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γW-exchange contributions in neutron β decay

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Outline

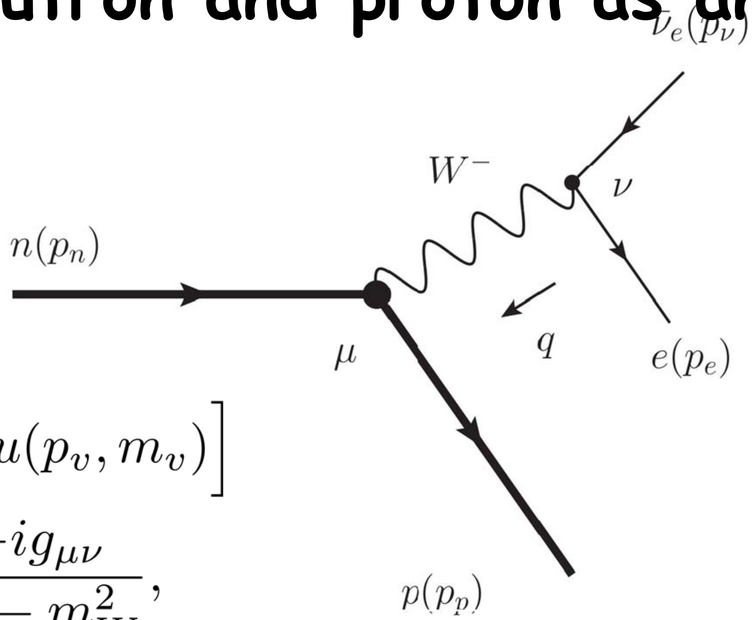
1. neutron β decay

2. radiative corrections and γW-exchange

3. a short summary

neutron β decay at tree level

At tree level, taking neutron and proton as an effective particles



$$\mathcal{M}^W = -\frac{ig^2 V_{ud}}{8}\left[\bar{u}(p_e, m_e)\gamma^\nu(1-\gamma_5)u(p_v, m_v)\right] \left[\bar{u}(p_p, m_p)\Gamma^\mu_{Wnp}(q)u(p_n, m_n)\right]\frac{-ig_{\mu\nu}}{q^2 - m_W^2},$$

one can extract V_{ud} from the lift time of neutron

$$|V_{ud}|^2 \propto \frac{1}{\tau_n(1+3\lambda^2)}\frac{1}{1+\mathrm{RC}}$$

$$\Gamma^\mu_{Wnp}(l) = \left(f_1(l^2)\gamma^\mu + i\frac{f_2(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{f_3(l^2)}{2m_N}l^\mu\right) + \left(f_4(l^2)\gamma^\mu + i\frac{f_5(l^2)}{2m_N}\sigma^{\mu\rho}l_\rho + \frac{f_6(l^2)}{2m_N}l^\mu\right)\gamma_5$$

radiative corrections in neutron βdecay

RCs in life time of neutron

$$\mathrm{RC} = \frac{\alpha}{2\pi}\bar{g}(E_m) + \Delta_R^V$$

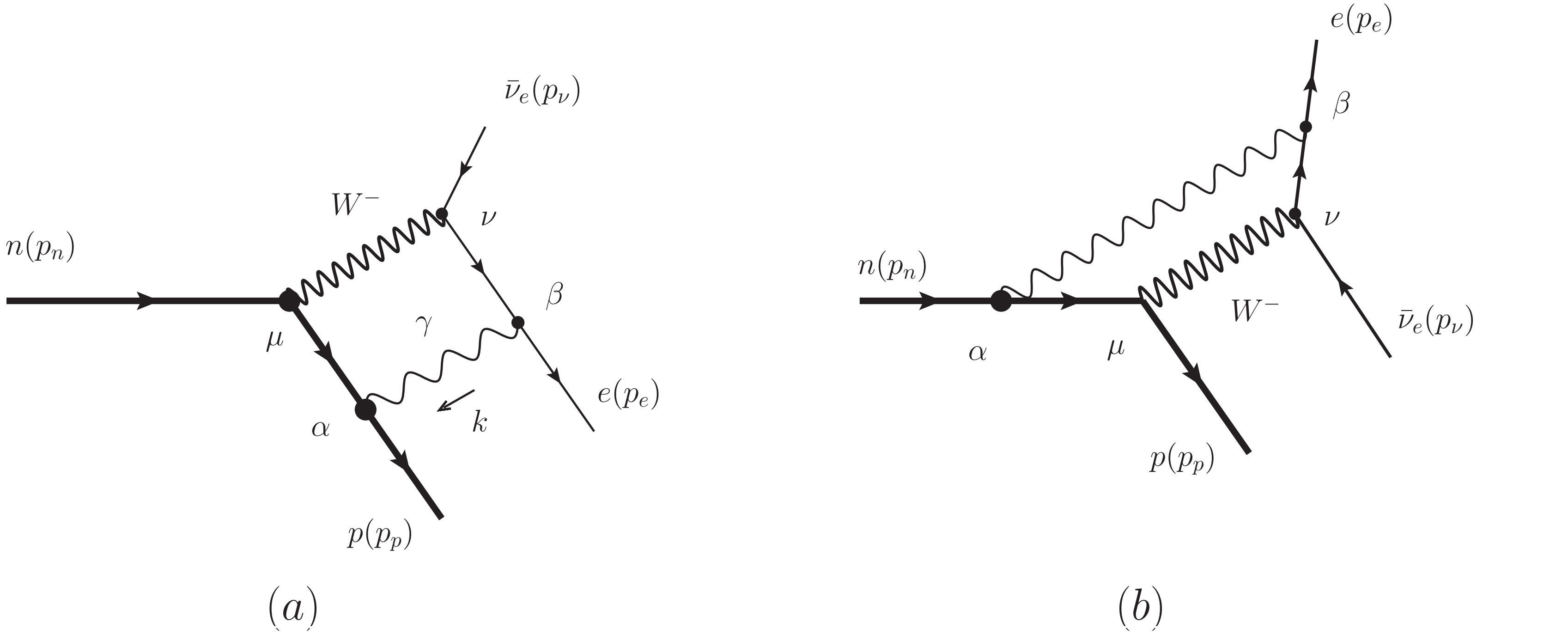
$$\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{m_Z}{m_\rho} + \ln\frac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{\mathrm{HD}}^{\mathrm{QED}} + 2\,\Box_{\gamma W}^V$$

$$\Box_{\gamma W}^V = \frac{\alpha}{2\pi}\left[\frac{1}{2}\ln\frac{m_W}{m_A} + C_{\mathrm{Born}} + \frac{1}{2}\mathrm{Ag}\right]$$

Among all the RCs, γW contribution plays special role

A. Sirlin, PR164 (1967) 1767, RMP50 (1978) 573 , W. J. Marciano and A. Sirlin, PRL 96 (2006) 032002

γW-exchange contributions



These contributions
(1) change the angle dependence, not only ratios.
(2) non-zero imaginary part.

$$\Gamma^{\mu}_{\gamma pp}(l) = ie\left[F_1^p(l^2)\gamma^{\mu} + i\frac{F_2^p(l^2)}{2m_p}\sigma^{\mu\nu}l_{\nu}\right], \quad \Gamma^{\mu}_{\gamma nn}(l) = ie\left[F_1^n(l^2)\gamma^{\mu} + i\frac{F_2^n(l^2)}{2m_n}\sigma^{\mu\nu}l_{\nu}\right]$$

γW-exchange in literatures

Calculations in literatures

(1) current algebra

(2) model FFs as input

(3) low energy effective theory (HBχPT)

(4) dispersion relations

Our opinion:
all these methods are equivalent in some sense.

C_{Born}:γW with elastci intermediate state

We use hadronic model in this work.
When only consider the elastic intermediate state, the following quantities are used

$$\Gamma^{\mu}_{\gamma pp}(l) = ie\Big[F_1^p(l^2)\gamma^{\mu} + i\frac{F_2^p(l^2)}{2m_p}\sigma^{\mu\nu}l_{\nu}\Big],$$

$$\Gamma^{\mu}_{\gamma nn}(l) = ie\Big[F_1^n(l^2)\gamma^{\mu} + i\frac{F_2^n(l^2)}{2m_n}\sigma^{\mu\nu}l_{\nu}\Big],$$

$$\Gamma^{\mu}_{Wnp}(l) = \Big(f_1(l^2)\gamma^{\mu} + i\frac{f_2(l^2)}{2m_N}\sigma^{\mu\rho}l_{\rho} + \frac{f_3(l^2)}{2m_N}l^{\mu}\Big) + \Big(g_1(l^2)\gamma^{\mu} + i\frac{g_2(l^2)}{2m_N}\sigma^{\mu\rho}l_{\rho} + \frac{g_3(l^2)}{2m_N}l^{\mu}\Big)\gamma_5,$$

C_{Born} in literatures

1st approximation: neglect some interactions

$$f_1 = g_V\,, f_2 = g_M\,, f_3 = g_S = 0,$$
$$f_4 = g_A, f_5 = g_T = 0, f_6 = g_P.$$

2nd approximation: forward limit (FWL) before loop

$$\bar{u}(p_e, m_e)\Gamma_L^{\omega\nu}(p_e, p_\nu, k)u(p_\nu, m_\nu) \approx \bar{u}(0,0)\Gamma_L^{\omega\nu}(0,0,k)u(0,0),$$
$$\bar{u}(p_p, m_p)\Gamma_H^{\rho\mu}(p_p, p_n, k)u(p_n, m_n) \approx \bar{u}(p, m_N)\Gamma_H^{\rho\mu}(p,p,k)u(p, m_N)$$

which means

$$\mathcal{M}^{(a)} \approx -i\int \frac{1}{(2\pi)^4}\bar{u}(0,0)\Gamma_L^{\omega\nu}(0,0,k)u(0,0)\bar{u}(p,m_N)\Gamma_H^{\rho\mu}(p,p,k)u(p,m_N)$$
$$\times\frac{-ig_{\mu\nu}}{k^2-m_W^2}\frac{-ig_{\rho\omega}}{k^2}\frac{1}{k^2+2p\cdot k}\frac{1}{k^2-m_e^2},$$

C_{Born} in literatures

3rd inner contributions and outer contributions, leptonic part

$$
\begin{aligned}
& \bar{u}(p_e, m_e)(-ie\gamma^\mu) S_F(p_e + k, m_e)(-i\gamma^\nu)(g_e^V - g_e^A\gamma_5) u(p_v, m_v) \\
= \quad & \frac{e}{(p_e+k)^2 - m_e^2} \bar{u}(p_e, m_e) \left[...... + i\epsilon^{\mu\nu\lambda\alpha} k_\lambda \gamma_\alpha \right] (g_e^V - g_e^A\gamma_5) u(p_\nu, m_\nu) \\
= \quad & \frac{e}{(p_e+k)^2 - m_e^2} \bar{u}(p_e, m_e) [i\epsilon^{\mu\nu\lambda\alpha} k_\lambda \gamma_\alpha (g_e^V - g_e^A\gamma_5)] u(p_v, m_v) + \text{outer} \\
\equiv \quad & \frac{e}{(p_e+k)^2 - m_e^2} i\epsilon^{\lambda\beta\nu\rho} k_\rho L_\lambda + \text{outer}
\end{aligned}
$$

$$
\gamma^\mu \gamma^\lambda \gamma^\nu = g^{\mu\lambda}\gamma^\nu - g^{\mu\nu}\gamma^\lambda + g^{\lambda\nu}\gamma^\mu - i\epsilon^{\mu\lambda\nu\alpha}\gamma_\alpha\gamma^5
$$

C_{Born} in literatures

4th FCC approximation for hadronic parts:
after applying the Dirac equation etc., only keep $\varepsilon_{\mu\nu\rho\sigma}$

$$\bar{u}(p,m_N)\Big[\gamma^{\mu}(\not{p}-\not{k}+m_N)\gamma^{\nu}\Big]u(p,m_N)$$
$$=\bar{u}(p,m_N)\Big[i\epsilon^{\mu\nu\rho\sigma}k_{\rho}\gamma_{\sigma}\gamma^{5}+......\Big]u(p,m_N)$$
$$\approx\ \bar{u}(p,m_N)\Big[i\epsilon^{\mu\nu\rho\sigma}k_{\rho}\gamma_{\sigma}\gamma^{5}\Big]u(p,m_N),$$

And also some other similar relations.

FCC approximation after FWL

our calculation: FCC is not unique. For example:

$$\bar{u}(p)\left[\sigma_{\mu\alpha}k^{\alpha}(\not{p}-\not{k}+m_N)\gamma_{\nu}u(p)\right] \xlongequal{\text{FCC}} 0,$$

$$\bar{u}(p)\left[\gamma_{\mu}(\not{p}-\not{k}+m_N)\sigma_{\nu\alpha}k^{\alpha}u(p)\right] \xlongequal{\text{FCC}} 2m_N\epsilon_{\mu\rho\nu\sigma}k^{\rho}\bar{u}(p)\gamma^{\sigma}\gamma^5u(p)$$

$$\left[\bar{u}(p)\sigma_{\mu\alpha}k^{\alpha}(\not{p}-\not{k}+m_N)\gamma_{\nu}\right]u(p) \xlongequal{\text{FCC}} 2m_N\epsilon_{\mu\rho\nu\sigma}k^{\rho}\bar{u}(p)\gamma^{\sigma}\gamma^5u(p),$$

$$\left[\bar{u}(p)\gamma_{\mu}(\not{p}-\not{k}+m_N)\sigma_{\nu\alpha}k^{\alpha}\right]u(p) \xlongequal{\text{FCC}} 0$$

our aim: γW-exchange contributions at amplitude level while not at cross section/decay width level.

$$\mathcal{M} \equiv \sum_{i=1}^{16} c_i O_i, \qquad c_i^{\gamma W} = \ ?$$

γW-exchange at amplitude level

In the practical calculation, when choosing the covariant form for O_i, it is very difficult to calculate the corresponding c_i, so we choose Pauli spinor form for O_i as

$$
\begin{aligned}
O_1 &\equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \eta_\nu] \\
O_2 &\equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \eta_\nu] \\
O_3 &\equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger \eta_\nu] \\
O_4 &\equiv [\xi_p^\dagger \xi_n][\xi_e^\dagger i\boldsymbol{\sigma}\cdot(\boldsymbol{n}_e\times\boldsymbol{n}_\nu)\eta_\nu] \\
O_5 &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \xi_n][\xi_e^\dagger \eta_\nu] \\
O_6 &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \eta_\nu] \\
O_7 &\equiv [\xi_p^\dagger i\boldsymbol{\sigma}\cdot(\boldsymbol{n}_e\times\boldsymbol{n}_\nu)\xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \eta_\nu] \\
O_8 &\equiv [\xi_p^\dagger i\boldsymbol{\sigma}\cdot(\boldsymbol{n}_e\times\boldsymbol{n}_\nu)\xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \eta_\nu]
\end{aligned}
$$

$$
\begin{aligned}
O_9 &\equiv [i\xi_p^\dagger \boldsymbol{\sigma}\xi_n]\times[\xi_e^\dagger \boldsymbol{\sigma}\eta_\nu]\cdot\boldsymbol{n}_e, \\
O_{10} &\equiv [i\xi_p^\dagger \boldsymbol{\sigma}\xi_n]\times[\xi_e^\dagger \boldsymbol{\sigma}\eta_\nu]\cdot\boldsymbol{n}_\nu, \\
O_{11} &\equiv [\xi_p^\dagger \boldsymbol{\sigma}_i \xi_n][\xi_e^\dagger \boldsymbol{\sigma}_i \eta_\nu] \\
O_{12} &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \eta_\nu] \\
O_{13} &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \eta_\nu] \\
O_{14} &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_e \eta_\nu] \\
O_{15} &\equiv [\xi_p^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \xi_n][\xi_e^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{n}_\nu \eta_\nu] \\
O_{16} &\equiv [\xi_p^\dagger i\boldsymbol{\sigma}\cdot(\boldsymbol{n}_e\times\boldsymbol{n}_\nu)\xi_n][\xi_e^\dagger \eta_\nu]
\end{aligned}
$$

γW-exchange at amplitude level

To compare the results in literature, we separate the amplitude as

$$\mathcal{M} \equiv \mathcal{M}^{\mathrm{Fermi}} + \mathcal{M}^{\mathrm{GT}}$$

$$\mathcal{M}^{\mathrm{Fermi}} \equiv \sum_{i=1}^{4} c_i^{\mathrm{Fermi}} O_i, \mathcal{M}^{\mathrm{GT}} \equiv \sum_{i=5}^{16} c_i^{\mathrm{GT}} O_i$$

coefficients c_i at tree level

Beyond the low energy limit, but taking E_e, E_ν, m_e, E_0 as small quantities comparing with m_n . Finally one has

$$c_{1,\mathrm{LO}}^{\mathrm{OBE}} = g_V \eta$$
$$c_{2,\mathrm{LO}}^{\mathrm{OBE}} = g_V$$
$$c_{3,\mathrm{LO}}^{\mathrm{OBE}} = -g_V [1 + \eta\beta]$$
$$c_{4,\mathrm{LO}}^{\mathrm{OBE}} = -g_V \eta$$
$$c_{5,\mathrm{LO}}^{\mathrm{OBE}} = g_A \eta$$
$$c_{6,\mathrm{LO}}^{\mathrm{OBE}} = g_A$$
$$c_{7,\mathrm{LO}}^{\mathrm{OBE}} = 0$$
$$c_{8,\mathrm{LO}}^{\mathrm{OBE}} = 0$$

$$c_{9,\mathrm{LO}}^{\mathrm{OBE}} = g_A \beta$$
$$c_{10,\mathrm{LO}}^{\mathrm{OBE}} = -g_A$$
$$c_{11,\mathrm{LO}}^{\mathrm{OBE}} = -g_A (1 - \eta\beta)$$
$$c_{12,\mathrm{LO}}^{\mathrm{OBE}} = 0$$
$$c_{13,\mathrm{LO}}^{\mathrm{OBE}} = -g_A \eta$$
$$c_{14,\mathrm{LO}}^{\mathrm{OBE}} = -g_A \eta$$
$$c_{15,\mathrm{LO}}^{\mathrm{OBE}} = 0$$
$$c_{16,\mathrm{LO}}^{\mathrm{OBE}} = g_A \eta$$

When recoil contributions are neglected, one has

$$\eta = 0, \beta = 0$$

$$N \equiv \frac{2m_n}{m_W^2} \sqrt{E_\nu (E_e + m_e)}, \qquad \eta \equiv \sqrt{\frac{E_e - m_e}{E_e + m_e}}, \qquad \beta = \boldsymbol{n}_e \cdot \boldsymbol{n}_\nu$$

γW-exchange at amplitude level

To calculate $c_i^{\gamma W}$, the following parameters are needed:

$$m_n, m_p, m_e, m_\nu, \alpha_e, F_{1,2}^{p,n}, f_i; \quad g, V_{ud}$$

assumed form factors

For EM FFs, we take

$$F_1^p(l^2) = F_{10}^p \sum_{j=1}^{N_1} a_{1j} G(l^2, \Lambda_{1j}^2, n_{1j}), \qquad F_2^p(l^2) = F_{20}^p \sum_{j=1}^{N_2} a_{2j} G(l^2, \Lambda_{2j}^2, n_{2j}),$$

$$F_1^n(l^2) = F_{10}^n \sum_{j=1}^{N_3} a_{3j} G(l^2, \Lambda_{3j}^2, n_{3j}), \qquad F_2^n(l^2) = F_{20}^n \sum_{j=1}^{N_4} a_{4j} G(l^2, \Lambda_{4j}^2, n_{4j}),$$

with $$G(l^2, \Lambda^2, n) \equiv \frac{(-1)^n}{(l^2 - \Lambda^2)^n}$$

For weak FFs, we take

$$f_i(l^2) = f_{i0} \sum_{j=1}^{\bar{N}_i} b_{ij} G(l^2, \bar{\Lambda}_{ij}^2, \bar{n}_{ij}), \qquad f_{1,2}(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2),$$

γW-exchange in the FW limit

For tree diagram, one has

$$c^W_{2,\mathrm{LO}} = -c^W_{3,\mathrm{LO}} = g_V\,, \quad c^W_{6,\mathrm{LO}} = ic^W_{10,\mathrm{LO}} = -c^W_{11,\mathrm{LO}} = g_A$$

For γW, one has

$$\begin{aligned} c^{\gamma W}_{2,\mathrm{LO}} =& g_A[d_{2,1}F^p_{10} + d_{2,2}F^p_{20} + d_{2,3}F^n_{10} + d_{2,4}F^n_{20}], \\ c^{\gamma W}_{6,\mathrm{LO}} =& g_V[d^V_{6,1}F^p_{10} + d^V_{6,2}F^p_{20} + d^V_{6,3}F^n_{10} + d^V_{6,4}F^n_{20}] \\ &+ g_M[d^M_{6,1}F^p_{10} + d^M_{6,2}F^p_{20} + d^M_{6,3}F^n_{10} + d^M_{6,4}F^n_{20}] \end{aligned}$$

$g_V \equiv f_{10}, g_M \equiv f_{20}, g_A \equiv f_{40}$

$d_{3,j} = -d_{2,j}\,, \quad d_{10,j} = -id_{6,j}\,, \quad d_{11,j} = -d_{6,j}$

γW-exchange in the FW limit

$$d_{2,i} = \sum_{j,k} \hat{\mathcal{F}}_{ij,4k}\Big[\frac{X_1(\Lambda_{ij},\Lambda_{4k})}{2m_N^2(\Lambda_{ij}^2-\Lambda_{4k}^2)} - \frac{\Lambda_{ij}Z_1(\Lambda_{4k})-\Lambda_{4k}Z_1(\Lambda_{ij})}{m_N^4\Lambda_{ij}\Lambda_{4k}(\Lambda_{ij}^2-\Lambda_{4k}^2)}\Big]$$

$$d_{6,1}^V = \sum_{j,k} \hat{\mathcal{F}}_{1j,1k}\Big[\frac{X_2(\Lambda_{1j},\Lambda_{1k})}{6m_N^2(\Lambda_{1j}^2-\Lambda_{1k}^2)} - \frac{2[\Lambda_{1j}Z_2(\Lambda_{1k})-\Lambda_{1k}Z_2(\Lambda_{1j})]}{3m_N^4\Lambda_{1j}\Lambda_{1k}(\Lambda_{1j}^2-\Lambda_{1k}^2)}\Big],$$

$$d_{6,2}^V = \sum_{j,k} \hat{\mathcal{F}}_{2j,1k}\Big[\frac{X_3(\Lambda_{2j},\Lambda_{1k})}{6m_N^2(\Lambda_{2j}^2-\Lambda_{1k}^2)} - \frac{2[\Lambda_{2j}Z_3(\Lambda_{1k})-\Lambda_{1k}Z_3(\Lambda_{2j})]}{3m_N^4\Lambda_{2j}\Lambda_{1k}(\Lambda_{2j}^2-\Lambda_{1k}^2)}\Big],$$

$$d_{6,3}^V = [d_{6,1}^V \text{ replacing the index } 1j, 1k \text{ by } 3j, 3k],$$

$$d_{6,4}^V = [d_{6,2}^V \text{ replacing the index } 2j, 2k \text{ by } 4j, 4k]$$

$$d_{6,1}^M = [d_{6,2}^V \text{ replacing the indexes } 2j \text{ and } 1k \text{ to } 1j \text{ and } 2k\text{, respectively}],$$

$$d_{6,2}^M = \sum_{j,k} \hat{\mathcal{F}}_{2j,2k}\Big[\frac{X_4(\Lambda_{2j},\Lambda_{2k})}{6m_N^2(\Lambda_{2j}^2-\Lambda_{2k}^2)} - \frac{\Lambda_{2j}Z_4(\Lambda_{2k})-\Lambda_{2k}Z_4(\Lambda_{2j})}{m_N^4(\Lambda_{2j}^2-\Lambda_{2k}^2)}\Big],$$

$$d_{6,3}^M = [d_{6,1}^M \text{ replacing the index } 1j \text{ by } 3j],$$

$$d_{6,4}^M = [d_{6,2}^M \text{ replacing the index } 2j \text{ by } 4j]$$

$$\hat{\mathcal{F}}_{ij,mk} \equiv a_{ij}b_{mk}\frac{(-1)^{n_{ij}+\bar{n}_{mk}}}{(n_{ij}-1)!\,(\bar{n}_{mk}-1)!}\frac{d^{n_{ij}-1}}{d(\Lambda_{ij}^2)^{n_{ij}-1}}\frac{d^{\bar{n}_{mk}-1}}{d(\bar{\Lambda}_{mk}^2)^{\bar{n}_{mk}-1}}$$

C_{Born}

then one has

$$\frac{\alpha_e}{2\pi}\delta_i \equiv \frac{\mathcal{C}_{i,\text{LO}}^{\gamma W}}{\mathcal{C}_{i,\text{LO}}^{W}}$$

$$C_{\text{Born}}^{\text{F}} = \delta_2 = \delta_3$$

$$C_{\text{Born}}^{\text{GT}} = \delta_6 = \delta_{10} = \delta_{11}$$

For comparison, we separate the corrections as

$$C_{\text{Born}}^{\text{F}} \equiv C_{\text{Born}}^{\text{F},g_A} + C_{\text{Born}}^{\text{F},g_M},$$

$$C_{\text{Born}}^{\text{GT}} \equiv C_{\text{Born}}^{\text{GT},g_V} + C_{\text{Born}}^{\text{GT},g_M}$$

FFs used in the practical numerical results

For f_4, we take the simple form as

$$\bar{N}_4 = 1, \bar{n}_{41} = 1, b_{41} = \Lambda_W^4, \bar{\Lambda}_{i1} = \Lambda_W = 1.09 \pm 0.05 \text{ GeV}$$

For EM FFs, we take three forms as examples
(I)

$$\begin{aligned}
&N_1 = 2, n_{1j} = 2, a_{11} = 0.152, \Lambda_{11} = 0.726, a_{12} = 1.270, \Lambda_{12} = 1.294, \\
&N_2 = 2, n_{2j} = 3, a_{21} = 0.359, \Lambda_{21} = 1.000, a_{22} = 0.656, \Lambda_{22} = 1.004, \\
&N_3 = 2, n_{3j} = 2, a_{31} = \Lambda_{31}^4, \Lambda_{31} = 1.288, a_{32} = -\Lambda_{32}^4, \Lambda_{32} = 1.378, F_{10}^n = 1, \\
&N_4 = 2, n_{4j} = 3, a_{41} = 0.041, \Lambda_{41} = 0.699, a_{42} = 2.087, \Lambda_{42} = 1.214, \quad \text{(typeI)}
\end{aligned}$$

FFs used in the practical numerical results

(II)

$$N_1 = 1, n_{11} = 2, a_{11} = \Lambda_{11}^4, \Lambda_{11} = 0.960,$$
$$N_2 = 1, n_{21} = 3, a_{21} = \Lambda_{21}^6, \Lambda_{21} = 1.003,$$
$$N_3 = 2, n_{3j} = 1, a_{31} = \Lambda_{31}^2, \Lambda_{31} = 0.847, a_{32} = -\Lambda_{32}^2, \Lambda_{32} = 0.914, F_{10}^n = 1,$$
$$N_4 = 1, n_{41} = 3, a_{41} = \Lambda_{41}^6, \Lambda_{41} = 1.038, \qquad \text{(typeII)}$$

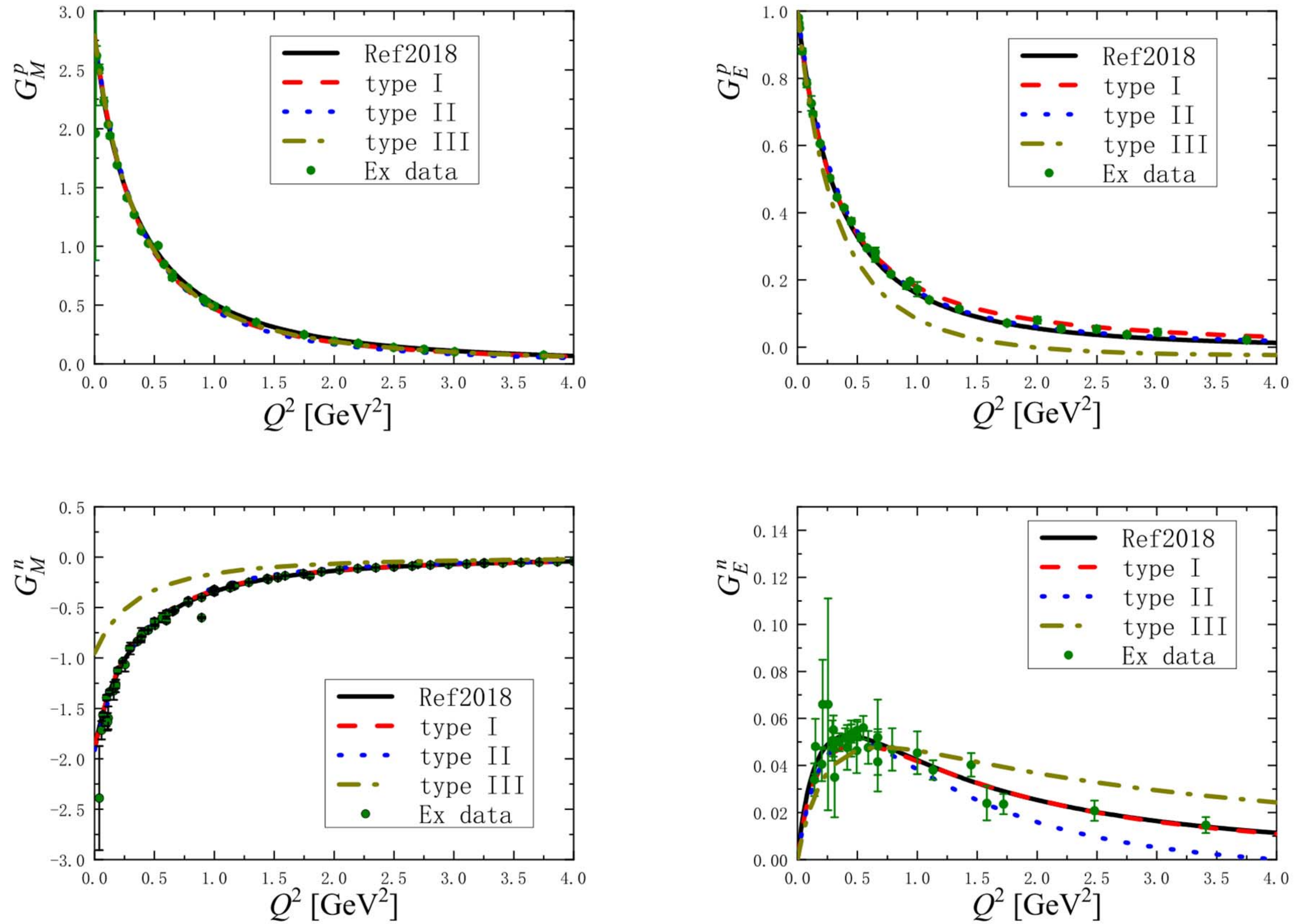
(III)

$$N_i = 1, n_{i1} = 2, a_{i1} = \Lambda_{i1}^4, \Lambda_{i1} = \Lambda_\gamma = 0.84, F_{10}^n = 0 \qquad \text{(typeIII)}$$

other used parameters

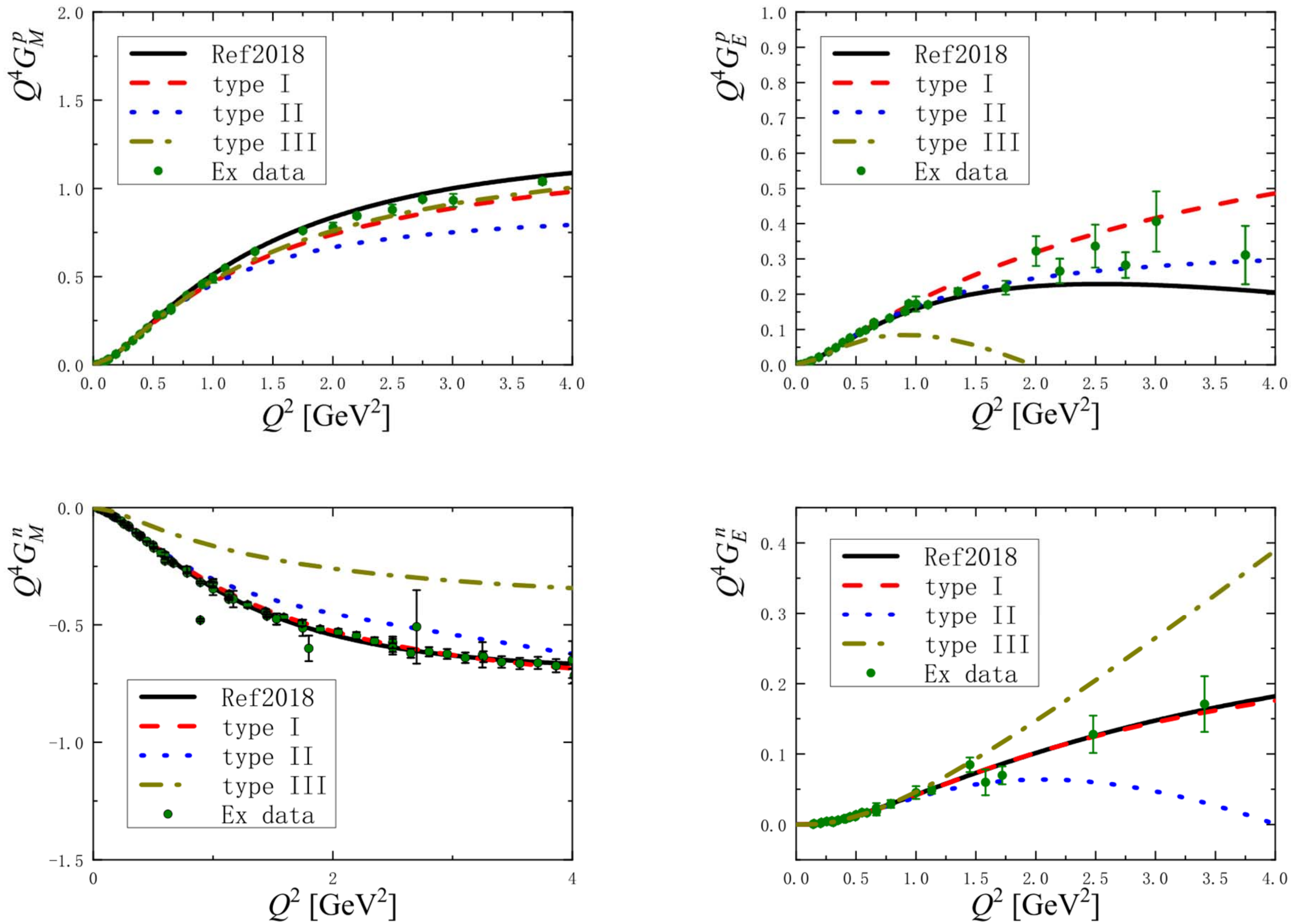
$$m_n = 939.56542 \text{ MeV}, m_p = 938.27209 \text{ MeV}, m_e = 0.51100 \text{ MeV},$$
$$F_{10}^p = 1, F_{20}^p = 1.793, F_{20}^n = -1.913,$$
$$g_V = 1, g_A = -1.26, g_M = F_{20}^p - F_{20}^n = 3.706$$

EM FFs vs. Ex-data

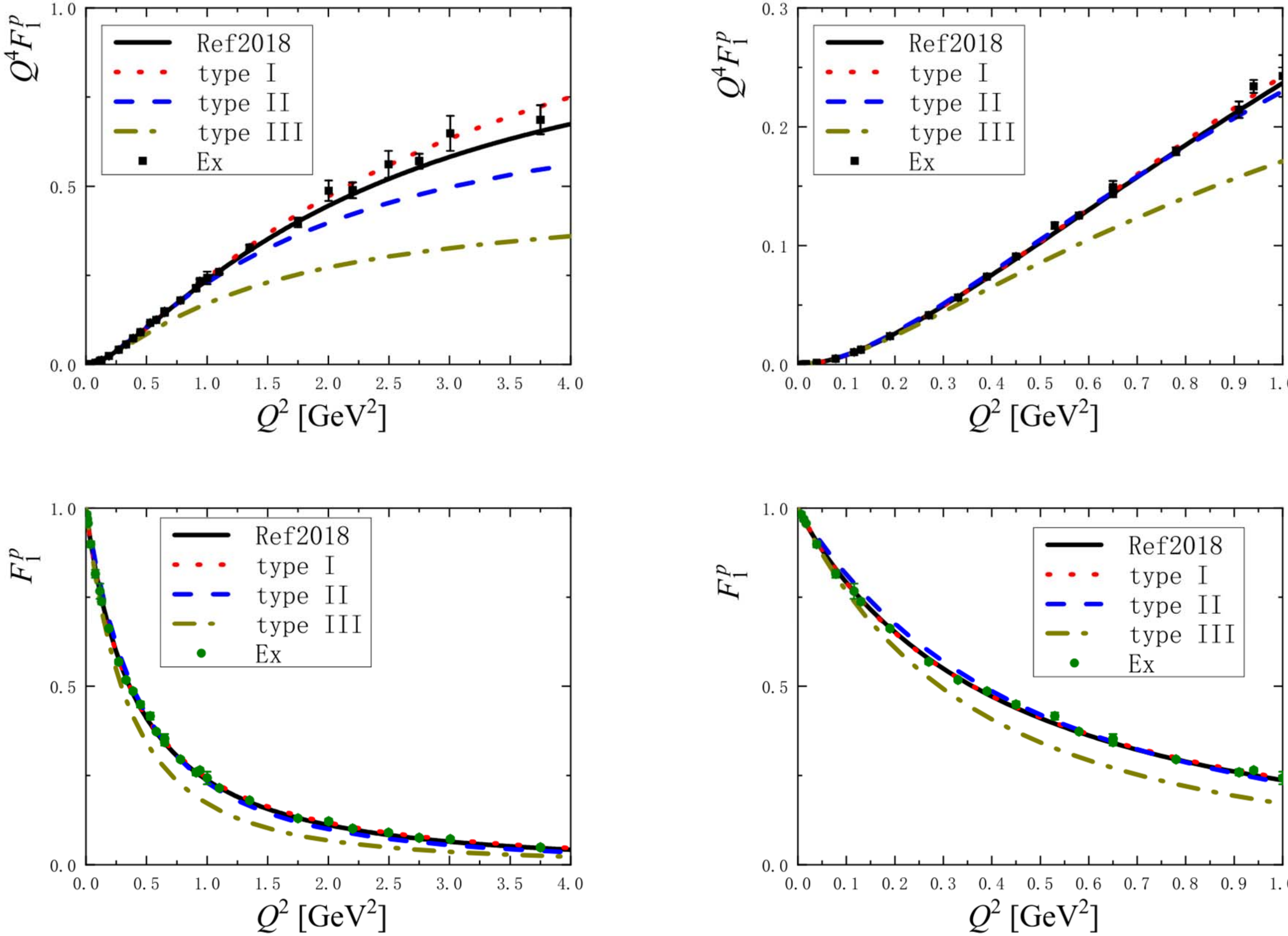


Zhihong Ye, etc. PLB777(2018)8

EM FFs vs. Ex-data



F_1^p vs. Ex-data



Numerical results with different FFs

$$
\text{type I}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 1.048F_{10}^{p} + 0.967F_{20}^{p} - 0.027F_{10}^{n} + 0.968F_{20}^{n} = 0.906, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.465F_{10}^{p} + 0.231F_{20}^{p} - 0.013F_{10}^{n} + 0.231F_{20}^{n} = 0.423, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.226F_{10}^{p} - 0.004F_{20}^{p} - 0.0058F_{10}^{n} - 0.005F_{20}^{n}]g_M = 0.825 \end{cases}
$$

$$
\text{type II}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 1.062F_{10}^{p} + 0.968F_{20}^{p} - 0.028F_{10}^{n} + 0.985F_{20}^{n} = 0.887, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.478F_{10}^{p} + 0.235F_{20}^{p} - 0.014F_{10}^{n} + 0.239F_{20}^{n} = 0.428, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.232F_{10}^{p} - 0.004F_{20}^{p} - 0.005F_{10}^{n} - 0.005F_{20}^{n}]g_M = 0.844 \end{cases}
$$

$$
\text{type III}: \begin{cases} C_{\text{Born}}^{\text{F},g_A} = 0.999F_{10}^{p} + 0.999F_{20}^{p} + 0.999F_{20}^{n} = 0.882, \\ C_{\text{Born}}^{\text{GT},g_V} = 0.414F_{10}^{p} + 0.223F_{20}^{p} + 0.223F_{20}^{n} = 0.388, \\ C_{\text{Born}}^{\text{GT},g_M} = [0.223F_{10}^{p} - 0.007F_{20}^{p} - 0.007F_{20}^{n}]g_M = 0.832 \end{cases}
$$

comparison with results in literatures

	FFs	$C_{\text{Born}}^{\text{F},g_A}$	$C_{\text{Born}}^{\text{GT},g_V}$	$C_{\text{Born}}^{\text{GT},g_M}$
Ref. [10]	Eqs. (25, 47, 48)	0.881 ± 0.014	no calculated	no calculated
Ref. [11]	Refs. [42, 43]	0.91(5)	0.39(1)	0.78(2)
type I	Eqs. (25, 47, 43)	0.906	0.423	0.825
type II	Eqs. (25, 47, 49)	0.887	0.428	0.843
type III	Eqs. (25, 47, 48)	0.882	0.388	0.832

The results for $C_{\text{Born}}^{\text{F},g_A}$ with type $\text{I}, \text{II}, \text{III}$ are consistent with those given in Refs within the error.

Different from the case $C_{\text{Born}}^{\text{F},g_A}$, our results (type I) for $C_{\text{Born}}^{\text{GT},g_V}$ and $C_{\text{Born}}^{\text{GT},g_M}$ are about 8% and 6% larger than those given by Ref, respectively.

[10]NPAA 540 (1992) 478, [11]PRD103 (2021) 113001

contributions from different parts

contributions/All	$C_{\rm Born}^{{\rm F},g_A}(f_4)$	$C_{\rm Born}^{{\rm GT},g_V}(f_1)$	$C_{\rm Born}^{{\rm GT},g_M}(f_2)$
F_1^p	115%	110%	102%
$F_2^{p,n}$	-13%	-7%	1%
F_1^n	-3%	-3%	-3%

Next step

1. the inner γW-exchange contributions with Born intermediate beyond the forward limit at the amplitude level (RC with recoil)
2. the outer contributions
3. dispersion relations beyond the forward limit
4. other processes such as weak decay of meson
5. ……

Short Summary

1. The inner γW-exchange contribution with Born intermediate in the forward limit are calculated at the amplitude level.
2. The numerical result for C^A is consistent with the previous results, while the results for $C^{V,M}$ are about 8% and 6% larger than the precious results.

Thanks!

any comments, suggestions, and discussions are **Welcome!**

请大家批评指正！

Expressions for some functions

$$X_1(x,y) \equiv x^2 \log \frac{m_N^2}{x^2} - y^2 \log \frac{m_N^2}{y^2} + 6m_N^2 \log \frac{x^2}{y^2},$$

$$X_2(x,y) \equiv x^2 \log \frac{m_N^2}{x^2} - y^2 \log \frac{m_N^2}{y^2} - 6m_N^2 \log \frac{x^2}{y^2},$$

$$X_3(x,y) \equiv x^2 \log \frac{m_N^2}{x^2} - y^2 \log \frac{m_N^2}{y^2},$$

$$X_4(x,y) \equiv 2x^2 \log \frac{m_N^2}{x^2} - 2y^2 \log \frac{m_N^2}{y^2} + m_N^2 \log \frac{x^2}{y^2},$$

$$Y(x) \equiv \log[\frac{x + \sqrt{-4m_N^2 + x^2}}{2m_N}],$$

$$Z_1(x) \equiv (-4m_N^2 + x^2)^{3/2} Y(x),$$

$$Z_2(x) \equiv (-4m_N^2 + x^2)^{1/2} (8m_N^2 + x^2) Y(x),$$

$$Z_3(x) \equiv (-4m_N^2 + x^2)^{1/2} (2m_N^2 + x^2) Y(x),$$

$$Z_4(x) \equiv (-4m_N^2 + x^2)^{1/2} x Y(x).$$