多夸克态和耦合道模型

吴佳俊 (中国科学院大学)

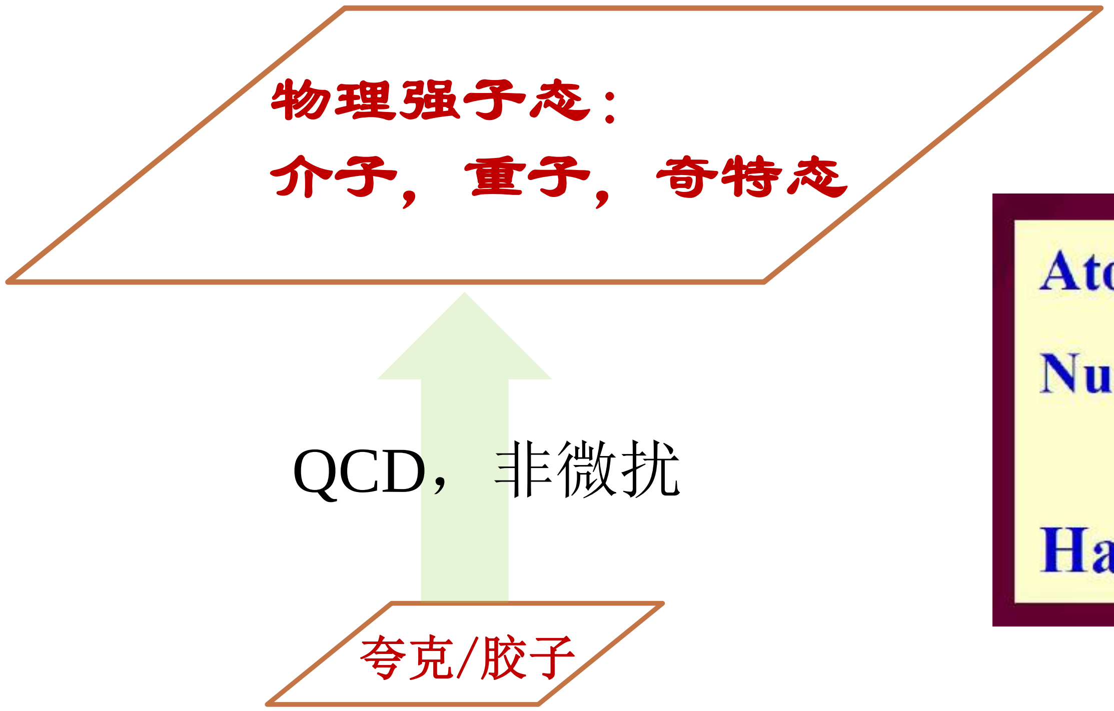
中高能核物理和强子物理研讨会　2025.1.19　北京 中国科学院高能物理所

祝张宗烨先生九十华诞

目录

- 背景介绍
- Tcc， X(3872)和Zc(3900)
- Pc(4457)的可能宇称
- 小结和展望

背景介绍



强子谱？强子结构---强子内部的成分？

Atomic spectroscopy → Atomic Quantum Theory

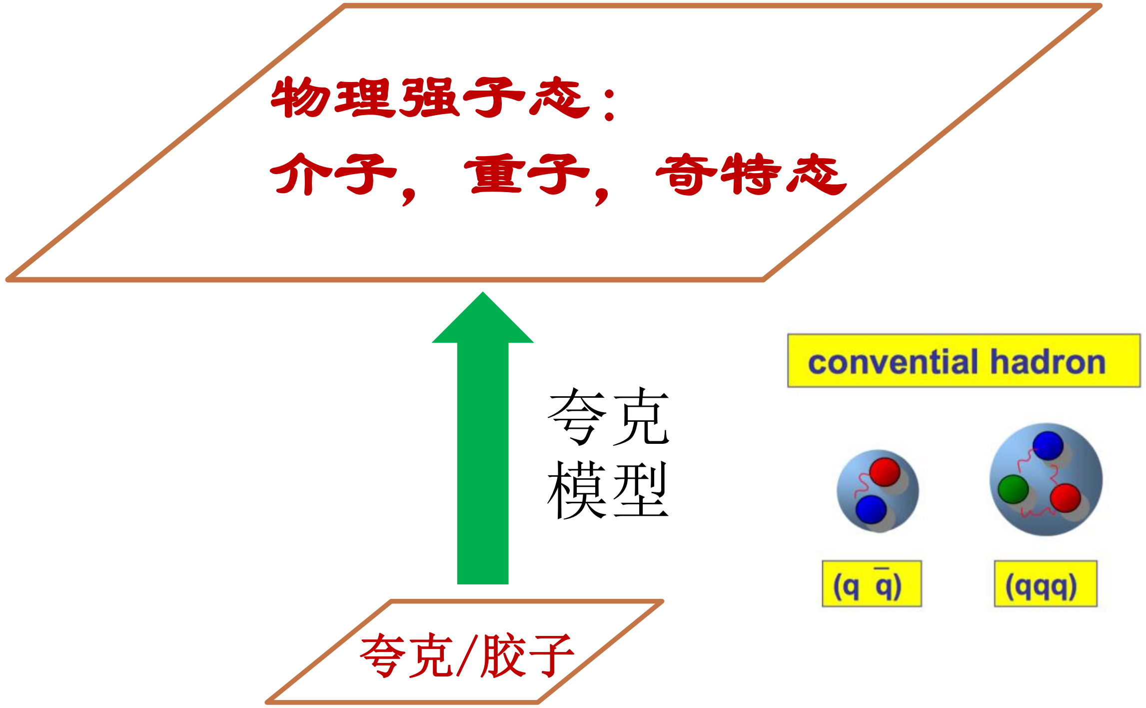
Nuclear spectroscopy → Shell Model &
Collective motion Model

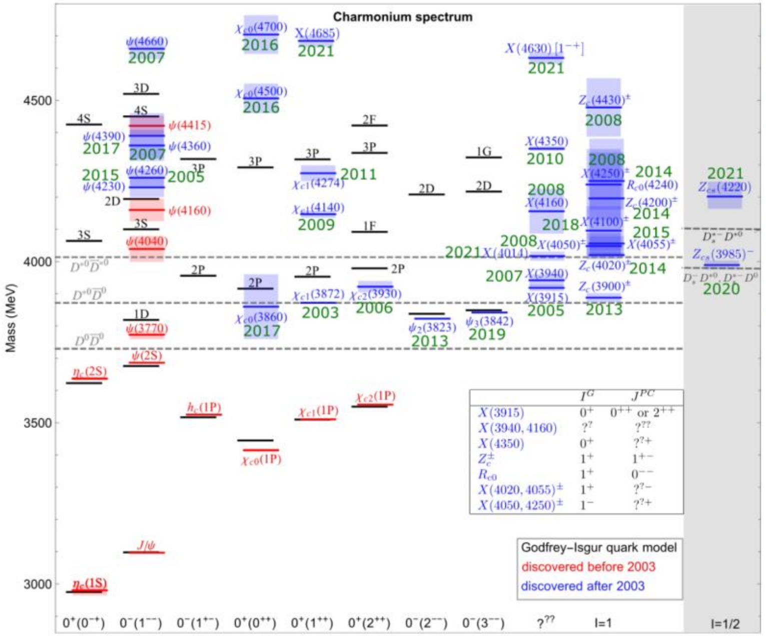
Hadron spectroscopy → ?

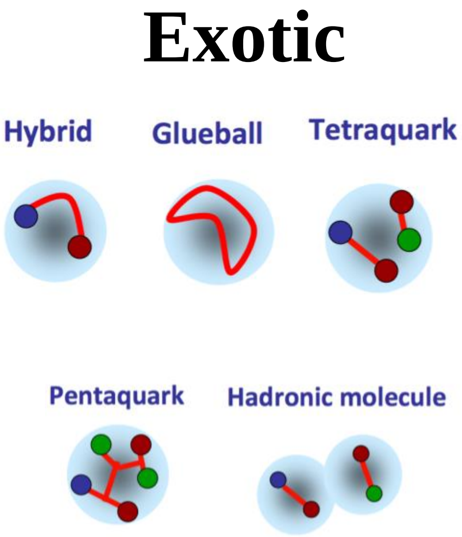
来自邹老师的PPT截图

背景介绍

问题：这些奇特态是由这些成分怎么组成的？







UNQUENCHED quark model

非淬火夸克模型

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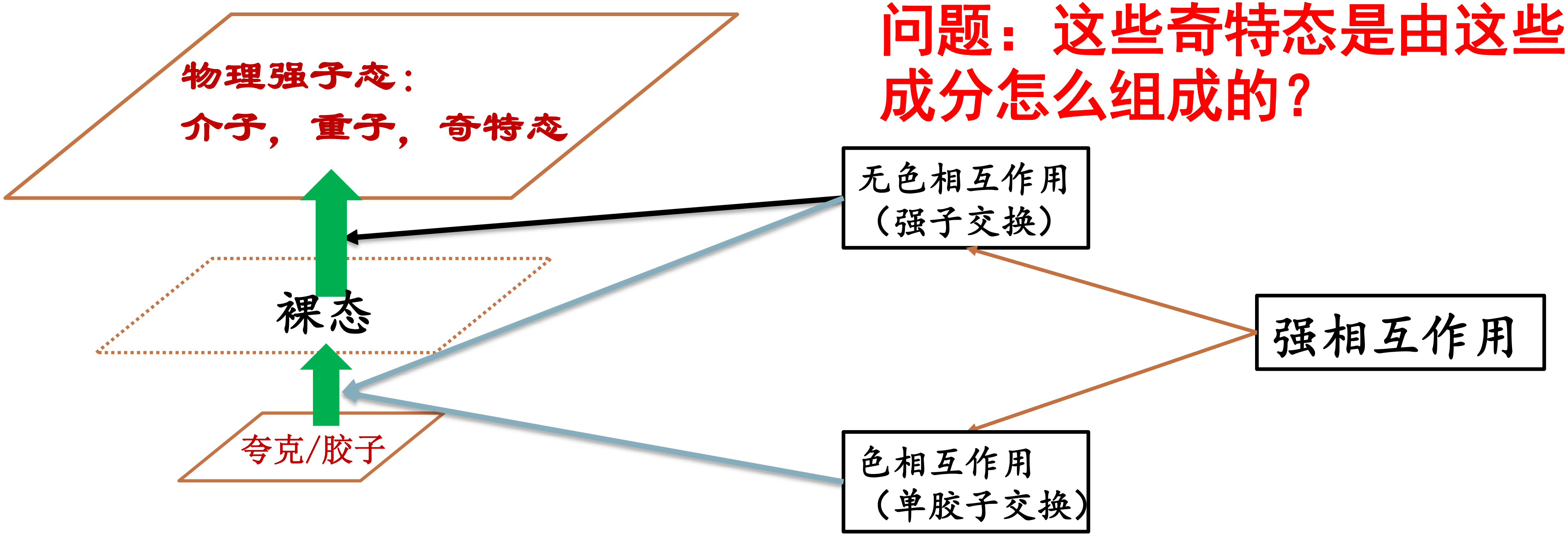

建立完整的、系统的模型描述强子

包括夸克层次和强子层次

解释一系列的强子态

耦合道模型是必须的

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- 背景介绍
- Tcc， X(3872)和Zc(3900) — Wang, Yang, Wu, Zhu, Oka Scib.2024.07.012; Yu, Wang, Yang, Wu arXiv 2409.10865
- Pc(4457)的可能宇称 — Wu, Pang, Wu CPL41 (2024) 9, 091201
- 小结和展望

$T_{cc}\ -\ X(3872)\ -\ Z_c(3900)$

	wave function	$I(J^{PC})$	$u-\text{channel}:\ \pi$	$u-\text{channel}:\ \rho/\omega$	$t-\text{channel}:\ \rho/\omega$
DD^*	$\frac{1}{\sqrt{2}}(D^+D^{*0}-D^0D^{*+})$	$0(1^+)\ [T_{cc}^+]$	$\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t+\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}(D^+D^{*0}+D^0D^{*+})$	$1(1^+)$	$\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t+\frac{1}{2}V_\omega^t$
$D\bar{D}^*$	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}]+[D^0\bar{D}^{*0}]\right)$	$0(1^{++})[X(3872)]$	$\frac{3}{2}V_\pi$	$-\frac{3}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}]-[D^0\bar{D}^{*0}]\right)$	$1(1^{++})$	$-\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\}+\{D^0\bar{D}^{*0}\}\right)$	$0(1^{+-})[h_c]$	$-\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\}-\{D^0\bar{D}^{*0}\}\right)$	$1(1^{+-})\ [Z_c(3900)]$	$\frac{1}{2}V_\pi$	$-\frac{1}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t-\frac{1}{2}V_\omega^t$

$$T_{cc}\ -\ X(3872)\ -\ Z_c(3900)$$

$T_{cc} \rightarrow DD^*$

$X(3872) \rightarrow D\bar{D}^* + c\bar{c}$

$Z_c(3900) \rightarrow D\bar{D}^* + J/\psi\pi + \eta_c\rho$

$T_{cc} - X(3872) - Z_c(3900)$

T_{cc} → $DD^*, D\bar{D}^*$ interaction → ($J/\psi\pi$, $\eta_c\rho$) → $Z_c(3900)$

Yu, Wang, Wu, Yang
arXiv 2409.10865

$DD^*, D\bar{D}^*$ interaction → ($c\bar{c}$ state) → X(3872)

Wang, Yang, Wu, Zhu, Oka
Scib.2024.07.012

Coupled channel
$c\bar{c}$ state and $D\bar{D}^*$

Question：Two interactions ?
Too many solutions of a+b=5

Study X(3872) from T_{cc}

Heavy Quark Symmetry

$D^{(*)}D^{(*)}$

$$H_a^{(Q)} = \frac{1+ \not{v}}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]$$

$$\bar{H}_a^{(Q)} \equiv \gamma_0 H^{(Q)\dagger}\gamma_0 = \left[P_a^{*\dagger\mu}\gamma_\mu + P_a^\dagger\gamma_5\right]\frac{1+ \not{v}}{2}$$

$$P = \left(D^0, D^+, D_s^+\right) \& P^* = \left(D^{*0}, D^{*+}, D_s^{*+}\right)$$

$$\mathcal{L}_{MH^{(Q)}H^{(Q)}} = ig\,\mathrm{Tr}\left[H_b^{(Q)}\gamma_\mu\gamma_5 A_{ba}^\mu \bar{H}_a^{(Q)}\right]$$

$$\mathcal{L}_{VH^{(Q)}H^{(Q)}} = i\beta\,\mathrm{Tr}\left[H_b^{(Q)} v_\mu \left(V_{ba}^\mu - \rho_{ba}^\mu\right)\bar{H}_a^{(Q)}\right] + i\lambda\,\mathrm{Tr}\left[H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\right]$$

$$H_a^{(\bar{Q})} \equiv C\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T C^{-1} = \left[P_{a\mu}^{(\bar{Q})*}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1- \not{v}}{2}$$

$$\bar{H}_a^{(\bar{Q})} \equiv \gamma_0 H_a^{(\bar{Q})\dagger}\gamma_0 = \frac{1- \not{v}}{2}\left[P_{a\mu}^{(\bar{Q})*\dagger}\gamma^\mu + P_a^{(\bar{Q})\dagger}\gamma_5\right]$$

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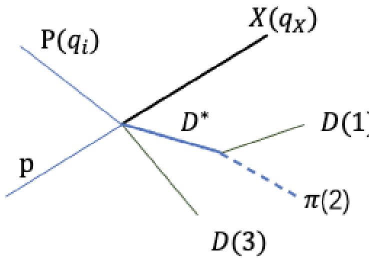
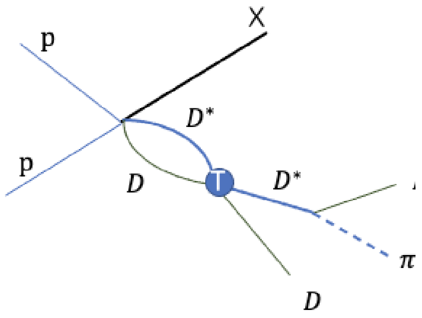
$D^{(*)}\overline{D^{(*)}}$

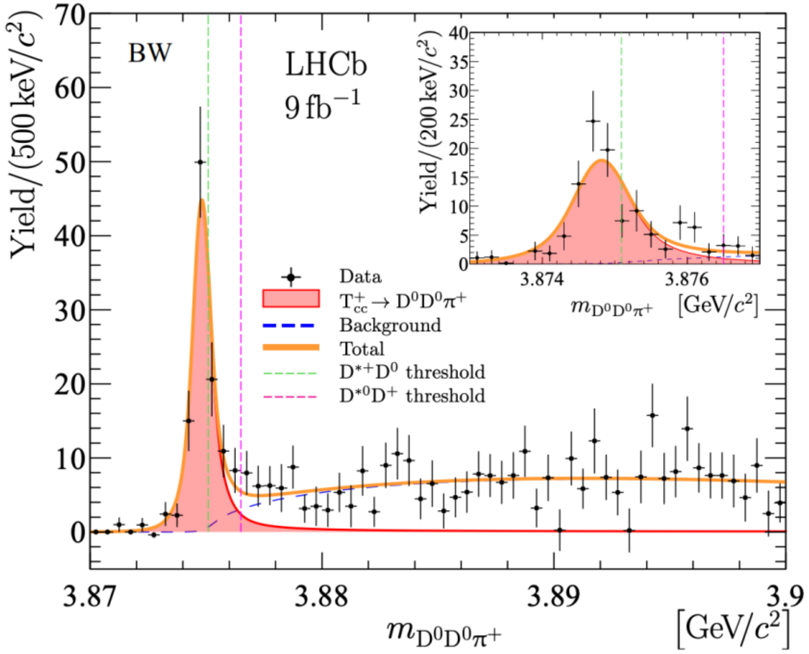
The generation of T_{cc}

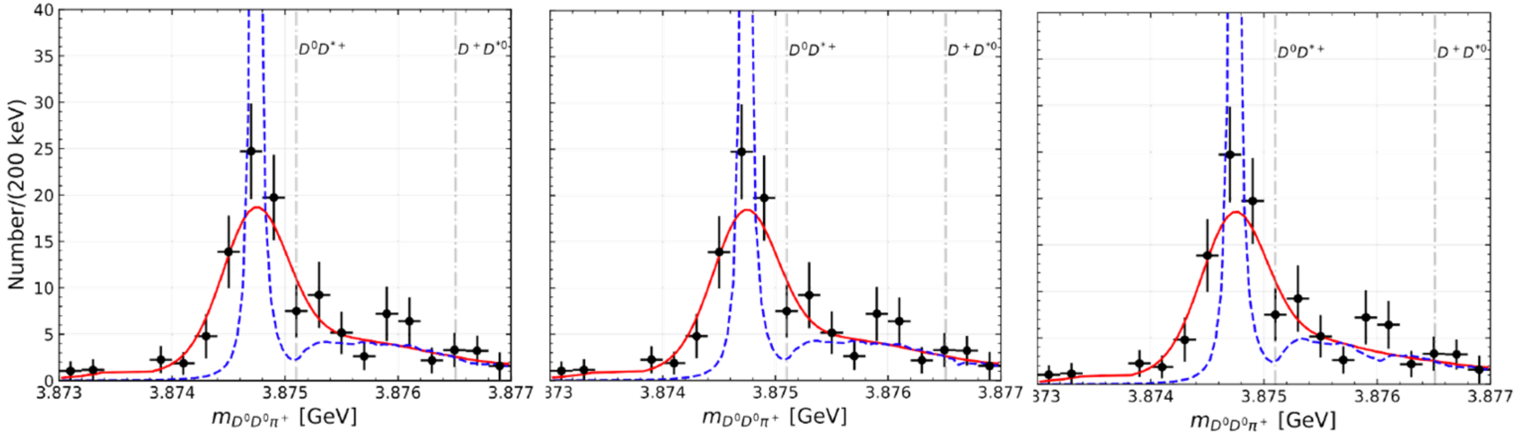
$pp \to X\, D^0 D^0 \pi^+$

p^t









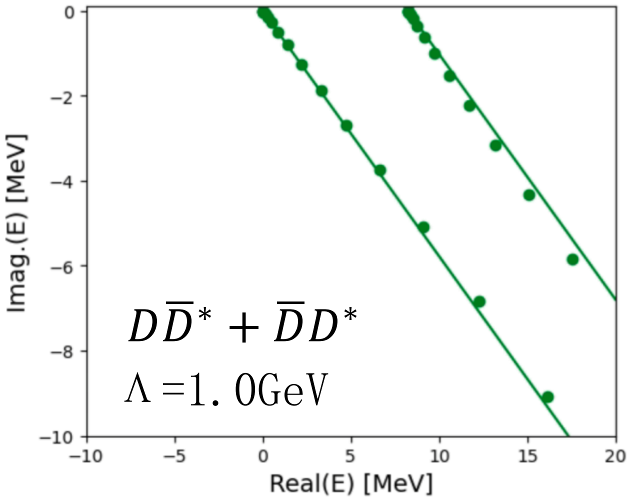
Λ (fixed)	λ（/GeV）	β
0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
1 GeV	0.683 ± 0.025	0.687 ± 0.017
1.2 GeV	0.587±0.21	0.550±0.12
1.17 GeV	0. 56	0. 9

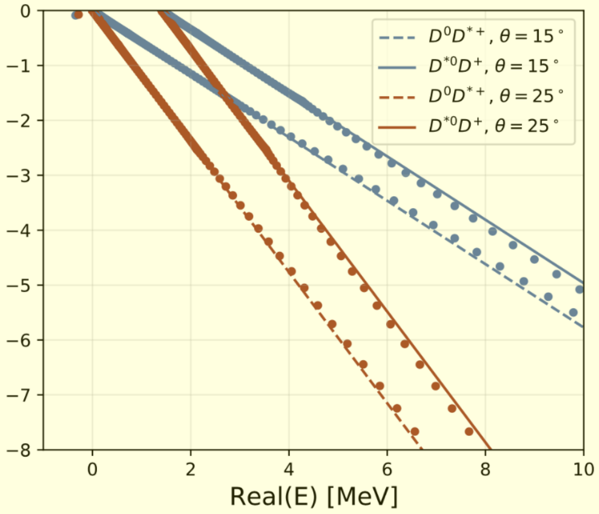
Λ (GeV)	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0D^{*+})$	$P(D^+D^{*0})$	$\frac{\mathrm{Res}(D^0D^{*+})}{\mathrm{Res}(D^+D^{*0})}$
0.8	-387.7	67.3	4.8 fm	95.8%	4.2%	70.0%	30.0%	$-1.063+0.001I$
1.0	-393.0	70.4	4.7 fm	95.8%	4.2%	70.0%	30.0%	$-1.055+0.001I$
1.2	-391.6	72.7	4.7 fm	95.7%	4.3%	70.3%	29.7%	$-1.052+0.001I$

Produce X(3872) with pure $D\bar{D}^* + \bar{D}D^*$

Complex scaling method

From the interaction of DD^* to obtain the interaction of $D\bar{D}^* + \bar{D}D^*$, check X(3872) exists or not by pure $D\bar{D}^* + \bar{D}D^*$ interaction, without $c\bar{c}$ state.





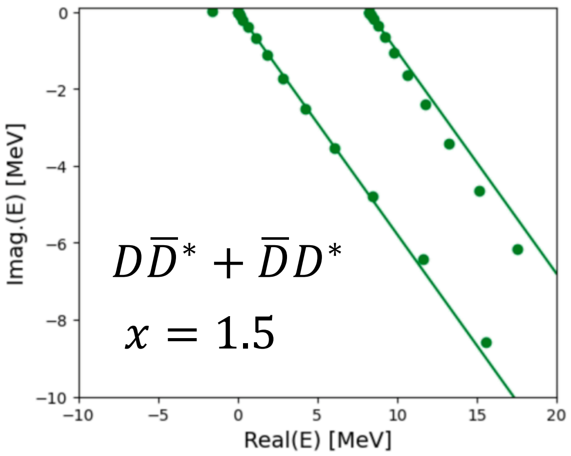
- **Bound state: T_{cc}**

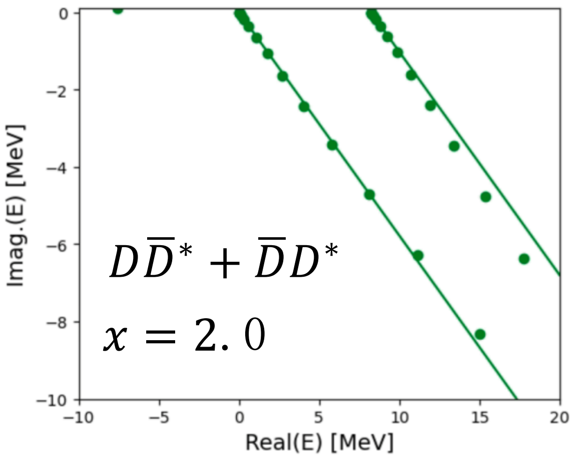
$m_{T_{cc}}$=3874.7 MeV,
$\Delta E = -393$ keV
$\Gamma_{T_{cc}} = 70.4$ keV

- $\sqrt{\langle r^2 \rangle} = 4.7\ fm$

- 70.0% $D^{*+}D^0$, 30.0% D^+D^{*0}

$$\mathbf{V'_{\overline{D^*D}} = x * V_{\overline{D^*D}}}$$



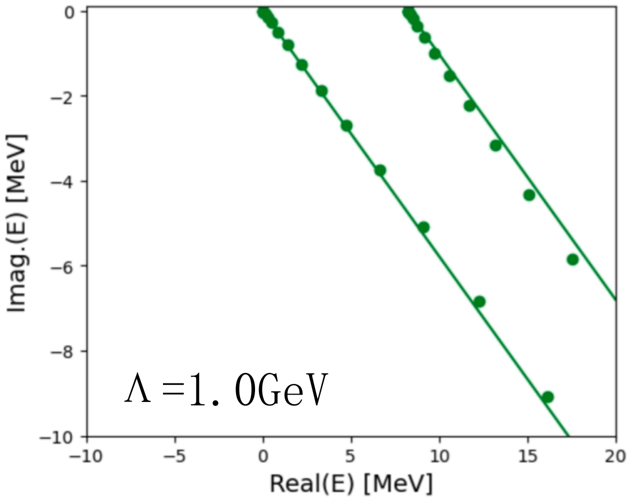


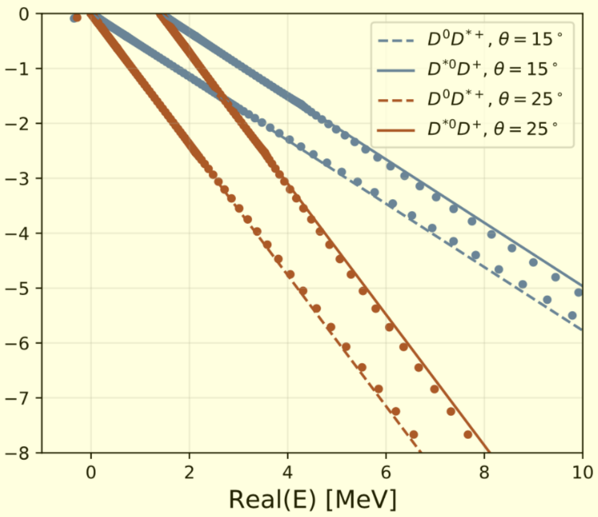
First of all, it is attractive interaction while it is not enough to form a bound state, while just a virtual state

3870.0 + 0.26 i MeV

Produce X(3872) with $D\bar{D}^* + \bar{D}D^*$ and $c\bar{c}$

From the interaction of DD^* to obtain the intraction of $D\bar{D}^* + \bar{D}D^*$, check X(3872) exists or not by pure $D\bar{D}^* + \bar{D}D^*$ interaction, without $c\bar{c}$ state.





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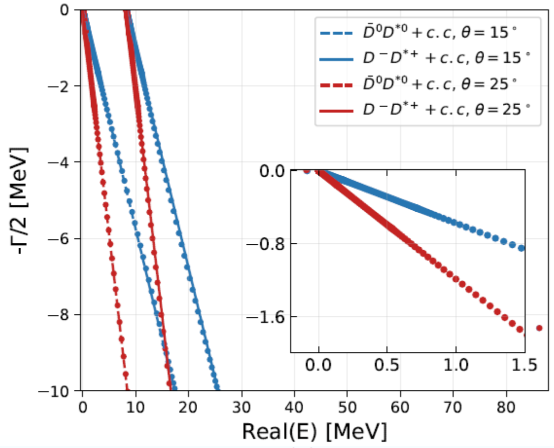
- 70.0% $D^{*+}D^0$, 30.0% D^+D^{*0}

Attractive interaction BUT not enough to form a bound state

- **A bare state shows the $c\bar{c}$ bare state component. χc1(2P, 3940) and its wave function, determined by the quark model.**
- **The interaction parameter γ = 4.69 for the 3P0 model is determined through ψ(3770) to $D\bar{D}$.**
- **Therefore the analysis of X(3872) does not introduce any additional parameters.**



(b)

- **Bound state for X(3872)**

$\Delta E = -80.4$ keV
$\Gamma_{T_{cc}} = 32.5$ keV

- $\sqrt{\langle r^2 \rangle} = 11.2\ fm$

- 94.0% $\bar{D}^{*0}D^0$, 4.8% $D^{*-}D^+$, 1.2% $c\bar{c}$

The nature of T_{cc} and X(3872)

- **T_{cc} bound state of $D^{*+}D^0$**

$\Delta E = -393$ keV $\quad \Gamma_{T_{cc}} = 70.4$ keV

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- $70.0\% D^{*+}D^0,\ \ 30.0\% D^+D^{*0}$

- **X(3872) bound state of $\bar{D}^{*0}D^0$**

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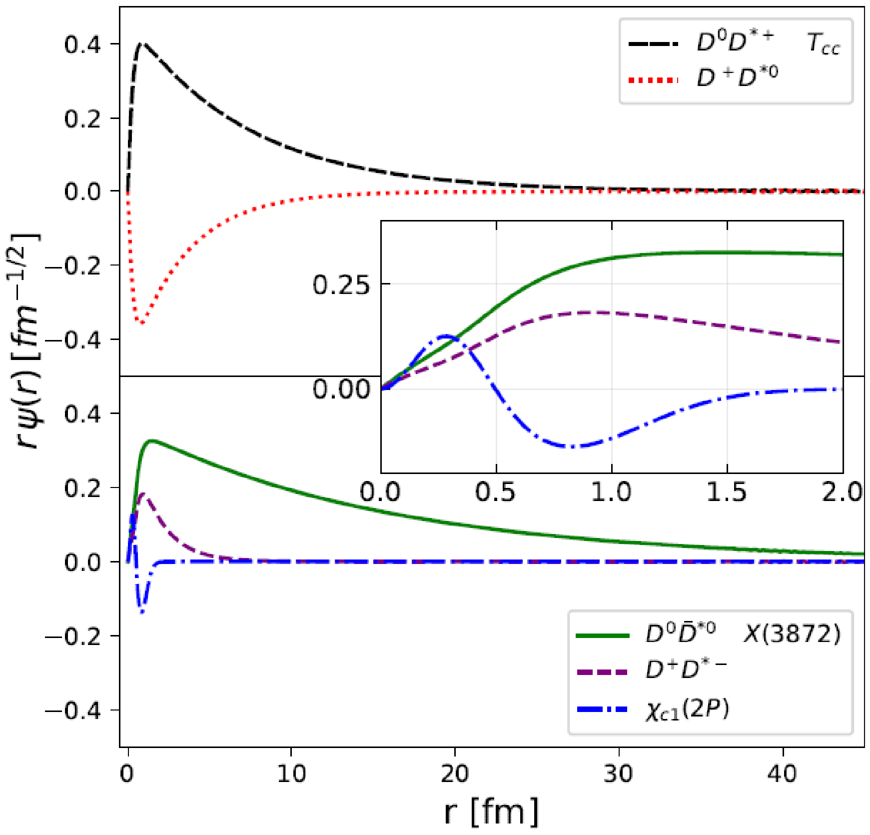
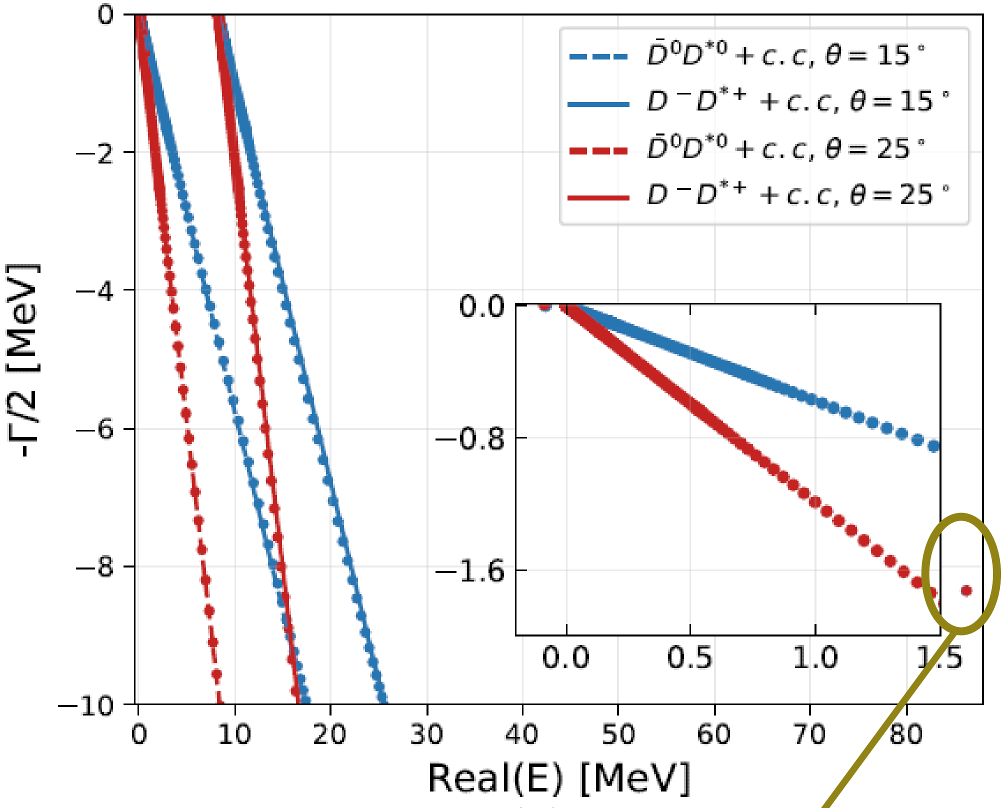


ΔE of **X(3872)** is extremely small, a significant $\boldsymbol{D^{*0}\bar{D}^0}$ wave function, dominates in long-range
the $\boldsymbol{c\bar{c}}$ component predominant in short-range, it is important to form this bound state

Prediction



(b)

- **χ_{c1}(2p)**

  M = 3957.9 MeV $\Gamma_{\chi_{c1(2P)}}$ = 16.7 MeV

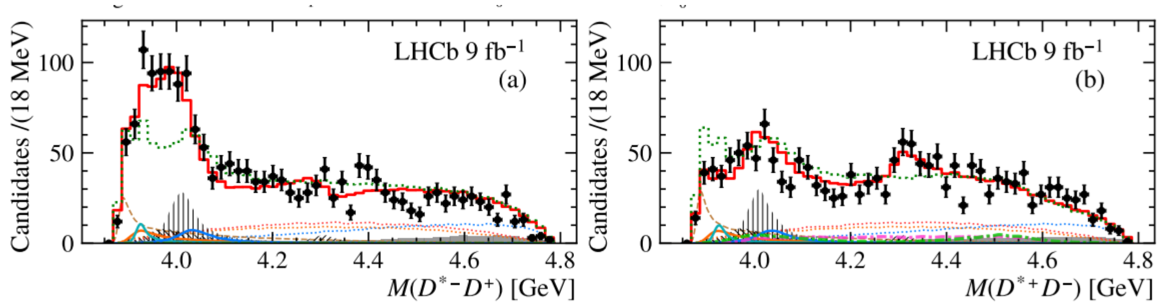
- Main decay channel： $\bar{D}^* D$

Observation of new charmonium(-like) states in $B^+ \to D^{*\pm} D^{\mp} K^+$ decays

LHCb Collaboration 2406.03156

$\chi_{c1}(4010)$ $J^{PC} = 1^{++}$

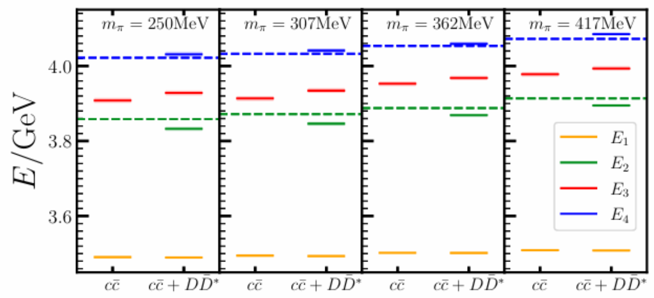
$m_0 = 4012.5^{+3.6}_{-3.9}{}^{+4.1}_{-3.7}$ $\Gamma_0 = 62.7^{+7.0}_{-6.4}{}^{+6.4}_{-6.6}$

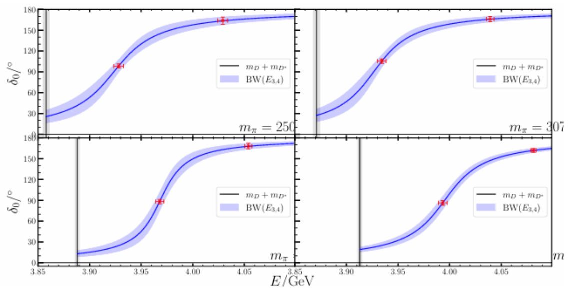


X(3872) Relevant $D\bar{D}^*$ Scattering in $N_f = 2$ Lattice QCD

H. Li, C. Shi, Y. Chen, M. Gong, J. Liang et al

CLQCD 2402.14541





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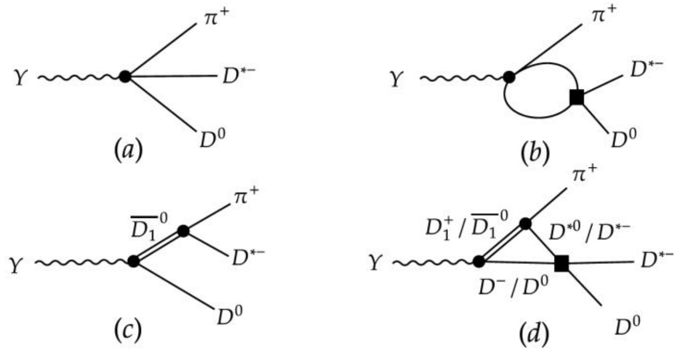
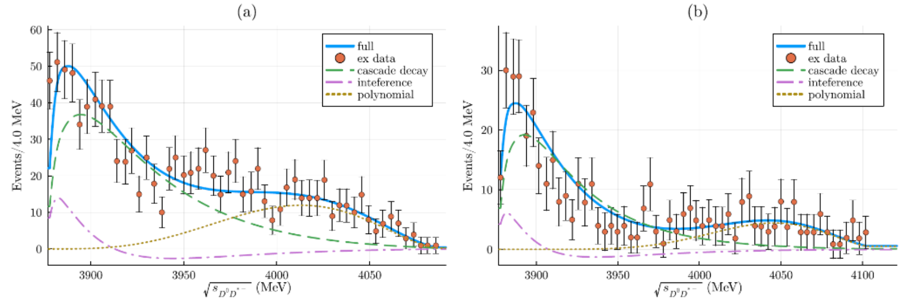
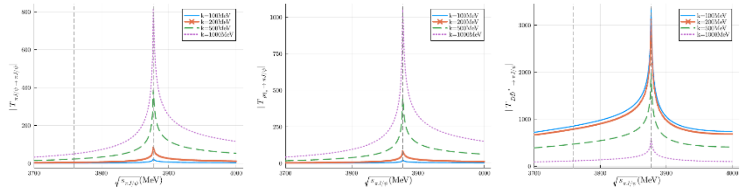
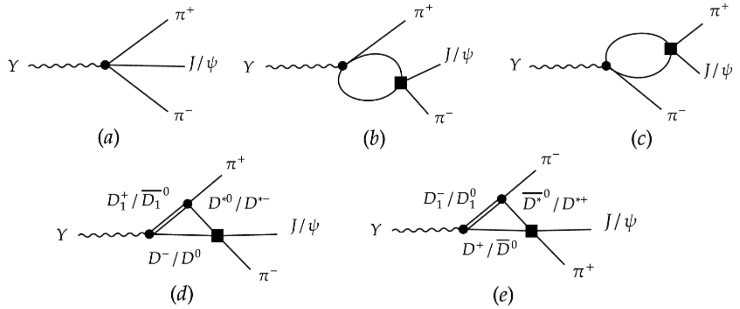
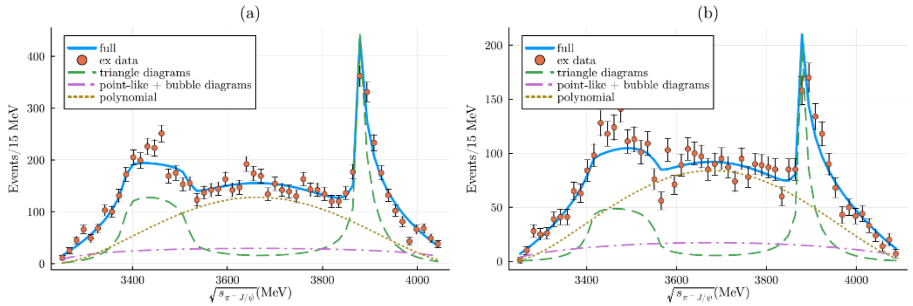
$T_{cc} - X(3872) - Z_c(3900)$

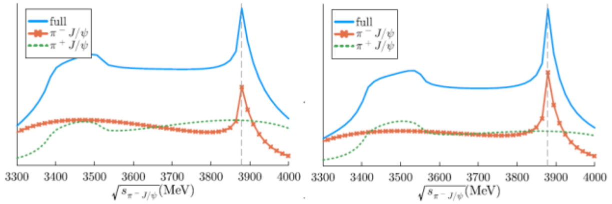


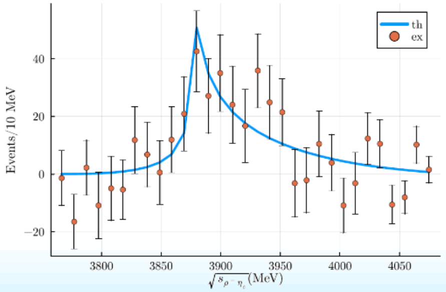


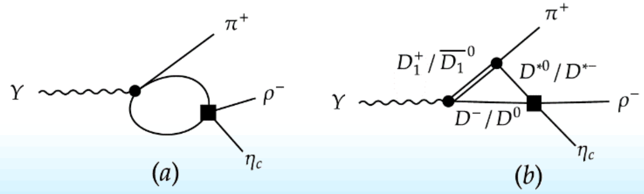










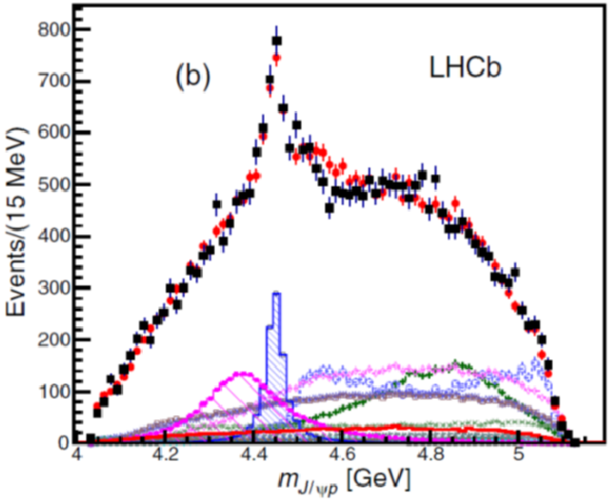


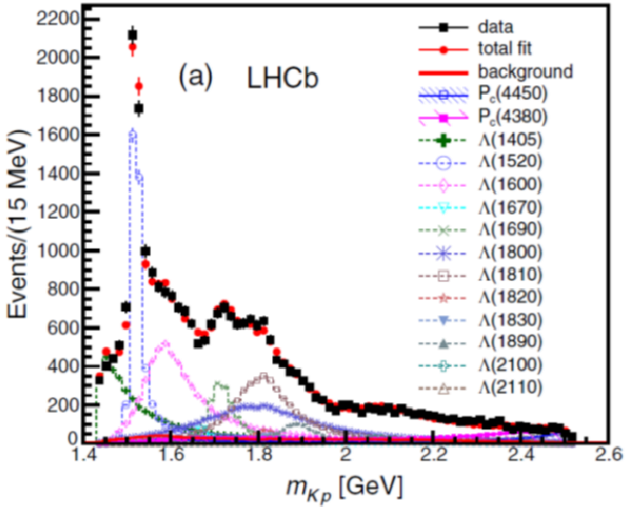
Coupled channel Constrain !

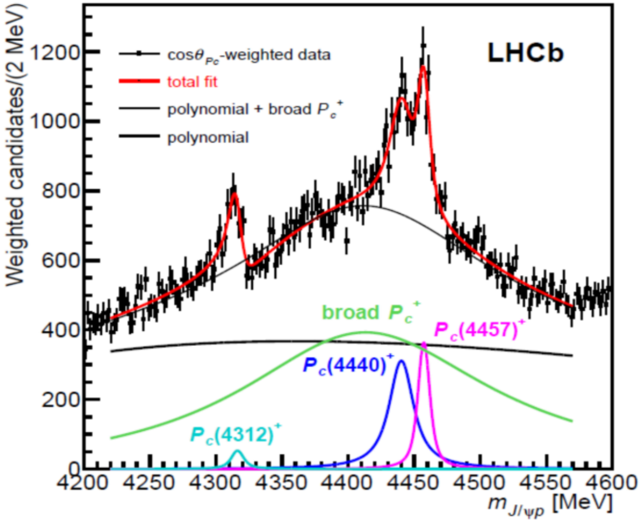
1. **Two-particle loops important ?**
2. **Triangle loop ?**
3. **Re-scattering T matrix ?**

Both play important role!

Introduction of Pc states

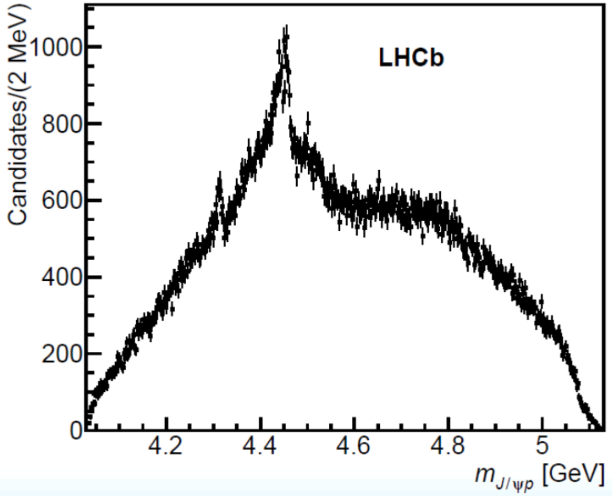


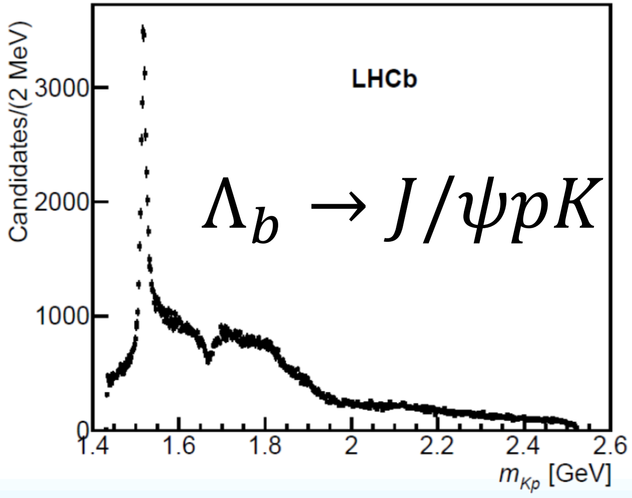




$\bar{D}\Sigma_c$ threshold: 4320 MeV
$\bar{D}^*\Sigma_c$ threshold: 4465 MeV

LHCb
PRL 115 (2015) 072001
1849 Citations
PRL 122 (2019) 222001
815 Citations





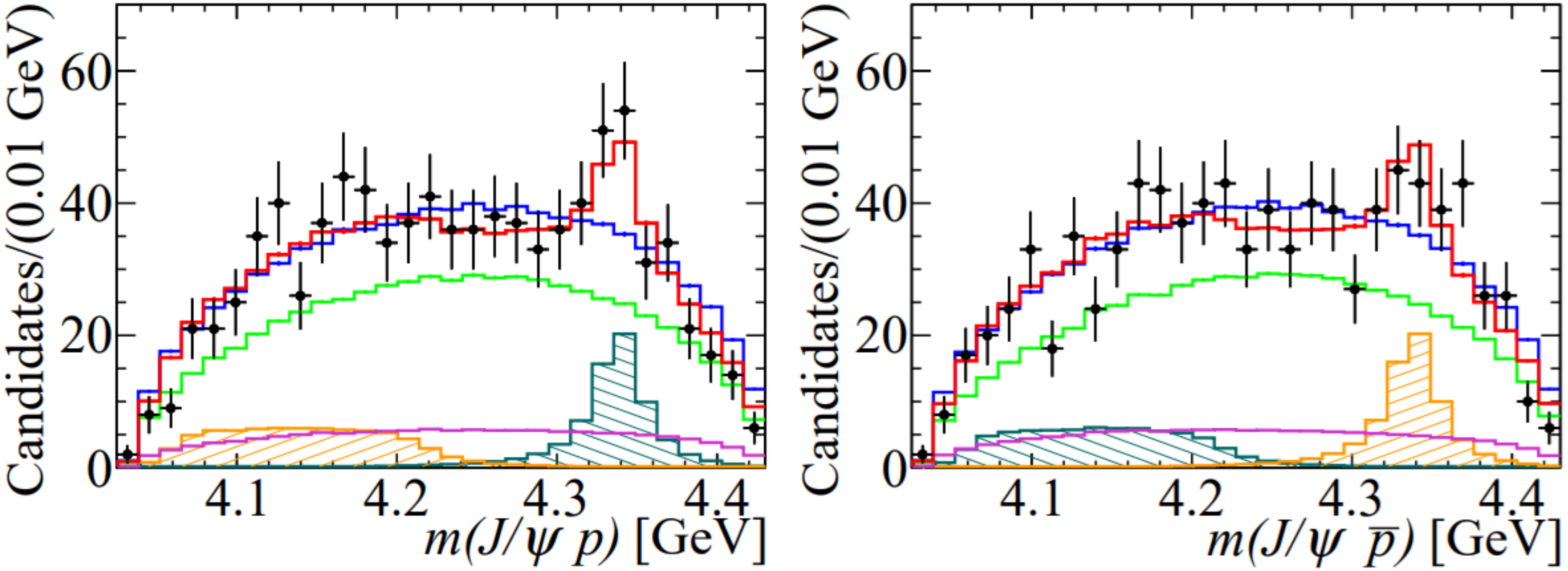
$\Lambda_b \to J/\psi p K$

Pc (4380)	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$
Pc (4450)	$4449.8 \pm 1.7 \pm 2.5$	$39 \pm 5 \pm 19$

Pc (4312)	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$
Pc (4440)	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.5}$
Pc (4457)	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$

Unit: MeV

Introduction of Pc states



LHCb PRL 128 (2022) 6, 062001
115 Citations

$\bar{D}\Sigma_c$ threshold: 4320 MeV

$$M_{P_c} = 4337\ ^{+7}_{-4}\ ^{+2}_{-2}\,\text{MeV},$$
$$\Gamma_{P_c} = 29\ ^{+26}_{-12}\ ^{+14}_{-14}\,\text{MeV},$$

$3.1 - 3.7\sigma$

$B_s^0 \to J/\psi p\bar{p}$

Introduction of Pc states

1) Molecular states: P_c -- $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$

……,too many papers !

2) Compact Pentaquark: cu+ $\bar{c}$(ud) **states**

Maiani,Polosa,Riquer, PLB749 (2015) 289;
Lebed, PLB749 (2015) 454;
Li,He,He, JHEP 1512 (2015) 128;
Zhu,Qiao, PLB756 (2016) 259;
Yuan, An, Wei, Zou, Xu,PRC87(2013) 025205;
Yuan, He, Xu, Zou, Eur.Phys.J.A 48 (2012) 61; ……

3) Kinematic triangle-singularity

Guo, Meißner, Wang, Yang, PRD92 (2015) 071502;
Liu, Wang, Zhao, PLB757 (2016) 231;
Bayar, Aceti, Guo and Oset, PRD94(2016) 074039; ……

Chen, Chen, Liu, and Zhu, PR 639, 1 (2016),1601.02092.

Dong, Faessler, and Lyubovitskij, PPNP 94, 282 (2017).

Guo, Hanhart, Meissner, Wang, Zhao, and Zou, RMP 90, 015004 (2018), 1705.00141.

Ali, Lange, and Stone, PPNP 97, 123 (2017), 1706.00610.

Introduction of Pc states

1) Molecular states: P_c—— $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$

……,too many papers !

- Valencia Model：

  Wu, Molina, Oset and Zou, PRL 105, 232001，PRC 84, 015202 (2010)
  Garcia-Recio, Nieves, Romanets, Salcedo, and Tolos, PRD87, 074034(2013)
  Xiao, Nieves, and Oset, PRD88, 056012(2013)
  Uchino, Liang, and Oset, EPJA 52, 43(2016)

  $\bar{D}\Sigma_c$ ~ 4.3 GeV $\bar{D}\Sigma_c^*$ ~ 4.35 GeV $\bar{D}^*\Sigma_c$ ~ 4.4 GeV $\bar{D}^*\Sigma_c^*$ ~ 4.5 GeV

- EBAC Model： **Wu, Lee and Zou, PRC 85, 044002 (2012)**

  $\bar{D}\Sigma_c$ ~ 4.3 GeV $\bar{D}^*\Sigma_c$ ~ 4.4 GeV

- Chiral constituent quark model & a resonating group method equation

  Wang, Huang, Zhang, Zou, PRC 84,015203(2011). $\bar{D}\Sigma_c$ ~ 4.3 GeV

- Schrödinger Equation & One boson exchange：

  Yang, Sun, He, Liu, Zhu, Chin.Phys. C36 (2012) 6-13

  $\bar{D}\Sigma_c$(I=3/2) ~ 4.3 GeV $\bar{D}^*\Sigma_c$ ~ 4.4 GeV

- Pentaquark Model： **Yuan, Wei, He, Xu and Zou, EPJA 48, 61(2012)**~ 4.3-4.5 GeV

$\bar{D}\Sigma_c - \eta_c N - \eta N$ coupled channel state ~ 3.5 GeV

J. Hofmann, M.F.M. Lutz, Nucl. Phys. A 763 (2005) 90

$c\bar{c}$-N bound states in topological soliton model **~ 3.9 GeV**

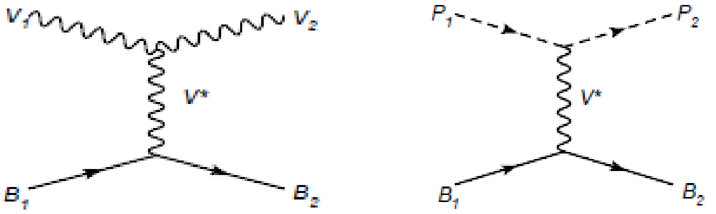
C. Gobbi, D.O. Riska, N.N. Scoccola, Phys. Lett. B 296 (1992) 166

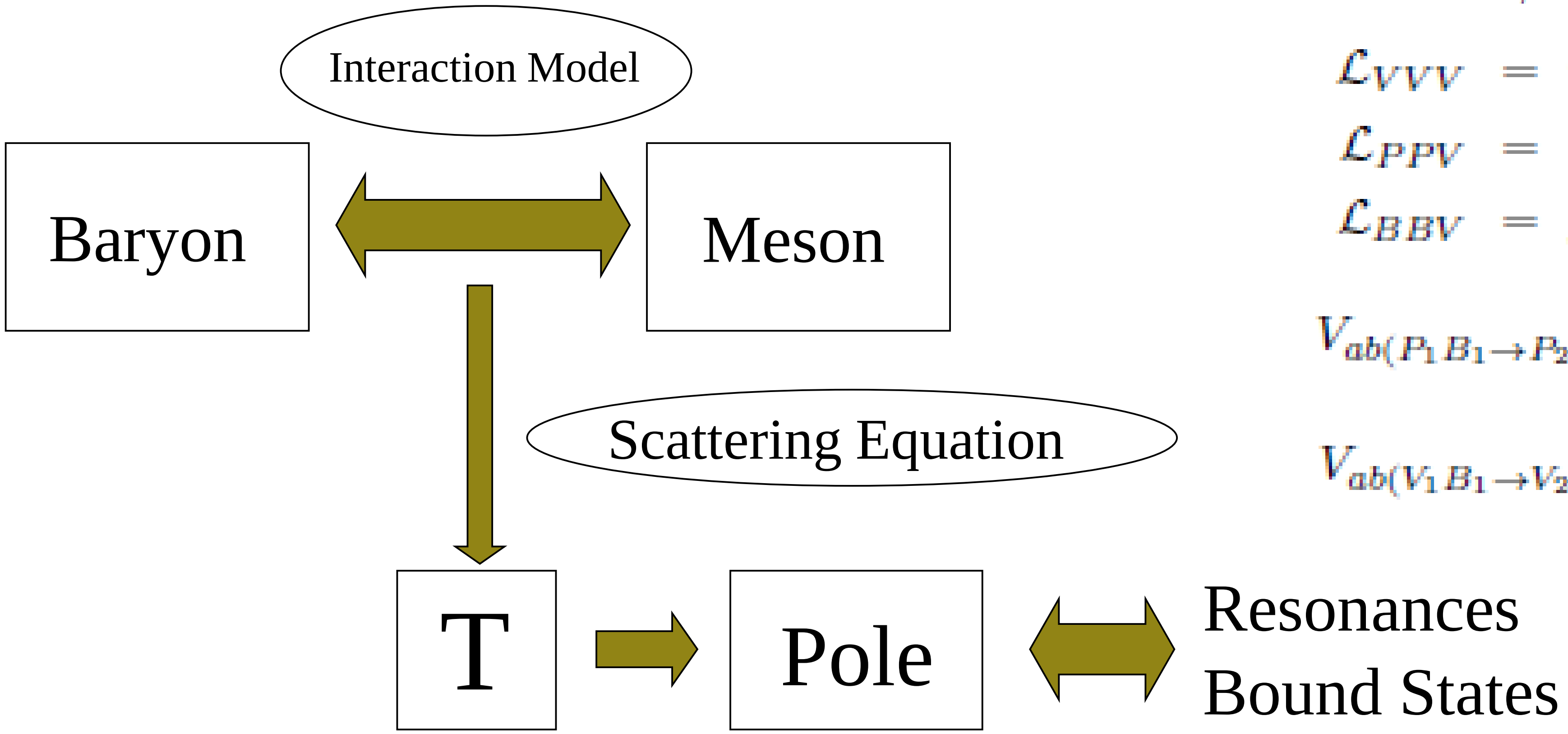
Introduction of Pc states

$J^P = 1/2^-,\ 3/2^-$

1) Molecular states: P_c -- $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$

Wu, Molina, Oset and Zou, PRL 105, 232001，PRC 84, 015202 (2010)





$$\mathcal{L}_{VVV} = ig\langle V^\mu[V^\nu, \partial_\mu V_\nu]\rangle$$

$$\mathcal{L}_{PPV} = -ig\langle V^\mu[P, \partial_\mu P]\rangle$$

$$\mathcal{L}_{BBV} = g(\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle + \langle \bar{B}\gamma_\mu B\rangle\langle V^\mu\rangle)$$

$$V_{ab(P_1 B_1 \to P_2 B_2)} = \frac{C_{ab}}{4f^2}(E_{P_1} + E_{P_2}),$$

$$V_{ab(V_1 B_1 \to V_2 B_2)} = \frac{C_{ab}}{4f^2}(E_{V_1} + E_{V_2})\vec{\epsilon}_1 \cdot \vec{\epsilon}_2,$$

$$T = [1 - VG]^{-1}V$$

$$T_{ab} = \frac{g_a g_b}{\sqrt{s} - z_R}$$

$\bar{D}\Lambda_{c1}(2595)$ channel

The threshold of $\bar{D}\Lambda_c(2595)$ is 1865+2595=4460 MeV, just above the Pc(4457)!

$\bar{D}\Lambda_{c1}(2595)$ channel

The threshold of $\bar{D}\Lambda_c(2595)$ is 1865+2595=4460 MeV, just above the Pc(4457)!

Before Pc(4457), just Pc(4450)

Geng, Lu, and Valderrama. PRD, 97 094036,

Scale invariance in heavy hadron molecules

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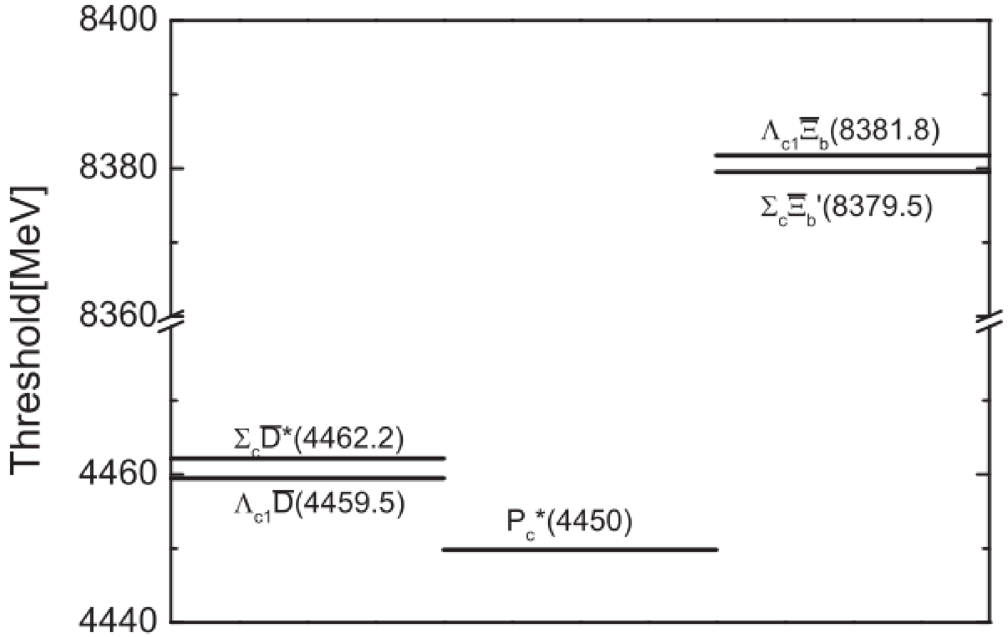
(Received 22 April 2017; published 31 May 2018)

We discuss a scenario in which the $P_c(4450)^+$ heavy pentaquark is a $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ molecule. The $\Lambda_{c1}\bar{D} \to \Sigma_c\bar{D}^*$ transition is mediated by the exchange of a pion almost on the mass shell that generates a long-range $1/r^2$ potential. This is analogous to the effective force that is responsible for the Efimov spectrum in three-boson systems interacting through short-range forces. The equations describing this molecule exhibit approximate scale invariance, which is anomalous and broken by the solutions. If the $1/r^2$ potential is strong enough this symmetry survives in the form of discrete scale invariance, opening the prospect of an Efimov-like geometrical spectrum in two-hadron systems. For a molecular pentaquark with quantum numbers $\frac{3}{2}^-$ the attraction is not enough to exhibit discrete scale invariance, but this prospect might very well be realized in a $\frac{1}{2}^+$ pentaquark or in other hadron molecules involving transitions between particle channels with opposite intrinsic parity and a pion near the mass shell. A very good candidate is the $\Lambda_c(2595)\bar{\Xi}_b - \Sigma_c\bar{\Xi}_b'$ molecule. Independently of this, the $1/r^2$ force is expected to play a very important role in the formation of this type of hadron molecule, which points to the existence of $\frac{1}{2}^+$ $\Sigma_c D^*$-$\Lambda_c(2595)D$ and 1^+ $\Lambda_c(2595)\Xi_b - \Sigma_c\Xi_b'$ molecules and $0^+/1^-$ $\Lambda_c(2595)\bar{\Xi}_b - \Sigma_c\bar{\Xi}_b'$ baryonia.



After Pc(4457)

University of Chinese Academy of Sciences

$\bar{D}\Lambda_{c1}(2595)$ channel

After Pc(4457)

Burns and Swanson PRD 100 114033,2019

Molecular interpretation of the $P_c(4440)$ and $P_c(4457)$ states

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(Received 23 August 2019; published 20 December 2019)

A molecular model of the $P_c(4457)$ and $P_c(4440)$ LHCb states is proposed. The model relies on channels coupled by long-range pion-exchange dynamics with features that depend crucially on the novel addition of the $\Lambda_c(2595)\bar{D}$ channel. A striking prediction of the model is the unusual combination of quantum numbers $J^P(4457) = 1/2^+$ and $J^P(4440) = 3/2^-$. Unlike in other models, a simultaneous description of both states is achieved without introducing additional short-range interactions. The model also gives a natural explanation for the relative widths of the states. We show that the usual molecular scenarios cannot explain the production rate of P_c states in Λ_b decays and that this can be resolved by including $\Lambda_c(2595)\bar{D}$ and related channels. Experimental tests and other states are discussed in the conclusions.

Yalikun, Lin, Guo, Kamiya, and Zou PRD 104,094039,2021.

Coupled-channel effects of the $\Sigma_c^{(*)}\bar{D}^{(*)}$–$\Lambda_c(2595)\bar{D}$ system and molecular nature of the P_c pentaquark states from one-boson exchange model

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The effects of the $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ coupled-channel dynamics and various one-boson-exchange (OBE) forces for the LHCb pentaquark states, $P_c(4440)$ and $P_c(4457)$, are reinvestigated. Both the pion and ρ-meson exchanges are considered for the $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ coupled-channel dynamics. It is found that the role of the $\Lambda_c(2595)\bar{D}$ channel in the descriptions of the $P_c(4440)$ and $P_c(4457)$ states is not significant with the OBE parameters constrained by other experimental sources. The naive OBE models with the short-distance $\delta(\vec{r})$ term of the one-pion exchange (OPE) keep failing to reproduce the $P_c(4440)$ and $P_c(4457)$ states simultaneously. The OPE potential with the full $\delta(\vec{r})$ term results in a too large mass splitting for the $J^P = 1/2^-$ and $3/2^-$ $\Sigma_c\bar{D}^*$ bound states with total isospin $I = 1/2$. The OBE model with only the OPE $\delta(\vec{r})$ term dropped may fit the splitting much better but somewhat underestimates the splitting. Since the $\delta(\vec{r})$ potential is from short-distance physics, which also contains contributions from the exchange of mesons heavier than those considered explicitly, we vary the strength of the $\delta(\vec{r})$ potential and find that the masses of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ can be reproduced simultaneously with the $\delta(\vec{r})$ term in the OBE model reduced by about 80%. While two different spin assignments are possible to produce their masses, in the preferred description, the spin parities of the $P_c(4440)$ and $P_c(4457)$ are $3/2^-$ and $1/2^-$, respectively.

$\bar{\mathrm{D}}\Lambda_{c1}(2595)$ channel

After Pc(4457)

Burns and Swanson PRD 100 114033,2019

Molecular interpretation of the $P_c(4440)$ and $P_c(4457)$ states

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(Received 23 August 2019; published 20 December 2019)

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Yaliku

Coupled-channel effects of the $\Sigma_c^{(*)}\bar{D}^{(*)}$–$\Lambda_c(2595)\bar{D}$ system and molecular nature of the P_c pentaquark states from one-boson exchange model

coupled-channel dynamics. It is found that the role of the $\Lambda_c(2595)\bar{D}$ channel in the descriptions of the $P_c(4440)$ and $P_c(4457)$ states is not significant with the OBE parameters constrained by other experimental sources. The naive OBE models with the short-distance $\delta(\vec{r})$

Kamiya, and Zou PRD 104,094039,2021.

Universität Bonn, D-53115 Bonn, Germany
[4] School of Physics and Electronics, Central South University, Changsha 410083, China

The effects of the $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ coupled-channel dynamics and various one-boson-exchange (OBE) forces for the LHCb pentaquark states, $P_c(4440)$ and $P_c(4457)$, are reinvestigated. Both the pion and ρ-meson exchanges are considered for the $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ coupled-channel dynamics. It is found that the role of the $\Lambda_c(2595)\bar{D}$ channel in the descriptions of the $P_c(4440)$ and $P_c(4457)$ states is not significant with the OBE parameters constrained by other experimental sources. The naive OBE models with the short-distance $\delta(\vec{r})$ term of the one-pion exchange (OPE) keep failing to reproduce the $P_c(4440)$ and $P_c(4457)$ states simultaneously. The OPE potential with the full $\delta(\vec{r})$ term results in a too large mass splitting for the $J^P = 1/2^-$ and $3/2^-$ $\Sigma_c\bar{D}^*$ bound states with total isospin $I = 1/2$. The OBE model with only the OPE $\delta(\vec{r})$ term dropped may fit the splitting much better but somewhat underestimates the splitting. Since the $\delta(\vec{r})$ potential is from short-distance physics, which also contains contributions from the exchange of mesons heavier than those considered explicitly, we vary the strength of the $\delta(\vec{r})$ potential and find that the masses of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ can be reproduced simultaneously with the $\delta(\vec{r})$ term in the OBE model reduced by about 80%. While two different spin assignments are possible to produce their masses, in the preferred description, the spin parities of the $P_c(4440)$ and $P_c(4457)$ are $3/2^-$ and $1/2^-$, respectively.

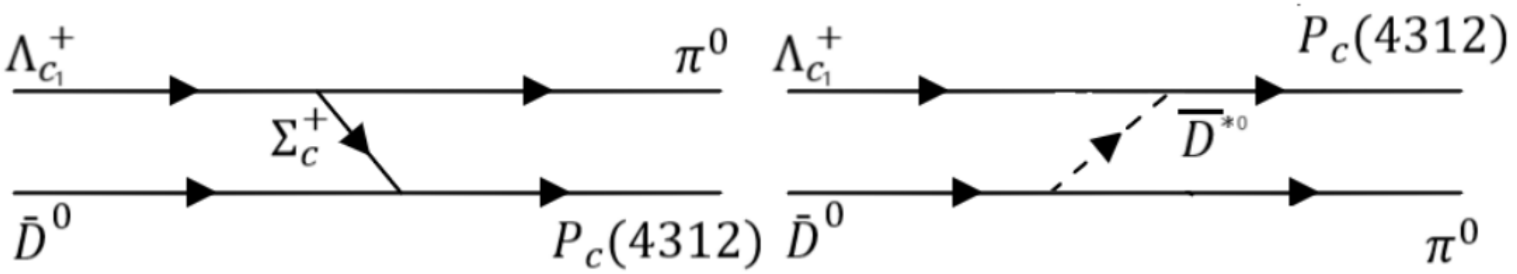
$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel

Another coupled channel $P_c(4312)\pi$, the threshold is 4312+135 = 4447 MeV, also very close to $P_c(4457)$.

Furthermore, the spin-parity is $1/2^-$ and 0^-, if we assume $P_c(4312)$ is bound state of $\bar{D}\Sigma_c$, then the quantum number of J^P for the S-wave state is also $1/2^+$.

Then we consider $\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ couple channel.

coupled-channel dynamics. It is found that the role of the $\Lambda_c(2595)\bar{D}$ channel in the descriptions of the $P_c(4440)$ and $P_c(4457)$ states is not significant with the OBE parameters constrained by other experimental sources. The naive OBE models with the short-distance $\delta(\vec{r})$

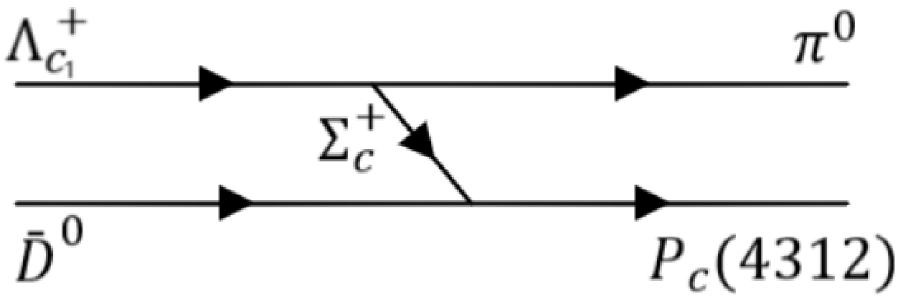


The diagonal term of potential is neglect! While the off-diagonal term will have two mechanisms.

Σ_C^+ is almost on-shell !

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$: $\bar{D}\pi\Sigma_c^+$ three-body

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel



$$\mathcal{U}_i(\boldsymbol{p},\lambda_i) = \sqrt{\frac{\omega_i(\boldsymbol{p})+m_i}{2m_i}}\begin{pmatrix}\Phi^{\lambda_i}\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\omega_i(\boldsymbol{p})+m_i}\Phi^{\lambda_i}\end{pmatrix},$$

$$\Gamma = g_1^2\frac{q_{on}}{4\pi}\frac{\left(\omega_{\Sigma_c^+}(q_{on})+m_{\Sigma_c^+}\right)}{m_{\Lambda_{c1}^+}}$$

$$g_2^2 = \frac{4\pi}{4m_{P_c}m_{\Sigma_c}}\frac{(m_{\Sigma_c}+m_D)^{\frac{5}{2}}}{(m_{\Sigma_c}m_D)^{\frac{1}{2}}}\sqrt{32\,|m_{\Sigma_c}+m_D-m_{P_c}|}$$

Weinberg PR 137(1965) B672
Baru, et.al. PLB586(2004) 53
Lin, Shen, Guo, and Zou. PRD95(2017) 114017

$$\mathcal{V}_{\alpha\beta}(\boldsymbol{p},\,\boldsymbol{q},\,\lambda_{\alpha_B},\,\lambda_{\beta_B}) = g_1 g_2 \bar{\mathcal{U}}_{\beta_B}(\boldsymbol{q},\lambda_{\beta_B})G^{\alpha\beta}_{\Sigma_c^+}(\boldsymbol{p},\boldsymbol{q})\mathcal{U}_{\alpha_B}(\boldsymbol{p},\lambda_{\alpha_B}).$$

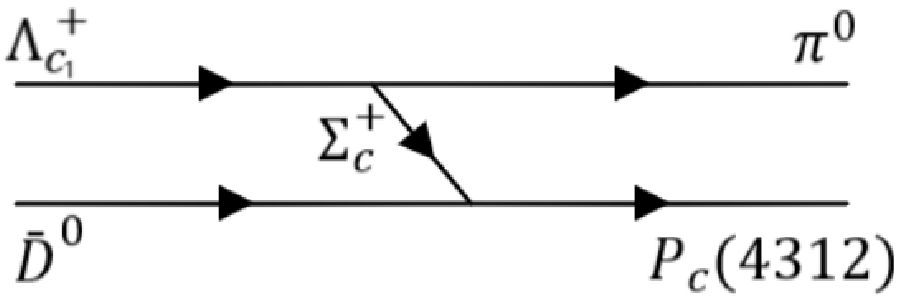
$$G^{\alpha\beta}_{\Sigma_c^+}(\boldsymbol{p},\boldsymbol{q},E) = \frac{1}{2}\left\{\frac{(\omega_{\alpha_B}(\boldsymbol{p})-\omega_{\beta_M}(\boldsymbol{q}))\gamma_0-(\boldsymbol{p}+\boldsymbol{q})\cdot\vec{\gamma}+m_{\Sigma_c^+}}{(\omega_{\beta_M}(\boldsymbol{q})-\omega_{\alpha_B}(\boldsymbol{p}))^2-\omega^2_{\Sigma_c^+}(\boldsymbol{p}+\boldsymbol{q})}+\frac{(\omega_{\beta_B}(\boldsymbol{q})-\omega_{\alpha_M}(\boldsymbol{p}))\gamma_0-(\boldsymbol{p}+\boldsymbol{q})\cdot\vec{\gamma}+m_{\Sigma_c^+}}{(\omega_{\beta_B}(\boldsymbol{q})-\omega_{\alpha_M}(\boldsymbol{p}))^2-\omega^2_{\Sigma_c^+}(\boldsymbol{p}+\boldsymbol{q})}\right\}.$$

Wu, Lee, and Zou. PRC85(2012) 044002

$$V_{\alpha\beta}(p,q) = \frac{F(p,q)}{2(2\pi)^3}\sqrt{\frac{m_{\Lambda_{c1}^+}m_{P_c}}{\omega_{\Lambda_{c1}^+}(p)\omega_{P_c}(q)2\omega_{D^0}(p)2\omega_\pi(q)}}\sum_{\lambda_{\alpha_B},\lambda_{\beta_B}}2\pi\int_{-1}^{1}d\cos\theta\, d^{1/2}_{\lambda_{\alpha_B}\lambda_{\beta_B}}(\theta)\,\mathcal{V}_{\alpha\beta}(\boldsymbol{p},\,\boldsymbol{q},\,\lambda_{\alpha_B},\,\lambda_{\beta_B}).$$

$$F(p,q) = \frac{\Lambda^2}{p^2+\Lambda^2}\frac{\Lambda^2}{q^2+\Lambda^2},$$

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel



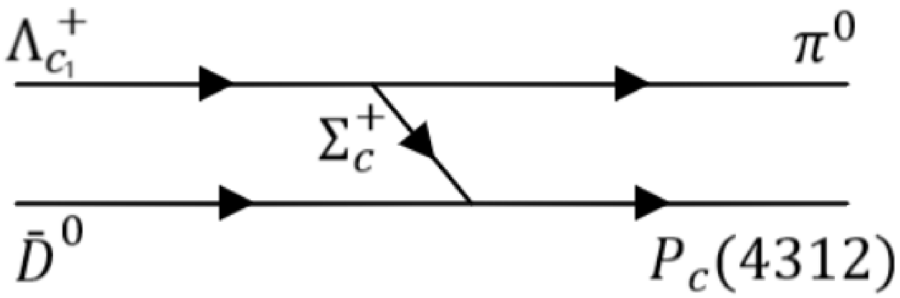
$$\mathcal{G}_\gamma(q;E) = \frac{1}{E - \omega_{\gamma_B}(q) - \omega_{\gamma_M}(q) + i\epsilon}$$

$$T_{\alpha\beta}(p,p';E) = V_{\alpha\beta}(p,p') + \sum_\gamma \int dq q^2 V_{\alpha\gamma}(p,q) G_\gamma(q;E) T_{\gamma\beta}(q,p';E)$$

Usual method: **change to a matrix equation ! Then we will find the routines of p and p' are the same as integral variable q !**

$$\det(\mathbb{I} - VG) = 0.$$

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel



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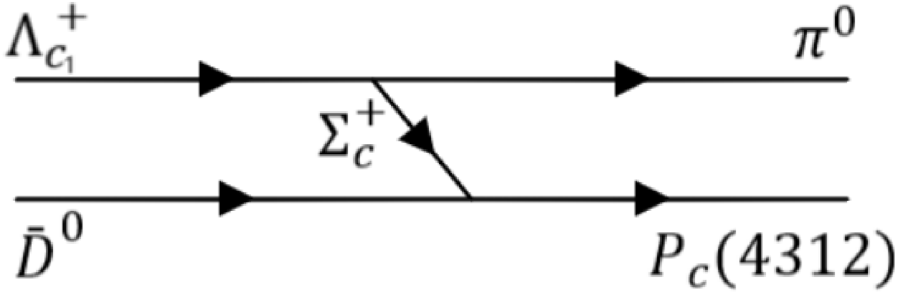
Usual method: change to a matrix equation ! Then we will find the routines of p and p' are the same as integral variable q !

Key problem: $V(p,q)$ will have a pole of integral variable q when p changed.

Left hand cut

$$\det(\mathbb{I} - VG) = 0.$$

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel



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Usual method: change to a matrix equation ! Then we will find the routines of p and p' are the same as integral variable q !

$$\det(\mathbb{I} - VG) = 0.$$

Key problem: $V(p,q)$ will have a pole of integral variable q when p changed.

Left hand cut

Solution: Find a special integral routine which will not touch the singularity because of integral routine.

$\bar{D}\Lambda_{c1}(2595)-P_c(4312)\pi$ coupled channel

Integral routine

Key problem: **$V(p,q)$ will have a pole of integral variable q when p changed.**

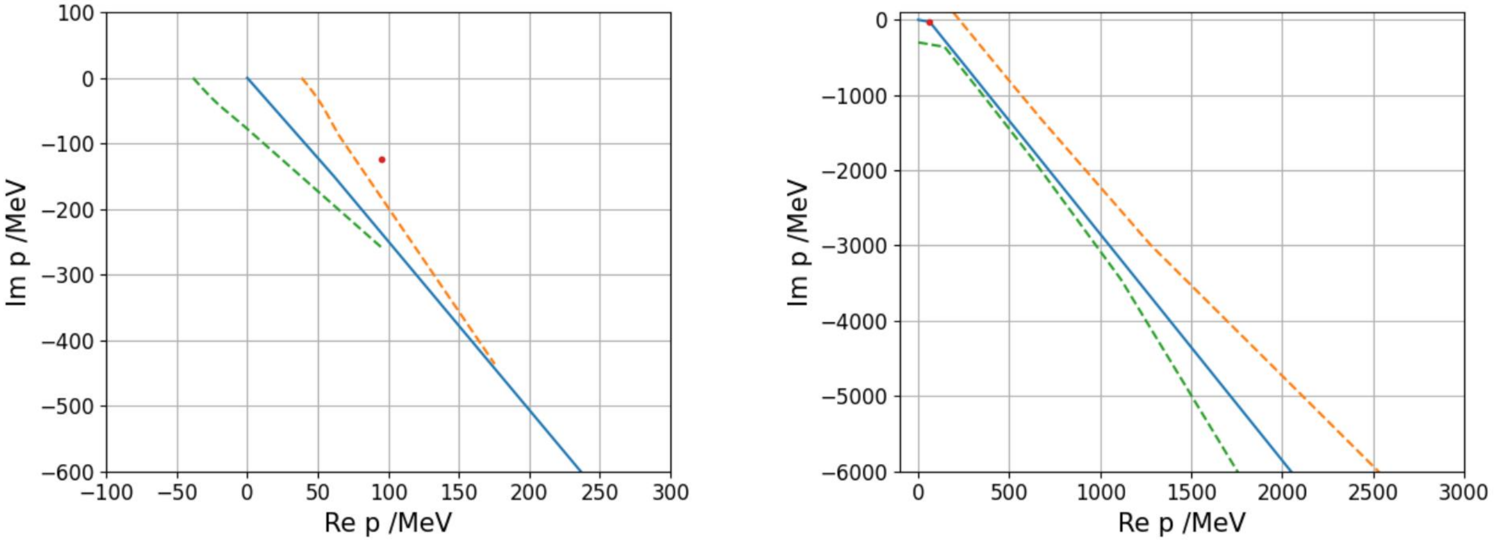


Figure 2: Two paths of integrate momenta for $\pi^0 P_c(4312)$(left) and $\bar{D}^0\Lambda_{c1}^+$(right).

Table 1. The pole position of $T-$matrix in the complex plain for different cutoffs.

	M_{P_c}/ MeV	$\Gamma_{P_c}/2$ MeV
$\Lambda = 0.8$ GeV	4456.7428	10.7337
$\Lambda = 1.0$ GeV	4456.7667	10.7293
$\Lambda = 1.2$ GeV	4456.7861	10.7238

the second Riemann sheet of $P_c(4312)\pi$
the first Riemann sheet of $\bar{D}\Lambda_{c1}(2595)$

A bound state of $\bar{D}\Lambda_{c1}(2595)$ with $J^P=\frac{1}{2}^+$

Typical property: Large decay width to $P_c(4312)\pi$.

$\bar{D}\Lambda_{c1}(2595) - P_c(4312)\pi$ coupled channel

Integral routine

Key problem: $V(p,q)$ will have a pole of integral variable q when p changed.

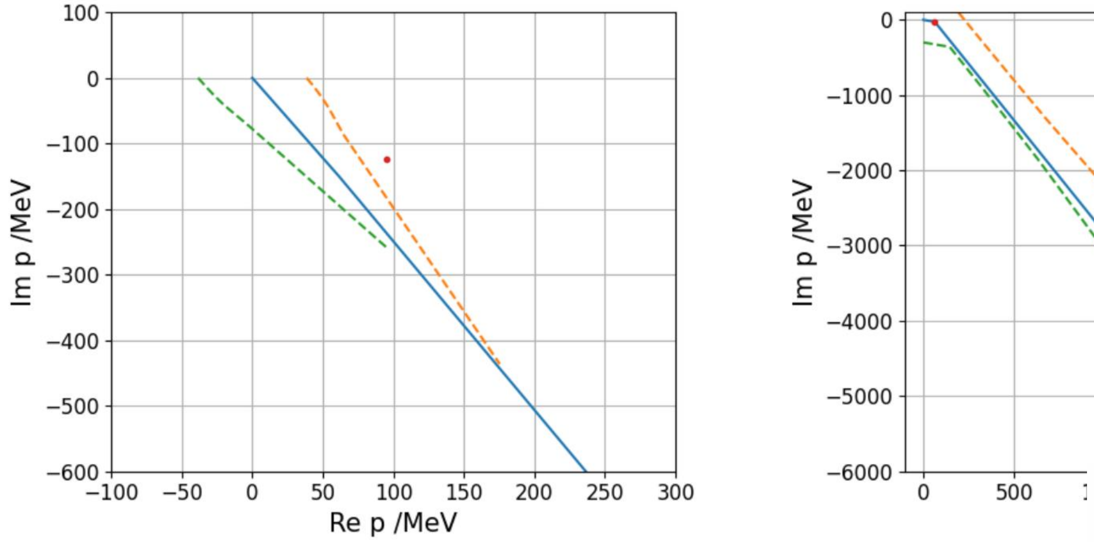


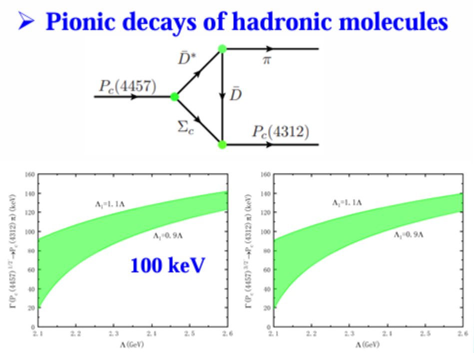
Figure 2: Two paths of integrate momenta for $\pi^0 P_c(4312)$(left)

Table 1. The pole position of T−matrix in the complex plain for different cutoffs.

	M_{P_c}/ MeV	$\Gamma_{P_c}/2$ MeV
$\Lambda = 0.8$ GeV	4456.7428	10.7337
$\Lambda = 1.0$ GeV	4456.7667	10.7293
$\Lambda = 1.2$ GeV	4456.7861	10.7238

the second Riemann sheet of $P_c(4312)\pi$
the first Riemann sheet of $\bar{D}\Lambda_{c1}(2595)$

A bound state of $\bar{D}\Lambda_{c1}(2595)$ with $J^P = \frac{1}{2}^+$

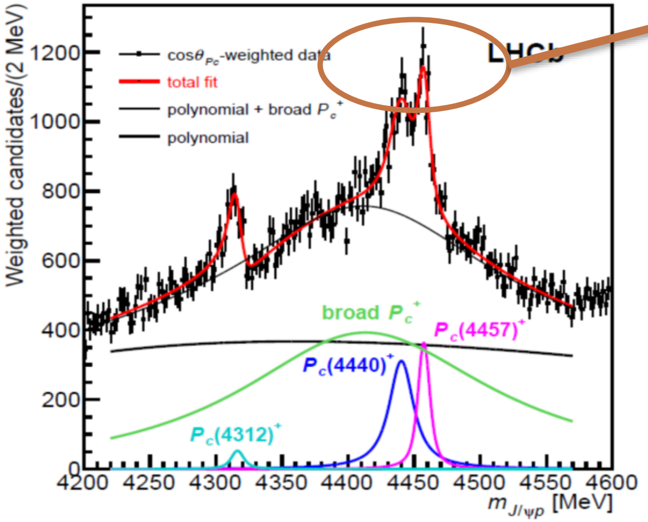


Typical property: **Large decay width to $P_c(4312)\pi$. While for $\frac{1}{2}^-$ state, the width of $Pc(4457) \rightarrow P_c(4312)\pi$ is only 100 keV.**

Minzhu Liu’ s talk at第九届手征有效场论研讨会

Discussion

$\Lambda_b^0 \rightarrow K^- P_c(4457) \rightarrow K^- J/\psi P$ √



How many states here ?
Maybe more !

Possible new processes

$$\Lambda_b^0 \rightarrow K^- P_c(4457) \rightarrow K^- P_c(4312)\pi^0 \rightarrow K^- J/\psi P \pi^0$$

$$\Lambda_b^0 \rightarrow K_s P_c^0(4457) \rightarrow K_s P_c(4312)\pi^- \rightarrow K_s J/\psi P \pi^-$$

$$\Lambda_b^0 \rightarrow K^- J/\psi P \pi^+ \pi^-$$

小结和展望

- 目前，耦合道模型是描述强子最佳方法。
- 基于DD*的相互作用，来理解$T_{cc}-X(3872)-Z_c(3900)$，试图回答它们是什么
- 利用 $\bar{D}\Lambda_{c1}(2595)-P_c(4312)\pi$耦合道对$P_c(4457)$提出新见解。

谢谢