Workshop on Multi-front Exotic phenomena in Particle and Astrophysics (MEPA 2025), Nanjing

Q-balls and boson stars

(Superradiance & multipolar stable solutions)

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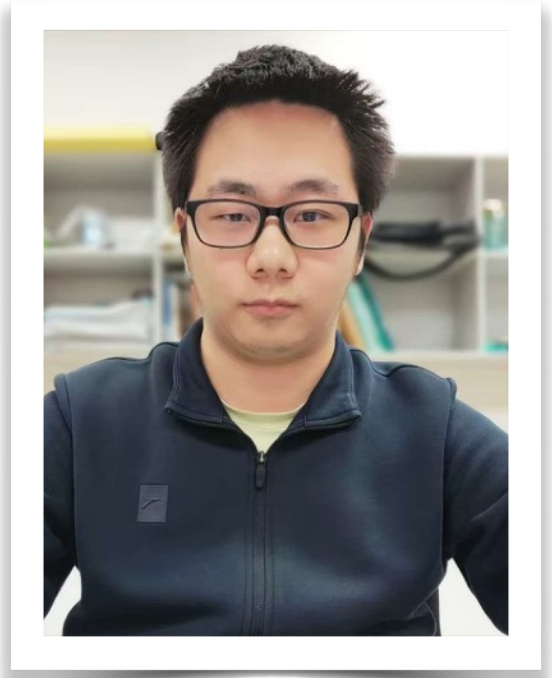
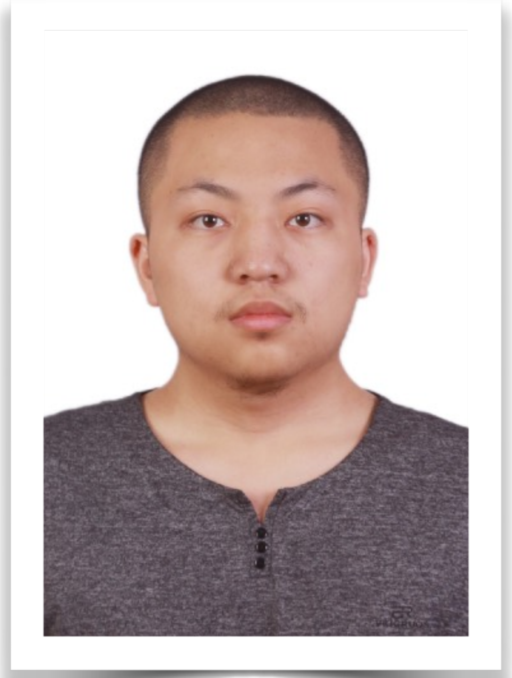
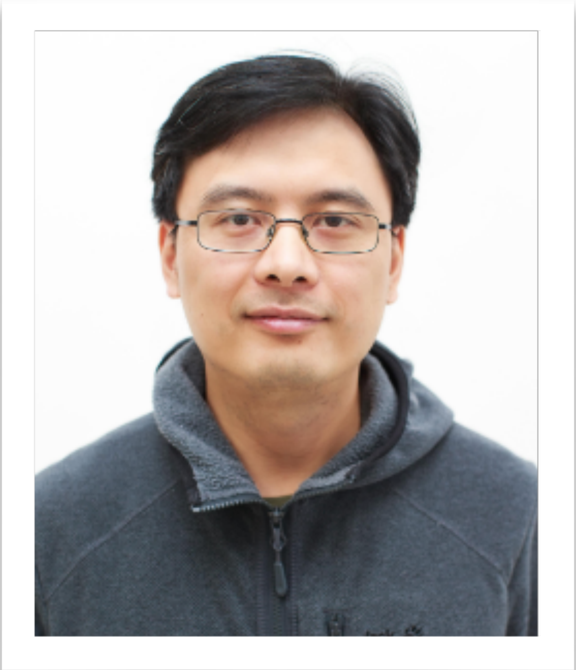




Fu-Ming Chang (常福明), He-Yu Gao (高鹤宇), Xin Meng (孟昕),



Shuang-Yong Zhou (周双勇).

Based in 2412.01894, 2407.12084 and 2411.08985

Outline

Q-balls & boson stars

Superradiance

Multipolar configurations


Q-balls &
boson stars

What is a Q-ball?

A Q-ball is a type of *nonperturbative* structure in a *relativistic field theory*.

In particular it is:

- A non-topological soliton contrary to topological solitons
- A localized object/configuration
- A time-dependent field conf. contrary to static solitons (which ∄ in 3+1 D)

And has a Noether charge Q contrary to periodic quasi-solitons (as oscillons)

Some authors add the requirement of being *stable* configurations.

In the simplest case, these are *classical* objects made of a U(1) symmetric scalar field

$$\mathscr{L}_{\text{field}} = -\nabla_\mu \Phi \nabla^\mu \Phi^* - U(\Phi); \qquad U(\Phi) = \mu^2 |\Phi|^2 + g|\Phi|^4 + h|\Phi|^6$$

Q-balls and quasi-solitons in cosmology

- Affleck-Dine condensate fragmentation (& GWs production)
- As dark matter
- Oscillons from preheating (& GWs production)

Q-balls in particle physics

Full discussion on properties, dynamics and applications in 2411.16604:

Reports on Progress in **Physics**

REVIEW

Non-topological solitons and quasi-solitons

Shuang-Yong Zhou

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What is a boson star?

A boson star is a type self-gravitating structure made of a complex scalar field.

In Einstein's gravity, e.g.: $S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R + \mathscr{L}_{\text{field}} \right], \quad \mathscr{L}_{\text{field}} = -\nabla_\mu \Phi \nabla^\mu \Phi^* - U(\Phi)$

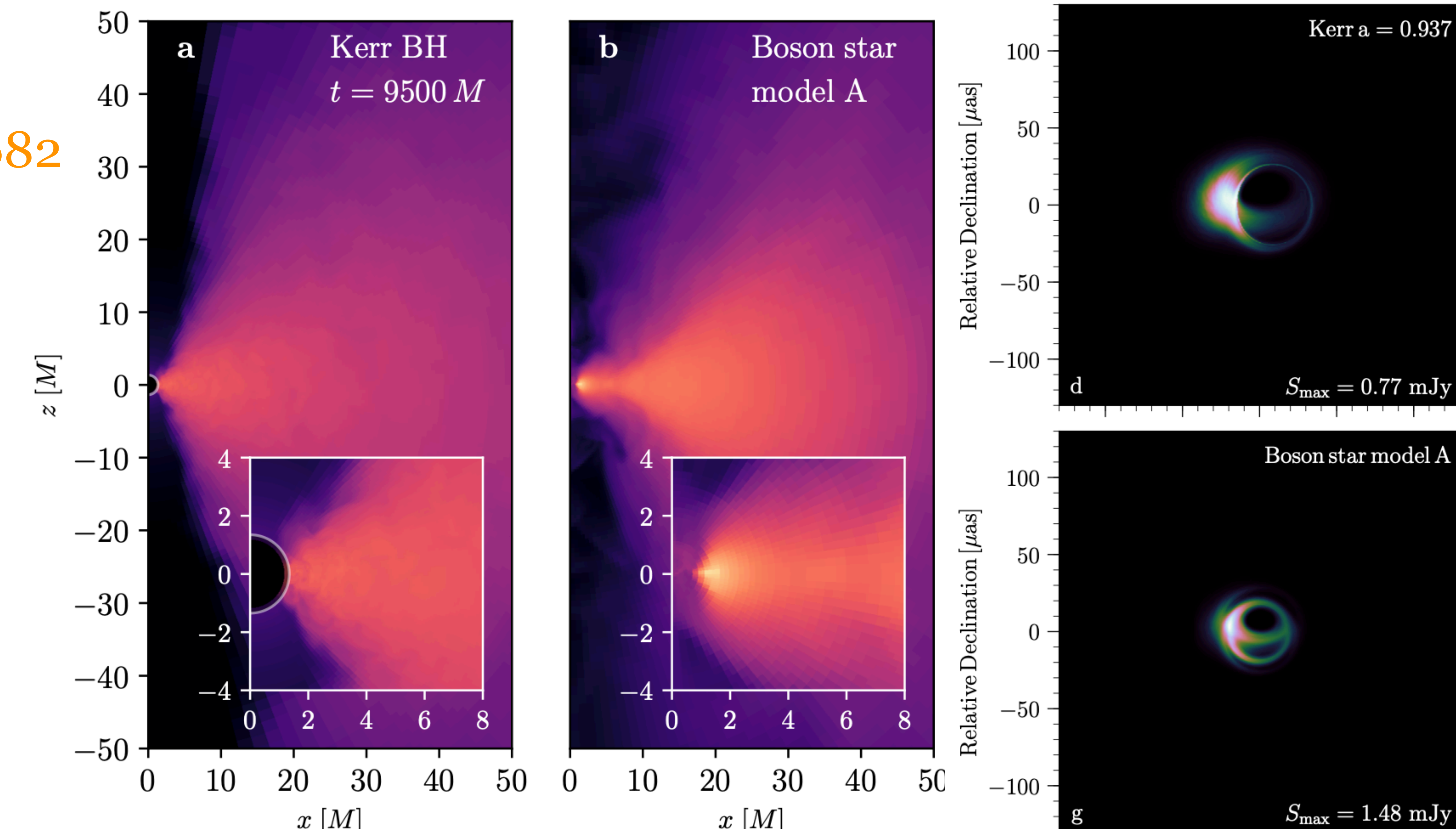
- They can be considered as the strong gravity "cousins" of Q-balls. The simplest solutions are **also**
  $\Phi = f(r)\, e^{i\omega t}$ (spherical, harmonic time-dep.), asymptotically flat, with a static $T_{\mu\nu}$ (hence $g_{\mu\nu}$)
- Gravitational attraction allows simpler potentials, as $U(\Phi) = \mu^2 |\Phi|^2$

Applications of boson stars:

A. Black hole mimickers → Olivares et al. 1809.08682

B. Dark matter halos (at the end of the talk) ULTRALIGHT SCALARS

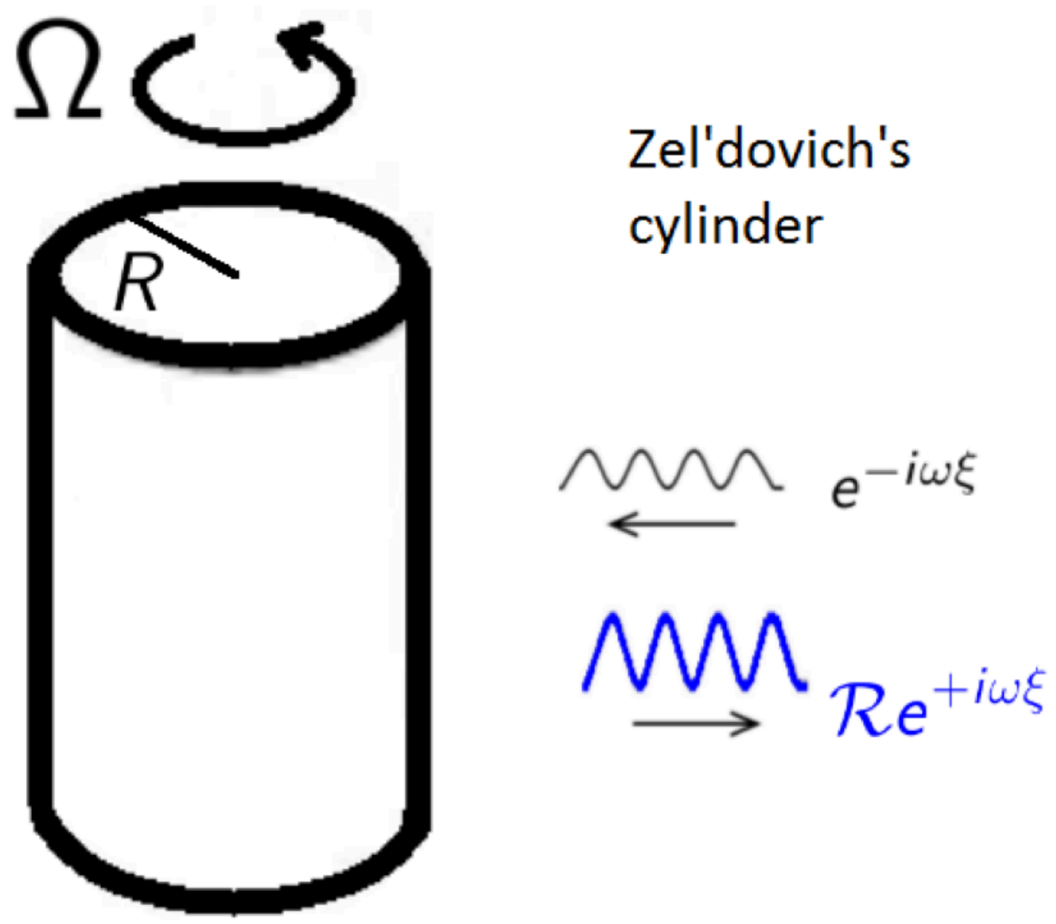
C. Workhorse



Superradiance

How do Q-balls superradiate?

Superradiance is a collection of phenomena where radiation gets amplified
[first studied by Ginzburg & Frank in the 40s]

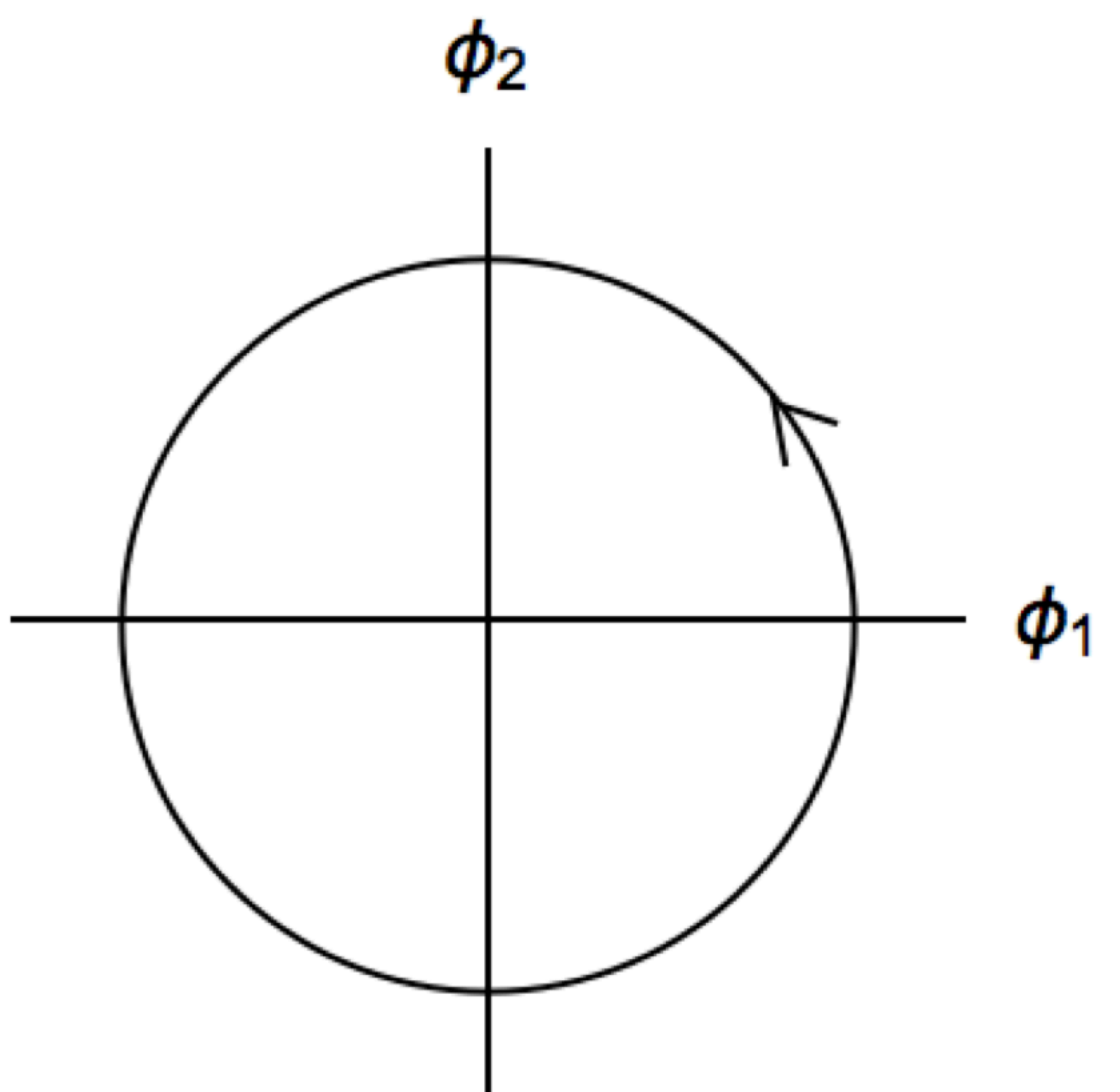


A very famous phenomena is *rotational superradiance*
(Zeldovich 70s) $\omega < m\Omega$
With applications in astrophysics and implications in particle detection, etc.

Recently Saffin, Xie & Zhou (2212.03269) showed that Q-balls can also radiate.

Q-balls coherently rotate in field space and can induce superradiance:

$$\Phi = f(r)e^{i\omega t} = \phi_1 + i\phi_2$$



How do Q-balls superradiate?

Perturbative waves scattering off a Q-ball (following Saffin, Xie & Zhou (2212.03269) or Cardoso, Vicente & Zhong (2307.13734)):

We start with the Klein-Gordon equation in flat spacetime $\partial^\mu\partial_\mu\Phi - \dfrac{\partial U}{\partial\Phi^\dagger} = 0$, and we take the field to be a parturbative wave Θ on top of a Q-ball background $\Phi_Q = f(r)e^{i\omega_Q t}$: $\Phi = \Phi_Q + \Theta$. The eq. Θ satisfy to leading order:

$$\partial^\mu\partial_\mu\Theta - Y(r)\Theta - e^{2i\omega_Q t}W(r)\Theta^\dagger = 0, \text{ with } Y(r) = \left.\frac{\partial^2 U}{\partial\Phi^\dagger\partial\Phi}\right|_{\Phi_Q} \text{ and } W(r) = \left.\frac{\partial^2 U}{\partial\Phi^\dagger\partial\Phi^\dagger}\right|_{\Phi_Q}$$

The minimal scattering solution contains the two modes:

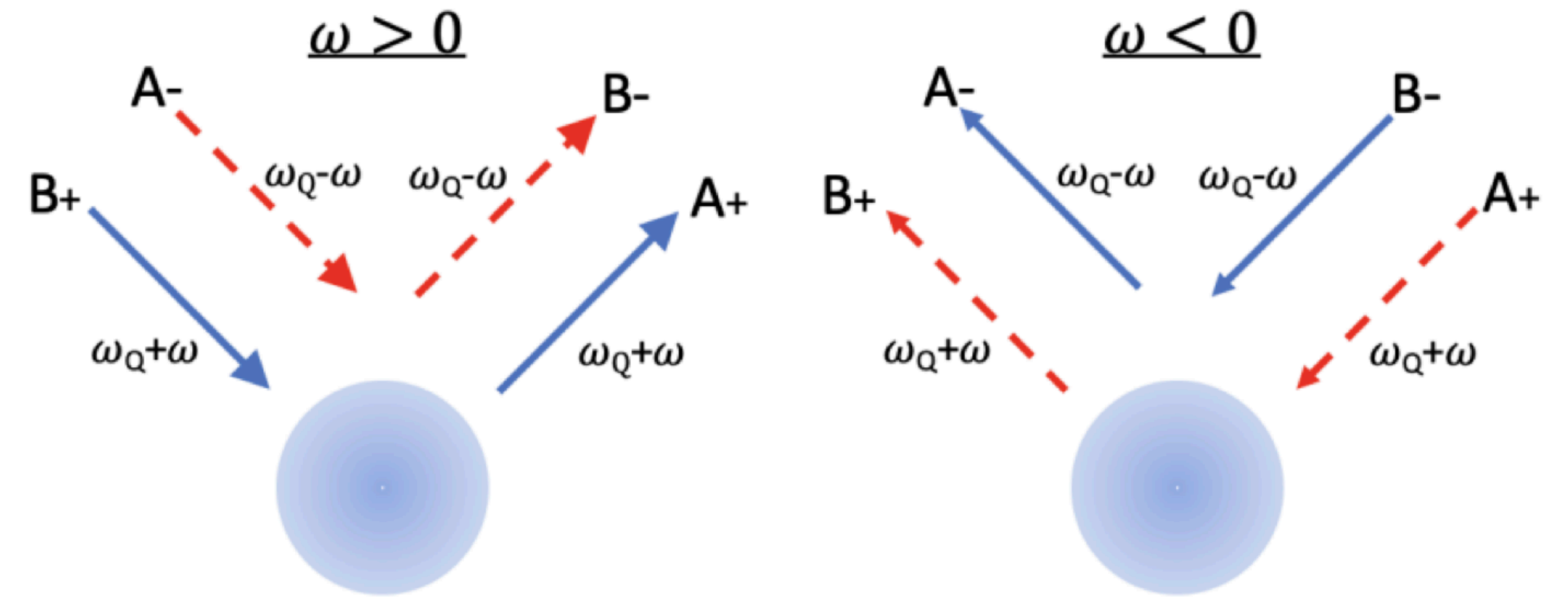
$$\Theta = \eta_+(r)e^{-i\omega_+ t} + \eta_-(r)e^{-i\omega_- t} \quad \text{with} \quad \omega_\pm = \omega_Q \pm \omega$$

How do Q-balls superradiate?

Then we insert the decomposition in the two modes $\eta_\pm$ back in the equation of the wave Θ and solve (numerically) for the *coupled system of ODE for* $\eta_\pm$.

Analyzing the asymptotic behavior -> understand the energy extraction

$r\eta_\pm \xrightarrow[r\to\infty]{} A_\pm e^{ik_\pm r} + B_\pm e^{-ik_\pm r}$ where $k_\pm^2 = \omega_\pm^2 - \mu^2$

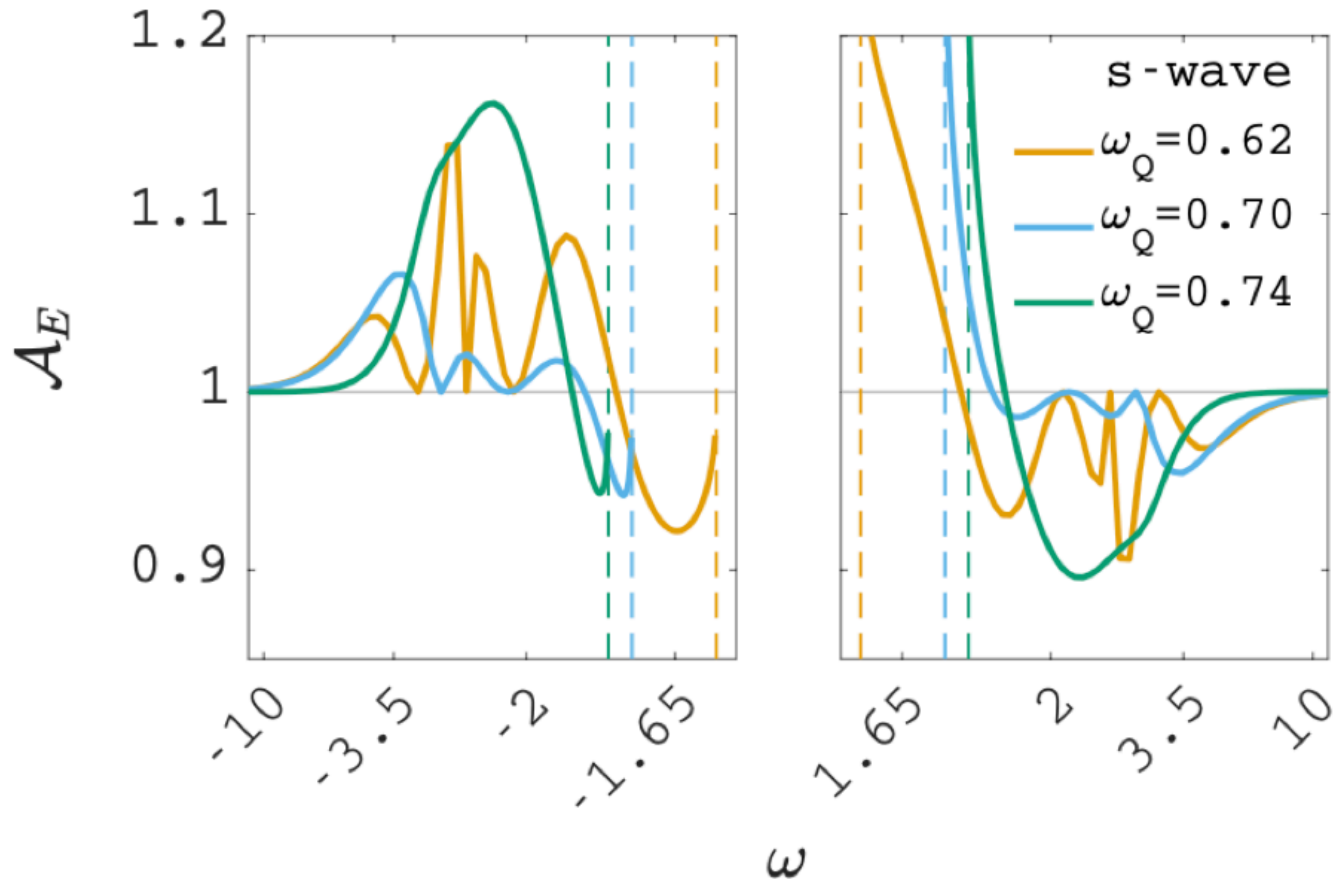


The coefficients $A_\pm$, $B_\pm$ allow to calculate the energy and energy flux amplification factors:

$$\mathscr{A}_E = \left(\frac{\frac{\omega_+^2}{k_+^2}\left|A_+\right|^2 + \frac{\omega_-^2}{k_-^2}\left|B_-\right|^2}{\frac{\omega_-^2}{k_-^2}\left|A_-\right|^2 + \frac{\omega_+^2}{k_+^2}\left|B_+\right|^2} \right)^{\text{sign}(\omega)} \qquad \mathscr{A}_{tr} = \left(\frac{\frac{\omega_+}{k_+}\left|A_+\right|^2 - \frac{\omega_-}{k_-}\left|B_-\right|^2}{\frac{\omega_+}{k_+}\left|B_+\right|^2 - \frac{\omega_-}{k_-}\left|A_-\right|^2} \right)^{\text{sign}}$$

Similar amplification factors $\mathscr{A}_L$ and $\mathscr{A}_{r\varphi}$ can be extracted if one allow either the Q-ball or the wave to *physically* rotate.

Q-ball superradiance results
& Recent developments



$$\Theta = \eta_+(r)e^{-i\omega_+ t} + \eta_-(r)e^{-i\omega_- t}$$

with $\omega_\pm = \omega_Q \pm \omega$

One-ingoing mode 3D static

After the discovery of this phenomena in Saffin, Xie & Zhou (2212.03269) where the 2D case was explored with and without rotation together with the static 3D case the analysis of superradiance of solitons in Minkowski spacetime **extended:**

A. Cardoso, Vicente & Zhong (2307.13734) [time-domain in 2D and insights about other solitons]

B. Zhang, Chang, Saffin, Xie, Zhou (2402.03193) [all 3D cases including rotating Q-ball]

Boson star superradiance

It is possible to include the effects of gravity of a Q-ball and explore the superradiance of Q-stars (boson star with a potential that supports Q-balls). Again consider the potential

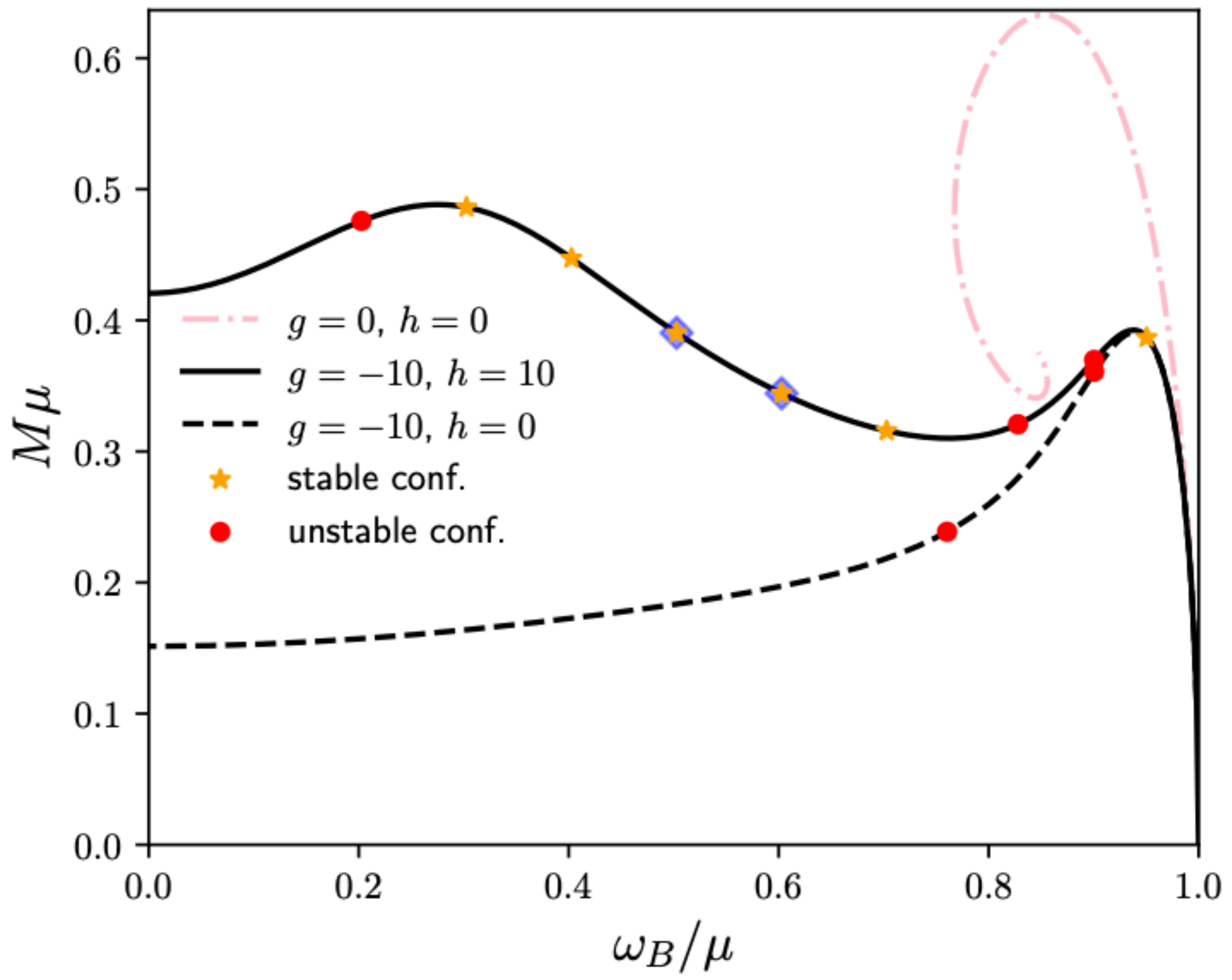
$$U(\Phi) = \mu^2 |\Phi|^2 + g|\Phi|^4 + h|\Phi|^6$$

And consider the system $$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2 R + \mathscr{L}_{\text{field}}\right] \qquad \mathscr{L}_{\text{field}} = -\nabla_\mu \Phi \nabla^\mu \Phi^* - U(\Phi)$$

In spherical symmetry a solution to the e.o.m. resulting from this system (the *Einstein-Klein -Gordon* system) is obtained after considering the static ansatz

$$\Phi_B = f(r)e^{i\omega t} \quad \&$$
$$ds^2 = -\alpha^2(r)dt^2 + \Psi^4(r)(dr^2 + r^2 d\Omega^2)$$

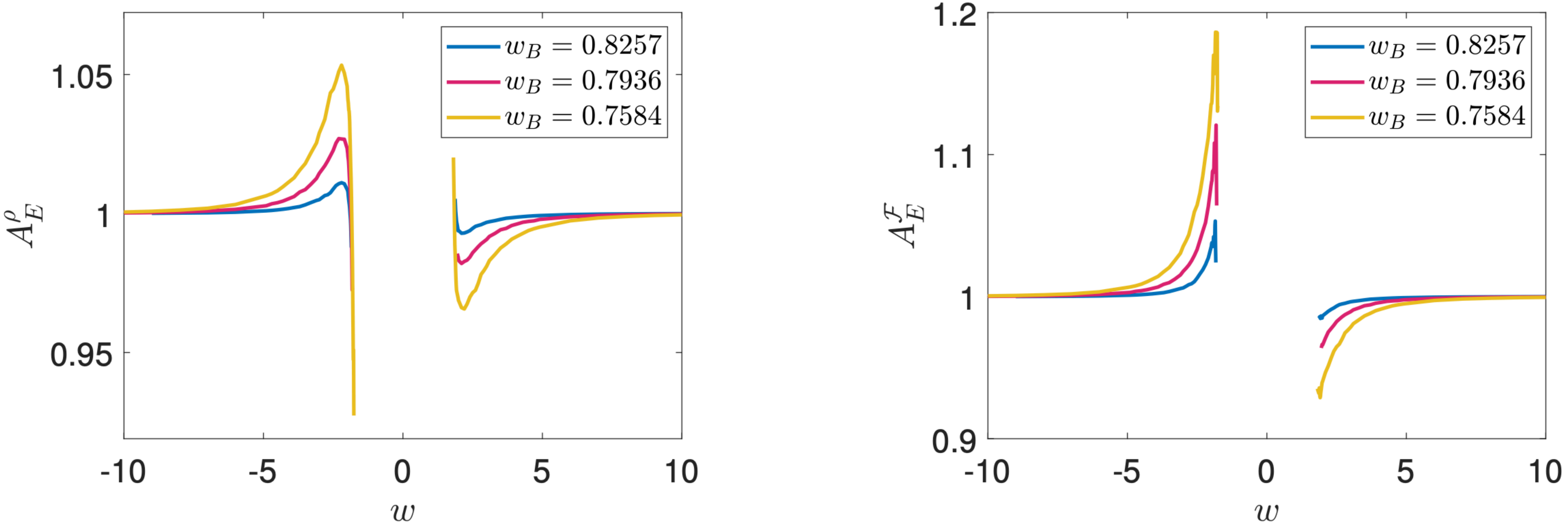
And solving for f, α and Ψ.



Boson star superradiance

Recently, Gao, Saffin, Wang, Xie & Zhou 2306.01868 explored the Q-star superradiance in spherical symmetry and for non-spinning waves obtaining that "[…] the same energy extraction mechanism still works for boson stars."

$$U(\Phi) = \mu^2|\Phi|^2 + g|\Phi|^4 + h|\Phi|^6, \quad S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_P^2 R + \mathscr{L}_{\text{field}}\right], \quad \mathscr{L}_{\text{field}} = -\nabla_\mu\Phi\nabla^\mu\Phi^* - U(\Phi)$$



Boson star superradiance

Chang, Gao, **VJ**, Meng, Zhou (2025) 2412.01894
"Boson star superradiance with spinning effects and in time domain"

- *Rotating* $U = \mu^2 |\Phi|^2$ boson stars are not stable, but *rotating* Q-stars are!

- Extended the linearized results (frequency domain) of Gao, Saffin, Wang, Xie & Zhou 2306.01868 to the case of a spinning boson star and/or spinning waves

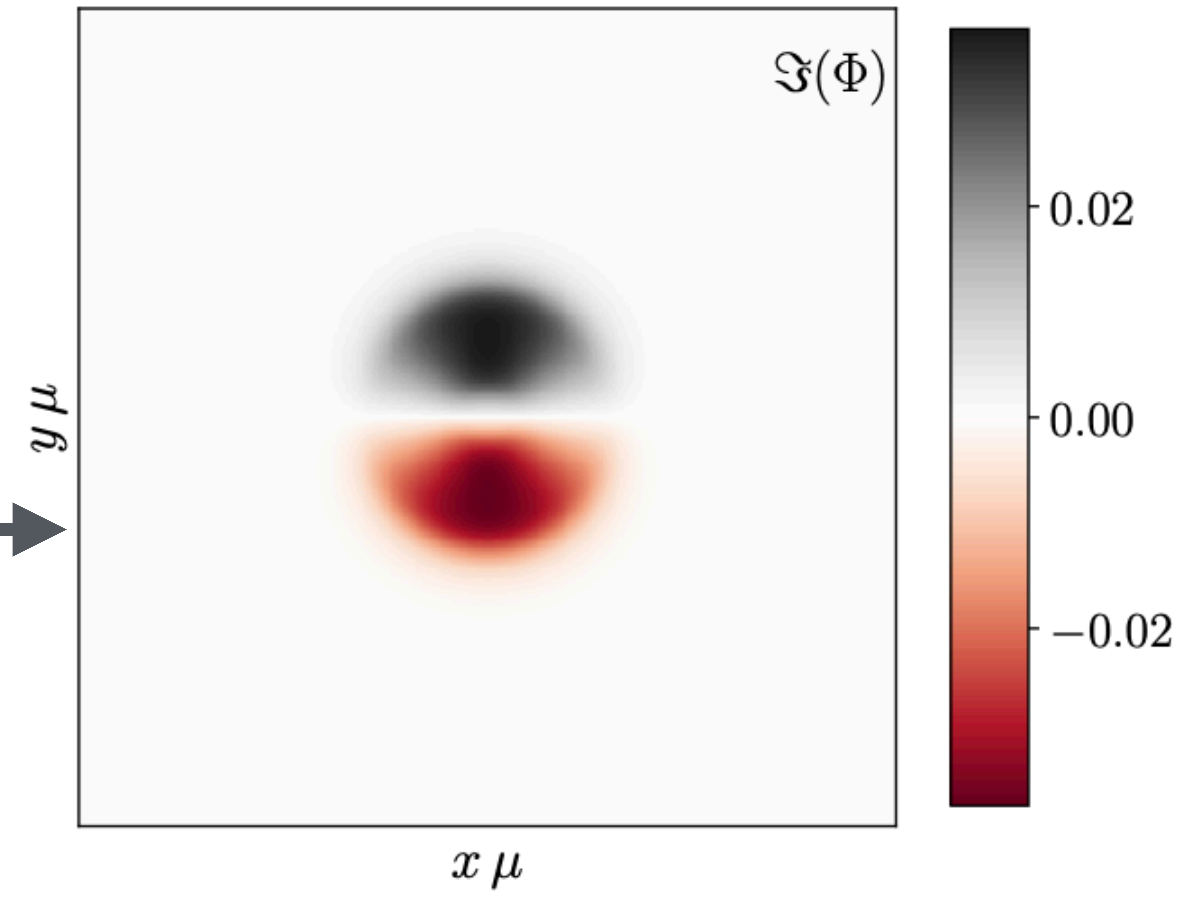
- Obtaining spinning boson stars requires to solve the elliptic PDE coming from the Einstein-Klein-Gordon system under the stationary ansatz

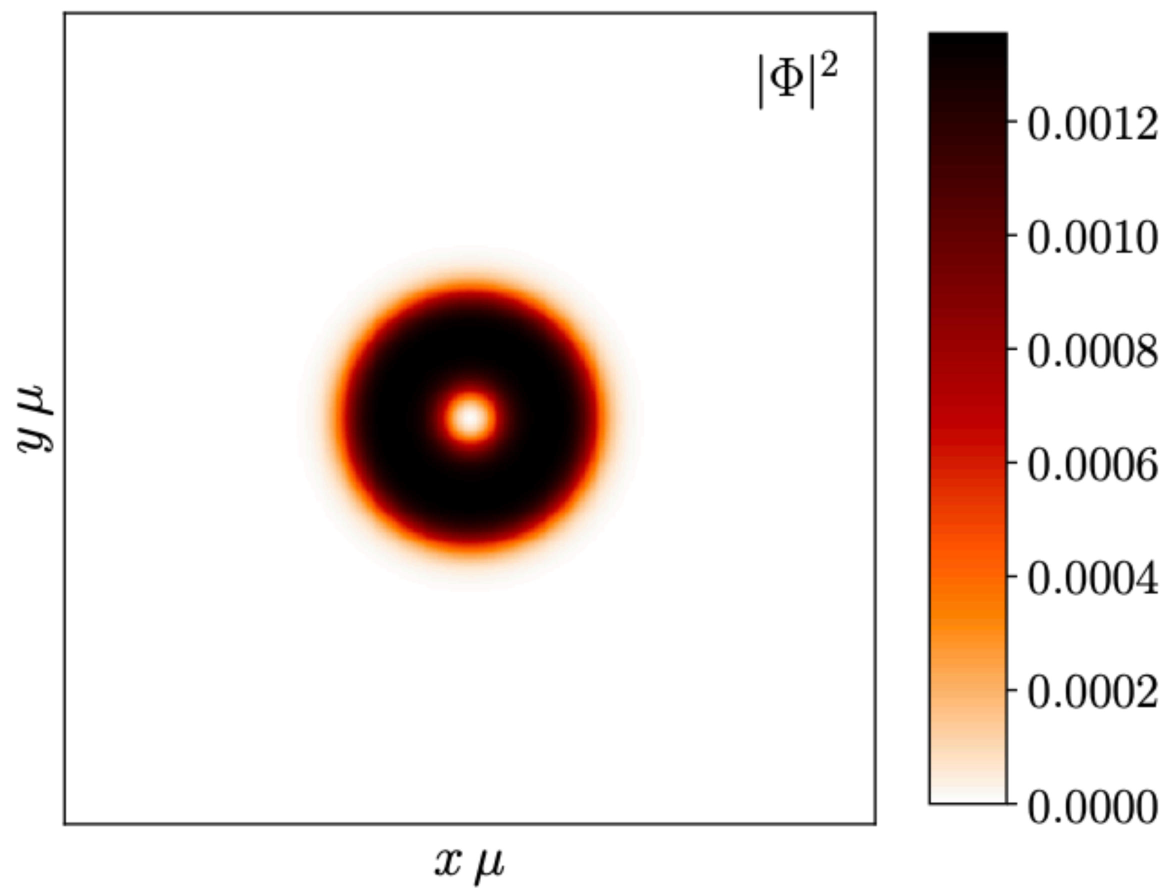
$$g_{\mu\nu}dx^\mu dx^\nu = e^{2F_0}dt^2 + e^{2F_1}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2}r^2\sin^2\theta\left(d\varphi - wdt\right)^2 , \qquad \Phi_B = \phi(r,\theta)e^{-i(\omega_B t - m_B\varphi)}$$

- Also the wave solution $\eta_\pm$ demand the solution of PDEs, to which we solve using spectral methods

KADATH SPECTRAL SOLVER

Typical solution in field space



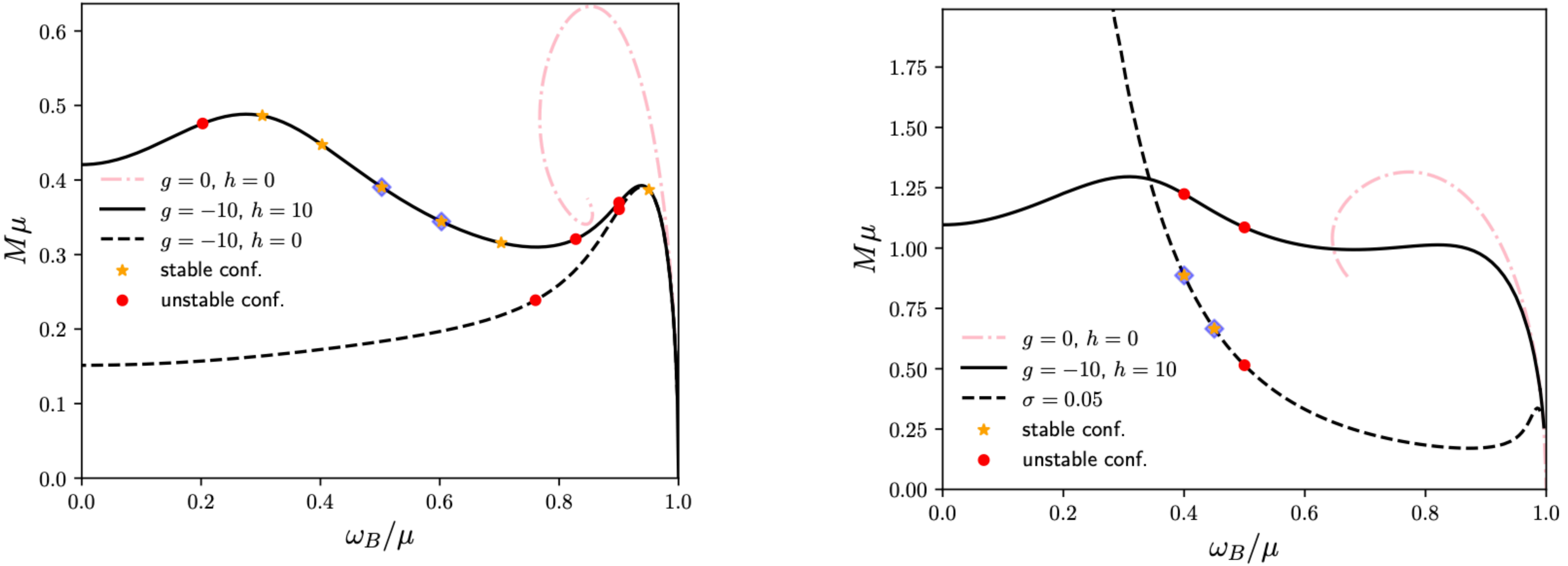


Chang, Gao, **VJ**, Meng, Zhou (2025) 2412.01894

"Boson star superradiance with spinning effects and in time domain"

$$g_{\mu\nu}dx^\mu dx^\nu = e^{2F_0}dt^2 + e^{2F_1}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2}r^2\sin^2\theta\left(d\varphi - w dt\right)^2, \qquad \Phi_B = \phi(r,\theta)e^{-i(\omega_B t - m_B\varphi)}$$

- Backgound sequence of solutions and stability tests for $m_B = 0$ & $m_B = 1$:



Chang, Gao, **VJ**, Meng, Zhou (2025) 2412.01894
"Boson star superradiance with spinning effects and in time domain"

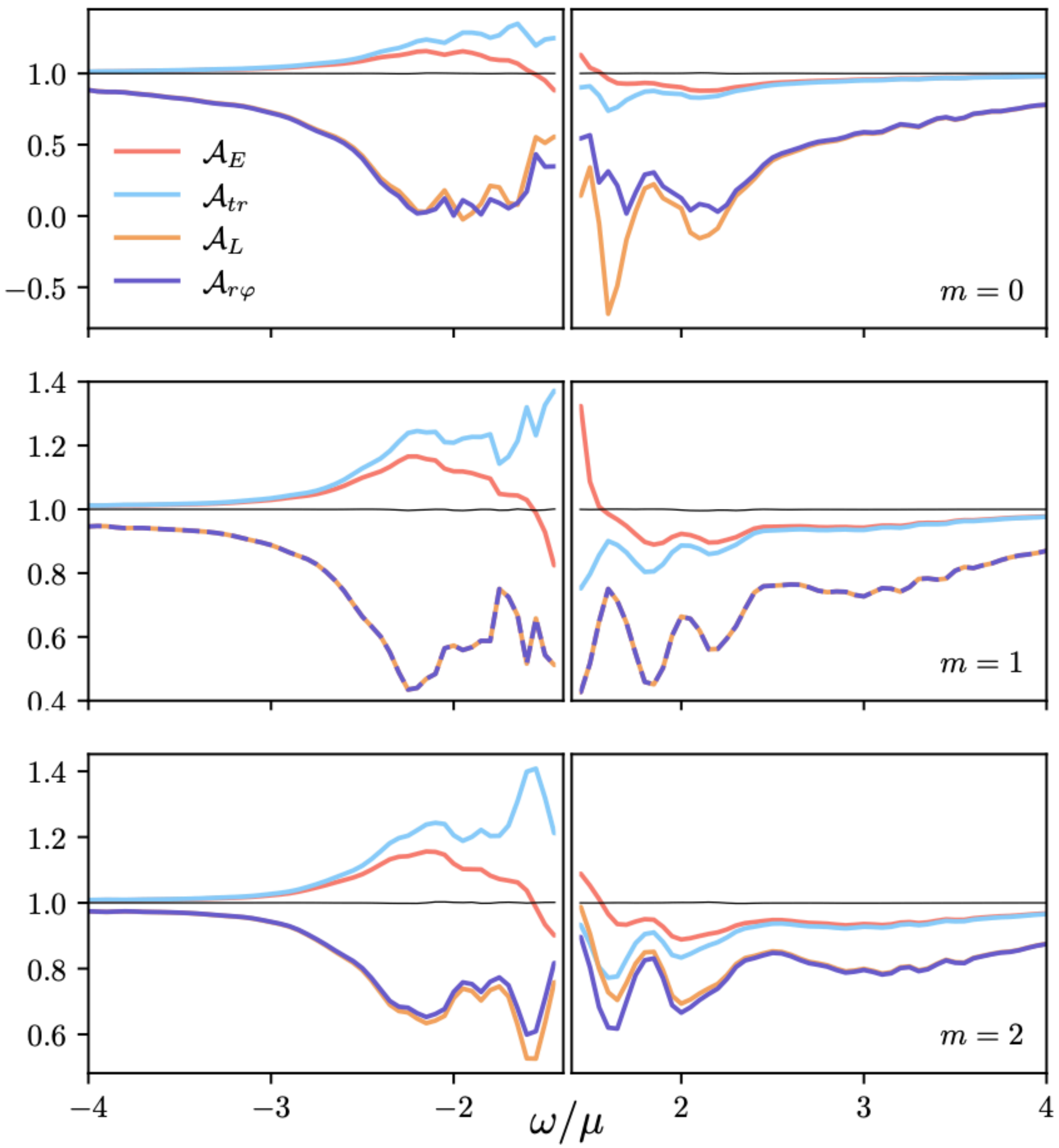
- Perturbative wave amplification factor results:

$$\Theta(t, \mathbf{r}) = \eta^+(r,\theta)\, e^{-i\omega_+ t + i m_+ \varphi} + \eta^-(r,\theta)\, e^{-i\omega_- t + i m_- \varphi}\,.$$

with

$$\omega_\pm = \omega_B \pm \omega, \quad m_\pm = m_B \pm m\,.$$

and *for example* a background boson star with $g = 10 = -h$ and $\omega_B = 0.40$



Boson star superradiance

Self-consistent, non-linear, 3D, **Numerical Relativity** evolutions to study a wavepacket interacting with a Q-star.

- Consistency check for small amplitudes
- Non-linear effects
- Possibility of analyze any superradiant instability

3+1 split of the spacetime metric:

$$ds^2 = -\left(\alpha^2 - \beta_i\beta^i\right)dt^2 + 2\beta_i dt dx^i + \gamma_{ij}dx^i dx^j$$

Initial data constraint equations
(Hamiltoninan & momentum constraints)

Kadath spectral solver

With Θ a (spinning) wave packet
-> Re-solve gravitational constraints for the new $T_{\mu\nu}$

$$\Phi = \Phi_B + \Theta$$

$$\Theta(r) = \frac{\delta\phi(0)}{r_0}\exp\left[-\frac{(r-r_0)^2}{2\sigma_r^2}\right] \times \exp\left[-is_{\omega_0}\sqrt{\omega_0^2-1}\,r - i\omega_0 t\right]_{t=0}$$

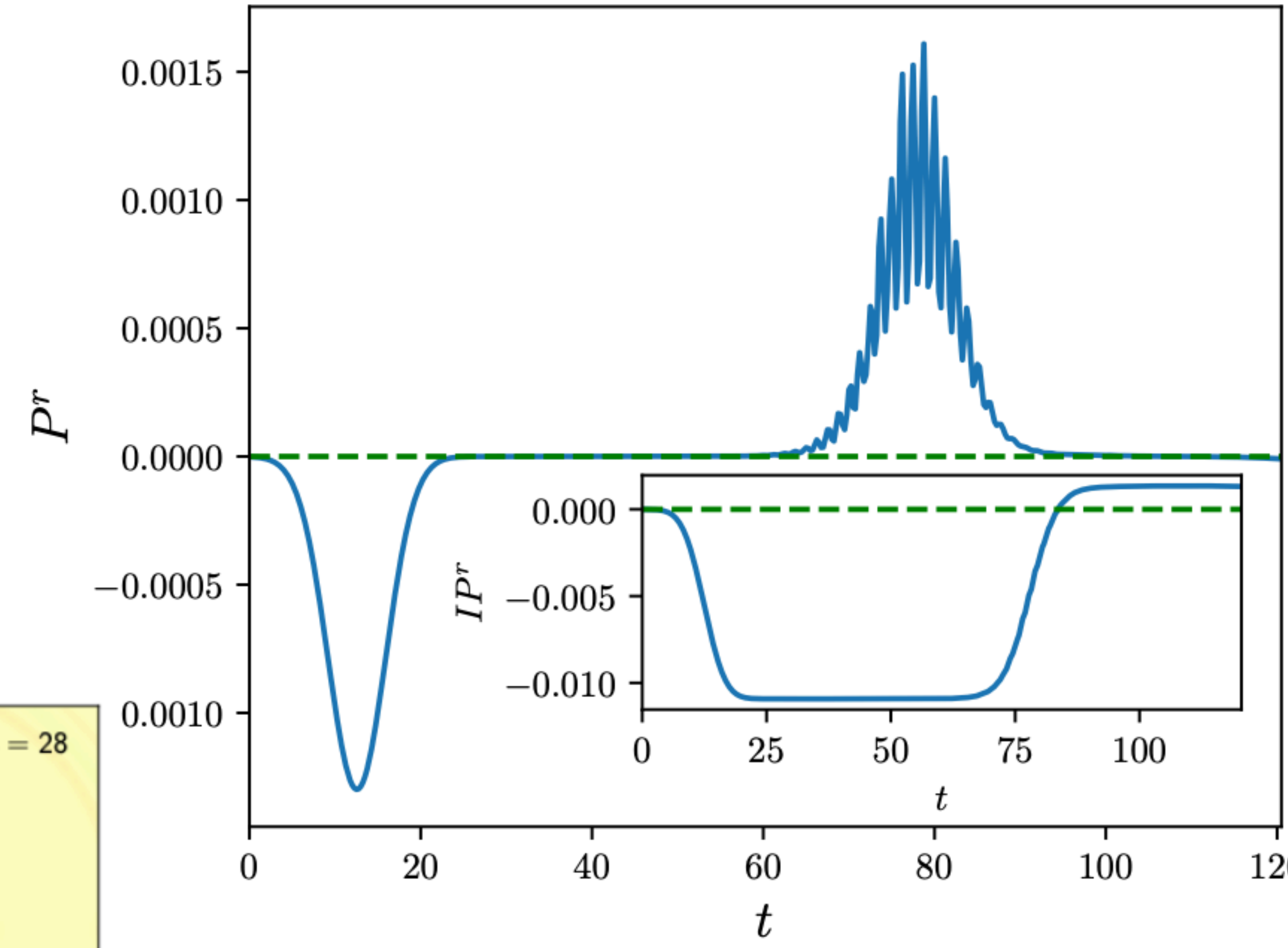
Evolution through the BSSN (well-posed) formulation

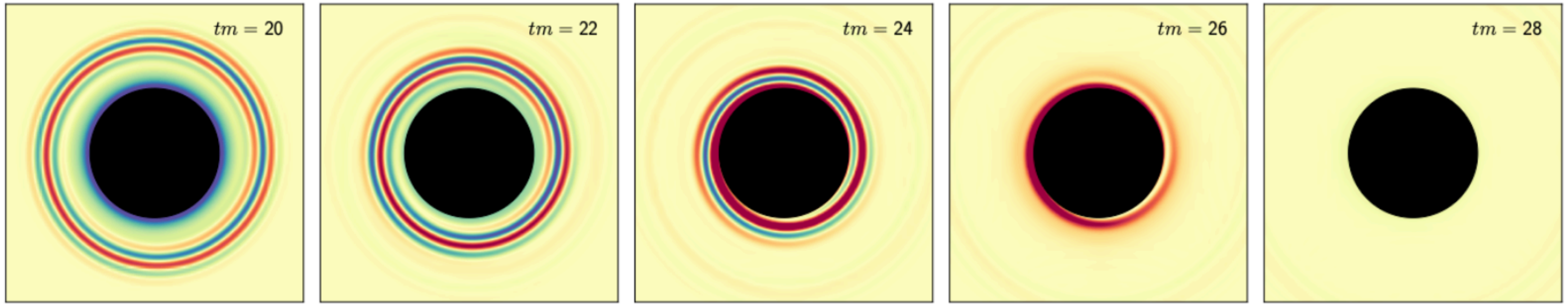
Einstein toolkit
(3D numerical relativity code)

Chang, Gao, **VJ**, Meng, Zhou (2025) 2412.01894
"Boson star superradiance with spinning effects and in time domain"

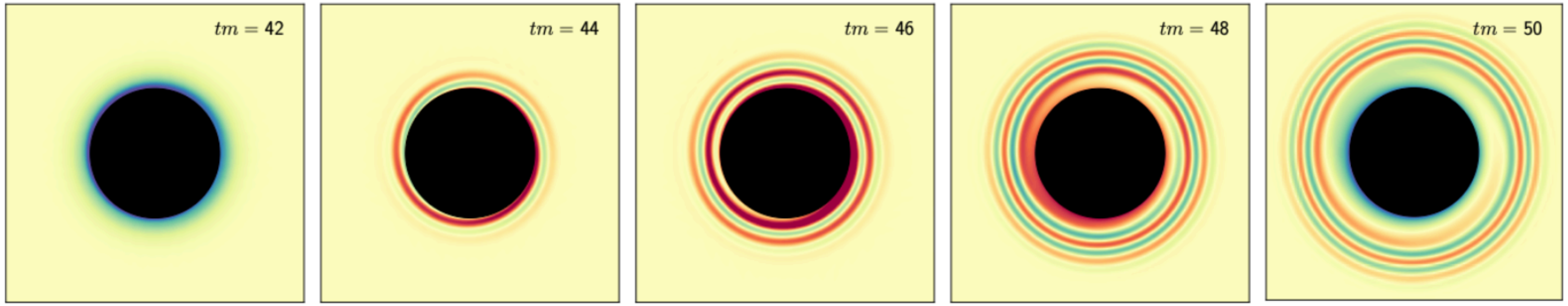
Numerical Relativity evolutions to study a wavepacket interacting with a Q-star.

- Consistency check for small amplitudes

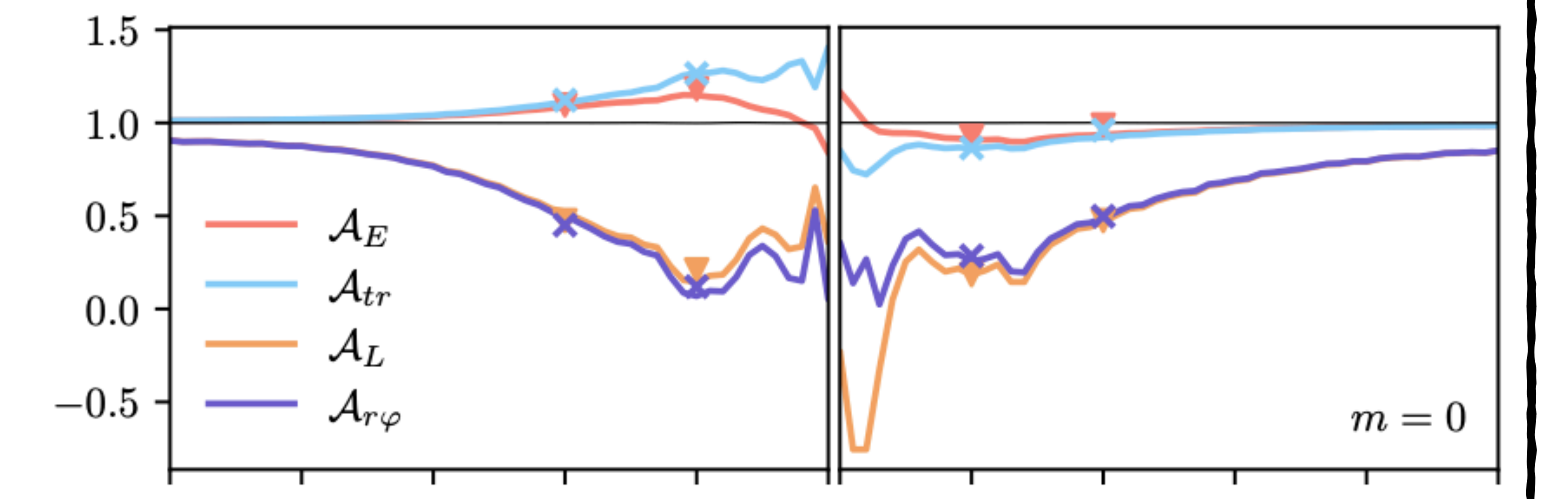




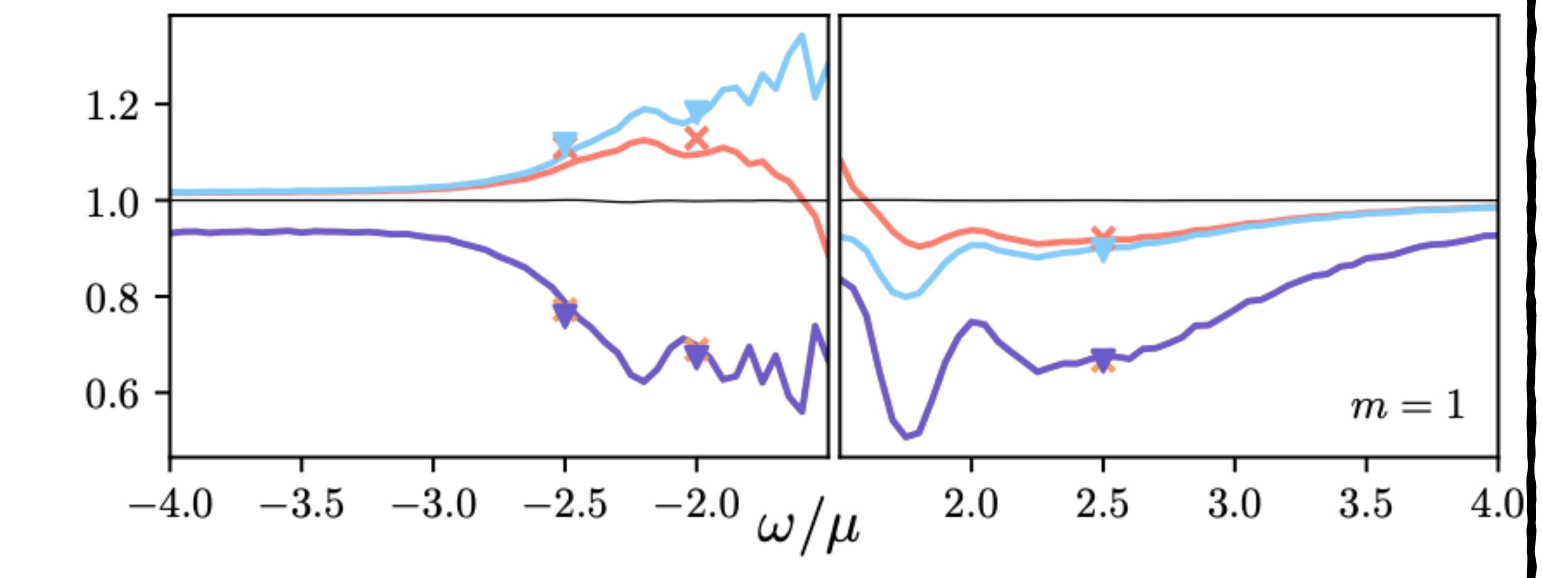
(a) Wavepacket entering the star, $\Re(\Phi)$ in the equatorial plane.



(b) Wavepacket leaving the star, $\Re(\Phi)$ in the equatorial plane.

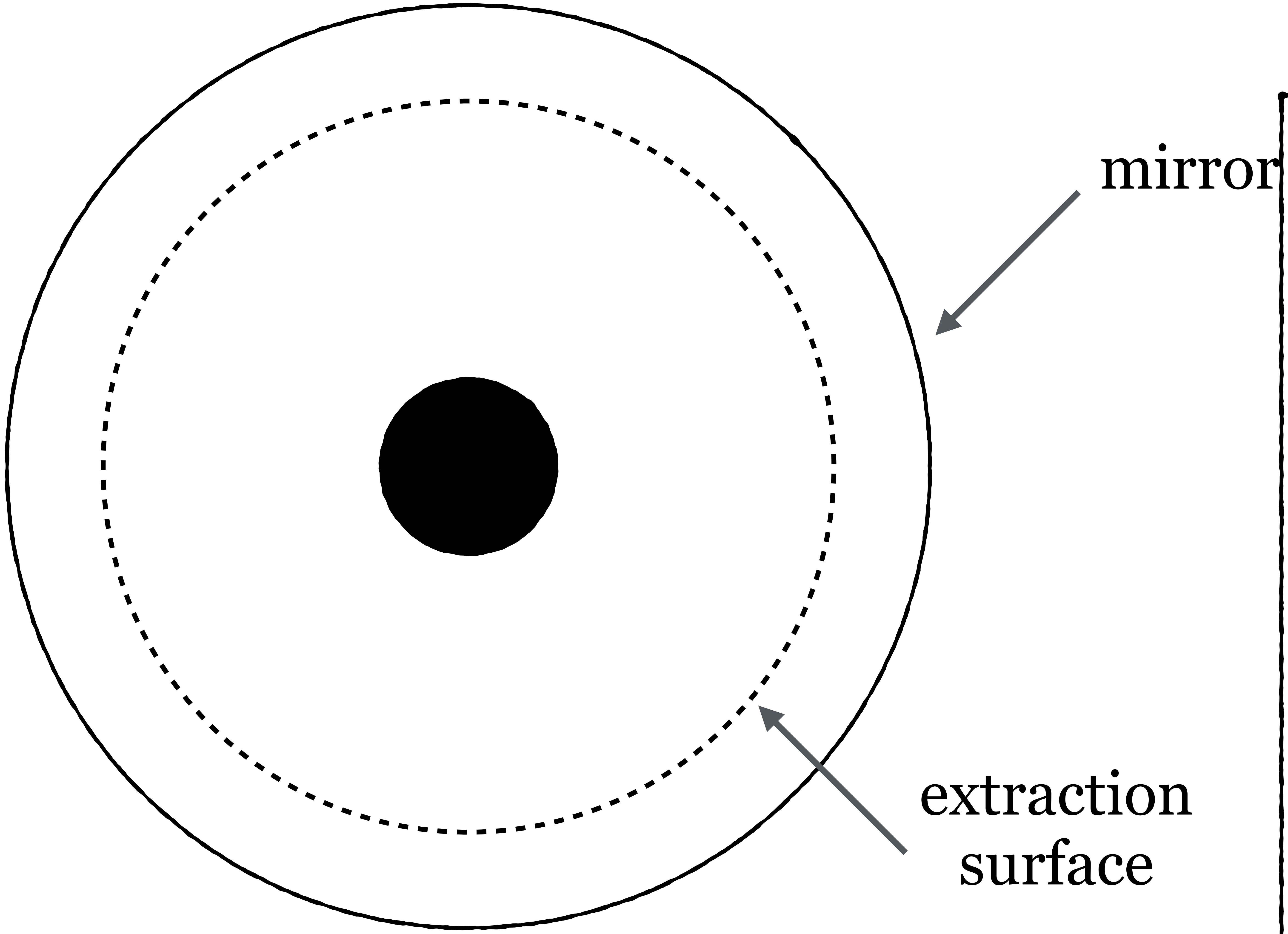


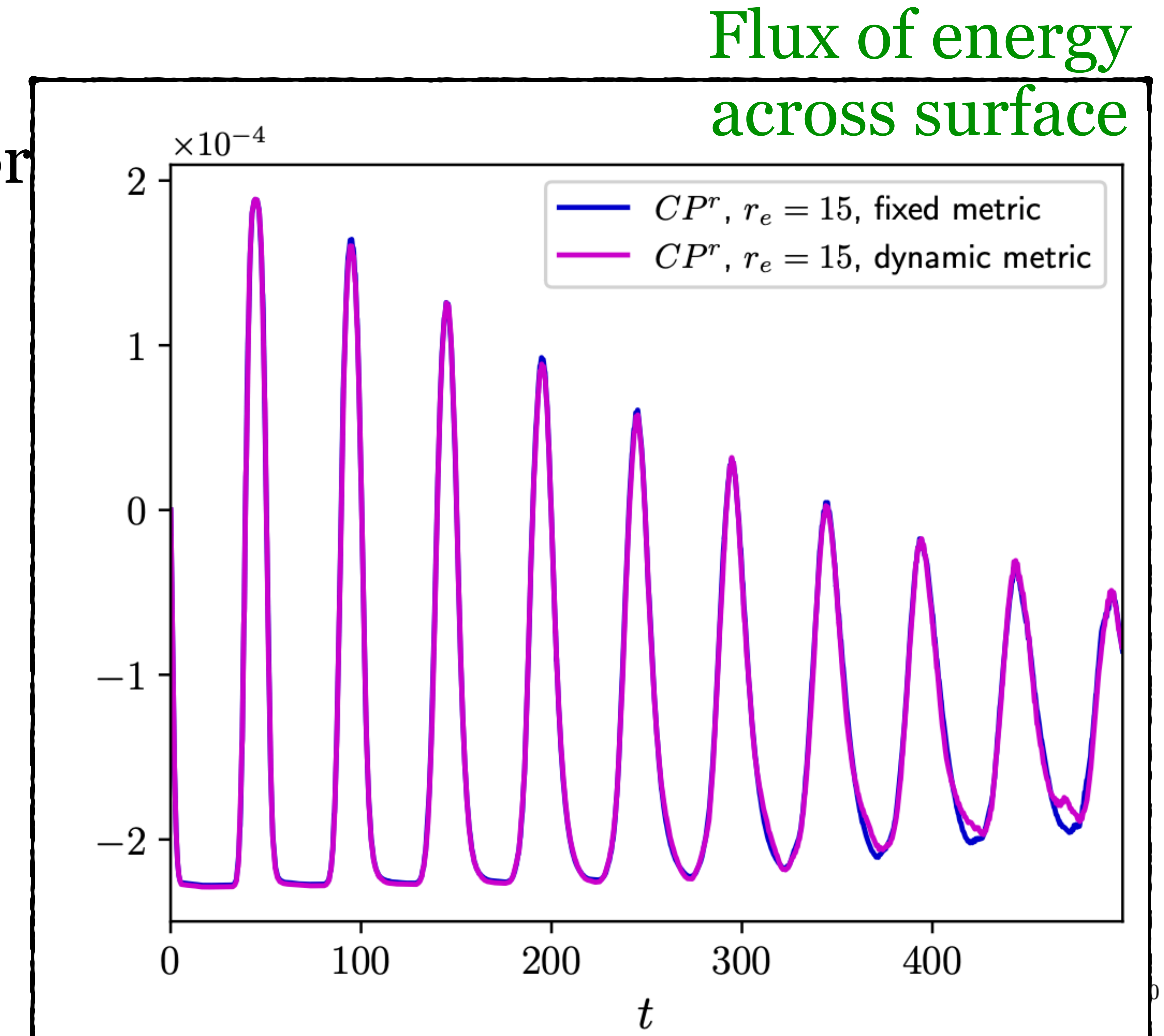
Consistent with linear

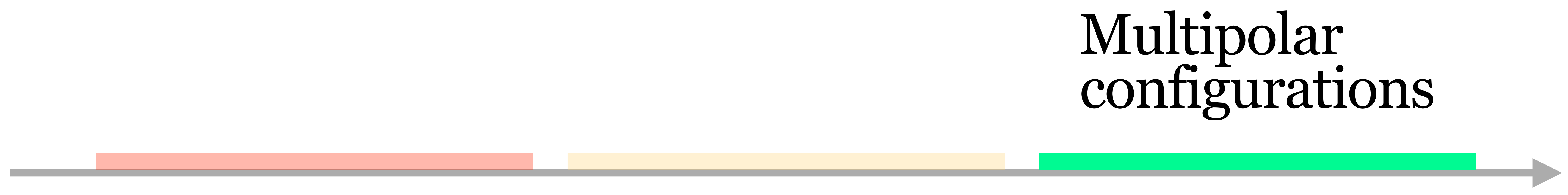


Chang, Gao, **VJ**, Meng, Zhou (2025) 2412.01894
"Boson star superradiance with spinning effects and in time domain"

Numerical Relativity evolutions inside a **cavity** with a boson star






Multipolar
configurations

Charge-swapping Q-balls

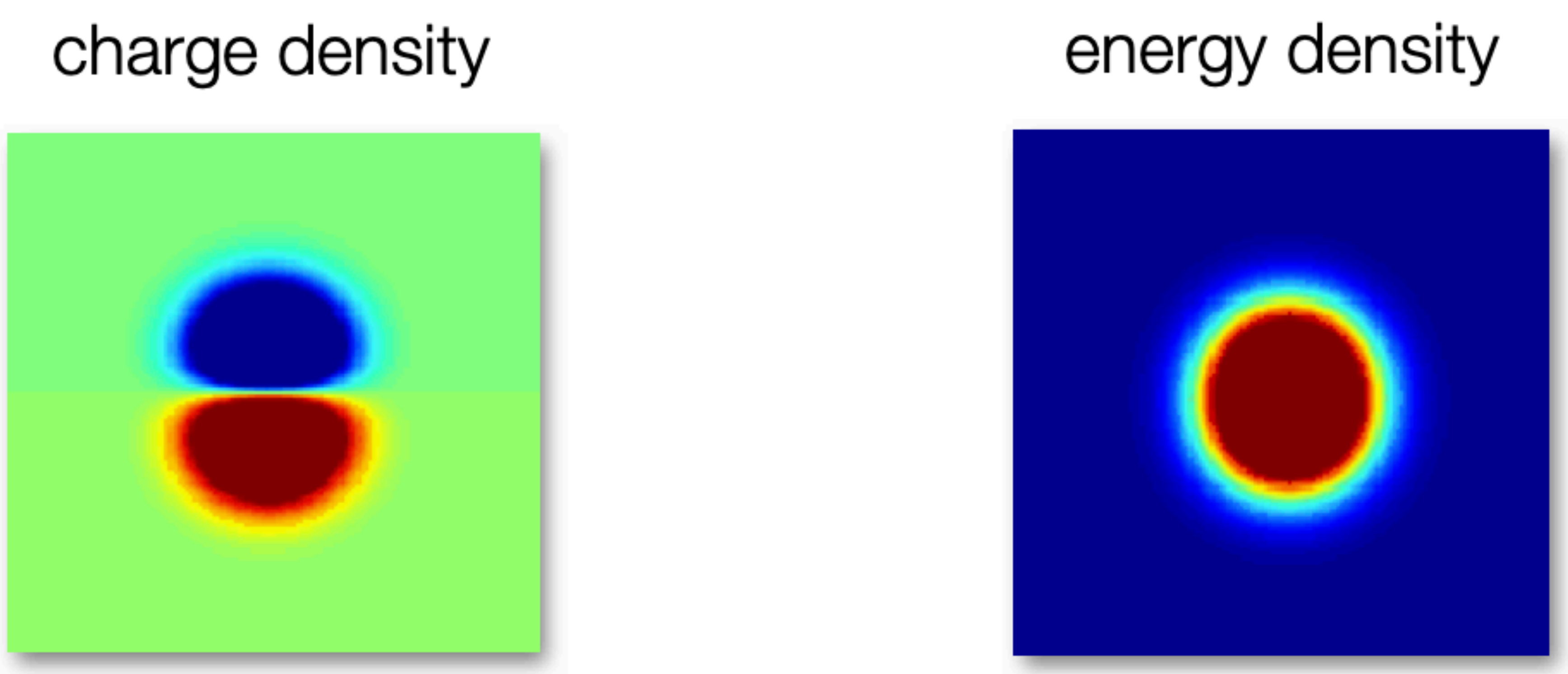
Q-balls exist beyond the static and spinning cases:

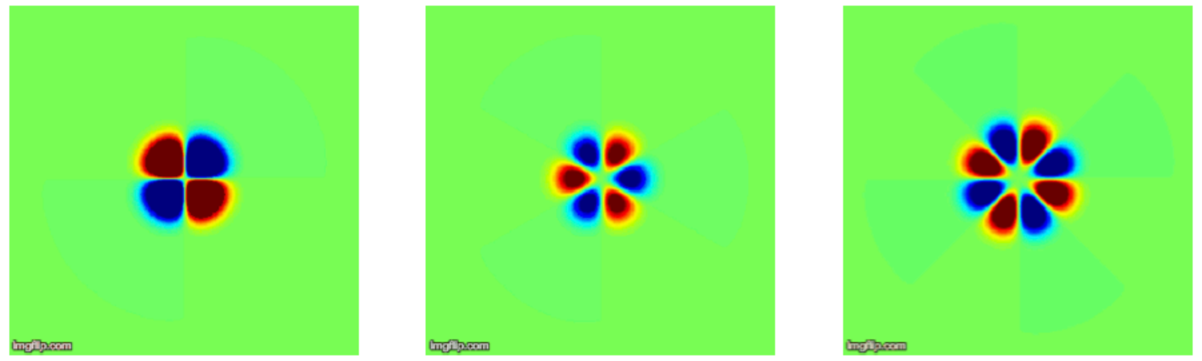
Charge swapping Q-balls are:

- *Quasi-stationary* configurations
Prepared (initially) from

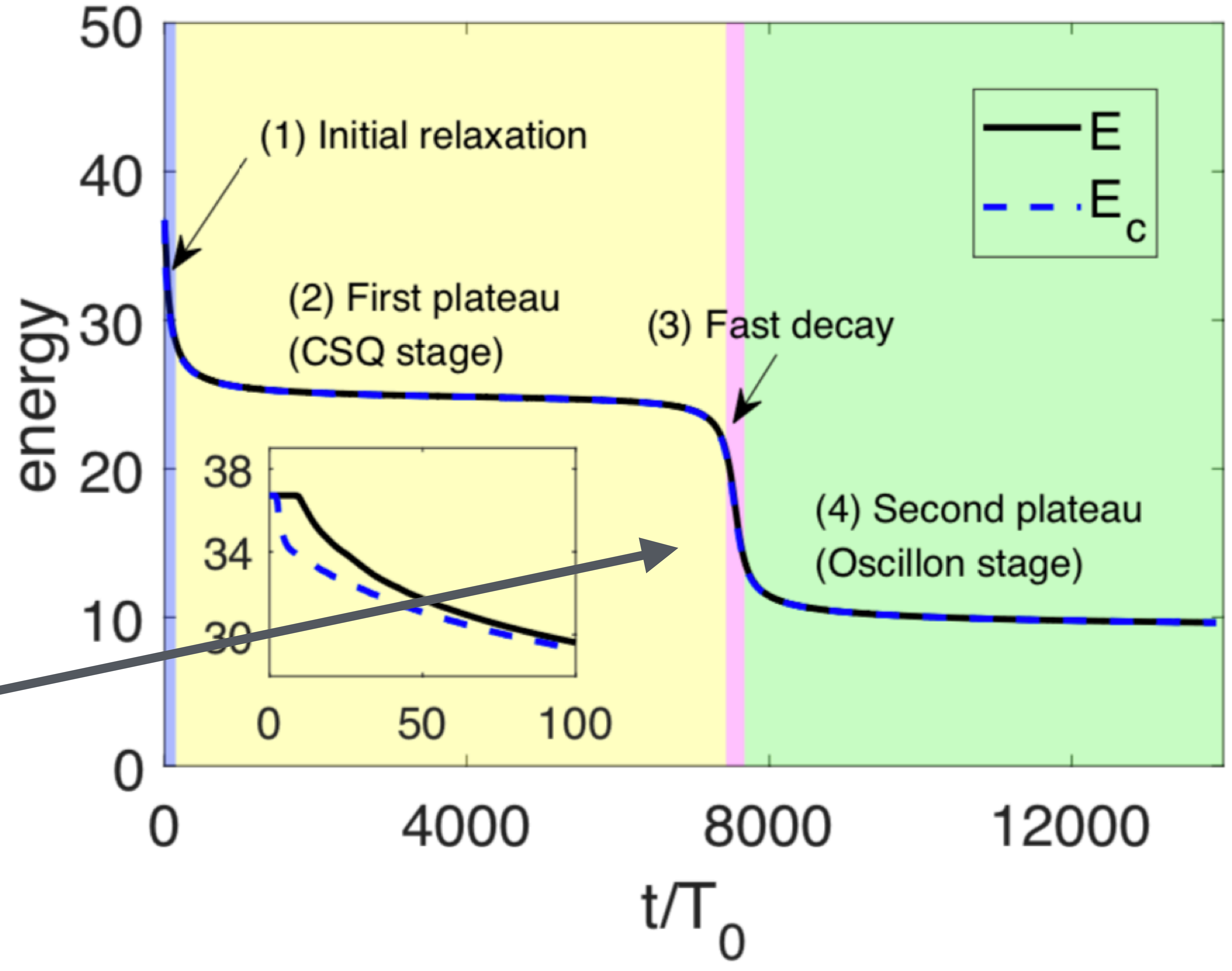
$$\Phi|_{t=0} = \Phi_Q(\mathbf{x}_1)e^{+i\omega_Q t} + \Phi_Q(\mathbf{x}_2)e^{-i\omega_Q t}$$

- Total Noether charge is equal to zero
- Attractor solutions and well-defined freqs.
- Exist for different scalar potentials U
- For the simplest Q-bal potential, they decay to oscillons





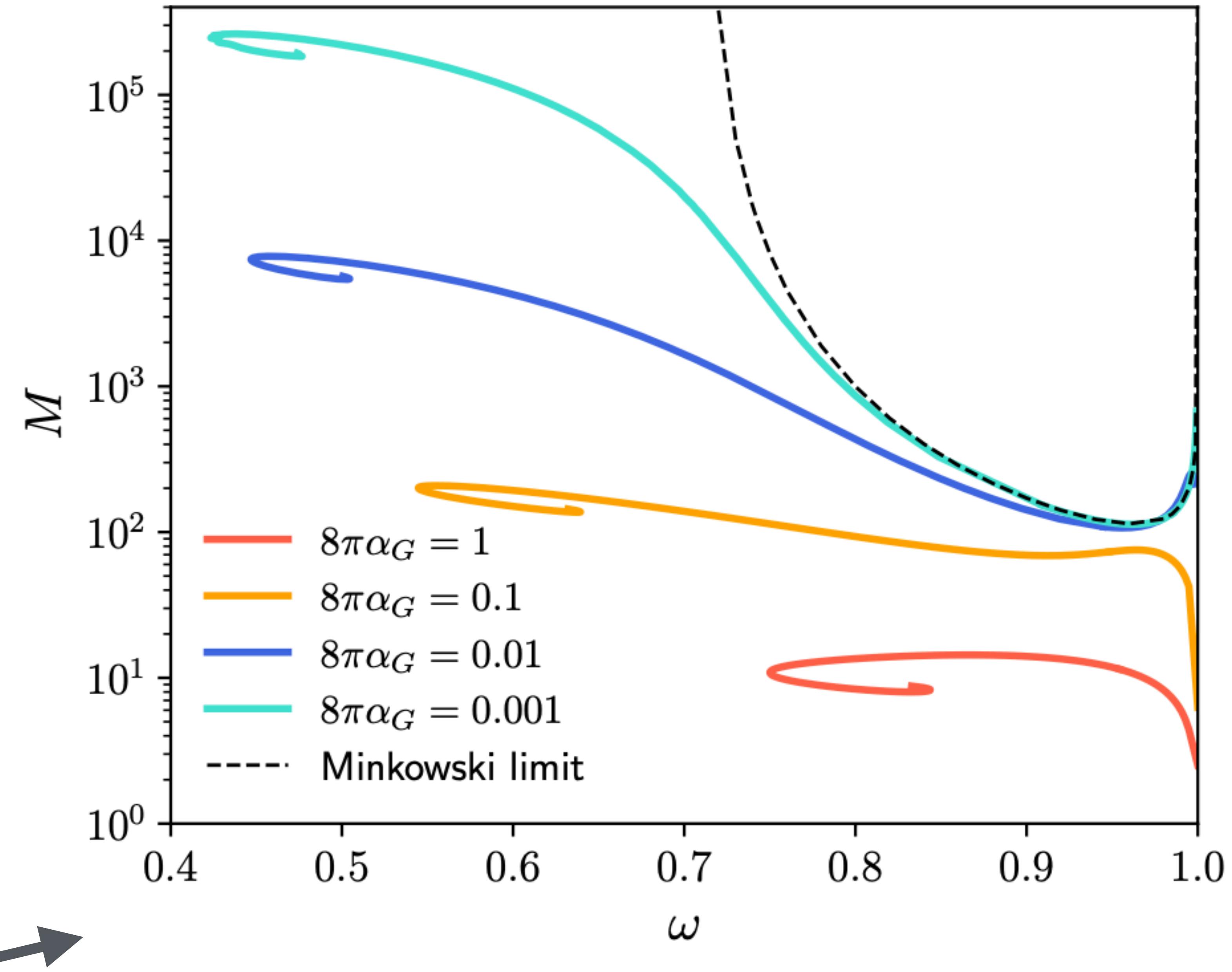
A tower of composite Q-balls exists



Multipolar boson stars

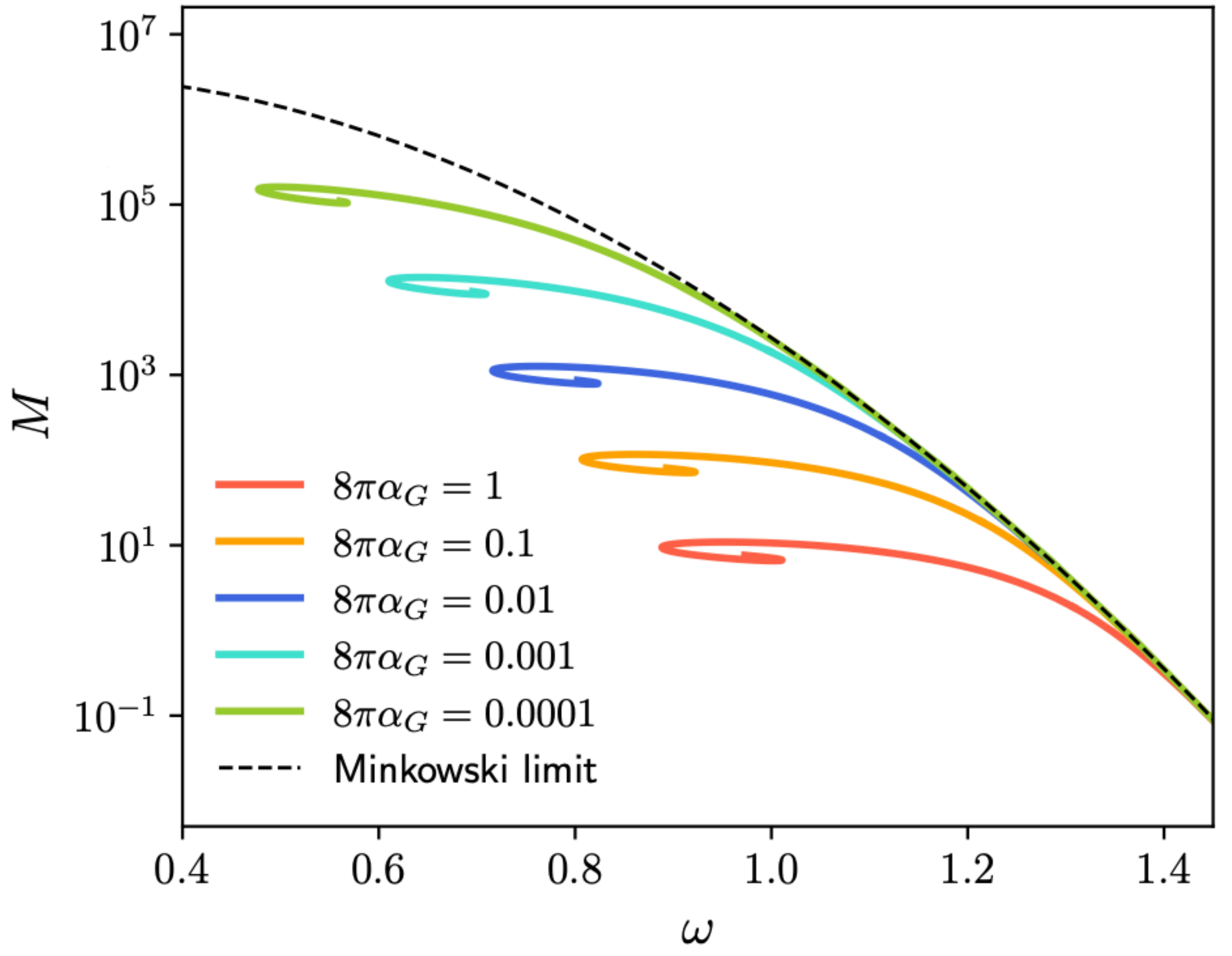
The self-gravitating version of charge-swapping Q-balls present qualitative differences with respect to their flat-spacetime case.

VJ, Zhou (2024) “Complex structures of boson stars and anisotropic distribution of satellite galaxies”



Point of start: boson stars in the scalar theories of the potentials

- $U(\Phi) = \mu^2|\Phi|^2 + g|\Phi|^4 + h|\Phi|^6$
- $U(\Phi) = |\Phi|^2\left(1 + K\ln\frac{|\Phi|^2}{M^2}\right)$



Once again, treating this objects self-consistently, when gravity is strong, demands the use of full numerical relativity.

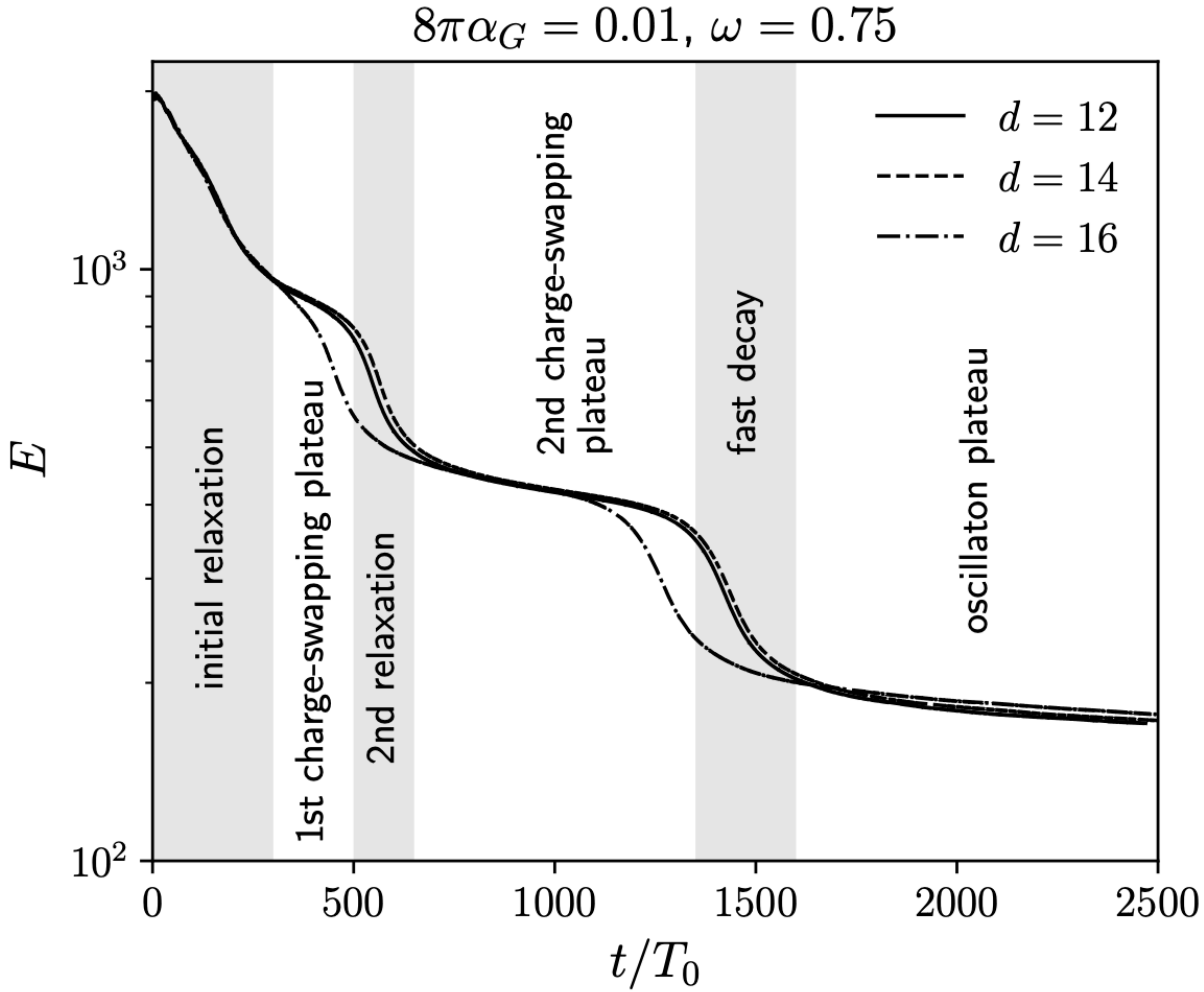
Preparation is similar to the flat-space case with the difference that one should now solve the *Hamiltonian constraint, which looks like this for the present situation*

$$\Delta_3\psi + \frac{\partial A \partial\psi}{A} + \frac{\psi}{4}\left(\frac{2\Delta_3 A}{A} - \frac{\partial A\partial A}{A^2}\right) + 2\pi\alpha_G\psi^5 A^2\rho = 0\,,$$

And evolve the system according to the BSSN equations →

$$\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\ \tilde\gamma_{ij} &= -2\alpha\tilde A_{ij}\,,\\
(\partial_t - \mathcal{L}_\beta)\ \chi &= \frac{2}{3}\alpha\chi K\,,\\
(\partial_t - \mathcal{L}_\beta)\ K &= \chi\tilde\gamma^{ij}D_jD_i\alpha\\
&\quad +\alpha\left(\tilde A_{ij}\tilde A^{ij} + \frac{1}{3}K^2\right) + 4\pi\alpha_G\alpha(\rho+S)\,,\\
(\partial_t - \mathcal{L}_\beta)\ \tilde A_{ij} &= [\ldots] - 8\pi\alpha_G\alpha\left(\chi S_{ij} - \frac{S}{3}\tilde\gamma_{ij}\right)\\
(\partial_t - \mathcal{L}_\beta)\ \tilde\Gamma^i &= [\ldots] - 16\pi\alpha_G\alpha\chi^{-1}P^i\,,\\
(\partial_t - \mathcal{L}_\beta)\ \phi &= -2\alpha K_\phi\,,\\
(\partial_t - \mathcal{L}_\beta)\ K_\phi &= \alpha\Big[KK_\phi - \frac{1}{2}\chi\tilde\gamma^{ij}\tilde D_i\partial_j\phi + \frac{1}{4}\tilde\gamma^{ij}\ldots\\
&\quad + \frac{1}{2}\left(1 - 2|\phi|^2 + 3\mathfrak{g}|\phi|^4\right)\phi\Big] - \frac{1}{2}\chi\tilde\gamma^{ij}\partial_i\alpha\partial_j\phi
\end{aligned}$$

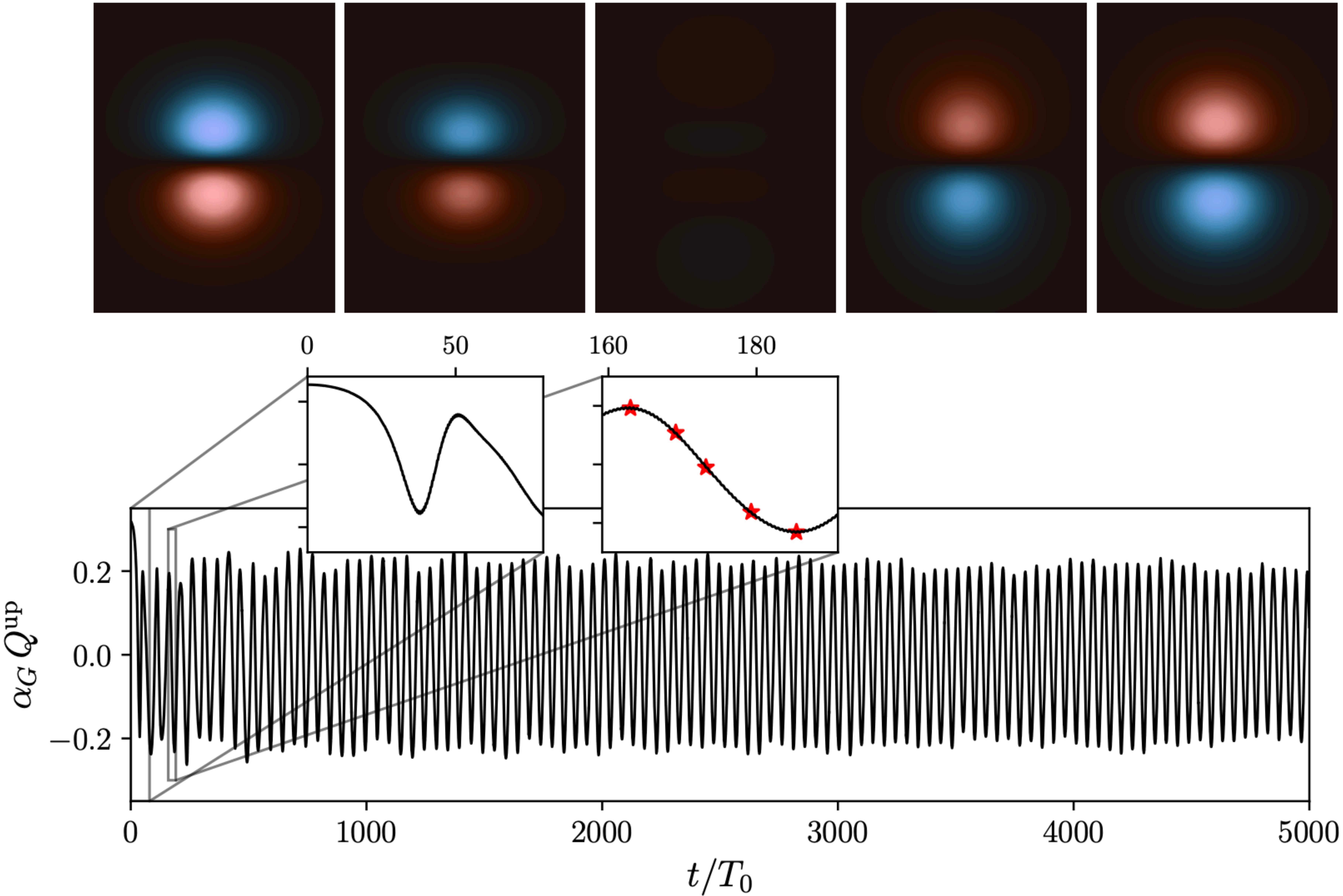
Dipolar boson stars with $U(\Phi) = \mu^2|\Phi|^2 + g|\Phi|^4 + h|\Phi|^6$



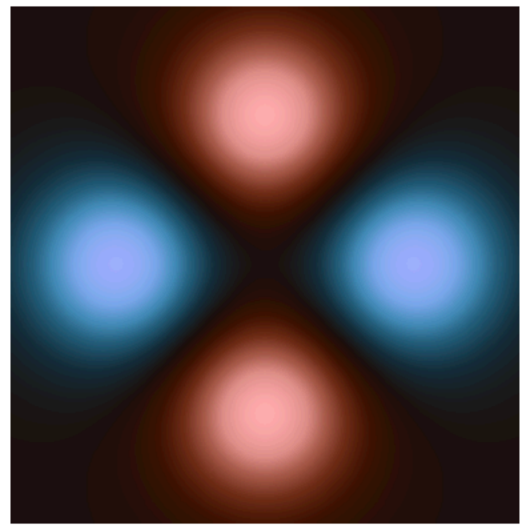
For the polynomial (sextic) potential we have a similar picture to the Q-balls

Multipolar boson stars

Dipolar boson stars with $U(\Phi) = \mu^2 |\Phi|^2$



Very long-lived!



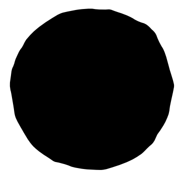
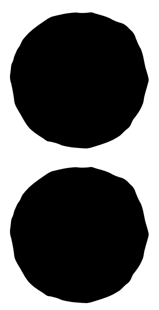
And similar results for the logarithmic potential...

An application for charge-swapping boson stars

Anisotropic distribution of satellite galaxies

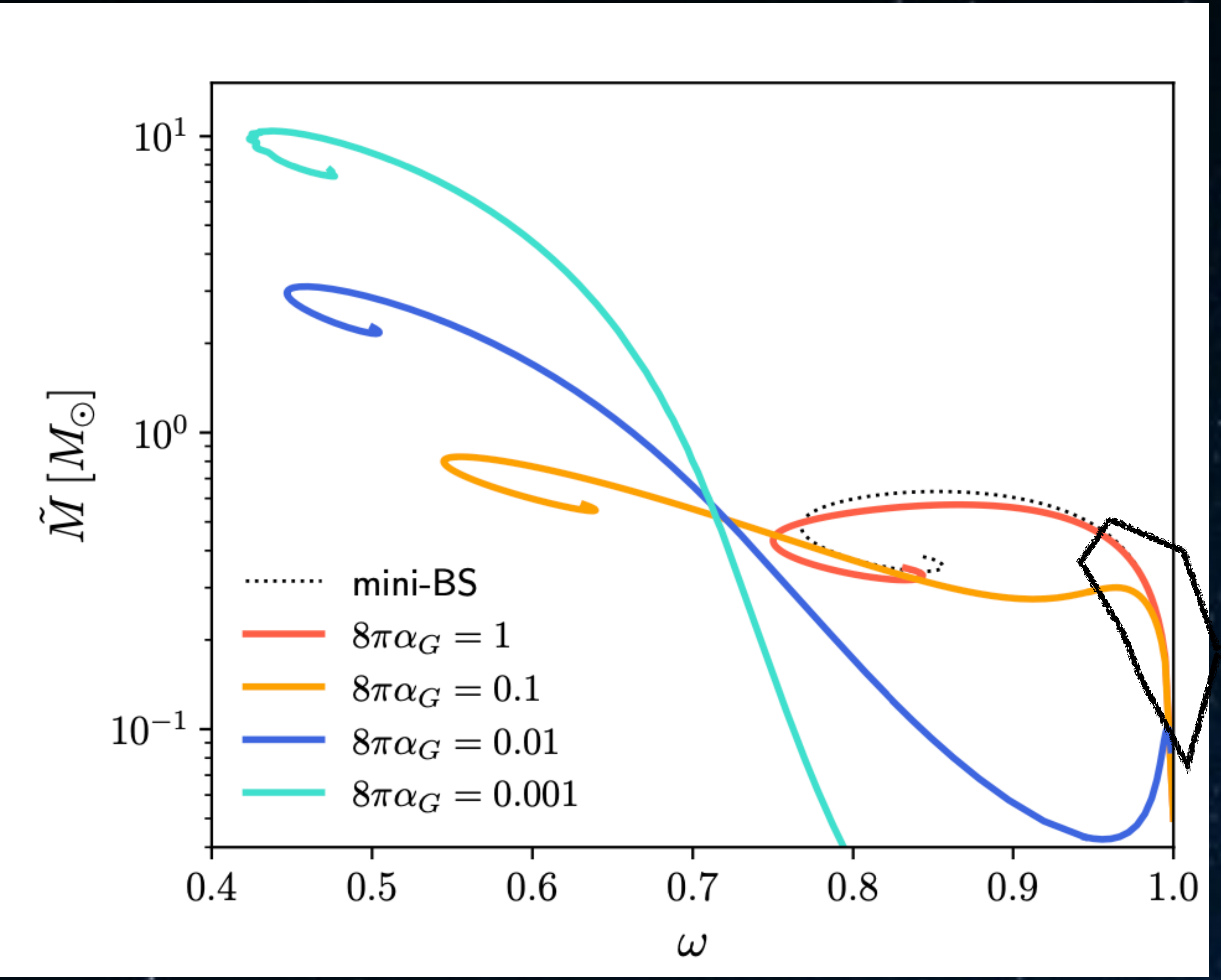
Multipolar boson stars

Dipolar boson stars with $U(\Phi) = \mu^2 |\Phi|^2$

- Peculiar morphology (in energy density): “a lot”  + “a little” 
- Known formation mechanism: collision of a boson vs. anti-boson star. 

Ideal objects to solve the
“Problem of anisotropic distribution of satellite Galaxies”
a.k.a. Vast polar structure, etc.

Halos



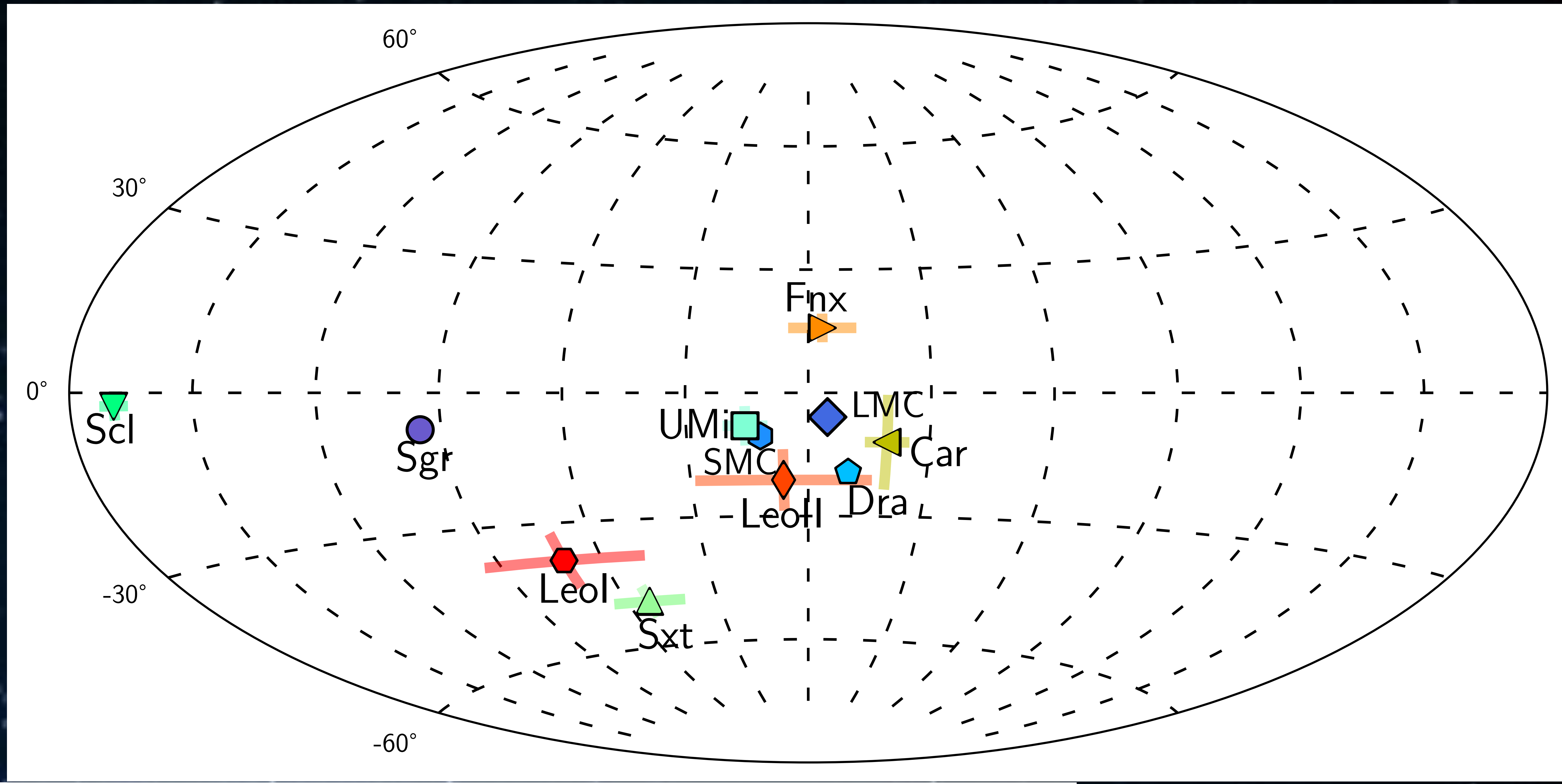
Newtonian-boson stars

$$i\partial_t \Psi_w = -\frac{1}{2}\nabla^2\Psi_w + U\Psi_w\,, \qquad \nabla^2 U = 4\pi\alpha_G |\Psi_w|^2\,.$$

$$\Psi_w = \sqrt{2}\exp(it)\phi\,,$$

Are the scalar field dark matter halos

Anisotropic distribution of satellite galaxies in the Milky Way



- Coherent observed orbital angular momentum of galaxies
- Improbable scenario in standard dark matter simulations
- This property is also observed in Andromeda and M31

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Scalar field dark matter as an alternative explanation for the anisotropic distribution of satellite galaxies

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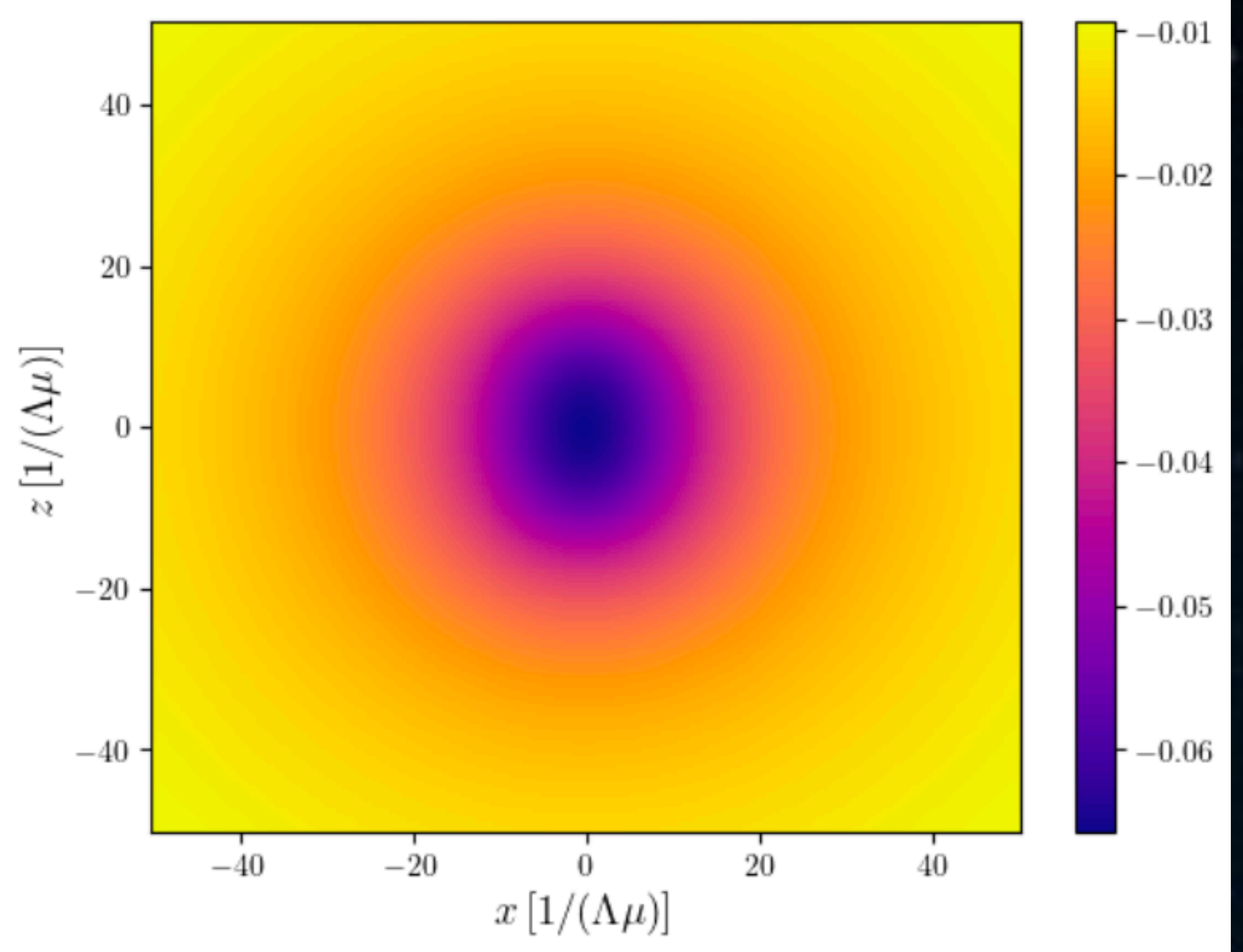
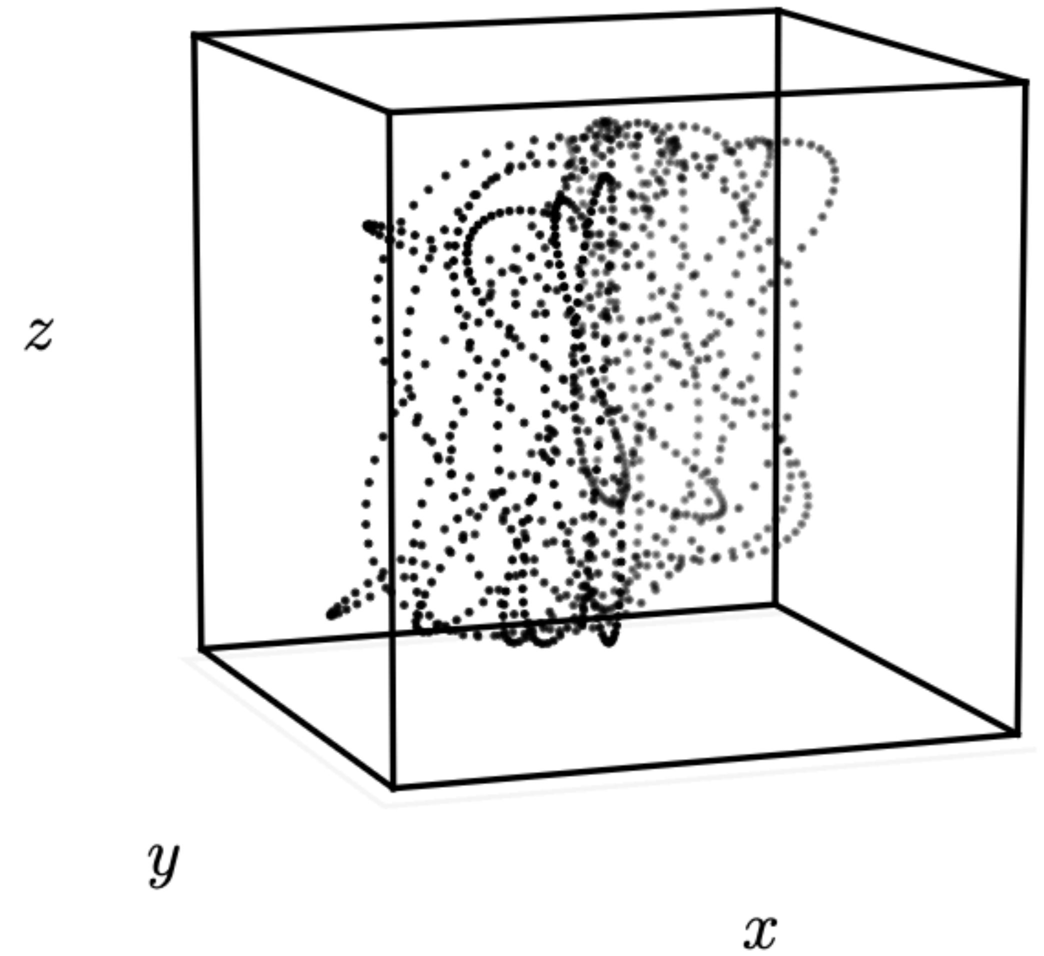
Charge-swapping stars to explain the Anisotropic distribution of satellite galaxies



In addition to the μ rescaling, Newtonian-boson stars have an additional rescaling

$$(\Psi_{\mathrm{w}}, U, x^i, t) \to (\Lambda^2\Psi_{\mathrm{w}}, \Lambda^2 U, \Lambda^{-1}x^i, \Lambda^{-2}t)\,.$$

We choose μ and Λ to be consistent with the rotation curve of the Milky Way for particles moving in the equatorial plane

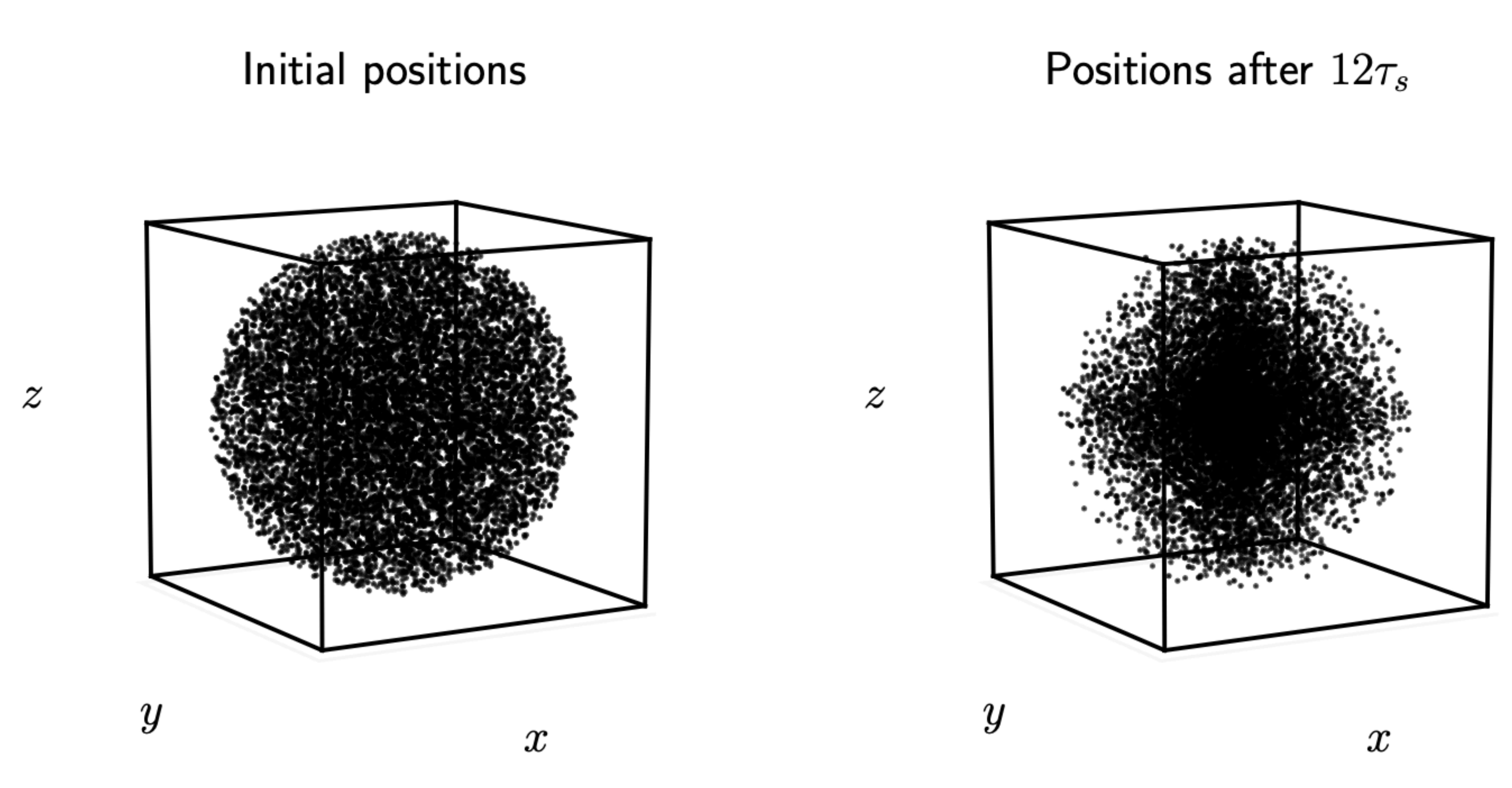


Example of a particle moving on top of a formed generic charge-swapping free field configuration

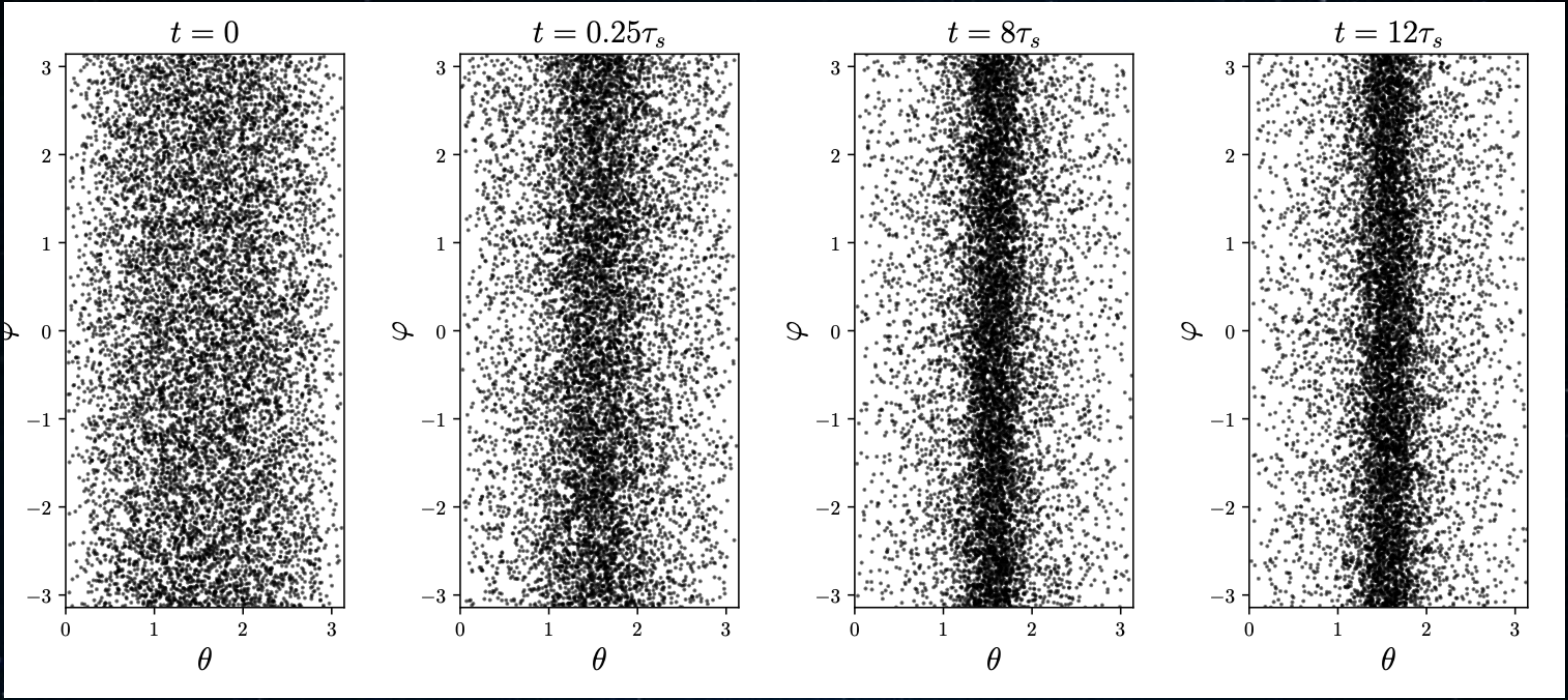
After that, we set 10000 particles initially placed inside a sphere of radius 300kpc,

with random velocities between 0 and 0.25 the escape velocity at 300kpc on the equatorial plane.

Defining the timescale τ_s as the time it takes a particle to circle around the halo. We evolve for tenths of τ_s



Obtaining that the orbital poles (orbital angular momentum) of the particles aligns with the galactic plane after a short time and then stabilizes



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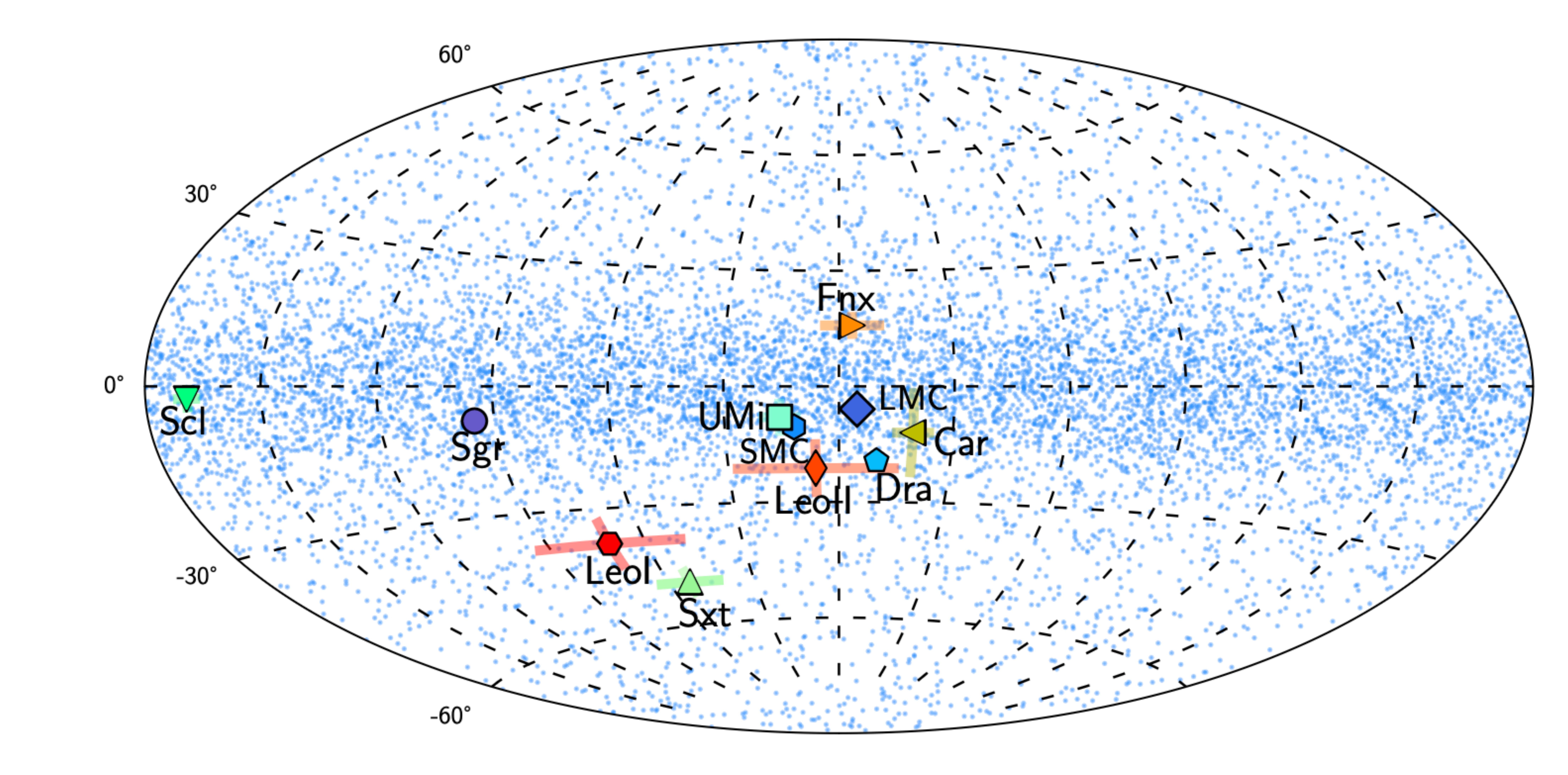
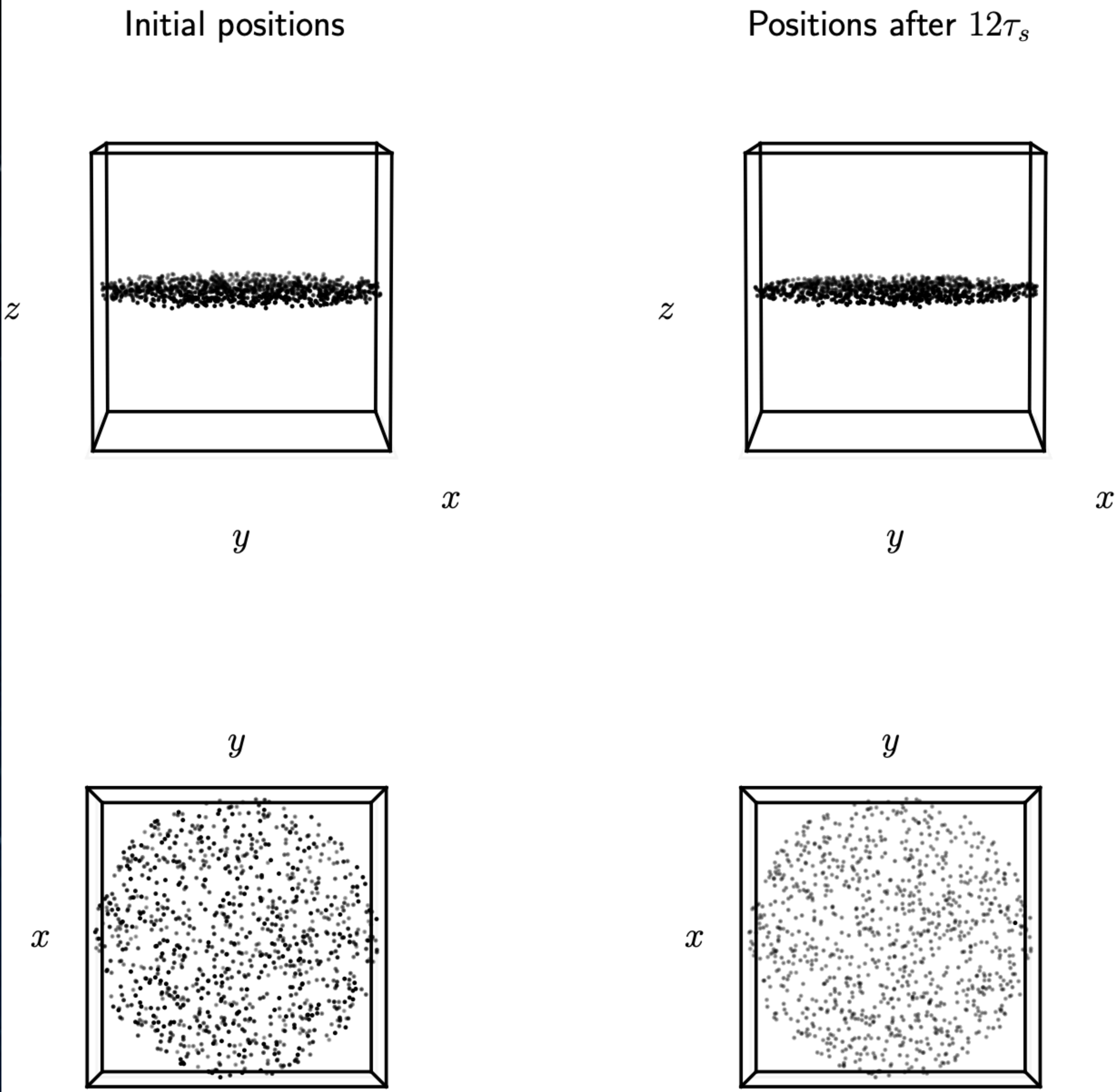


Multipolar boson stars

Simple test to check the stability of a disk moving in the charge-swapping boson star gravitational potential

More details can be found in arXiv:2407.12084



Conclusions and outlook

- Boson stars exhibit phenomena analogous to Q-balls, with gravity introducing a “qualitatively jump” in their features.
- Scalar field solitons are capable of undergoing superradiant amplification, leading to (observable) energy emission in astrophysical settings.
- Halos composed of ultralight scalar fields are a potential explanation for the anisotropic distribution of satellite galaxies around host galaxies.
- Both superradiant processes and charge-swapping dynamics in boson stars are promising sources of gravitational waves.

Thank you