胶球的格点 QCD 研究

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Outline

Introduction

The radiative decay of scalar glueball

Mixing between glueballs and mesons

summary

Hadron

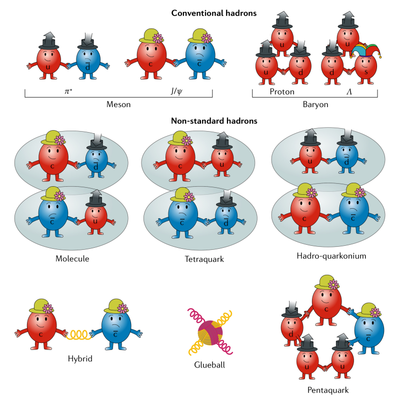


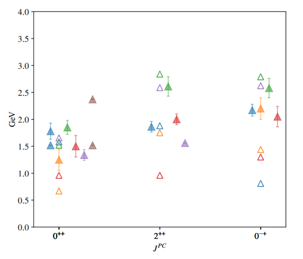
Figure: Various types of hadrons.[10.1038/s42254-019-0082-y]

Why study glueballs?

- It provides a deep understanding of the behavior of pure gluons, helping to distinguish the interaction mechanisms between quarks and gluons.
- The existence of glueballs offers an important test of the validity of quantum chromodynamics.
- Research on glueballs helps to understand the non-perturbative characteristics of strong interactions.
- Glueballs may be associated with new physical phenomena such as dark matter in phenomenological models, providing potential candidates for dark matter.

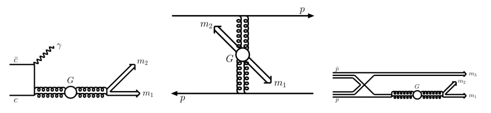
Theory study

- the phenomenological models,the constituent gluon model,MIT bag model ...
- the analytical models, QCD sum rules, the Bethe-Salpeter equation...



How to search for glueballs

- The decay process should exhibit flavor symmetry.
- There exist supernumerary isoscalar state in the $q\bar{q}$ nonets.
- The ideal search process.
  - $N\bar{N}$ annihilation processes
  - Central production experiments
  - J/ψ radiative decay processes, etc.



Glueball candidates

- scalar glueball
  - $f_0(1710), f_0(1500), f_0(1370)$
- pseudoscalar glueball
  - $\eta(1405), \eta(1475)$
  - X(2370)
- tensor glueball
  - $f_J(2220)$, $f_2(2340)$
- We still have not fully identified the glueball and need more efforts in both experimental and theoretical aspects.

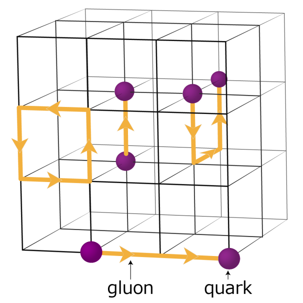
Latttice QCD

- Path integral quantization

$$Z = \int DAD\psi D\bar{\psi} e^{iS[A,\psi,\bar{\psi}]}$$

$$\rightarrow \int DU \det M[U] e^{-S_g[U]}$$

$$\langle \widehat{O} \rangle = \frac{1}{Z} \int DU \det M[U] e^{-S_g[U]} \hat{O}[U]$$



- Perform a Wick rotation to Euclidean spacetime, and solve it using Monte Carlo methods after discretization.

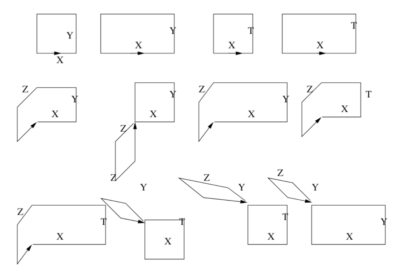
$$\langle \widehat{\mathcal{O}}[U, \psi, \bar{\psi}] \rangle = \frac{1}{N} \sum_i \mathcal{O}\,[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$

Glueball operator

- Use optimized operators to extract the glueball mass.

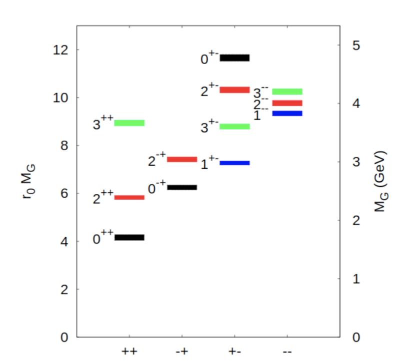
$$C(\vec{p}, t) = \frac{1}{T} \sum_{\tau} \langle \Phi(\vec{p}, t+\tau) \Phi^{\dagger}(\vec{p}, \tau) \rangle$$

$$\approx \frac{|\langle 0|\Phi(\vec{p}, 0)|S(\vec{p})\rangle|^2}{2E_S V_3} e^{-E_S t} \approx e^{-E_S t},$$



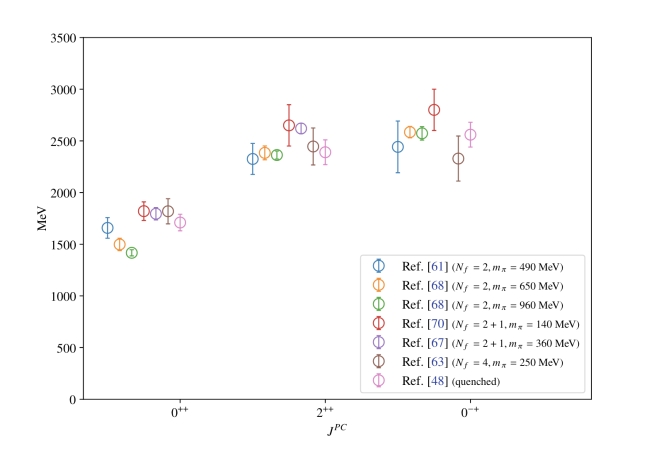
Quenched approximation

- Under the quenched lattice QCD , the scalar (0^{++}) glueball has a mass of approximately ($1.5 \sim 1.7$ GeV), the tensor (2^{++}) glueball has a mass of approximately $2.2 \sim 2.4$ GeV, and the pseudoscalar (0^{-+})glueball has a mass of approximately $2.4 \sim 2.6$ GeV.[PhysRevD.73.014516]



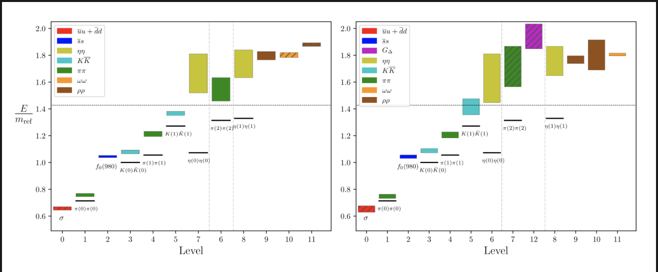
Unquenched approximation

- In recent years, there have been many studies investigating the glueball spectrum using pure glueball operators in the unquenched approximation.[arXiv:2305.04869)]



Beyond gluon operators

- Some studies have attempted to construct correlation functions using $\bar{q}q$,multi-quark operators, and gluon operators together, and analyze the light meson spectrum through variational methods.[AIP Conf. Proc., 2249(1):030032, 2020.]。



The production rate of glueballs

- $J/\psi \to \gamma G_s$: [PRL110,021601],

$$\Gamma(J/\psi \to \gamma G_s) = 0.35(8) keV$$

$$Br(J/\psi \to \gamma G_s) = 3.8(9) \times 10^{-3}$$

- $J/\psi \to \gamma G_T$: [PRL111,091601],

$$\Gamma(J/\psi \to \gamma G_T) = 1.01(22)(10) keV$$

$$Br(J/\psi \to \gamma G_T) = 1.1(2)(1) \times 10^{-2}$$

- $J/\psi \to \gamma G_{Ps}$: [PRD100,054511],

$$\Gamma(J/\psi \to \gamma G_{ps}) = 0.0215(74) keV$$

$$Br(J/\psi \to \gamma G_{ps}) = 2.31(80) \times 10^{-4}$$

Glueball

- However, so far, we still cannot provide a definitive answer regarding the existence of glueballs.
  - There is a lack of sufficient understanding of the properties of glueballs, such as their production, decay, and other related information.
  - There is a strong likelihood of mixing between glueballs and isospin singlet mesons, making them difficult to distinguish.
- On one hand, there is a need for more comprehensive experimental measurements of glueball candidates; on the other hand, it is essential to provide richer theoretical insights.

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The radiative decay of glueball

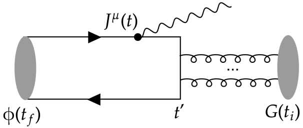


Figure: The radiative decay of glueball.

- Radiative processes provide an ideal probe to understand the internal structure of particles.
- Theoretical estimates of various pure state decay processes offer important theoretical inputs for calculating their mixing matrix.
- The combined radiative decay and two-photon decay processes can provide information on the stickiness parameter of particles.

Formulas

- Radiative decay width

$$\Gamma(i \to \gamma f) = \frac{1}{2J_i + 1} \frac{1}{32\pi^2} \int d\Omega_q \frac{|\vec{q}|}{M_i^2} \sum_{r_i, r_j, r_\gamma} \left| \mathcal{M}_{r_i, r_j, r_\gamma} \right|^2$$

- Decay amplitude

$$\mathcal{M}_{r_i, r_f, r_\gamma} = \epsilon_\mu^* (q, r_\gamma) \left\langle f(p_f, r_f) \left| j_{em}^\mu(0) \right| i(p_i, r_i) \right\rangle$$

- Multi-pole expanding

$$\left\langle f(p_f, r_f) \left| j_{em}^\mu(0) \right| i(p_i, r_i) \right\rangle = \sum_k \alpha_k^\mu (p_f, p_i, \epsilon_f, \epsilon_i) F_k(Q^2)$$

Three point function

- Three point function

$$\Gamma^{(3)}_{i,\mu,j}(\vec{q};t_f,t) = \frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}} e^{-i\vec{q}\vec{y}}\langle \Phi^{(i)}(t_f+\tau)$$
$$\times J_\mu(\vec{y},t+\tau)O_{V,j}(\vec{0},\tau)\rangle$$
$$= \sum_{T,V,r}\frac{e^{-M_T(t_f-t)}e^{-E_V(\vec{q})t}}{2M_T V_3 2E_V(\vec{q})}$$
$$\times \langle 0|\Phi^{(i)}(0)|G_i\rangle\langle G_i|J_\mu(0)|V(\vec{q},r)\rangle$$
$$\times \langle V(\vec{q},r)|O^\dagger_{V,j}(0)|0\rangle$$

Lattice setup

TABLE I. The configuration parameters and mass spectrum. The spatial lattice spacing a_s is determined from r_0^{-1} = 0.410(20) GeV by calculating the static potential.

β	ξ	a_s(fm)	La_s(fm)	$L^3 \times T$	$N_{\rm conf}$	$m[\eta_s(0^{-+})]$ (GeV)	$m[\phi(1^{--})]$ (GeV)	$m[f_0^{(s)}(0^{++})]$ (GeV)	$m[G(0^{++})]$ (GeV)
2.4	5	0.222(2)	2.66	$12^3 \times 192$	4000	0.7025(19)	1.0241(17)	1.569(22)	1.372(27)
2.8	5	0.138(1)	2.21	$16^3 \times 192$	4000	0.7064(12)	1.0287(20)	1.549(29)	1.495(54)
3.0	5	0.110(1)	1.76	$16^3 \times 192$	4000	0.6946(27)	1.0214(22)	1.593(24)	1.612(63)
∞						0.7044(20)	1.0252(23)	1.582(28)	1.635(62)

[Sci. China Phys. Mech. Astron. 67,111012 (2024)]

Glueball and ϕ

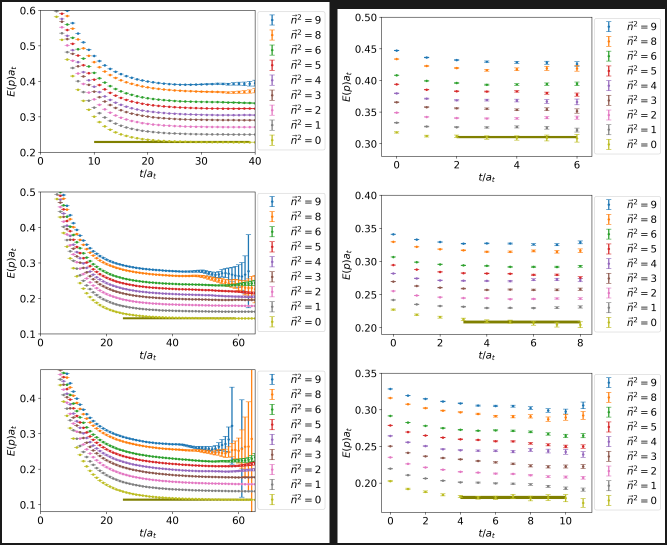


Figure: The effective mass of glueball and ϕ.

Formfactor

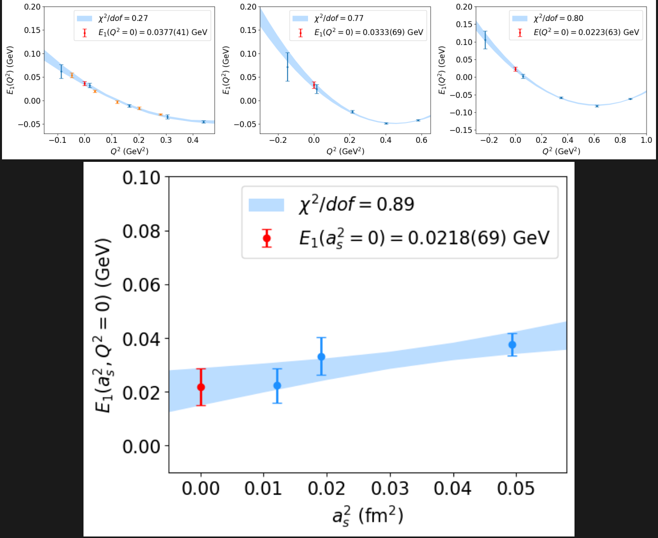


Figure: The form factor of $G \to \gamma\phi$.

$G_s \to \gamma\phi$

- The decay width of $G_s \to \gamma\phi$

$$\Gamma_{G\to\gamma\phi} = 0.074(47)\ \text{keV}$$

- Assuming the decay width of the glueball is $\sim \mathcal{O}(100\ \text{MeV})$, we can obtain

$$\text{Br}(J/\psi \to \gamma G, G \to \gamma\phi) \sim \mathcal{O}\left(10^{-9}\right).$$

This can explain why the $f_0(1710)$ particle has not been observed in the experimental process $J/\psi \to \gamma\gamma\phi$.[arxiv:2401.00918]

$G_s \to \gamma\gamma$

- Additionally, based on the VMD model, we estimate the width of $G_s \to \gamma\gamma$ to be

$$\Gamma(G \to \gamma\gamma) \approx 0.52(33)\text{eV},$$

which allows us to determine the stickiness parameter of the glueball as follows:

$$S(G) = C\left(\frac{m_G}{q_\gamma}\right)\frac{\Gamma(J/\psi \to \gamma G)}{\Gamma(G \to \gamma\gamma)} \sim \mathcal{O}\left(10^4\right)$$

$J/\psi \to \gamma G_s$

- Additionally, we have recalculated the process $J/\psi \to \gamma G_s$.

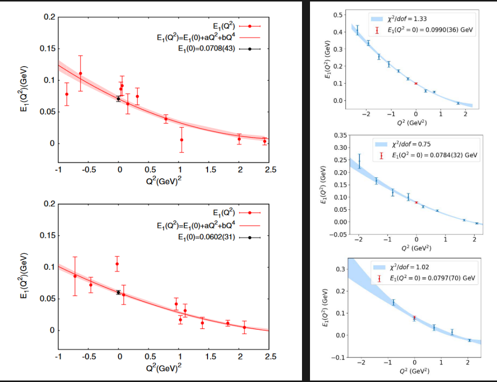


Figure: The left figure shows our previous calculation results for the $J/\psi \to \gamma G_S$, while the right figure presents the most recently updated calculation results.

- The decay width and branch ratio of $J/\psi \to \gamma G_s$

$$\Gamma_{J/\psi \to \gamma G} = 0.578(86)\ \text{keV}$$

$$\text{Br}(J/\psi \to \gamma G) = 6.2(9) \times 10^{-3}$$

- The new results reconsider the calculation of the current renormalization coefficient. This result is consistent with coupled-channel analysis based on BESIII data, which gives a yield of 5.8×10^{-3} for the scalar glueball in the J/ψ radiative decay process.[Phys.Lett.B 816,136227 (2021)]

Outline

Mixing

- In the presence of dynamical quarks, glueballs can mix with meson states that have the same quantum numbers.

$$\begin{pmatrix} |g\rangle \\ |f_0\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |G\rangle \\ |s\bar{s}\rangle \end{pmatrix}$$

- Using a basis composed of pure glueball states $|G\rangle$ and pure $|\bar{s}s\rangle$ states, the Hamiltonian can be expressed as:

$$\hat{H} = \begin{pmatrix} m_{G_1} & x_1 \\ x_1 & m_{(s\bar{s})_1} \end{pmatrix} \oplus \dots$$

- The relationship between the mixing angle θ and the mixing energy x can be expressed as:

$$\sin\theta = \frac{x}{\Delta} + \mathcal{O}\left(\frac{x^3}{\Delta^3}\right),$$

$$\Delta = m_G - m_{(s\bar{s})}$$

Mixing

- The mixing energy can be extracted from the two-point correlation function of the glueball and the $\bar{s}s$:

$$C_{G\mathbb{1}}(t) \approx \sqrt{Z_G Z_{\mathbb{1}}} \frac{x}{m_{s\bar{s}} - m_G} \left(e^{-m_{f_0} t} - e^{-m_g t}\right)$$

- By combining the two-point correlation functions $C_{GG}(t)$ (for the glueball) and $C_{\mathbb{1}\mathbb{1}}(t)$ (for the $s\bar{s}$ state), one can obtain the normalization constants Z_G and $Z_{\mathbb{1}}$, as well as the masses m_{f_0} and m_G.
- With these parameters, one can fit the $C_{G\mathbb{1}}(t)$ to extract the mixing energy and the corresponding mixing angle.

Mixing

- We have applied this method using configurations with a single flavor of sea quark, specifically setting $m_{sea} = m_c$, to compute the mixing of the pseudoscalar glueball with η_c[Phys.Lett.B 827 (2022) 136960]:

ensemble	Γ	$[t_l, t_h]_{CC}$	$[t_l, t_h]_{GG}$	$[t_l, t_h]_{GC}$	χ^2/dof	m_{η_1}(MeV)	m_{g_1}(MeV)	θ_1	x_1(MeV)
	γ_5	[10, 25]	[2, 18]	[2, 25]	1.1	2705(2)	2289(50)	6.8(9)°	49(9)
I	$\gamma_5\gamma_4$	[10, 25]	[2, 18]	[2, 30]	0.98	2701(1)	2283(51)	6.5(9)°	48(9)
	avg.	–	–	–	–	2703(1)	2286(50)	6.6(9)°	48(9)
	γ_5	[13, 30]	[3, 15]	[2, 20]	1.1	3028(8)	2261(74)	4.5(6)°	60(10)
II	$\gamma_5\gamma_4$	[13, 30]	[2, 15]	[1, 30]	1.1	3031(3)	2348(47)	3.9(3)°	47(5)
	avg.	–	–	–	–	3031(3)	2323(55)	4.3(4)°	49(6)

Mixing

- In the literature [Phys. Rev. D 107, 094510 (2023)], a similar method was employed using two-flavor dynamical configurations to compute the mixing of the pseudoscalar glueball with the η meson.

Γ	$[t_l, t_h]_\Gamma$	$[t_l, t_h]_{GG}$	$[t_l, t_h]_{G\Gamma}$	χ^2/dof	$m_\eta a_t$	$m_G a_t$	$\lvert x_1 \rvert a_t$	$\lvert\theta\rvert$
γ_5	[9, 30]	[1, 14]	[3, 30]	0.96	0.10358(84)	0.3607(75)	0.0155(22)	3.46(46)°
$\gamma_4\gamma_5$	[5, 30]	[1, 14]	[0, 30]	0.15	0.10358(84)	0.3607(75)	0.0112(55)	2.5(1.2)°

- How about the scalar case?

Lattice setup

- Therefore, we recently used two-flavor anisotropic configurations to calculate the mixing of the scalar glueball with the scalar $s\bar{s}$ meson.

$L^3 \times T$	β	a_t^{-1}GeV	ξ	m_π(MeV)	N_{cfg}
16×128	2.4	6.66	5.0	686	4893

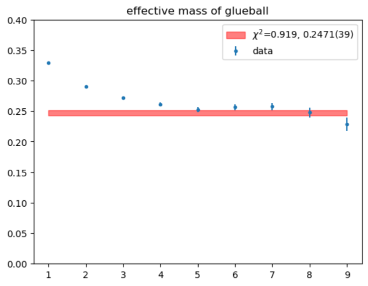
Table: The parameters of configurations

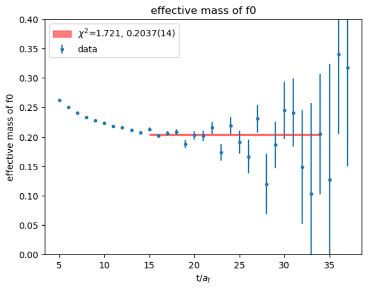
Two-point function

- The correlation functions of glueball and scalar $s\bar{s}$:

$$
\begin{aligned}
C_{GG}(\vec{p},t) &= \langle \mathcal{O}_G(t,\vec{p})\mathcal{O}_G^\dagger(0,\vec{p})\rangle \\
&= \sum_n Z_G(e^{-E_G t} + e^{-E_G(T-t)}) \\
C_{SS}(\vec{p},t) &= \langle \mathcal{O}_{\mathbb{1}}(t)\mathcal{O}_{\mathbb{1}}(0)\rangle \\
&= -\langle G^{(s)}(0,t;\vec{p})\mathbb{1}G^{(s)}(t,0;\vec{p})\mathbb{1}\rangle \\
&+ \mathrm{Tr}\langle G^{(s)}(0,0;\vec{p})\mathbb{1}\rangle \mathrm{Tr}\langle G^{(s)}(t,t;\vec{p})\mathbb{1}\rangle \\
&= C_{ss}^{con.}(\vec{p},t) + C_{ss}^{disc.}(\vec{p},t)
\end{aligned}
$$

Two-point function



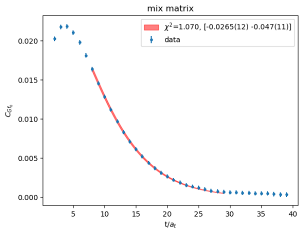


The mixing matrix

$$C_{G\mathbb{1}}(t) = \langle \mathcal{O}_{\mathbb{1}}(t)\mathcal{O}_G^\dagger(0)\rangle$$
$$= \langle G_{ss}(t, t, \vec{p})\mathbb{1}\mathcal{O}_G^\dagger(0, \vec{p})\rangle$$

The mixing angle was obtained as

$$\theta = -35.0(1.5)^\circ.$$

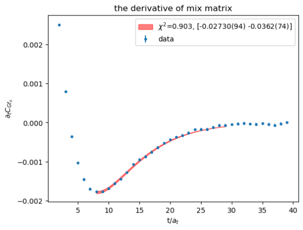


The mixing matrix

$$\partial_t C_{G\mathbb{1}}(t) \equiv \frac{1}{2a}\left(C_{G\mathbb{1}}(t+1) - C_{G\mathbb{1}}(t-1)\right)$$

The mixing angle was determined by fitting $\partial_t C_{G\mathbb{1}}(t)$

$$\theta = -36.1(1.2)^{\circ}$$



Mixing angle

- In the literature [Phys. Lett. B 826, 136906 (2022)], the mixing angles of various f_0 mesons with glueballs were obtained based on experimental data fitting. The mixing angles for $f_0(1770)$, $f_0(1710)$ with the glueball are given as follows:

$$\phi^G_{2H} = -(29 \pm 6)^\circ, \quad \phi^G_{2L} = -(20 \pm 5)^\circ$$

- Our results currently only take into account the mixing between the glueball and the pure $s\bar{s}$ meson.

Outline

summary

- Currently, the glueball mass from most lattice calculations is quite consistent. The results for quenched and unquenched cases are also not significantly different.
- We are the first to calculate the radiative decay of the scalar glueball to ϕ based on lattice QCD and estimate the stickiness parameter of the scalar glueball to be $\mathcal{O}(10^4)$.
- We have obtained the mixing angle between the scalar glueball and the scalar $s\bar{s}$ particle for the first time under the unquenched lattice QCD.
- How is a glueball defined in the unquenched approximation?
- What can be calculated on the lattice to help determine glueballs?

祝张老师生日快乐，身体健康！