Workshop on Multi-front Exotic phenomena in Particle and Astrophysics (MEPA 2025)
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Recent progress on monopoles and gravitational waves

Ye-Ling Zhou

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國科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

Contents

- Monopole as an extension of electromagnetic theory
- Monopole as a topological defect in QFT
- Monopole’s motivation on inflated GWs via GUT phase transition
- Monopole’s influence on GW via cosmic strings

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Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

In the absence of source

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot (\mathbf{E} + i\mathbf{B}) = 0$$
$$\nabla \times (\mathbf{E} + i\mathbf{B}) - i\frac{\partial}{\partial t}(\mathbf{E} + i\mathbf{B}) = 0$$

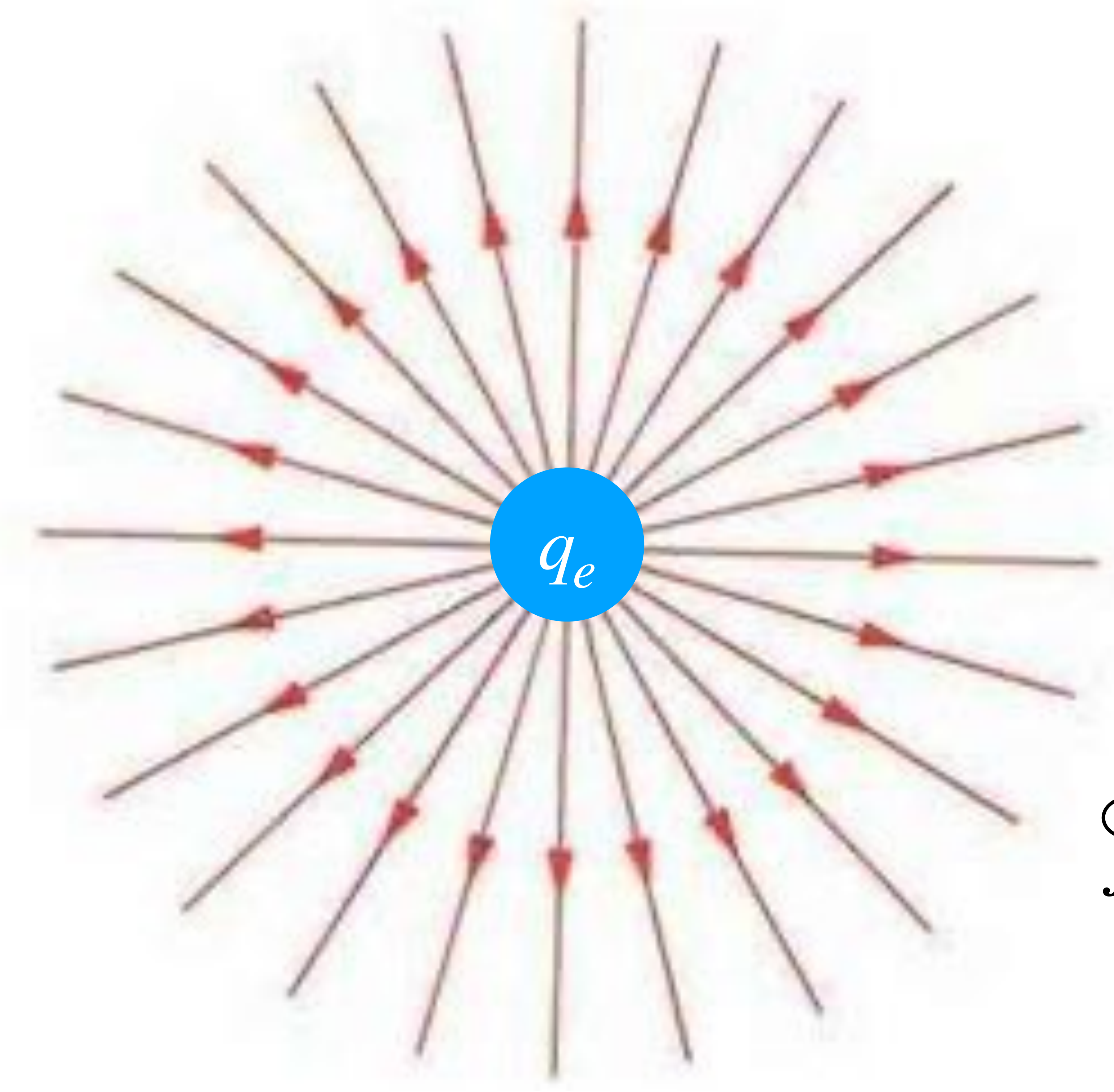
Enlarged $U(1)$ symmetry

$$\mathbf{E} + i\mathbf{B} \to e^{i\alpha}(\mathbf{E} + i\mathbf{B})$$

This symmetry is broken in the presence of electric-charged particles

If there are magnetic-charged particles, the symmetry can be restored

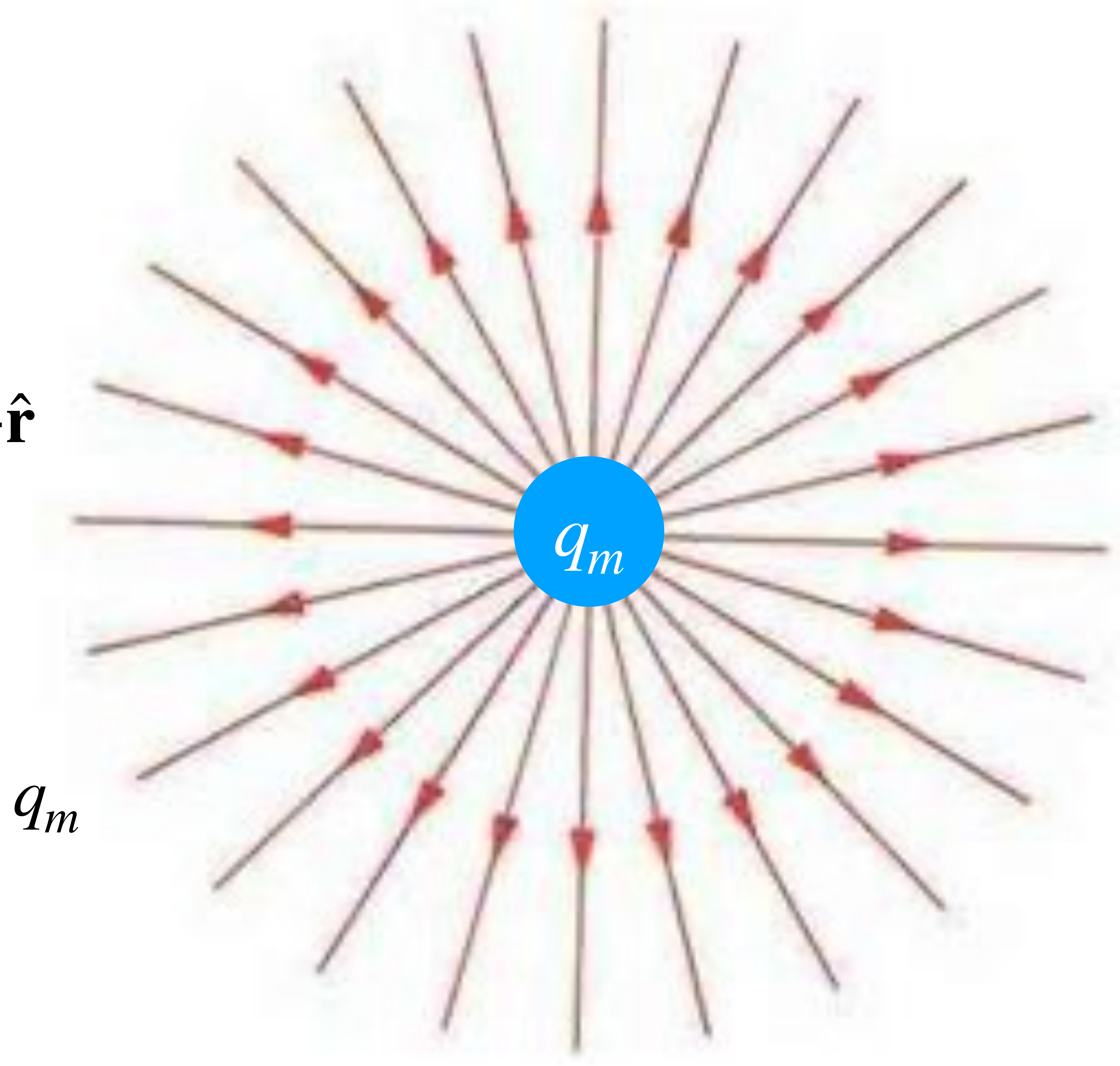
Magnetic fields in a Coulomb-like potential



Parity

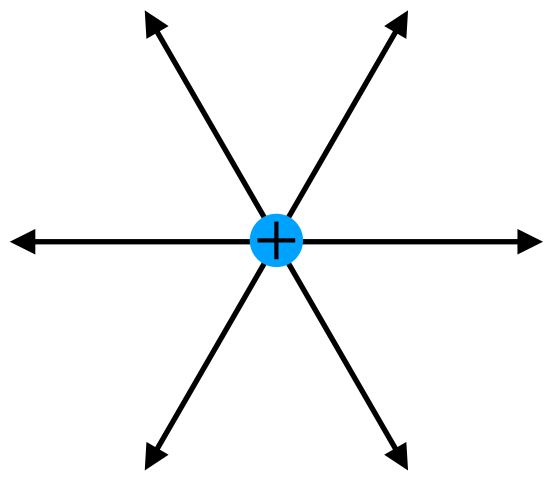
$$\mathbf{E} = \frac{q_e}{4\pi r^2}\hat{\mathbf{r}} \quad \Longleftrightarrow \quad \mathbf{B} = \frac{q_m}{4\pi r^2}\hat{\mathbf{r}}$$

$$\oiint \mathbf{E} \cdot \mathrm{d}\mathbf{S} = q_e \qquad \oiint \mathbf{B} \cdot \mathrm{d}\mathbf{S} = q_m$$



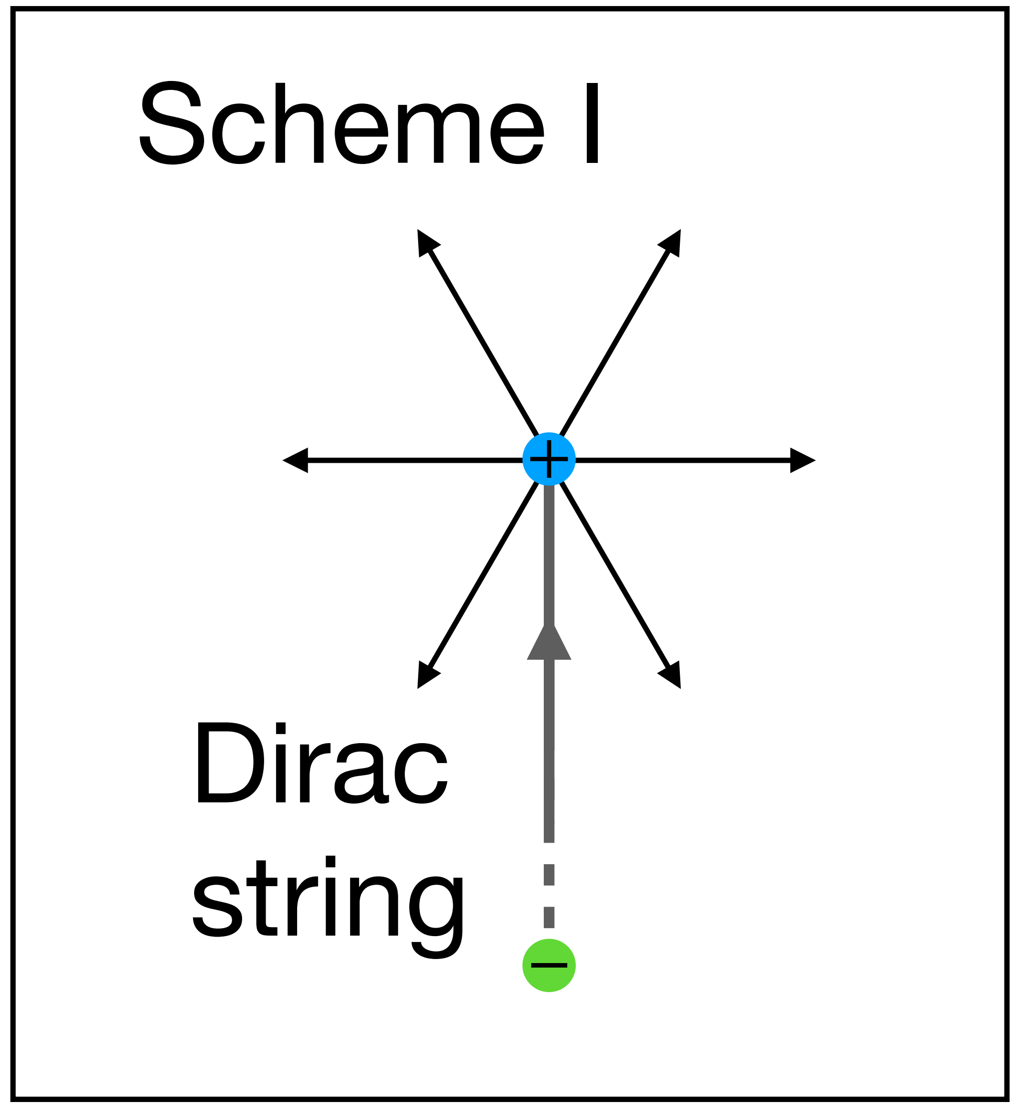
Dirac's monopole, 1931

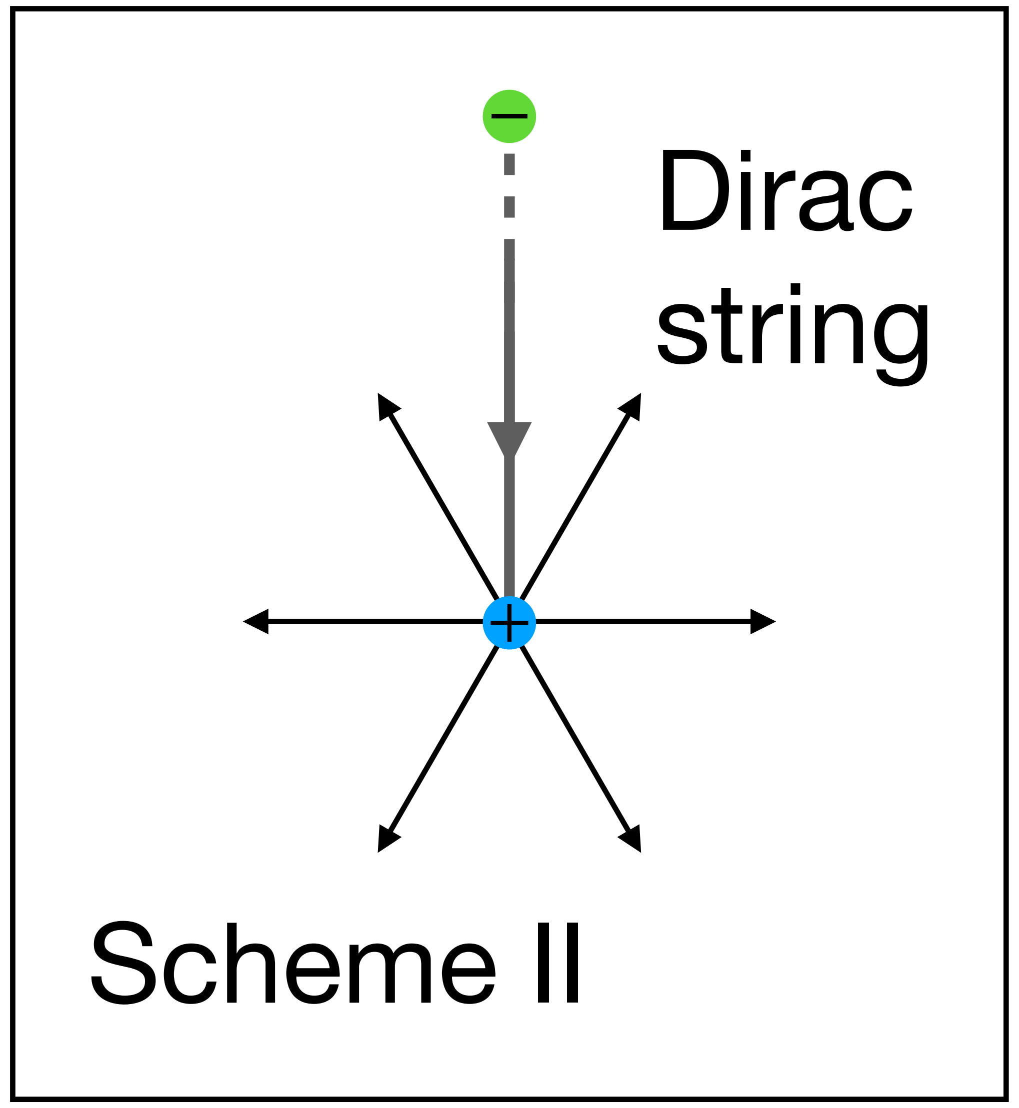
Given a monopole with charge q_m, the magnetic field distributes as



$$\mathbf{B} = \frac{g}{r^2}\hat{\mathbf{r}} \qquad g = \frac{q_m}{4\pi}$$

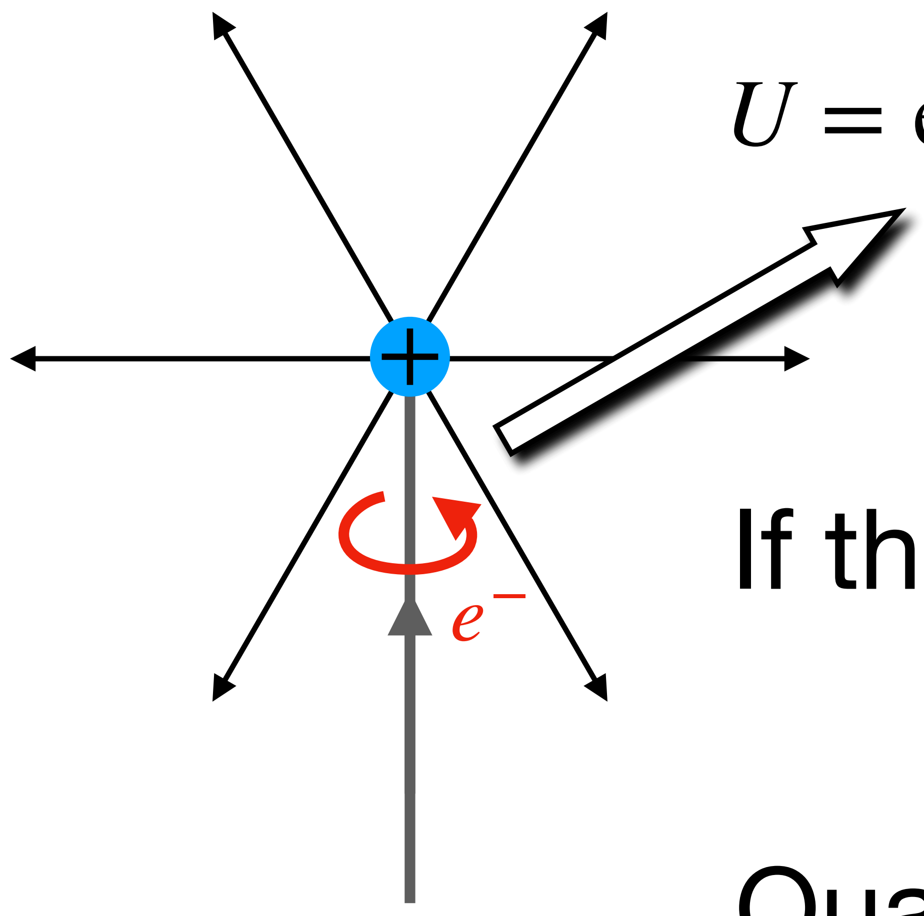
We calculate the vector potential $\mathbf{A}$ via $\mathbf{B} = \nabla \times \mathbf{A}$





$$\mathbf{A}^{\mathrm{I}} = g(\cos\theta - 1)\mathbf{e}_\theta \qquad \mathbf{A}^{\mathrm{II}} = g(\cos\theta + 1)\mathbf{e}_\theta$$

Gauge difference $\mathbf{A}^{\mathrm{II}} - \mathbf{A}^{\mathrm{I}} = \nabla(2g\phi)$



$$U = \exp\left(\mathrm{i}\, e \oint \mathrm{d}\mathbf{x} \cdot \mathbf{A}\right) = \exp\left(\mathrm{i}4\pi\, eg\right)$$

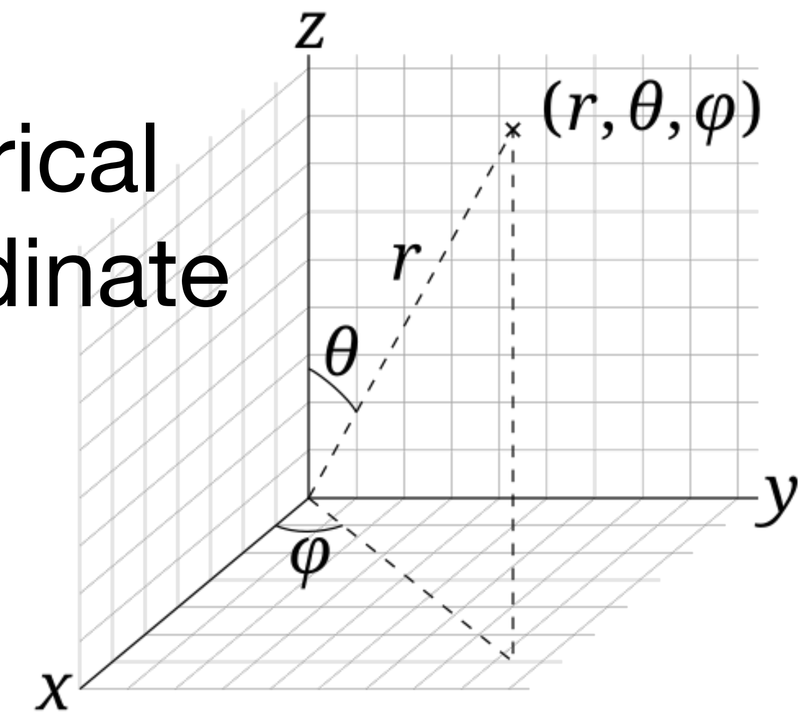
If the Dirac string has no physical meaning

$$4\pi\, eg = 2\pi, 4\pi, \cdots$$

Quantisation of electric charge $\quad e_{\min} = \dfrac{1}{2g_{\max}}$

Dirac magnetic charge $\quad g_D = \dfrac{1}{2e}$

spherical coordinate



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Soliton solution

- A global U(1) theory with a complex scalar $\phi = \frac{1}{\sqrt{2}}(h + ia)$

$$\mathscr{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi, \phi^*) \qquad V(\phi, \phi^*) = -\mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$$

VEV: $\langle \phi \rangle = e^{i\alpha} v \qquad v = \sqrt{\mu^2/\lambda}$

- A two-dimensional soliton solution

EOM $\partial^2\phi + \frac{\partial V}{\partial \phi} = 0$ (time-independent solution) $\Rightarrow \nabla^2 \phi = \frac{\partial V}{\partial \phi}$

Parameterisation $\phi = \frac{1}{\sqrt{2}} f(r) e^{i\,\alpha(\theta)} \qquad \alpha(\theta) = \theta \qquad S_1 \to S_1$

ODE for $f(r)$ $\quad f'' + \frac{1}{r} f' - \frac{f}{r^2} + \lambda(v^2 - f^2) f = 0$

With Boundary conditions $f(0) = 0$ & $f(\infty) = v$

E. Weinberg, Classical solutions in quantum field theories, 2012

Soliton solution

- A global SO(3) theory with a triplet scalar $\phi = (\phi_1, \phi_2, \phi_3)^T$

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - V(\phi) \qquad V(\phi) = -\frac{\mu^2}{2}(\phi^T\phi) + \frac{\lambda}{4}(\phi^T\phi)^2$$

VEV: $$\langle \phi \rangle = O_{3\times 3}\begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

- A three-dimensional soliton solution $$\nabla^2 \phi = \frac{\partial V}{\partial \phi}$$

Parameterisation $\phi_a = \hat{\mathbf{r}}_a h(r)$ $\qquad S_2 \to S_2$

ODE for $h(r)$ $$h'' + \frac{2}{r}h' - \frac{2}{r^2}h + \lambda(v^2 - h^2)h = 0$$

With boundary conditions $h(0) = 0$ & $h(\infty) = v$

E. Weinberg, Classical solutions in quantum field theories, 2012

’t Hooft-Polyakov monopole

’t Hooft, NPB, 74; Polyakov, JEPT Lett., 74

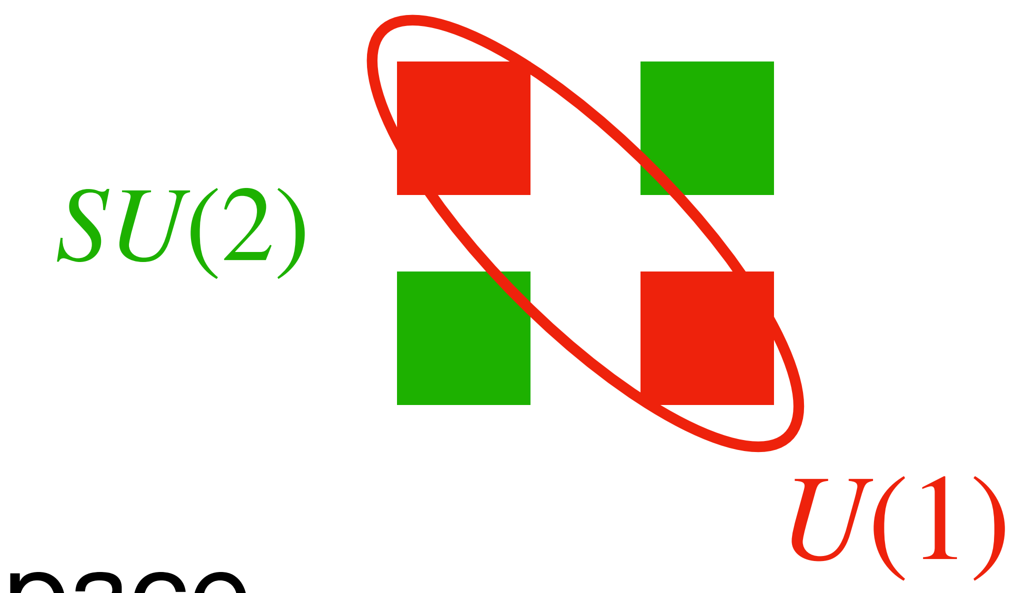
- $SU(2)$ gauge theory with an adjoint scalar

$$\Phi = \frac{1}{2}\tau^a\Phi^a = \frac{1}{2}\begin{bmatrix} \Phi^3 & \Phi^1 - i\Phi^2 \\ \Phi^1 - i\Phi^2 & -\Phi^3 \end{bmatrix}$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\, D_\mu\Phi D^\mu\Phi - V(\Phi)$$

- After Φ gains the VEV, $SU(2)$ is spontaneously broken to $U(1)$

$$\langle\Phi\rangle = U(\Omega)\frac{1}{2}\begin{bmatrix} v & 0 \\ 0 & -v \end{bmatrix} U^{-1}(\Omega)$$



- The Φ VEV does not have to be globally diagonal in the gauge space

$$\begin{aligned} A_i^a &= \epsilon_{iam}\hat{r}^m\left[\frac{1-u(r)}{er}\right] \\ \Phi^a &= \hat{r}^a h(r) \end{aligned}$$

$$\begin{aligned} 0 &= h'' + \frac{2}{r}h' - \frac{2u^2h}{r^2} + \lambda(v^2 - h^2)h \\ 0 &= u'' - \frac{u(u^2-1)}{r^2} - e^2uh^2 \end{aligned}$$

- This field configuration leads to a energy condensation, which we call mass of monopole

$$M_{\mathrm{mono}} = \frac{4\pi v}{e} f(\lambda/e^2)$$

In the BPS limit, $\lambda/e^2 \to 0,\ f(\lambda/e^2) \to 1$

Most energy is restricted in the narrow radius $R \sim 1/v$

Basic feature of a monopole

- A soliton solution in 3D spatial space.
- It arises from spontaneous breaking of non-abelian symmetry
- It appears as an object of a certain mass M and most mass restricted in a radius R from the centre
- It has a “magnetic” charge (does not need to be real magnetic in QED)

Homotopy group ⇔ topological defects

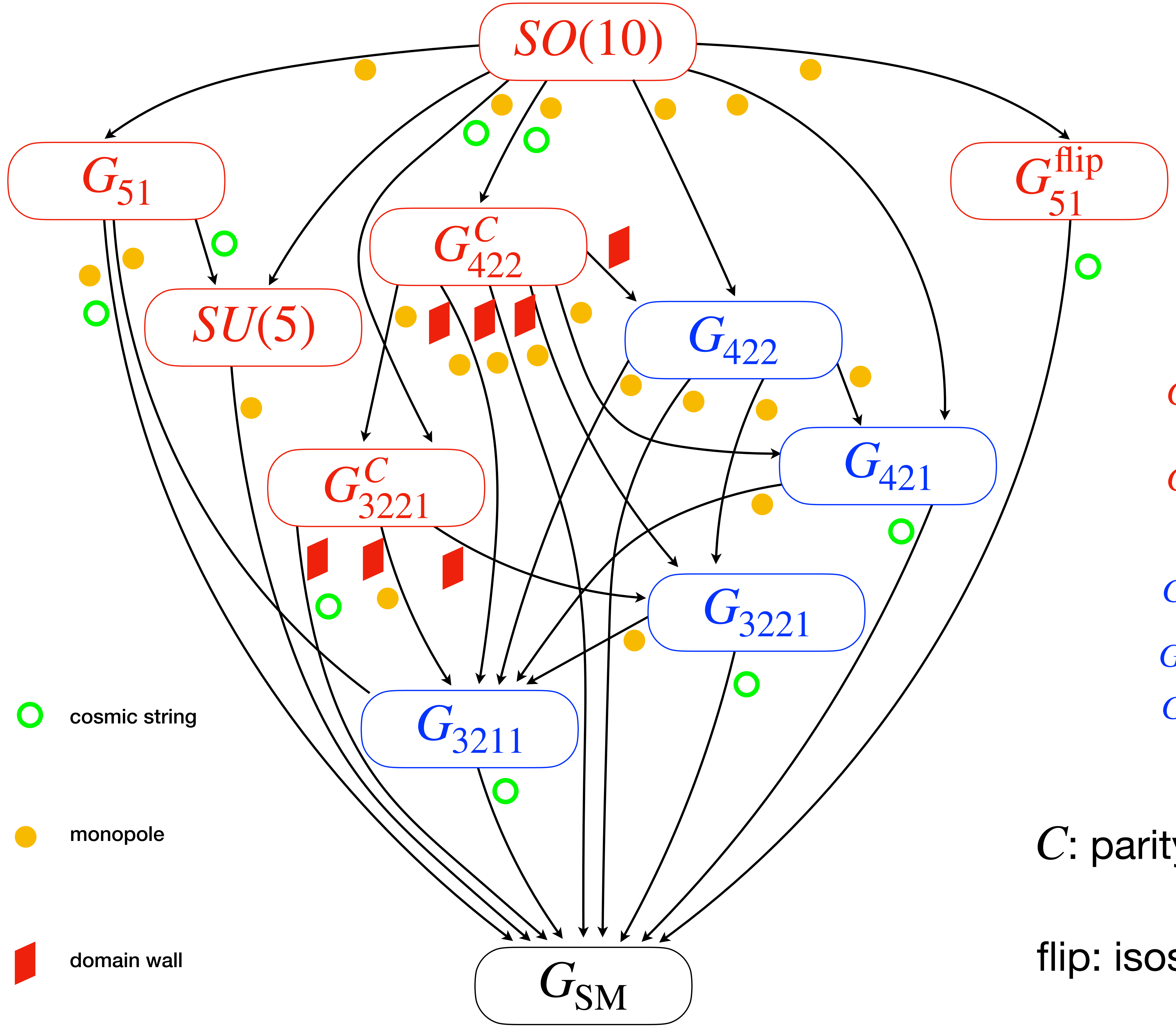
- Kibble mechanism, 1976
  —— originally proposed for defects generated in a continuous phase transition.

- Topological defects depend on the homotopy groups (同伦群) of the manifold of degenerate vacua.

For symm breaking $G \to H$, degenerate vacua form a manifold $\mathscr{M} = G/H$, a homotopy group $\pi_k(\mathscr{M})$ is defined by the set of mapping $\mathscr{M} \to S^k$ (k-dim sphere)

- If $\pi_k(G/H) \neq 1$, $(2-k)$-dim topological defects defects form
  - $k = 2 \Rightarrow$ monopoles (单极子), 0-dim point in the core
  - $k = 1 \Rightarrow$ cosmic strings (宇宙弦), 1-dim string in the core
  - $k = 0 \Rightarrow$ domain walls (畴壁), 2-dim surface in the core

Monopoles in grand unified theories



$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$

$G_{51} = SU(5) \times U(1)$

$G_{421} = SU(4)_c \times SU(2)_L \times U(1)_R$

$G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$G_{3211} = SU(3)_c \times SU(2)_L \times SU(1)_Y \times U(1)_{B-L}$

C: parity $\psi_L \leftrightarrow \psi_R^C$

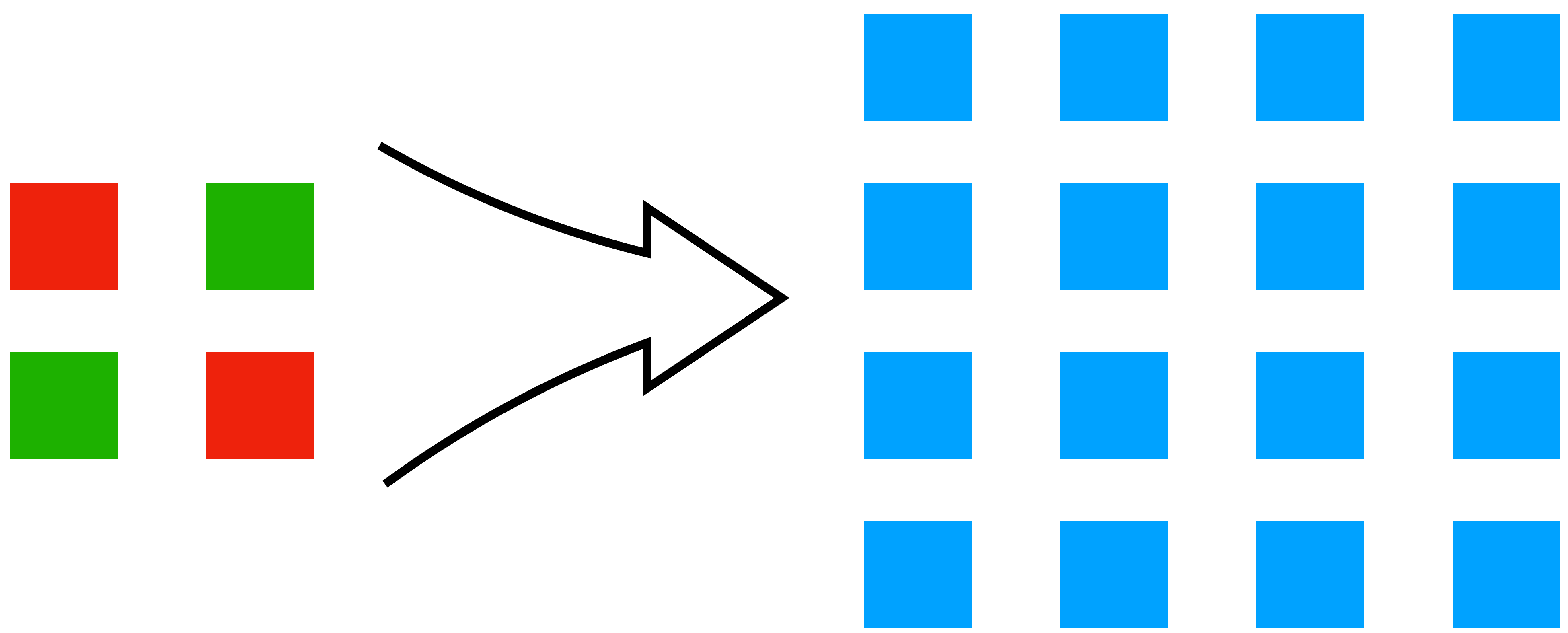
flip: isospin flipping $u \leftrightarrow d, \nu \leftrightarrow e$

Monopoles in grand unified theories

Preskill, Ann. Rev. Nucl. Part. Sci., 1984

Pati-Salam monopoles

- Gauge symmetry $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$
- $SU(4)_c$ is spontaneously broken to $SU(3)_c \times U(1)_{B-L}$
- The breaking of $SU(2) \rightarrow U(1)$ can be embedded into $SU(4) \rightarrow SU(3) \times (1)$
- Monopole arises from the breaking of Pati-Salam symmetry.
  Its property is determined by the embedding of $SU(2)$ into the gauge space of $SU(4)$



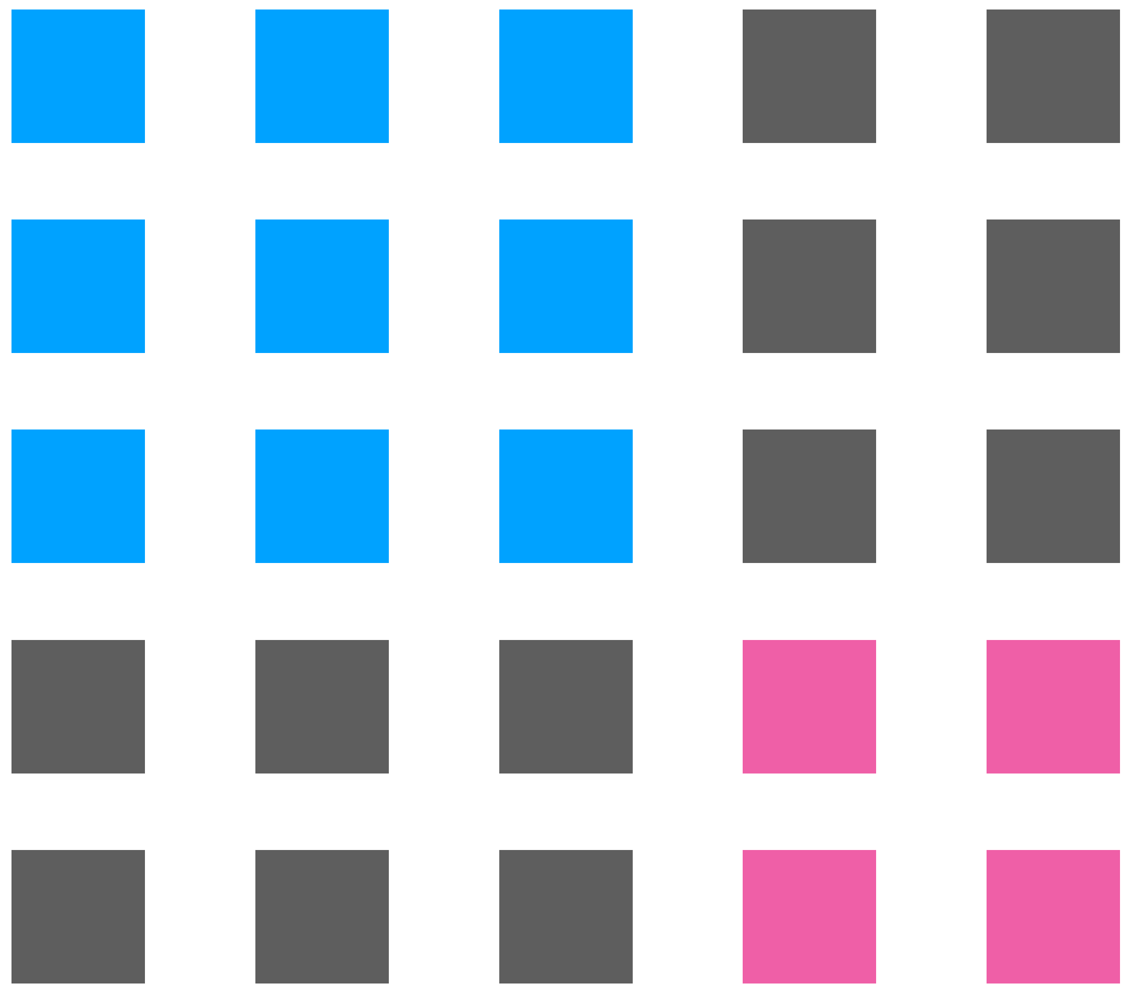
Since $SU(3)_c$ is unbroken, there is only one type of monopole

Monopoles in grand unified theories

Preskill, Ann. Rev. Nucl. Part. Sci., 1984

SU(5) monopoles

- Gauge symmetry $SU(5)$, broken to SM gauge symmetry directly via an adjoint 24-plet Higgs



$$SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

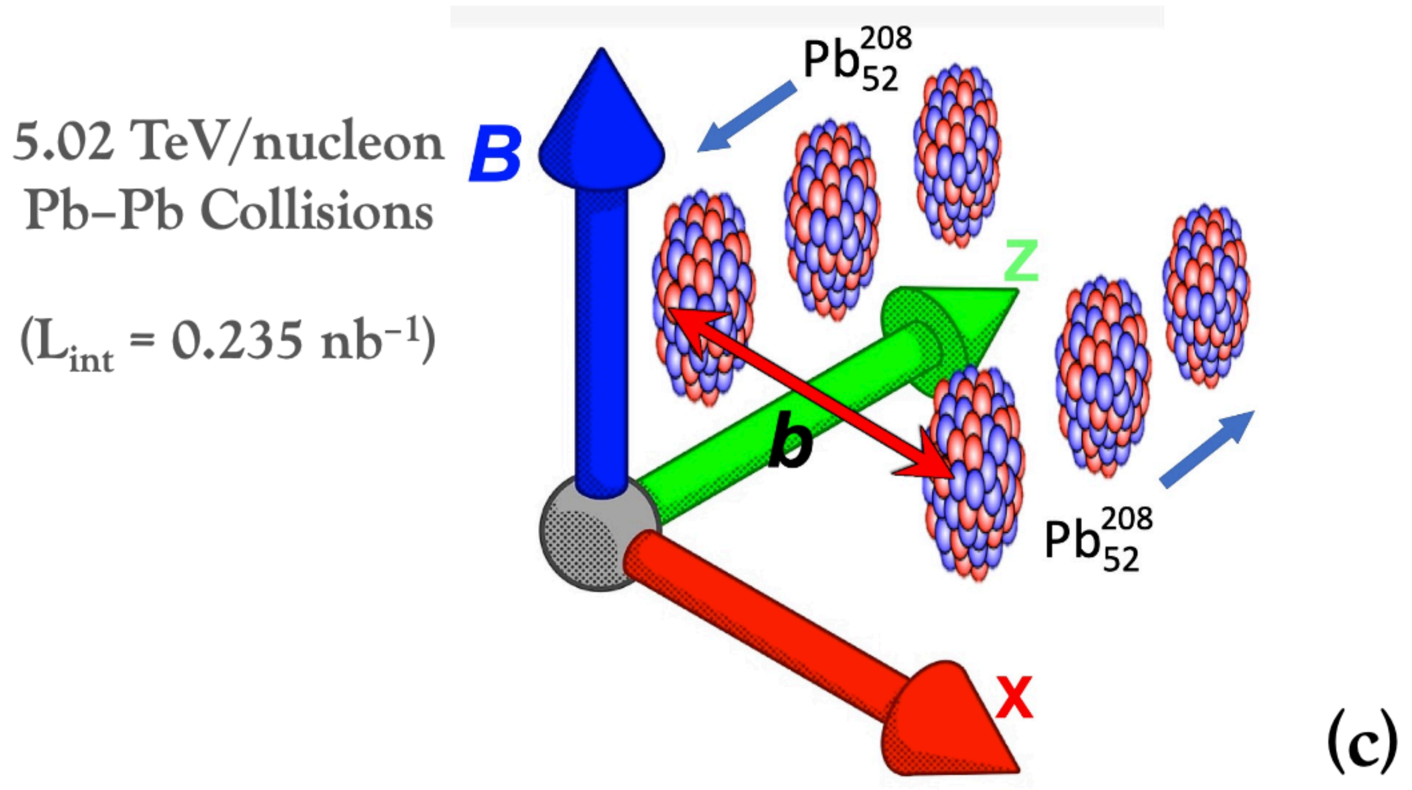
SO(10) monopoles: two types

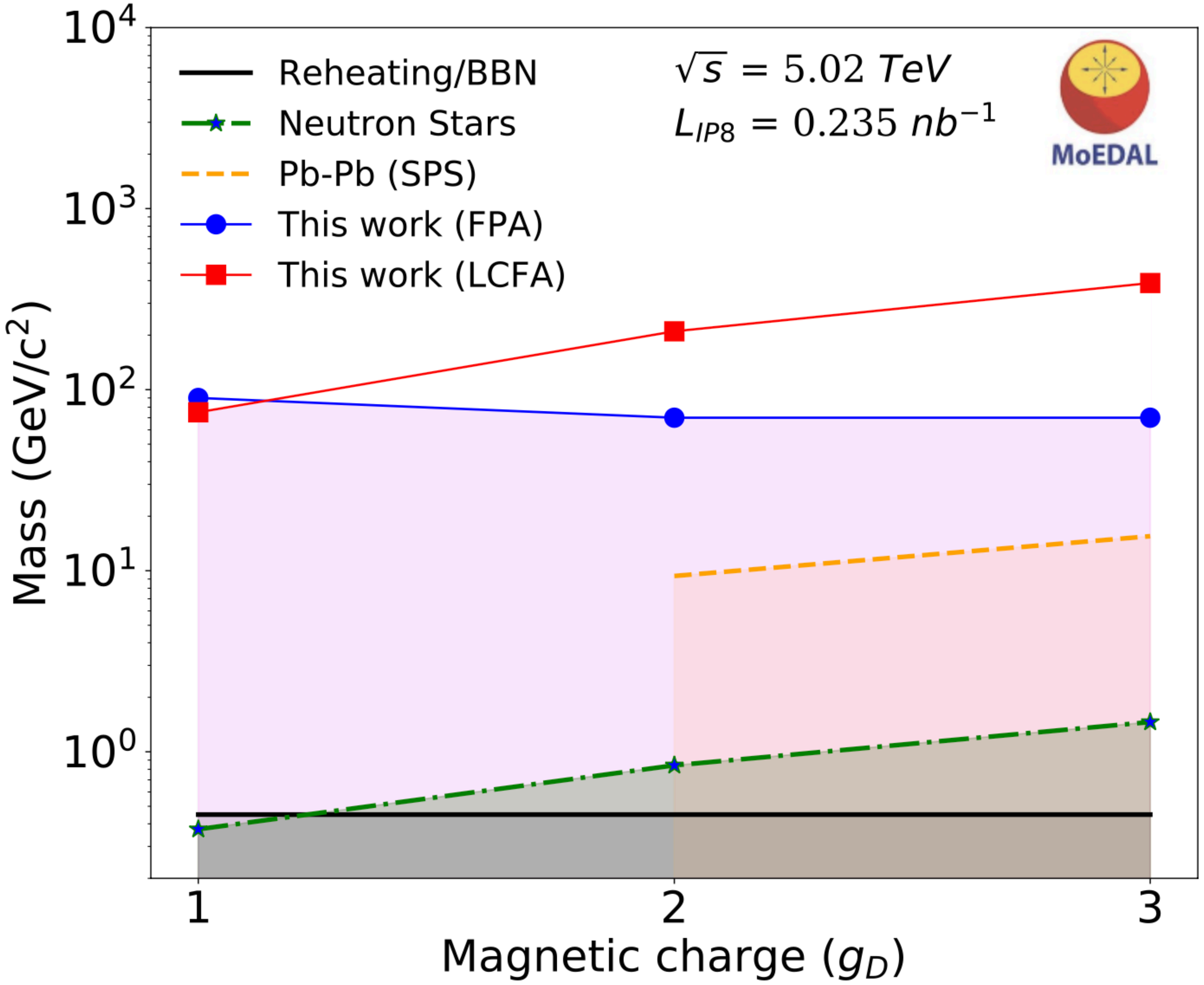
$$SU(10) \rightarrow SU(4)_c \times SU(2)_R \times SU(2)_L$$

$$SU(4)_c \rightarrow SU(3)_c \times U(1)_{B-L}$$

Monopole searches at colliders

Measuring magnetic monopoles via Schwinger mechanism in Pb-Pb heavy-ion collisions at the LHC





MoEDAL, 2106.11933, Nature 602, 63 (2022)

Altas in 2408.11035 excludes monopole with mass below 80-120 GeV.

GUT monopole problem

- GUT monopoles are produced after the breaking of GUTs, with masses naturally around the GUT scale $M_{\text{mono}} > 10^{15}$ GeV and number density $n_\star = H_\star^3$.

- Monopoles, once they are produced, evolve as matter during Hubble expansion. The number density today is given by

$$n_{\text{mono}}(t_0) = \left(\frac{a(t_\star)}{a(t_0)}\right)^3 n_\star$$

- Their energy density fraction $\Omega_{\text{mono}} = M_{\text{mono}} n(t_0)/\rho_c$ is given by

$$\Omega_{\text{mono}} = \frac{8\pi G M_{\text{mono}} H_\star^3}{3H_0^2(1+z_{\text{Rh}})^3} \sim 10^{40} \left(\frac{T_{\text{Rh}}}{10^{15}\ \text{GeV}}\right)^4 \qquad \gg 1$$

Monopole as one of the main motivations of inflation

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

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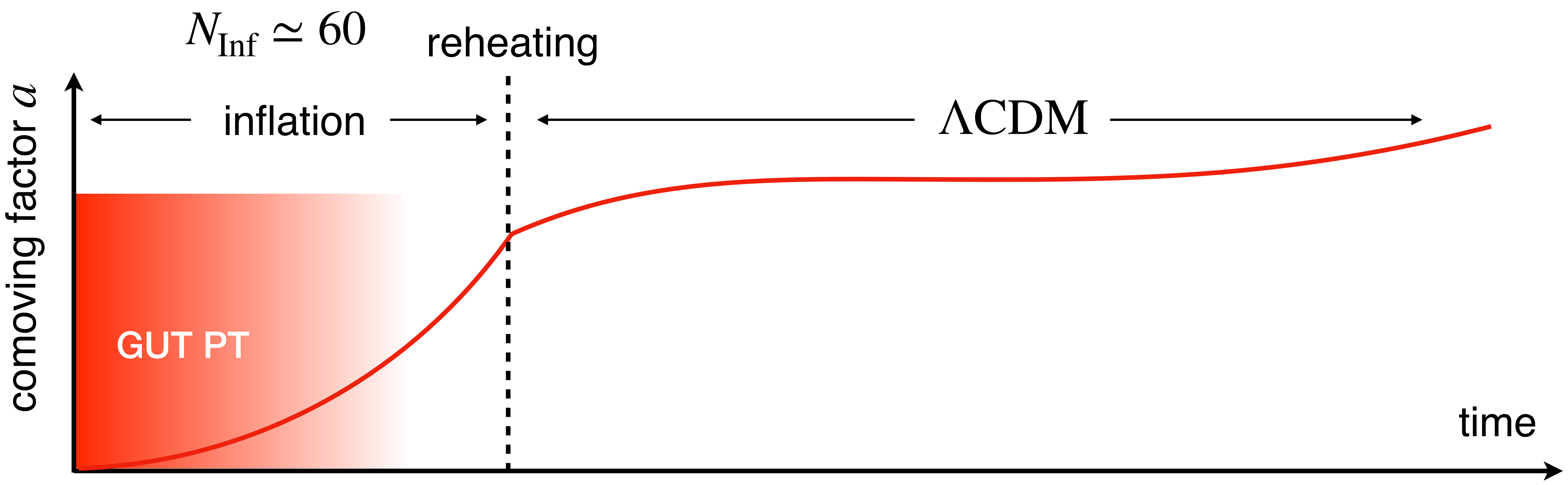
GUT inflation

- Most GUT inflation models assume GUT phase transition (PT) in the beginning of or smoothing during inflation



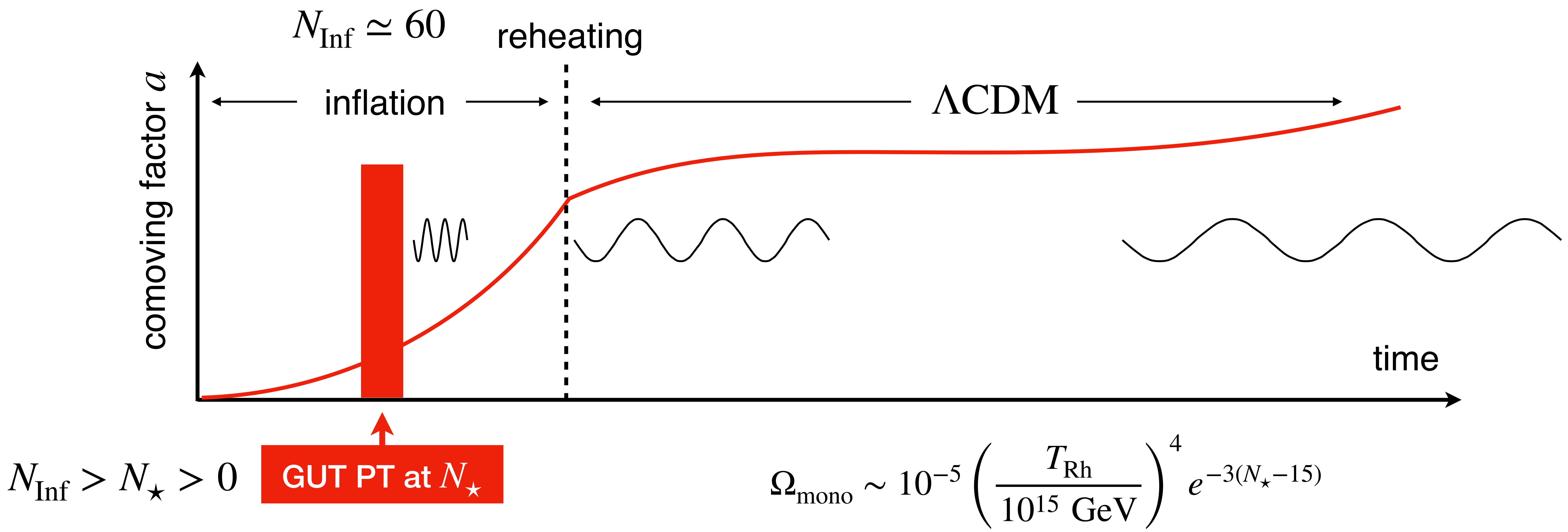
SU(5) inflation, Vilenki, Shafi, PRL, 1984

Smooth hybrid inflation, Lazarides, Panagiotakopoulos, hep-ph/9506325

Shifted hybrid inflation in PS model, Jeannerot, Khalil, Lazarides, Shafi, hep-ph/0002151

GUT breaking during inflation

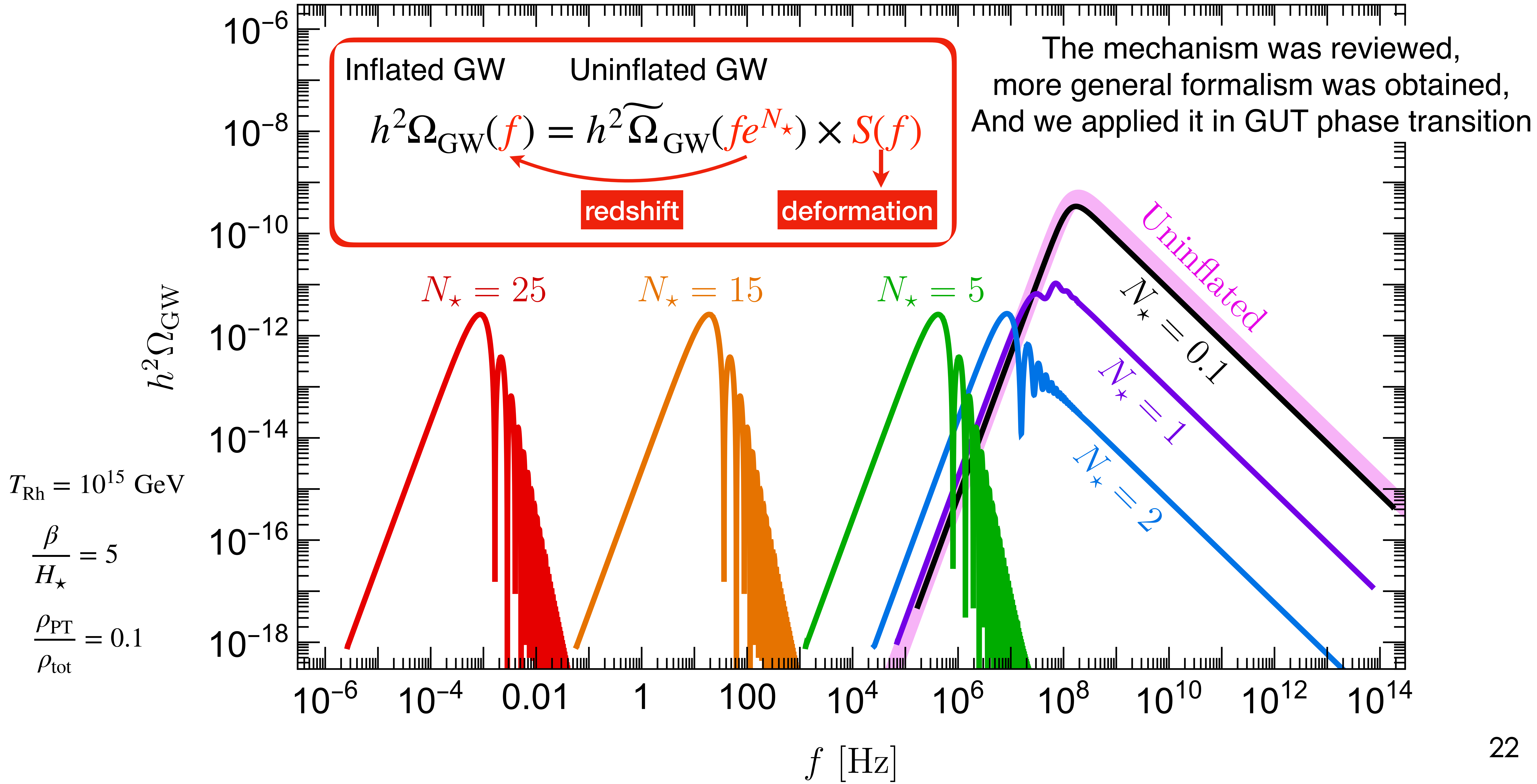
- Locating the GUT PT during inflation can also solve the problem.
  And if the PT is first order …



- GWs via phase transition during inflation can generate distinguishable features in the GW spectra, as pointed out by Haipeng An's group An, Lyu, Wang, Zhou, 2009.12381, 2201.05171

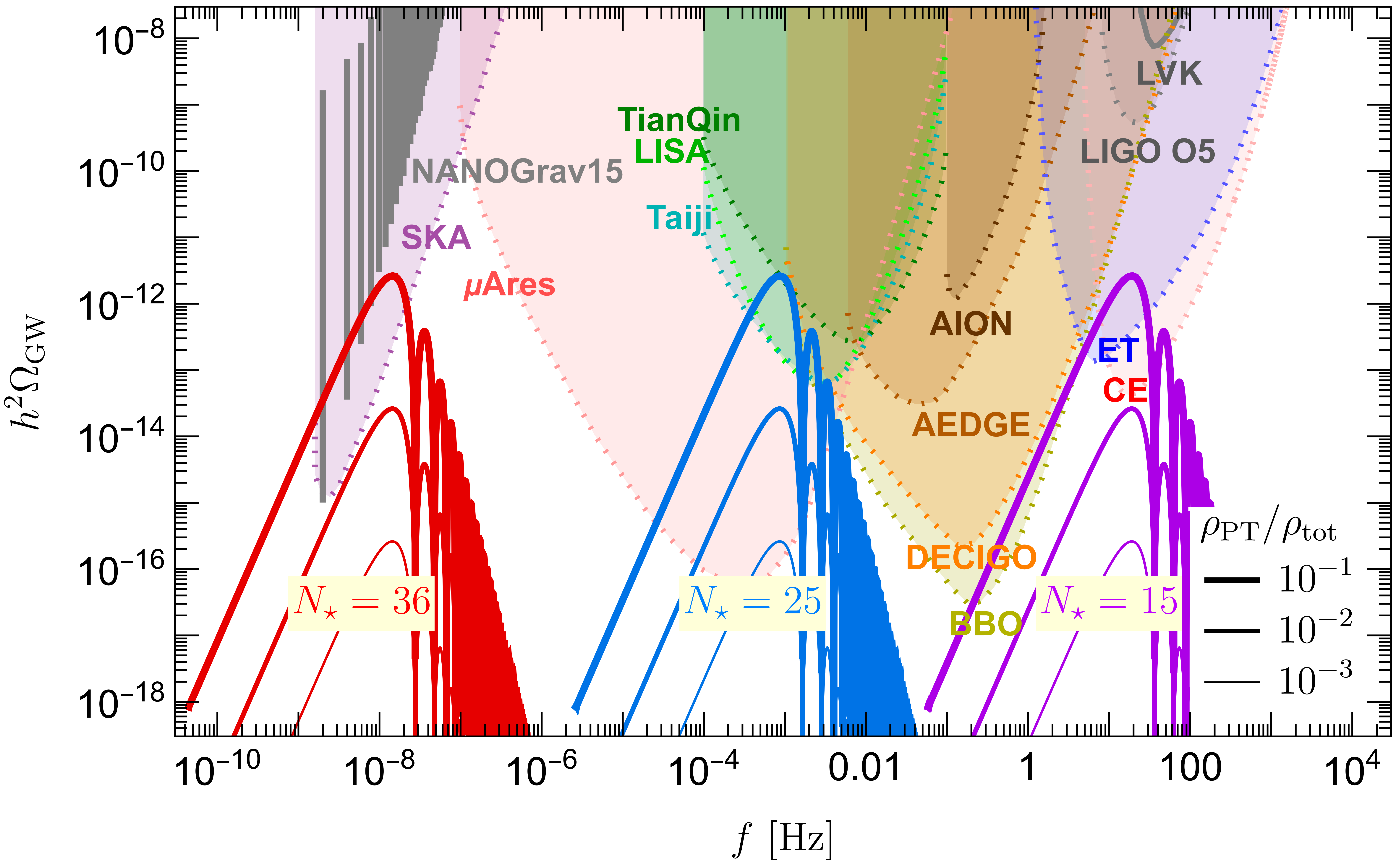
Inflated GWs via GUT phase transition

X.H. He, YLZ, 2501.01491

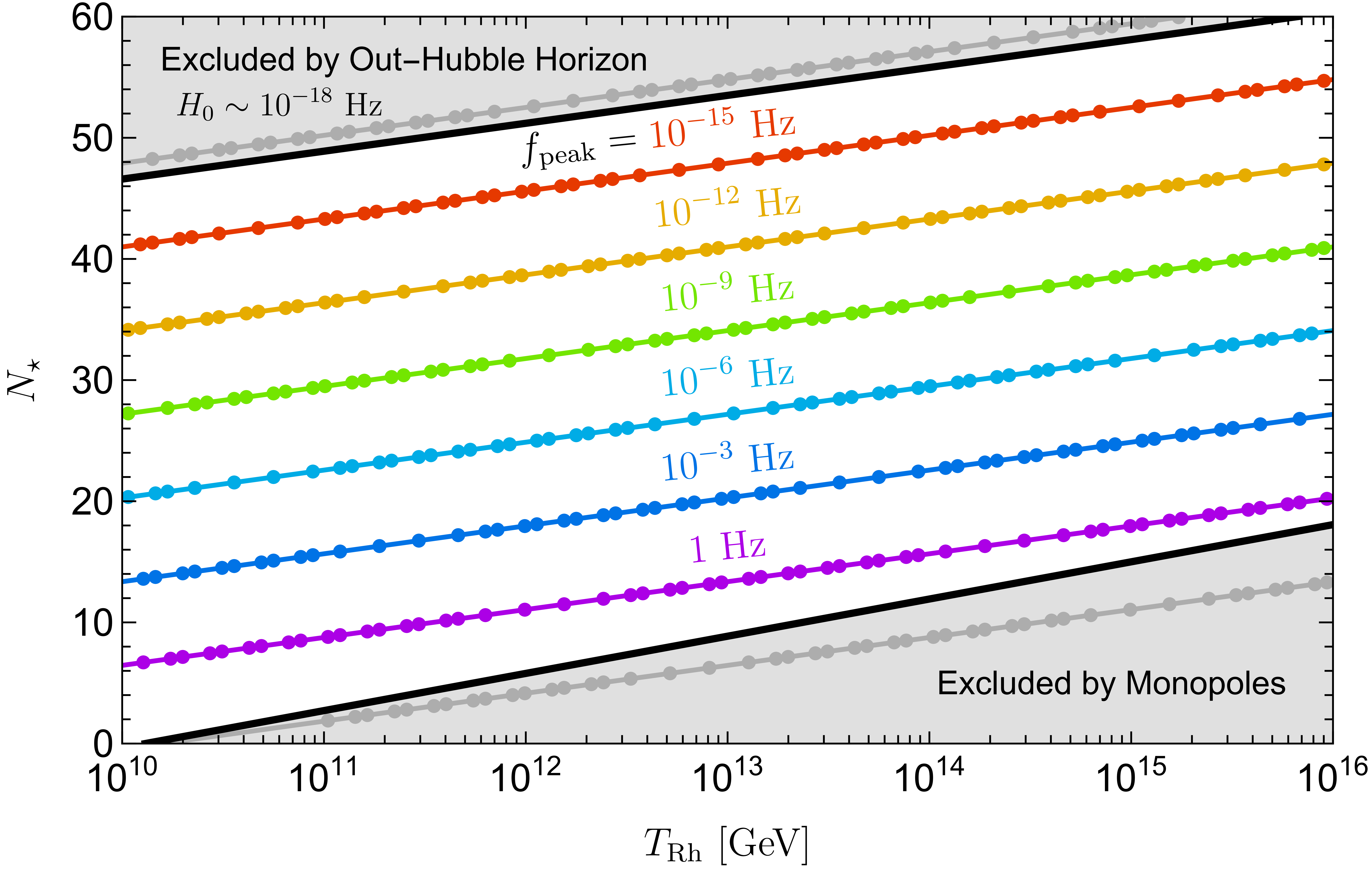


Inflated GWs via GUT phase transition

X.H. He, YLZ, 2501.01491



Inflated GWs via phase transition below the GUT scale



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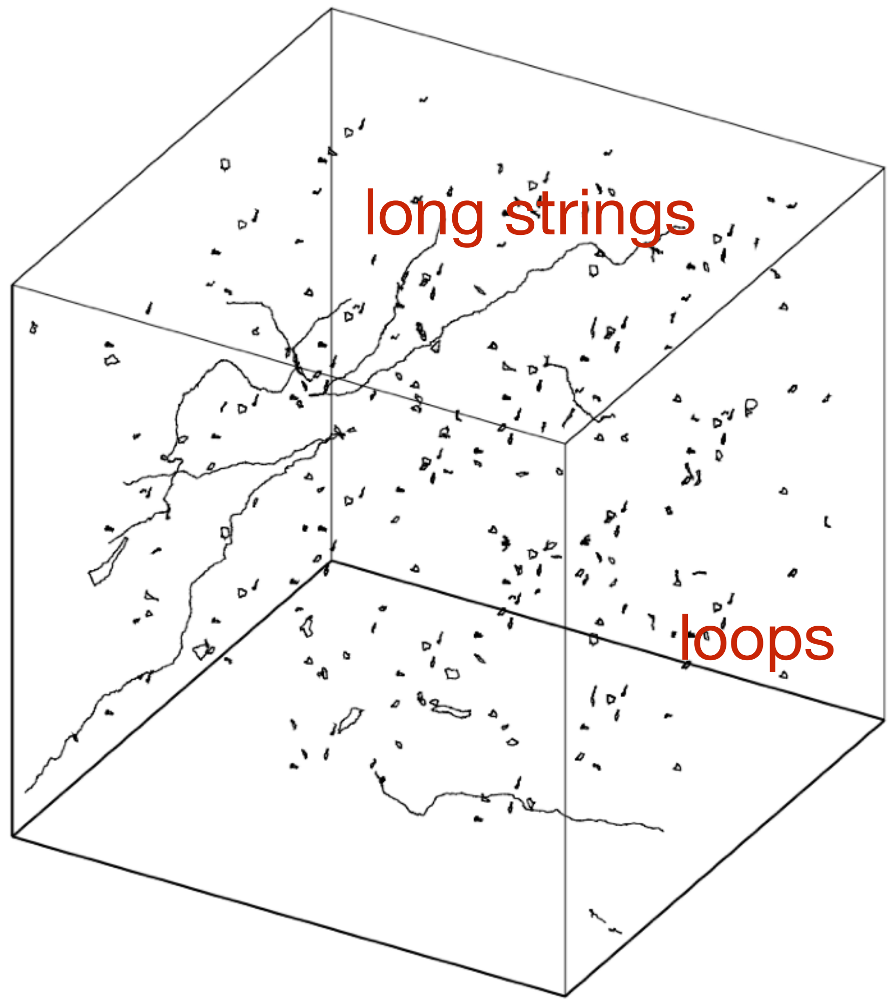
Standard picture of GWs via cosmic strings

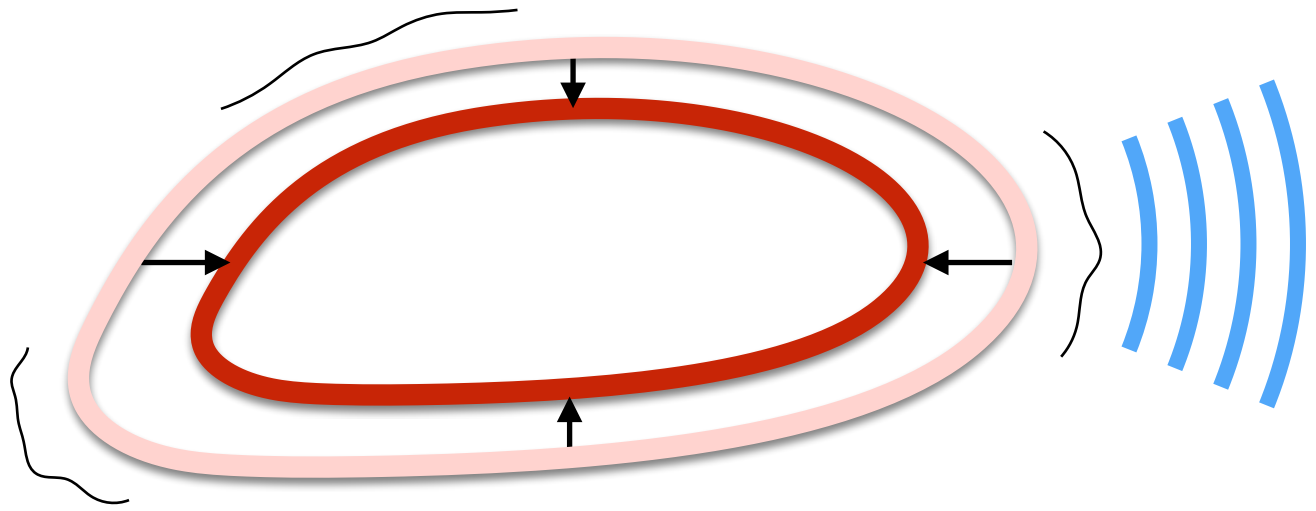
GW via cosmic strings

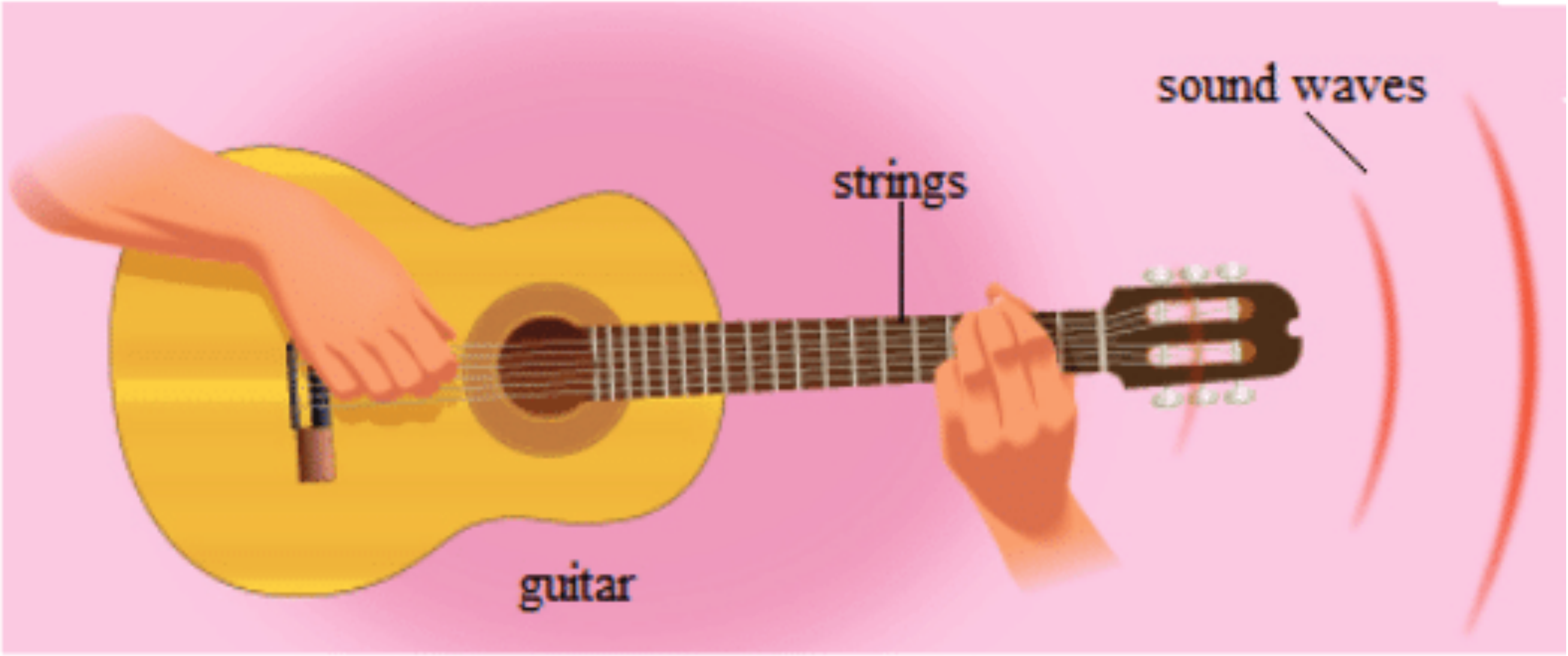
$$\pi_1(U(1)) = Z$$

- Most GUTs include a $U(1)_{B-L}$ symmetry.
- Spontaneous breaking of this U(1) generates cosmic strings.
- Strings intersect and intercommute to form loops and cusps
- Loops oscillate via gravitational radiation



Vanchurin, Olum, Vilenkin, 0511159

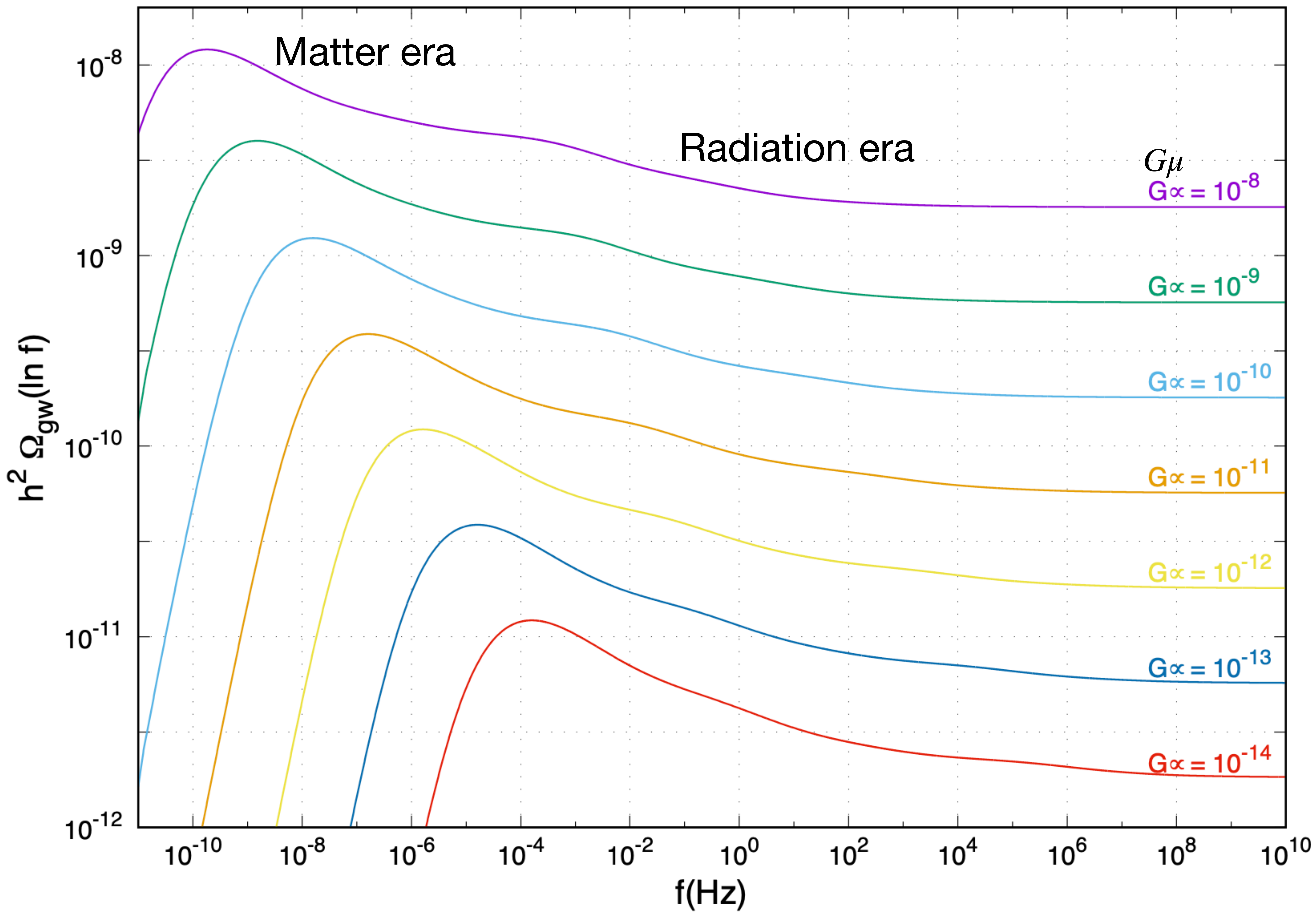




Another mechanism: GW via GUT phase transition?

——require technique developments to measure high-frequency GW, see e.g., 2011.12414, 2310.06607

Standard picture of GWs via cosmic strings



Plateau

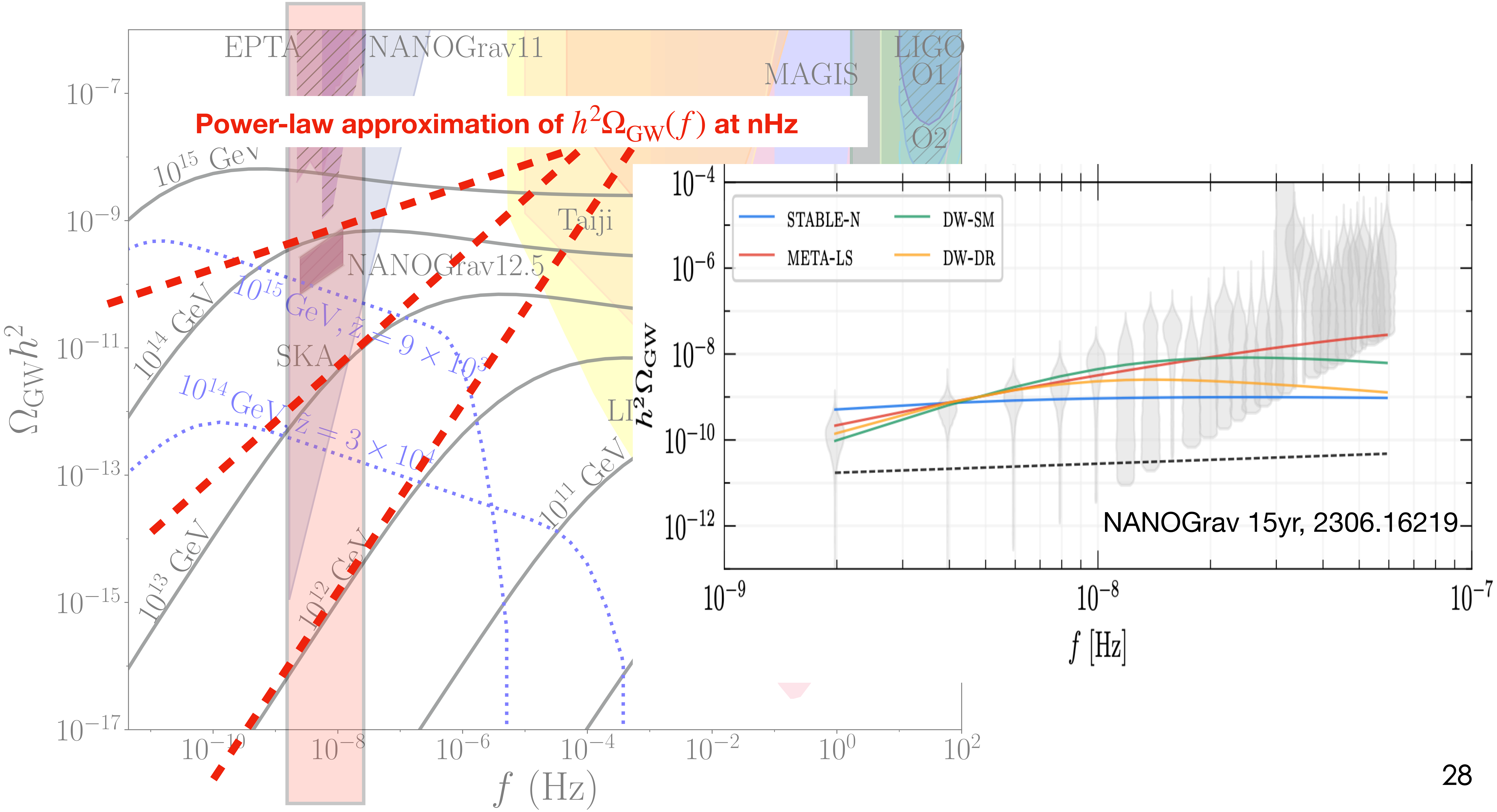
$\Omega_{\rm GW} h^2 \sim 5 \times 10^{-5} \sqrt{G\mu}$

$\propto \dfrac{M_{B-L}}{M_{\rm Planck}}$ in GUTs

$G = M_{\rm Planck}^{-2}$ Newton constant

Blanco-Pillado, Olum
1709.02693

Tensions between NANOGrav and GWs via Nambu-Goto strings



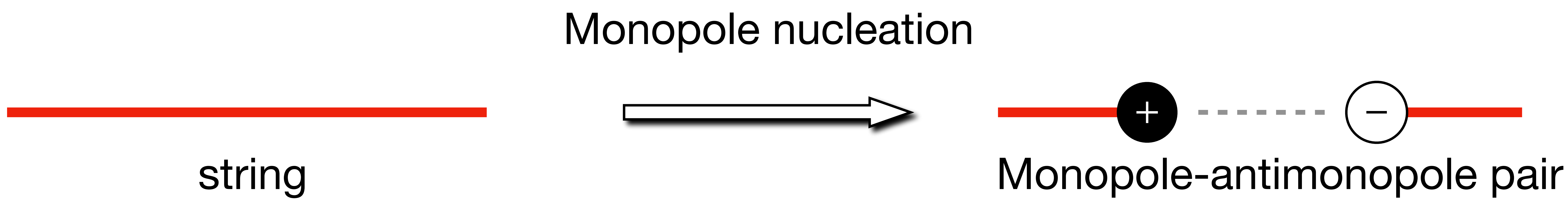
Way out: metastable strings

- In GUTs beyond SU(5), GUT is broken to SM via several steps, e.g.,

$$SO(10) \longrightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

monopole cosmic string

- If monopole mass scale is not far away from the string tension scale, string can decay to monopole-antimonopol pairs and become metastable

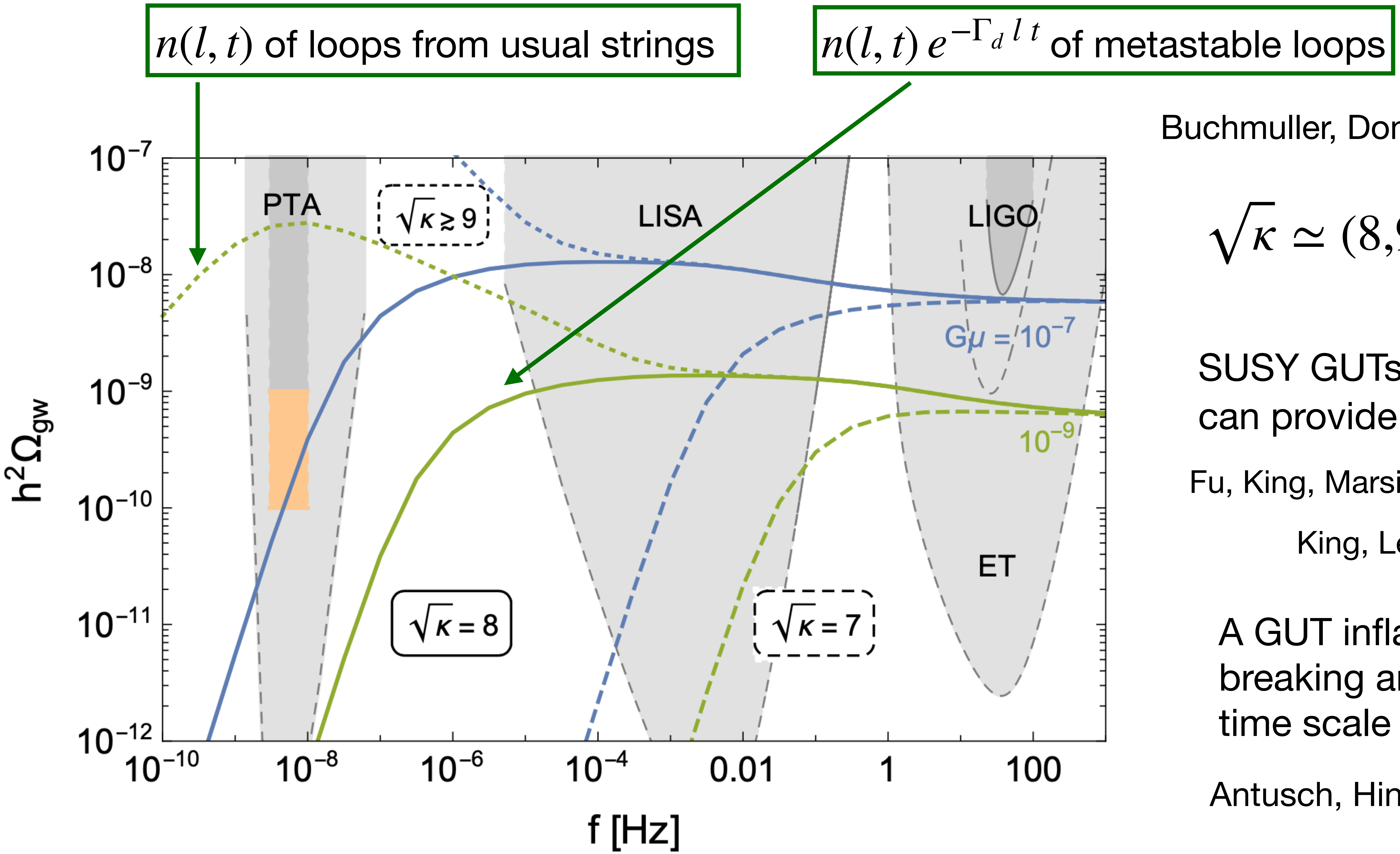


- Decay rate is calculated via bounce action, which is effectively parametrised to be

$$\Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa} \qquad \kappa = \frac{M^2_{\rm mono}}{\mu_{\rm string}}$$

Preskill, Vilenkin, hep-ph/9209210;
Leblond, Shlaer, Siemens, 0903.4686;
Monin, Voloshin, 0808.1693

GW spectrum via metastable strings



$$\sqrt{\kappa} \sim \alpha_{\rm GUT}^{-1/2} \frac{M_{\rm GUT}}{M_{B-L}}$$

Buchmuller, Domcke, Schmitz, 2307.04691

$$\sqrt{\kappa} \simeq (8{,}9) \Rightarrow M_{\rm GUT} \sim M_{B-L}$$

SUSY GUTs and flipped SU(5) GUT can provide such κ in this regime

Fu, King, Marsili, Pascoli, Turner, Zhou, 2308.05799

King, Leotaris, Zhou, 2311.11857

A GUT inflation separates the GUT breaking and B-L breaking in the time scale is required.

Antusch, Hinze, Saad, Steiner, 2307.04595

Summary

- Monopole as an extension of electromagnetic theory

  enlarge the symmetry of electromagnetism;
  explanation of quantisation of electric charge via Dirac's argument

- Monopole as a topological defect in QFT

  a type of soliton solution in QFT,
  universally predicted in GUTs,
  motivation of inflation

- Monopole's motivation on inflated GWs via GUT phase transition

  Solving monopole problem leaves room for GUT phase transition during inflation.
  If GUT phase transition, an inflated GWs, with spectrum redshifted and deformed, might be observed.
  This feature can be used to prove GUT and inflation

- Monopole's influence on GW via cosmic strings

  cosmic strings can decay to monopoles if their generating scales are not far away.
  It can explain NANOGrav-15 data.

Thank you very much!