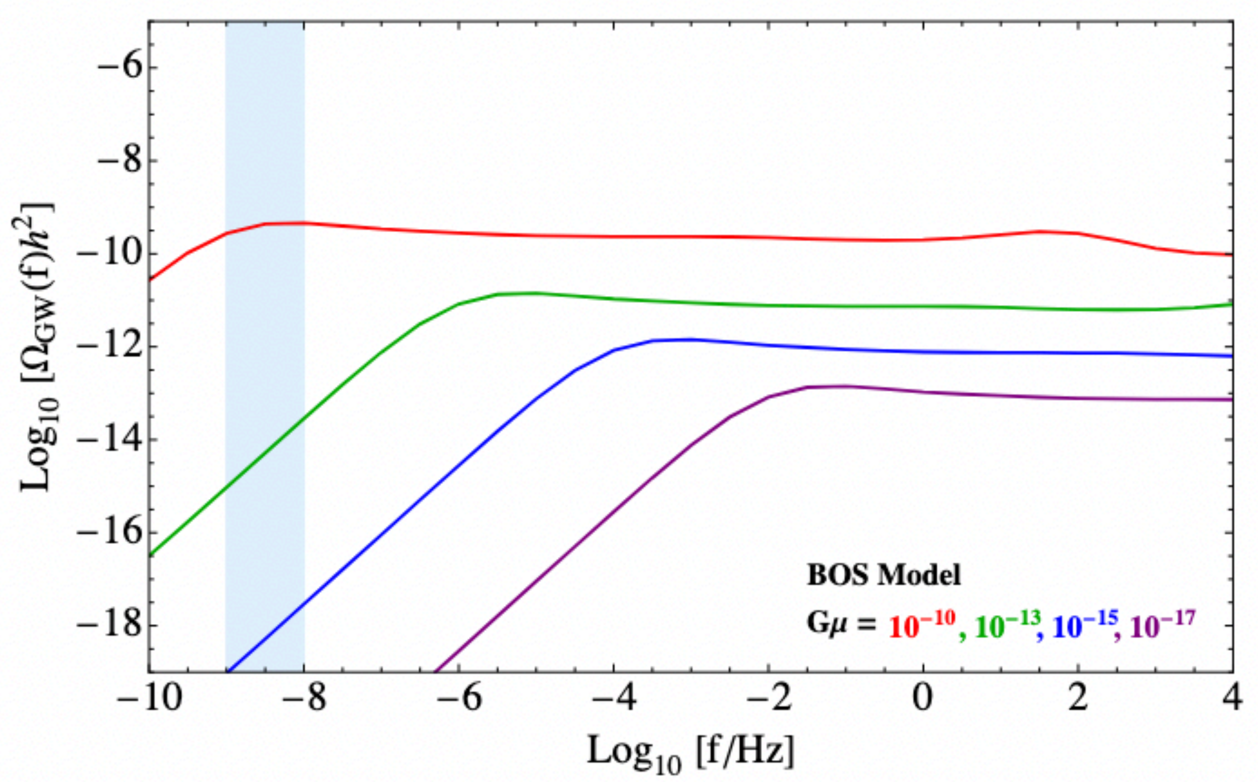
**引力波辐射**

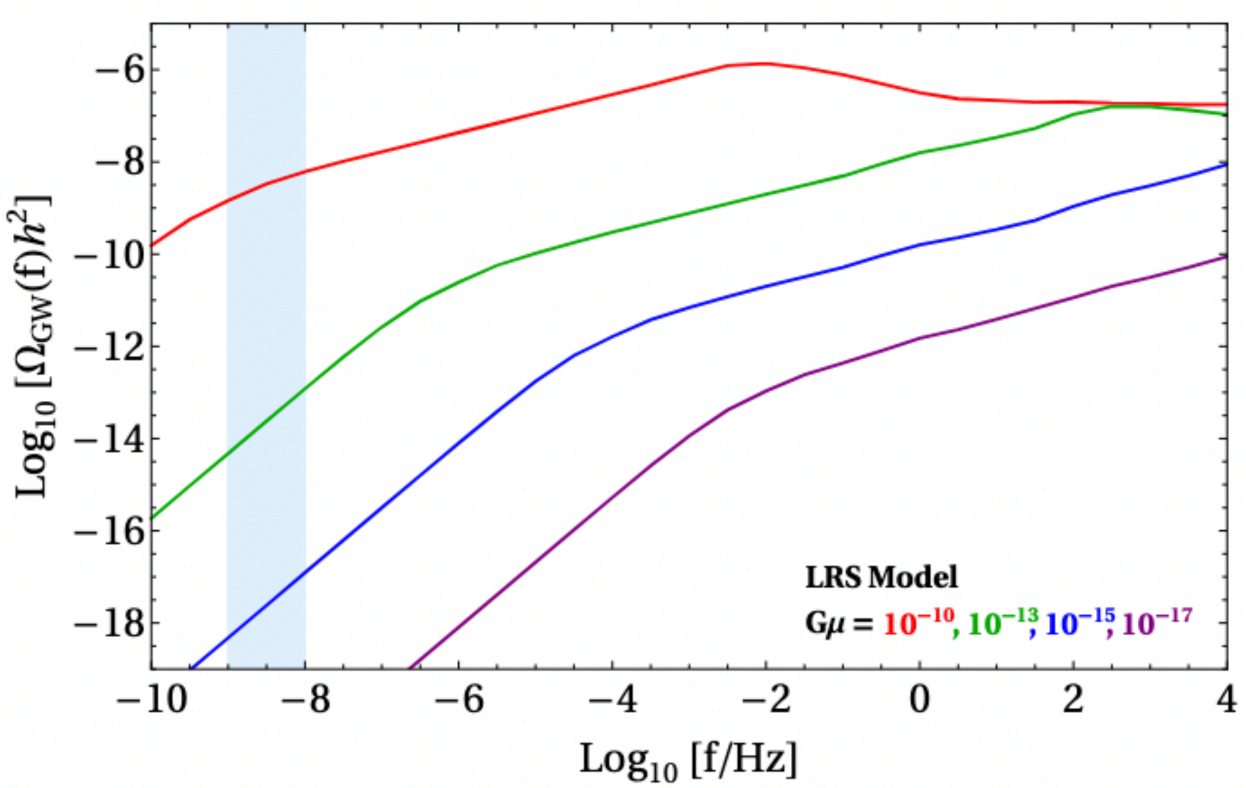



Yann Gouttenoire *et al* JCAP07(2020)032

# ▸ Nambu-Goto宇宙弦引力波

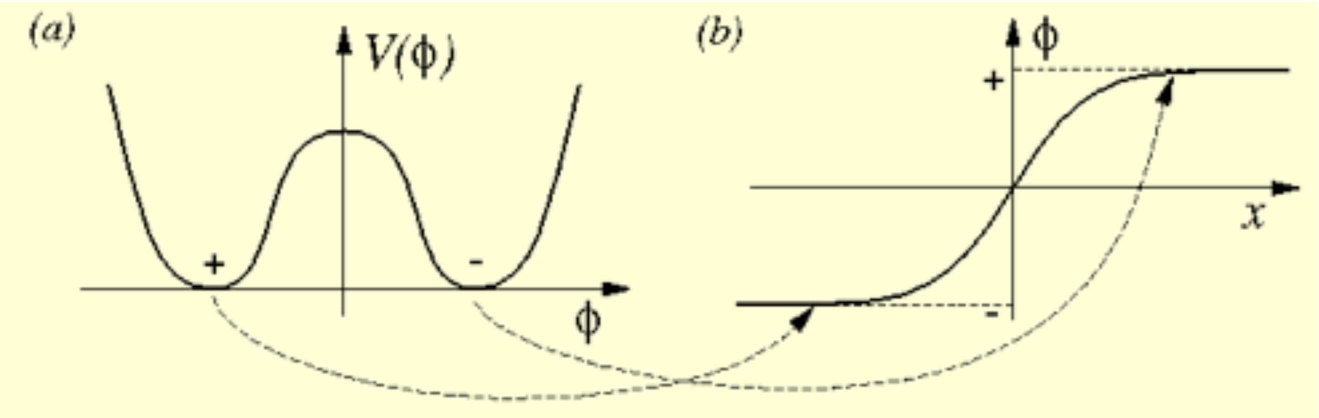
Nambu-Goto (NG) action is the leading-order approximation when the curvature scale of the strings is much larger than their width

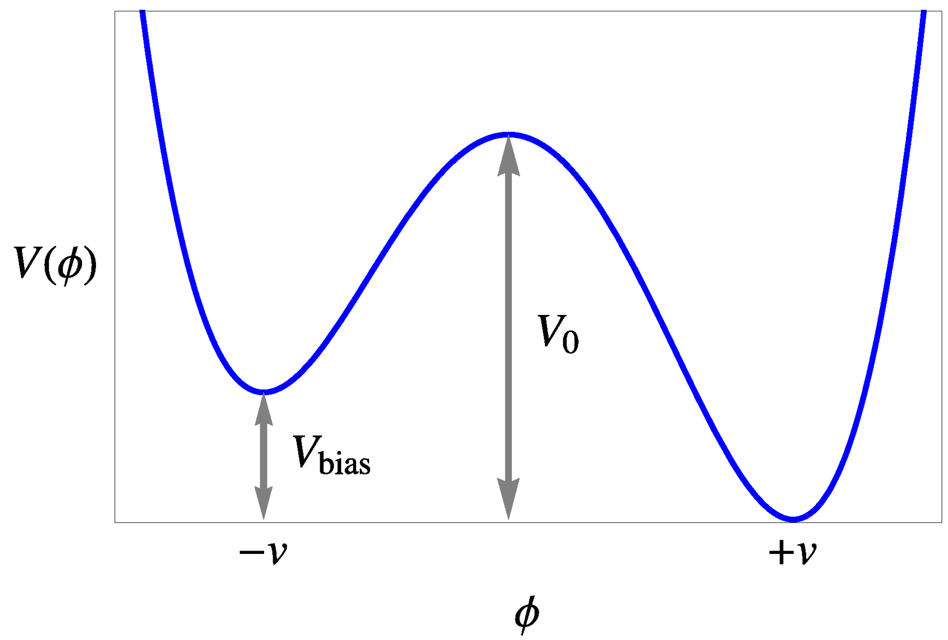


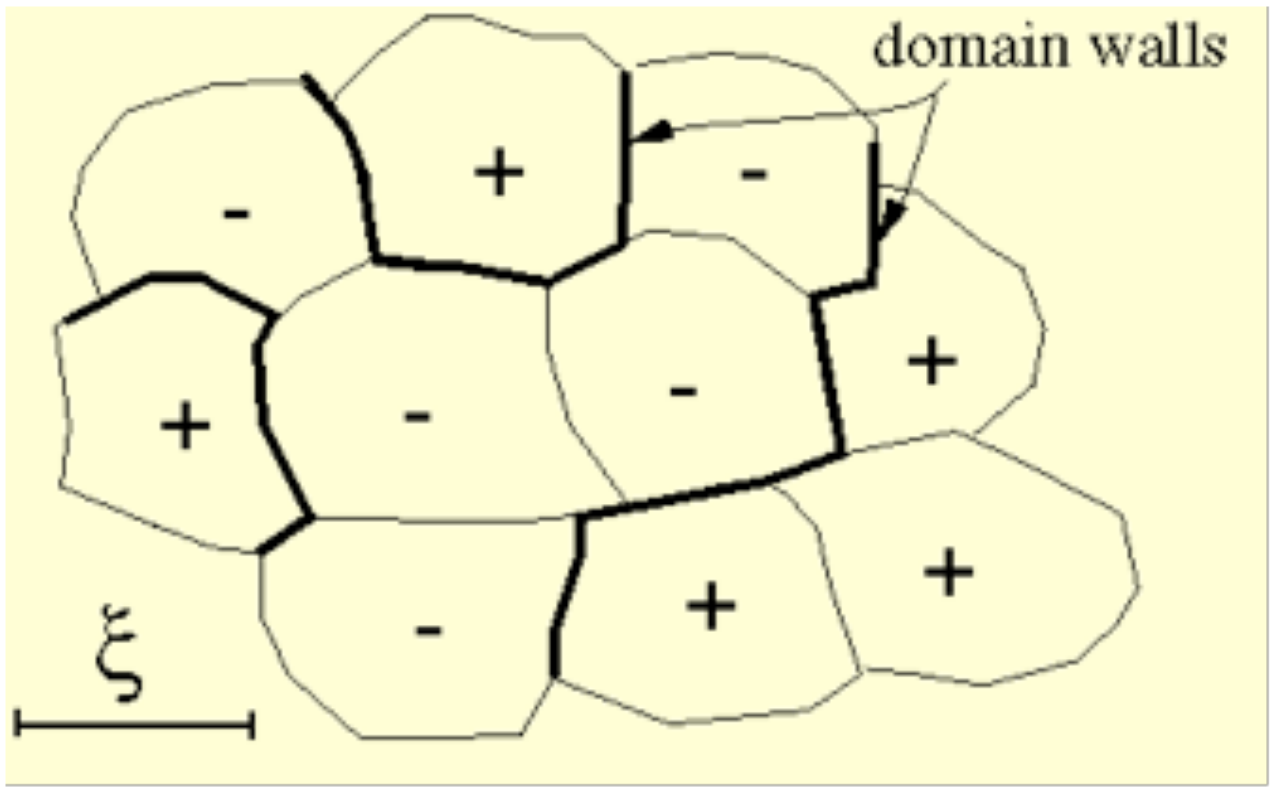




loop production function is numerically obtained

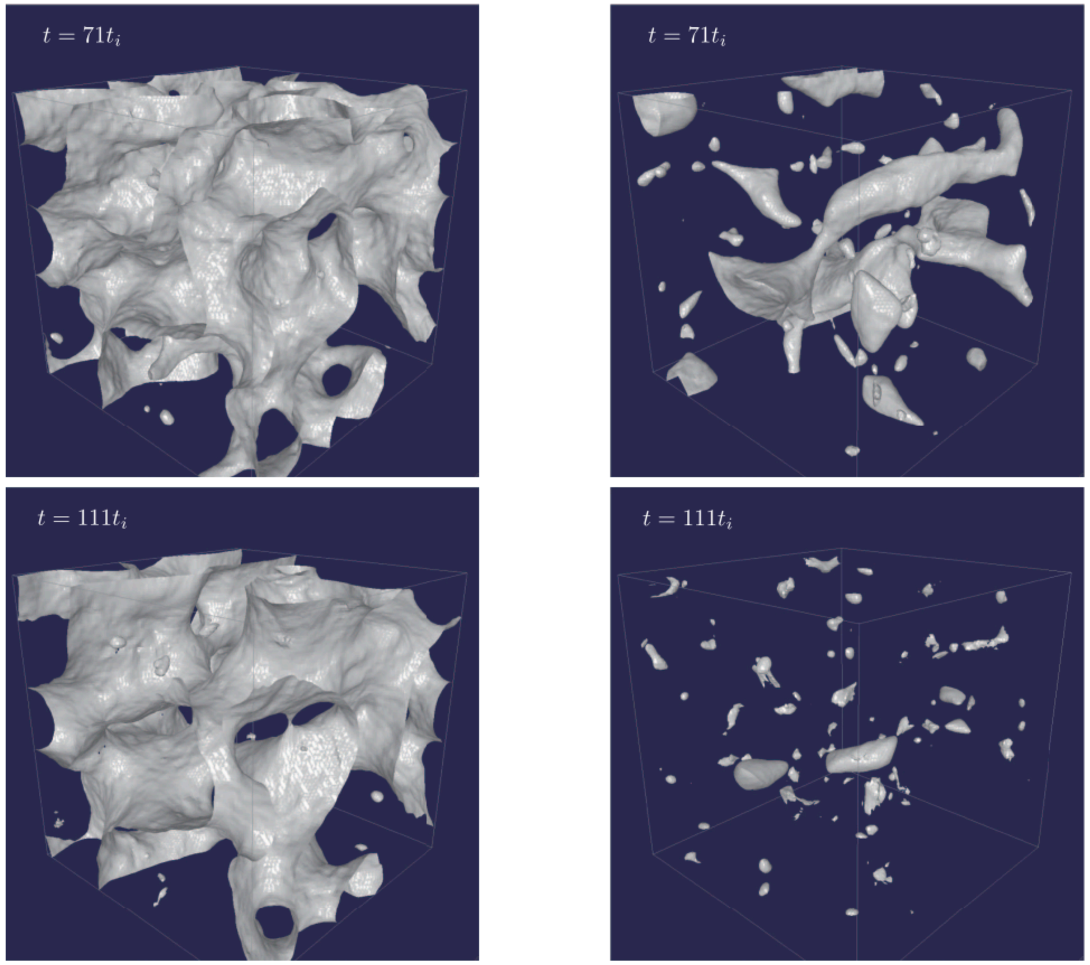
loop distribution of the scaling loops is directly extracted from numerical simulation



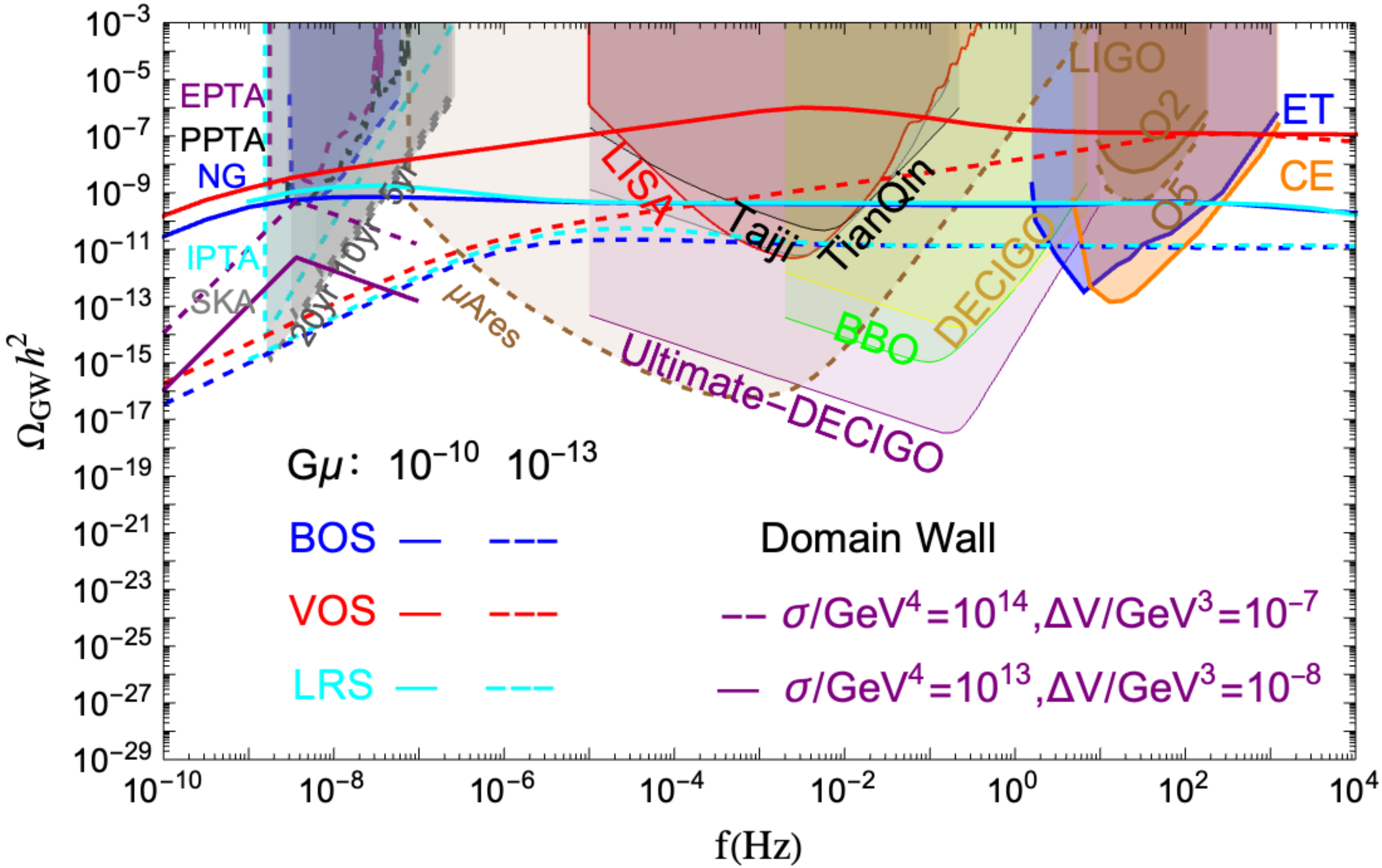




Kibble mechanism



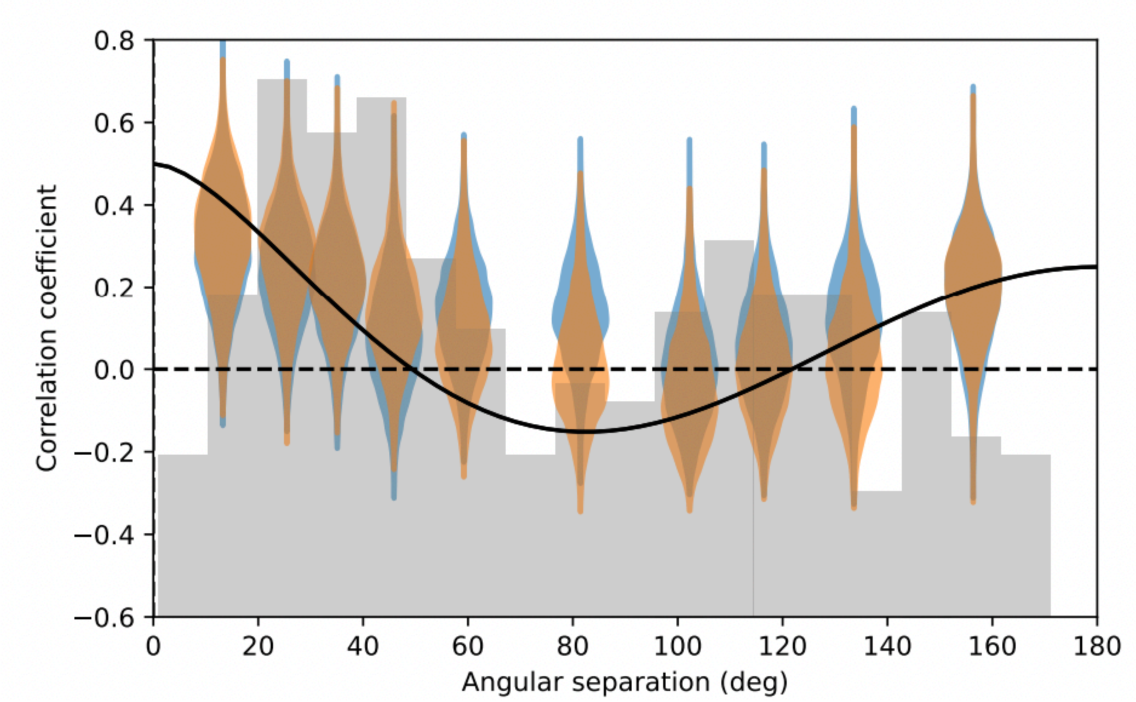
1002.1555

# 引力波源

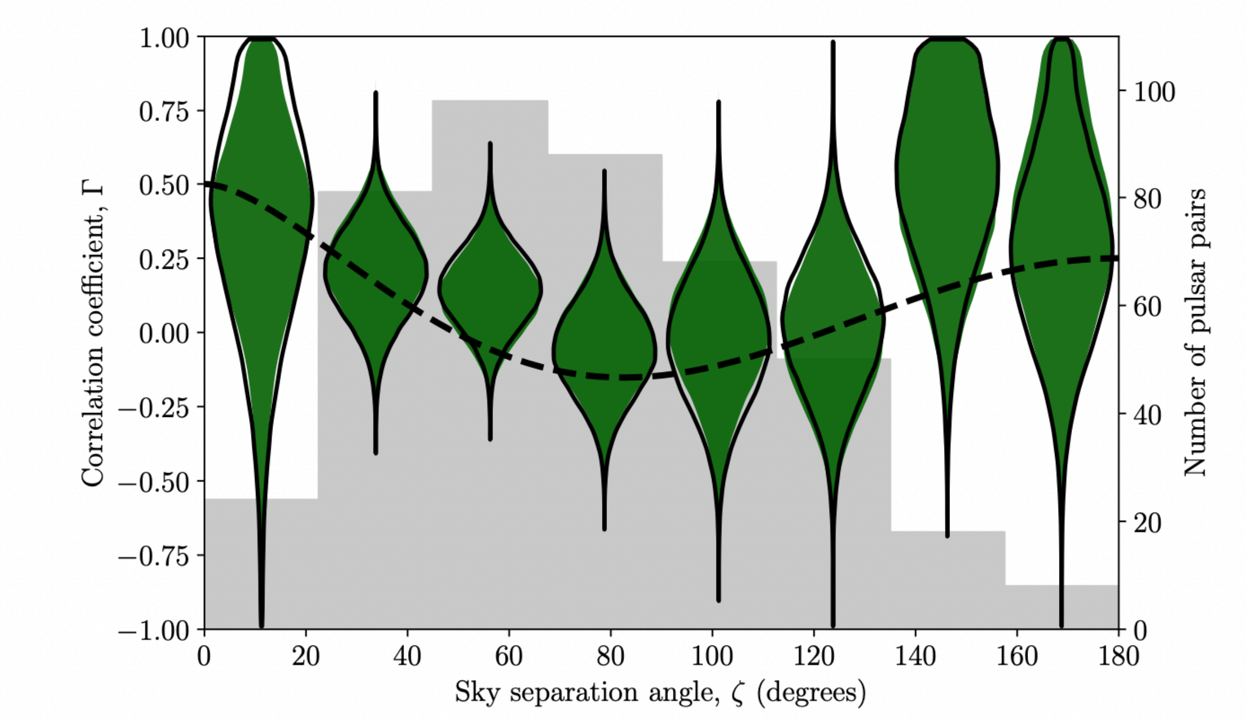



Bian,Pi,Wang, Sci.Sin.Phys.Mech.Astro. 55 (2025) 3, 230405

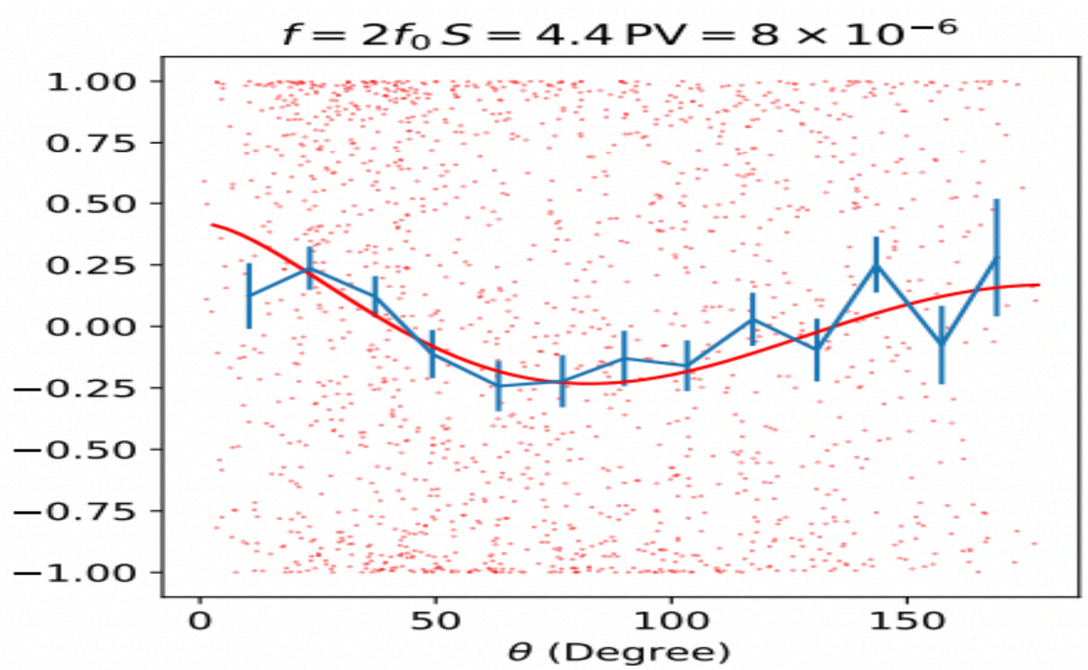
# New dataset from PTAs



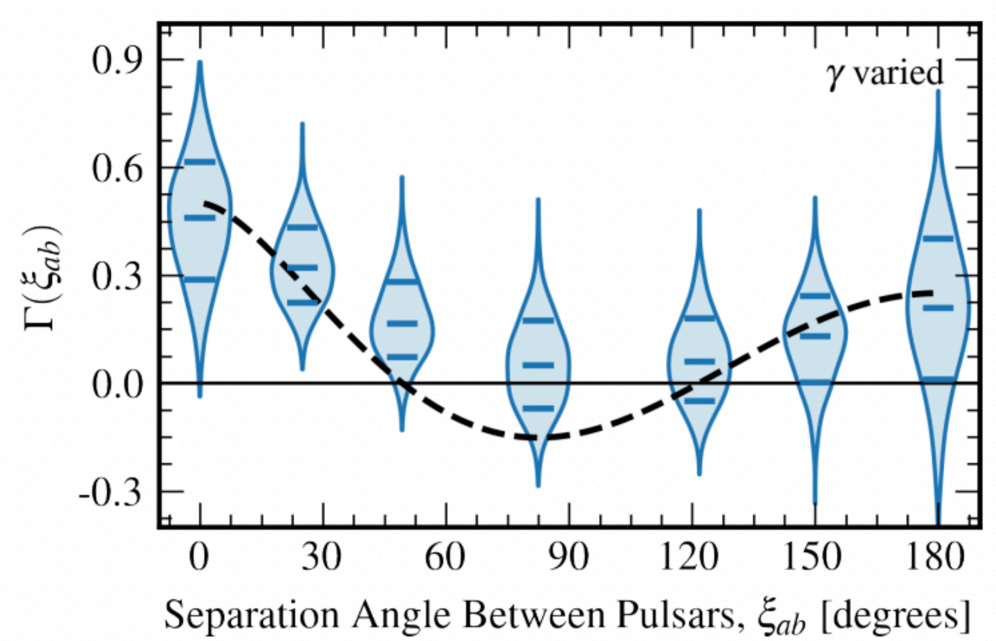

EPTA,2306.16214




PPTA,2306.16215

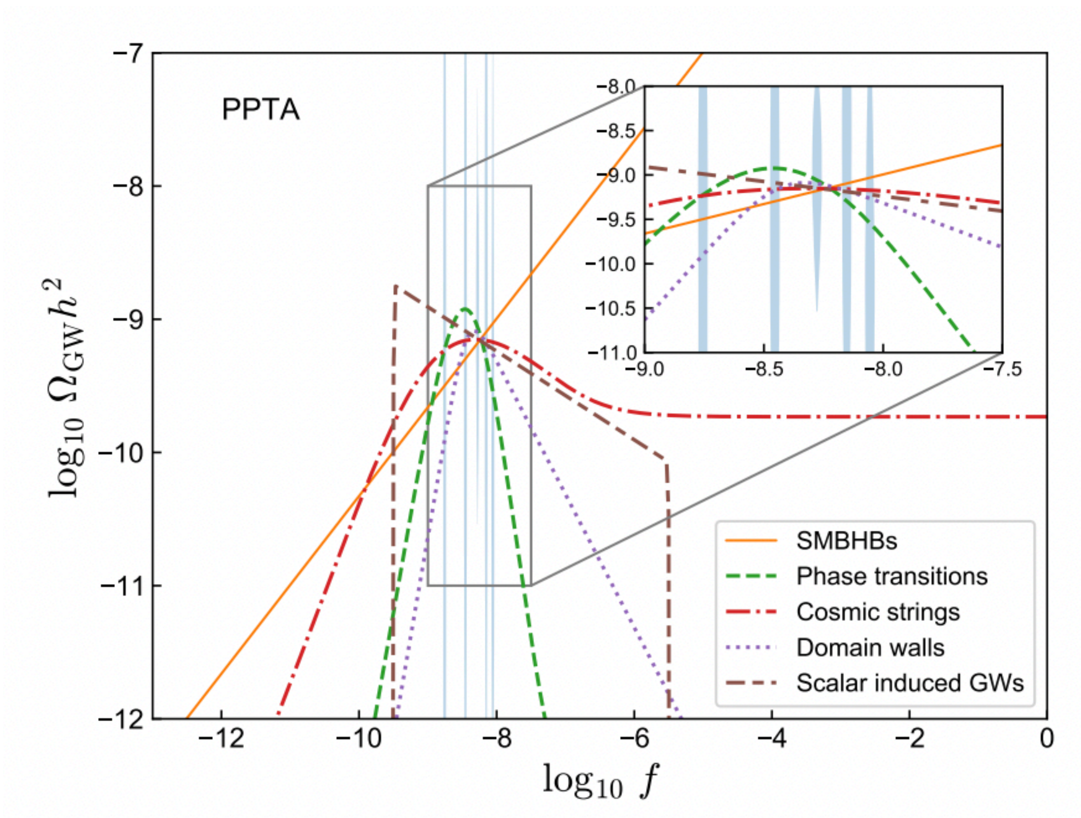


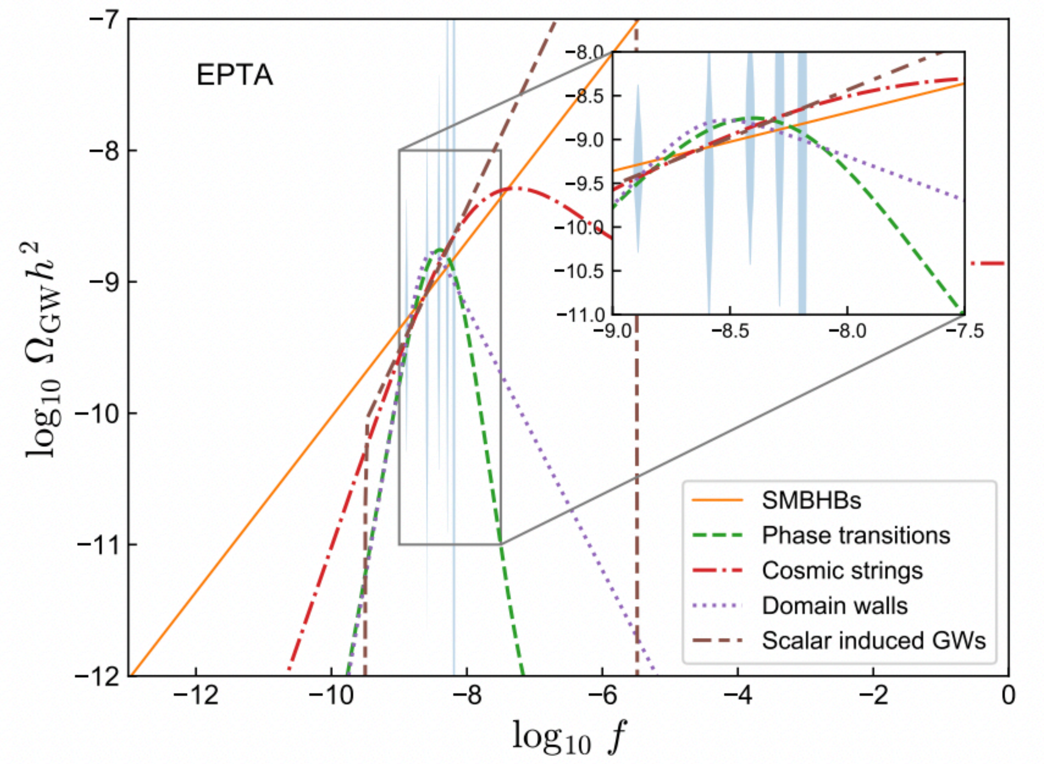

CPTA ,2306.16216

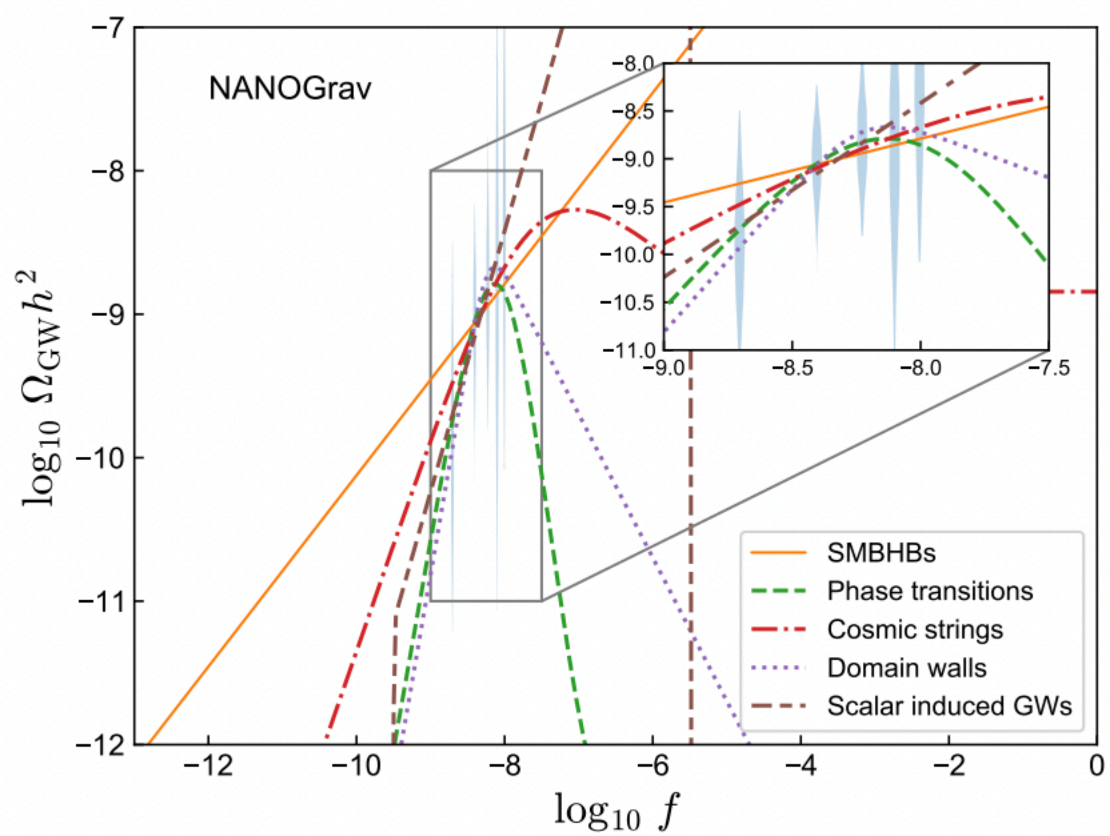



NANOGrav,2306.16213

# Gravitational wave sources for Pulsar Timing Arrays




Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

# DFSZ轴子模型

## The action in expanding universe with spatially flat FLRW metric:

$$S = -\int d^4x\sqrt{-g}\left(g^{\mu\nu}\frac{1}{2}\partial_\mu\varphi^*\partial_\nu\varphi + V(\varphi)\right)$$

PQ complex scalar: $\varphi = \phi_1 + i\phi_2$

$$V(\varphi) = \frac{1}{4}\lambda(|\varphi|^2 - v^2)^2 + \frac{\lambda}{6}T^2|\varphi|^2 + \frac{m^2(T)v^2}{N_{DW}^2}(1-\cos(N_{DW}\theta)) - \Xi v^3(\varphi e^{-i\delta} + \text{h.c.})$$

$m^2(T)$: $\min\left[\frac{\alpha_a\Lambda^4}{f_a^2(T/\Lambda)^{6.68}}, m_a^2\right]$; $\theta$: axion field; $\Xi$ term: bias term

**PQ era**, PQ symmetry broken, second order phase transition, $T_c \sim 10^9$-$10^{11}$GeV

→ Axion(global) strings form and enters the scaling regime

**QCD era**, axion acquires a non-zero mass due to the QCD non-perturbative effect, $T \sim 100$MeV

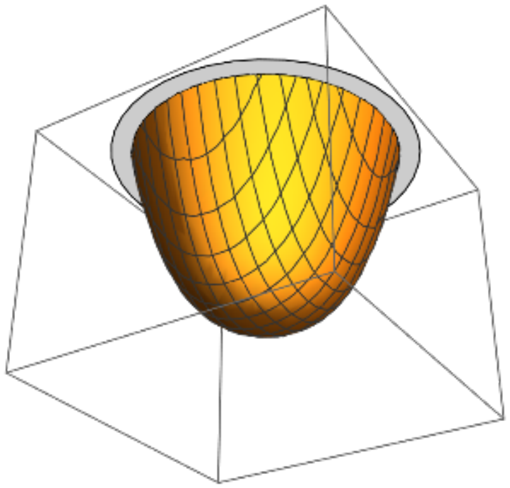
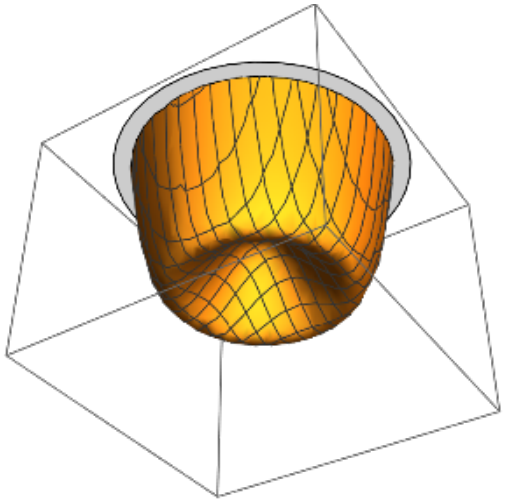
→ String-domain wall hybrid networks form and eventually decay

Gravitational waves and axion radiated by topological defects of two eras → Detection of axion dark matter

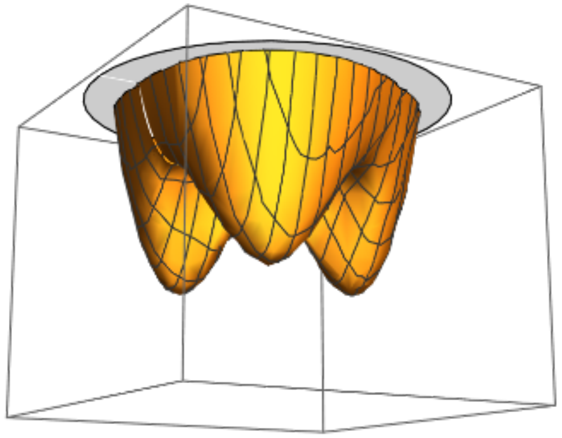
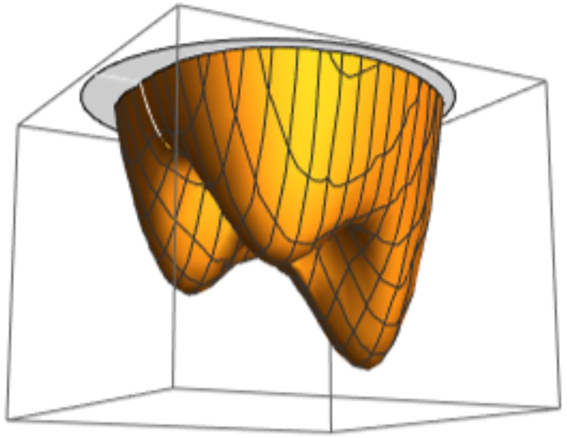
Yang Li, **Ligong Bian**, Rong-Gen Cai, Jing Shu, 2311.02011

# ▸ Shape of potential

**PQ era:**

before PQ transition →  after PQ transition → 

**QCD era:**

with nonzero axion mass without bias term →  with bias term → 

# 模拟方案

## Equations of motion

**PQ era-the first stage**

$$\begin{cases} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2[\lambda\phi_1(\phi_1^2+\phi_2^2-v^2+\frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{DW}^2}(\cos\theta\cos N_{DW}\theta + N_{DW}\sin\theta\sin N_{DW}\theta) - 2\Xi v^3\cos\delta] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2[\lambda\phi_2(\phi_1^2+\phi_2^2-v^2+\frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{DW}^2}(\sin\theta\cos N_{DW}\theta - N_{DW}\cos\theta\sin N_{DW}\theta) - 2\Xi v^3\sin\delta] \end{cases}$$

## Initial condition

**thermal spectrum**

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \quad \mathcal{P}_{\dot\phi_1}(k) = \mathcal{P}_{\dot\phi_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1}$$

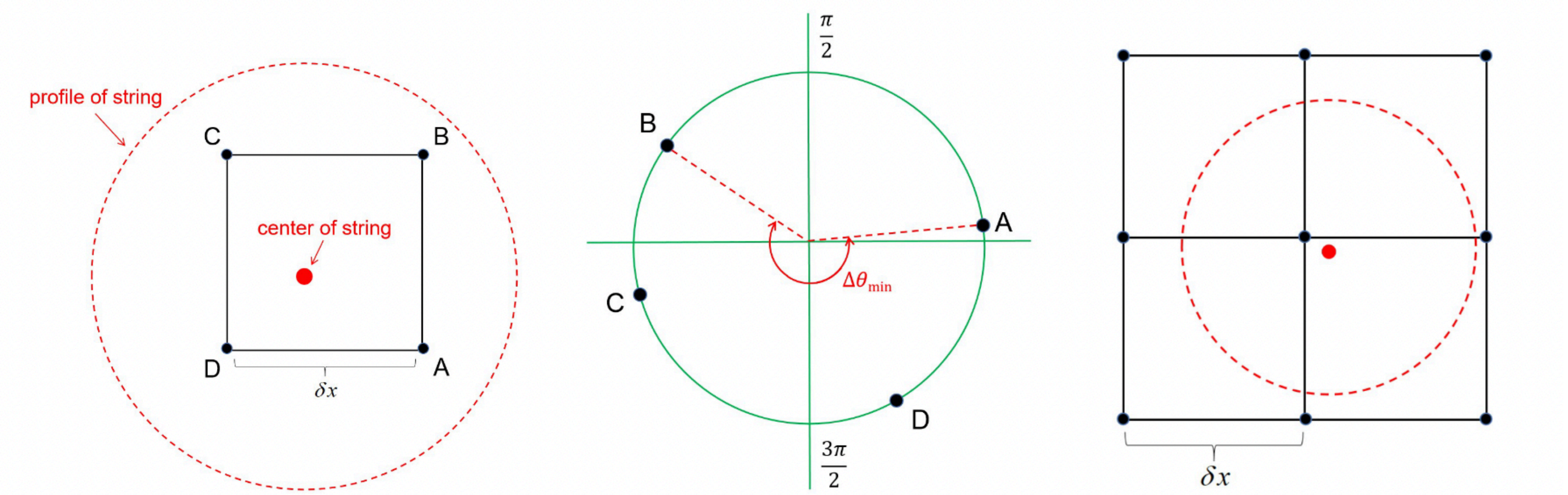
$$w_k = \sqrt{k^2/R^2 + m_{\rm eff}^2} \qquad m_{\rm eff}^2 = \lambda(T^2/3 - v^2)$$

**two-point correlation functions**

$$\langle\phi_i(\mathbf{k})\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\dot\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot\phi}(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = 0\,.$$

$$\langle|\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\phi_i}(k), \qquad \langle\phi_i(\mathbf{k})\rangle = 0,$$
$$\langle|\dot\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot\phi_i}(k), \qquad \langle\dot\phi_i(\mathbf{k})\rangle = 0,$$

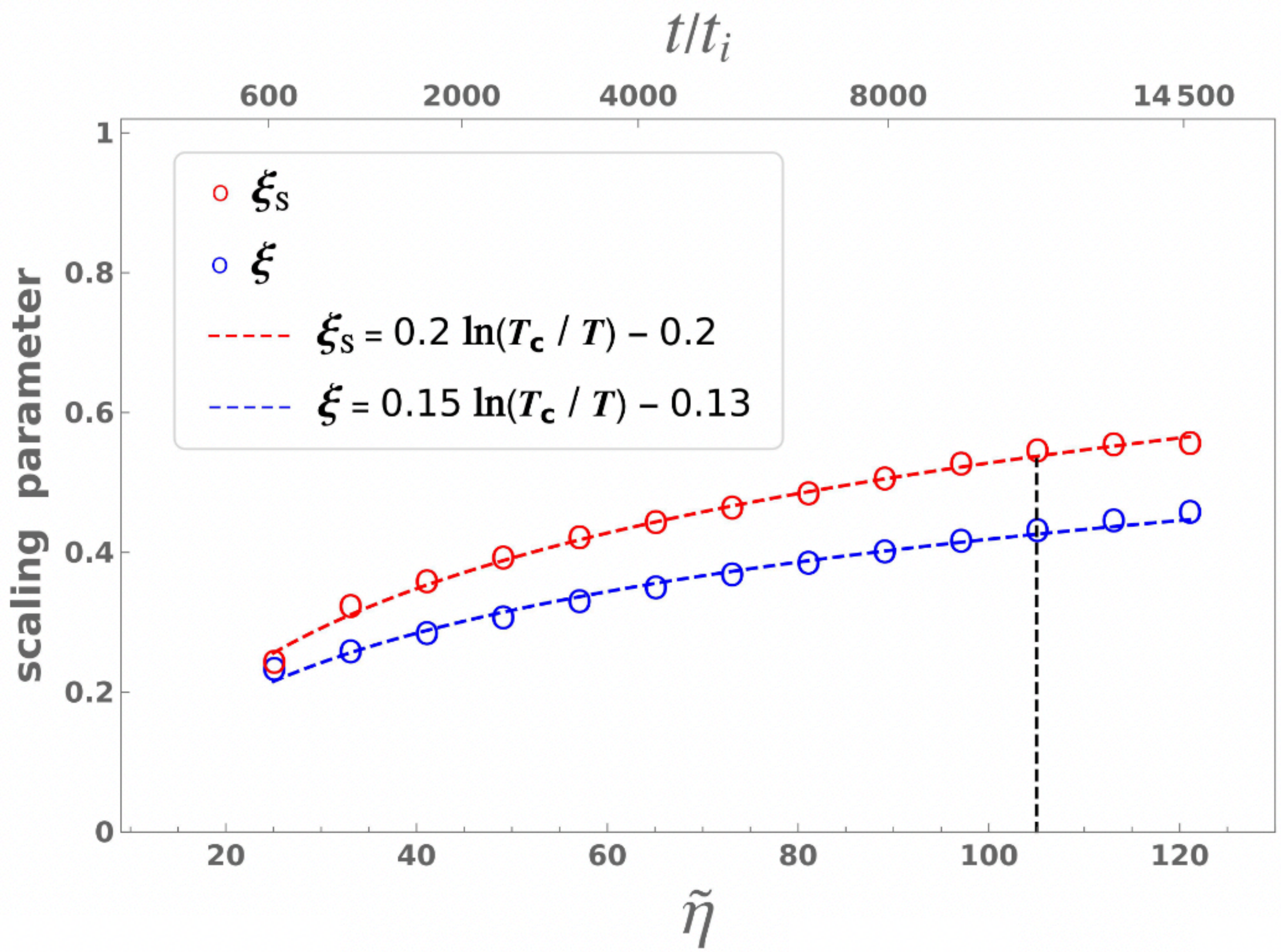
String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously



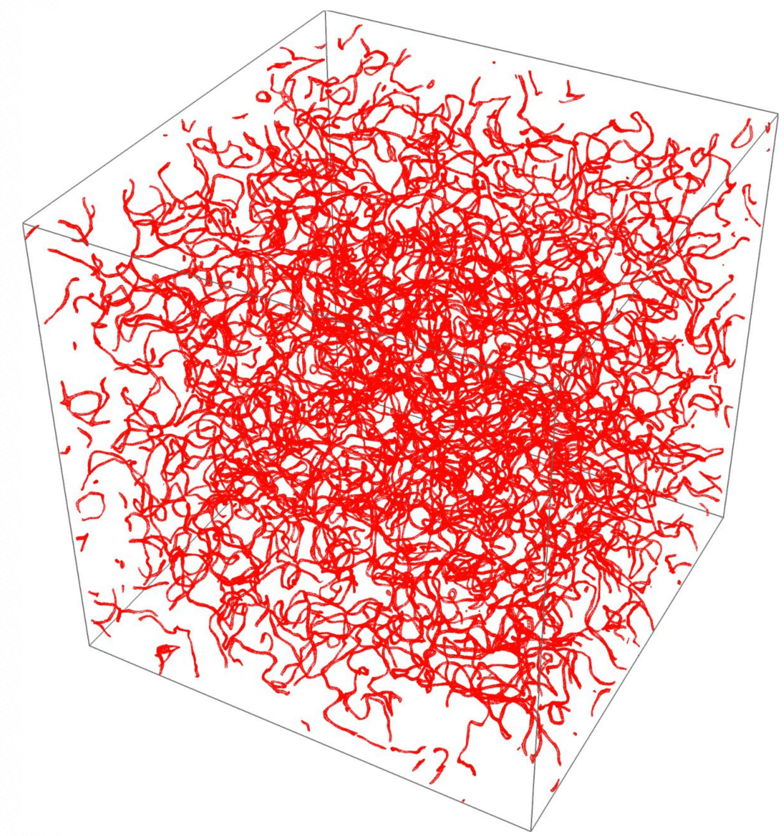

For a specific square loop, assuming that the minimum phase at four points is θmin

(1) $\theta_{\min} < \pi$.
(2) There exists at least one phase at another point minus $\theta_{\min}$ is greater than $\pi$.
(3) There exists at least one phase at another point minus $\theta_{\min}$ is smaller than $\pi$.
(4) Denote the phase closest to $\pi$ in all phases greater than $\pi$ as $\theta_a$, and denote the phase closest to $\pi$ in all phases smaller than $\pi$ as $\theta_b$, it is required to meet $\theta_a - \theta_b < \pi$.
(5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.

# ▸ 宇宙弦的scaling




Axion string network in the final moment of PQ era



$$\xi = \frac{\rho_{st} t^2}{\mu_{st}}, \quad \text{with } \rho_{st} = \frac{\mu_{st} l}{R^2 V} \Longrightarrow \xi = \frac{l t^2}{R^2 V}$$

$$\xi_s = \frac{l_{phy} t^2}{V_{phy}} = \frac{t^2}{L_m^2} = \frac{1}{\kappa^2}\frac{t^2}{(t-t_0)^2} \to \frac{1}{\kappa^2}$$

string tension: $\mu_{st} \simeq \pi v^2 \ln(t/\delta_{st})$

string core width: $\delta_{st} = 1/\sqrt{\lambda(v^2 - \frac{1}{3}T^2)}$

$l$ : comoving string length

V: comoving volume

Mean string separation $L_m = \sqrt{V_{phy}/l_{phy}}$

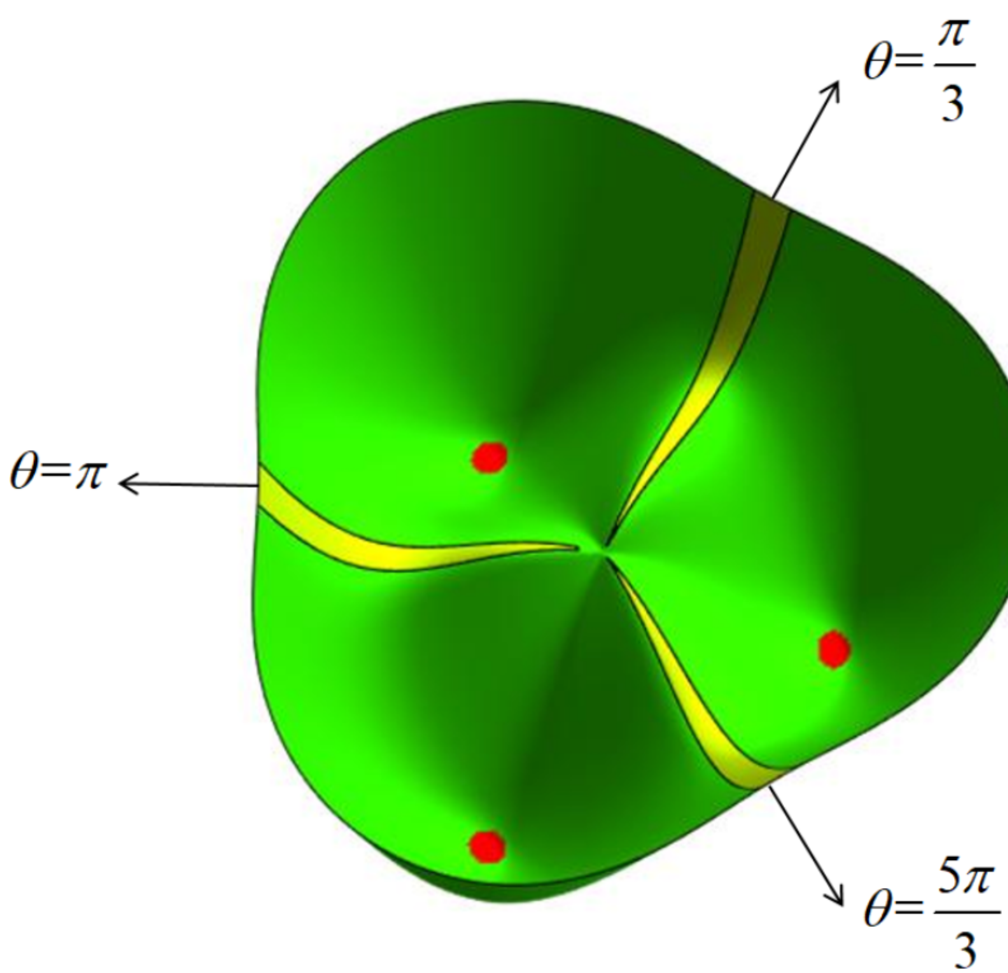
physical string length $l_{phy} = (2/3) n_c (R\delta x)$

nc: squre loops

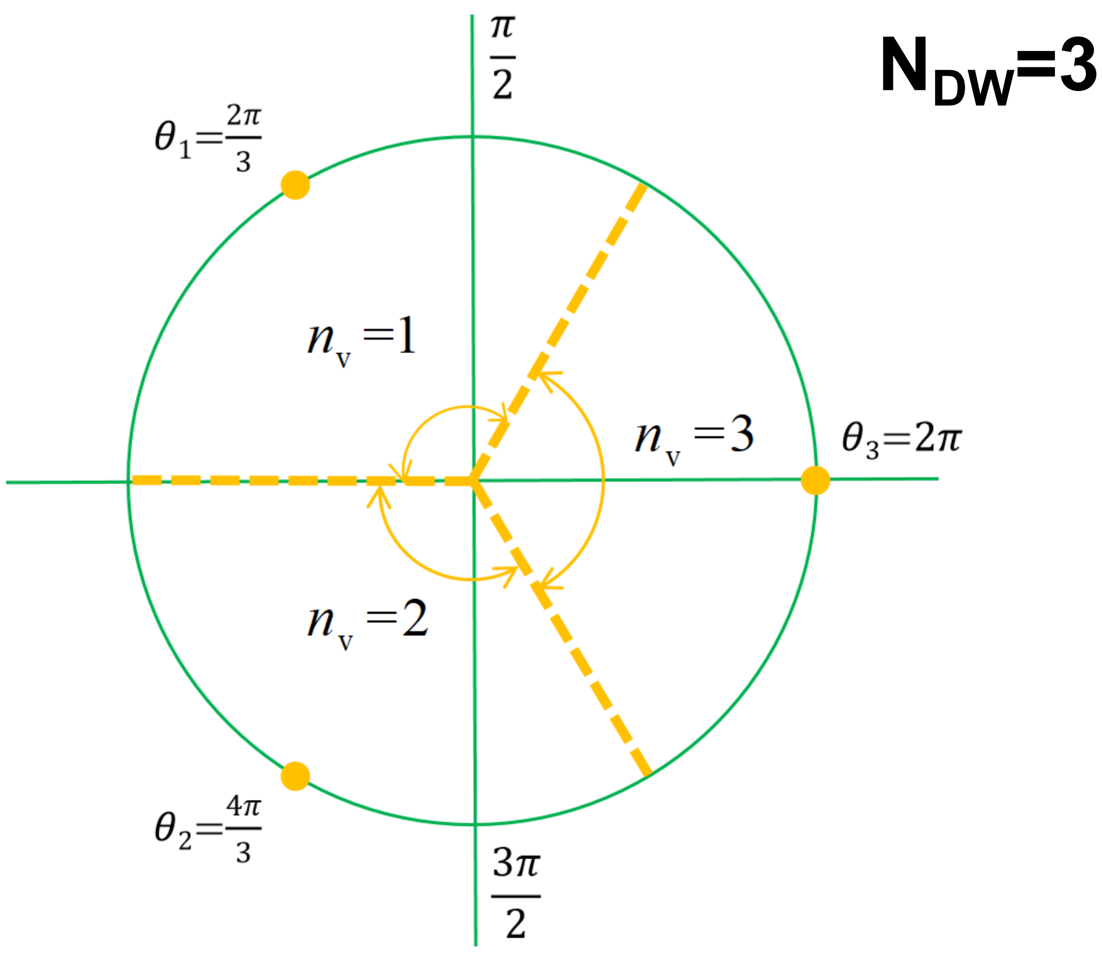
In the scaling regime, Lm increases linearly with t, and the scaling parameter tends to be a constant. The same logarithmic increase behavior, as 1809.0924, 1906.00967, 1806.0467,1806.05566

# ▸ 畴壁（Domain wall）识别

Top view of the shape of potential energy




The distribution of fields in phase space




Comoving area density

$$A/V = C\sum_{\text{links}}\delta\frac{|\nabla\theta|}{|\theta_{,x}|+|\theta_{,y}|+|\theta_{,z}|}$$

θ,i (i = x, y, z)： spatial derivatives of the dimensionless axion field θ(x)

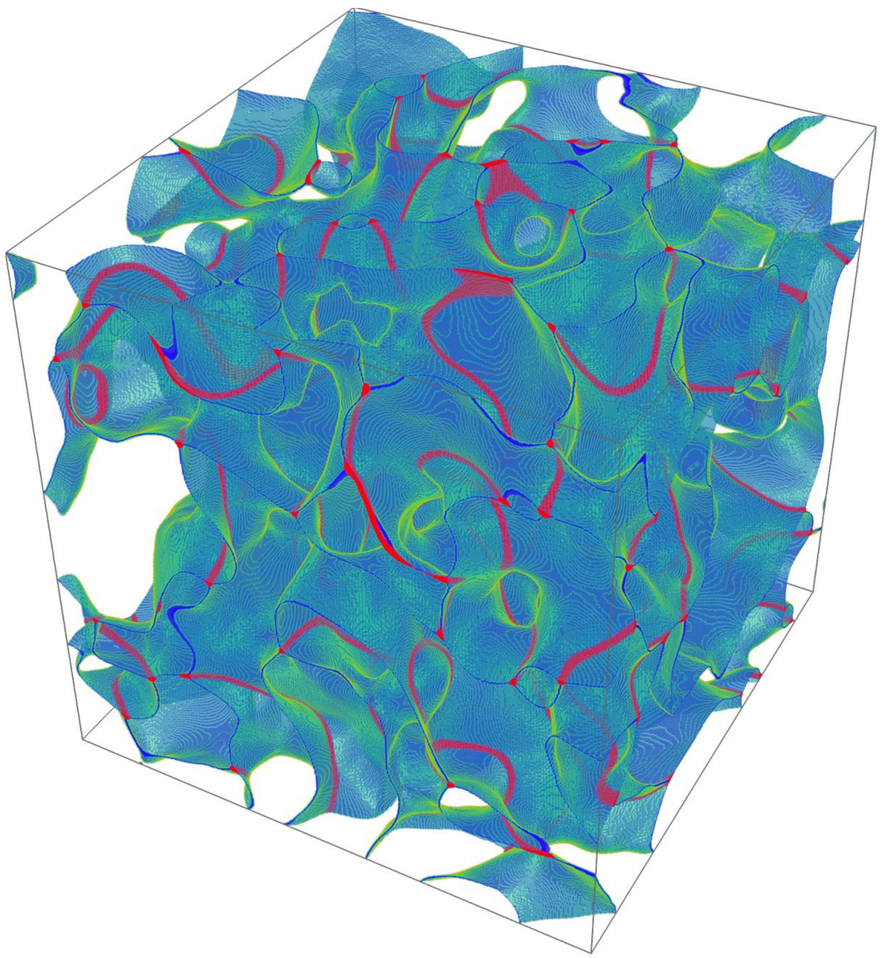
Area parameter A of DW (scaling parameter of DW) $\xi_{dw} \equiv \mathcal{A} = \frac{\rho_{\text{wall}}}{\sigma_{\text{wall}}}t,\ \text{with}\ \rho_{\text{wall}} = \frac{\sigma_{\text{wall}}A}{R(t)V}$

$$A = \Delta A\sum_{\text{links}}\delta\frac{|\nabla\theta|}{|\theta_{,x}|+|\theta_{,y}|+|\theta_{,z}|}$$

$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

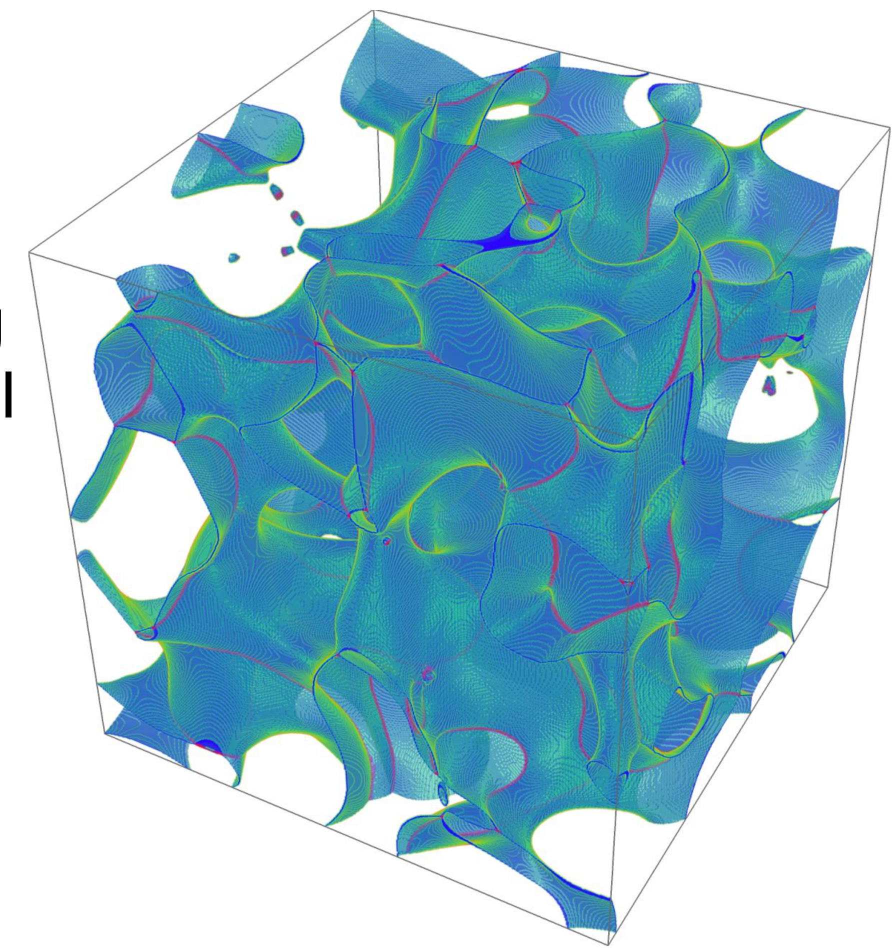
## ▸ String-wall evolution

Axion string-domain wall hybrid network in our simulation



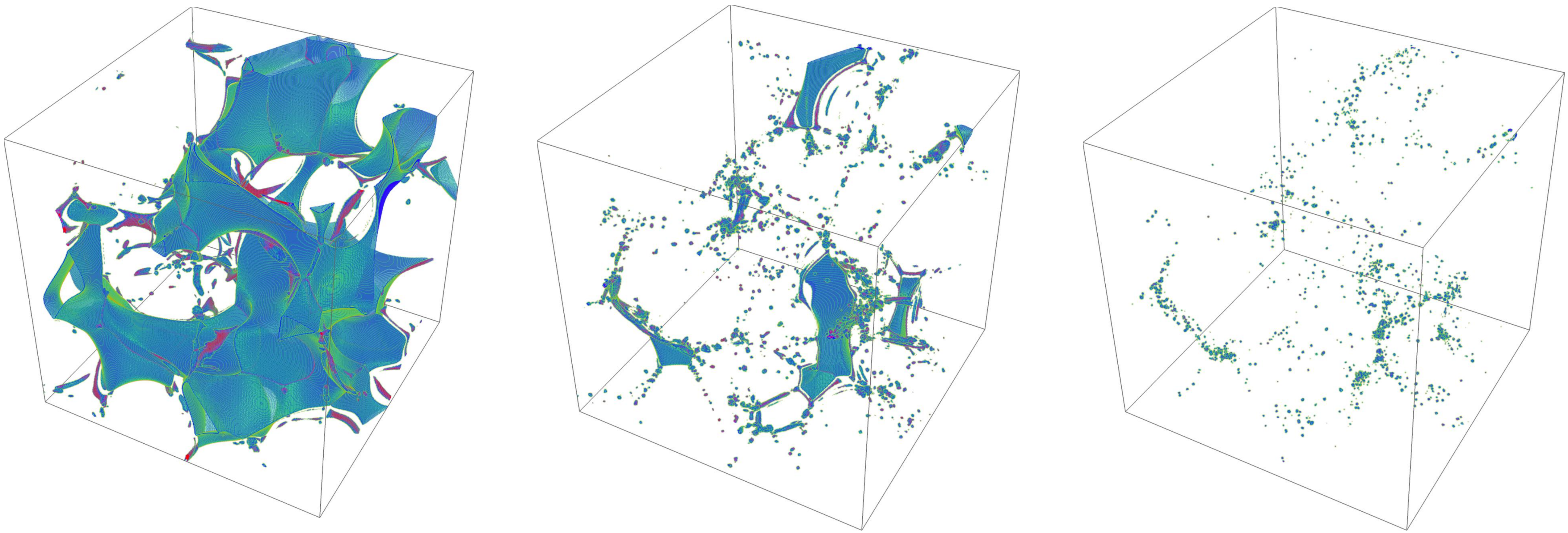
Red region → Axion string
Blue region → Domain wall

$N_{DW} = 3$



$\eta = 4.6$ $\eta = 10$

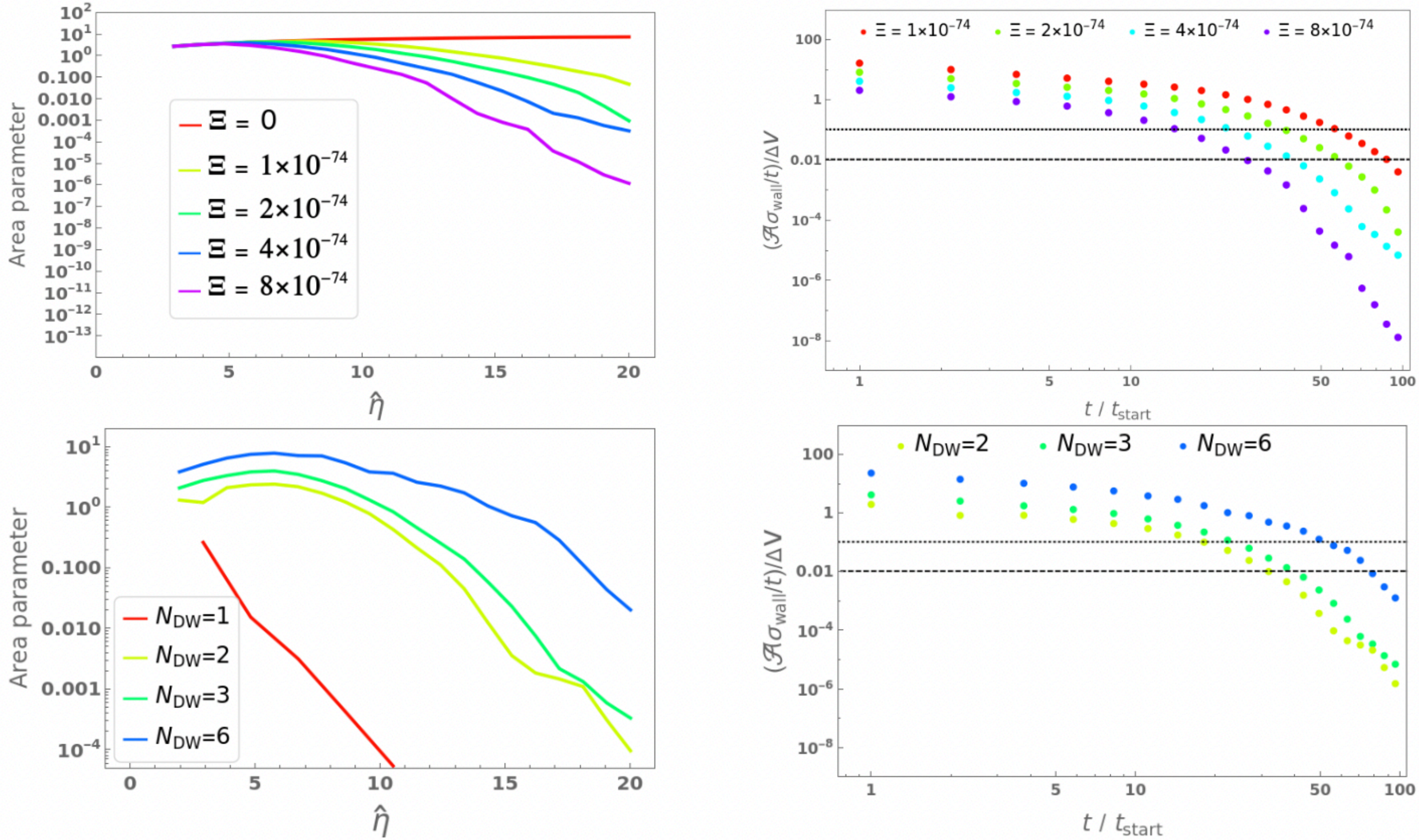
# String-wall evolution



$\eta=15.4$ $\eta=20.8$ $\eta=26.2$

Axion string-domain wall hybrid network destruction with gravitational waves and axions emitted during this process

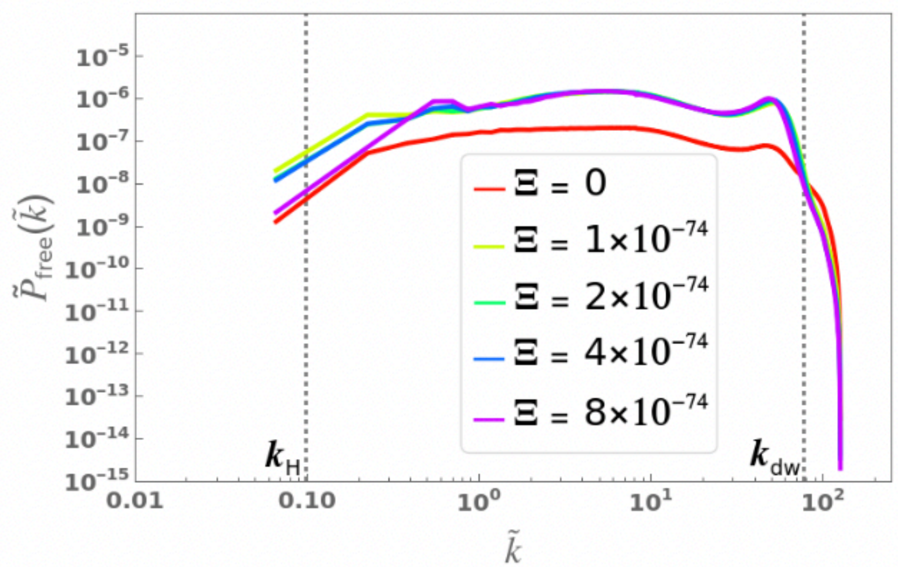
# ▸ Domain wall Area parameter

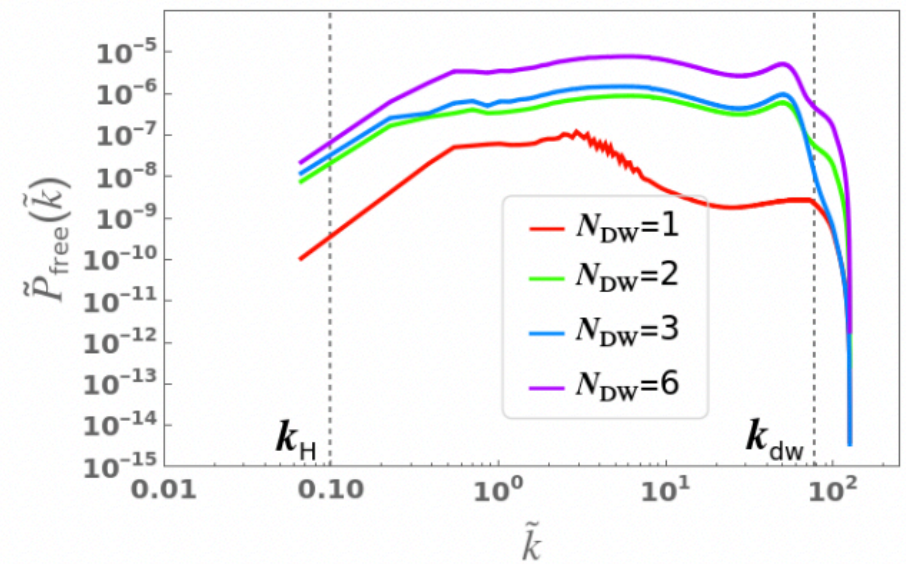



**DW decay**

$$\Delta V = 2\Xi v^4(1-\cos(2\pi/N_{\rm DW})) \sim \rho_{\rm wall} = \mathcal{A}\sigma_{\rm wall}/t \Rightarrow t_{\rm dec} = t_{\rm form}\left(\frac{C_d\mathcal{A}_{\rm form}\sigma_{\rm wall}}{t_{\rm form}\Xi N_{\rm DW}^4 f_a^4(1-\cos(2\pi/N_{\rm DW}))}\right)^{1/p}$$

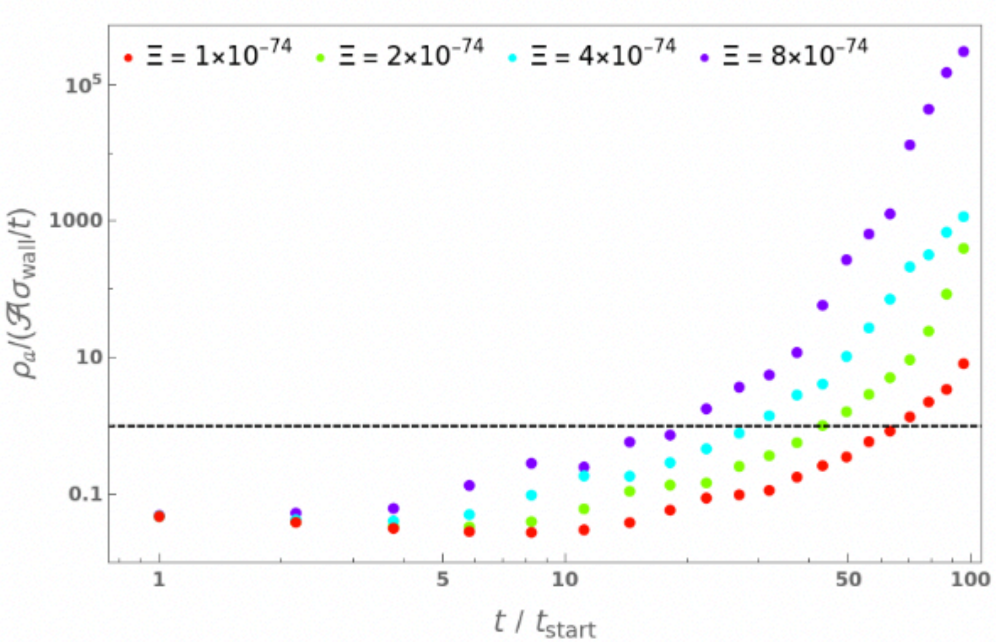
with $p = 1 + \ln C_d / \ln(t_{\rm dec}/t_{\rm form})$ $\quad C_d = \Delta V/(\mathcal{A}\sigma_{\rm wall}/t) \sim \mathcal{O}(10^2)$

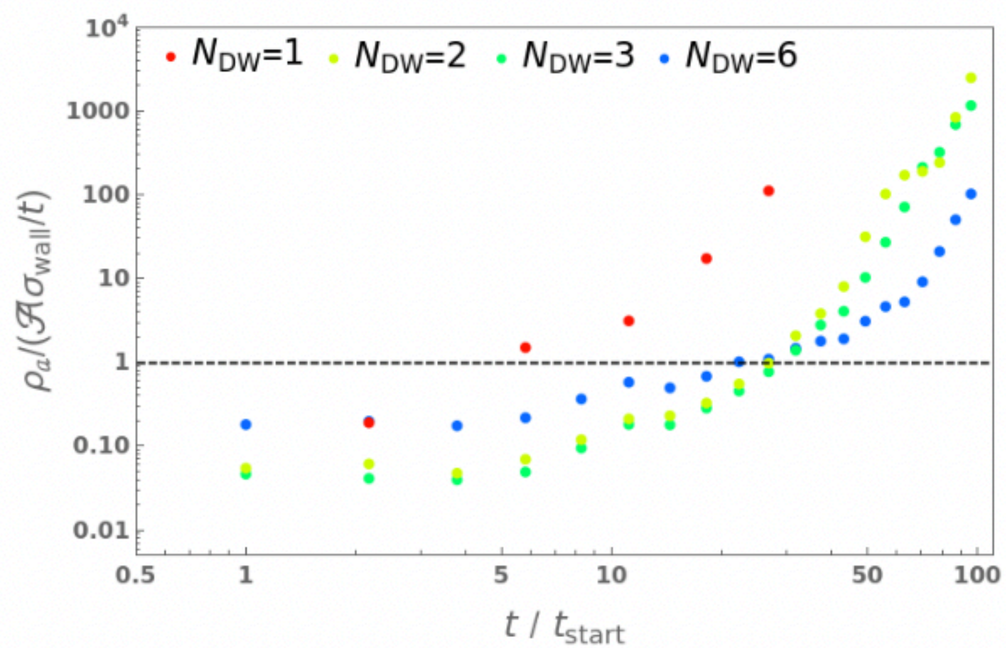



$$\frac{1}{2}\langle \dot{a}_{\rm free}(\mathbf{k})^* \dot{a}_{\rm free}(\mathbf{k}')\rangle = (2\pi)^3 \frac{2\pi^2}{k^3} P_{\rm free}(k)\delta(\mathbf{k}-\mathbf{k}') \qquad W(\mathbf{x}) = W_{\rm st}(\mathbf{x}) \times W_{\rm dw}(\mathbf{x}), \qquad \dot{a}_{\rm free}(\mathbf{x}) = W(\mathbf{x})\dot{a}(\mathbf{x}), \qquad \dot{a}(\mathbf{x}) = f_a \frac{\phi_1\dot{\phi}_2 - \dot{\phi}_1\phi_2}{\phi_1^2+\phi_2^2}$$

Window function Wdw(x) will take 0 near the DW core and 1 near the true vacuum

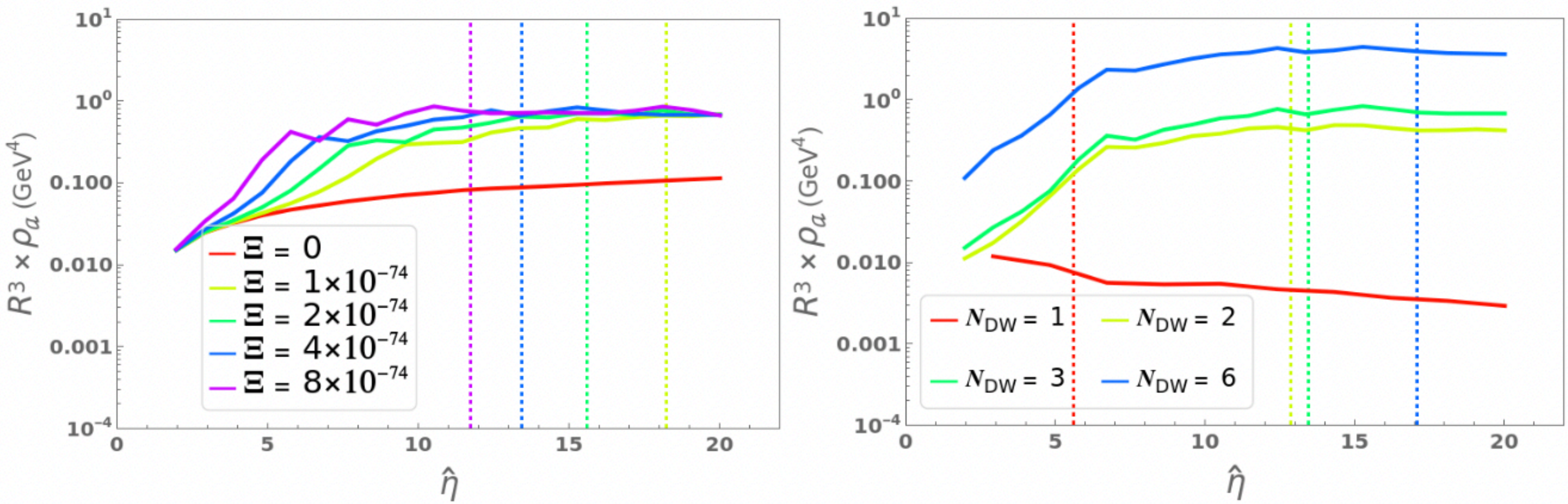



$\rho_a \simeq 2 \times \frac{1}{2}\langle \dot{a}^2_{\rm free}(\mathbf{x})\rangle$

$\rho_a \sim \mathcal{A}\sigma_{\rm wall}/t \sim 8\mathcal{A} m f_a^2/t_{\rm dec}$

# ▸ Domain wall decay to free axions




$$\rho_a(t_{\rm dec}) \simeq \frac{2(2p-1)f_a^4 N_{\rm DW}^4}{(3-2p)C_d}\,\Xi\,\sin^2(\pi/N_{\rm DW})$$

Axion energy density tends to be a constant at the final moment, and the energy density of the radiated free axion is (almost) proportional to the bias term and $N_{DW}$

## ▸ Domain wall decay to axion Dark matter

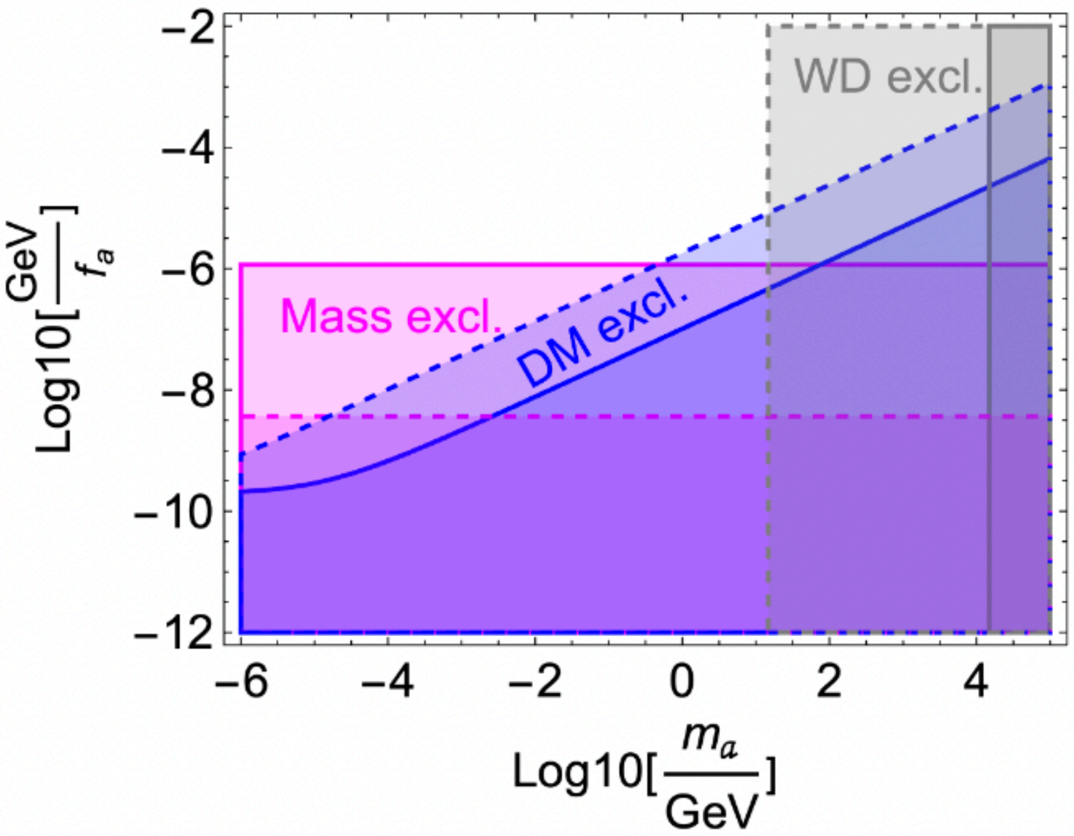
$$\Omega_{a,\mathrm{new}}(t_0)h^2 = \Omega_a^{\mathrm{DW}} h^2 + \Omega_{a,0}(t_0)h^2 + \Omega_{a,\mathrm{st}}(t_0)h^2$$

Axion DM from Wall decay:

$$\Omega_a h^2 = \frac{\rho_a(t_0)}{\rho_{0,c}/h^2} = 1.024\times10^{-19}\left(\frac{8}{3}\right)^{3/(2p)}\left(\frac{2p-1}{3-2p}\right)C_d^{3/(2p)-1}\left(\frac{m}{\mathrm{GeV}}\right)^{3/p-3/2}\left(\frac{f_a}{\mathrm{GeV}}\right)^{4-3/p}\Xi^{1-3/(2p)}\left(\frac{\csc(\pi/N_{\mathrm{DW}})}{N_{\mathrm{DW}}^2}\right)^{3/p-2}$$

Misalignment mechanism: $(\Omega_{a,0}(t_0)h^2 \simeq 4.63\times10^{-3}(f_{a,\mathrm{new}}/10^{10}\mathrm{GeV})^{1.19})$

Axion DM from pure axion string: $\Omega_{a,\mathrm{st}}(t_0)h^2 = 2.0\times(f_a/10^{12}\,\mathrm{GeV})^{1.19}$




**WD excl.**:

$t_{\mathrm{dec}} < t_{\mathrm{WD}}$ $\quad t_{\mathrm{WD}} = \frac{3}{16\pi G\sigma_{\mathrm{wall}}}$

**Mass excl.**:
bias term dominating over the QCD instanton effect in the contributions to axion mass term

$$\Xi < 2\times10^{-45}N_{\mathrm{DW}}^{-2}\left(\frac{10^{10}\mathrm{GeV}}{f_a}\right)^4$$

**DM excl.**

$$\Omega_a(t_0)h^2 \leq 0.12$$

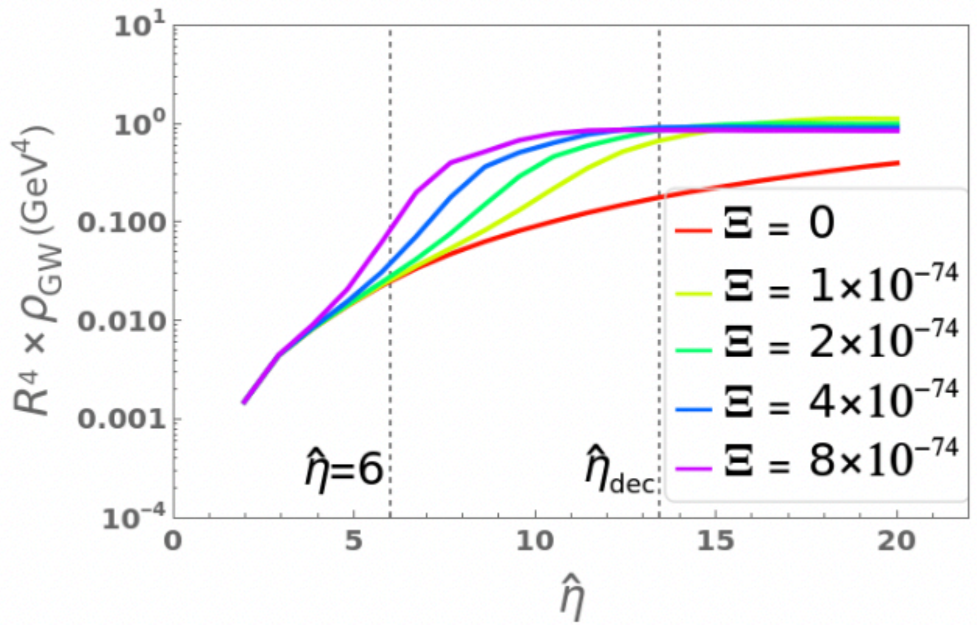
# ▸ String-wall 引力波计算

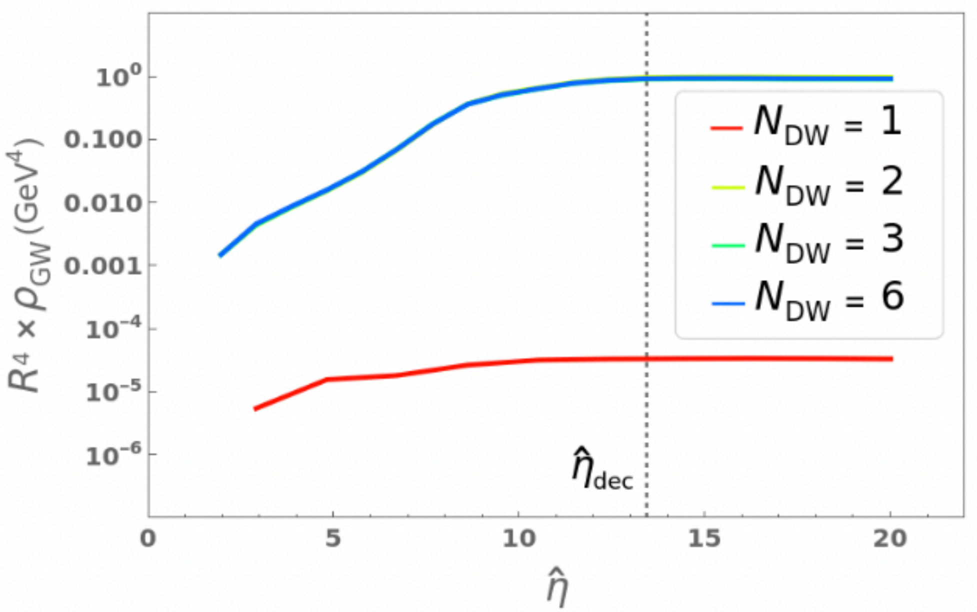
$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \frac{\nabla^2}{R^2}h_{ij} = \frac{16\pi G}{R^2}\Pi_{ij}^{TT}$$

$$\Pi_{ij} \equiv T_{ij} - p g_{ij} = T_{ij} - \frac{\delta_{ij}}{3}\sum_{l} T_{ll}$$

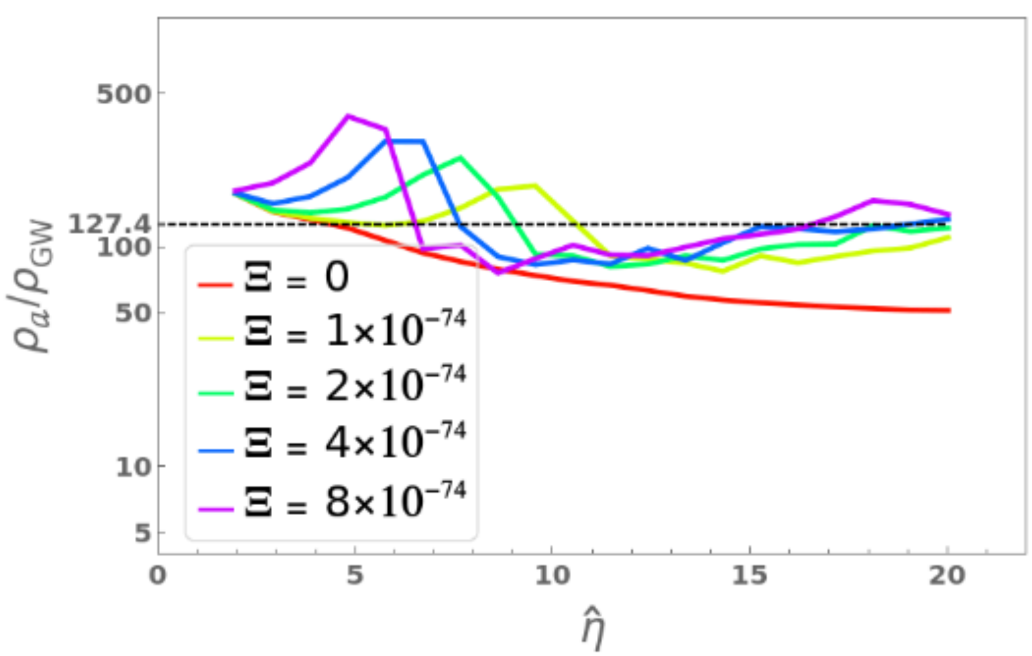
$$\rho_{GW}(t) = t_{00} = \frac{1}{32\pi G}\langle \partial_\mu h_{ij}^{TT}(\mathbf{x},t)\partial_\nu h^{TT,ij}(\mathbf{x},t)\rangle|_{\mu=\nu=0} = \frac{1}{32\pi G}\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\rangle$$

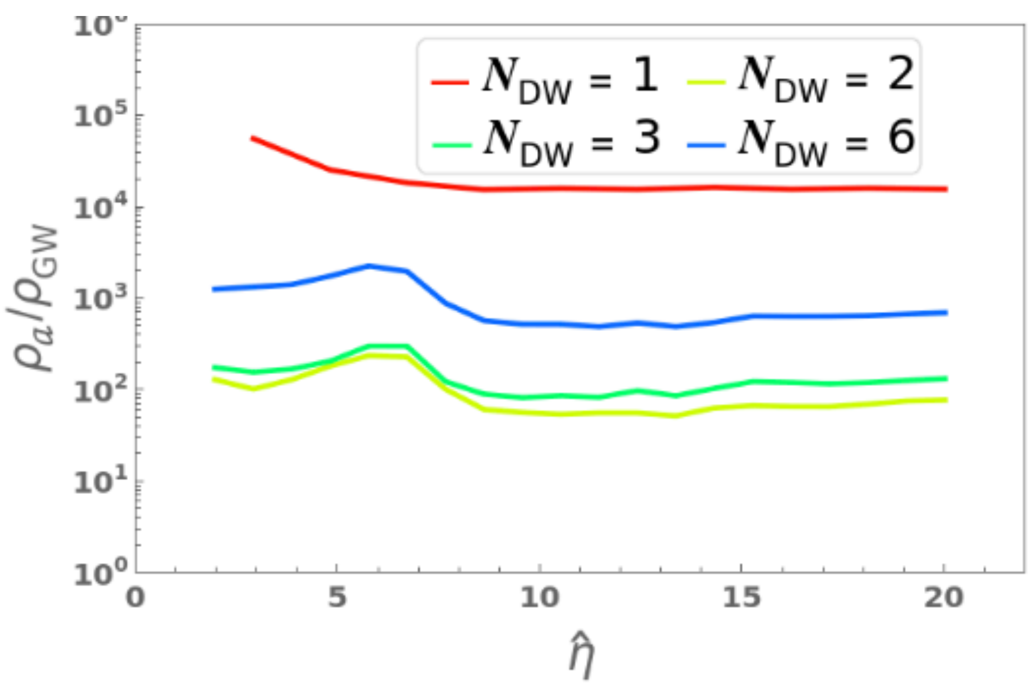
$$= (\partial_i\phi_1\partial_j\phi_1 + \partial_i\phi_2\partial_j\phi_2) - \frac{\delta_{ij}}{3}\sum_{l}(\partial_l\phi_1\partial_l\phi_1 + \partial_l\phi_2\partial_l\phi_2),$$



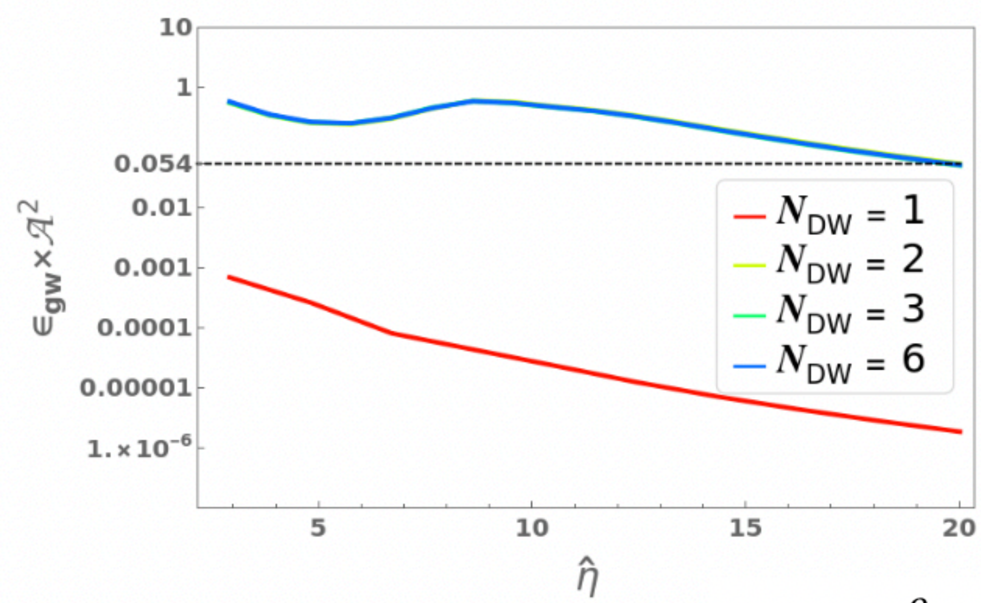

引力波能量密度几乎不依赖于bias term 系数和$N_{DW}$， $\rho_{gw} = \varepsilon_{gw} G A^2 \sigma_{wall}^2 \propto \varepsilon_{gw} G A^2 m^2 f_a^4$

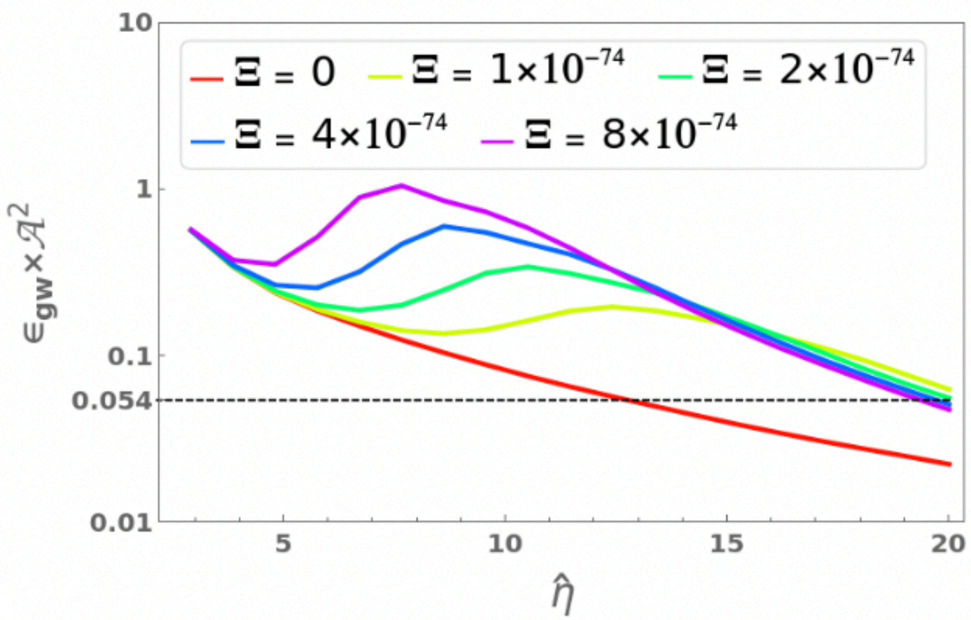


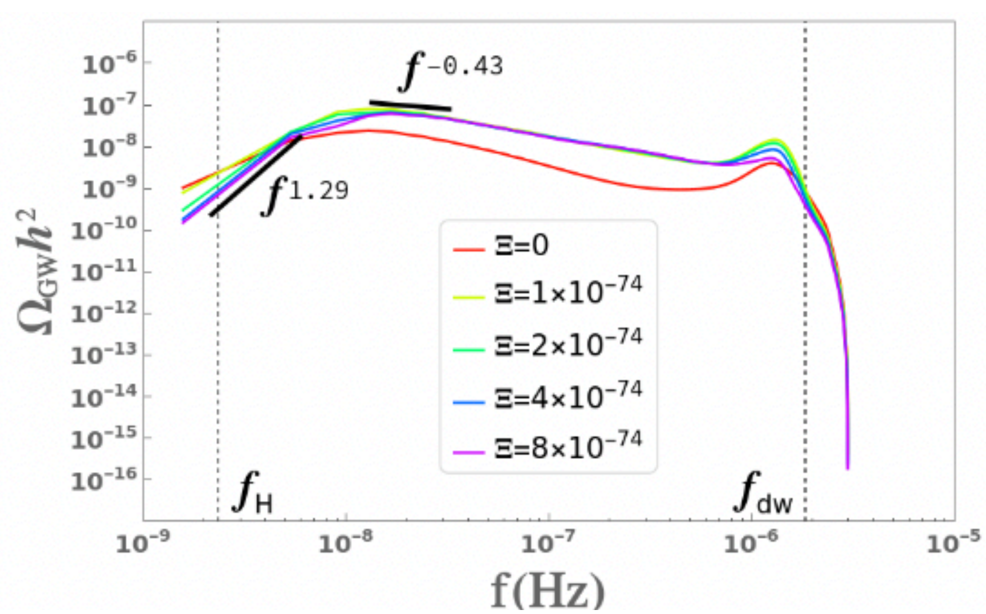

**axion string-wall networks mainly decay to axion**

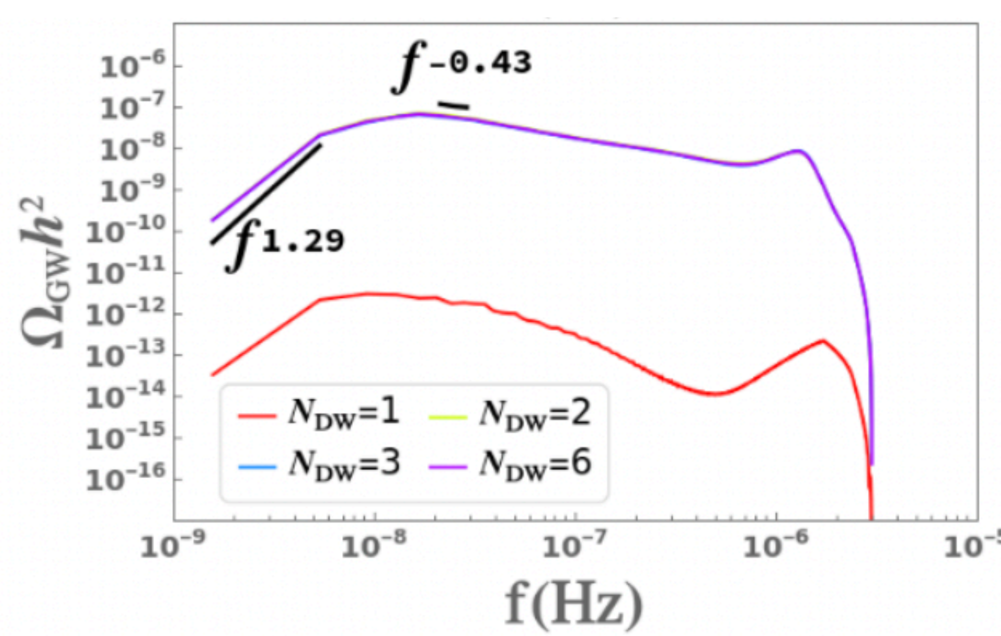
# ▸ String-wall 引力波计算




Efficiency parameter $\epsilon_{\rm gw}\mathcal{A}^2 = \frac{\rho_{\rm gw}}{G\sigma_{\rm wall}^2}$ 趋于一个常数， $\rho_{\rm gw} \propto G m^2 f_a^4$



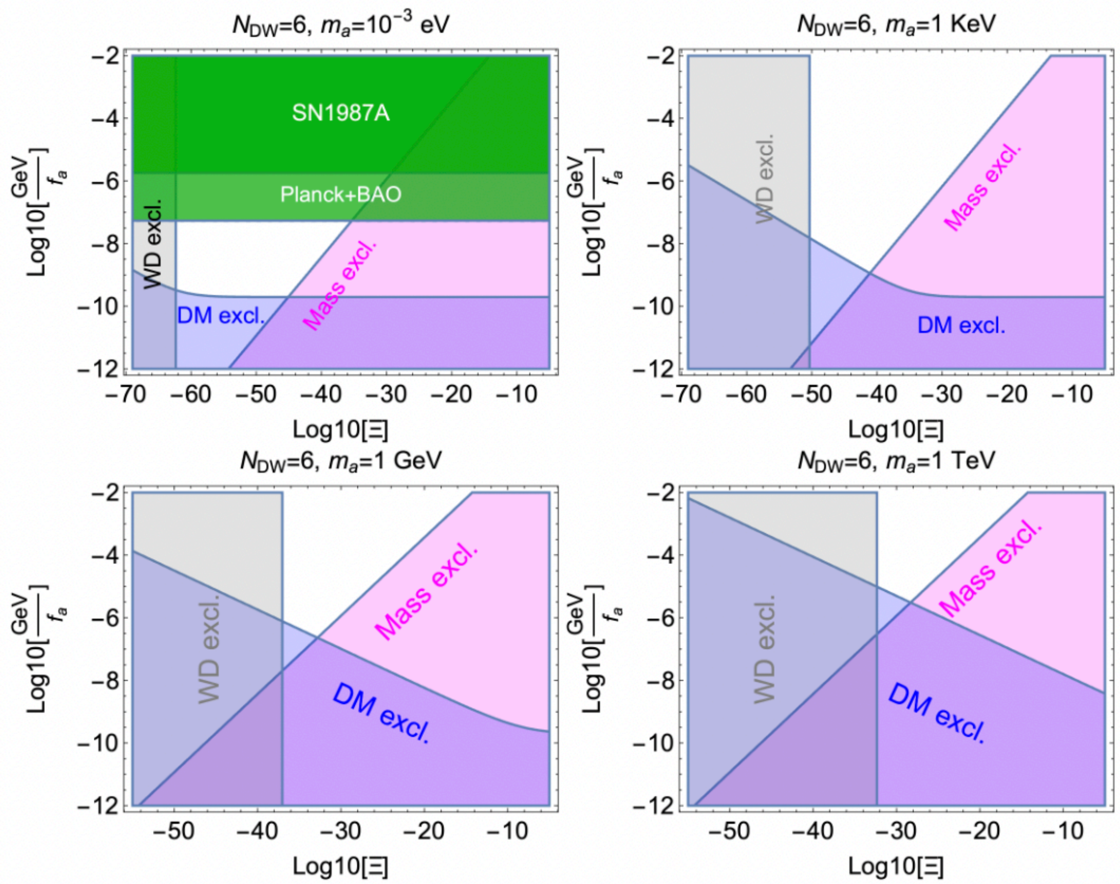



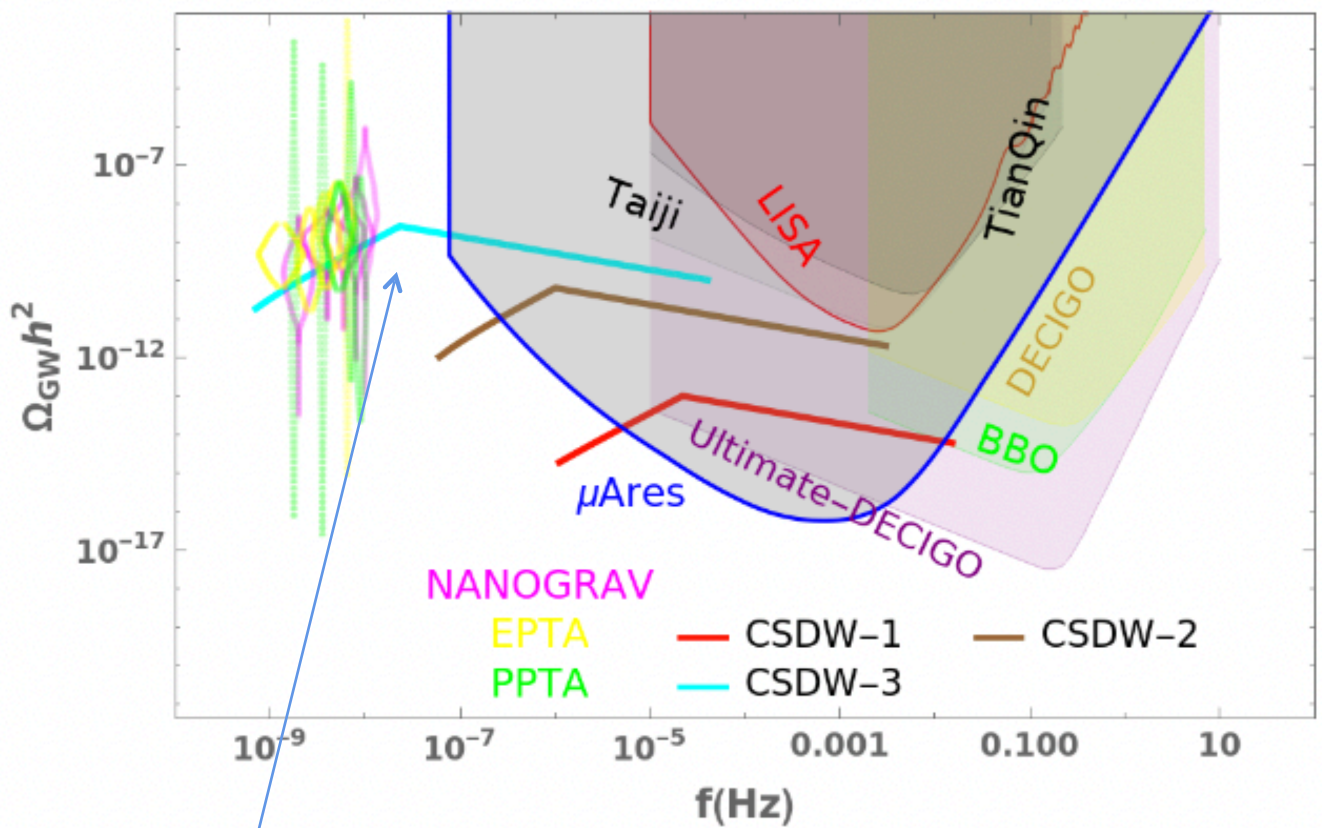

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k} = \frac{1}{32\pi G}\frac{1}{(2\pi)^3 V}\int d\Omega |\mathbf{k}|^3 |\dot{h}_{ij}(\mathbf{k},t)|^2$$

$$\Omega_{\rm GW,0}h^2 = \frac{h^2}{\rho_{c,0}}\frac{d\rho_{\rm GW,0}}{d\ln k_{\rm p,0}} = \Omega_{\rm rad,0}h^2\left(\frac{g_0}{g_{\rm e}}\right)^{1/3}\left\{\frac{1}{\rho_{c,\rm e}}\frac{d\rho_{\rm GW,e}}{d\ln k_{\rm co,e}}\right\}$$

# ▸ Axion DM and GW from String-wall

Our research provides the possibility of searching for axion models or constraining model parameters through GW detection experiments in the future




Simultaneously explanation of DM and GWs with axion string-wall networks, only for $N_{DW}>1$ of ALP scenarios.

For $N_{DW} = 1$, the GW energy density appears undetectable for QCD axions and axion-like particles.

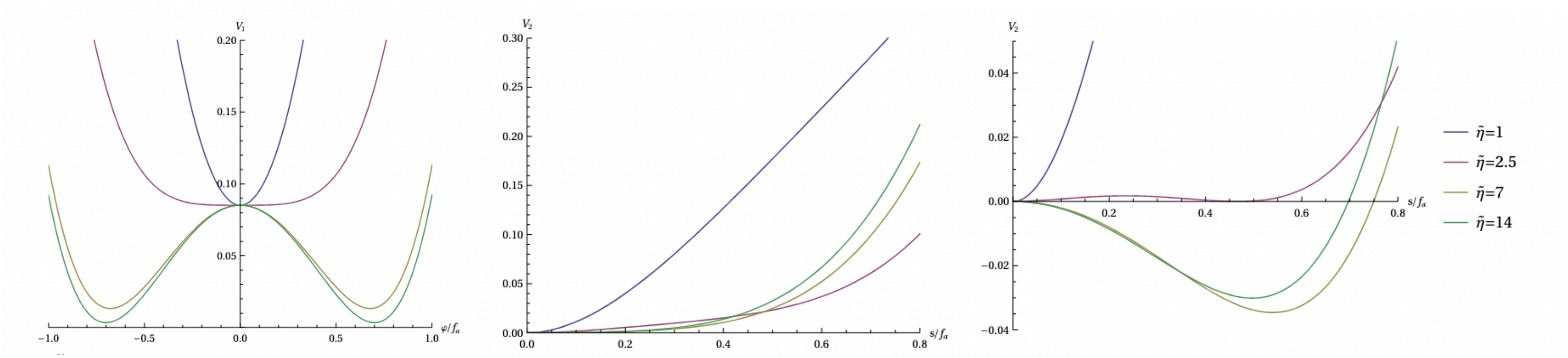
## The action in expanding universe with spatially flat FLRW metric:

$$S = -\int dx^4 \sqrt{-g}\left(\partial_\mu \varphi^* \partial^\mu \varphi + \frac{1}{2}\partial_\mu h \partial^\mu h + V(\varphi, h, T)\right)$$

### Thermal effective potential

$$V(\varphi, h, T) = V_1(\varphi, T) + V_2(\varphi, h, T)$$

$$V_1(\varphi, T) = \lambda_\varphi \left(|\varphi|^2 - \frac{f_a^2}{2}\right)^2 + \left(\frac{\lambda_\varphi}{3} + \frac{\lambda_{\varphi h}}{6}\right) T^2 |\varphi|^2 \qquad V_2(\varphi, s, T) = \frac{1}{2}\gamma\left(T^2 - T_0^2\right) h^2 + \frac{1}{3} A T h^3 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\lambda_{\varphi h} |\varphi|^2 h^2$$




$$T_c = \sqrt{\lambda_\phi / (\lambda_\phi/3 + \lambda_{\phi h})} f_a$$

### Potential Shape

Yongtao Jia, **Ligong Bian**, 2412.04218

# ▸ 模拟方案

## Equations of motion

## Only PQ era

$$\begin{cases} \tilde{\varphi}'' - \tilde{\nabla}^2\tilde{\varphi} + 2\frac{a'}{a}\tilde{\varphi}' = -a^2\frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}\frac{\tilde{\varphi}}{2|\tilde{\varphi}|}\,, \\ \tilde{h}'' - \tilde{\nabla}^2\tilde{h} + 2\frac{a'}{a}\tilde{h}' = -a^2\frac{\partial\tilde{V}}{\partial\tilde{h}}\,, \end{cases}$$

$$\tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{h} = \frac{h}{f_*} \qquad f_* = f_a$$

$$\tilde{V} = \frac{V\left(f_*\tilde{\varphi},, f_*\tilde{h}, T\right)}{f_*^2\omega_*^2} \qquad \omega_* = a_i H_i$$

## Initial condition

**thermal spectrum**

$$\mathcal{P}_{\phi_i}(k) = \frac{1}{\omega_k(e^{\omega_k/T}-1)} \quad \mathcal{P}_{\pi_{\phi_i}}(k) = \frac{\omega_k}{e^{\omega_k/T}-1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\rm eff}^2} \qquad m_\varphi^2 = (\lambda_\varphi/3 + \lambda_{\phi h}/6)T^2 - \lambda_\phi v_\phi^2$$

**two-point correlation functions**

$$\begin{aligned} \langle\phi_i(\mathbf{k})\phi_j(\mathbf{k}')\rangle &= (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij}, \\ \langle\dot{\phi}_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}')\rangle &= (2\pi)^3\mathcal{P}_{\dot{\phi}}(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij}, \\ \langle\phi_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}')\rangle &= 0\,. \end{aligned}$$

$$\langle|\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\phi_i}(k), \qquad \langle\phi_i(\mathbf{k})\rangle = 0,$$

$$\langle|\dot{\phi}_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot{\phi}_i}(k), \qquad \langle\dot{\phi}_i(\mathbf{k})\rangle = 0,$$

# ▸ 场的空间构型

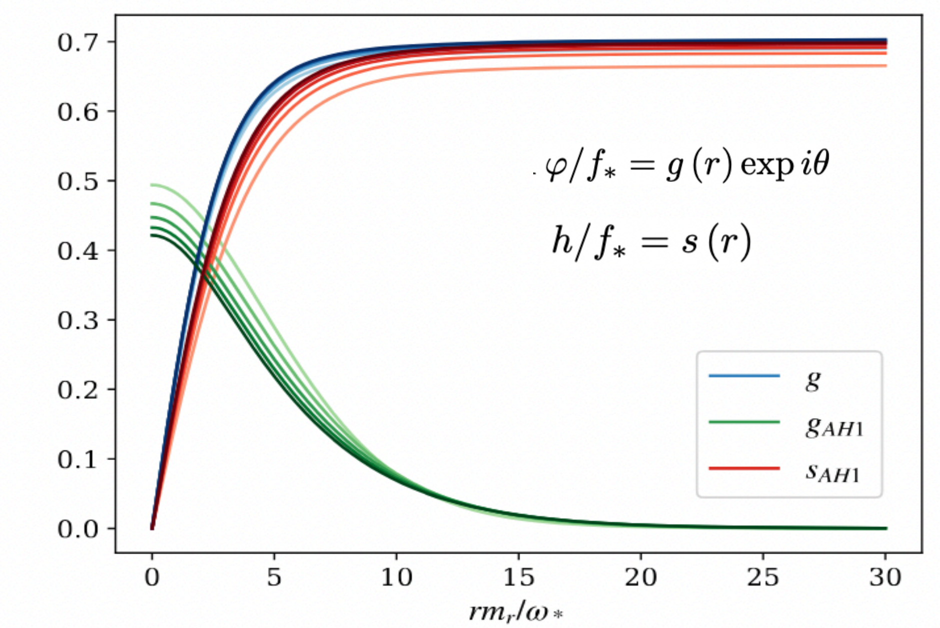
$$g''(r)+\frac{1}{r}g'(r)-\frac{1}{r^2}g(r)=\lambda_\varphi\frac{f_*^2}{\omega_*^2}g(g^2-\frac{1}{2})+(\frac{\lambda_\varphi}{3}+\frac{\lambda_{\varphi h}}{6})\frac{T^2}{\omega_*^2}g+\frac{\lambda_{\phi h}}{2}\frac{f_*^2}{\omega_*^2}gs^2\,,$$

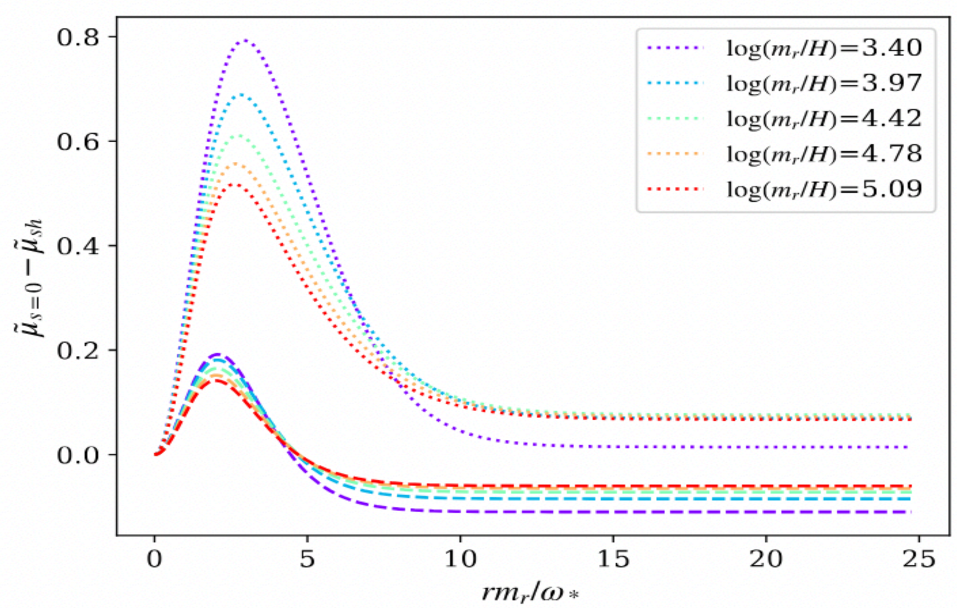
$$s''(r)+\frac{1}{r}s'(r)=\lambda_{\phi h}\frac{f_*^2}{\omega_*^2}g^2s+\gamma\frac{T^2-T_0^2}{\omega_*^2}s+AT\frac{f_*}{\omega_*^2}s^2+\lambda_h\frac{f_*^2}{\omega_*^2}s^3\,.$$

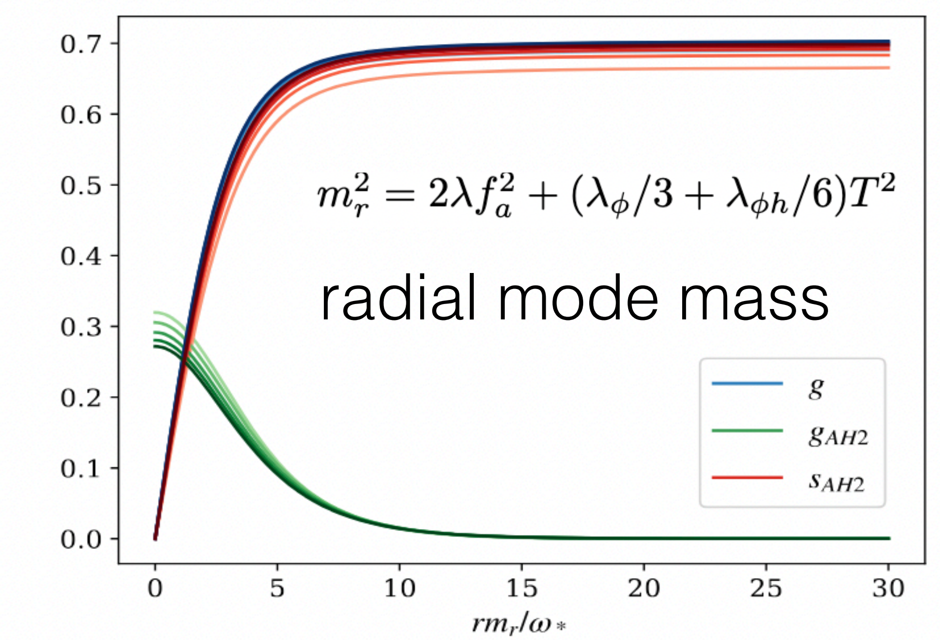
## Surface tension

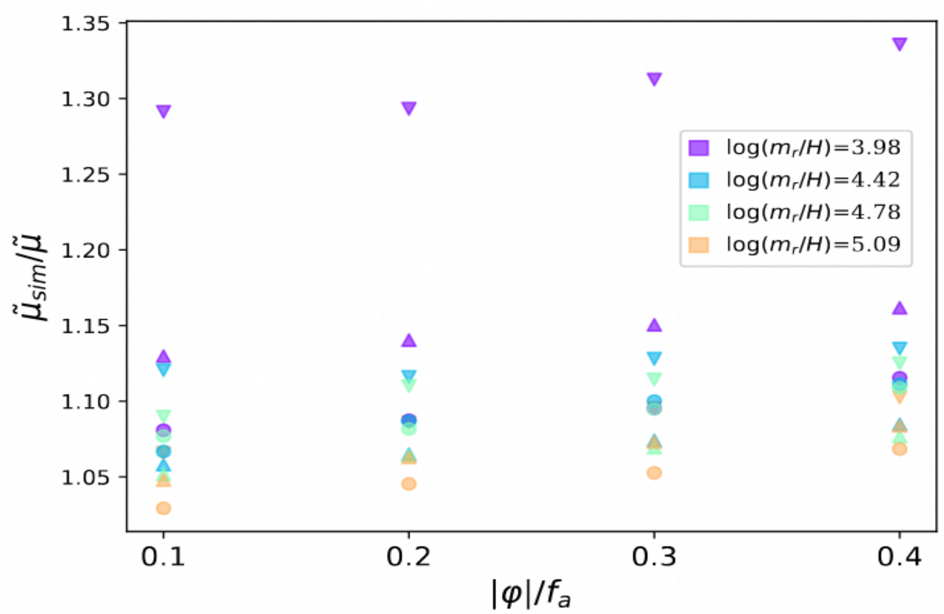
$$\tilde{\mu}=\mu/f_*^2=\int\tilde{\rho}r\mathrm{d}r\mathrm{d}\theta$$

$$\tilde{\mu}=2\pi\int\left((g'^2+\frac{g^2}{r^2}+(s'^2+\tilde{V}(g,s)\right)r\mathrm{d}r$$

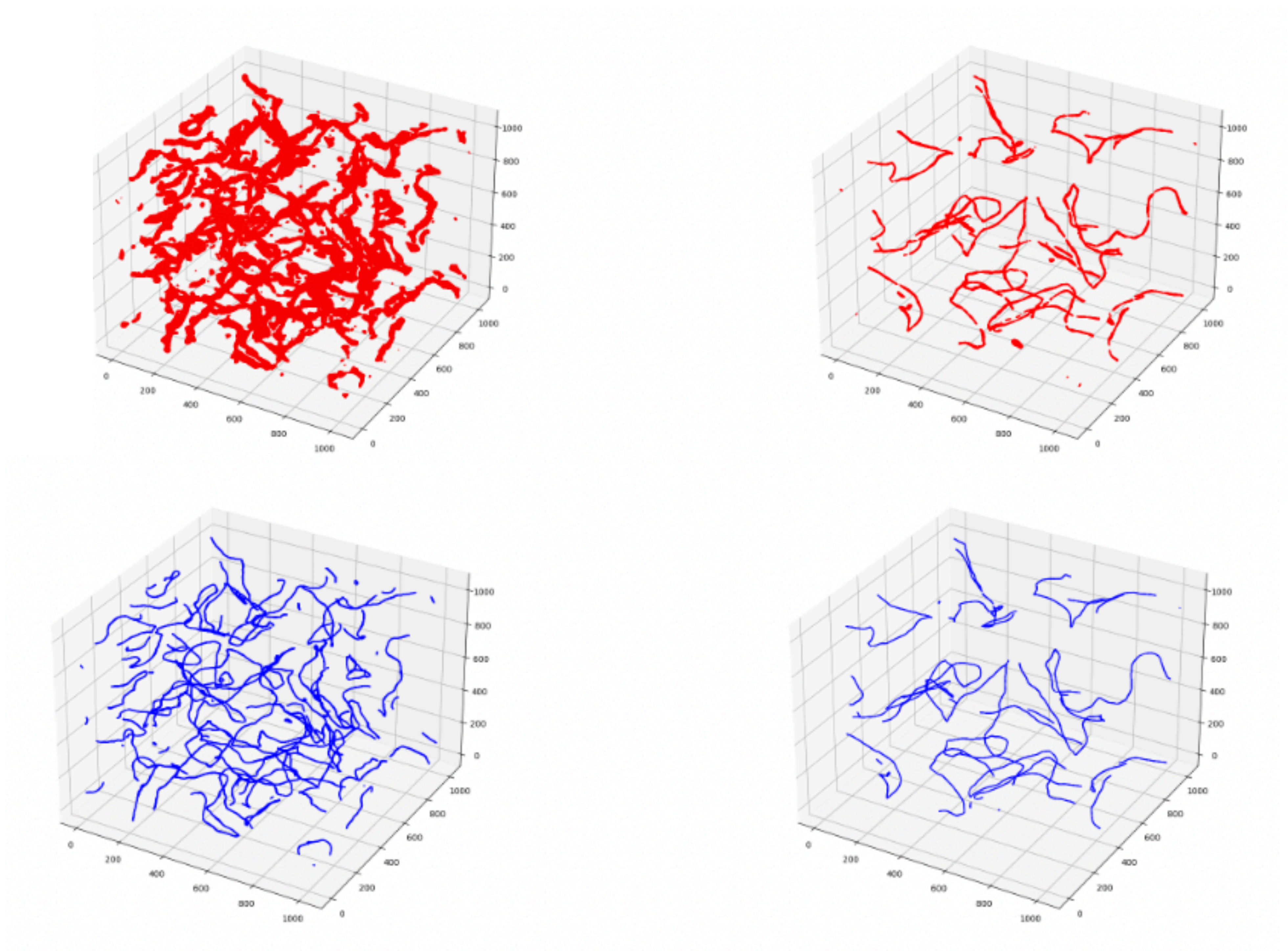


$\tilde{\eta} = 9.0$ $\tilde{\eta} = 14.0$

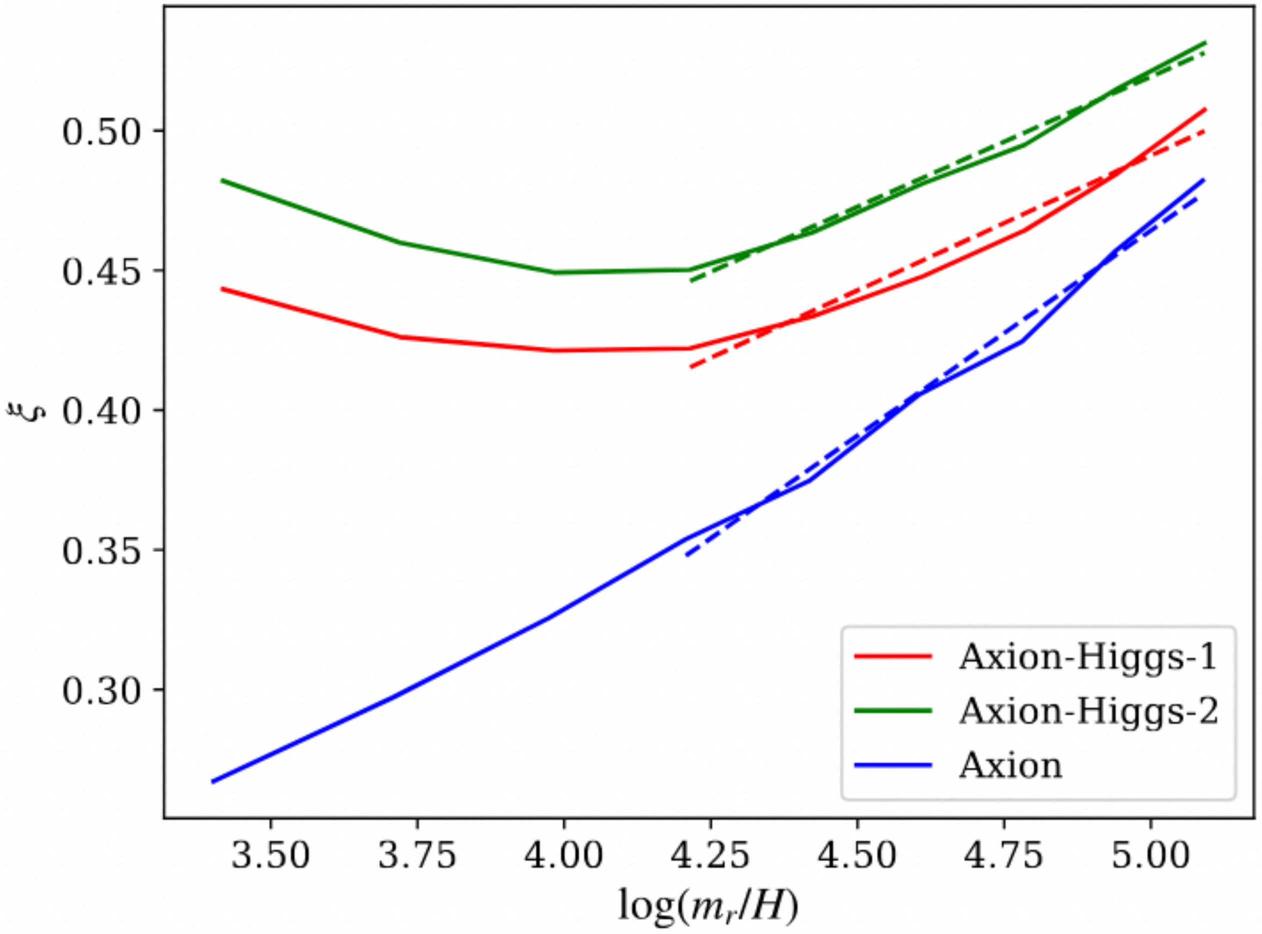



$s > 0.35 f_a$

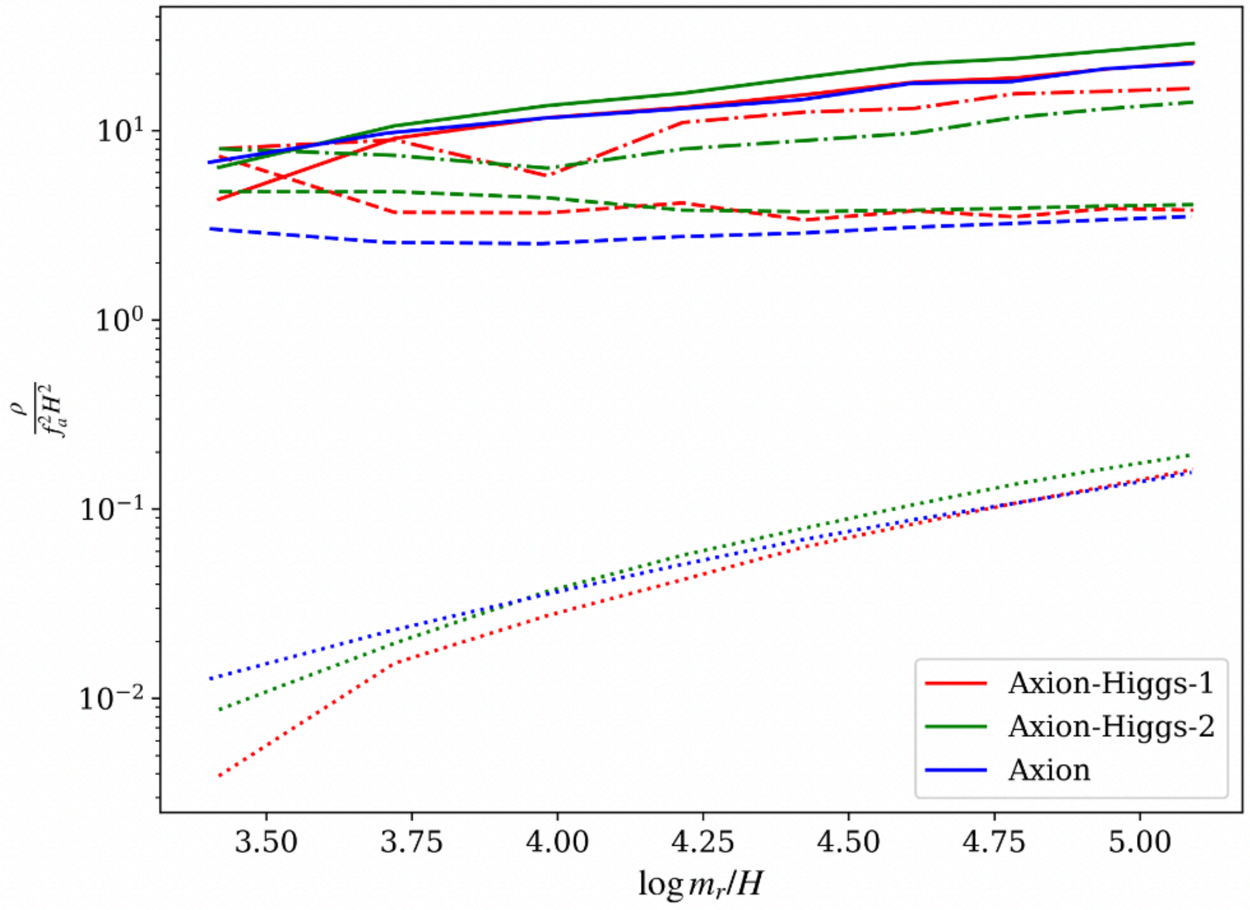
$|\varphi| < 0.1 f_a$

# ▸ 宇宙弦的scaling

scaling parameters




Energy density




$$\xi = \alpha \log(m_r/H) + \beta$$

$$\alpha = 0.096, 0.093, 0.15$$

$$\beta = -0.009, 0.054, -0.27$$

linear growth of log(mr/H),
same as arXiv:2007.04990, 1806.04677

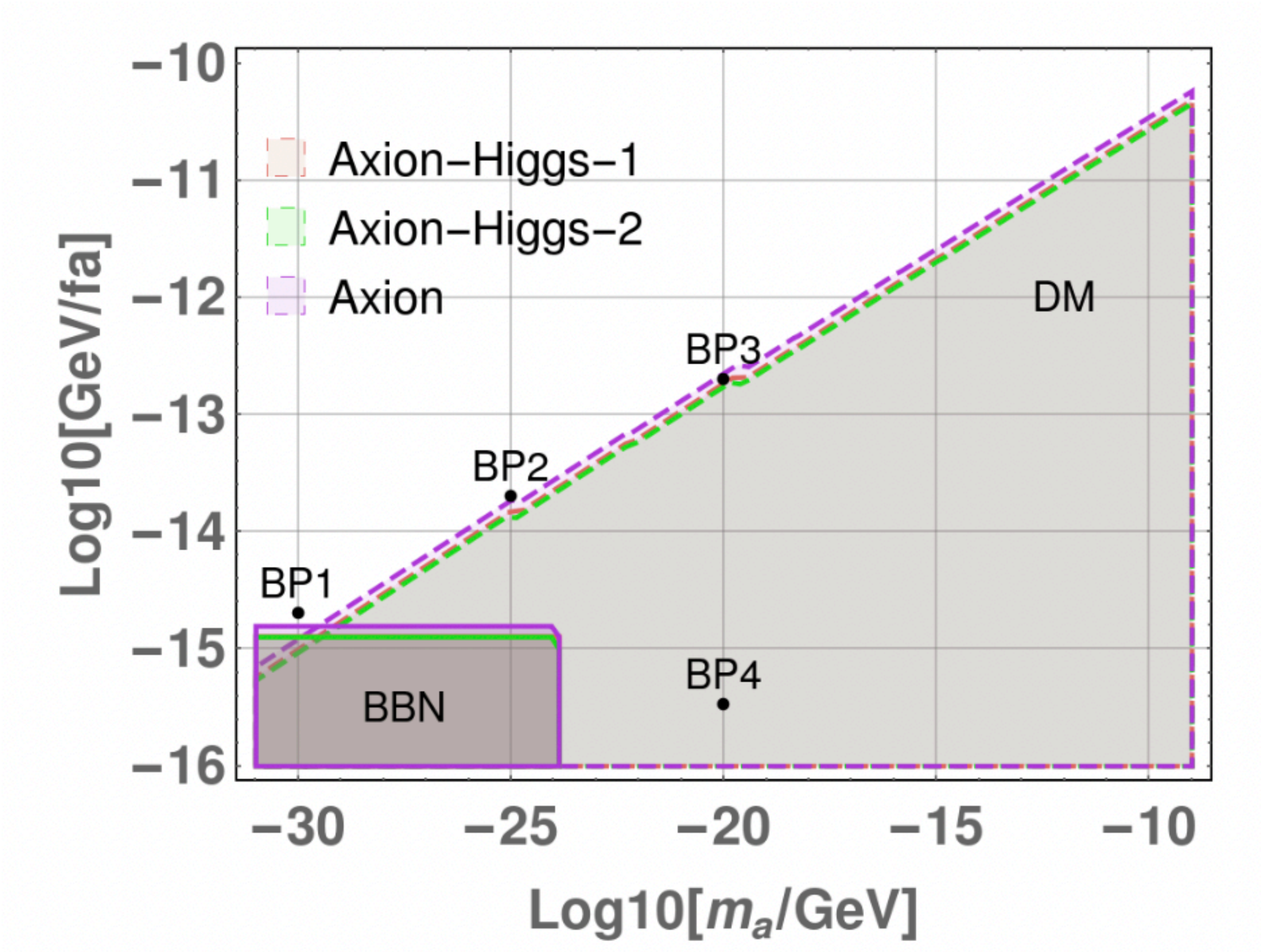
$$\frac{\mathrm{d}\rho_{str}(t)}{\mathrm{d}t} = -2H(t)\rho_{str}(t) - \left[\frac{\mathrm{d}\rho_{str}(t)}{\mathrm{d}t}\right]_{emi},$$

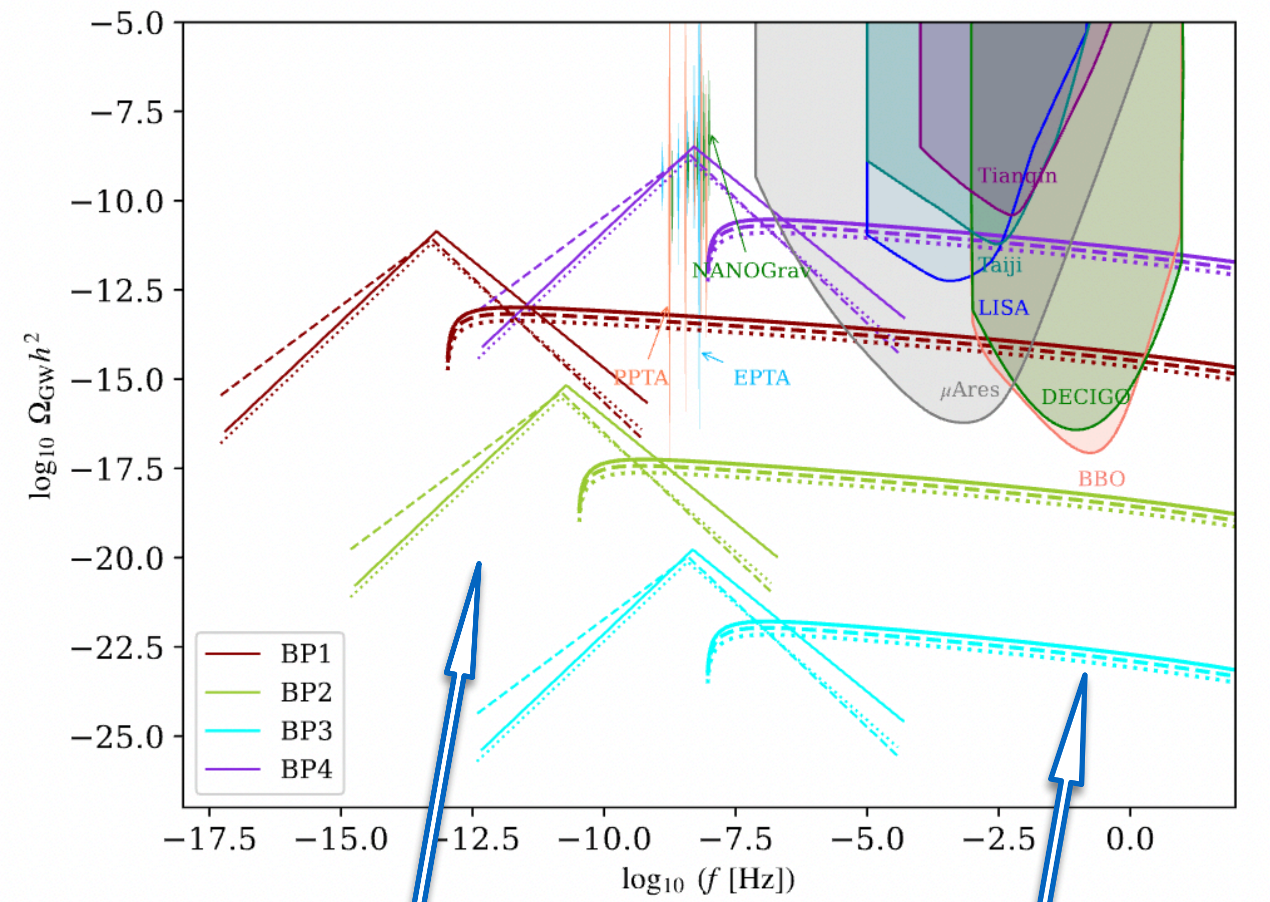
$$\frac{\mathrm{d}\rho_a(t)}{\mathrm{d}t} = -4H(t)\rho_a(t) + \left[\frac{\mathrm{d}\rho_{str}(t)}{\mathrm{d}t}\right]_{emi1},$$

$$\frac{\mathrm{d}\rho_g(t)}{\mathrm{d}t} = -4H(t)\rho_g(t) + \left[\frac{\mathrm{d}\rho_{str}(t)}{\mathrm{d}t}\right]_{emi2}.$$

$$\rho_{gw}/\rho_a \sim \mathcal{O}(10^{-3} - 10^{-2})$$

# ▶ 宇宙弦的观测




Dark Matter relic abundance

$$\Omega_{DM}h^2 \approx 1.6\times 10^9 f_a^2 m_a^2 \xi(-2.25+\log\frac{f_a}{m_a\xi^{1/2}}) \times \epsilon^{-1}(m_a^2 m_{pl}^2 g_*^{1/3})^{-\frac{3}{4}}\ .$$

Dark Radiation

$$\Delta N_{eff} = \xi(\frac{f_a}{10^{15}\mathrm{GeV}})^2\Big(-1.21\times 10^{-3} + 1.67\times 10^{-5}\log^2(\frac{1.12\times 10^{39}\frac{f_a}{10^{15}\mathrm{GeV}}}{\xi^{1/2}})\Big)$$

$$(\mathrm{d}(\mathrm{a}^4\rho_\mathrm{g})/\mathrm{dt}/(\mathrm{d}(\mathrm{a}^4\rho_\mathrm{a})/\mathrm{dt}) = \gamma G\mu^2/f_a^2$$

Method 1

$$F_g(x,y) = \frac{H}{\Gamma_g}\frac{1}{a^3}\frac{\partial}{\partial t}\left(a^3\frac{\rho_g}{\partial k}\right)$$

$$\Gamma_g \simeq 8\pi\xi r G f_a^4 H^3\log^3 m_r/H$$

Method 2

**Gravitational waves provide a new window to probe/constrain beyond standard model physics**

## ❖Axion Dark Matter

- Updated a semi-realistic DM relic abundance from two-stage simulation of axion string wall
- GW of ALP particles in the DFSZ case can be probed by GW detectors
- GW of Axion/ALP dark matter scenario in KSVZ cannot be probed
- Axion minihalo? Axion star?

## ❖Lattice simulation

- Nucleation/Sphaleron  rate simulations
- PT-GW simulation
- Topological defects: Magnetic monopoles, cosmic strings, domain walls, string-wall
- PBH
- Magnetic fields

# 谢谢！