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SHANDONG UNIVERSITY

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手征夸克模型和 重子手征拉氏量中的低能常数关系

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Based on:

J. Jiang, S.Z. Jiang, S.Y. Li, Y.R. Liu, Z.G. Si, H.Q. Wang, Phys. Rev. D 106, 054023 (2022)

J. Jiang, S.Z. Jiang, S.Y. Li, Y.R. Liu, Z.G. Si, H.Q. Wang, Eur. Phys. J. C 83, 296 (2023)

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1. LECs in baryon ChPT and chiral quark model

2. LEC relations in baryon ChPT to $\mathcal{O}(p^3)$

3. The case including $\Delta(1232)$

4. Summary

- Various systems and constructed chiral Lagrangians:

Pseudoscalar mesons $q\bar{q}$: $\mathcal{O}(p^8)$

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)} + \dots$$

Light quark baryons qqq : $\mathcal{O}(p^4)$

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \dots$$

$$\left\{ \begin{array}{l} SU(2) : \mathcal{L}_{\pi NN} \\ SU(3) : \mathcal{L}_{MBB} \\ SU(2) : \mathcal{L}_{\pi\Delta\Delta}, \mathcal{L}_{\pi N\Delta} \\ SU(3) : \mathcal{L}_{MTT}, \mathcal{L}_{MBT} \end{array} \right.$$

M: meson octet
B: baryon octet
T: baryon decuplet

Heavy quark hadrons $Q\bar{q}$, Qqq , QQq : $\mathcal{O}(p^4)$

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \dots$$

- Increasing number of LECs in the SU(3) light hadron case

Pseudoscalar mesons:

\mathcal{L}_2	\mathcal{L}_4	\mathcal{L}_6	\mathcal{L}_8
2	12	94	1254

Light quark baryons

$$\mathcal{L}_{MBB}: 2, 16, 78, 540_?$$

S.Z. Jiang et al., PRD 95, 014012 (2017)

$$\mathcal{L}_{MTT}: 1, 13, 55, 548?$$

$$\mathcal{L}_{MBT}: 1, 5, 67, 611?$$

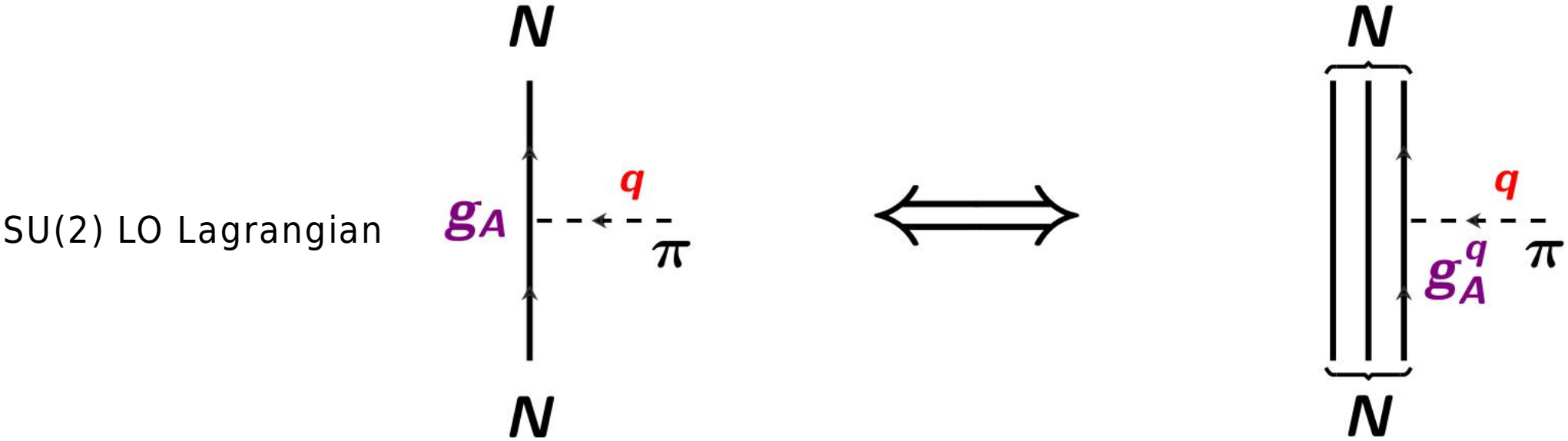
S.Z. Jiang et al., PRD 97, 054031 (2018)

- LEC \rightarrow hadron masses, hadron-hadron scattering, phase shifts, ...
- Difficult to determine LECs from QCD
- Fitting experimental data, lattice QCD, resonance saturation, quark model, ...
- Here: focus on **LEC relations** using chiral quark model (**χ QM**)
- Fields between Λ_{QCD} and $\Lambda_{\chi SB}$ in χ QM: quarks, gluons, and goldstone bosons [A.Manohar, H.Georgi, Nucl.Phys.B 234, 189 (1984)]

$$\mathcal{L}_{\chi QM} = \sum_{q=u,d} \bar{\psi}_q (i\not{D} + \not{Y}) \psi_q + g_A^q \bar{\psi}_q \not{A} \gamma_5 \psi_q + \frac{1}{4} F_\pi^2 \langle u^\mu u_\mu \rangle - \frac{1}{4} \langle \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \rangle + \dots$$

$$A_\mu = \frac{i}{2} (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger), \quad V_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \quad u = \exp\left(\frac{i\phi}{2F_\pi}\right), \quad \phi = \sum_{i=1}^3 \tau_i \pi_i \text{ or } \sum_{i=1}^8 \lambda_i \pi_i$$

- Coupling constant in χ QM

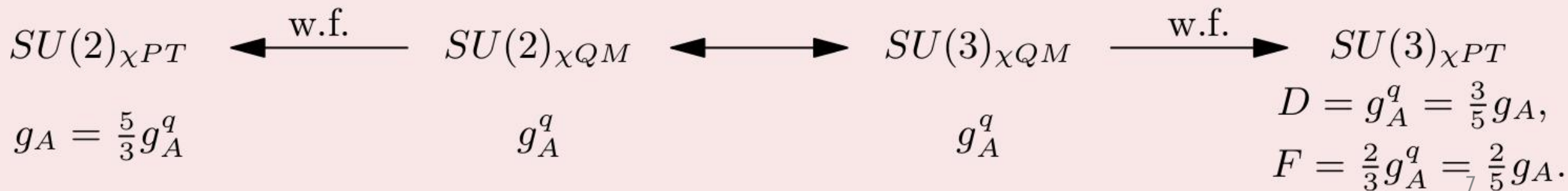


$$\left. \begin{aligned}
 \langle t \rangle_{\text{hadron}} &= \frac{g_A}{2} q_3 \langle p_\uparrow | \sigma_3 \tau_3 | p_\uparrow \rangle = \frac{g_A}{2} q_3 \\
 \langle t \rangle_{\text{quark}} &= \frac{g_A^q}{2} q_3 \langle p_\uparrow | \sum_{i=1}^3 \sigma_3^{(i)} \tau_3^{(i)} | p_\uparrow \rangle = \frac{g_A^q}{2} q_3 \frac{5}{3}
 \end{aligned} \right\} \Rightarrow g_A = \frac{5}{3} g_A^q$$

$\mathcal{O}(p^1)$ 阶 χ PT 和 χ QM 的拉氏量

J. Gasser, Nucl. Phys. B, 307, 779 (1988)

$SU(2)_{\chi PT/\chi QM}$	$SU(3)_{\chi QM}$	$SU(3)_{\chi PT}$	
g_A/g_A^q	g_A^q	$D + F$	$D - F$
$\frac{1}{2}\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$\frac{1}{2}\bar{\Psi}u^\mu\gamma_\mu\gamma_5\Psi$	$\frac{1}{2}\langle\bar{B}u^\mu\gamma_\mu\gamma_5 B\rangle$	$\frac{1}{2}\langle\bar{B}\gamma_\mu\gamma_5 Bu^\mu\rangle$



- Extend relation determination to higher chiral orders
- Purpose: **constrain SU(3) from SU(2)**
- Treating χ QM as quark-level description of ChPT

手征夸克模型方法 \Rightarrow 不同体系拉氏量低能常数间关系

八重态重子

- SU(2) $\mathcal{L}_{\pi NN} \iff \mathcal{L}_{\pi qq}$
- SU(3) $\mathcal{L}_{MBB} \iff \mathcal{L}_{Mqq}$
- χ QM $\mathcal{L}_{Mqq} \iff \mathcal{L}_{\pi qq}$
- χ PT $\mathcal{L}_{MBB} \iff \mathcal{L}_{\pi NN}$

十重态重子

- SU(3) \mathcal{L}_{MTT} 、 \mathcal{L}_{MBT}
- SU(2) $\mathcal{L}_{\pi\Delta\Delta}$ 、 $\mathcal{L}_{\pi N\Delta}$
- $\mathcal{L}_{\pi\Delta\Delta} \iff \mathcal{L}_{\pi qq}$
- $\mathcal{L}_{\pi N\Delta} \iff \mathcal{L}_{\pi qq}$

- Final: LEC relations between **hadron-level** Lagrangians

$$\mathcal{O}(p^1) \Rightarrow \mathcal{O}(p^3)$$

- Need high-order Lagrangians in χ QM (bridging role)
- Results of **QM symmetry**: SU(2), SU(3), and different representations for quarks and baryons
- A systematic approach to get LEC relations
- **Grouping** is needed according to **operator structures**

$\mathcal{O}(p^2)$ 阶 χ PT 和 χ QM 的拉氏量

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)

J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)

$SU(2)_{\chi PT/\chi QM}$	$SU(3)_{\chi QM}$	$SU(3)_{\chi PT}$	
α_i/β_i	c_i	d_i	d_i
1 $\bar{\psi}\langle u^\mu u_\mu\rangle\psi$	1 $\bar{\Psi}\langle u^\mu u_\mu\rangle\Psi$	1 $\langle\bar{B}u^\mu u_\mu B\rangle$	3 $\langle\bar{B}B u^\mu u_\mu\rangle$
	2 $\bar{\Psi}u^\mu u_\mu\Psi$	2 $\langle\bar{B}u^\mu B u_\mu\rangle$	4 $\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle$
2 $i\bar{\psi}u^\mu u^\nu\sigma_{\mu\nu}\psi$	3 $i\bar{\Psi}u^\mu u^\nu\sigma_{\mu\nu}\Psi$	5 $i\langle\bar{B}u^\mu u^\nu\sigma_{\mu\nu}B\rangle$	7 $i\langle\bar{B}u^\mu\rangle\langle u^\nu\sigma_{\mu\nu}B\rangle$
		6 $i\langle\bar{B}\sigma_{\mu\nu}B u^\mu u^\nu\rangle$	
3 $\bar{\psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\psi$	4 $\bar{\Psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\Psi$	8 $\langle\bar{B}u^\mu u^\nu D_{\mu\nu}B\rangle$	10 $\langle\bar{B}D_{\mu\nu}B u^\mu u^\nu\rangle$
	5 $\bar{\Psi}u^\mu u^\nu D_{\mu\nu}\Psi$	9 $\langle\bar{B}u^\mu D_{\mu\nu}B u^\nu\rangle$	11 $\langle\bar{B}D_{\mu\nu}B\rangle\langle u^\mu u^\nu\rangle$
4 $\bar{\psi}f_+^{\mu\nu}\sigma_{\mu\nu}\psi$	6 $\bar{\Psi}f_+^{\mu\nu}\sigma_{\mu\nu}\Psi$	12 $\langle\bar{B}f_+^{\mu\nu}\sigma_{\mu\nu}B\rangle$	13 $\langle\bar{B}\sigma_{\mu\nu}B f_+^{\mu\nu}\rangle$
5 $\bar{\psi}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu}\psi \rightarrow 0$			
6 $\bar{\psi}\tilde{\chi}_+\psi$	7 $\bar{\Psi}\tilde{\chi}_+\Psi$	14 $\langle\bar{B}\tilde{\chi}_+B\rangle$	16 $\langle\bar{B}B\rangle\langle\chi_+\rangle$
7 $\bar{\psi}\langle\chi_+\rangle\psi$	8 $\bar{\Psi}\langle\chi_+\rangle\Psi$	15 $\langle\bar{B}B\tilde{\chi}_+\rangle$	

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^3)$ 阶 χ PT 和 χ QM 的拉氏量第 1 到第 4 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)

J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)

Jiang et al., Phys. Rev. D, 106, 054023 (2022)

$SU(2)_{\chi PT/\chi QM}$		$SU(3)_{\chi QM}$		$SU(3)_{\chi PT}$			
α_i/β_i		c_i		d_i			
1	$\bar{\psi}\langle u^\mu u_\mu\rangle u^\nu \gamma_\nu \gamma_5 \psi$	1	$\bar{\Psi}\langle u^\mu u_\mu\rangle u^\nu \gamma_\nu \gamma_5 \Psi$	1	$[\langle \bar{B}u^\mu u_\mu u^\nu \gamma_\nu \gamma_5 B \rangle]_+$	7	$\langle \bar{B} \gamma_\nu \gamma_5 B u^\mu u^\nu u_\mu \rangle$
2	$\bar{\psi}\langle u^\mu u^\nu\rangle u_\mu \gamma_\nu \gamma_5 \psi$	2	$\bar{\Psi}\langle u^\mu u^\nu\rangle u_\mu \gamma_\nu \gamma_5 \Psi$	2	$\langle \bar{B}u^\mu u^\nu u_\mu \gamma_\nu \gamma_5 B \rangle$	8	$[\langle \bar{B} \gamma_\nu \gamma_5 B u^\nu u^\mu u_\mu \rangle]_+$
		3	$\bar{\Psi}\langle u^\mu u_\mu u^\nu\rangle \gamma_\nu \gamma_5 \Psi$	3	$\langle \bar{B}u^\mu u_\mu \gamma_\nu \gamma_5 B u^\nu \rangle$	9	$\langle \bar{B} \gamma_\nu \gamma_5 B u^\nu \rangle \langle u^\mu u_\mu \rangle$
		4	$[\bar{\Psi}u^\mu u_\mu u^\nu \gamma_\nu \gamma_5 \Psi]_+$	4	$[\langle \bar{B}u^\nu u^\mu \gamma_\nu \gamma_5 B u_\mu \rangle]_+$	10	$\langle \bar{B} \gamma_\nu \gamma_5 B \rangle \langle u^\nu u^\mu u_\mu \rangle$
				5	$\langle \bar{B}u^\nu \gamma_\nu \gamma_5 B u^\mu u_\mu \rangle$	11	$[\langle \bar{B}u^\mu u_\mu \rangle \langle u^\nu \gamma_\nu \gamma_5 B \rangle]_+$
				6	$[\langle \bar{B}u_\mu \gamma_\nu \gamma_5 B u^\mu u^\nu \rangle]_+$		
3	$\epsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle u^\mu u^\nu u^\lambda\rangle D^\rho \psi$	5	$\epsilon_{\mu\nu\lambda\rho} \bar{\Psi}\langle u^\mu u^\nu u^\lambda\rangle D^\rho \Psi$	12	$\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}u^\mu u^\nu u^\lambda D^\rho B \rangle$	15	$\epsilon_{\mu\nu\lambda\rho} \langle \bar{B} D^\rho B u^\mu u^\nu u^\lambda \rangle$
		6	$\epsilon_{\mu\nu\lambda\rho} \bar{\Psi}u^\mu u^\nu u^\lambda D^\rho \Psi$	13	$\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}u^\mu u^\nu D^\rho B u^\lambda \rangle$	16	$\epsilon_{\mu\nu\lambda\rho} \langle \bar{B} D^\rho B \rangle \langle u^\mu u^\nu u^\lambda \rangle$
				14	$\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}u^\lambda D^\rho B u^\mu u^\nu \rangle$		
4	$\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \psi$	7	$\bar{\Psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi$	17	$[\langle \bar{B}u^\mu u^\nu u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle]_+$	23	$[\langle \bar{B} \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\mu u^\nu u^\lambda \rangle]_+$
5	$\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\lambda \gamma_5 D_{\mu\nu} \psi$	8	$\bar{\Psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi$	18	$\langle \bar{B}u^\nu u^\mu u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle$	24	$\langle \bar{B} \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\nu u^\mu u^\lambda \rangle$
		9	$\bar{\Psi}\langle u^\mu u^\nu u^\lambda\rangle \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi$	19	$\langle \bar{B}u^\nu u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\mu \rangle$	25	$\langle \bar{B} \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle \langle u^\mu u^\nu u^\lambda \rangle$
		10	$[\bar{\Psi}u^\mu u^\nu u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi]_+$	20	$[\langle \bar{B}u^\mu u^\nu \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\lambda \rangle]_+$	26	$\langle \bar{B} \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\mu \rangle \langle u^\nu u^\lambda \rangle$
				21	$[\langle \bar{B}u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\mu u^\nu \rangle]_+$	27	$[\langle \bar{B}u^\nu u^\lambda \rangle \langle u^\mu \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle]_+$
				22	$\langle \bar{B}u^\mu \gamma_\mu \gamma_5 D_{\nu\lambda} B u^\nu u^\lambda \rangle$		
6	$[\psi u_\mu h^{\mu\nu} D_\nu \psi]_+$	11	$[\bar{\Psi}u_\mu h^{\mu\nu} D_\nu \Psi]_+$	28	$[\langle \bar{B}u_\mu h^{\mu\nu} D_\nu B \rangle]_+$	30	$[\langle \bar{B}h^{\mu\nu} \rangle \langle u_\mu D_\nu B \rangle]_+$
				29	$[\langle \bar{B}D_\nu B u_\mu h^{\mu\nu} \rangle]_+$		

LECs in baryon ChPT and chiral quark model

$O(p^3)$ 阶 χ PT 和 χ QM 的拉氏量第 5 到第 11 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)

J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)

Jiang et al., Phys. Rev. D, 106, 054023 (2022)

	$SU(2)_{\chi PT/\chi QM}$		$SU(3)_{\chi QM}$	$SU(3)_{\chi PT}$			
	α_i/β_i		c_i	d_i	d_i		
7	$[\bar{\psi}u^\mu h^{\nu\lambda} D_{\mu\nu\lambda}\psi]_+$	12	$[\bar{\Psi}u^\mu h^{\nu\lambda} D_{\mu\nu\lambda}\Psi]_+$	31	$[\langle \bar{B}u^\mu h^{\nu\lambda} D_{\mu\nu\lambda}B \rangle]_+$	33	$[\langle \bar{B}h^{\mu\nu} \rangle \langle u^\lambda D_{\mu\nu\lambda}B \rangle]_+$
				32	$[\langle \bar{B}D_{\mu\nu\lambda}Bu^\mu h^{\nu\lambda} \rangle]_+$		
8	$i\bar{\psi}\langle u^\mu h^{\nu\lambda} \rangle \sigma_{\mu\nu} D_\lambda \psi$	13	$i\bar{\Psi}\langle u^\mu h^{\nu\lambda} \rangle \sigma_{\mu\nu} D_\lambda \Psi$	34	$[i\langle \bar{B}u^\mu h^{\nu\lambda} \sigma_{\mu\nu} D_\lambda B \rangle]_+$	36	$[i\langle \bar{B}\sigma_{\mu\nu} D_\lambda Bu^\mu h^{\nu\lambda} \rangle]_+$
		14	$[i\bar{\Psi}u^\mu h^{\nu\lambda} \sigma_{\mu\nu} D_\lambda \Psi]_+$	35	$i\langle \bar{B}u^\mu \sigma_{\mu\nu} D_\lambda B h^{\nu\lambda} \rangle$	37	$i\langle \bar{B}\sigma_{\mu\nu} D_\lambda B \rangle \langle u^\mu h^{\nu\lambda} \rangle$
9	$[i\bar{\psi}f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 \psi]_+$	15	$[i\bar{\Psi}f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 \Psi]_+$	38	$[i\langle \bar{B}f_+^{\mu\nu} u_\mu \gamma_\nu \gamma_5 B \rangle]_+$	40	$[i\langle \bar{B}u_\mu \rangle \langle f_+^{\mu\nu} \gamma_\nu \gamma_5 B \rangle]_+$
				39	$[i\langle \bar{B}\gamma_\nu \gamma_5 B f_+^{\mu\nu} u_\mu \rangle]_+$		
10	$i\epsilon_{\mu\nu\lambda\rho} \bar{\psi} \langle f_+^{\mu\nu} \rangle u^\lambda D^\rho \psi \rightarrow 0$	16	$i\epsilon_{\mu\nu\lambda\rho} \bar{\Psi} \langle f_+^{\mu\nu} u^\lambda \rangle D^\rho \Psi$	41	$[i\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}f_+^{\mu\nu} u^\lambda D^\rho B \rangle]_+$	44	$[i\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}D^\rho B f_+^{\mu\nu} u^\lambda \rangle]_+$
11	$i\epsilon_{\mu\nu\lambda\rho} \bar{\psi} \langle f_+^{\mu\nu} u^\lambda \rangle D^\rho \psi$	17	$[i\epsilon_{\mu\nu\lambda\rho} \bar{\Psi} f_+^{\mu\nu} u^\lambda D_\rho \Psi]_+$	42	$i\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}f_+^{\mu\nu} D^\rho Bu^\lambda \rangle$	45	$i\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}D^\rho B \rangle \langle f_+^{\mu\nu} u^\lambda \rangle$
				43	$i\epsilon_{\mu\nu\lambda\rho} \langle \bar{B}u^\lambda D^\rho B f_+^{\mu\nu} \rangle$		
12	$i\bar{\psi} \nabla_\mu f_+^{\mu\nu} D_\nu \psi$	18	$i\bar{\Psi} \nabla_\mu f_+^{\mu\nu} D_\nu \Psi$	46	$i\langle \bar{B} \nabla_\mu f_+^{\mu\nu} D_\nu B \rangle$	47	$i\langle \bar{B} D_\nu B \nabla_\mu f_+^{\mu\nu} \rangle$
13	$i\bar{\psi} \langle \nabla_\mu f_+^{\mu\nu} \rangle D_\nu \psi \rightarrow 0$						
14	$[i\bar{\psi} f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \psi]_+$	19	$[i\bar{\Psi} f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \Psi]_+$	48	$[i\langle \bar{B}f_+^{\mu\nu} u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle]_+$	50	$[i\langle \bar{B}u^\lambda \rangle \langle f_+^{\mu\nu} \gamma_\mu \gamma_5 D_{\nu\lambda} B \rangle]_+$
				49	$[i\langle \bar{B}\gamma_\mu \gamma_5 D_{\nu\lambda} B f_+^{\mu\nu} u^\lambda \rangle]_+$		
15	$[\bar{\psi}u_\mu f_-^{\mu\nu} D_\nu \psi]_+$	20	$[\bar{\Psi}u_\mu f_-^{\mu\nu} D_\nu \Psi]_+$	51	$[\langle \bar{B}u_\mu f_-^{\mu\nu} D_\nu B \rangle]_+$	53	$[\langle \bar{B}f_-^{\mu\nu} \rangle \langle u_\mu D_\nu B \rangle]_+$
				52	$[\langle \bar{B}D_\nu Bu_\mu f_-^{\mu\nu} \rangle]_+$		

LECs in baryon ChPT and chiral quark model

$\mathcal{O}(p^3)$ 阶 χ PT 和 χ QM 的拉氏量第 12 到第 16 组

N. Fettes, U.G. Meissner, M. Mojzis, Annl. Phys. 283, 273 (2000)

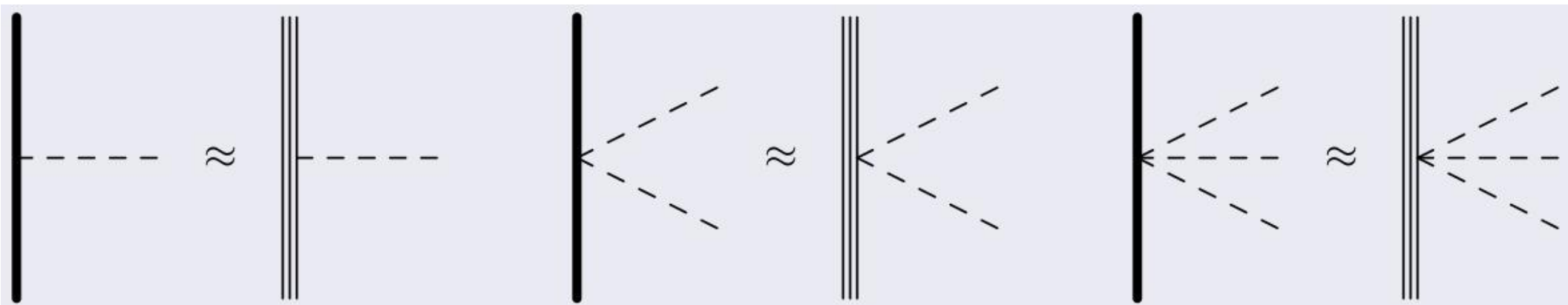
J.A. Oller, M. Verbeni, J. Prades, JHEP 09, 079 (2006)

Jiang et al., Phys. Rev. D, 106, 054023 (2022)

$SU(2)_{\chi PT/\chi QM}$		$SU(3)_{\chi QM}$		$SU(3)_{\chi PT}$			
$\rightarrow \alpha_i/\beta_i$		c_i		d_i			
16	$i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\psi$	21	$i\bar{\Psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\Psi$	54	$[i\langle\bar{B}u^\mu f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda B\rangle]_+$	59	$i\langle\bar{B}u^\mu\sigma_{\nu\lambda}D_\mu Bf_-^{\nu\lambda}\rangle$
17	$i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\psi$	22	$i\bar{\Psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\Psi$	55	$[i\langle\bar{B}u^\mu f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu B\rangle]_+$	60	$[i\langle\bar{B}\sigma_{\mu\nu}D_\lambda Bu^\mu f_-^{\nu\lambda}\rangle]_+$
		23	$[i\bar{\Psi}u^\mu f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda\Psi]_+$	56	$i\langle\bar{B}f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu Bu^\mu\rangle$	61	$[i\langle\bar{B}\sigma_{\nu\lambda}D_\mu Bu^\mu f_-^{\nu\lambda}\rangle]_+$
		24	$[i\bar{\Psi}u^\mu f_-^{\nu\lambda}\sigma_{\nu\lambda}D_\mu\Psi]_+$	57	$i\langle\bar{B}f_-^{\nu\lambda}\sigma_{\mu\nu}D_\lambda Bu^\mu\rangle$	62	$i\langle\bar{B}\sigma_{\mu\nu}D_\lambda B\rangle\langle u^\mu f_-^{\nu\lambda}\rangle$
				58	$i\langle\bar{B}u^\mu\sigma_{\mu\nu}D_\lambda Bf_-^{\nu\lambda}\rangle$	63	$i\langle\bar{B}\sigma_{\nu\lambda}D_\mu B\rangle\langle u^\mu f_-^{\nu\lambda}\rangle$
18	$\bar{\psi}\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\psi$	25	$\bar{\Psi}\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\Psi$	64	$\langle\bar{B}\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5 B\rangle$	65	$\langle\bar{B}\gamma_\mu\gamma_5 B\nabla_\nu f_-^{\mu\nu}\rangle$
19	$\bar{\psi}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5\psi$	26	$\bar{\Psi}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5\Psi$	66	$[\langle\bar{B}u^\mu\tilde{\chi}_+\gamma_\mu\gamma_5 B\rangle]_+$	70	$\langle\bar{B}\gamma_\mu\gamma_5 B\rangle\langle u^\mu\tilde{\chi}_+\rangle$
20	$\bar{\psi}\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\psi$	27	$\bar{\Psi}\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\Psi$	67	$\langle\bar{B}\tilde{\chi}_+\gamma_\mu\gamma_5 Bu^\mu\rangle$	71	$\langle\bar{B}u^\mu\gamma_\mu\gamma_5 B\rangle\langle\chi_+\rangle$
		28	$[\bar{\Psi}u^\mu\tilde{\chi}_+\gamma_\mu\gamma_5\Psi]_+$	68	$\langle\bar{B}u^\mu\gamma_\mu\gamma_5 B\tilde{\chi}_+\rangle$	72	$\langle\bar{B}\gamma_\mu\gamma_5 Bu^\mu\rangle\langle\chi_+\rangle$
				69	$[\langle\bar{B}\gamma_\mu\gamma_5 Bu^\mu\tilde{\chi}_+\rangle]_+$		
21	$i\bar{\psi}\tilde{\chi}_-^\mu\gamma_\mu\gamma_5\psi$	29	$i\bar{\Psi}\tilde{\chi}_-^\mu\gamma_\mu\gamma_5\Psi$	73	$i\langle\bar{B}\tilde{\chi}_-^\mu\gamma_\mu\gamma_5 B\rangle$	75	$i\langle\bar{B}\gamma_\mu\gamma_5 B\rangle\langle\chi_-^\mu\rangle$
22	$i\bar{\psi}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\psi$	30	$i\bar{\Psi}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\Psi$	74	$i\langle\bar{B}\gamma_\mu\gamma_5 B\tilde{\chi}_-^\mu\rangle$		
23	$[i\bar{\psi}u^\mu\tilde{\chi}_-D_\mu\psi]_+$	31	$[i\bar{\Psi}u^\mu\tilde{\chi}_-D_\mu\Psi]_+$	76	$[i\langle\bar{B}u^\mu\tilde{\chi}_-D_\mu B\rangle]_+$	78	$[i\langle\bar{B}\tilde{\chi}_-\rangle\langle u^\mu D_\mu B\rangle]_+$
				77	$[i\langle\bar{B}D_\mu Bu^\mu\tilde{\chi}_-\rangle]_+$		

- Nonrelativistic reduction and adopted approximation

$$\begin{aligned}
 \bar{\psi}\psi &\rightarrow \psi_H^\dagger\psi_H \\
 \bar{\psi}\gamma^\mu\gamma_5\psi \approx 2\bar{\psi}_\nu S^\mu\psi_\nu &\rightarrow \begin{cases} 0, & (\mu = 0) \\ \psi_H^\dagger\sigma^k\psi_H, & (\mu = k) \end{cases} \\
 \bar{\psi}\sigma^{\mu\nu}\psi \approx 2\epsilon^{\mu\nu\alpha\beta}v_\alpha\bar{\psi}_\nu S_\beta\psi_\nu &\rightarrow \begin{cases} \epsilon^{ijk}\psi_H^\dagger\sigma^k\psi_H, & (\mu = i, \nu = j) \\ 0, & (\text{other}) \end{cases} \\
 \text{Pauli-Lubanski 自旋矢量} &: S^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu, \psi_\nu = \frac{1 + \not{v}}{2}\psi
 \end{aligned}$$



two-quark
spectator

• Structure correspondences

D. Drechsel, Nuovo Cimento A 76, 388 (1983)

$SU(2)$ 情形

$$1 \rightarrow \alpha_i = 3\beta_i, \quad \sigma \rightarrow \alpha_i = \beta_i, \quad \tau \rightarrow \alpha_i = \beta_i, \quad \tau \otimes \sigma \rightarrow \alpha_i = \frac{5}{3}\beta_i.$$

$SU(3)$ 情形

$$\begin{aligned} \langle \bar{B}B \rangle \rightarrow \bar{\Psi}\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = 3(\text{若干 } c \text{ 的线性组合}), \\ \langle \bar{B}\lambda B \rangle \rightarrow \bar{\Psi}\lambda\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = (\text{若干 } c \text{ 的线性组合}), \\ \langle \bar{B}\sigma B \rangle \rightarrow \bar{\Psi}\sigma\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = (\text{若干 } c \text{ 的线性组合}), \\ \langle \bar{B}\lambda\sigma B \rangle \rightarrow \bar{\Psi}\lambda\sigma\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = \frac{5}{3}(\text{若干 } c \text{ 的线性组合}), \\ \langle \bar{B}\sigma B\lambda \rangle \rightarrow \bar{\Psi}\lambda\sigma\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = \frac{1}{3}(\text{若干 } c \text{ 的线性组合}), \\ \langle \bar{B}B\lambda \rangle \rightarrow \bar{\Psi}\lambda\Psi &\Rightarrow (\text{若干 } d \text{ 的线性组合}) = -(\text{若干 } c \text{ 的线性组合}). \end{aligned}$$

$$\begin{aligned}
& (d_1 - d_2 + d_{11}) \langle \bar{B} (u^\mu \widetilde{u}_\mu u^\nu + u^\nu \widetilde{u}^\mu u_\mu) \gamma_\nu \gamma^5 B \rangle + (d_8 - d_7 + d_{11}) \langle \bar{B} \gamma_\nu \gamma^5 B (u^\mu \widetilde{u}_\mu u^\nu + u^\nu \widetilde{u}^\mu u_\mu) \rangle \\
& + d_4 \langle \bar{B} (\widetilde{u}^\mu u^\nu + \widetilde{u}^\nu u^\mu) \gamma_\nu \gamma^5 B u_\mu \rangle + d_6 \langle \bar{B} u_\mu \gamma_\nu \gamma^5 B (\widetilde{u}^\mu u^\nu + \widetilde{u}^\nu u^\mu) \rangle \\
& + (d_3 + d_{11}) \langle \bar{B} \widetilde{u}^\mu u_\mu \gamma_\nu \gamma^5 B u^\nu \rangle + (d_5 + d_{11}) \langle \bar{B} u^\nu \gamma_\nu \gamma^5 B \widetilde{u}^\mu u_\mu \rangle \\
& + \left(d_2 + \frac{2}{3} d_6 \right) \langle \bar{B} u_\mu \gamma_\nu \gamma^5 B \rangle \langle u^\mu u^\nu \rangle + \left(d_7 + \frac{2}{3} d_4 \right) \langle \bar{B} \gamma_\nu \gamma^5 B u_\mu \rangle \langle u^\mu u^\nu \rangle \\
& + \left(\frac{1}{2} d_2 + \frac{1}{3} d_5 - \frac{2}{3} d_{11} \right) \langle \bar{B} u^\nu \gamma_\nu \gamma^5 B \rangle \langle u^\mu u_\mu \rangle + \left(\frac{1}{3} d_3 + \frac{1}{2} d_7 + d_9 - \frac{2}{3} d_{11} \right) \langle \bar{B} \gamma_\nu \gamma^5 B u^\nu \rangle \langle u^\mu u_\mu \rangle \\
& + \left(\frac{2}{3} d_1 + \frac{1}{3} d_2 + \frac{1}{3} d_7 + \frac{2}{3} d_8 + d_{10} + \frac{1}{3} d_{11} \right) \langle \bar{B} \gamma_\nu \gamma^5 B \rangle \langle u^\mu u_\mu u^\nu \rangle,
\end{aligned}$$

八重态重子自旋-味道波函数

$$\begin{aligned}
 p_{\uparrow} &= \frac{1}{3\sqrt{2}} [udu \otimes (2 \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + duu \otimes (2 \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - uud \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 n_{\uparrow} &= \frac{1}{3\sqrt{2}} [udd \otimes (\uparrow\downarrow\uparrow - 2 \downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) + dud \otimes (\downarrow\uparrow\uparrow - 2 \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow) + ddu \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 \Sigma_{\uparrow}^{+} &= -\frac{1}{3\sqrt{2}} [usu \otimes (2 \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + suu \otimes (2 \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - uus \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 \Sigma_{\uparrow}^{-} &= \frac{1}{3\sqrt{2}} [dsd \otimes (2 \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + sdd \otimes (2 \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) - dds \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 \Xi_{\uparrow}^{0} &= -\frac{1}{3\sqrt{2}} [uss \otimes (\uparrow\downarrow\uparrow - 2 \downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) + sus \otimes (\downarrow\uparrow\uparrow - 2 \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow) + ssu \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 \Xi_{\uparrow}^{-} &= \frac{1}{3\sqrt{2}} [sds \otimes (\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow - 2 \uparrow\downarrow\uparrow) + dss \otimes (\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2 \downarrow\uparrow\uparrow) + ssd \otimes (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow)], \\
 \Sigma_{\uparrow}^{0} &= \frac{1}{6} [(dsu + usd) \otimes (2 \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + (sdu + sud) \otimes (2 \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) \\
 &\quad + (dus + uds) \otimes (-\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow + 2 \uparrow\uparrow\downarrow)], \\
 \Lambda_{\uparrow} &= -\frac{1}{2\sqrt{3}} [(dsu - usd) \otimes (\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + (sdu - sud) \otimes (\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) + (uds - dus) \otimes (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)].
 \end{aligned}$$

LEC relations in baryon ChPT: ChPT \Leftrightarrow χ QM

	Group	$SU(2)_{\chi PT} \Leftrightarrow SU(2)_{\chi QM}$
$\mathcal{O}(p^1)$	1	$g_A = \frac{5}{3}g_A^q$.
$\mathcal{O}(p^2)$	1	$\alpha_1 = 3\beta_1$;
	2	$\alpha_2 = \frac{5}{3}\beta_2$;
	3	$\alpha_3 = 3\beta_3$;
	4	$\alpha_4 = \frac{5}{3}\beta_4$;
	5	$\alpha_6 = \beta_6, \alpha_7 = 3\beta_7$.
$\mathcal{O}(p^3)$	1	$\alpha_1 = \frac{5}{3}\beta_1, \alpha_2 = \frac{5}{3}\beta_2$;
	2	$\alpha_3 = 3\beta_3$;
	3	$\alpha_4 = \frac{5}{3}\beta_4, \alpha_5 = \frac{5}{3}\beta_5$;
	4	$\alpha_6 = \beta_6$;
	5	$\alpha_7 = \beta_7$;
	6	$\alpha_8 = \beta_8$;
	7	$\alpha_9 = \frac{5}{3}\beta_9$;
	8	$\alpha_{11} = 3\beta_{11}$;
	9	$\alpha_{12} = \beta_{12}$;
	10	$\alpha_{14} = \frac{5}{3}\beta_{14}$;
	11	$\alpha_{15} = \beta_{15}$;
	12	$\alpha_{16} = \beta_{16}, \alpha_{17} = \beta_{17}$;
13	$\alpha_{18} = \frac{5}{3}\beta_{18}$;	
14	$\alpha_{19} = \beta_{19}, \alpha_{20} = \frac{5}{3}\beta_{20}$;	
15	$\alpha_{21} = \frac{5}{3}\beta_{21}, \alpha_{22} = \beta_{22}$;	
16	$\alpha_{23} = \beta_{23}$.	

	Group	$SU(3)_{\chi PT} \Leftrightarrow SU(3)_{\chi QM}$
$\mathcal{O}(p^1)$	1	$D = g_A^q, F = \frac{2}{3}g_A^q$.
$\mathcal{O}(p^2)$	1	$d_1 = -d_3 = c_2, d_2 = 0, d_4 = 3c_1 + c_2$;
	2	$d_5 = 5d_6 = \frac{5}{3}c_3, d_7 = 0$;
	3	$d_8 = -d_{10} = c_5, d_9 = 0, d_{11} = 3c_4 + c_5$;
	4	$d_{12} = 5d_{13} = \frac{5}{3}c_6$;
	5	$d_{14} = -d_{15} = c_7, d_{16} = 3c_8$.
$\mathcal{O}(p^3)$	1	$d_1 = \frac{5}{6}(2c_1 + c_2 + 2c_4), d_2 = 5d_7 = \frac{5}{3}c_2, d_3 = d_5 = -\frac{5}{4}d_9 = -d_{11} = \frac{5}{6}(2c_1 - c_2),$ $d_4 = d_6 = 0, d_8 = \frac{1}{6}(10c_1 - 3c_2 + 2c_4), d_{10} = \frac{1}{6}(-10c_1 - 7c_2 + 6c_3 - 4c_4)$;
	2	$d_{12} = -d_{15} = c_6, d_{13} = d_{14} = 0, d_{16} = 3c_5 + c_6$;
	3	$d_{17} = \frac{5}{6}(c_7 + 2c_8 + 2c_{10}), d_{18} = 5d_{24} = \frac{5}{3}c_7, d_{19} = d_{22} = -\frac{5}{4}d_{26} = -d_{27} = \frac{5}{6}(-c_7 + 2c_8),$ $d_{20} = d_{21} = 0, d_{23} = \frac{1}{6}(-3c_7 + 10c_8 + 2c_{10}), d_{25} = \frac{1}{6}(-7c_7 - 10c_8 + 6c_9 - 4c_{10})$;
	4	$d_{28} = -d_{29} = c_{11}, d_{30} = 0$;
	5	$d_{31} = -d_{32} = c_{12}, d_{33} = 0$;
	6	$d_{34} = 5d_{36} = \frac{5}{3}c_{14}, d_{35} = 0, d_{37} = c_{13} - \frac{2}{3}c_{14}$;
	7	$d_{38} = 5d_{39} = \frac{5}{3}c_{15}, d_{40} = 0$;
	8	$d_{41} = -d_{44} = c_{17}, d_{42} = d_{43} = 0, d_{45} = 3c_{16} + 2c_{17}$;
	9	$d_{46} = -d_{47} = c_{18}$;
	10	$d_{48} = 5d_{49} = \frac{5}{3}c_{19}, d_{50} = 0$;
	11	$d_{51} = -d_{52} = c_{20}, d_{53} = 0$;
	12	$d_{54} = 5d_{60} = \frac{5}{3}c_{23}, d_{55} = 5d_{61} = \frac{5}{3}c_{24}, d_{56} = d_{57} = d_{58} = d_{59} = 0,$ $d_{62} = c_{21} - \frac{2}{3}c_{23}, d_{63} = c_{22} - \frac{2}{3}c_{24}$;
	13	$d_{64} = 5d_{65} = \frac{5}{3}c_{25}$;
	14	$d_{66} = 5d_{69} = \frac{5}{3}c_{28}, d_{67} = d_{68} = 0, d_{70} = c_{26} - \frac{2}{3}c_{28}, d_{71} = 5d_{72} = \frac{5}{3}c_{27}$;
	15	$d_{73} = 5d_{74} = \frac{5}{3}c_{29}, d_{75} = c_{30}$;
	16	$d_{76} = -d_{77} = c_{31}, d_{78} = 0$.

LEC relations in baryon ChPT: χ QM \Leftrightarrow χ QM, ChPT \Leftrightarrow ChPT

nc.: SU(2) gives no constraint

Group	$SU(3)_{\chi\text{QM}} \Leftrightarrow SU(2)_{\chi\text{QM}}$
$\mathcal{O}(p^1)$	1 $g_A^q = g_A^q$
$\mathcal{O}(p^2)$	1 $\beta_1 = c_1 + \frac{1}{2}c_2$;
	2 $\beta_2 = c_3$;
	3 $\beta_3 = c_4 + \frac{1}{2}c_5$;
	4 $\beta_4 = c_6$;
$\mathcal{O}(p^3)$	1 $\beta_1 = c_1 + c_4, \beta_2 = c_2, c_3$ (nc.);
	2 $\beta_3 = c_5 + \frac{1}{2}c_6$;
	3 $\beta_4 = c_7, \beta_5 = c_8 + c_{10}, c_9$ (nc.);
	4 $\beta_6 = c_{11}$;
	5 $\beta_7 = c_{12}$;
	6 $\beta_8 = c_{13} + c_{14}$;
	7 $\beta_9 = c_{15}$;
	8 $\beta_{11} = c_{16} + c_{17}$;
	9 $\beta_{12} = c_{18}$;
	10 $\beta_{14} = c_{19}$;
	11 $\beta_{15} = c_{20}$;
	12 $\beta_{16} = c_{21} + c_{23}, \beta_{17} = c_{22} + c_{24}$;
	13 $\beta_{18} = c_{25}$;
	14 $\beta_{19} = c_{26} + c_{28}, \beta_{20} = c_{27} + \frac{1}{3}c_{28}$;
	15 $\beta_{21} = c_{29}, \beta_{22} = \frac{1}{6}c_{29} + c_{30}$;
	16 $\beta_{23} = c_{31}$.

Group	With χ QM	Without χ QM
$\mathcal{O}(p^1)$	1 $D = \frac{3}{5}g_A, F = \frac{2}{5}g_A$.	$D + F = g_A$.
$\mathcal{O}(p^2)$	1 $\alpha_1 = \frac{1}{2}d_1 + d_4 = -\frac{1}{2}d_3 + d_4, d_2 = 0$;	$\alpha_1 = \frac{1}{2}d_1 + d_4$;
	2 $\alpha_2 = d_5 = 5d_6, d_7 = 0$;	$\alpha_2 = d_5$;
	3 $\alpha_3 = \frac{1}{2}d_8 + d_{11} = -\frac{1}{2}d_{10} + d_{11}, d_9 = 0$;	$\alpha_3 = \frac{1}{2}d_8 + d_{11}$;
	4 $\alpha_4 = d_{12} = 5d_{13}$;	$\alpha_4 = d_{12}$;
	5 $\alpha_6 = d_{14} = -d_{15}, \alpha_7 = \frac{1}{2}d_{14} + d_{16} = -\frac{1}{2}d_{15} + d_{16}$.	$\alpha_6 = d_{14}, \alpha_7 = \frac{1}{2}d_{14} + d_{16} - \frac{1}{3}(d_{14} + d_{15})$.
$\mathcal{O}(p^3)$	1 $\alpha_1 + \frac{1}{2}\alpha_2 = d_1 = 5d_8 - 4d_3, d_3 = d_5 = -\frac{5}{4}d_9 = -d_{11},$ $\alpha_2 = d_2 = 5d_7, d_4 = d_6 = 0, d_{10}$ (nc.);	$\alpha_1 = d_1 - \frac{1}{2}d_2, \alpha_2 = d_2$;
	2 $\alpha_3 = \frac{1}{2}d_{12} + d_{16} = -\frac{1}{2}d_{15} + d_{16}, d_{13} = d_{14} = 0$;	$\alpha_3 = \frac{1}{2}d_{12} - d_{16}$;
	3 $\alpha_4 = d_{18} = 5d_{24}, \alpha_5 + \frac{1}{2}\alpha_4 = d_{17} = 5d_{23} - 4d_{19},$ $d_{19} = d_{22} = -\frac{5}{4}d_{26} = -d_{27}, d_{20} = d_{21} = 0, d_{25}$ (nc.);	$\alpha_4 = d_{18}, \alpha_5 = d_{17} - \frac{1}{2}d_{18}$;
	4 $\alpha_6 = d_{28} = -d_{29}, d_{30} = 0$;	$\alpha_6 = d_{28}$;
	5 $\alpha_7 = d_{31} = -d_{32}, d_{33} = 0$;	$\alpha_7 = d_{31}$;
	6 $\alpha_8 = d_{34} + d_{37} = 5d_{36} + d_{37}, d_{35} = 0$;	$\alpha_8 = d_{34} + d_{37}$;
	7 $\alpha_9 = d_{38} = 5d_{39}, d_{40} = 0$;	$\alpha_9 = d_{38}$;
	8 $\alpha_{11} = d_{41} + d_{45} = -d_{44} + d_{45}, d_{42} = d_{43} = 0$;	$\alpha_{11} = d_{41} + d_{45}$;
	9 $\alpha_{12} = d_{46} = -d_{47}$;	$\alpha_{12} = d_{46}$;
	10 $\alpha_{14} = d_{48} = 5d_{49}, d_{50} = 0$;	$\alpha_{14} = d_{48}$;
	11 $\alpha_{15} = d_{51} = -d_{52}, d_{53} = 0$;	$\alpha_{15} = d_{51}$;
	12 $\alpha_{16} = d_{54} + d_{62} = 5d_{60} + d_{62}, \alpha_{17} = d_{55} + d_{63} = 5d_{61} + d_{63},$ $d_{56} = d_{57} = d_{58} = d_{59} = 0$;	$\alpha_{16} = d_{54} + d_{62}, \alpha_{17} = d_{55} + d_{63}$;
	13 $\alpha_{18} = d_{64} = 5d_{65}$;	$\alpha_{18} = d_{64}$;
	14 $\alpha_{19} = d_{66} + d_{70} = 5d_{69} + d_{70}, d_{67} = d_{68} = 0,$ $\alpha_{20} = \frac{1}{3}d_{66} + d_{71} = \frac{5}{3}d_{69} + d_{71} = \frac{1}{3}d_{66} + 5d_{72}$;	$\alpha_{19} = d_{66} + d_{70}, \alpha_{20} = \frac{1}{3}d_{66} - \frac{1}{3}d_{68} + d_{71}$;
	15 $\alpha_{21} = d_{73} = 5d_{74}, \alpha_{22} = \frac{1}{10}d_{73} + d_{75} = \frac{1}{2}d_{74} + d_{75}$;	$\alpha_{21} = d_{73}, \alpha_{22} = \frac{1}{6}d_{73} - \frac{1}{3}d_{74} + d_{75}$;
	16 $\alpha_{23} = d_{76} = -d_{77}, d_{78} = 0$.	$\alpha_{23} = d_{76}$. 19

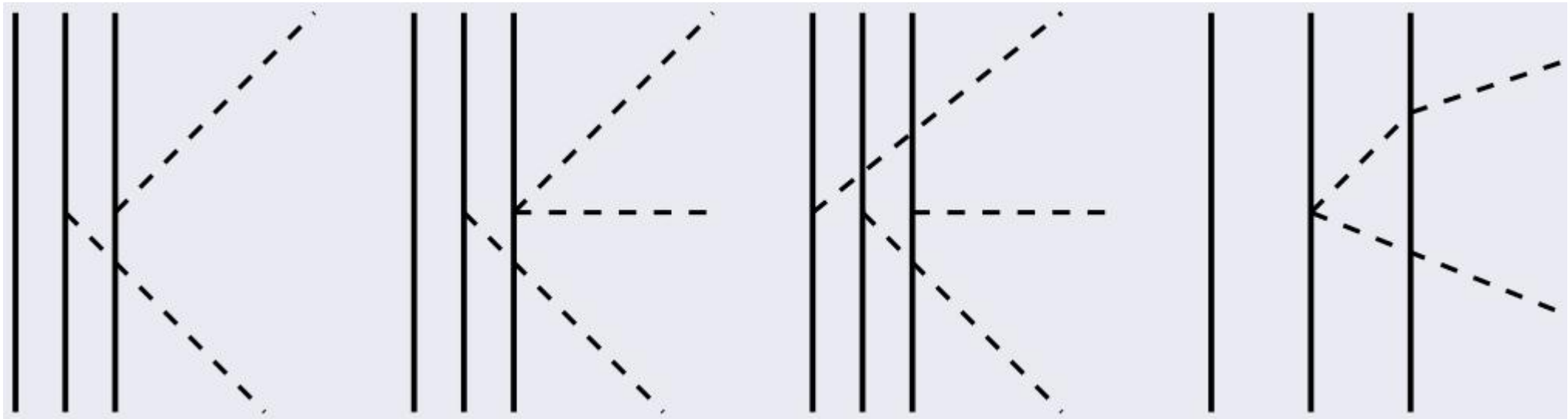
LEC relations in baryon ChPT

$$\left\{ \begin{array}{l} \bar{p}\pi^0 p \\ \bar{n}\pi^0 n \end{array} \right\} \Rightarrow D + F = \frac{5}{3}g_A^q$$

$$\left\{ \begin{array}{l} \bar{\Sigma}^+\pi^0\Sigma^+ \\ \bar{\Sigma}^-\pi^0\Sigma^- \end{array} \right\} \left\{ \begin{array}{l} \bar{\Sigma}^0\pi^+\Sigma^- \\ \bar{\Sigma}^-\pi^-\Sigma^0 \end{array} \right\} \Rightarrow F = \frac{2}{3}g_A^q$$

← s quark as spectator

Numerical analysis: corrections become important for high-order LEC relations



$$\begin{aligned}(i\not{\partial} - m_\Delta)\Psi_\mu &= 0, \\ \gamma^\mu\Psi_\mu &= 0, \\ \partial^\mu\Psi_\mu &= 0,\end{aligned}$$

• Rarita-Schwinger field

$$\Psi_\mu(x) = \sum_{s_\Delta} \int \frac{d^3p M_\Delta}{(2\pi)^3 E} \left[b(\mathbf{p}, s_\Delta) u_\mu(\mathbf{p}, s_\Delta) e^{-ip \cdot x} + d^\dagger(\mathbf{p}, s_\Delta) v_\mu(\mathbf{p}, s_\Delta) e^{ip \cdot x} \right].$$

其中 $u_\mu(\mathbf{p}, s_\Delta)$ 是 Rarita-Schwinger 旋量，自旋-1 矢量和自旋-1/2 的旋量的耦合

$$u_\mu(\mathbf{p}, s_\Delta) = \sum_{\lambda, s} \langle 1\lambda \frac{1}{2}s | \frac{3}{2}s_\Delta \rangle e_\mu(\mathbf{p}, \lambda) u(\mathbf{p}, s)$$

$$e^\mu(\mathbf{p}, \lambda) = \left(\frac{\hat{e}_\lambda \cdot \mathbf{p}}{M_\Delta}, \hat{e}_\lambda + \frac{\mathbf{p}(\hat{e}_\lambda \cdot \mathbf{p})}{M_\Delta(p_0 + M_\Delta)} \right), \quad u(\mathbf{p}, s) = \sqrt{\frac{E + M_\Delta}{2M_\Delta}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \chi_s \end{pmatrix}$$

其中单位矢量 \hat{e}_λ , $\lambda = 0, \pm 1$ 的球面表示 $\hat{e}_+ = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$ $\hat{e}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\hat{e}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$

The case including $\Delta(1232)$

自旋-3/2 的 Δ 场的全对称张量形式

$$T^{111} = \Delta^{++}, \quad T^{222} = \Delta^{-},$$

$$T^{112} = T^{121} = T^{211} = \frac{\Delta^+}{\sqrt{3}},$$

$$T^{122} = T^{212} = T^{221} = \frac{\Delta^0}{\sqrt{3}}.$$

$\mathcal{O}(p^1)$ 阶拉氏量和 LEC 关系

Hemmert et al., J. Phys. G 24, 1831 (1998)

$\mathcal{L}_{\pi NN/\pi qq}$ g_A/g_A^q $\frac{1}{2}\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$\mathcal{L}_{\pi\Delta\Delta}$ $e_i^{(1)}$ $\bar{T}^{abc,\lambda}u^{ad,\mu}\gamma_\mu\gamma_5T_\lambda^{bcd}$	$\mathcal{L}_{\pi N\Delta}$ $f_i^{(1)}$ $\epsilon^{ab}\bar{N}^c u^{ad,\mu}T_\mu^{bcd} + h.c.$
--	--	---

$$\mathcal{L}_{\pi NN}$$

$$g_A = \frac{5}{3}g_A^q$$

w.f. ←

$$\mathcal{L}_{\pi qq}$$

$$g_A^q$$

$$\mathcal{L}_{\pi\Delta\Delta}$$

$$e_1^{(1)} = -\frac{3}{2}g_A^q$$

$$\mathcal{L}_{\pi N\Delta}$$

$$f_1^{(1)} = k_1 g_A^q$$

Introduced multiplication factor

The case including $\Delta(1232)$

非相对论约化

Define transition spin $S_{t\mu}$:

$$T_{\mu}^{abc} \equiv S_{t\mu} T^{abc} = \sqrt{\frac{1}{3} \left(\frac{9}{2} - a - b - c \right)^2 + \frac{1}{4} \Phi_{\mu}^{abc}} = \sqrt{\frac{1}{3} \left(\frac{9}{2} - a - b - c \right)^2 + \frac{1}{4} S_{t\mu} \Phi^{abc}}$$

取静态极限, $S_t^{\mu} = (0, \vec{S}_t)$, Δ 重子自旋波函数的第三分量如下

$$\Phi_{s_z=3/2} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Phi_{s_z=1/2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \Phi_{s_z=-1/2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \Phi_{s_z=-3/2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

使用 RS 旋量表达式 $\Phi_{\mu}(s_z) = \sum_{\lambda, s} \langle 1\lambda \frac{1}{2}s | \frac{3}{2}s_z \rangle e_{\mu}(\lambda) u(s)$

$$S_t^3 = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}.$$

The case including $\Delta(1232)$

$\sigma_{RS}^\mu \equiv -\bar{S}_{t\rho}\gamma^\mu\gamma^5 S_t^\rho \rightarrow \vec{\sigma}_{RS} = (S_t^\dagger)^j \vec{\sigma}(S_t)^j$: spin operator of RS field

$$\sigma_{RS}^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g_A^q)\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$-e_1^{(1)}T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f_1^{(1)}\epsilon^{ab}\bar{N}^c u^{ad,\mu}S_{t,\mu}T^{bcd} + h.c.$

$$\begin{aligned} k_1 S_t^\mu &\rightarrow \gamma^\mu \gamma^5, \\ \sigma_{RS}^\mu &\equiv -\bar{S}_{t\rho}\gamma^\mu\gamma^5 S_t^\rho \rightarrow \gamma^\mu \gamma^5. \end{aligned} \quad \longrightarrow \quad \begin{aligned} f_1^{(1)} &= k_1 g_A^q, \\ e_1^{(1)} &= -\frac{3}{2} g_A^q. \end{aligned}$$

$$k_1 = ?$$

The case including $\Delta(1232)$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g_A^q)\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$-e_1^{(1)}T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f_1^{(1)}\epsilon^{ab}\bar{N}^c u^{ad,\mu}S_{t,\mu}T^{bcd} + h.c.$

Value of k_1 :

$$g_{\pi N\Delta} = \sqrt{2}f_1^{(1)}, \quad f_1^{(1)} = k_1 g_A^q, \quad g_A = \frac{5}{3}g_A^q \Rightarrow g_{\pi N\Delta} = \frac{3\sqrt{2}}{5}k_1 g_A$$

- 当不引入 k_1 时, $g_{\pi N\Delta}^{QM}/g_{\pi NN}^{QM} \approx 1.7$, 与实验数据 $g_{\pi N\Delta}^{\text{expt}}/g_{\pi NN}^{\text{expt}} \approx 2.21$ 不一致
Hemmert et al., J. Phys. G 24, 1831 (1998)

- 当 $k_1 = \frac{5}{4}$ 时, $g_{\pi N\Delta}^{QM}/g_{\pi NN}^{QM} \approx 2.13$, 与实验相符

Bernard et al., Int. J. Mod. Phys. E 4, 193 (1995)

- 考虑到 S_t^3 中的数值, 我们取 $k_1 = \sqrt{\frac{3}{2}} \approx 1.225$

$$S_t^3 = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}$$

$$\begin{aligned} f_1^{(1)} &= k_1 g_A^q, \\ e_1^{(1)} &= -\frac{3}{2}g_A^q. \end{aligned}$$

The case including $\Delta(1232)$

$\mathcal{L}_{\pi NN/\pi qq}$	$\mathcal{L}_{\pi\Delta\Delta}$	$\mathcal{L}_{\pi N\Delta}$
$\frac{1}{2}(g_A/g_A^q)\bar{\psi}u^\mu\gamma_\mu\gamma_5\psi$	$-e_1^{(1)}T^{abc,\dagger}u^{ad,\mu}\sigma_{RS,\mu}T^{bcd}$	$f_1^{(1)}\epsilon^{ab}\bar{N}^c u^{ad,\mu}S_{t,\mu}T^{bcd} + h.c.$

Phenomenological perspective

- J=1/2 case operator correspondence $\vec{\sigma}_{hadron} \leftrightarrow \vec{\sigma}_{quark}$

$|\frac{1}{2}, \frac{1}{2}\rangle_{hadron} = \sigma_{hadron}^z |\frac{1}{2}, \frac{1}{2}\rangle_{hadron} : \sigma_{hadron}^z (\sigma_{quark}^z)$ transits $|\frac{1}{2}, \frac{1}{2}\rangle$ to the same state.

- Present operator correspondence $\sqrt{\frac{3}{2}}\vec{S}_t \rightarrow \vec{\sigma}_{quark}$

$|\frac{1}{2}, \frac{1}{2}\rangle_{hadron} = \sqrt{\frac{3}{2}}S_{t,hadron}^z |\frac{3}{2}, \frac{1}{2}\rangle_{hadron} : \sqrt{\frac{3}{2}}S_t^z \stackrel{=1}{=} \text{transits } |\frac{3}{2}, \frac{1}{2}\rangle \text{ to } |\frac{1}{2}, \frac{1}{2}\rangle.$

- Present operator correspondence $\vec{\sigma}_{RS} \rightarrow \vec{\sigma}_{quark}$

$|\frac{3}{2}, \frac{3}{2}\rangle = \sigma_{RS}^z |\frac{3}{2}, \frac{3}{2}\rangle : \sigma_{RS}^z$ transits $|\frac{3}{2}, \frac{3}{2}\rangle$ to the same state.

The case including $\Delta(1232)$

$\mathcal{O}(p^1)$ 和 $\mathcal{O}(p^2)$ 阶的结构对应关系

$$\begin{aligned}
 k_1 S_t^\mu &\rightarrow \gamma^\mu \gamma^5, \\
 k_2 \sigma_t^{\mu\nu} &\equiv ik_2(\gamma^\mu \gamma^5 S_t^\nu - \gamma^\nu \gamma^5 S_t^\mu) = k_2 \epsilon^{\mu\nu\rho\lambda} \gamma_\rho S_{t\lambda} \rightarrow \sigma^{\mu\nu}; \\
 -\bar{S}_{t\rho} S_t^\rho &= 1 \rightarrow 1, \\
 \sigma_{RS}^\mu &\equiv -\bar{S}_{t\rho} \gamma^\mu \gamma^5 S_t^\rho \rightarrow \gamma^\mu \gamma^5, \\
 \sigma_{RS}^{\mu\nu} &\equiv -i(\bar{S}_t^\mu S_t^\nu - \bar{S}_t^\nu S_t^\mu) = -\bar{S}_{t\rho} \sigma^{\mu\nu} S_t^\rho \rightarrow \sigma^{\mu\nu}.
 \end{aligned}$$

$\mathcal{O}(p^3)$ 阶的结构对应关系

$$\begin{aligned}
 k_3 \sigma^{\mu\nu} S_t^\lambda &\rightarrow i\left(g^{\nu\lambda} \gamma^\mu \gamma^5 - g^{\mu\lambda} \gamma^\nu \gamma^5 - i\epsilon^{\mu\nu\lambda\rho} \gamma_\rho\right) \\
 &\doteq i\left(g^{\nu\lambda} \gamma^\mu \gamma^5 - g^{\mu\lambda} \gamma^\nu \gamma^5 + \frac{1}{m_\Delta} \epsilon^{\mu\nu\lambda\rho} D_\rho\right); \\
 k_4(\gamma^\mu \gamma_5 S_t^\nu + \gamma^\nu \gamma_5 S_t^\mu) &\rightarrow g^{\mu\nu}; \\
 -k_5(\bar{S}_t^\mu S_t^\nu + \bar{S}_t^\nu S_t^\mu) &\rightarrow g^{\mu\nu}, \\
 -k_6(\bar{S}_t^\mu \gamma^\lambda \gamma_5 S_t^\nu + \bar{S}_t^\nu \gamma^\lambda \gamma_5 S_t^\mu) &\rightarrow (g^{\lambda\mu} \gamma^\nu \gamma_5 + g^{\lambda\nu} \gamma^\mu \gamma_5 - g^{\mu\nu} \gamma^\lambda \gamma^5), \\
 -k_7(\bar{S}_t^\mu \gamma^\lambda \gamma_5 S_t^\nu - \bar{S}_t^\nu \gamma^\lambda \gamma_5 S_t^\mu) &\rightarrow i\epsilon^{\mu\nu\lambda\rho} \gamma_\rho \doteq -\frac{1}{m_\Delta} \epsilon^{\mu\nu\lambda\rho} D_\rho.
 \end{aligned}$$

$\mathcal{O}(p^2)$ 阶拉氏量

$\mathcal{L}_{\pi NN/\pi qq}$ $\alpha_i^{(2)}/\beta_i^{(2)}$	$\mathcal{L}_{\pi\Delta\Delta}$ $\hat{e}_i^{(2)}$	$\mathcal{L}_{\pi N\Delta}$ $\hat{f}_i^{(2)}$
1 $\bar{\psi}\langle u^\mu u_\mu\rangle\psi$	1 $\bar{T}^{abc,\lambda}\langle u^\mu u_\mu\rangle T_\lambda^{abc}$ 2 $\bar{T}^{abc,\lambda} u^{ag,\mu} u_\mu^{be} T_\lambda^{cge}$ 3 $\bar{T}_\mu^{abc}\langle u^\mu u^\nu\rangle T_\nu^{abc}$ 4 $\bar{T}_\mu^{abc} u^{ag,\mu} u^{be,\nu} T_\nu^{cge}$	1 $[\epsilon^{bc}\bar{N}^a u^{ag,\mu} u^{be,\nu}(\gamma_\mu\gamma_5 T_\nu^{cge} + \gamma_\nu\gamma_5 T_\mu^{cge})]_+$
2 $i\bar{\psi}u^\mu u^\nu\sigma_{\mu\nu}\psi$	5 $i\bar{T}^{abc,\lambda}(u^\mu u^\nu)^{ae}\sigma_{\mu\nu} T_\lambda^{bce}$	2 $[\epsilon^{ab}\bar{N}^c(u^\mu u^\nu)^{ae}(\gamma_\mu\gamma_5 T_\nu^{bce} - \gamma_\nu\gamma_5 T_\mu^{bce})]_+$
3 $\bar{\psi}\langle u^\mu u^\nu\rangle D_{\mu\nu}\psi$	6 $\bar{T}^{abc,\lambda}\langle u^\mu u^\nu\rangle D_{\mu\nu} T_\lambda^{abc}$ 7 $\bar{T}^{abc,\lambda} u^{ag,\mu} u^{be,\nu} D_{\mu\nu} T_\lambda^{cge}$	
4 $\bar{\psi}\tilde{f}_+^{\mu\nu}\sigma_{\mu\nu}\psi$	8 $\bar{T}^{abc,\lambda}(\tilde{f}_+^{\mu\nu})^{ae}\sigma_{\mu\nu} T_\lambda^{bce}$	3 $[i\epsilon^{ab}\bar{N}^c(\tilde{f}_+^{\mu\nu})^{ae}(\gamma_\mu\gamma_5 T_\nu^{bce} - \gamma_\nu\gamma_5 T_\mu^{bce})]_+$
5 $\bar{\psi}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu}\psi$	9 $\bar{T}^{abc,\lambda}\langle f_+^{\mu\nu}\rangle\sigma_{\mu\nu} T_\lambda^{abc}$	
6 $\bar{\psi}\tilde{\chi}_+\psi$	10 $\bar{T}^{abc,\lambda}\tilde{\chi}_+^{ae} T_\lambda^{bce}$	
7 $\bar{\psi}\langle\chi_+\rangle\psi$	11 $\bar{T}^{abc,\lambda}\langle\chi_+\rangle T_\lambda^{abc}$	

The case including $\Delta(1232)$

$\mathcal{O}(p^3)$ 阶拉氏量第 1 到第 5 组

$\mathcal{L}_{\pi NN/\pi qq}$ $\alpha_i^{(3)}/\beta_i^{(3)}$	$\mathcal{L}_{\pi\Delta\Delta}$ $\hat{e}_i^{(3)}$	$\mathcal{L}_{\pi N\Delta}$ $\hat{f}_i^{(3)}$
1 $\bar{\psi}\langle u^\mu u_\mu\rangle u^\nu \gamma_\nu \gamma_5 \psi$	1 $\bar{T}^{abc,\lambda}\langle u^\mu u_\mu\rangle u^{ae,\nu} \gamma_\nu \gamma_5 T_\lambda^{bce}$	1 $[\epsilon^{bc} \bar{N}^d \langle u^\mu u_\mu\rangle u^{be,\nu} T_\nu^{cde}]_+$
2 $\bar{\psi}\langle u^\mu u^\nu\rangle u_\mu \gamma_\nu \gamma_5 \psi$	2 $\bar{T}^{abc,\lambda}\langle u^\mu u^\nu\rangle u_\mu^{ae} \gamma_\nu \gamma_5 T_\lambda^{bce}$	2 $[\epsilon^{bc} \bar{N}^d \langle u^\mu u^\nu\rangle u_\mu^{be} T_\nu^{cde}]_+$
3 $\epsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle u^\mu u^\nu u^\lambda\rangle D^\rho \psi$	3 $\epsilon_{\mu\nu\lambda\rho} \bar{T}^{abc,\eta}\langle u^\mu u^\nu u^\lambda\rangle D^\rho T_\eta^{abc}$	3 $[\epsilon^{bc} \bar{N}^d (u^\mu u^\nu)^{be} u_\mu^{df} T_\nu^{cef}]_+$
	4 $\bar{T}^{abc,\lambda} u^{ad,\mu} u_\mu^{be} u^{cf,\nu} \gamma_\nu \gamma_5 T_\lambda^{def}$	4 $[i\epsilon^{bc} \bar{N}^d (u^\mu u^\nu)^{be} u^{df,\lambda} \sigma_{\mu\nu} T_\lambda^{cef}]_+$
	5 $\bar{T}^{abc}_\rho u^{ad,\rho} u^{be,\nu} u^{cf,\lambda} \gamma_\nu \gamma_5 T_\lambda^{def}$	5 $[i\epsilon^{bc} \bar{N}^d \langle u^\mu u^\lambda\rangle u^{be,\nu} \sigma_{\mu\nu} T_\lambda^{cde}]_+$
	6 $\bar{T}^{abc}_\rho \langle u^\rho u^\lambda\rangle u^{ae,\nu} \gamma_\nu \gamma_5 T_\lambda^{bce}$	
	7 $[\bar{T}^{abc}_\rho \langle u^\rho u^\nu\rangle u^{ae,\lambda} \gamma_\nu \gamma_5 T_\lambda^{bce}]_+$	
	8 $\bar{T}^{abc}_\rho [u^\rho, u^\lambda]^{ad} u^{be,\nu} \gamma_\nu \gamma_5 T_\lambda^{cde}$	
4 $\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\mu \gamma_5 D_{\nu\lambda} \psi$	9 $\bar{T}^{abc,\rho}\langle u^\mu u^\nu\rangle u^{ae,\lambda} \gamma_\mu \gamma_5 D_{\nu\lambda} T_\rho^{bce}$	6 $[\epsilon^{bc} \bar{N}^d \langle u^\mu u^\nu\rangle u^{be,\lambda} D_{\nu\lambda} T_\mu^{cde}]_+$
5 $\bar{\psi}\langle u^\mu u^\nu\rangle u^\lambda \gamma_\lambda \gamma_5 D_{\mu\nu} \psi$	10 $\bar{T}^{abc,\rho}\langle u^\mu u^\nu\rangle u^{ae,\lambda} \gamma_\lambda \gamma_5 D_{\mu\nu} T_\rho^{bce}$	7 $[\epsilon^{bc} \bar{N}^d \langle u^\mu u^\nu\rangle u^{be,\lambda} D_{\mu\nu} T_\lambda^{cde}]_+$
	11 $\bar{T}^{abc,\rho} u^{ad,\mu} u^{be,\nu} u^{cf,\lambda} \gamma_\mu \gamma_5 D_{\nu\lambda} T_\rho^{def}$	8 $[\epsilon^{bc} \bar{N}^d (u^\mu u^\nu)^{be} u^{df,\lambda} D_{\mu\lambda} T_\nu^{cef}]_+$
6 $[\bar{\psi} u_\mu h^{\mu\nu} D_\nu \psi]_+$	12 $[\bar{T}^{abc,\rho} (u_\mu h^{\mu\nu})^{ae} D_\nu T_\rho^{bce}]_+$	
7 $[\bar{\psi} u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} \psi]_+$	13 $[\bar{T}^{abc,\rho} (u^\mu h^{\nu\lambda})^{ae} D_{\mu\nu\lambda} T_\rho^{bce}]_+$	
8 $i\bar{\psi}\langle u^\mu h^{\nu\lambda}\rangle \sigma_{\mu\nu} D_\lambda \psi$	14 $i\bar{T}^{abc,\rho}\langle u^\mu h^{\nu\lambda}\rangle \sigma_{\mu\nu} D_\lambda T_\rho^{abc}$	9 $[\epsilon^{bc} \bar{N}^d (u^\mu h^{\nu\lambda})^{be} D_\lambda (\gamma_\mu \gamma_5 T_\nu^{cde} + \gamma_\nu \gamma_5 T_\mu^{cde})]_+$
		10 $[\epsilon^{bc} \bar{N}^d u^{be,\mu} h^{df,\nu\lambda} \gamma_\mu \gamma_5 D_\lambda T_\nu^{cef}]_+$
9 $i\bar{\psi}[\tilde{f}_+^{\mu\nu}, u_\mu] \gamma_\nu \gamma_5 \psi$	15 $i\bar{T}^{abc,\rho}[\tilde{f}_+^{\mu\nu}, u_\mu]^{ae} \gamma_\nu \gamma_5 T_\rho^{bce}$	11 $[i\epsilon^{bc} \bar{N}^d [\tilde{f}_+^{\mu\nu}, u_\mu]^{be} T_\nu^{cde}]_+$
10 $i\epsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle \tilde{f}_+^{\mu\nu}\rangle u^\lambda D^\rho \psi$	16 $i\epsilon_{\mu\nu\lambda\rho} \bar{T}^{abc,\eta}\langle \tilde{f}_+^{\mu\nu}\rangle u^{ae,\lambda} D^\rho T_\eta^{bce}$	
11 $i\epsilon_{\mu\nu\lambda\rho} \bar{\psi}\langle \tilde{f}_+^{\mu\nu} u^\lambda\rangle D^\rho \psi$	17 $i\epsilon_{\mu\nu\lambda\rho} \bar{T}^{abc,\eta}\langle \tilde{f}_+^{\mu\nu} u^\lambda\rangle D^\rho T_\eta^{abc}$	
	18 $i\bar{T}^{abc}_\mu \langle \tilde{f}_+^{\mu\nu} u^\lambda\rangle \gamma_\lambda \gamma_5 T_\nu^{abc}$	12 $[i\epsilon^{bc} \bar{N}^d \tilde{f}_+^{be,\mu\nu} u_\mu^{df} T_\nu^{cef}]_+$
	19 $[i\bar{T}^{abc}_\lambda [\tilde{f}_+^{\mu\nu}, u^\lambda]^{ae} \gamma_\mu \gamma_5 T_\nu^{bce}]_+$	13 $f_{27}^{(3)} : [i\epsilon^{bc} \bar{N}^d \langle \tilde{f}_+^{\mu\nu}\rangle u_\mu^{be} T_\nu^{cde}]_+$
	20 $i\epsilon_{\mu\nu\lambda\rho} \bar{T}^{abc,\eta} \tilde{f}_+^{ad,\mu\nu} u^{be,\lambda} D^\rho T_\eta^{cde}$	14 $[\epsilon^{bc} \bar{N}^d \langle \tilde{f}_+^{\mu\nu}\rangle u^{be,\lambda} \sigma_{\mu\nu} T_\lambda^{cde}]_+$
	21 $i\bar{T}^{abc}_\mu \tilde{f}_+^{ad,\mu\nu} u^{be,\lambda} \gamma_\lambda \gamma_5 T_\nu^{cde}$	15 $[\epsilon^{bc} \bar{N}^d \tilde{f}_+^{be,\mu\nu} u^{df,\lambda} \sigma_{\mu\nu} T_\lambda^{cef}]_+$
	22 $i\bar{T}^{abc}_\mu \langle \tilde{f}_+^{\mu\nu}\rangle u^{ae,\lambda} \gamma_\lambda \gamma_5 T_\nu^{bce}$	16 $[\epsilon^{bc} \bar{N}^d [\tilde{f}_+^{\mu\nu}, u^\lambda]^{be} \sigma_{\mu\nu} T_\lambda^{cde}]_+$
12 $i\bar{\psi} \nabla_\mu \tilde{f}_+^{\mu\nu} D_\nu \psi$	23 $i\bar{T}^{abc,\rho} (\nabla_\mu \tilde{f}_+^{\mu\nu})^{ae} D_\nu T_\rho^{bce}$	
13 $i\bar{\psi} \langle \nabla_\mu \tilde{f}_+^{\mu\nu}\rangle D_\nu \psi$	24 $i\bar{T}^{abc,\rho} \langle \nabla_\mu \tilde{f}_+^{\mu\nu}\rangle D_\nu T_\rho^{abc}$	

$\mathcal{O}(p^3)$ 阶拉氏量第 6 到第 11 组

$\mathcal{L}_{\pi NN/\pi qq}$ $\alpha_i^{(3)}/\beta_i^{(3)}$	$\mathcal{L}_{\pi\Delta\Delta}$ $\hat{e}_i^{(3)}$	$\mathcal{L}_{\pi N\Delta}$ $\hat{f}_i^{(3)}$
14 $[i\bar{\psi}\tilde{f}_+^{\mu\nu}u^\lambda\gamma_\mu\gamma_5D_{\nu\lambda}\psi]_+$	25 $[i\bar{T}^{abc,\rho}(\tilde{f}_+^{\mu\nu}u^\lambda)^{ae}\gamma_\mu\gamma_5D_{\nu\lambda}T_\rho^{bce}]_+$	17 $[i\epsilon^{bc}\bar{N}^d[\tilde{f}_+^{\mu\nu},u^\lambda]^{be}D_{\nu\lambda}T_\mu^{cde}]_+$ 18 $[i\epsilon^{bc}\bar{N}^d\tilde{f}_+^{be,\mu\nu}u^{df,\lambda}D_{\nu\lambda}T_\mu^{cef}]_+$ 19 $[i\epsilon^{bc}\bar{N}^d\langle f_+^{\mu\nu}\rangle u^{be,\lambda}D_{\nu\lambda}T_\mu^{cde}]_+$
15 $[\bar{\psi}u_\mu f_-^{\mu\nu}D_\nu\psi]_+$	26 $[\bar{T}^{abc,\rho}(u_\mu f_-^{\mu\nu})^{ae}D_\nu T_\rho^{bce}]_+$	20 $[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cde} - \gamma_\nu\gamma_5 T_\mu^{cde})]_+$ 21 $[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\mu(\gamma_\nu\gamma_5 T_\lambda^{cde} - \gamma_\lambda\gamma_5 T_\nu^{cde})]_+$ 22 $[\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cde} + \gamma_\nu\gamma_5 T_\mu^{cde})]_+$ 23 $[\epsilon^{bc}\bar{N}^d u^{be,\mu} f_-^{df,\nu\lambda} D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cef} - \gamma_\nu\gamma_5 T_\mu^{cef})]_+$ 24 $[\epsilon^{bc}\bar{N}^d u^{be,\mu} f_-^{df,\nu\lambda} D_\mu(\gamma_\nu\gamma_5 T_\lambda^{cef} - \gamma_\lambda\gamma_5 T_\nu^{cef})]_+$ 25 $[\epsilon^{bc}\bar{N}^d u^{be,\mu} f_-^{df,\nu\lambda} D_\lambda(\gamma_\mu\gamma_5 T_\nu^{cef} + \gamma_\nu\gamma_5 T_\mu^{cef})]_+$ 26 $[\epsilon_{\mu\nu\lambda\rho}\epsilon^{bc}\bar{N}^d(u^\mu f_-^{\nu\lambda})^{be}T^{cde,\rho}]_+$ 27 $[\epsilon_{\mu\nu\lambda\rho}\epsilon^{bc}\bar{N}^d u^{be,\mu} f_-^{df,\nu\lambda} T^{cef,\rho}]_+$
16 $i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda\psi$	27 $i\bar{T}^{abc,\rho}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\mu\nu}D_\lambda T_\rho^{abc}$	
17 $i\bar{\psi}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu\psi$	28 $i\bar{T}^{abc,\rho}\langle u^\mu f_-^{\nu\lambda}\rangle\sigma_{\nu\lambda}D_\mu T_\rho^{abc}$	
	29 $[\bar{T}_\mu^{abc}[u^\mu, f_-^{\nu\lambda}]^{ae}D_\lambda T_\nu^{bce}]_+$	
	30 $i\bar{T}^{abc,\rho}u^{ad,\mu}f_-^{be,\nu\lambda}\sigma_{\mu\nu}D_\lambda T_\rho^{cde}$	
	31 $i\bar{T}^{abc,\rho}u^{ad,\mu}f_-^{be,\nu\lambda}\sigma_{\nu\lambda}D_\mu T_\rho^{cde}$	
18 $\bar{\psi}\nabla_\mu f_-^{\mu\nu}\gamma_\nu\gamma_5\psi$	32 $\bar{T}^{abc,\rho}(\nabla_\mu f_-^{\mu\nu})^{ae}\gamma_\nu\gamma_5 T_\rho^{bce}$	
19 $\bar{\psi}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5\psi$	33 $\bar{T}^{abc,\rho}\langle u^\mu\tilde{\chi}_+\rangle\gamma_\mu\gamma_5 T_\rho^{abc}$	30 $[\epsilon^{bc}\bar{N}^d\langle\chi_+\rangle u^{be,\mu}T_\mu^{cde}]_+$ 31 $[\epsilon^{bc}\bar{N}^d(u^\mu\tilde{\chi}_+)^{be}T_\mu^{cde}]_+$ 32 $[\epsilon^{bc}\bar{N}^d u^{be,\mu}\tilde{\chi}_+^{df}T_\mu^{cef}]_+$
20 $\bar{\psi}\langle\chi_+\rangle u^\mu\gamma_\mu\gamma_5\psi$	34 $\bar{T}^{abc,\rho}\langle\chi_+\rangle u^{ae,\mu}\gamma_\mu\gamma_5 T_\rho^{abc}$	
	35 $\bar{T}^{abc,\rho}u^{ad,\mu}\tilde{\chi}_+^{be}\gamma_\mu\gamma_5 T_\rho^{cde}$	
21 $i\bar{\psi}\tilde{\chi}_-^\mu\gamma_\mu\gamma_5\psi$	36 $i\bar{T}^{abc\rho}(\tilde{\chi}_-^\mu)^{ae}\gamma_\mu\gamma_5 T_\rho^{bce}$	33 $[i\epsilon^{bc}\bar{N}^d(\tilde{\chi}_-^\mu)^{be}T_\mu^{cde}]_+$
22 $i\bar{\psi}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5\psi$	37 $i\bar{T}^{abc,\rho}\langle\chi_-^\mu\rangle\gamma_\mu\gamma_5 T_\rho^{abc}$	
23 $[i\bar{\psi}u^\mu\tilde{\chi}_-D_\mu\psi]_+$	38 $[i\bar{T}^{abc,\rho}(u^\mu\tilde{\chi}_-)^{ae}D_\mu T_\rho^{bce}]_+$	

强子与夸克层次 LEC 关系

Chiral order	Group	$\mathcal{L}_{\pi\Delta\Delta} \Leftrightarrow \mathcal{L}_{\pi qq}$	$\mathcal{L}_{\pi N\Delta} \Leftrightarrow \mathcal{L}_{\pi qq}$
$\mathcal{O}(p^1)$	1	$e_1^{(1)} = -\frac{3}{2}g_A^q$	$f_1^{(1)} = k_1 g_A^q$
$\mathcal{O}(p^2)$	1	$\hat{e}_1^{(2)} = -3\beta_1^{(2)}, \hat{e}_{2,4}^{(2)} = 0, \hat{e}_3^{(2)} = -6k_5\beta_1^{(2)};$	$\hat{f}_1^{(2)} = 0.$
	2	$\hat{e}_5^{(2)} = -3\beta_2^{(2)};$	$\hat{f}_2^{(2)} = 2k_2\beta_2^{(2)}$
	3	$\hat{e}_6^{(2)} = -3\beta_3^{(2)}, \hat{e}_7^{(2)} = 0;$	$\hat{f}_3^{(2)} = 2k_2\beta_4^{(2)}.$
	4	$\hat{e}_8^{(2)} = -3\beta_4^{(2)}, \hat{e}_9^{(2)} = -3\beta_5^{(2)};$	
	5	$\hat{e}_{10}^{(2)} = -3\beta_6^{(2)}, \hat{e}_{11}^{(2)} = -3\beta_7^{(2)}.$	
$\mathcal{O}(p^3)$	1	$\hat{e}_1^{(3)} = -3\beta_1^{(3)}, \hat{e}_2^{(3)} = -3\beta_2^{(3)}, \hat{e}_3^{(3)} = -3\beta_3^{(3)}, \hat{e}_{4,5,8}^{(3)} = 0,$ $\hat{e}_6^{(3)} = -3k_6\beta_2^{(3)}, \hat{e}_7^{(3)} = -3k_6(\beta_1^{(3)} + \frac{1}{2}\beta_2^{(3)});$	$\hat{f}_1^{(3)} = 2k_1\beta_1^{(3)}, \hat{f}_2^{(3)} = 2k_1\beta_2^{(3)},$ $\hat{f}_{3,4}^{(3)} = 0, \hat{f}_5^{(3)} = k_3(\beta_1^{(3)} - \beta_2^{(3)});$
	2	$\hat{e}_9^{(3)} = -3\beta_4^{(3)}, \hat{e}_{10}^{(3)} = -3\beta_5^{(3)}, \hat{e}_{11}^{(3)} = 0;$	$\hat{f}_6^{(3)} = 2k_1\beta_4^{(3)}, \hat{f}_7^{(3)} = 2k_1\beta_5^{(3)}, \hat{f}_8^{(3)} = 0;$
	3	$\hat{e}_{12}^{(3)} = -3\beta_6^{(3)};$	$\hat{f}_9^{(3)} = 4k_4\beta_6^{(3)}, \hat{f}_{10}^{(3)} = 0;$
	4	$\hat{e}_{13}^{(3)} = -3\beta_7^{(3)};$	
	5	$\hat{e}_{14}^{(3)} = -3\beta_8^{(3)};$	
	6	$\hat{e}_{15}^{(3)} = -3\beta_9^{(3)}, \hat{e}_{16}^{(3)} = -3\beta_{10}^{(3)}, \hat{e}_{17}^{(3)} = -3\beta_{11}^{(3)},$ $\hat{e}_{18}^{(3)} = 6k_7 m_\Delta \beta_{11}^{(3)}, \hat{e}_{19}^{(3)} = -\frac{3}{2}k_6\beta_9^{(3)},$ $\hat{e}_{20,21}^{(3)} = 0, \hat{e}_{22}^{(3)} = 6k_7 m_\Delta \beta_{10}^{(3)};$	$\hat{f}_{11}^{(3)} = 2k_1\beta_9^{(3)}, \hat{f}_{12,13,15}^{(3)} = 0,$ $\hat{f}_{14}^{(3)} = 2k_3 m_\Delta \beta_{10}^{(3)}, \hat{f}_{16}^{(3)} = -k_3\beta_9^{(3)};$
	7	$\hat{e}_{23}^{(3)} = -3\beta_{12}^{(3)}, \hat{e}_{24}^{(3)} = -3\beta_{13}^{(3)};$	$\hat{f}_{17}^{(3)} = 2k_1\beta_{14}^{(3)}, \hat{f}_{18,19}^{(3)} = 0;$
	8	$\hat{e}_{25}^{(3)} = -3\beta_{14}^{(3)};$	
	9	$\hat{e}_{26}^{(3)} = -3\beta_{15}^{(3)}, \hat{e}_{27}^{(3)} = -3\beta_{16}^{(3)},$ $\hat{e}_{28}^{(3)} = -3\beta_{17}^{(3)}, \hat{e}_{29}^{(3)} = -3k_5\beta_{15}^{(3)}, \hat{e}_{30,31}^{(3)} = 0;$	$\hat{f}_{20,21,23-27}^{(3)} = 0, \hat{f}_{22}^{(3)} = 4k_4\beta_{15}^{(3)};$
	10	$\hat{e}_{32}^{(3)} = -3\beta_{18}^{(3)};$	$\hat{f}_{28}^{(3)} = 2k_1\beta_{18}^{(3)}, \hat{f}_{29}^{(3)} = 2k_3\beta_{18}^{(3)};$
	11	$\hat{e}_{33}^{(3)} = -3\beta_{19}^{(3)}, \hat{e}_{34}^{(3)} = -3\beta_{20}^{(3)}, \hat{e}_{35}^{(3)} = 0;$	
	12	$\hat{e}_{36}^{(3)} = -3\beta_{21}^{(3)}, \hat{e}_{37}^{(3)} = -3\beta_{22}^{(3)};$	$\hat{f}_{30}^{(3)} = 2k_1\beta_{20}^{(3)}, \hat{f}_{31,32}^{(3)} = 0;$
	13	$\hat{e}_{38}^{(3)} = -3\beta_{23}^{(3)}.$	$\hat{f}_{33}^{(3)} = 2k_1\beta_{21}^{(3)}$

- ◆ With structure correspondences, obtain LEC relations up to $\mathcal{O}(p^3)$ in baryon ChPT using χ QM.
- ◆ Find several structure correspondences for the case including $\Delta(1232)$ and multiplication factors (k_i) are introduced.
- ◆ In progress: heavy quark hadron case and further studies of k_i .

$$k_1 = \sqrt{\frac{3}{2}}?$$

Thanks for your attention!