A novel method for the Tau decay and its possible applications

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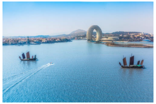

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HUZHOU UNIVERSITY

湖州师范学院
Huzhou University

依然历历在目高能所读博期间的点点滴滴，师恩难忘 。。。

岁月凝香，祝张宗烨老师九十华诞快乐。

Outline

1. $\tau \to \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons

LRD, Pavao, Sakai, Oset, EPJA55(2019)20 arXiv:1805.04573

2. Polarization amplitudes in $\tau \to \nu_\tau VP$ decay beyond the Standard Model (BSM)

LRD, Oset, EPJA54(2018)219 arXiv:1809.02510

3. Triangle singularity in $\tau \to \nu_\tau \pi f_0(980)$ ($a_0(980)$) decays

LRD, Yu, Oset, PRD99(2019)016021 arXiv:1809.11007

4. τ decay into a pseudoscalar and an axial-vector meson

LRD, Roca, Oset, PRD99(2019)096003 arXiv:1811.06875

5. Tau decay into ν_τ and $a_1(1260)$, $b_1(1235)$, and two $K_1(1270)$

LRD, Roca, Oset, EPJC80(2020)673 arXiv: 2005.02653

G-parity plays an important role in these reactions

$$G = (-1)^{L+S+I}$$

1. $\tau \to \nu_\tau M_1 M_2$, with M_1, M_2 pseudoscalar or vector mesons

arXiv:1805.04573 EPJA55(2019)20

meson		J
pseudoscalar	P	1
vector	V	0

motivation

- Tau lepton decays have been instrumental to learn about weak interaction as well as strong interaction. (τ mass: 1776.93 ± 0.09 MeV)
- Several modes are well measured, $\tau \to \nu_\tau PP$ and $\tau \to \nu_\tau PV$.
- Surprisingly, there are no $\tau \to \nu_\tau VV$. (see pdg)

Naturely wondering whether there is some fundamental reason for this experimental fact?

meson ($q\bar{q}$) in the quark model

a) P and V mesons differ only by the spin arrangement of the quarks
$\Rightarrow$ possible to relate the rates of decay for $\tau^- \to \nu_\tau PP, PV, VV$
b) one important issue is charge symmetry [S. Weinberg, PR112(1958)1375]
one interesting reaction $\tau \to \nu_\tau \pi\eta(\eta')$ $\Leftarrow$ forbidden by G-parity [Leroy & Pestieau, PLB72(1978)398]

The G-parity plays an important role in these reactions. $\Rightarrow$ we offer a new perspective into this issue.

what we did? establish a relationship [EPJA55(2019)20]

- by using the basic weak interaction and angular momentum algebra to relate the different processes.
- different interpretation of the role played by G-parity in these decays.

Algebra method

The derivation requires some patience, but we succeed using Racah Algebra.

no any free parameter
relate the different processes

relevant form factors would be the same
the structures can be very different for the produced P or V

we obtained the analytical amplitudes for each reaction

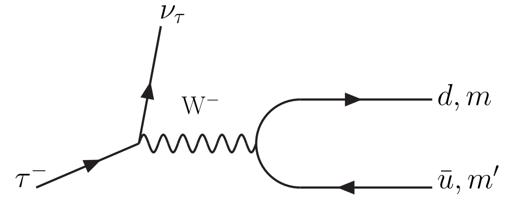
- The evaluation of the invariant mass distributions and branching ratios of rates for PP, PV&VV cases
- good agreement with experimental data
- make some predictions

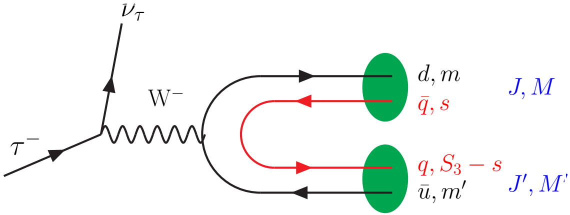
[EPJA55(2019)20] $JJ' = 00, 01, 10, 11$

Appendix A. Evaluation of the matrix elements for the operators "1" and σ_i

Appendix B. Evaluation of $\overline{\sum}\sum |t|^2$

The derivation - Cabibbo-favored $d\bar{u}$ production and Hadronization





$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$d\bar{u} \to \sum_{i=1}^{3} d\ \bar{q}_i\ q_i\ \bar{u} = M_{2i}\ M_{i1} = (M \cdot M)_{21}$$

$$M \Rightarrow P \text{ or } V$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix},$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu$$

For the hadronization, we use the $^3P_0$ model, which has been widely used in the literature and recently it has been found very instrumental to address different problems in hadron physics.

for $^3P_0$ model

1) L. Micu, NPB10(1969)521 cites 577
2) A. Le Yaouanc, et. al., PRD8(1973)2223 cites 737
3) F. E. Close, An Introduction to Quark and Partons, Academic Press, 1979

Cabibbo-favored $d\bar{u}$ productions and hadronization $\Rightarrow$ h_i and $\bar{h}_i$

$$d\bar{u} \to \sum_{i=1}^{3} d\ \bar{q}_i\ q_i\ \bar{u} = M_{2i}\, M_{i1} = (M\cdot M)_{21}$$

$$\begin{aligned}
(P\cdot P)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\pi^0 - \pi^0\pi^-) + \frac{1}{\sqrt{3}}(\pi^-\eta + \eta\pi^-) + \frac{1}{\sqrt{6}}(\pi^-\eta' + \eta'\pi^-) + K^0K^-\,, \\
(P\cdot V)_{21} &= \frac{1}{\sqrt{2}}(\pi^-\rho^0 + \pi^-\omega) - \frac{\pi^0\rho^-}{\sqrt{2}} + \frac{\eta\rho^-}{\sqrt{3}} + \frac{\eta'\rho^-}{\sqrt{6}} + K^0K^{*-}\,, \\
(V\cdot P)_{21} &= \frac{\rho^-\pi^0}{\sqrt{2}} + \frac{\rho^-\eta}{\sqrt{3}} + \frac{\rho^-\eta'}{\sqrt{6}} + \frac{1}{\sqrt{2}}(-\rho^0\pi^- + \omega\pi^-) + K^{*0}K^-\,, \\
(V\cdot V)_{21} &= \frac{1}{\sqrt{2}}(\rho^-\rho^0 - \rho^0\rho^-) + \frac{1}{\sqrt{2}}(\rho^-\omega - \omega\rho^-) + K^{*0}K^{*-}\,. \qquad (1)
\end{aligned}$$

Similarly for Cabibbo-suppressed $s\bar{u}$ production and hadronization

$$s\bar{u} \to \sum_{i=1}^{3} s\ \bar{q}_i\ q_i\ \bar{u} = M_{3i}\ M_{i1} = (M \cdot M)_{31}$$

$$\begin{aligned}
(P \cdot P)_{31} &= K^- \frac{\pi^0}{\sqrt{2}} + \bar{K}^0 \pi^- + \left(K^- \frac{\eta}{\sqrt{3}} - \frac{\eta}{\sqrt{3}} K^- \right) + \left(K^- \frac{\eta'}{\sqrt{6}} + \frac{2\eta'}{\sqrt{6}} K^- \right), \\
(P \cdot V)_{31} &= K^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \bar{K}^0 \rho^- + \left(-\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \right) K^{*-}, \\
(V \cdot P)_{31} &= K^{*-} \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) + \bar{K}^{*0} \pi^- + \phi K^-, \\
(V \cdot V)_{31} &= K^{*-} \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \bar{K}^{*0} \rho^- + \phi K^{*-}.
\end{aligned} \tag{2}$$

Weak matrix elements in Standard Model (SM)

$$H = \mathcal{C} L^{\mu} Q_{\mu}$$

where $\mathcal{C}$ containing weak interaction constants and radial matrix elements.

L^{μ} is the leptonic current

$$L^{\mu} = \langle \bar{u}_{\nu} | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | u_{\tau} \rangle$$

Q^{μ} is the quark current

$$Q^{\mu} = \langle \bar{u}_{d} | \gamma^{\mu} - \gamma^{\mu}\gamma_5 | v_{\bar{u}} \rangle$$

In the evaluation of Q_{μ} matrix element we assume that quark spinors are at rest in that frame

$$u_r = \begin{pmatrix} \chi_r \\ 0 \end{pmatrix}, v_r = \begin{pmatrix} 0 \\ \chi_r \end{pmatrix}$$

The amplitudes

$$Q_0 = \langle \chi' | 1 | \chi \rangle \equiv M_0 \,, \qquad Q_i = \langle \chi' | \sigma_i | \chi \rangle \equiv N_i$$

Denoting for simplicity: $\overline{L}^{\mu\nu} = \overline{\sum} \sum L^{\mu} L^{\nu\dagger}$

$$\begin{aligned} \overline{\sum} \sum |t|^2 &= \overline{\sum} \sum L^{\mu} L^{\nu\dagger} Q_{\mu} Q_{\nu}^* \\ &= \overline{L}^{00} M_0 \, M_0^* + \overline{L}^{0i} M_0 \, N_i^* + \overline{L}^{i0} N_i \, M_0^* + \overline{L}^{ij} N_i \, N_j^* \end{aligned}$$

where we sum over the final polarizations of the mesons produced.

$\overline{L}^{\mu\nu}$ can be easily evaluated in PRD92(2015)014031

$$\overline{\sum} \sum L^{\mu} L^{\nu\dagger} = \frac{1}{m_{\nu} m_{\tau}} \left(p'^{\mu} p^{\nu} + p'^{\nu} p^{\mu} - g^{\mu\nu} p' \cdot p + i\epsilon^{\alpha\mu\beta\nu} p'_{\alpha} p_{\beta} \right)$$

Matrix elements in p-wave for M_0

PP	$J=0, J'=0$	$M_0 = 0$
PV	$J=0, J'=1$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\, \delta_{M0}$
VP	$J=1, J'=0$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{6}}\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})\, \delta_{M'0}$
VV	$J=1, J'=1$	$M_0 = (-1)^{-M-M'} \frac{1}{\sqrt{3}} \mathcal{C}(111; M, M', M+M')$
		$\times\, q\, Y_{1,-(M+M')}(\hat{\boldsymbol{q}})$

By inspecting the change when we permute particle 1 and 2, taking into account that in this permutation $Y_{1,\nu}(\hat{\boldsymbol{q}}) = Y_{1,\nu}(\widehat{\boldsymbol{p_1} - \boldsymbol{p_2}})$ goes to $Y_{1,\nu}(\widehat{\boldsymbol{p_2} - \boldsymbol{p_1}}) = (-)^1 Y_{1,\nu}(\widehat{\boldsymbol{p_1} - \boldsymbol{p_2}})$

Signs change of M_0 amplitude by permuting the order of mesons

	PP	PV	VP	VV
M_0	0	$-$	$-$	$+$

Matrix elements in p-wave for N_μ

PP	$J=0, J'=0$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu}(\hat{\boldsymbol{q}})\, \delta_{M0}\, \delta_{M'0}$
PV	$J=0, J'=1$	$N_\mu = (-1)^{1-M'} \frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M'}(\hat{\boldsymbol{q}}) \mathcal{C}(111; M', -\mu, M'-\mu)\, \delta_{M0}$
VP	$J=1, J'=0$	$N_\mu = (-1)^{-M} \frac{1}{\sqrt{3}}\, q\, Y_{1,\mu-M}(\hat{\boldsymbol{q}}) \mathcal{C}(111; M, -\mu, M-\mu)\, \delta_{M'0}$
VV	$J=1, J'=1$	$N_\mu = \frac{1}{\sqrt{6}}\, q\, Y_{1,\mu-M-M'}(\hat{\boldsymbol{q}}) \{(-1)^{-M'} \delta_{\mu M} + 2\,(-1)^{-M}$
		$\times \mathcal{C}(111; M, -\mu, M-\mu) \mathcal{C}(111; M', -M-M'+\mu, -M+\mu)\}$

Signs change of N_μ amplitude by permuting the order of mesons

	PP	PV	VP	VV
N_μ	$-$	$+$	$+$	$-$

from a different perspective that $\pi^-\eta$ and $\pi^-\eta'$ are forbidden by G-parity

in coincidence with results obtained through different methods [C. Leroy, J. Pestieau, PLB72(1978)398

1) for the $\pi^-\pi^0$ channel

It comes with the combination $\pi^-\pi^0 - \pi^0\pi^-$. As a consequence N_μ adds for the two terms and we have a weight $2\frac{1}{\sqrt{2}}$ for the $\pi^-\pi^0$ channel

2) for $\pi^-\eta$ channel

It comes with the combinations $\pi^-\eta + \eta\pi^-$, and then the combination of the two terms cancels $\Rightarrow$ do not have $\pi^-\eta$ production

Weights for the different channels and Contributions

Channels	h_i (for M_0)	$\overline{h}_i$ (for N_μ)	M_0	N_μ
$\pi^0\pi^-$	0	$\sqrt{2}$	0	$\times$
$\pi^-\eta$	0	0	0	0
$\pi^-\eta'$	0	0	0	0

M_0 and N_μ carry negative and positive G-parity, respectively.

EPJA55(2019)20 in p-wave

Table 5. Branching ratios for PP case in p-wave normalize by $\tau^- \to \nu_\tau K^- K^0$.

Decay process	BR (Theo.)	BR (Exp.)
$^1\tau^- \to \nu_\tau \pi^0 \pi^-$ [a]	2.48×10^{-2}	$(3.0 \pm 3.2) \times 10^{-3}$
$^1\tau^- \to \nu_\tau \eta \pi^-$	0	$< 9.9 \times 10^{-5}$
$^1\tau^- \to \nu_\tau \eta' \pi^-$	0	$< 4.0 \times 10^{-6}$
$^2\tau^- \to \nu_\tau \eta K^-$ [b]	8.17×10^{-5}	$(1.55 \pm 0.08) \times 10^{-4}$
$^2\tau^- \to \nu_\tau \eta' K^-$	3.26×10^{-7}	$< 2.4 \times 10^{-6}$
$^2\tau^- \to \nu_\tau \pi^0 K^-$	1.29×10^{-4}	$(2.7 \pm 1.1) \times 10^{-4}$
$^1\tau^- \to \nu_\tau K^- K^0$	fit to the Exp.	$(1.48 \pm 0.05) \times 10^{-3}$
$^2\tau^- \to \nu_\tau \pi^- \bar{K}^0$	2.52×10^{-4}	$(5.4 \pm 2.1) \times 10^{-4}$

[a] Means Cabibbo-allowed.
[b] Means Cabibbo-suppressed.

VV case not shown on the right table, due to no experimental data

Table 6. The same as table 5 but for PV and VV cases. The results here in p-wave are only to support that they are in clear contradiction with experiment. Our real predictions for these cases are in table 7 and 8.

Decay process	BR (Theo.)	BR (Exp.)
$^1\tau^- \to \nu_\tau \pi^- \rho^0$	3.90×10^{-3}	
$^1\tau^- \to \nu_\tau \pi^- \omega$	5.31×10^{-3}	$(1.95 \pm 0.06)\%$
$^1\tau^- \to \nu_\tau \pi^0 \rho^-$	3.95×10^{-3}	
$^1\tau^- \to \nu_\tau \eta \rho^-$	4.32×10^{-4}	
$^1\tau^- \to \nu_\tau \eta' \rho^-$	8.25×10^{-9}	
$^1\tau^- \to \nu_\tau K^0 K^{*-}$	2.51×10^{-4}	
$^1\tau^- \to \nu_\tau K^{*0} K^-$	2.49×10^{-4}	$(2.1 \pm 0.4) \times 10^{-3}$
$^2\tau^- \to \nu_\tau K^- \rho^0$	2.18×10^{-5}	$(1.4 \pm 0.5) \times 10^{-3}$
$^2\tau^- \to \nu_\tau K^- \omega$	2.04×10^{-5}	$(4.1 \pm 0.9) \times 10^{-4}$
$^2\tau^- \to \nu_\tau \bar{K}^0 \rho^-$	4.22×10^{-5}	$(2.2 \pm 0.5) \times 10^{-3}$
$^2\tau^- \to \nu_\tau \eta K^{*-}$	3.70×10^{-6}	$(1.38 \pm 0.15) \times 10^{-4}$
$^2\tau^- \to \nu_\tau \eta' K^{*-}$	0	
$^2\tau^- \to \nu_\tau \pi^0 K^{*-}$	6.37×10^{-5}	
$^2\tau^- \to \nu_\tau \bar{K}^{*0} \pi^-$	1.22×10^{-4}	$(2.2 \pm 0.5) \times 10^{-3}$
$^2\tau^- \to \nu_\tau \phi K^-$	2.40×10^{-6}	$(4.4 \pm 1.6) \times 10^{-5}$

We develop formalism for s-wave production for VP and VV case

The matrix elements in s-wave production

$$
\begin{array}{lll}
PP & J=0, J'=0 & M_0 = 0 \\
PV & J=0, J'=1 & M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi} \\
VP & J=1, J'=0 & M_0 = \frac{1}{\sqrt{6}}\frac{1}{4\pi} \\
VV & J=1, J'=1 & M_0 = \frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, M', M+M')
\end{array}
$$

$$
\begin{array}{lll}
PP & J=0, J'=0 & N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\,\delta_{M0}\,\delta_{M'0}\,(-1)^{-\mu} \\
PV & J=0, J'=1 & N_\mu = -(-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M', -\mu, M'-\mu)\,\delta_{M0} \\
VP & J=1, J'=0 & N_\mu = (-1)^{-\mu}\frac{1}{\sqrt{3}}\frac{1}{4\pi}\mathcal{C}(111; M, -\mu, M-\mu)\,\delta_{M'0} \\
VV & J=1, J'=1 & N_\mu = \frac{1}{\sqrt{6}}\frac{1}{4\pi}\Big\{\delta_{M\mu} + 2\,(-1)^{-\mu-M'}\mathcal{C}\,(111; M, -\mu, M-\mu) \\
 & & \times\mathcal{C}\,(111; M', -M-M'+\mu, -M+\mu)\Big\}
\end{array}
$$

Signs by permuting the order of the mesons in s-wave production

	PP	PV	VP	VV
M_0	0	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

We obtain the analytical amplitudes in s-wave production EPJA55(2019)20

1) $PP(J=0, J'=0)$

$$\overline{\sum}\sum|t|^2=\frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left(E_\tau E_\nu-\frac{\boldsymbol{p}^2}{3}\right)\frac{1}{2}\overline{h}_i^2 \quad \text{(2-a)}$$

2) $PV(J=0, J'=1)$; $VP(J=1, J'=0)$

$$\overline{\sum}\sum|t|^2=\frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left[\left(E_\tau E_\nu+\boldsymbol{p}^2\right)\frac{1}{2}h_i^2+\left(E_\tau E_\nu-\frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \quad \text{(2-b)}$$

3) $VV(J=1, J'=1)$

$$\overline{\sum}\sum|t|^2=\frac{1}{m_\tau m_\nu}\left(\frac{1}{4\pi}\right)^2\left[\left(E_\tau E_\nu+\boldsymbol{p}^2\right)h_i^2+\frac{7}{2}\left(E_\tau E_\nu-\frac{\boldsymbol{p}^2}{3}\right)\overline{h}_i^2\right] \quad \text{(2-c)}$$

p is the momentum of the τ or ν given by $p=\frac{\lambda^{1/2}(m_\tau^2,m_\nu^2,M_{\rm inv}^2(M_1M_2))}{2M_{\rm inv}(M_1M_2)}$

h_i and $\bar{h}_i$ coefficients in s-wave production

channels	h_i (for M_0)	$\bar{h}_i$ (for N_μ)	
$\pi^-\rho^0$	0	$\sqrt{2}$	
$\pi^-\omega$	$\sqrt{2}$	0	
$\pi^0\rho^-$	0	$-\sqrt{2}$	
$\eta\rho^-$	$\frac{2}{\sqrt{3}}$	0	
$\eta'\rho^-$	$\frac{2}{\sqrt{6}}$	0	
$K^{*0}K^-$	1	1	Scalar resonances
K^0K^{*-}	1	1	
$\rho^-\omega$	0	$\sqrt{2}$	
$K^{*0}K^{*-}$	1	1	Axial-vector resonances
$\rho^-\rho^0$	$\sqrt{2}$	0	
ηK^{*-}	0	$-\frac{2}{\sqrt{3}}\tan\theta_c$	
$\eta' K^{*-}$	$\frac{3}{\sqrt{6}}\tan\theta_c$	$\frac{1}{\sqrt{6}}\tan\theta_c$	

The final differential mass distribution and width

$$\frac{d\Gamma}{dM_{\text{inv}}(M_1 M_2)} = \frac{2\, m_\tau 2\, m_\nu}{(2\pi)^3} \frac{1}{4m_\tau^2}\, p_\nu \widetilde{p}_1 \overline{\sum} \sum |t|^2 \qquad (3)$$

where p_ν is the neutrino momentum in the tau rest frame, and $\widetilde{p}_1$ the momentum of M_1 in the M_1, M_2 rest frame.

$$p_\nu = \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, M_{\text{inv}}^2(M_1 M_2))}{2M_\tau}, \qquad \widetilde{p}_1 = \frac{\lambda^{1/2}(M_{\text{inv}}^2(M_1 M_2), m_{M_1}^2, m_{M_2}^2)}{2M_{\text{inv}}(M_1 M_2)}$$

Then by integrating Eq. (3) over the $M_1 M_2$ invariant mass, we obtain the width.

EPJA55(2019)20 in s-wave

Table 8. The same as table 7 but with convolution.

Decay process	*BR* (Theo.)	*BR* (Exp.)
$\tau^- \to \nu_\tau \pi^- \rho^0$	7.81×10^{-2}	
$\tau^- \to \nu_\tau \pi^- \omega$	5.56×10^{-2}	$(1.95 \pm 0.06)\%$
$\tau^- \to \nu_\tau \pi^0 \rho^-$	7.91×10^{-2}	
$\tau^- \to \nu_\tau \eta \rho^-$	5.34×10^{-3}	
$\tau^- \to \nu_\tau \eta' \rho^-$	2.96×10^{-5}	
$\tau^- \to \nu_\tau K^0 K^{*-}$	4.91×10^{-3}	
$\tau^- \to \nu_\tau K^{*0} K^-$	4.87×10^{-3}	$(2.1 \pm 0.4) \times 10^{-3}$
$\tau^- \to \nu_\tau K^- \rho^0$	3.82×10^{-4}	$(1.4 \pm 0.5) \times 10^{-3}$
$\tau^- \to \nu_\tau K^- \omega$	3.10×10^{-4}	$(4.1 \pm 0.9) \times 10^{-4}$
$\tau^- \to \nu_\tau \bar{K}^0 \rho^-$	7.44×10^{-4}	$(2.2 \pm 0.5) \times 10^{-3}$
$\tau^- \to \eta K^{*-} \nu_\tau$	fit to the Exp.	$(1.38 \pm 0.15) \times 10^{-4}$
$\tau^- \to \nu_\tau \eta' K^{*-}$	1.21×10^{-10}	
$\tau^- \to \nu_\tau \pi^0 K^{*-}$	1.03×10^{-3}	
$\tau^- \to \nu_\tau \bar{K}^{*0} \pi^-$	1.99×10^{-3}	$(2.2 \pm 0.5) \times 10^{-3}$
$\tau^- \to \nu_\tau \phi K^-$	6.54×10^{-5}	$(4.4 \pm 1.6) \times 10^{-5}$

predictions for VV case

$\tau^- \to \nu_\tau \rho^- \rho^0$	3.31×10^{-3}
$\tau^- \to \nu_\tau \rho^- \omega$	5.82×10^{-3}
$\tau^- \to \nu_\tau K^{*0} K^{*-}$	8.18×10^{-6}
$\tau^- \to \nu_\tau K^{*-} \rho^0$	2.96×10^{-5}
$\tau^- \to \nu_\tau K^{*-} \omega$	6.0×10^{-6}
$\tau^- \to \nu_\tau \bar{K}^{*0} \rho^-$	5.46×10^{-5}
$\tau^- \to \nu_\tau K^{*-} \phi$	0

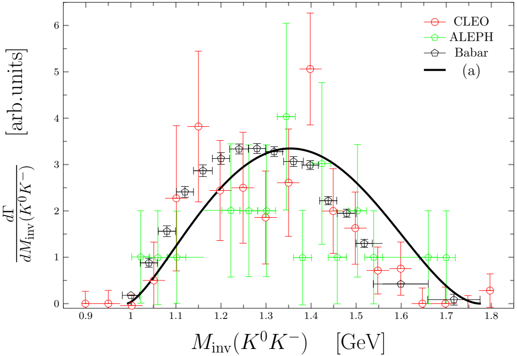
by comparisons with experiments for rates, we find that

a) *PP* case, p-wave production

b) *PV* and *VV* cases, s-wave production

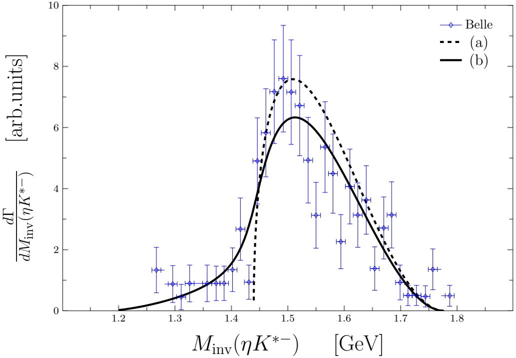
Comparisons with experiments for invariant mass distributions

PP case



p-wave production

PV case



s-wave production

we find that our predictions are in line with the results of other theoretical approaches

for examples:
Kühn, Santamaria, ZPC48(1990)445;
Barish, troynowski, Phys Rep 157(1988) 1 ;
Li, PRD52(1995)5165; Li, PRD52(1995)5184 (vector meson dominance)
Volkov, Kostunin, Phys Part Nucl Lett 10 (2013)7

discussions in section 7 (comparison with other approaches):
EPJA55(2019)20

2. Polarization amplitudes in $\tau \to \nu_\tau VP$ decay beyond the Standard Model (BSM) arXiv:1809.02510 EPJA54,219

- $\tau \to \nu_\tau VP$
- project over spin components
- M, M' are the third components of the K^{*0} and K^-, respectively,

K^{*0}	$J = 1$	$M = 0, \pm 1$
K^-	$J' = 0$	$M' = 0$

- The quantization axis is taken along the direction of the neutrino in the τ rest frame.

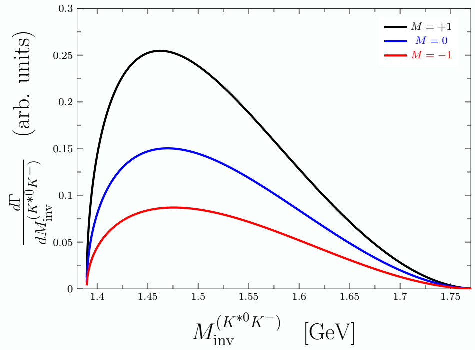
we obtain:

1) first The τ decay amplitude for different spin M components
2) then obtain the final differential width for each M component and ratios divided by the total differential width

$$d\Gamma/dM_{\rm inv}^{(K^{*0}K^-)} = \frac{2\,m_\tau 2\,m_\nu}{(2\pi)^3}\frac{1}{4m_\tau^2}\,p_\nu \widetilde{p}_1 \overline{\sum}\sum|t|^2$$

where p_ν is the momentum of neutrino in τ rest frame and $\widetilde{p}_1$ of K^{*0} in the $K^{*0}K^-$ rest frame.

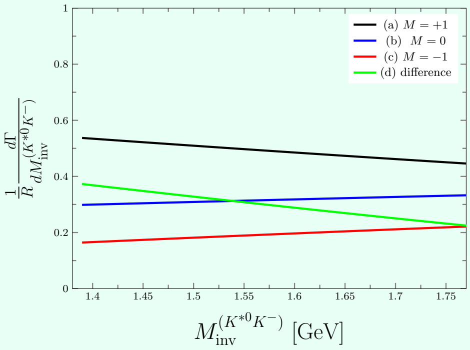
The differential width for each different M



$R = \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1} + \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=0} + \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=-1}$

difference: $\frac{1}{R}\left(\frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1} - \frac{d\Gamma}{dM_{\text{inv}}^{(K^{*0}K^-)}}\Big|_{M=+1}\right)$

difference of $M = \pm 1$



a big sensitivity of magnitude

we propose to measure the difference

Extension to the consideration of right-handed quark currents
The new differential widths (BSM)

$$a(\gamma^\mu - \gamma^\mu\gamma_5) + b(\gamma^\mu + \gamma^\mu\gamma_5) = \gamma^\mu - \alpha\gamma^\mu\gamma_5 \, .$$

we will study the distributions for different M' as a function of α.

1) $M = 0$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{\left(E_\tau E_\nu + p^2\right) + 2\alpha^2\left(E_\tau E_\nu - p^2\right)\right\}$$

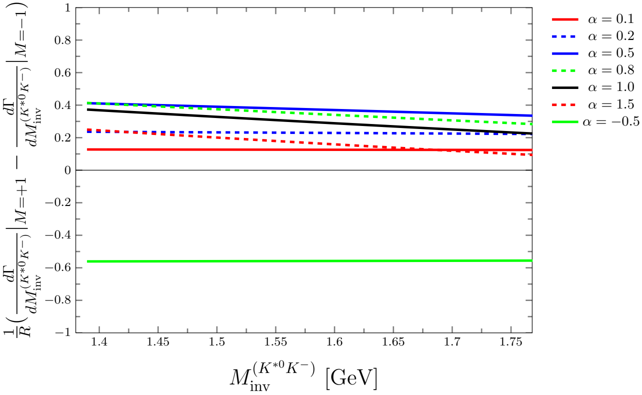
2) $M = 1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) + 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu + (E_\nu - E_\tau)p\right]\alpha^2\right\}$$

3) $M = -1$

$$\overline{\sum}\sum |t|^2 = \frac{1}{m_\tau m_\nu}\frac{1}{6}\frac{1}{(4\pi)^2}\left\{(E_\tau E_\nu + p^2) - 2\alpha(E_\nu + E_\tau)p + \left[2E_\tau E_\nu - (E_\nu - E_\tau)p\right]\alpha^2\right\}$$

suggest to measure the difference experimentally



It is seen that the difference of M=+1 and M=-1 contributions depend strongly on different α
$\Rightarrow$ a big sensitivity of magnitude

this magnitude should be easy to differentiate experimentally.

Test on the nature of scalar and axial-vector resonances in the tau decay

Scalar and axial-vector resonances in the chiral unitary approach

G-Parity

π	$f_0(980)$	$a_0(980)$
$-$	$+$	$-$

G-Parity

π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$

two poles for $K_1(1270)$

- scalar resonances as dynamically generated from the pseudoscalar-pseudoscalar interaction
  Oller, Oset, NPA620,438; NPA652, 407 (Erratum)
  Oller, Oset, Pelaez, PRD59, 074001;PRD60,099906 (Erratum);PRD75,099903 (Erratum)
  Kaiser, EPJA 3,307; Locher, Markushin, Zheng, EPJC4,317
  Nieves, Arriola, NPA 679,57; Pelaez, Rios, PRL97,242002
- axial-vector resonances as dynamically generated from the vector-pseudoscalar interaction
  Lutz, Kolomeitsev, NPA 730, 392
  Roca, Oset, Singh, PRD 72, 014002
  Geng, Oset, Roca, Oller, PRD 75, 014017
  Zhou, Ren, Chen, Geng, PRD 90, 014020

3. Triangle singularity in $\tau \to \nu_\tau \pi f_0(980)$ ($a_0(980)$) decays

arXiv:1809.11007 PRD99,016021

$\tau \to \nu_\tau K^{*0} K^-$ for $J = 1, J' = 0$ case

the experimental branching ratio $\mathcal{B}(\tau \to \nu_\tau K^{*0} K^-) = (2.1 \pm 0.4) \times 10^{-3}$

Signs resulting in the M_0 and N_μ amplitudes by permuting the order of the mesons in s-wave production

	PP	PV	VP	VV
M_0	$-$	$+$	$+$	$-$
N_μ	$+$	$-$	$-$	$+$

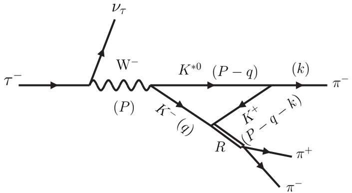
G-Parity

π	$f_0(980)$	$a_0(980)$
$-$	$+$	$-$

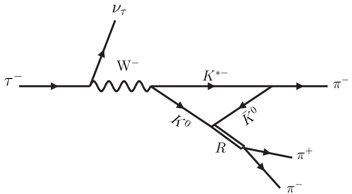
- while M_0 is the same for VP and PV productions, N_i changes sign which is essential for the conservation of G-parity in the reaction.
- there is no simultaneous contribution of the two terms in these reactions
  $\pi^- f_0(980)$ will proceed with the N_i amplitude while $\pi^- a_0(980)$ proceeds with the M_0 term

Diagram for the $\tau \to \nu_\tau K^{*0} K^-$ decay

$\tau \to \nu_\tau \pi^- \pi^+ \pi^-$ for $f_0(980)$

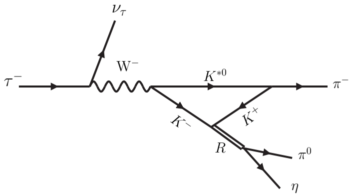


(a)

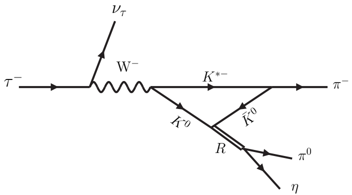


(b)

$\tau \to \nu_\tau \pi^- \pi^0 \eta$ for $a_0(980)$



(a)



(b)

Explicit filter of G-parity states

For the production of $\pi^- f_0(980)$ $\Rightarrow$ negative G-parity

$$\overline{\sum}\sum |t|^2 = \bar{L}^{ij}\widetilde{N}_i\,\widetilde{N}_j^*\,g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu - \frac{1}{3}p^2\right)\frac{1}{3}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^+\pi^-}|^2$$

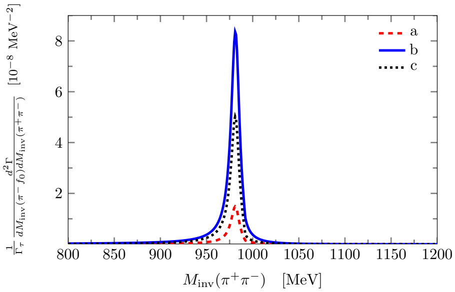
For the production of $\pi^- a_0(980)$ $\Rightarrow$ positive G-parity

$$\overline{\sum}\sum |t|^2 = \bar{L}^{00}\widetilde{M}_0\,\widetilde{M}_0^*\,g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$
$$= \frac{\mathcal{C}^2}{m_\tau m_\nu}\left(E_\tau E_\nu + p^2\right)\frac{1}{6}\frac{1}{(4\pi)^2}\,k^2|t_L|^2\,g^2\,|\,2\,t_{K^+K^-,\pi^0\eta}|^2$$

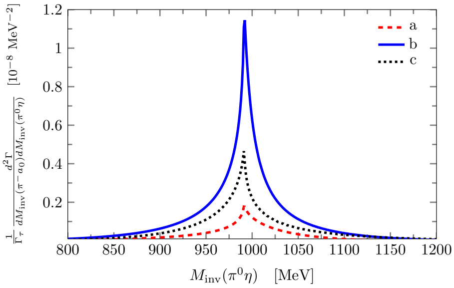
The double differential mass distribution as a function of $M_{\text{inv}}(R)$

Chiral Unitary Approach + Triangle Singularity

$\tau \to \nu_\tau \pi^- \pi^+ \pi^-$ for $f_0(980)$



$\tau \to \nu_\tau \pi^- \pi^0 \eta$ for $a_0(980)$

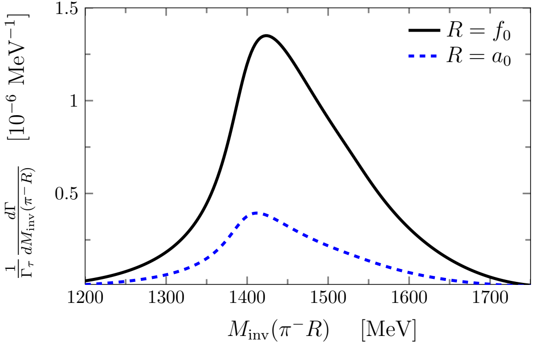


$M_{\text{inv}}(\pi^- R)$ at 1317 MeV (Line a), 1417 MeV (Line b), and 1517 MeV (Line c)

- The distribution with largest strength is near $M_{\text{inv}}(\pi^- R)$=1417 MeV
- A strong peak in the $\pi^+\pi^-$ mass distribution around 980 MeV corresponding to the $f_0(980)$
- The distinctive cusp in the $\pi^0\eta$ mass distribution around 990 MeV corresponding to the $a_0(980)$

Integrating $\frac{d\Gamma}{dM_{\text{inv}}(\pi^- R)}$ over $M_{\text{inv}}(\pi^- R)$ we obtain the branching fractions

It is found that these numbers are within measurable range!



$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- f_0(980);\ f_0(980) \to \pi^+\pi^-) = (2.6 \pm 0.5) \times 10^{-4}$$

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- a_0(980);\ a_0(980) \to \pi^0\eta) = (7.1 \pm 1.4) \times 10^{-5}$$

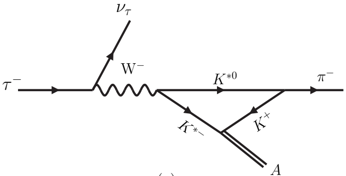
4. τ decay into a pseudoscalar and an axial-vector meson

arXiv:1811.06875 PRD99, 096003

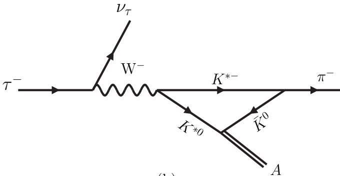
Diagrams for the decay of $\tau^- \to \nu_\tau \pi^- A$ (A axial vectors)

G-Parity

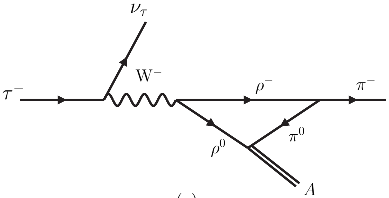
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$



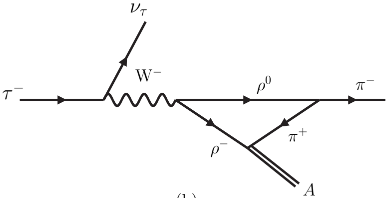
(a)



(b)



(a)



(b)

Explicit filter of G-parity states

G-Parity		
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$

there is no simultaneous contribution in these reactions.

$\pi^- f_1(1285)$ and $\pi^- b_1(1235)$ will proceed with the N_i amplitude.

$\pi^- h_1(1170)$, $\pi^- h_1(1380)$ and $\pi^- a_1(1260)$ proceed with the M_0 term.

- for G-parity negative axial states

$$\overline{\sum}\sum|t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\,\frac{1}{(4\pi)^2}\,\frac{1}{3}\,(E_\tau E_\nu + p^2)\,g^2\,k^2\,|(-1)g_{A,K^*\bar{K}}\,t_L(K^*\bar{K}^*) - 2\,D(-1)g_{A,\rho\pi}\,t_L(\rho\rho)|^2$$

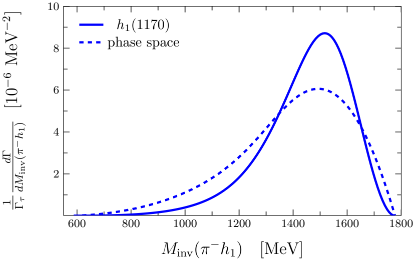
- for G-parity positive axial states

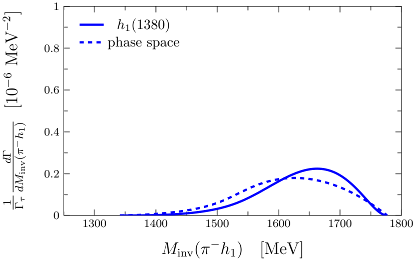
$$\overline{\sum}\sum|t|^2 = \frac{\mathcal{C}^2}{m_\tau m_\nu}\,\frac{1}{(4\pi)^2}\,\frac{7}{6}(E_\tau E_\nu - \frac{1}{3}p^2)g^2k^2|g_{A,K^*\bar{K}}|^2\,|t_L(K^*\bar{K}^*)|^2$$

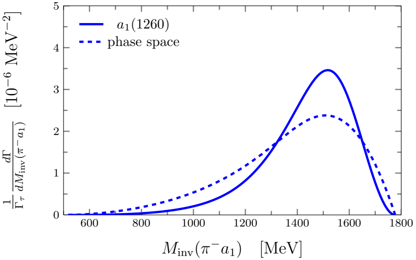
- (only input) from the experimental branching ratio to obtain $\frac{\mathcal{C}^2}{\Gamma_\tau}$

$$\mathcal{B}(\tau\to\nu_\tau K^{*0}K^{*-}) = \frac{1}{\Gamma_\tau}\Gamma(\tau\to\nu_\tau K^{*0}K^{*-}) = (2.1\pm 0.5)\times 10^{-3}\ \Rightarrow\ \frac{\mathcal{C}^2}{\Gamma_\tau} = (5.0)\times 10^{-4}\,\mathrm{MeV}^{-1}.$$

invariant mass distributions and branching ratios for G-parity positive states



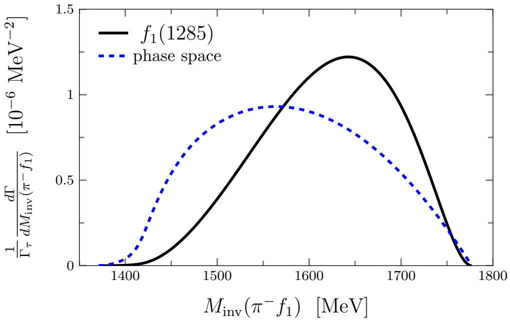


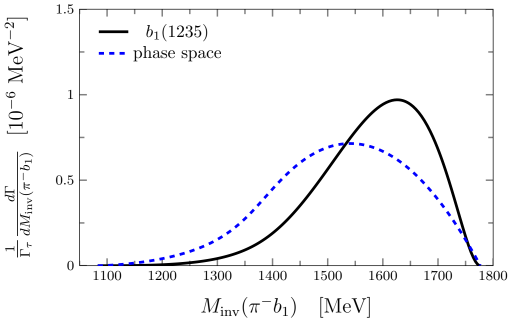


A	$\mathcal{B}(\tau \to \nu_\tau \pi A)$
$h_1(1170)$	3.1×10^{-3}
$a_1(1260)$	1.3×10^{-3}
$b_1(1235)$	2.4×10^{-4}

These numbers are within measurable range.

invariant mass distributions and branching ratios for G-parity negative states

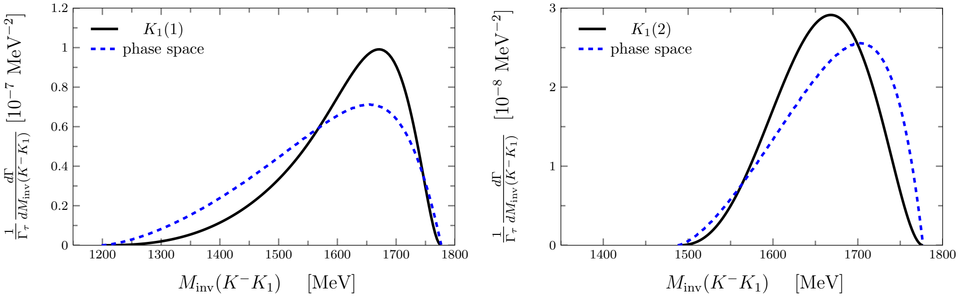




A	$\mathcal{B}(\tau \to \nu_\tau \pi A)$
$f_1(1285)$	2.4×10^{-4}
$h_1(1380)$	3.8×10^{-5}

These numbers are within measurable range.

invariant mass distributions and branching ratios for two $K_1(1270)$ states



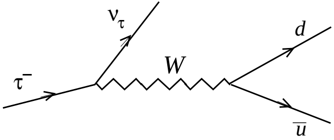
	$\mathcal{B}(\tau^- \to \nu_\tau K^- K_1)$
$K_1(1)$	2.1×10^{-5}
$K_1(2)$	4.1×10^{-6}

These numbers are within measurable range.

5. Tau decay into ν_τ and $a_1(1260)$, $b_1(1235)$, and two $K_1(1270)$

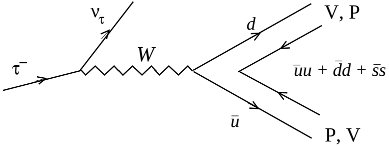
arXiv:2005.02653 EPJC80,673

Cabibbo-favored τ decay to quark-antiquark



$$d\bar{u} \to \sum_{i=1}^{3} d\bar{q}_i q_i \bar{u} = \sum_{i=1}^{3} M_{2i} M_{i1} = (M^2)_{21}$$

Hadronization of the primary $q\bar{q}$ pair (with the quantum numbers of the vacuum) to produce a vector and pseudoscalar meson.



$$(P \cdot V)_{21} = \pi^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) + \left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) \rho^- + K^0 K^{*-} \,,$$

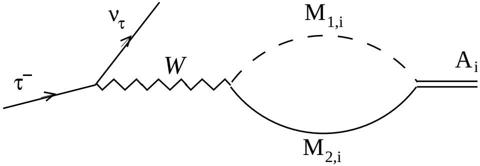
$$(V \cdot P)_{21} = \rho^- \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} \right) + \left(-\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \right) \pi^- + K^{*0} K^- \,.$$

Cabibbo-suppressed: proceed analogously but the hadronization of $s\bar{u}$ gives rise to the matrix element $(M^2)_{31}$ two $K_1(1270)$ with $I = \frac{1}{2}$

$$(P \cdot V)_{31} = K^- \left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right) + \bar{K}^0\rho^- + \left(-\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}}\right) K^{*-} ,$$

$$(V \cdot P)_{31} = K^{*-} \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}}\right) + \bar{K}^{*0}\pi^- + \phi K^- .$$

final state interaction



Mechanism for the production of the dynamically generated axial-vector resonance A_i through the coupled channels $M_{1,i}$, $M_{2,i}$, of pseudoscalar and vector mesons

axial-vector resonance A_i in s-wave in the chiral unitary approach

Roca, Oset, Singh, PRD 72(2005)014002; Geng, Oset, Roca, Oller, PRD75(2007)014017

an example: $\tau \to \nu_\tau a_1(1260)$

Table: Weights h_i' ($\bar{h}_i'$) of the different $I=1$, $G=-1$, VP components for the M_0 (N_i) amplitudes.

	$\rho^0\pi^-$	$\rho^-\pi^0$	$K^{*0}K^-$	$K^{*-}K^0$
h_i'	0	0	1	1
$\bar{h}_i'$	$\sqrt{2}$	$-\sqrt{2}$	-1	1

Table: Couplings of the $I=1$, $G=-1$, axial-vector resonance $a_1(1260)$ to the different VP channels

	$\rho^0\pi^-$	$\rho^-\pi^0$	$K^{*0}K^-$	$K^{*-}K^0$
g_i	$\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}$	$-\frac{1}{\sqrt{2}}g_{a_1,\rho\pi}$	$\frac{1}{\sqrt{2}}g_{a_1,K^*\bar{K}}$	$-\frac{1}{\sqrt{2}}g_{a_1,K^*\bar{K}}$

The amplitude EPJA55(2019)20 for the (final state interaction) mechanism is readily obtained simply by substituting

$$h_j' \to h_j'' = \sum_i h_i' G_i(M_{A_j}) g_{A_j,i}$$
$$\bar{h}_j' \to \bar{h}_j'' = \sum_i \bar{h}_i' G_i(M_{A_j}) g_{A_j,i}$$

here A_j and G_i (loop function) for $a_1(1260)$.

Results $\Gamma(\tau \to \nu A)$

$$\Gamma(\tau \to \nu A) = \frac{2\, m_\tau 2\, m_\nu}{8\pi} \frac{1}{m_\tau^2}\, p_\nu \overline{\sum} \sum |t|^2$$

Table: Branching ratios (in %) for creation of the different axial-vector resonances.

mode	Decay channel	$\mathcal{B}$ (%)
Cabibbo-favored	$\tau^- \to \nu_\tau a_1(1260)$	18 ± 7 (exp)
	$\tau^- \to \nu_\tau b_1(1235)$	10 ± 4
Cabibbo-suppressed	$\tau^- \to \nu_\tau K_1(A)$	0.63 ± 0.25
	$\tau^- \to \nu_\tau K_1(B$	0.65 ± 0.26

for Cabibbo-favored mode, we find that the unmeasured rates for $b_1(1235)$ production are similar to those of the $a_1(1260)$.

for Cabibbo-suppressed mode, two $K_1(1270)$ states are produced with smaller rates, since they are Cabibbo suppressed by a factor about $\tan\theta_c^2 \simeq 1/20$, but they have very distinct decay modes, which we propose to differentiate.

Suggestions for looking for the two K_1 states

So far experiments look at the $\bar{K}\pi\pi$ invariant mass distribution, which contains both $K^*\pi$ and ρK.

$$\Gamma(\tau \to \nu_\tau A \to \nu_\tau VP) = \Gamma(\tau \to \nu_\tau A)\frac{\Gamma(A \to VP)}{\Gamma_A}, \qquad \Gamma(A \to VP) = \frac{|g_{A,VP}|^2}{8\pi M_A^2} q$$

Table: Branching ratios (in %) for the $\Gamma(\tau \to \nu_\tau A \to \nu_\tau VP)$ process for the $I = 1/2$ intermediate axial-vector resonances. The results have an uncertainty of 40%.

Decay channel	$K^{*-}\pi^0$	$\bar{K}^{*0}\pi^-$	$\rho^0 K^-$	$\rho^- \bar{K}^0$	ωK^-	ϕK^-	$K^{*-}\eta$
$\tau^- \to \nu_\tau K_1(1) \to \nu_\tau VP$	0.12	0.23	0.005	0.010	0.007	0	0
$\tau^- \to \nu_\tau K_1(2) \to \nu_\tau VP$	0.019	0.037	0.085	0.17	0.012	0	0.007

we suggest to measure: (the channels to which this resonance couple most strongly)

- the $K^*\pi$ decay mode to see the $K_1(1)$ state
- the ρK decay mode to see the $K_1(2)$ state

These measurements should shed light on the existence of these two K_1 states.

further comparison with experiments of EPJC81(2021)226 for branching ratios for three pseudoscalar decay modes (or pseudoscalar plus ω or ϕ)

Decay channel	Experimental BR (%)	This work, BR (%)
$K^-\pi^0\pi^0\nu_\tau$	0.0585 ± 0.0027	0.046
$\pi^-\bar{K}^0\pi^0\nu_\tau$	0.3807 ± 0.0124	0.36
$K^-\pi^0 K^0\nu_\tau$	0.1494 ± 0.0070	0.17
$\pi^-K^0\bar{K}^0\nu_\tau$	0.1516 ± 0.0247	0.17
$K^-\pi^+\pi^-\nu_\tau$	0.2923 ± 0.0067	0.27
$\pi^-K^-K^+\nu_\tau$	0.1431 ± 0.0027	0.17
$K^-\pi^0\eta\nu_\tau$	0.00483 ± 0.00116	0.0023
$\pi^-\bar{K}^0\eta\nu_\tau$	0.00936 ± 0.00149	0.0047
$K^-\omega\nu_\tau$	0.0410 ± 0.0092	0.019
$K^-\phi\nu_\tau$	0.0044 ± 0.0016	0
$\pi^-\omega\nu_\tau$	1.955 ± 0.063	2.3

we find a very good overall agreement with data. This gives further strength to our predictions and should be an extra stimulus to find the resonant contributions that we have evaluated.

Summary

- **proposed an algebra method to study the tau decay**

  arXiv:1805.04573 EPJA55(2019)20

  $\Rightarrow$ analytical amplitudes $\Rightarrow$ relate

  only one input parameter decided by experiment data

  from a different perspective to show that $\pi^-\eta(\eta')$ are forbidden by G-parity.

  we find that

  a) *PP* case, p-wave production

  b) *PV* and *VV* cases, s-wave production

- **Polarization amplitudes and test BSM in the tau decay**

  arXiv:1809.02510 EPJA54,219

  finding that the difference of $M = +1$ and $M = -1$ contributions depend strongly on different α ($\alpha = 1$ for SM)

  this magnitude should be easy to differentiate experimentally.

- **Test on the nature of scalar resonances in the tau decay**
  arXiv:1809.11007 PRD99,016021

G-parity plays an important role in these reactions

explicit filter of different G-parity states

$$\begin{cases} f_0(980) & I=0 \quad G=+ \\ a_0(980) & I=1 \quad G=- \end{cases}$$

$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- f_0(980); f_0(980) \to \pi^+\pi^-) = (2.6 \pm 0.5) \times 10^{-4}$$
$$\mathcal{B}(\tau^- \to \nu_\tau \pi^- a_0(980); a_0(980) \to \pi^0\eta) = (7.1 \pm 1.4) \times 10^{-5}$$

These numbers are within measurable range.

• Test on the nature of axial-vector resonances in the tau decay

arXiv:1811.06875 PRD99, 096003

G-parity plays an important role in these reactions.

	G-Parity	
π	$f_1(1285)$	$b_1(1235)$
$-$	$+$	$+$
$h_1(1170)$	$h_1(1385)$	$a_1(1260)$
$-$	$-$	$-$

Table: The branching ratios for $\tau \to \pi^- A, K^- K_1$ decays. An error of the order of 40% to the values is expected (PLB782,332 for $f_1(1285)$)

	$\mathcal{B}$
$h_1(1170)$	3.1×10^{-3}
$a_1(1260)$	1.3×10^{-3}
$b_1(1235)$	2.4×10^{-4}
$f_1(1285)$	2.4×10^{-4}
$h_1(1380)$	3.8×10^{-5}
$K_1(1)$	2.1×10^{-5}
$K_1(2)$	4.1×10^{-6}

only this $\tau^- \to \nu_\tau \pi^- f_1(1285)$ has been experimentally measured giving $(3.8 \pm 1.4) \times 10^{-4}$

$\Rightarrow$ which compares well with the value we obtain for $\tau^- \to \nu_\tau \pi^- f_1(1285)$ within uncertainties.

- **further test on the nature of axial-vector resonances in the $\Gamma(\tau \to \nu A)$**
  arXiv:2005.02653 EPJC80,673

Table: Branching ratios $\Gamma(\tau \to \nu A)$

mode	Decay channel	$\mathcal{B}$ (%)
Cabibbo-favored	$\tau^- \to \nu_\tau a_1(1260)$	18 ± 7 (exp)
	$\tau^- \to \nu_\tau b_1(1235)$	10 ± 4
Cabibbo-suppressed	$\tau^- \to \nu_\tau K_1(1)$	0.63 ± 0.25
	$\tau^- \to \nu_\tau K_1(2)$	0.65 ± 0.26

for Cabibbo-favored mode, we find that the unmeasured rates for $b_1(1235)$ production are similar to those of the $a_1(1260)$.

for Cabibbo-suppressed mode, two $K_1(1270)$ states which we propose to differentiate.

- the $K^*\pi$ decay mode to see the $K_1(1)$ state
- the ρK decay mode to see the $K_1(2)$ state

further comparison with experiments of EPJC81(2021)226 for branching ratios for three pseudoscalar decay modes (or pseudoscalar plus ω or ϕ) we find a very good overall agreement with data.

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