开味四夸克态$T_{c\bar{s}0}$(2900)及其自旋伴随态的理论研究

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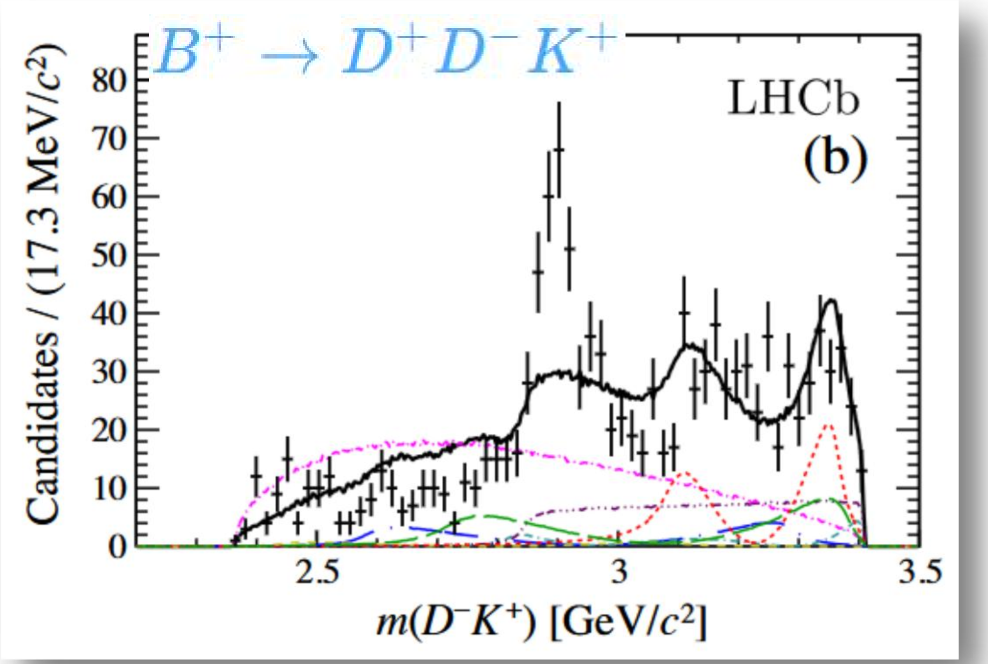
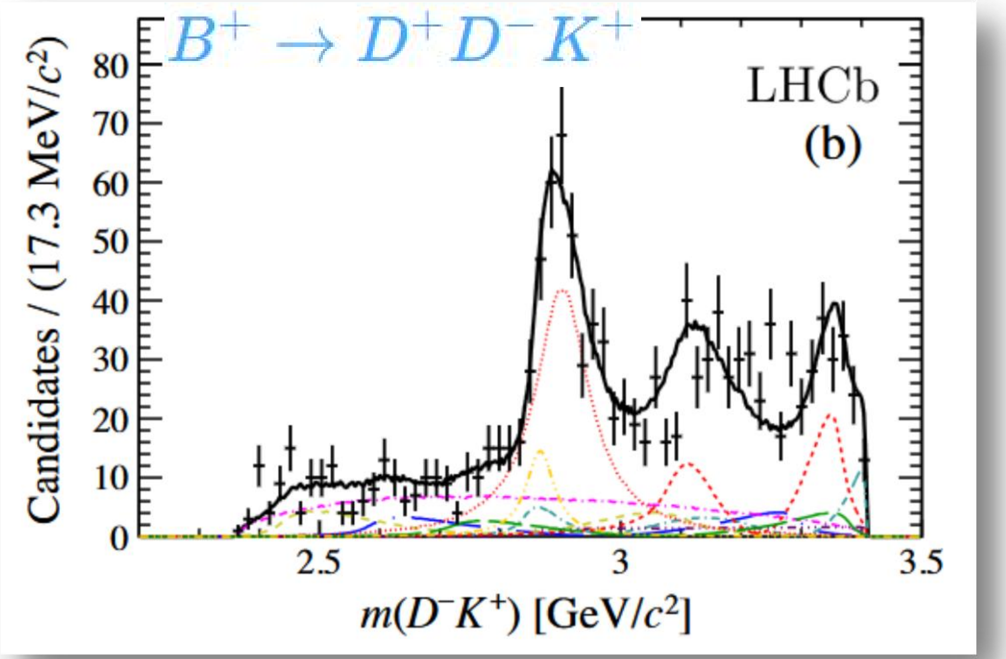
$X_{0,1}(2900)$ at LHCb

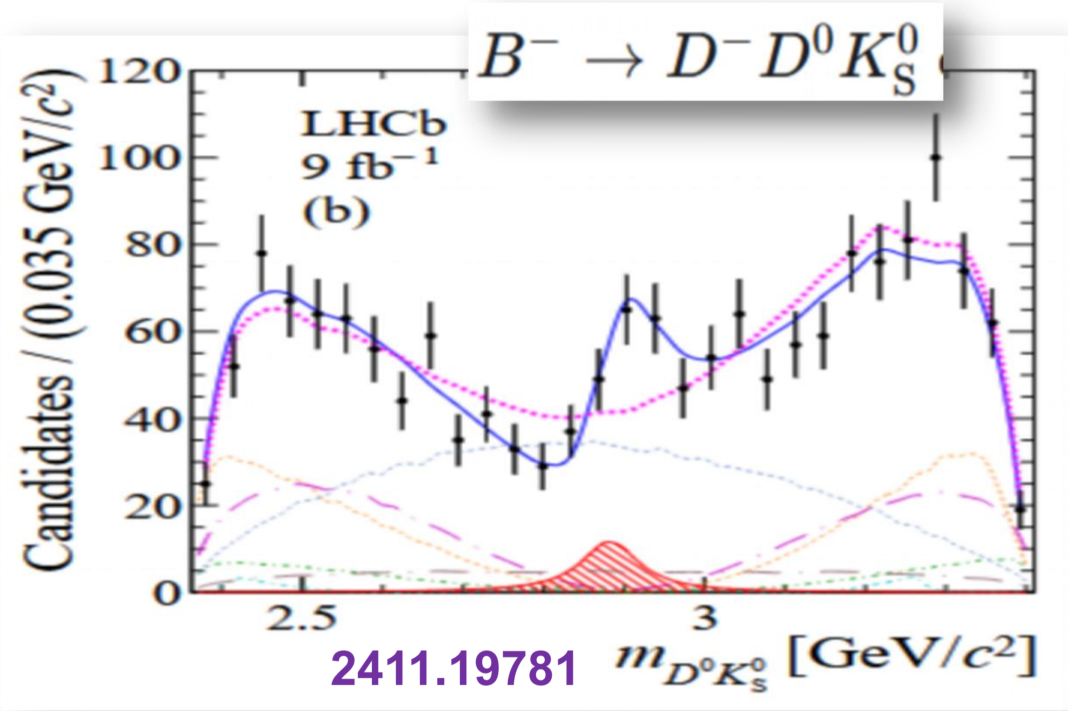




$X_0(2900)$	$2.866 \pm 0.007 \pm 0.002$	$57 \pm 12 \pm 4$
$X_1(2900)$	$2.904 \pm 0.005 \pm 0.001$	$110 \pm 11 \pm 4$

PRL125(2020)242001
PRD102(2020)112003

$X_{0,1}(2900)$ (T_{cs}) at LHCb



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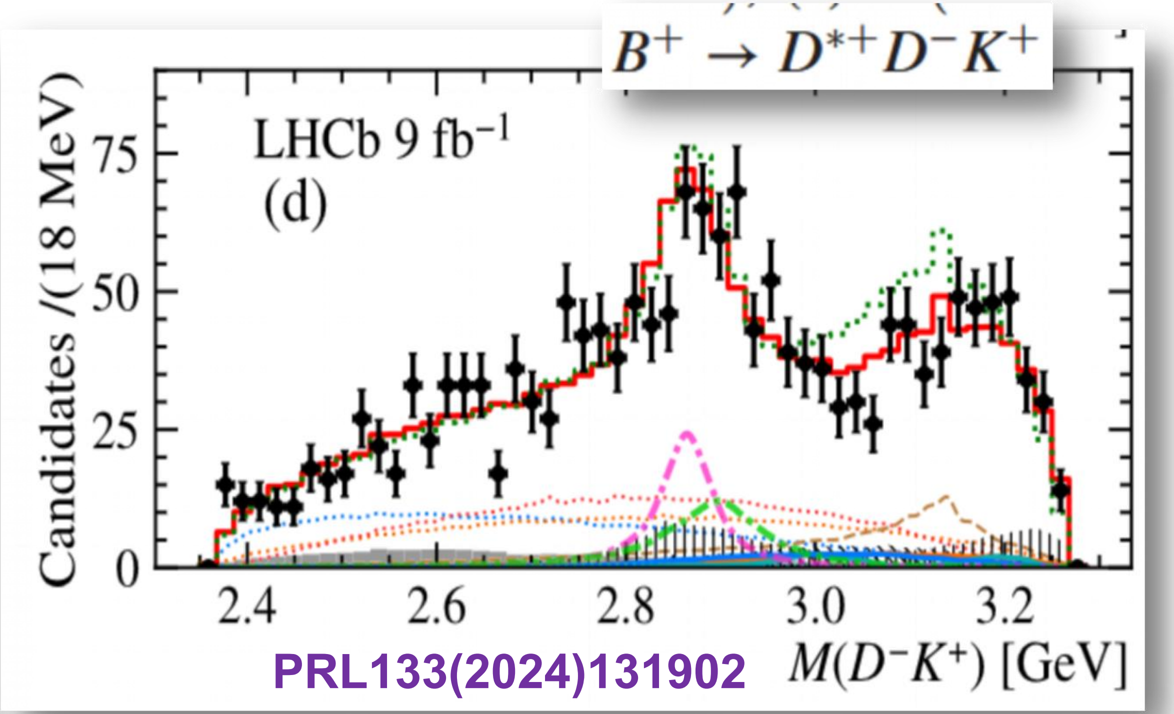
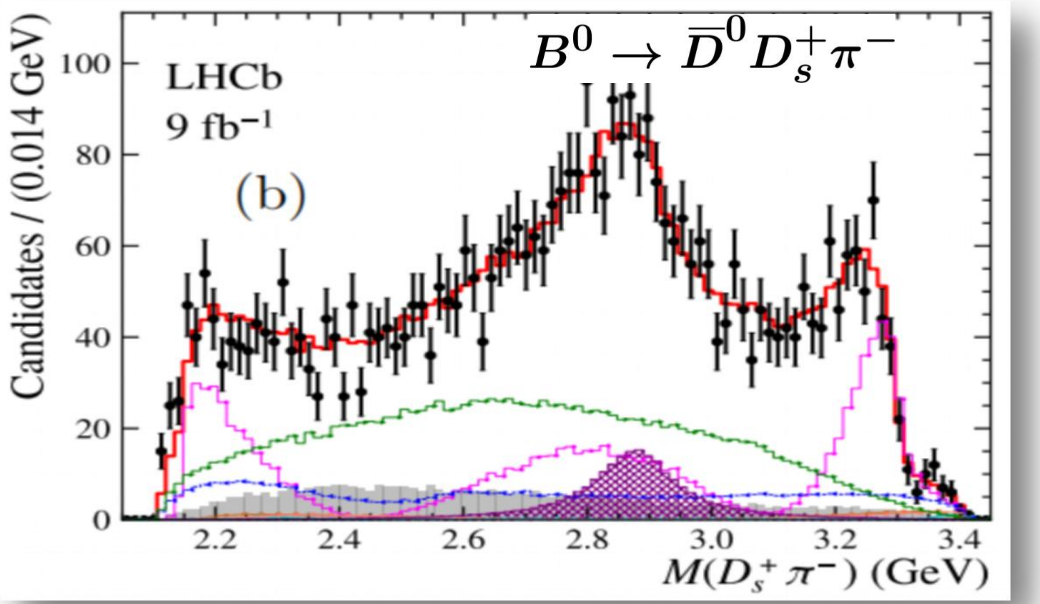


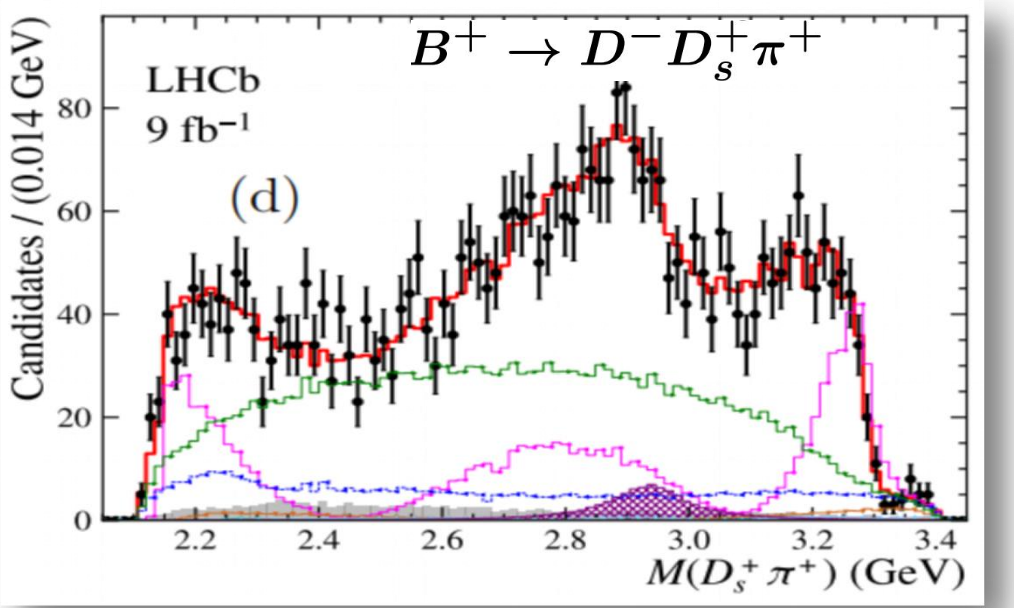
PRL133(2024)131902

$$M(T^{*0}_{cs0}) = 2883 \pm 11 \pm 7\,\mathrm{MeV}/c^2,$$
$$\Gamma(T^{*0}_{cs0}) = 87^{+22}_{-47} \pm 6\,\mathrm{MeV},$$
$$\mathrm{FF}(T^{*0}_{cs0} \to D^0 K^0_S) = (2.6 \pm 1.2 \pm 0.2)\%,$$

	This work	Previous work
$T^*_{\bar{c}\bar{s}0}(2870)^0$, mass [MeV]	$2914 \pm 11 \pm 15$	2866 ± 7
$T^*_{\bar{c}\bar{s}0}(2870)^0$, width [MeV]	$128 \pm 22 \pm 23$	57 ± 13
$T^*_{\bar{c}\bar{s}1}(2900)^0$ mass [MeV]	$2887 \pm 8 \pm 6$	2904 ± 5
$T^*_{\bar{c}\bar{s}1}(2900)^0$ width [MeV]	$92 \pm 16 \pm 16$	110 ± 12

$T_{c\bar{s}0}(2900)$ at LHCb





$$m_{T_{c\bar{s}0}(2900)^0} = (2892 \pm 14 \pm 15)\ \text{MeV},$$
$$\Gamma_{T_{c\bar{s}0}(2900)^0} = (119 \pm 26 \pm 13)\ \text{MeV},$$

$$m_{T_{c\bar{s}0}(2900)^{++}} = (2921 \pm 17 \pm 20)\ \text{MeV},$$
$$\Gamma_{T_{c\bar{s}0}(2900)^{++}} = (137 \pm 32 \pm 17)\ \text{MeV},$$

$$m_{T_{c\bar{s}0}(2900)} = (2908 \pm 11 \pm 20)\ \text{MeV},$$
$$\Gamma_{T_{c\bar{s}0}(2900)} = (136 \pm 23 \pm 13)\ \text{MeV}.$$

PRL131(2023)041902
PRD108(2023)012017

V-V interaction

□ Hidden-gauge formalism Molina-Branz-Oset, PRD82(2010)014010

$$\mathcal{L} = -\frac{1}{4}\langle \bar{V}_{\mu\nu}\bar{V}^{\mu\nu}\rangle + \frac{1}{2}M_v^2\langle [V_\mu - (i/g)\Gamma_\mu]^2\rangle, \qquad g = \frac{M_V}{2f},$$

$$\bar{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu],$$

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^\dagger\},$$

□ *V/P* mesons in SU(4) flavor space

$$u^2 = U = \exp(\frac{i\sqrt{2}\Phi}{f})$$

$$V_\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu . \qquad \Phi = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}.$$

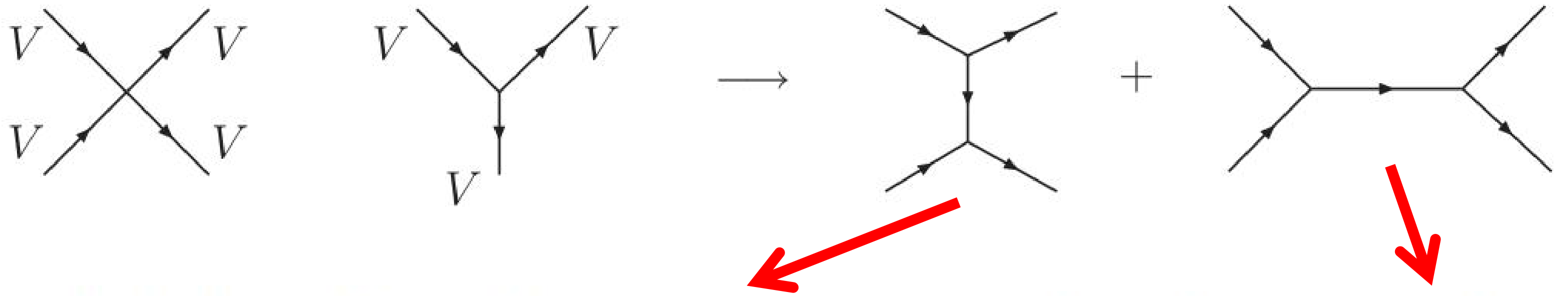
V-V interaction

□The Lagrangian of four-vector and three-vector contact terms

$$\mathcal{L}_{\mathrm{VVVV}} = \tfrac{1}{2} g^2 \langle [V_\mu, V_\nu] V^\mu V^\nu \rangle,$$

$$\begin{aligned}\mathcal{L}_{VVV} &= ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle \\ &= ig\langle V^\mu \partial_\nu V_\mu V^\nu - \partial_\nu V_\mu V^\mu V^\nu \rangle \\ &= ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle\rangle.\end{aligned}$$

$$\vec{k}/M_V \sim 0,$$



$$(k_1 + k_3)\cdot(k_2 + k_4)\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4, \qquad (k_1 + k_4)\cdot(k_2 + k_3)\epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3.$$

V-V interaction

- spin-projection operators

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu$$

$$\mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu)$$

$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3}\epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu\}.$$

$$(k_1 + k_3) \cdot (k_2 + k_4)\epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4,$$

$$(k_1 + k_4) \cdot (k_2 + k_3)\epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3.$$

$$(k_1 + k_3) \cdot (k_2 + k_4) \quad \text{for } J = 0, 1, 2,$$

$$(k_1 + k_4) \cdot (k_2 + k_3) \quad \text{for } J = 0, 2,$$

$$-(k_1 + k_4) \cdot (k_2 + k_3) \quad \text{for } J = 1,$$

$$k_1 \cdot k_2 = \frac{s - M_1^2 - M_2^2}{2}$$

$$k_1 \cdot k_3 = k_1^0 k_3^0 - \vec{p} \cdot \vec{q} \to \frac{(s + M_1^2 - M_2^2)(s + M_3^2 - M_4^2)}{4s},$$

Bethe-Salpeter equation

□Bethe-Salpeter equation

$$T = (\hat{1} - VG)^{-1}V.$$

$$G_i = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - M_1^2 + i\epsilon}\frac{1}{(P-q)^2 - M_2^2 + i\epsilon},$$

$$G_i = \frac{1}{16\pi^2}\left(\alpha + \log\frac{M_1^2}{\mu^2} + \frac{M_2^2 - M_1^2 + s}{2s}\log\frac{M_2^2}{M_1^2} + \frac{p}{\sqrt{s}}\left(\log\frac{s - M_2^2 + M_1^2 + 2p\sqrt{s}}{-s + M_2^2 - M_1^2 + 2p\sqrt{s}} + \log\frac{s + M_2^2 - M_1^2 + 2p\sqrt{s}}{-s - M_2^2 + M_1^2 + 2p\sqrt{s}}\right)\right),$$

$$G_i = \int_0^{q_{\max}} \frac{q^2 dq}{(2\pi)^2}\frac{\omega_1 + \omega_2}{\omega_1\omega_2[(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon]},$$

$$G_i^{II}(\sqrt{s}) = G_i^{I}(\sqrt{s}) + i\frac{p}{4\pi\sqrt{s}} \qquad \mathrm{Im}(p) > 0,$$

$$\tilde{G}(s) = \frac{1}{N}\int_{(M_1-2\Gamma_1)^2}^{(M_1+2\Gamma_1)^2} d\tilde{m}_1^2\left(-\frac{1}{\pi}\right)\mathrm{Im}\frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} \times G(s, \tilde{m}_1^2, M_2^2), \qquad (21)$$

$$N = \int_{(M_1-2\Gamma_1)^2}^{(M_1+2\Gamma_1)^2} d\tilde{m}_1^2\left(-\frac{1}{\pi}\right)\mathrm{Im}\frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1},$$

$$q_{\max} = 1.2\ \mathrm{GeV}$$

Pole and coupling

TABLE II. $C = 1$; $S = 1$; $I = 0$. Quantum numbers, pole positions, and couplings g_i in units of MeV for $I = 0$. Here $\alpha = -1.6$.

$I[J^P]$	$\sqrt{s}_{\text{pole}}$ (MeV)	$g_{D^*K^*}$	$g_{D_s^*\omega}$	$g_{D_s^*\phi}$
$0[0^+]$	2683	15 635	−4035	6074
$0[1^+]$	2707	14 902	−5047	4788
$0[2^+]$	2572	18 252	−7597	7257

$X_0(2900)$	$2.866 \pm 0.007 \pm 0.002$
$X_1(2900)$	$2.904 \pm 0.005 \pm 0.001$

Molina-Branz-Oset, PRD82(2010)014010

2 (see Table XIV). In fact, we only obtain a pole for $J = 2$. For $J = 0$ and 1 we only observe a cusp in the $D_s^*\rho$ threshold. In Table III we show the pole position and

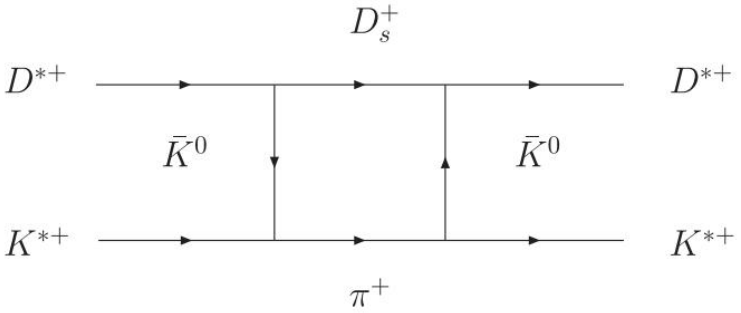
$$m_{T_{c\bar{s}0}(2900)} = (2908 \pm 11 \pm 20)\ \text{MeV},$$
$$\Gamma_{T_{c\bar{s}0}(2900)} = (136 \pm 23 \pm 13)\ \text{MeV}.$$

TABLE III. $C = 1$; $S = 1$; $I = 1$. Quantum numbers, pole positions, and couplings g_i in units of MeV. Here $\alpha = -1.6$.

$I^G[J^{PC}]$	$\sqrt{s}_{\text{pole}}$ (MeV)	$g_{D^*K^*}$	$g_{D_s^*\rho}$
$1[2^+]$	2786	11 041	11 092

Spin partner $T_{c\bar{s}2}$

◻ Molina-Oset, PRD107(2023)056015



$I[J^P]$	$\sqrt{s_0}$	Γ_0	Experiment
$1[0^+]$	2920 (Cusp)	130	$m = 2908 \pm 11 \pm 20$ $\Gamma = 136 \pm 23 \pm 11$
$1[1^+]$	2923 (Cusp)	145	...
$1[2^+]$	2834	19	...

◻ Du-Guo-Chen-Wang, PRD108(2023)074006

TABLE II. The pole positions and effective couplings evaluated for $I = 1$, $J = 0$ on different RSs with $\mu = 1500$ MeV. The threshold of $D_s^*\rho$ is 2887 MeV.

RS	α	$\sqrt{s}_{\text{pole}}$ [MeV]	$\lvert g_{D^*K^*}\rvert$ [MeV]	$\lvert g_{D_s^*\rho}\rvert$ [MeV]
{1, 1}	$-1.65 \sim -1.60$	$2885 \sim 2887$	$5531 \sim 2198$	$5379 \sim 2082$
{2, 1}	$-1.60 \sim -1.55$	$2887 \sim 2885$	$1755 \sim 8202$	$1650 \sim 7348$
{1, 2}	$-1.39 \sim -1.35$	$2885 \sim 2887$	$6587 \sim 1625$	$7886 \sim 1865$
{2, 2}	$-1.35 \sim -1.28$	$2887 \sim 2885$	$1415 \sim 4202$	$1613 \sim 4672$

TABLE III. The pole positions evaluated in the sectors of $I = 1$, $J = 1$ and $I = 1$, $J = 2$ on the different RSs with $-1.65 < \alpha < -1.55$ and $-1.39 < \alpha < -1.28$. The "—" indicates that no pole is found. In the present work, we only consider the energy region safe from the left-hand cut, i.e., $\sqrt{s} > 2780$ MeV.

	$I = 1, J = 1$		$I = 1, J = 2$	
RS	α	$\sqrt{s}_{\text{pole}}$ [MeV]	α	$\sqrt{s}_{\text{pole}}$ [MeV]
{1, 1}	$-1.65 \sim -1.61$	$2886 \sim 2887$	$-1.31 \sim -1.28$	$2780 \sim 2806$
{2, 1}	$-1.61 \sim -1.55$	$2887 \sim 2883$	...	...
{1, 2}	$-1.39 \sim -1.36$	$2886 \sim 2887$	...	...
{2, 2}	$-1.36 \sim -1.28$	$2887 \sim 2885$	...	...

Spin partner $T_{c\bar{s}2}$ with mass around 2.8GeV was predicted!

Interpretation of $T_{c\bar{s}0}(2900)$

□Virtual state or cusp

- PRD107(2023)056015, PRD108(2023)074006

□Compact tetraquark

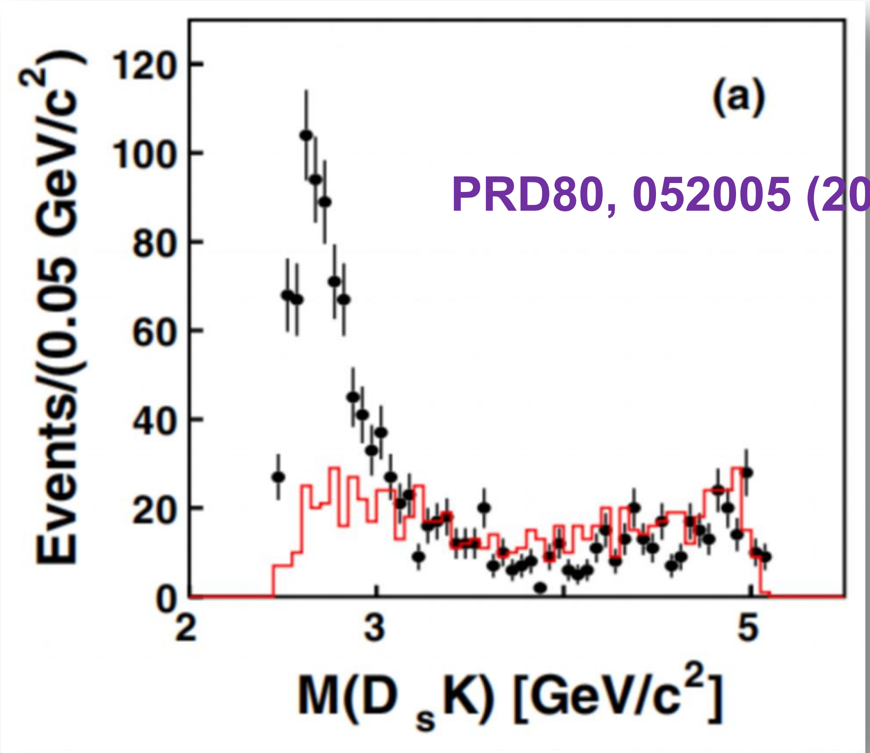
- IJMPA38(2023)2350056, PRD108(2023)114016
- EPJC84(2024)1
- Symmetry 2023, 15, 695

□Searching for the $T_{c\bar{s}0}(2900)$

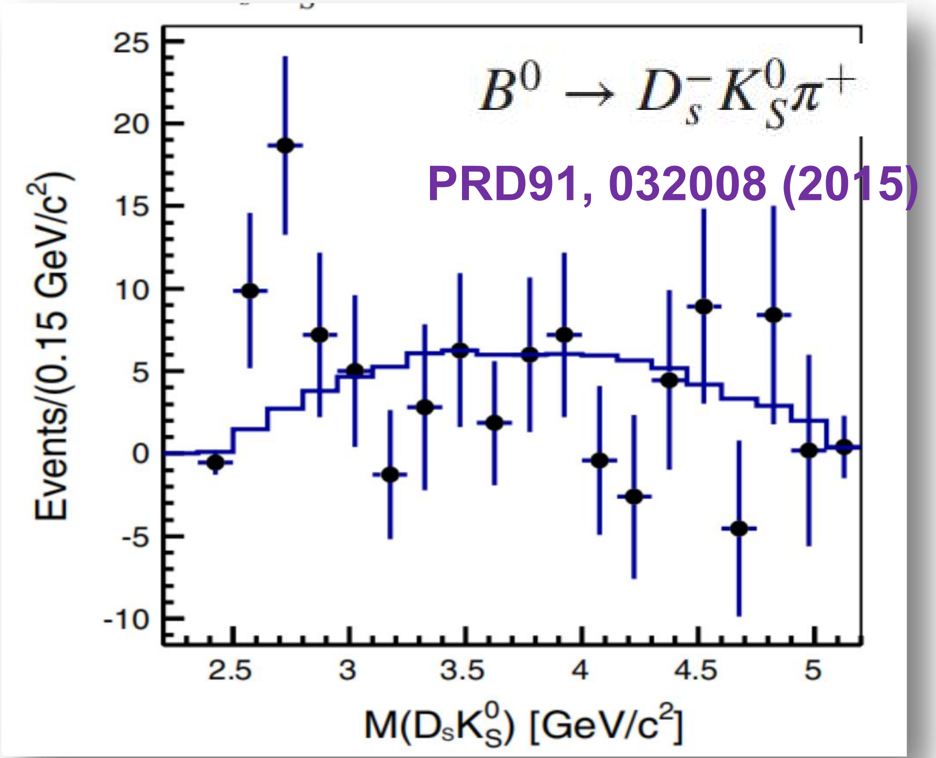
- $\Lambda_b \to \Lambda D^0 K^0$, 2310.11139
- $\bar{B}_s \to K^0 D^0 \pi^0$, 2408.11454
- $B \to D_s^+ \pi^- K^-$, PRD109(2024)014008
- $B^+ \to K^+ D^+ D^-$ EPJC84(2024)681

$B^- \to D_s^+ K^- \pi^-$

□ Belle data



PRD80, 052005 (2009)



PRD91, 032008 (2015)

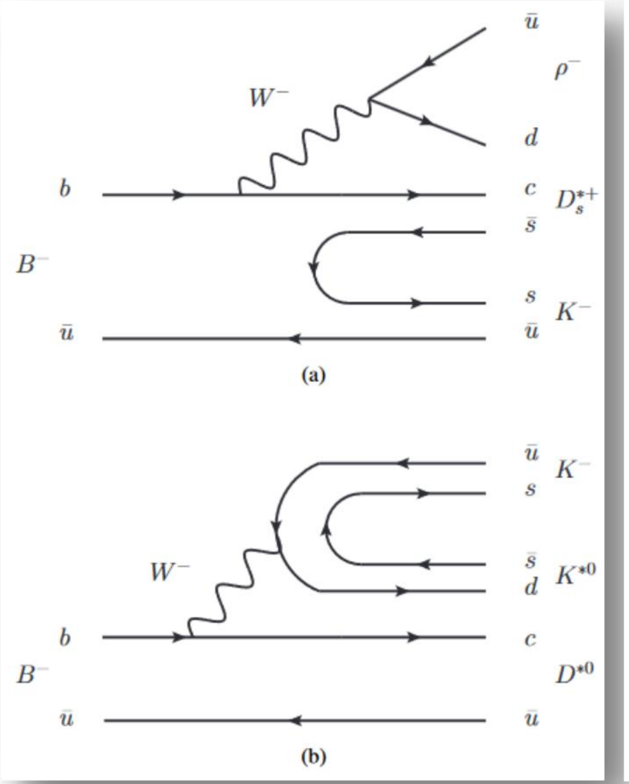
$$\mathcal{B}(B^+ \to D_s^- K^+ \pi^+) = (1.71^{+0.08}_{-0.07}(\text{stat})^{+0.20}_{-0.20}(\text{syst}) \pm 0.15(\mathcal{B}_{\text{int}})) \times 10^{-4},$$

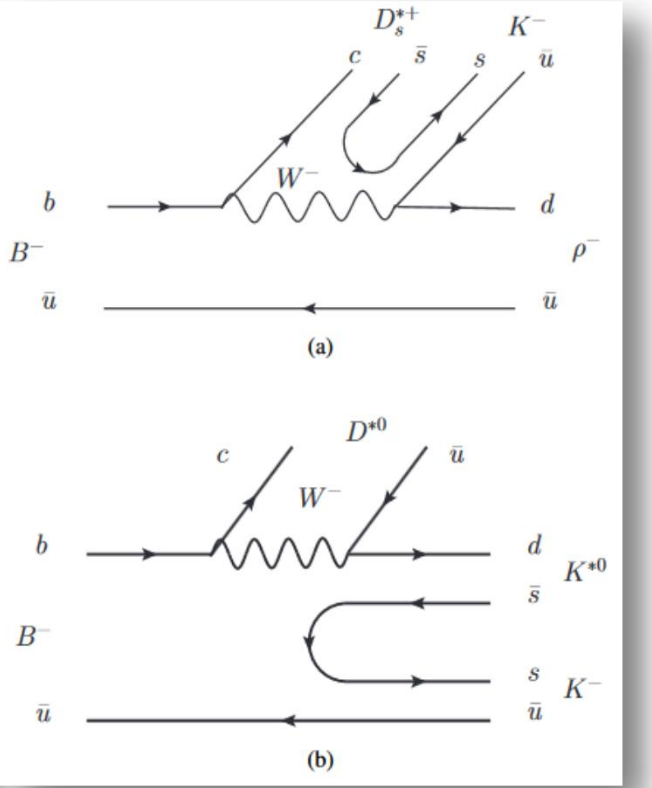
$$\mathcal{B}(B^0 \to D_s^- K_S^0 \pi^+) = [0.47 \pm 0.06(\text{stat}) \pm 0.05(\text{syst})] \times 10^{-4}$$

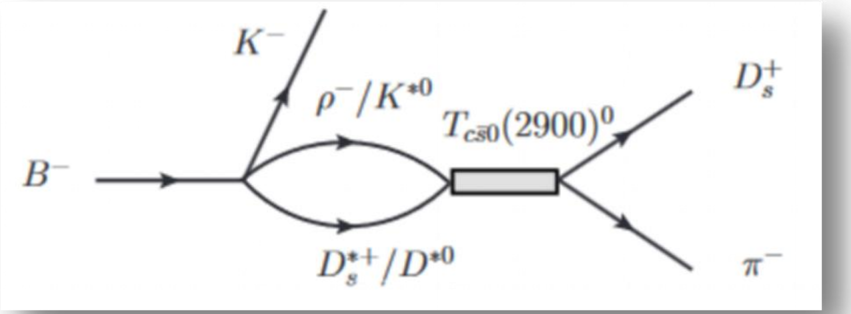
$B^- \to D_s^+ K^- \pi^-$

PRD109(2024)014008

□Mechanisms







$$\tilde{\mathcal{T}} = Q(C+1)\vec{\epsilon}(V_1)\cdot\vec{\epsilon}(V_2),$$

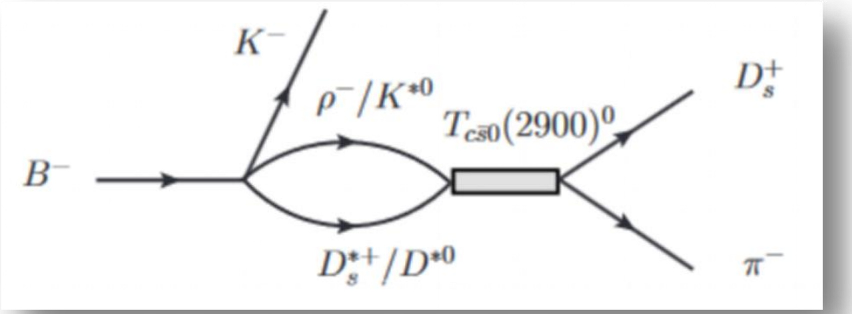
$$Q^2 \approx \frac{\Gamma_{B^-}\mathcal{B}(B^- \to D^{*0}K^{*0}K^-)}{\int \frac{3}{(2\pi)^3}\frac{(C+1)^2}{4M_{B^-}^2} p_{K^*}\tilde{p}_K dM_{\rm inv}(D^{*0}K^{*0})}.$$

$$\mathcal{B}(B^- \to D^{*0}K^{*0}K^-) = (1.5 \pm 0.4)\times 10^{-3}$$

$$\mathcal{T}^{T^0_{c\bar{s}0}} = Q\vec{\epsilon}(V_1)\cdot\vec{\epsilon}(V_2) \times(C+1)\left[G_{\rho^- D_s^{*+}} t_{\rho^- D_s^{*+}\to D_s^+\pi^-} + G_{D^{*0}K^{*0}} t_{D^{*0}K^{*0}\to D_s^+\pi^-}\right],$$

coupling constants

□Transition amplitudes



$$t_{\rho^- D_s^{*+}\to D_s^+\pi^-} = \frac{g_{T^0_{c\bar{s}0},\rho^- D_s^{*+}}\, g_{T^0_{c\bar{s}0},D_s^+\pi^-}}{M^2_{D_s^+\pi^-} - m^2_{T^0_{c\bar{s}0}} + i m_{T^0_{c\bar{s}0}}\Gamma_{T^0_{c\bar{s}0}}},$$

$$t_{D^{*0}K^{*0}\to D_s^+\pi^-} = \frac{g_{T^0_{c\bar{s}0},D^{*0}K^{*0}}\, g_{T^0_{c\bar{s}0},D_s^+\pi^-}}{M^2_{D_s^+\pi^-} - m^2_{T^0_{c\bar{s}0}} + i m_{T^0_{c\bar{s}0}}\Gamma_{T^0_{c\bar{s}0}}},$$

$$g^2_{T^0_{c\bar{s}0},D^*K^*} = 16\pi(m_{D^*} + m_{K^*})^2\tilde{\lambda}^2\sqrt{\frac{2\Delta E}{\mu}},$$

$$\Gamma_{T^0_{c\bar{s}0}\to\rho^- D_s^{*+}} = \frac{3}{8\pi}\frac{1}{m^2_{T^0_{c\bar{s}0}}}|g_{T^0_{c\bar{s}0},\rho^- D_s^{*+}}|^2|\vec{q}_\rho|, \qquad \Gamma_{T^0_{c\bar{s}0}\to\rho^- D_s^{*+}} = 4.13$$

$$\Gamma_{T^0_{c\bar{s}0}\to D_s^+\pi^-} = \frac{1}{8\pi}\frac{1}{m^2_{T^0_{c\bar{s}0}}}|g_{T^0_{c\bar{s}0},D_s^+\pi^-}|^2|\vec{q}_{\pi^-}|, \qquad \Gamma_{T^0_{c\bar{s}0}\to D_s^+\pi^-} = 4.45\ \text{MeV}$$

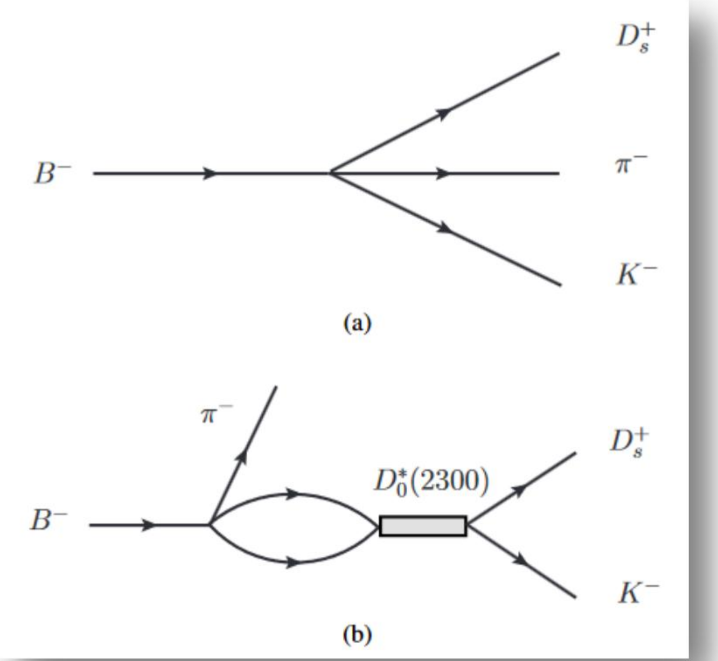
DYChen, PRD107, 034018 (2023)

$\lambda = 1$ gives the probability to find the molecular component in the physical states

S. Weinberg, PR137, B672 (1965)
Baru, PLB 586, 53-61 (2004)

$D(2300)$-D_sK interaction

□Mechanisms

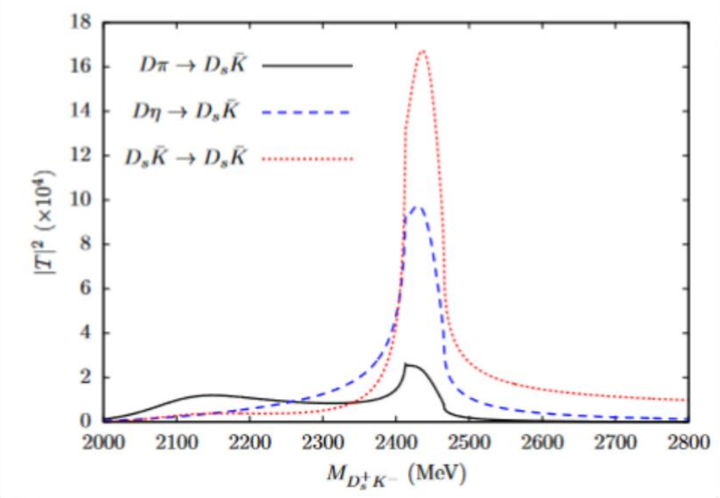


$$T = [1 - VG]^{-1} V.$$

$$\mathcal{T}^{D_0^*(2300)} = Q'(C+1)(h_{D_s\bar{K}} + \sum_i h_i G_i t_{i\to D_s\bar{K}}) = \mathcal{T}^{\text{tree}} + \mathcal{T}^{S},$$

$$\mathcal{B}(B^- \to D_s^+ K^- \pi^-) = (1.80 \pm 0.22) \times 10^{-4}$$

$$\Gamma_{B^-}\mathcal{B}(B^- \to D_s^+ K^- \pi^-) = Q'^2 \int \frac{1}{(2\pi)^3} \frac{(C+1)^2}{4M_{B^-}^2} p_\pi \tilde{p}_K \times |h_{D_s\bar{K}} + \sum_i h_i G_i t_{i\to D_s\bar{K}}|^2 dM_{\text{inv}}(D_s^+ K^-).$$



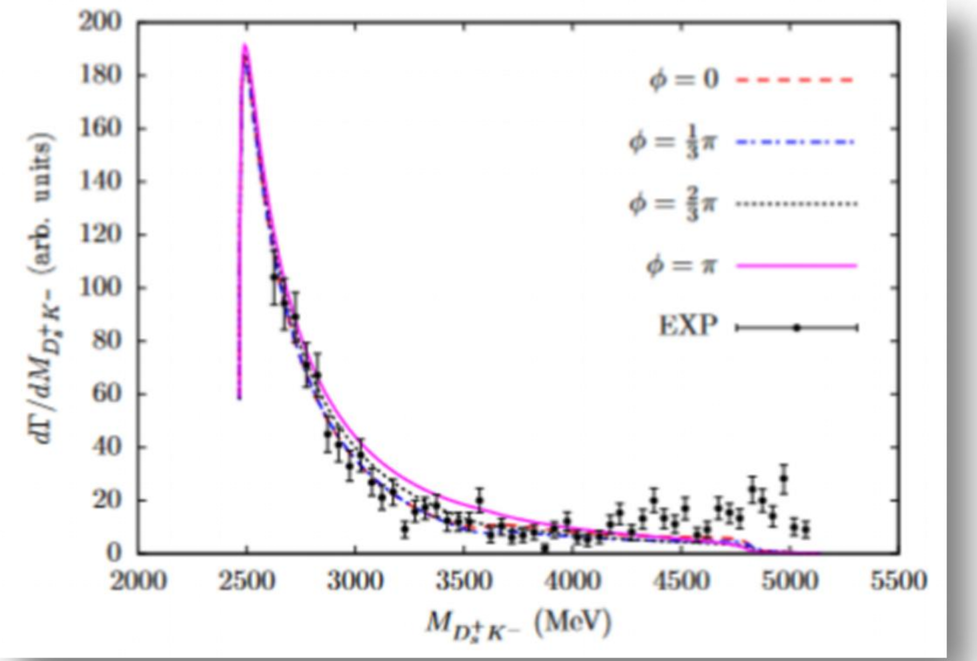
$$\left|\mathcal{T}^{\text{total}}\right|^2 = \left|\mathcal{T}^{T^0_{c\bar{s}0}} e^{i\phi} + \mathcal{T}^{D_0^*(2300)}\right|^2,$$

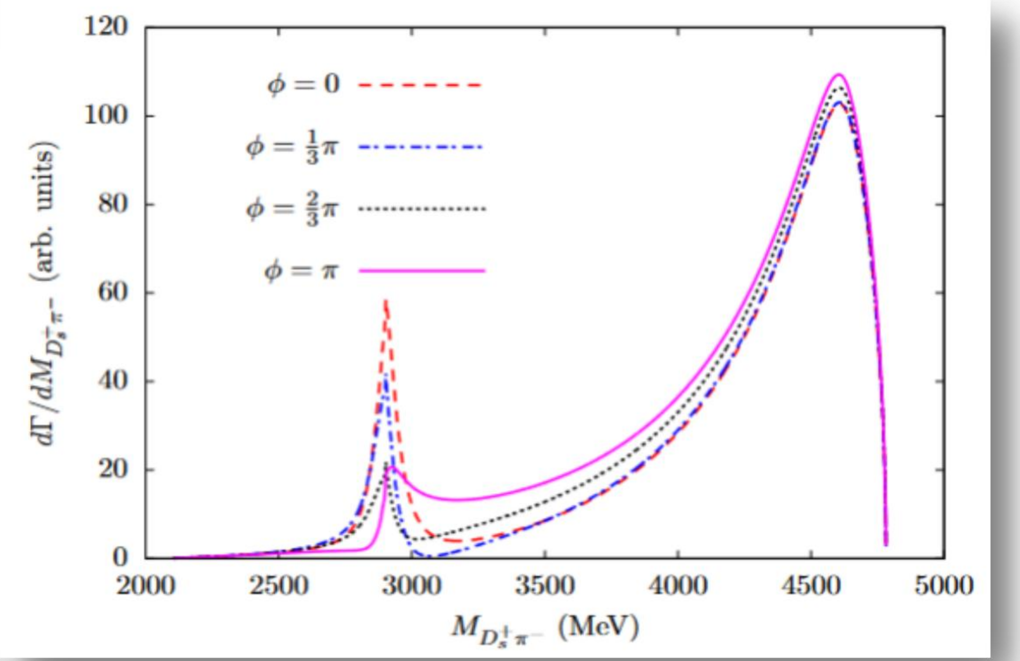
$$\frac{d^2\Gamma}{dM_{D_s^+K^-} dM_{D_s^+\pi^-}} = \frac{1}{(2\pi)^3} \frac{2M_{D_s^+K^-} 2M_{D_s^+\pi^-}}{32 M_{B^-}^3} |\mathcal{T}^{\text{total}}|^2,$$

Phys. Rev. D 102, 096020 (2020)
Phys. Rev. D 87, 014508 (2013)

Results for $B^- \rightarrow D_s^+ K^- \pi^-$

□Mass distributions



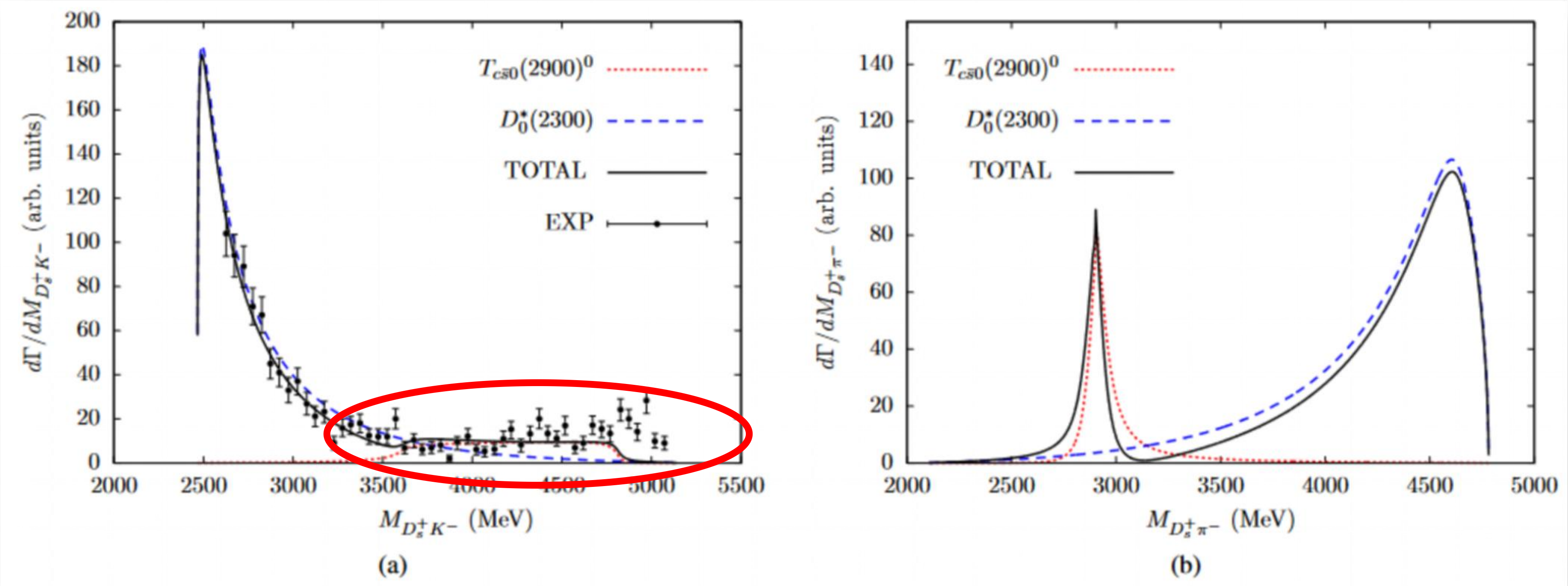


Belle: PRD80 (2009)052005

$$\left|\mathcal{T}^{\text{total}}\right|^2 = \left|\mathcal{T}^{T^0_{c\bar{s}0}} e^{i\phi} + \mathcal{T}^{D_0^*(2300)}\right|^2,$$

Results with fitted parameters

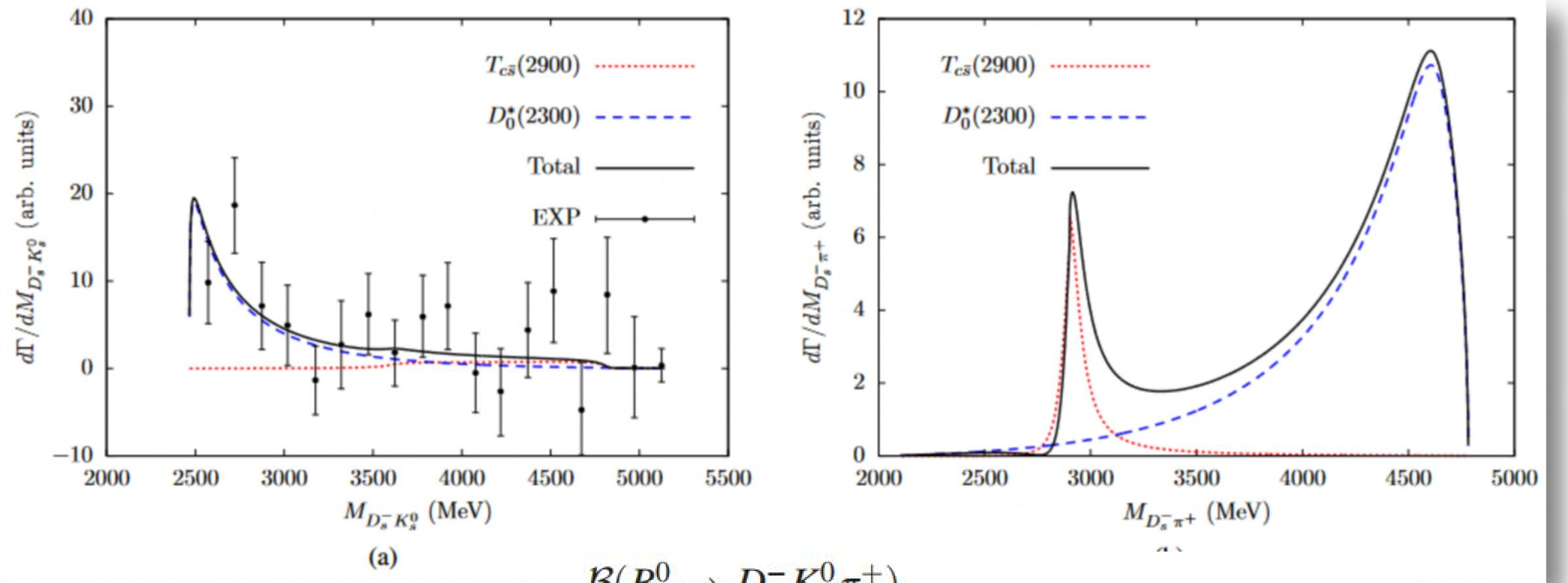
☐Fitted results



$$\Gamma_{T^0_{c\bar{s}0}\to D_s^+\pi^-} = 10.45\ \text{MeV and } \phi = 0.35\pi,$$

Results-Fitted parameters

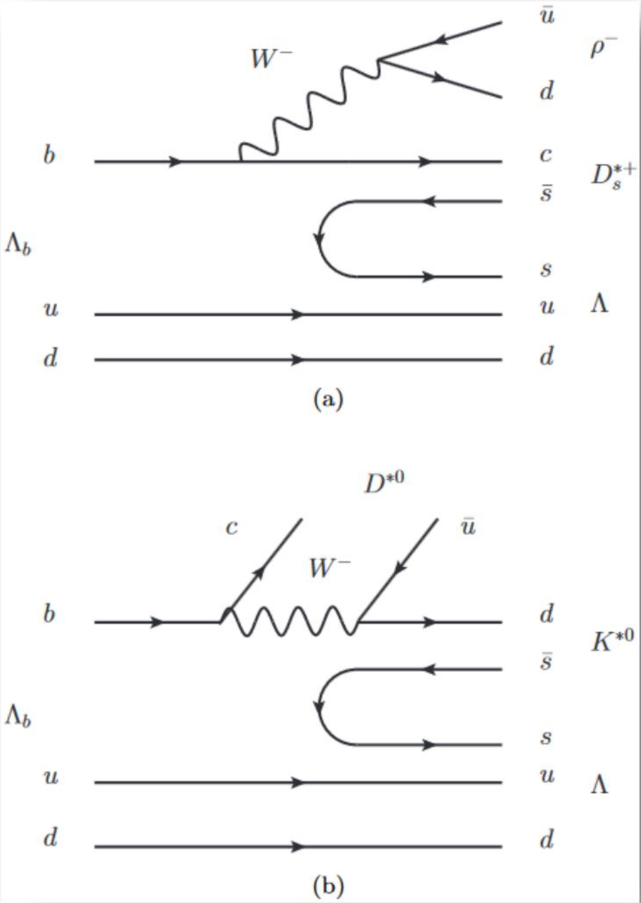
□ **Belle:** $B^0 \to D_s^- K_S^0 \pi^+$, PRD91, 032008(2015)



$$\mathcal{B}(B^0 \to D_s^- K_S^0 \pi^+) = [0.47 \pm 0.06(\text{stat}) \pm 0.05(\text{syst})] \times 10^{-4}$$

$\Lambda_b \to K^0 D^0 \Lambda$ 2310.11139, EPJC

□Mechanisms



$$
\begin{aligned}
|\Lambda_b\rangle &= \frac{1}{\sqrt{2}} b\,(ud - du) \\
&\Rightarrow W^- c \frac{1}{\sqrt{2}} (ud - du) \\
&\Rightarrow \bar{u} d c\,(\bar{s}s) \frac{1}{\sqrt{2}} (ud - du) \\
&\Rightarrow \rho^- c\,(\bar{s}s) \frac{1}{\sqrt{2}} (ud - du) \\
&\Rightarrow \rho^- D_s^{*+} \frac{1}{\sqrt{2}} s\,(ud - du) \\
&\Rightarrow -\sqrt{\frac{2}{3}} \rho^- D_s^{*+} \Lambda,
\end{aligned}
$$

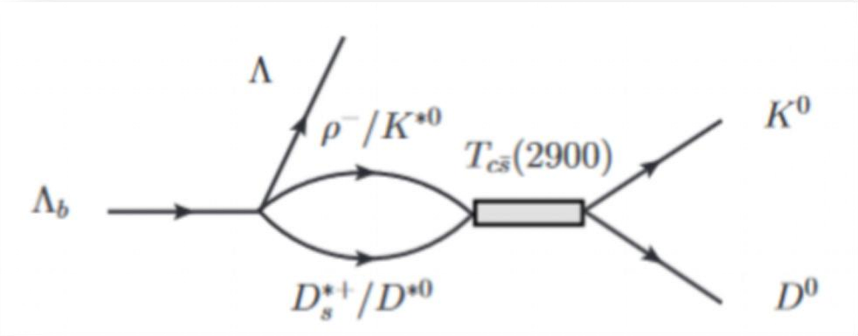
$$
\begin{aligned}
|\Lambda_b\rangle &= \frac{1}{\sqrt{2}} b\,(ud - du) \\
&\Rightarrow c W^- \frac{1}{\sqrt{2}} (ud - du) \\
&\Rightarrow c \bar{u} d\,(\bar{s}s) \frac{1}{\sqrt{2}} (ud - du) \\
&\Rightarrow -\sqrt{\frac{2}{3}} D^{*0} K^{*0} \Lambda.
\end{aligned}
$$

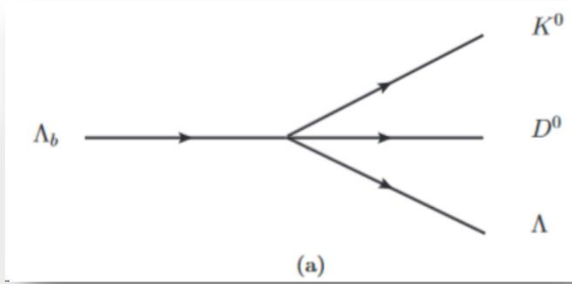
$$
\Lambda_b = \frac{1}{\sqrt{2}} |b(ud - du)\rangle\,,
$$

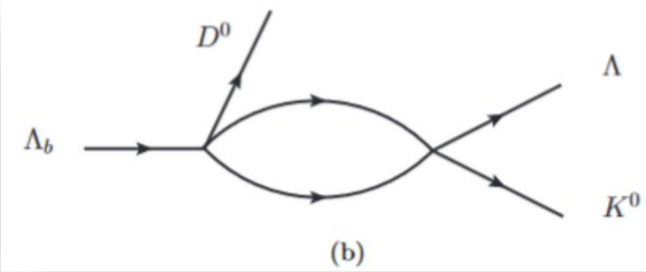
$$
\Lambda = \frac{1}{\sqrt{12}} |u(ds - sd) + d(su - us) - 2s(ud - du)\rangle
$$

Formalism

□Final state interaction







$$\mathcal{T}^{T_{c\bar{s}}} = -\sqrt{\frac{2}{3}} V_p \Big[C \times G_{D_s^{*+}\rho^-} t_{D_s^{*+}\rho^- \to D^0 K^0} + G_{D^{*0}K^{*0}} t_{D^{*0}K^{*0} \to D^0 K^0} \Big],$$

$$t_{D_s^{*+}\rho^- \to D^0 K^0} = \frac{g_{T_{c\bar{s}},D_s^{*+}\rho^-} g_{T_{c\bar{s}},D^0 K^0}}{M^2_{D^0 K^0} - m^2_{T_{c\bar{s}}} + i m_{T_{c\bar{s}}} \Gamma_{T_{c\bar{s}}}},$$

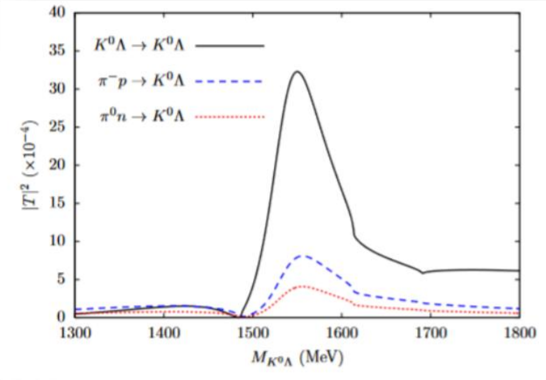
$$t_{D^{*0}K^{*0} \to D^0 K^0} = \frac{g_{T_{c\bar{s}},D^{*0}K^{*0}} g_{T_{c\bar{s}},D^0 K^0}}{M^2_{D^0 K^0} - m^2_{T_{c\bar{s}}} + i m_{T_{c\bar{s}}} \Gamma_{T_{c\bar{s}}}},$$

$$|H> = \pi^- p - \frac{1}{\sqrt{2}} \pi^0 n + \frac{1}{\sqrt{3}} \eta n - \sqrt{\frac{2}{3}} K^0 \Lambda,$$

$$\mathcal{T}^{S-wave} = V_{p'} (h_{K^0\Lambda} + \sum h_i \tilde{G}_i t_{i \to K^0 \Lambda}),$$

$$T = [1 - VG]^{-1} V.$$

Inone Oset, PRC65, 035204 (2002)

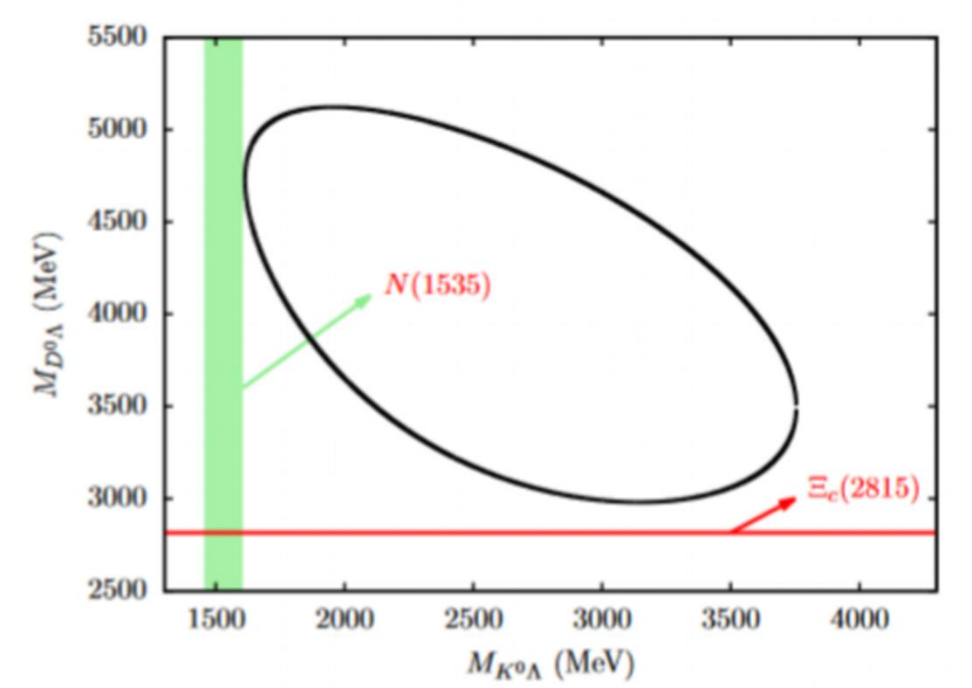


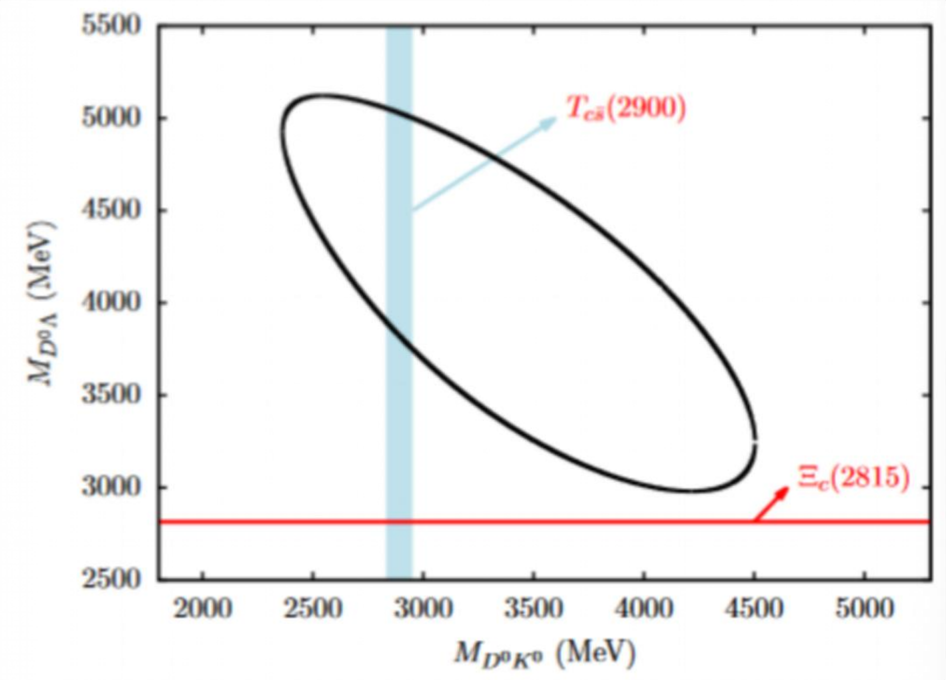
$$\frac{d^2\Gamma}{dM_{K^0\Lambda} dM_{D^0 K^0}} = \frac{1}{(2\pi)^3} \frac{2M_{K^0\Lambda} 2M_{D^0K^0}}{32 M^3_{\Lambda_b}} |\mathcal{T}^{\rm Total}|^2,$$

$$\mathcal{T}^{\rm Total} = \mathcal{T}^{S-wave} + \mathcal{T}^{T_{c\bar{s}}}.$$

ΛD interaction

□Dalitz plots



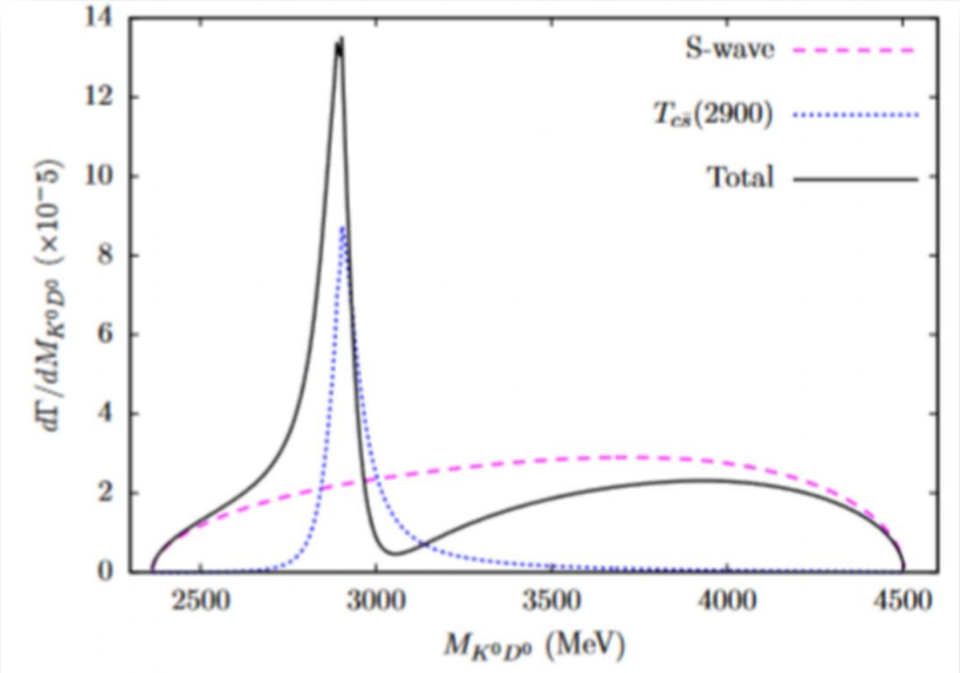


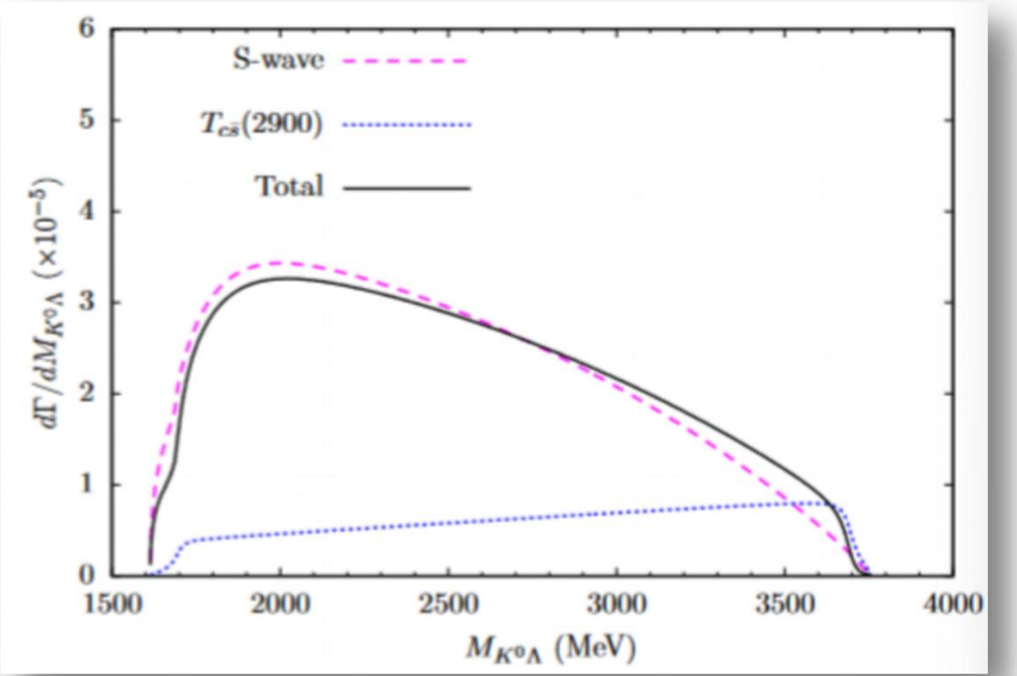
Contribution from ΛD interaction is neglected!

PRD 85 (2012), 114032

Results for $\Lambda_b \to K^0 D^0 \Lambda$

□Mass distributions





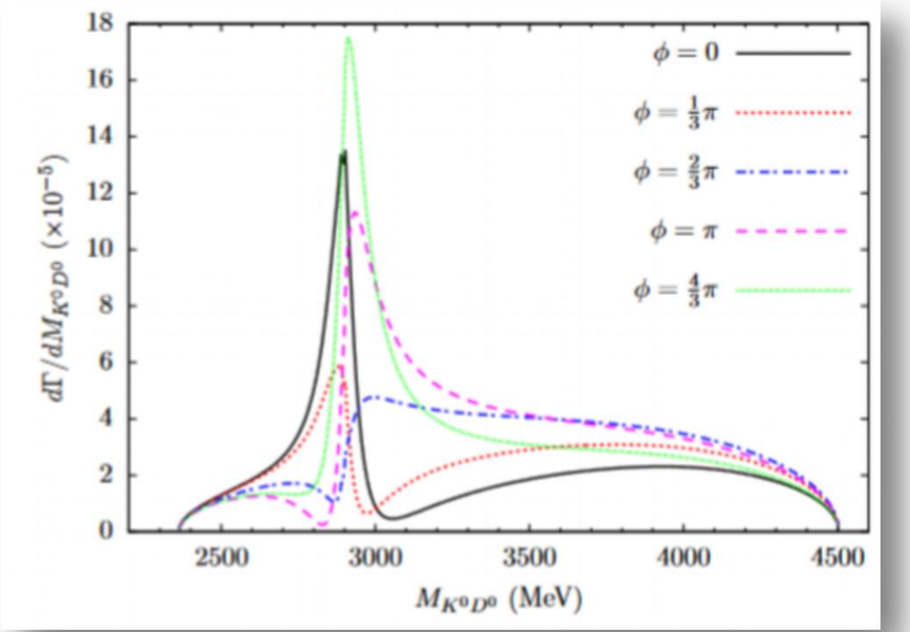
$$\mathcal{T}^{T_{c\bar{s}}} = -\sqrt{\frac{2}{3}} V_p \left[C \times G_{D_s^{*+}\rho^-} t_{D_s^{*+}\rho^- \to D^0 K^0} + G_{D^{*0}K^{*0}} t_{D^{*0}K^{*0} \to D^0 K^0} \right],$$

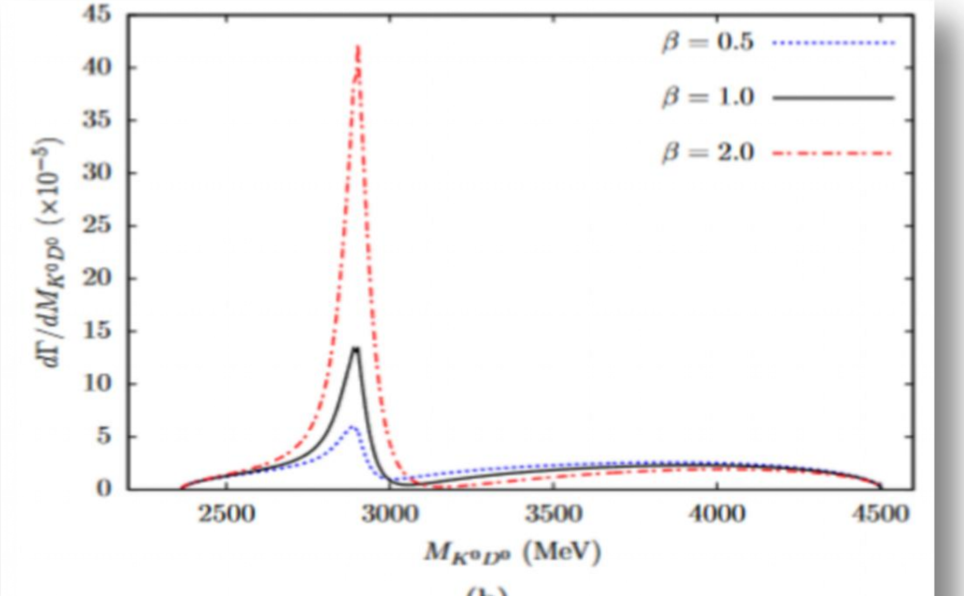
$$\mathcal{T}^{S-wave} = V_{p'} (h_{K^0\Lambda} + \sum_i h_i \tilde{G}_i t_{i \to K^0\Lambda}),$$

$$V_p = V_{p'},$$

Results

□Interference



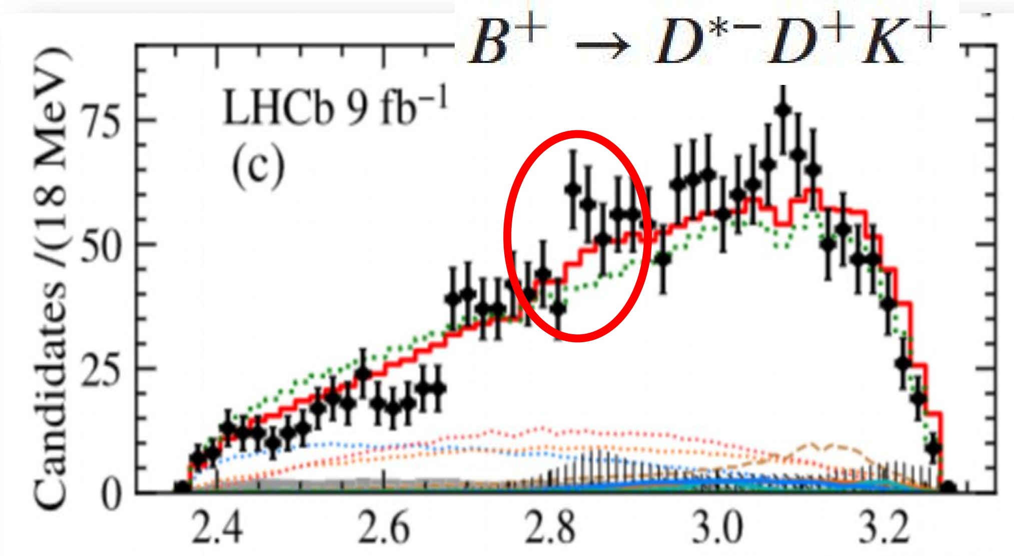


(b)

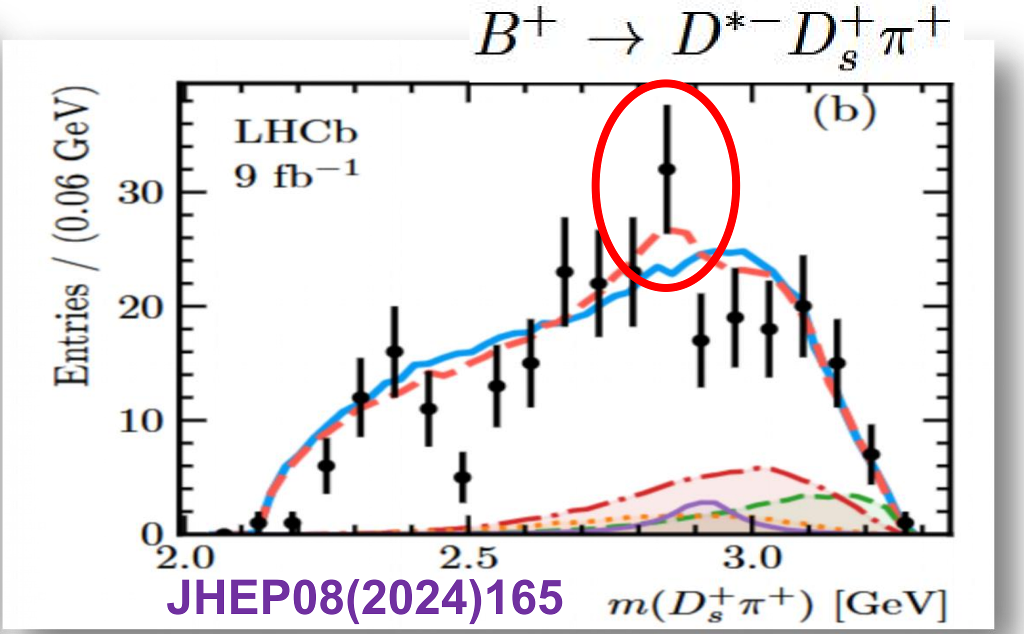
$$\mathcal{T}^{T_{c\bar{s}}} = -\sqrt{\frac{2}{3}} V_p \Big[C \times G_{D_s^{*+}\rho^-} t_{D_s^{*+}\rho^- \to D^0 K^0} + G_{D^{*0}K^{*0}} t_{D^{*0}K^{*0} \to D^0 K^0} \Big],$$

$$\mathcal{T}^{\mathrm{Total}} = \mathcal{T}^{S-wave} + \mathcal{T}^{T_{c\bar{s}}}.$$

Some hints of $T_{c\bar{s}2}$



PRL133(2024)131902



JHEP08(2024)165

fractions and phases for the components of the amplitude are presented in table 6. No strong evidence of exotic contributions in the $D^{*-}D_s^+$ or $D_s^+\pi^+$ channels is observed. The fit fraction of the scalar state $T^*_{c\bar{s}0}(2900)^{++}$ in the $D_s^+\pi^+$ channel observed in the $B^+ \to D^-D_s^+\pi^+$ analysis [5, 6] is found to be less than 2.3% at 90% CL.

$I[J^P]$	$\sqrt{s_0}$		Γ_0	Experiment
$1[0^+]$	2920	(Cusp)	130	$m = 2908 \pm 11 \pm 20$ $\Gamma = 136 \pm 23 \pm 11$
$1[1^+]$	2923	(Cusp)	145	$\cdots$
$1[2^+]$	2834		19	$\cdots$

$B^+ \to D^{*-} D^+ K^+$

EPJC84 (2024) 12, 1302

□Formalism

$$\frac{d\Gamma}{dM_{\rm inv}(D^+K^+)d\tilde{\Omega}} = \frac{1}{(2\pi)^4}\frac{1}{8M_B^2} p_{D^{*-}}\, \tilde{k} \sum |t|^2,$$

$$t = \epsilon_\mu(D^{*-})\, P_B^\mu\, (aY_{00} + bY_{20} + cY_{10}),$$

$$\sum |t|^2 = \left(\frac{M_{B^+}}{M_{D^{*-}}}\right)^2 \mathbf{p}_{D^{*-}}^2 \Big(|a|^2 Y_{00}^2 + |b|^2 Y_{20}^2 + |c|^2 Y_{10}^2 + 2Re(ab^*)Y_{00}Y_{20} + 2Re(ac^*)Y_{00}Y_{10} + 2Re(bc^*)Y_{20}Y_{10}\Big).$$

$$\int d\Omega Y^*_{l_3 m_3} Y_{l_2 m_2} Y_{l_1 m_1} = \left[\frac{(2l_1+1)(2l_2+1)}{4\pi(2l_3+1)}\right]^{\frac{1}{2}} \mathcal{C}(l_1 l_2 l_3; m_1 m_2 m_3) \times \mathcal{C}(l_1 l_2 l_3; 000),$$

Angular moments

$$\frac{d\Gamma_l}{dM_{\rm inv}} = \int d\tilde{\Omega} \frac{d\Gamma}{dM_{\rm inv} d\tilde{\Omega}} Y_{l0},$$

$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right], \quad \frac{d\,\Gamma}{dM_{\rm inv}} = \sqrt{4\pi}\frac{d\,\Gamma_0}{dM_{\rm inv}}.$$

$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$

$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$

$$FAC = \frac{1}{\sqrt{4\pi}}\frac{1}{(2\pi)^4}\frac{1}{8M_{B^+}^2}\mathbf{p}_{D^{*-}}^2\, p_{D^{*-}}\, \tilde{k} \left(\frac{M_{B^+}}{M_{D^{*-}}}\right)^2$$

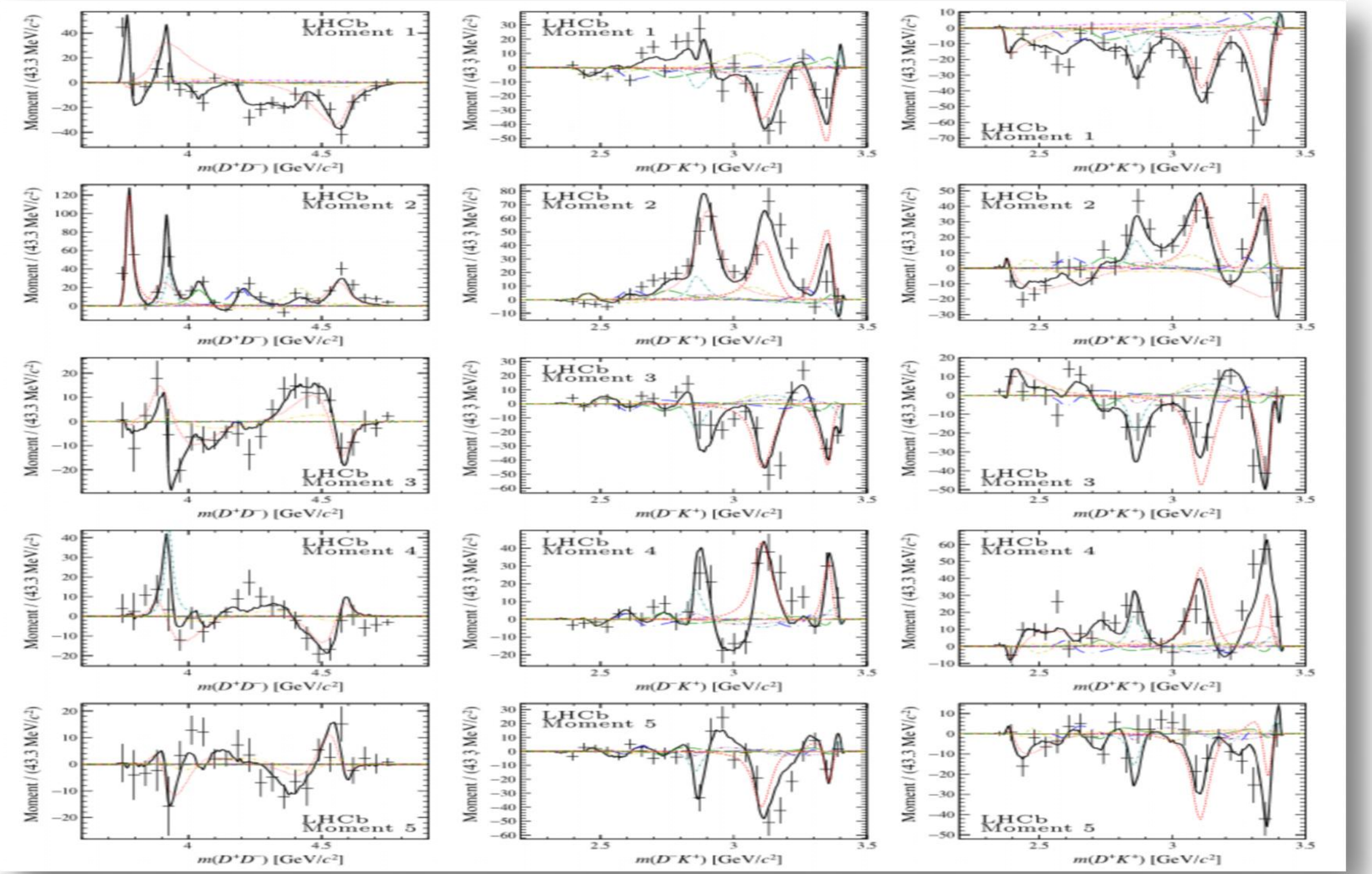
Angular moments

- LHCb: PRD102(2020)112003

$$B^+ \to D^+ D^- K^+$$

- BESIII: PRD104(2021)012016
- BABAR: PRD83(2011)052001
- EW-DMLi, PLB821(2021)136617

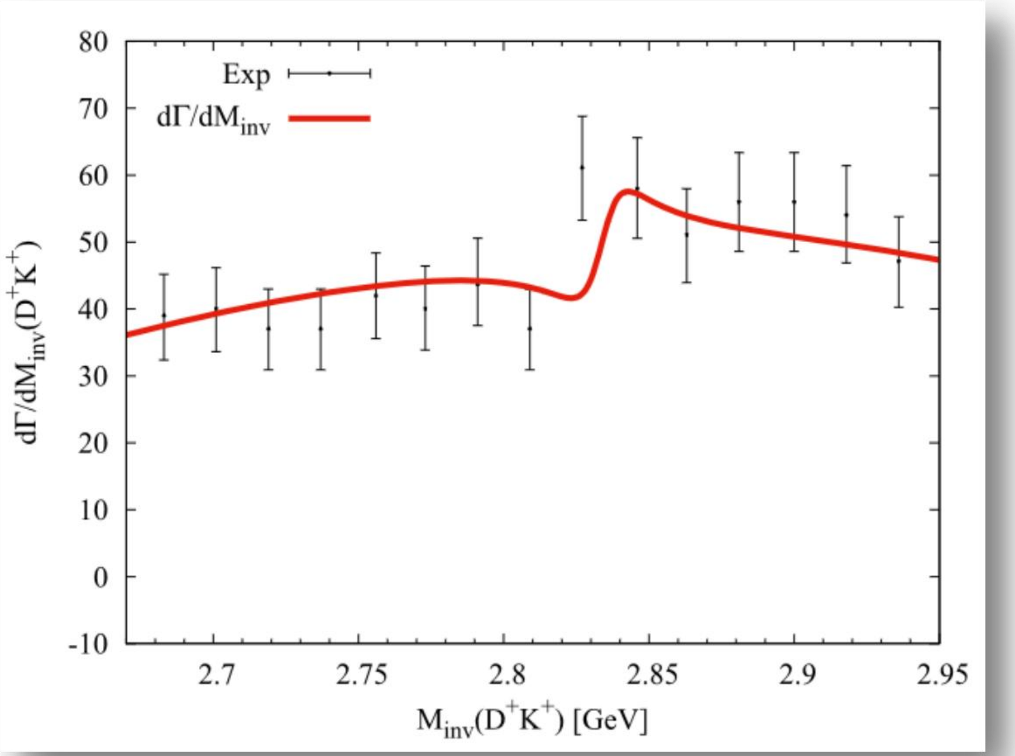
$$D_s^+ \to K^+ K^- \pi^+$$



Results for J=0

$$t=\epsilon_\mu(D^{*-})\,P_B^\mu\,(aY_{00}+bY_{20}+cY_{10}),\qquad a_i\to\tilde{a}_i\frac{\vec{k}}{M_B}$$

$$aY_{00}=\left(a_0+a_0'\frac{M_B^2}{M_{\rm inv}^2(D^+K^+)-M_R^2+iM_R\Gamma_R}\right)Y_{00},$$



$$\frac{d\,\Gamma_0}{dM_{\rm inv}}=FAC\left[|a|^2+|b|^2+|c|^2\right],$$

$$\frac{d\,\Gamma_1}{dM_{\rm inv}}=FAC\left[2\,Re(ac^*)+\frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

$$\frac{d\,\Gamma_2}{dM_{\rm inv}}=FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2+\frac{2}{5}\sqrt{5}\,|c|^2+2\,Re(ab^*)\right],$$

$$\frac{d\,\Gamma_3}{dM_{\rm inv}}=FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

$$\frac{d\,\Gamma_4}{dM_{\rm inv}}=FAC\,\frac{6}{7}|b|^2,$$

Results for J=1

$$aY_{00} + cY_{10}$$

$$= a_1 Y_{00} + c' \frac{M_B \tilde{k}}{M^2_{\text{inv}}(D^+K^+) - M_R^2 + iM_R\Gamma_R} Y_{10}$$

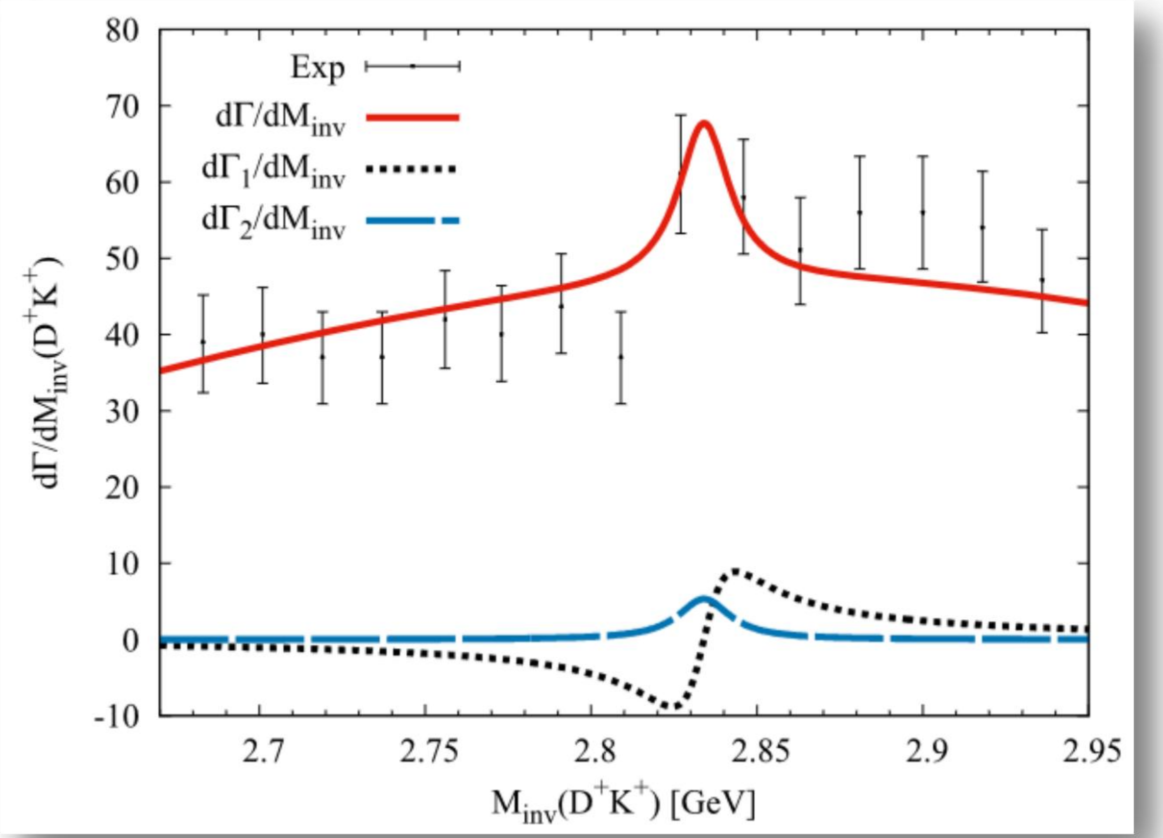
$$\frac{d\,\Gamma_0}{dM_{\text{inv}}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$

$$\frac{d\,\Gamma_1}{dM_{\text{inv}}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

$$\frac{d\,\Gamma_2}{dM_{\text{inv}}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$

$$\frac{d\,\Gamma_3}{dM_{\text{inv}}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

$$\frac{d\,\Gamma_4}{dM_{\text{inv}}} = FAC\,\frac{6}{7}|b|^2,$$



Results for J=2

$$aY_{00} + bY_{20}$$

$$= a_2 Y_{00} + b' \frac{\tilde{k}^2}{M_{\rm inv}^2(D^+K^+) - M_R^2 + i M_R \Gamma_R} Y_{20},$$

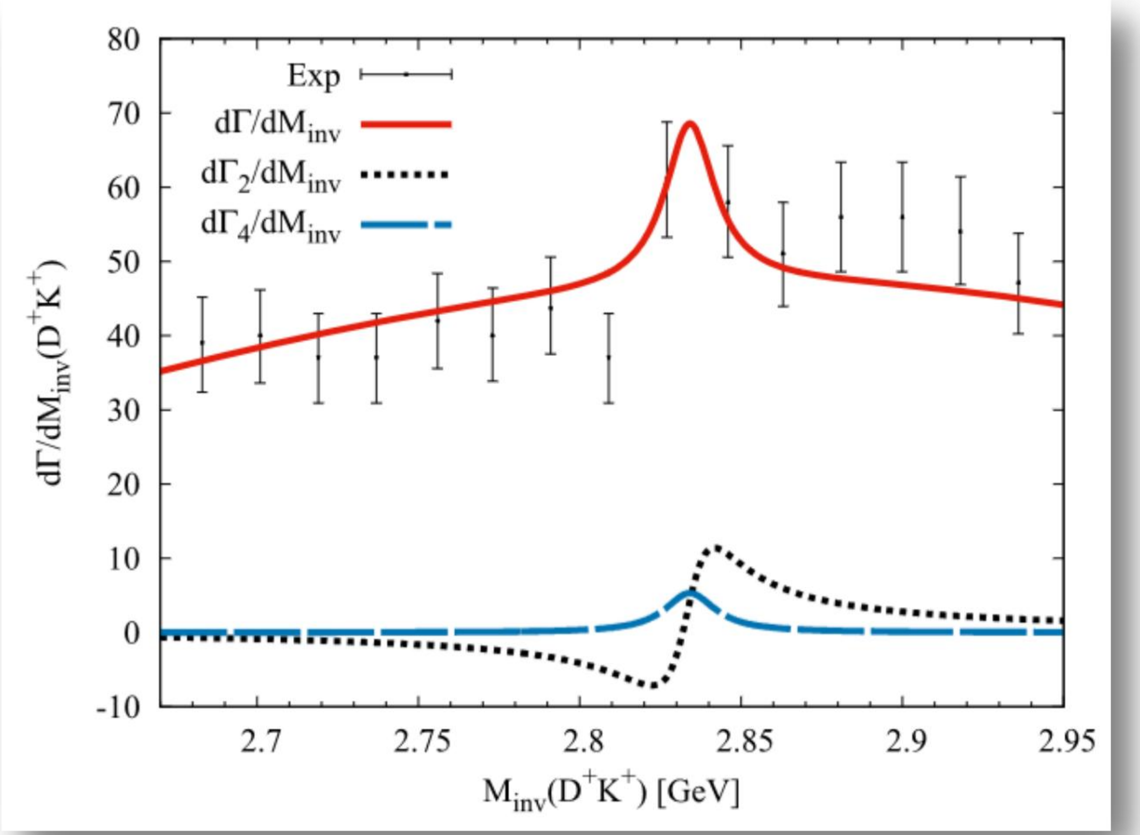
$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$

$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

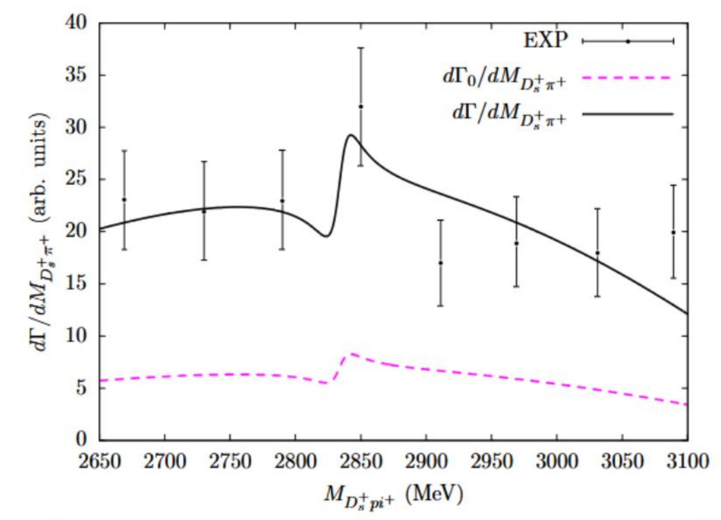
$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$

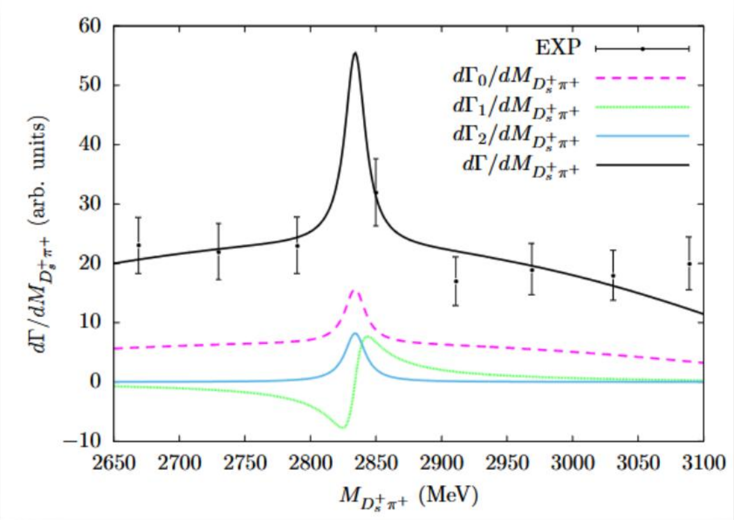
$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

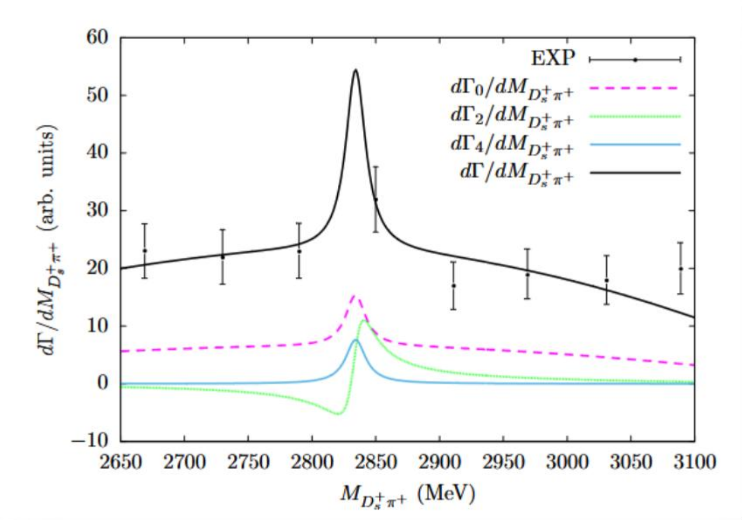
$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$



$B^+ \to D^{*-} D_s^+ \pi^+$ 2501.02839







$$aY_{00} = \left(a_0 + a_0' \frac{M_{B^+}^2}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R}\right) Y_{00}$$

$$aY_{00} + cY_{10} = a_1 Y_{00} + c' \frac{M_{B^+}\tilde{k}}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R} Y_{10}$$

$$aY_{00} + bY_{20} = a_2 Y_{00} + b' \frac{\tilde{k}^2}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R} Y_{20}$$

Summary

- $T_{c\bar{s}0}(2900)$ could be explained as the virtual state within the hidden-gauge formalism.
- Spin partner $T_{c\bar{s}2}$ is expected be more bound than $T_{c\bar{s}0}(2900)$.
- The measurements of $B^+ \to D^{*-}D^+K^+$ and $B^+ \to D^{*-}D_s^+\pi^+$ show some hints for the predicted $T_{c\bar{s}2}$.
- The angular mass distributions are very different for each of the spin assumptions $J = 0, 1, 2$.

Thanks for your attention!