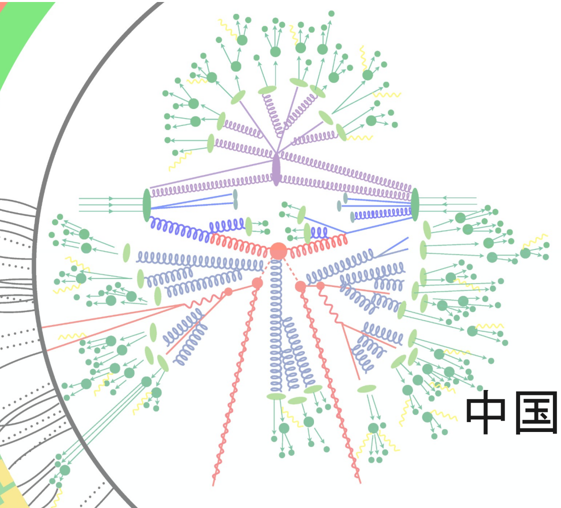
Funded by the European Union

Simulating Physics with Quantum Computing

Wenyang Qian

University of Santiago de Compostela

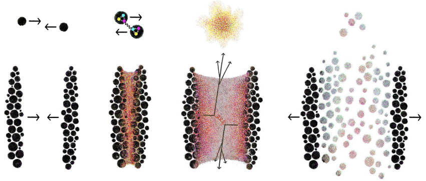


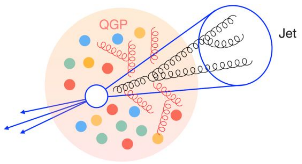
Dec 20, 2024,
中国科学院大学高能核物理课题组前沿讲座, UCAS

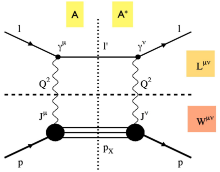
Experiments

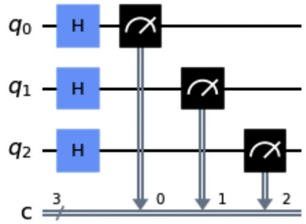
Theory
+
Phenomenology

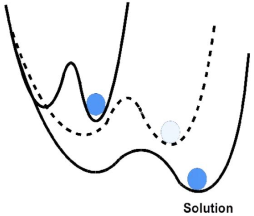
New
Technology

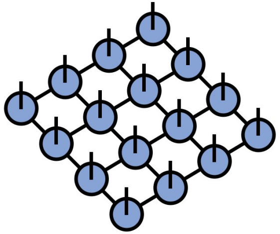












Quantum computing

The Idea: *“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical”*

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

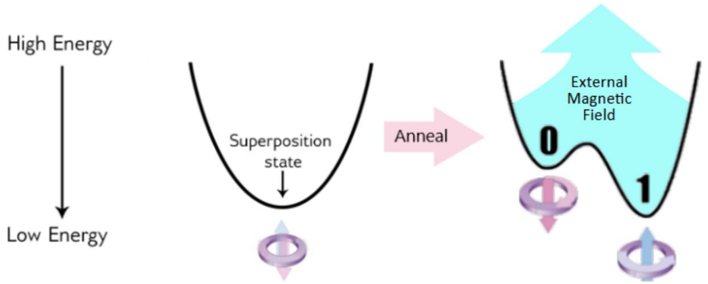
Quantum computing (QC) is a rapidly-emerging technology that harnesses the laws of quantum mechanics to solve problems.

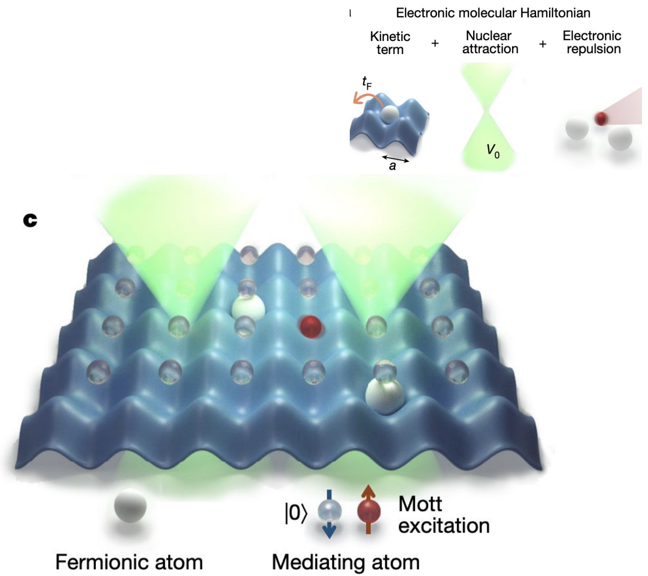
After 40+ years, we are almost there with various emerging QC technology…

Analog quantum computer

Basic idea: Mimic with physical systems using continuous variables

Quantum annealing





Successfully for optimization problems, ~5000 quantum bits (qubits)

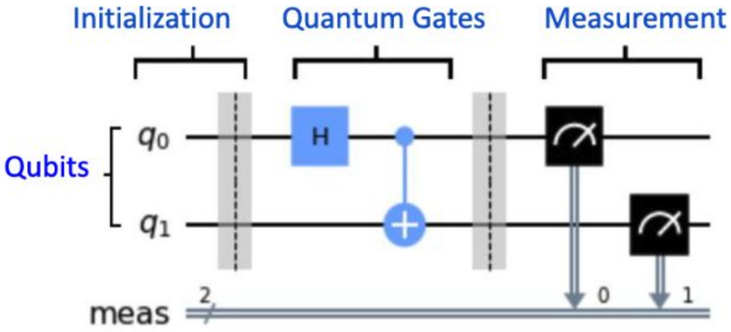
Argüello-Luengo et al, 1807.09228
ultracold atoms in optical lattices + cavity QED

Not a universal approach

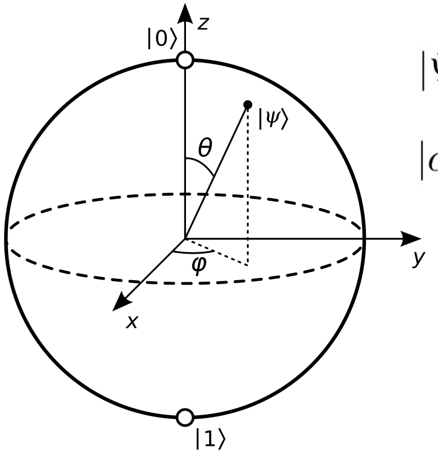
Schematic: D-Wave

Digital quantum computer

Basic idea: Spin chain = qubits (lines) + unitary gates (operators)



Qubits = Digitalization



$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

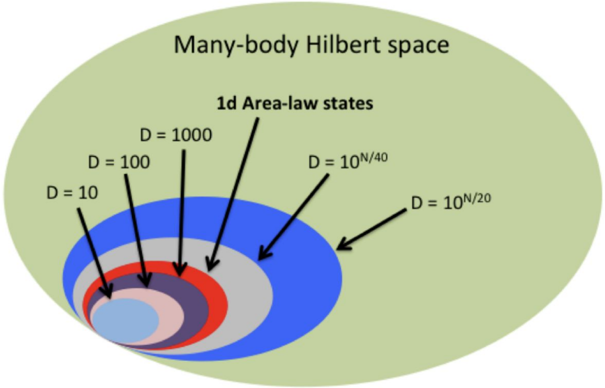
Conceptually clean for universal simulation

Noisy, intermediate-scale (NISQ) era, ~100 qubits

Tensor network (classical)

Basic idea: Efficient local representation of Hilbert space obeying the area-law

$S \sim \log D$



Very suitable for 1+1 problems

Fail for long time simulation or high entanglement

Quantum computing

Here, we focus on the **universal**, **digital** quantum computing.

Classical computing

- classical bit: 0, 1
- classical gates: logic gates
- deterministic

Quantum computing

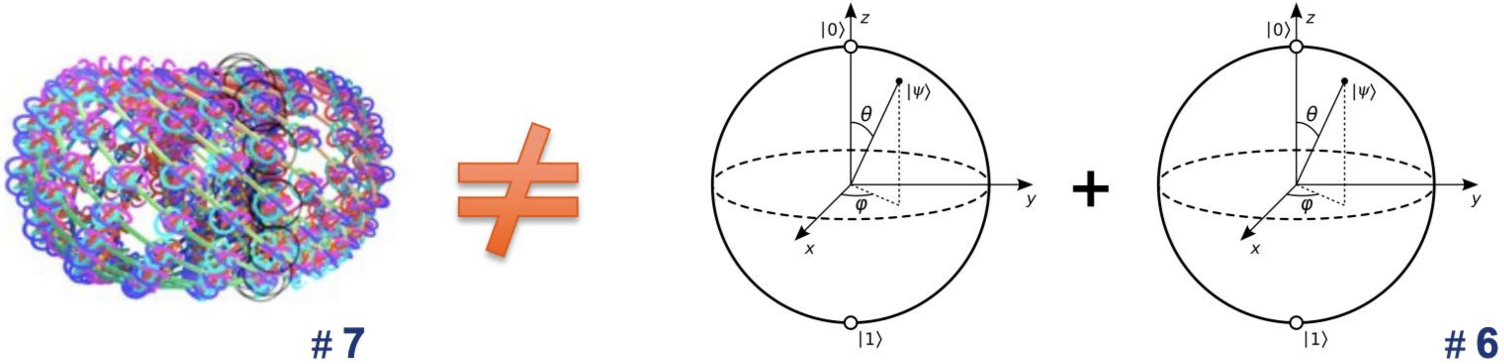
- quantum bit (qubit): $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- quantum gates: **unitary operators**
- probabilistic (entanglement & superposition)

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Quantum computing

QC really **outclass** CC in multiple qubit states due to entanglement!



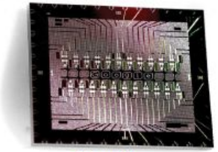
D.o.f. for n qubits:

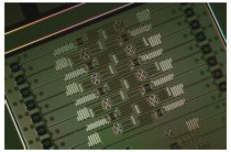
$$2^n\text{(variables)} \times 2\text{(complex)} - 1\text{(normalization constraint)} = \mathbf{2^{n+1}\text{-}1} \gg 3n$$

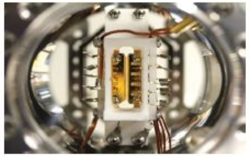
Quantum computing

We have really come a long way in past 40 years since Feynman! Feynman, "Simulating Physics with Computers" (1981)

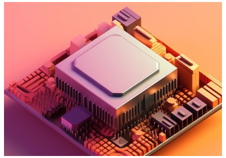
State-of-the-art: Noisy intermediate-scale quantum (NISQ) era = substantially imperfect and insufficient qubits. However, this can change fast!


Google


IBM


Rigetti


Intel


USTC

Qiskit

Cirq

XANADU

…

…

Optimist's prediction:

Neven's law = Double-exponential scaling

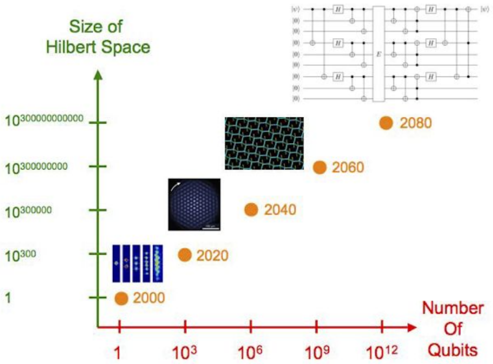
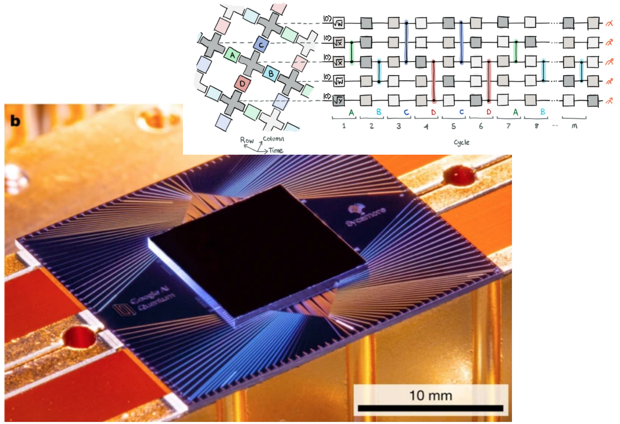


Chart: Quantum Pundit

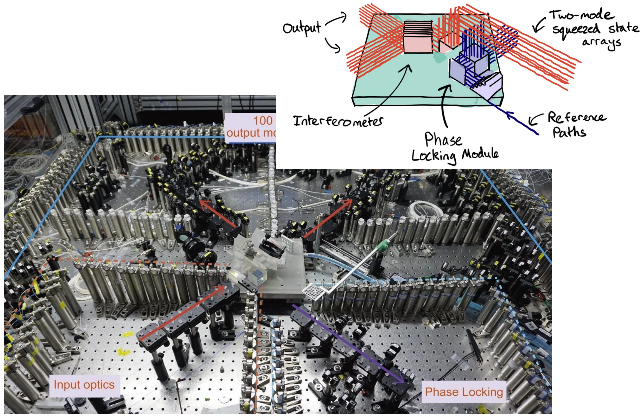
Quantum supremacy

Quantum supremacy = **anything** with a quantum device “cannot” be performed classically

Specific evidence for supremacy are found in sampling distributions!



random circuit 53 qubits, Google Quantum, 1910.11333

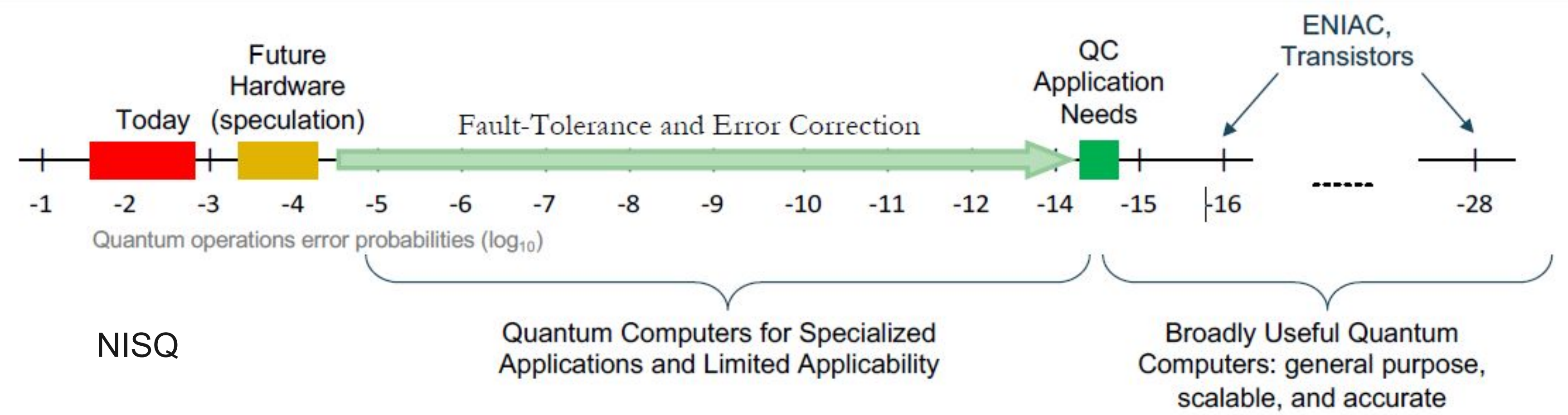


GBS 76 qubits, UTSC, 2012.01625
Schematic: Pennylane

Quantum advantage = **sth useful** involving a quantum device “cannot” be performed classically

Quantum supremacy

Simply a matter of time before QC revolutionizes the modern research (**sth useful**)



NISQ

Schematic: De Jong

2. Quantum computing in physics

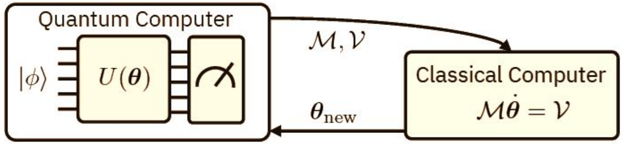
[brief highlights of several directions taking for HEP and NP]

Applications in HEP/NP

Proof of principles

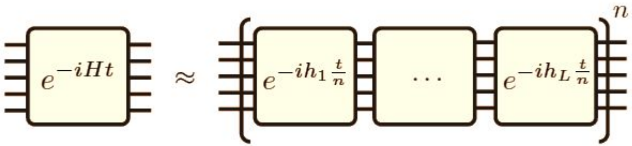
Roadmap: Bauer et al, 2204.03381; Meglio et al, 2307.03236

- **Variational Approaches**: Quantum Eigensolver, Hadronic spectrum, Partonic structure functions, Quantum Machine Learning



Peruzzo et al, 1304.3061 (2013)
Kreshchuk et al, 2011.13443
WQ et al, 2112.01927
Li et al, 2106.03865 & 2301.04179
…

- **Decomposition Approaches**: Quantum simulation algorithm, Lattice Gauge Theory, Light-front Hamiltonian, Open quantum system, Thermal field theory



Wiesner, 9603028 (1996); Zalka, 9603026; JLP, 1111.3633
De Jong et al, 2010.03571 & 2106.08394
Czajka et al, 2112.03944 & 2210.03062
Bañuls et al, 2409.16996
WQ et al., 2411.09762, 2307.01792, 2208.06750
…

- **Quantum Search Approaches**: Grover search, Amplitude estimation, Quantum walk

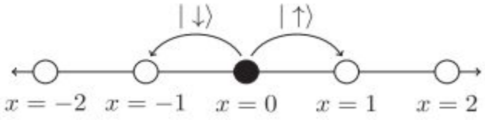


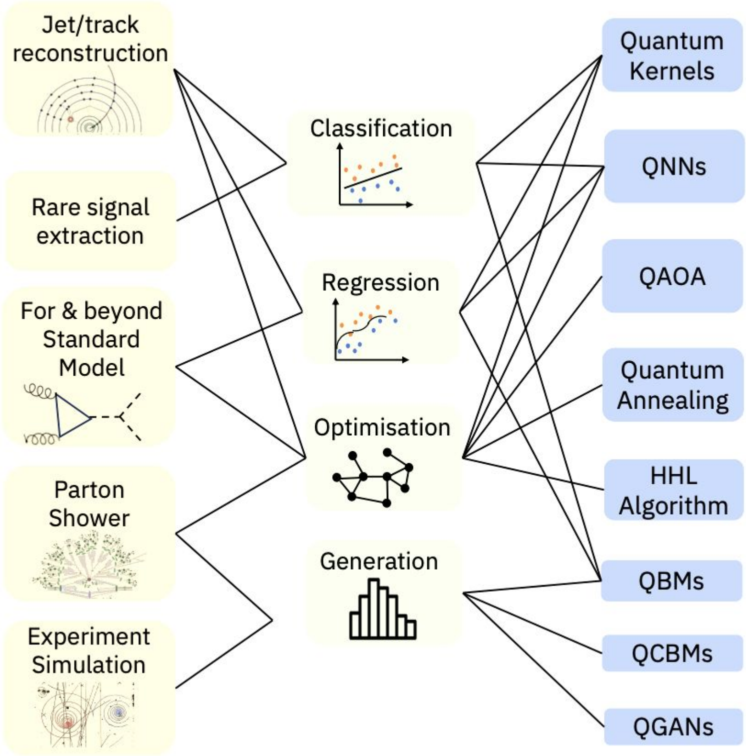
Image: Miessen (2022)

Grover, 9605043 (1996)
Wei et al, 1908.08949; Delgado & Thaler, 2205.02814
Williams et al, 2109.13975, 2207.10694
Du & WQ, 2312.16294
…

QC for Experimental Physics

Several motivations:

- LHC Physics involves large data processing
- Quantum search algorithm provides theoretical speedup



See recent/comprehensive review:

Delgado et al 2203.08805
Di Meglio et al, 2307.03236

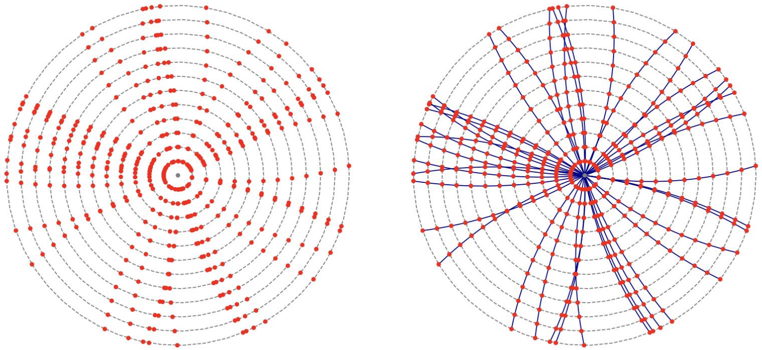
Tracking particles

Track reconstruction with Quadratic Unconstrained Binary Optimization (QUBO) using quantum annealing to High Luminosity LHC

Zlokapa1 et al, 1908.04475

Quantum speedup to recover charge particle trajectories using quantum search algorithm

Magano et al, 2104.11583



(a) Input. (b) Output.

Q-Search principle:
Grover, 9605043 (1996)
Brassard et al, 0005055 (2000)

Jet clustering

Digital quantum algorithm to tackle event reconstruction and jet clustering

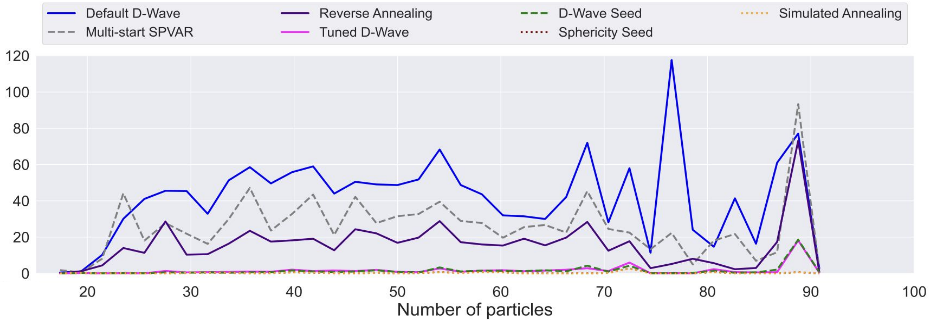
Jet algorithm for thrust via

Wei et al, 1908.08949; Delgado, Thaler, 2205.02814

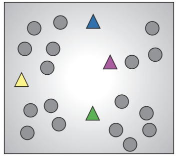
- QUBO formulation (quantum annealing)
- Grover search (digital)

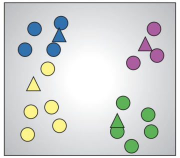
$$T(\hat{n}) = \frac{\sum_{i=1}^{N} |\hat{n} \cdot \vec{p_i}|}{\sum_{i=1}^{N} |\vec{p_i}|}$$

$e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$



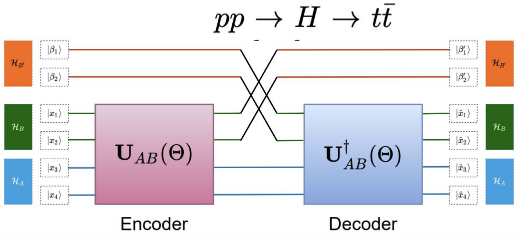
Quantum k-means Pires et al, 2101.05618





Quantum machine learning

Extending classical ML with quantum data encoding



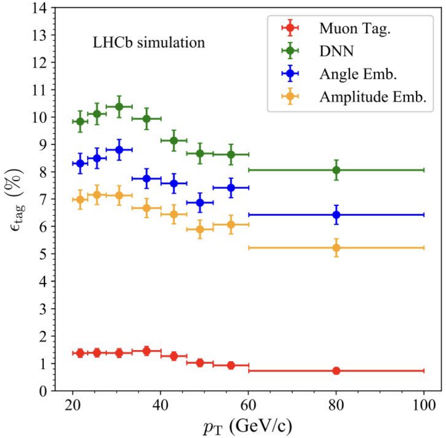
Anomaly detection with parameterized circuits (PQC) and autoencoder

Alvi, Bauer, Nachman, 2206.08391
Ngairangbam, Spannowsky, Takeuchi, 2112.04958

Quantum Generative Adversarial Networks   Chang et al, 2101.11132

B-jet charge tagging in LHCb simulation   Gianelle et al, 2202.13943

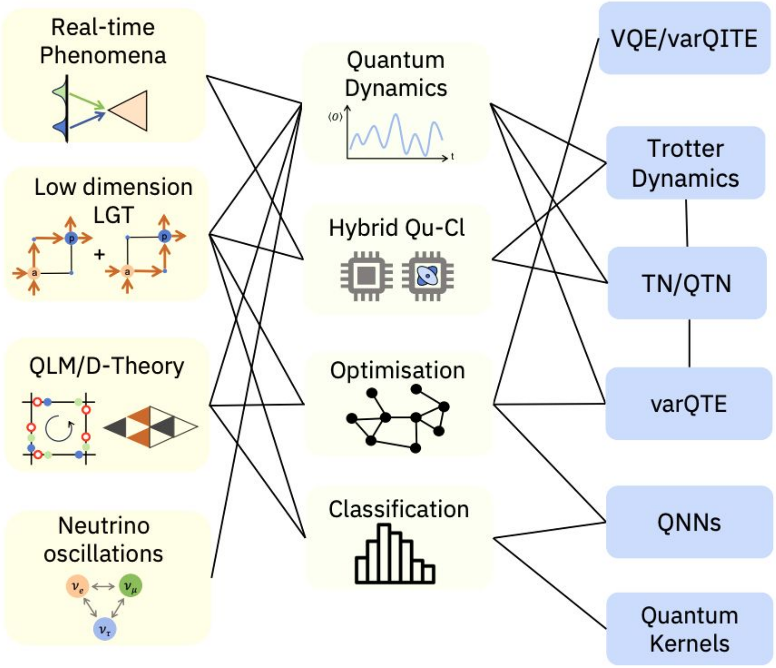
$$\epsilon_{\text{tag}} = \epsilon_{\text{eff}}(2a-1)^2$$



QC for Theories & Phenomenologies

Several motivations:

- Classical simulation encounters inherent problems and high problem complexity
- Quantum simulation algorithm provides an ultimate path to simulate quantum field theory



See recent/comprehensive review:

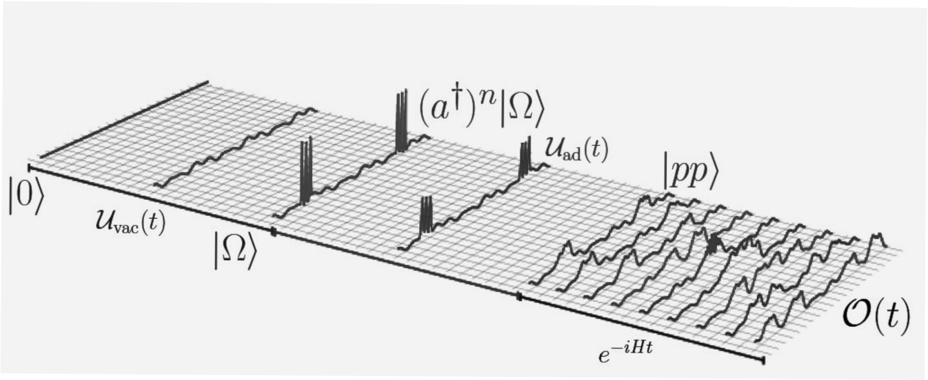
Bauer et al, 2204.03381
Di Meglio et al, 2307.03236

Prototypical task

Quantum simulation of quantum field theory = perform “**ideal experiments**” on quantum computer

Jordan, Lee, & Preskill,
1111.3633, 1401.7115, 1703.00454

- Prepare initial state in quantum computer
- Evolve state forward in time using Hamiltonian, for some specified time interval
- Measure observables by simulating measurement performed in idealized lab

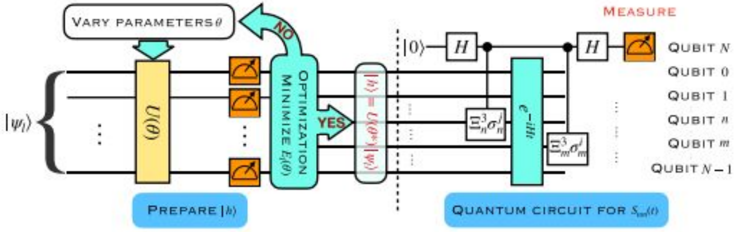


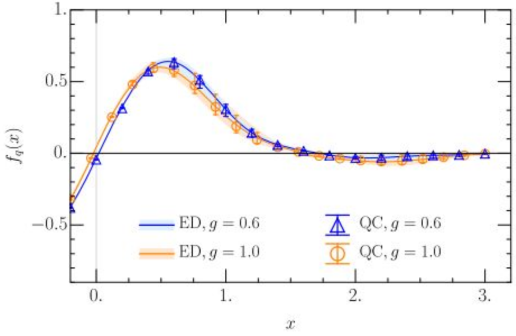
Extracting partonic functions

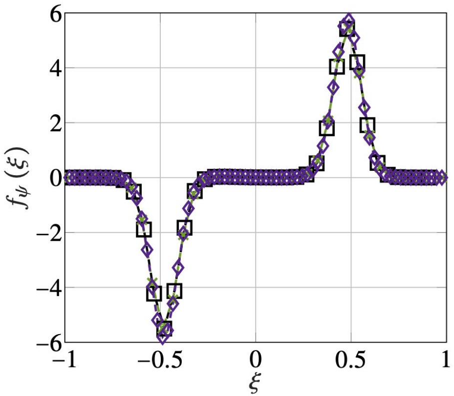
Quark parton distribution function (PDF) evaluated from flavored hadronic states

Lamm et al, 1908.10439
Mueller, Tarasov, Venugopalan, 1908.07051
Li et al, 2106.03865, 2207.13258, 2406.05683
Banuls et al, 2409.16996

$$f_{q/h}(x) = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h|e^{iHt}\bar{\psi}(0,-z)e^{-iHt}\gamma^+\psi(0,0)|h\rangle$$







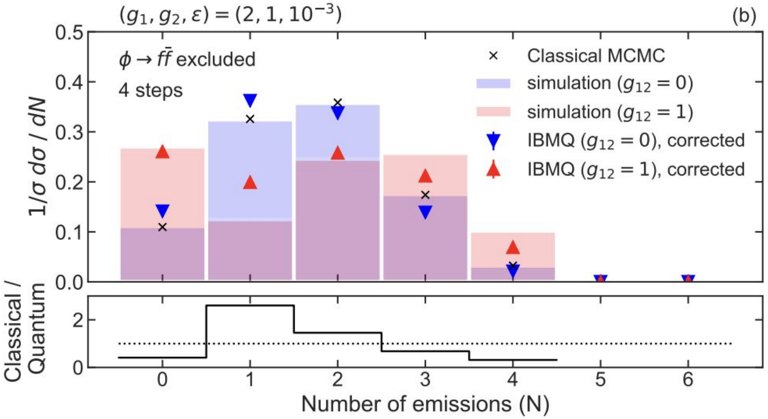
Hadron state preparation using Variational approaches (VQE)
Other methods include Adiabatic and Tensor Networks

Simulating parton showers

Quantum algorithm for HEP simulation of parton shower to include quantum interference

Nachman et al, 1904.03196

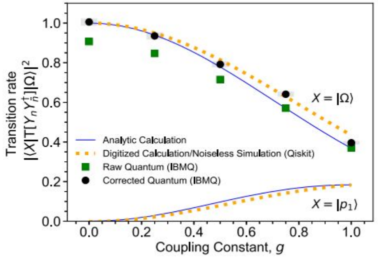
$$\mathcal{L} = \bar{f}_1(i\partial\!\!\!/ + m_1)f_1 + \bar{f}_2(i\partial\!\!\!/ + m_2)f_2 + (\partial_\mu\phi)^2 + g_1\bar{f}_1 f_1\phi + g_2\bar{f}_2 f_2\phi + g_{12}\left[\bar{f}_1 f_2 + \bar{f}_2 f_1\right]\phi$$



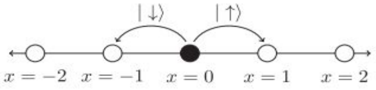
Simulating soft functions from EFT

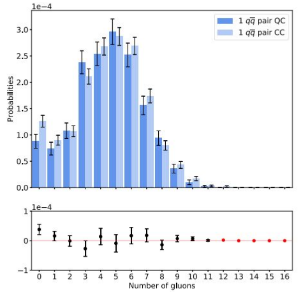
Bauer et al, 2102.05044

$$\sigma = H \otimes J_1 \otimes \ldots \otimes J_n \otimes \boxed{S}.$$



Quantum walk approach to simulate parton showers

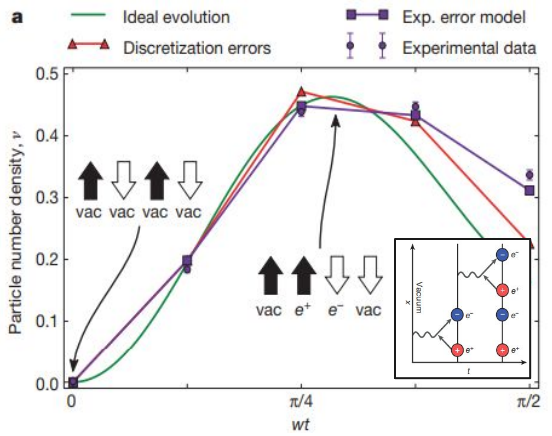




Williams et al, 2109.13975, 2207.10694

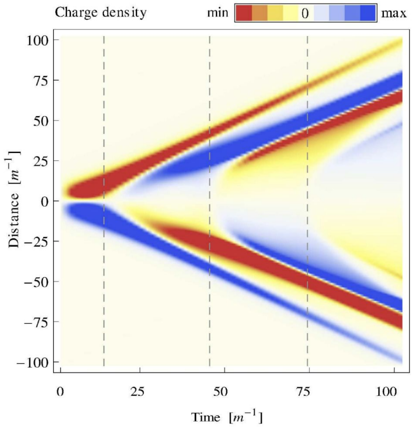
Pair production and more

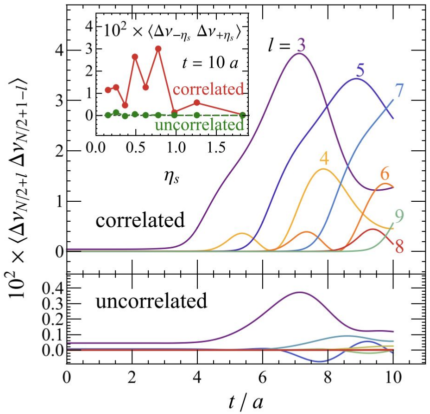
Pair-production from vacuum



Martinez et al, 1605.04570

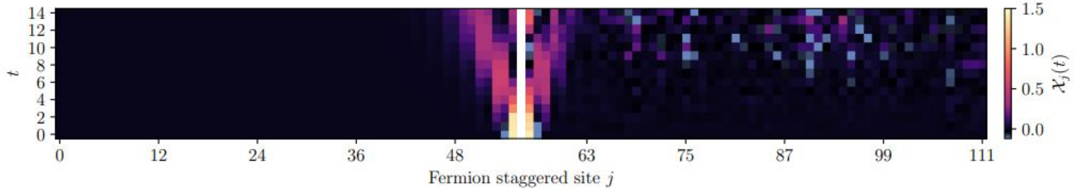
QCD string breaking and external source modification





Hebenstreit, Berges, Gelfand, 1307.4619
Kasper et al, 1506.01238

Florio et al, 2301.11991



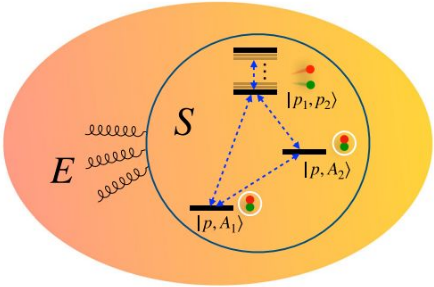
Hadron state preparation and evolution on 112 qubits Farrell et al, 2401.08044

Non-equilibrium dynamics at finite temperature

Simulating hard probes in QGP as open system via Lindblad equation — De Jong et al, 2010.03571, 2106.08394

Open quantum system formulation for quarkonia, jets, etc — Blaizot & Escobedo, 1711.10812, 1803.07996

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -i\left[H_{S1}(t) + H_L, \rho_S(t)\right] + \sum_{j=1}^{m}\left(L_j\rho_S(t)L_j^\dagger - \frac{1}{2}\left\{L_j^\dagger L_j, \rho_S(t)\right\}\right)$$



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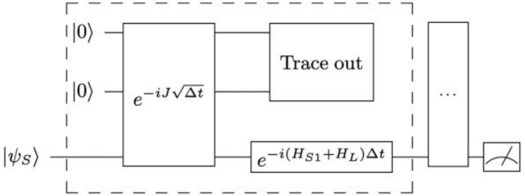
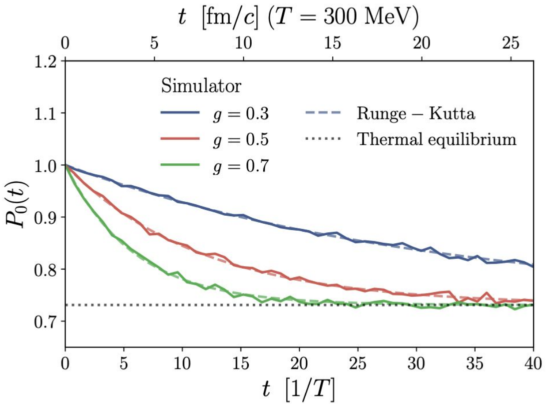


Cleve & Wang, 1612.09512



3. Quantum simulation of hadron structures

[from 3+1 effective theory to 1+1 field theory in continuum]

WQ, Jia, Li, Vary, Phys..Rev.C, 2005.13806
WQ, Basili, Pal, Luecke, Vary; Phys.Rev.Research, 2112.01927
Kang, Moran, Nguyen, WQ (to appear)
…

Effective Hamiltonian for hadron

One may also consider effective (3+1)d Hamiltonian on the light front.

Valence $|q\bar{q}\rangle$ for light mesons

WQ, Jia, Li, Vary, 2005.13806

$$H_{\mathrm{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_{\mathrm{eff}}} + V_g + H_{\gamma_5}$$

Effective potentials lifted with higher Fock sectors see Xingbo's group, CAS

For given set of basis states, the Hamiltonian operator can be written in creation and annihilation operators for those modes:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \cdots = \sum_{ij} h_{ij}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{4}\sum_{ijkl} h_{ijkl}\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l + \cdots$$

Qubit encoding: One-hot encoding and Binary encoding

$O(N)$ $O(\log N)$

$$(N, N) = (2^n, 2^n) \rightarrow H_q = \sum_\alpha c_\alpha P_\alpha$$

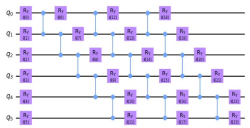
Jordan & Wigner (1928); Kreshchuk et al, 2002.04016

Variational approaches are used to solve the hadronic mass spectrum

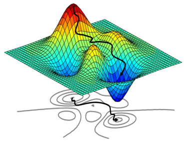
Variational quantum algorithms

Peruzzo, et al, 1304.3061 (2014)
Nakanishi, Mitarai, Fujii, 1810.09434 (2019)

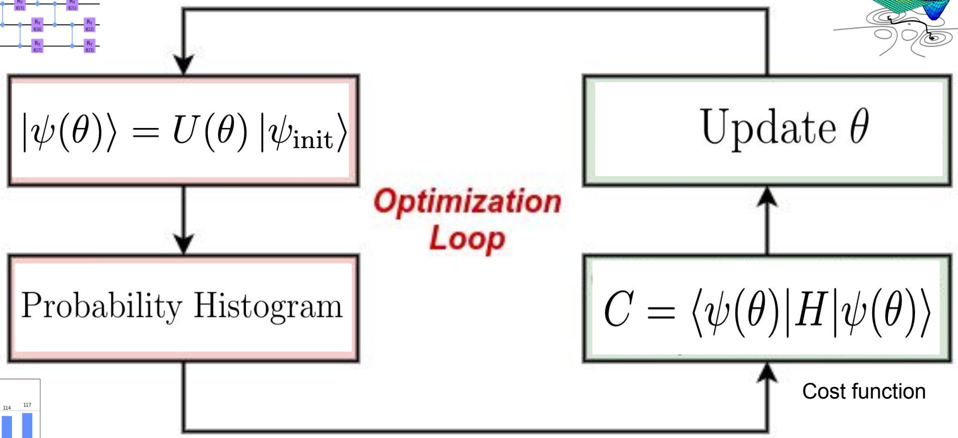
Based on variational principle, build parameterized wave functions $\langle\psi(\theta)|H|\psi(\theta)\rangle \geq \langle E_0\rangle = \langle\psi_0|H|\psi_0\rangle$



Problem-inspired
Heuristic ansatz

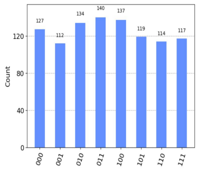


Quantum simulation



Classical optimization

Cost function



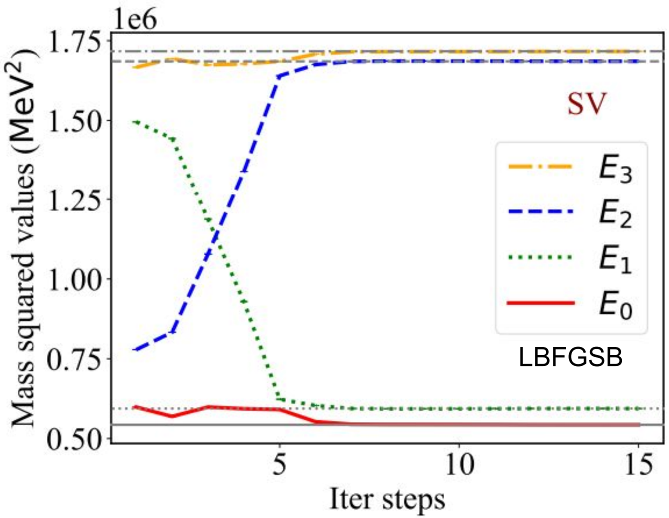
Hybrid approach:
Complementary Advantage for QC and CC

Variational quantum eigensolver (VQE) for ground state
Subspace-search VQE (SSVQE) for full spectrum

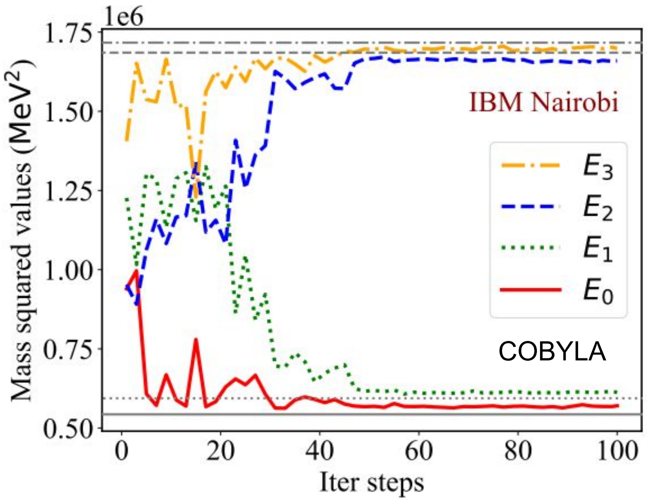
Hadronic spectrum

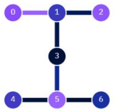
WQ, Basili, Pal, Luecke, Vary; 2112.01927

Noiseless simulation (2 qubits)



IBM Quantum devices **with Noise-Mitigation**

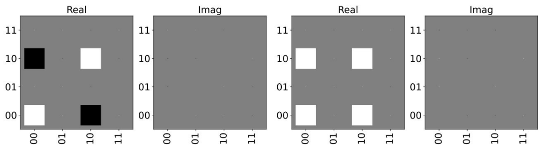




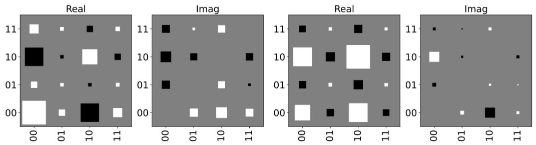
IBM Nairobi:

7 qubit
32 QV

Qubit representation (density matrix) for lowest two states: $D_{ij} = |\psi_i\rangle \langle\psi_j|$



(a) SV, π meson (b) SV, ρ meson



(e) Nairobi, π meson (f) Nairobi, ρ meson

Hadronic observables

Li, Maris, Vary, 1704.06968
WQ, Basili, Pal, Luecke, Vary, 2112.01927

Using light-front wave functions (LFWF) on qubits to compute various observables with projection ops

Difficulty of simulating Wilson line is lifted for using light-cone gauge

Collins (2011)
Kreshchuk et al, 2002.04016

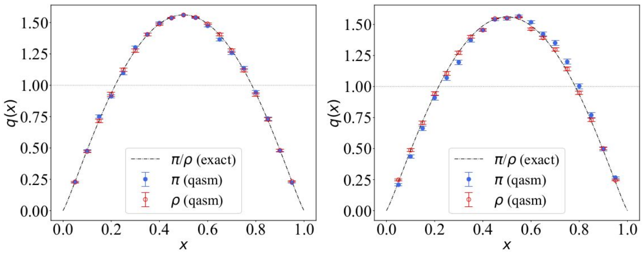
$$q(x;\mu) = \frac{1}{x(1-x)}\sum_{s\bar{s}}\int\frac{\mathrm{d}^2\boldsymbol{k}_\perp}{2(2\pi)^3}|\psi_{s\bar{s}}^{(m_j=0)}(x,\boldsymbol{k}_\perp)|^2$$
$$\equiv \frac{1}{4\pi}\sum_{s\bar{s}}\sum_{nm}\sum_{l\bar{l}}\tilde{\psi}_{s\bar{s}}^{*(m_j=0)}(n,m,\bar{l})\tilde{\psi}_{s\bar{s}}^{(m_j=0)}(n,m,l)\chi_l(x)\chi_{\bar{l}}(x)$$

$$q(x) = \sum_{s\bar{s}}\sum_{nm}\sum_{l\bar{l}}\langle\psi(\vec{\theta})|\,\hat{O}_{\mathrm{pdf}}(x)\,|\psi(\vec{\theta})\rangle$$

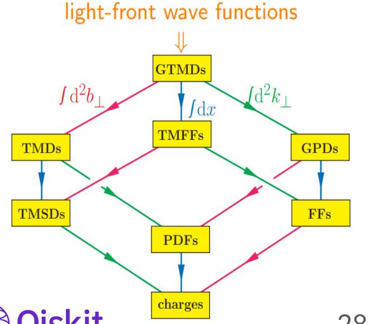
$$f_{\mathrm{P,V}} = 2\sqrt{2N_c}\int_0^1\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^3}\psi_{\uparrow\downarrow\mp\downarrow\uparrow}^{(m_j=0)}(x,\boldsymbol{k}_\perp)$$
$$\equiv \frac{\kappa\sqrt{N_c}}{\pi}\sum_{nl}(-1)^n C_l(m_q,\kappa)\left(\tilde{\psi}_{\uparrow\downarrow}^{(m_j=0)}(n,0,l)\mp\tilde{\psi}_{\downarrow\uparrow}^{(m_j=0)}(n,0,l)\right)$$

$$f_{\mathrm{P,V}} \propto \;|\langle\nu_{\mathrm{P,V}}|\psi(\vec{\theta})\rangle|$$

$N_{\max}$	$L_{\max}$	Decay constants	Exact result (MeV)	qasm sim (MeV)
1	1	f_π	178.18	178.17 ± 1.97
		f_ρ	178.18	178.17 ± 1.97
4	1	f_π	193.71	194.28 ± 15.49
		f_ρ	231.00	225.72 ± 13.44



(a) PDFs at $N_{\max} = L_{\max} = 1$
(b) PDFs at $N_{\max} = 4, L_{\max} = 1$

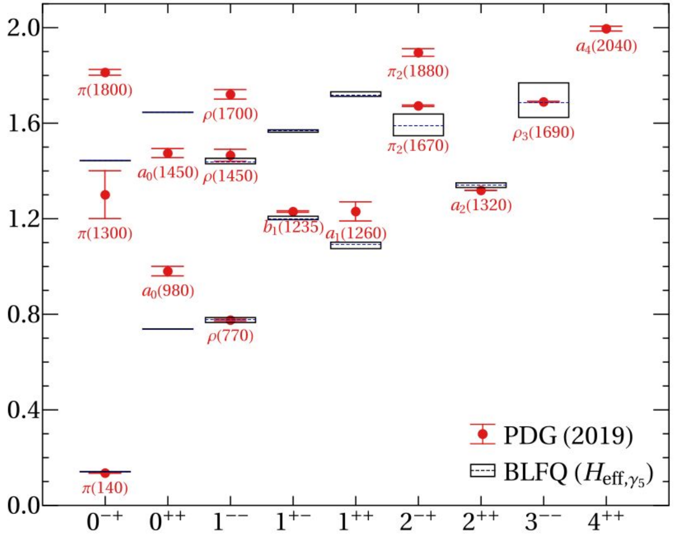


Qiskit

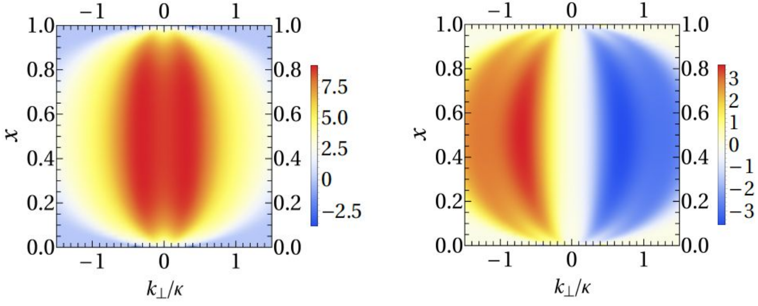
Go beyond classical results

WQ, Jia, Li, Vary, 2005.13806

Meson spectrum

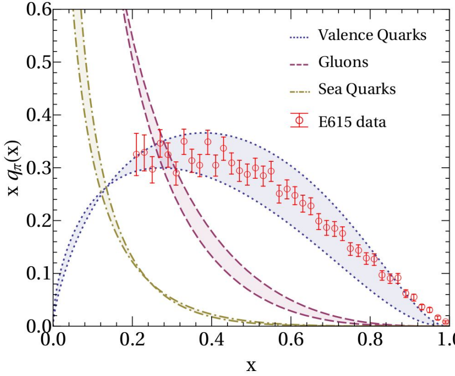


Pion Wave Functions

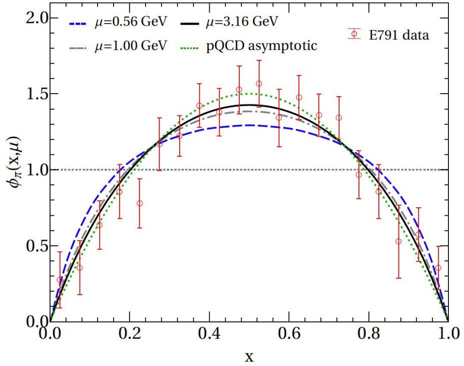


(a) π: $\psi_{\uparrow\downarrow-\downarrow\uparrow}(k_\perp, x)$ (b) π: $\psi_{\uparrow\uparrow}(k_\perp, x) = \psi_{\downarrow\downarrow}(k_\perp, x)$

Parton Distribution Function



Distribution Amplitude



Recent applications:
Anisotropic flow, Li, WQ, Wu, Zhang, 2304.06557

Parton structure functions

Parton distribution functions (PDF) is highly relevant to the core QCD programs.

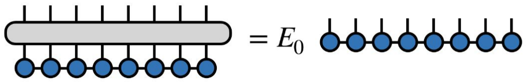
Direct computation of the PDFs remains difficult as they are non-perturbative quantities defined as real-time correlators of quark and gluon fields.

$$f_\psi(\xi) = \int_{-\infty}^{\infty} \frac{dt}{4\pi} e^{-it\xi\vec{n}\cdot\vec{P}} \langle P | \overline{\psi}(t\vec{n}) W(t\vec{n} \leftarrow \vec{0}) \vec{n} \cdot \vec{\gamma} \psi(\vec{0}) | P \rangle_c$$

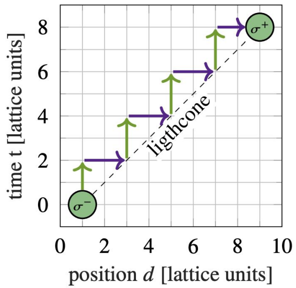
Two main difficulties

- Hadron state preparation
- **Wilson line**

Similarly for the light-cone distribution amplitude, etc



Density Matrix Renormalization Group Algorithm (DMRG)
White, 1992



Banuls, Cichy, Lin, Schneider, 2409.16996

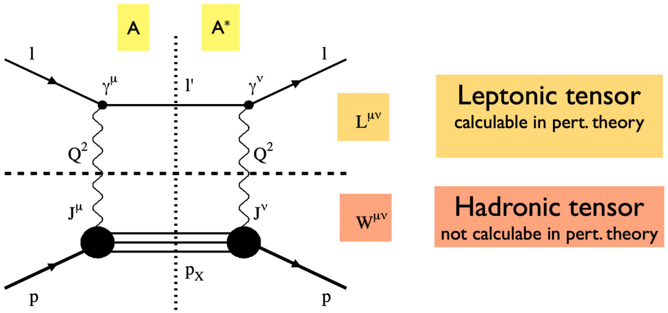
Hadronic tensor

Schienbein, CTEQ-MCnet 2021

Hadronic tensor (HT) is another important non-perturbative quantity. It is responsible for computing the cross section.

$$d\sigma \sim |A|^2 \sim L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu}(P,q) = \frac{1}{4\pi} \int d^2y e^{iq\cdot y} \langle \Psi(P)| J^\mu(y) J^\nu(0) | \Psi(P) \rangle \,,$$

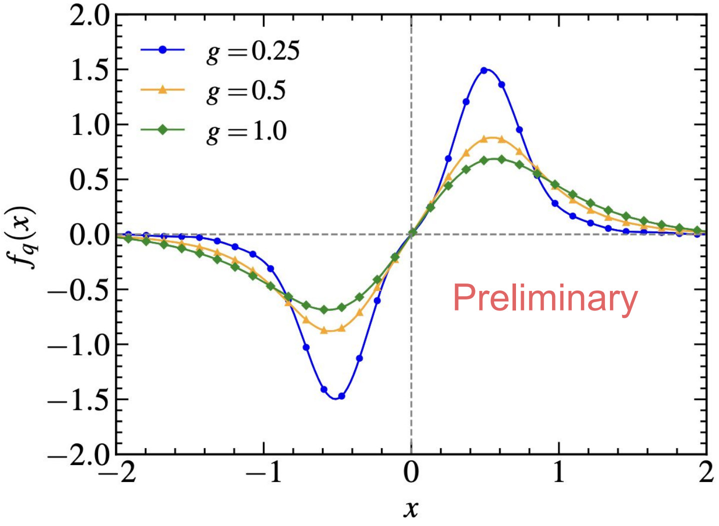


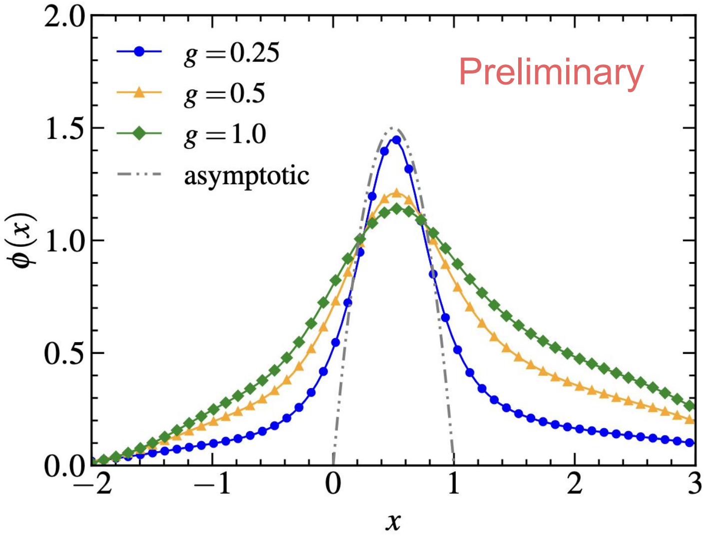
Its calculation directly relates long/trans structure function

We are currently working on PDA, LCDA, HT in the same setting with tensor network + quantum computing.

Tensor network simulation results

Simulation results on PDA and LCDA in the same setting with tensor network on 102 qubits.





Exploratory calculation in 1+1 Nambu–Jona-Lasinio with **ALL** Fock sectors

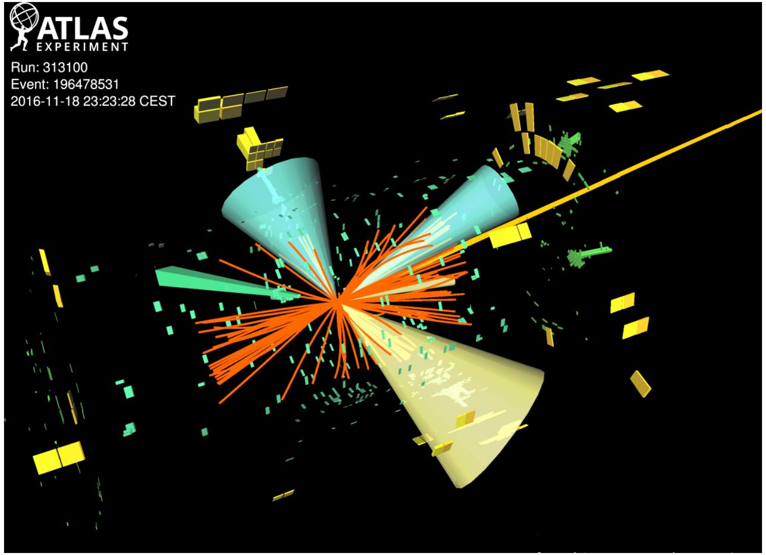
Ongoing work with Zhongbo, Peter, Noah

4. Quantum simulation of jets in heavy-ion collisions

[using (3+1) light-front Hamiltonian formalism]

Barata, Du, Li, WQ, Salgado, Phys.Rev.D, 2208.06750
Barata, Du, Li, WQ, Salgado, Phys.Rev.D, 2307.01792
WQ, Li, Kreshchuk, Salgado, 2411.09762
…

Quantum jet simulation: Big picture



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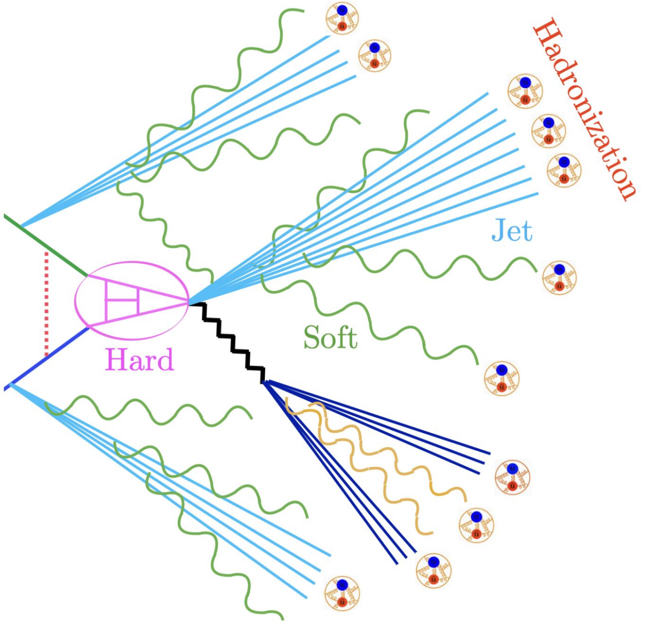


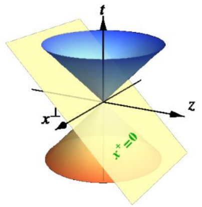
TeV → MeV

In QGP medium

Quantum jet simulation: Method

Light-front QCD Hamiltonian + Classical background field

- First-principle method formulated in the front form
- Hamiltonian is used to study hadron structure and time evolution alike



front form

$x^+ \triangleq x^0 + x^3$

Natural to extend from classical simulation to quantum simulation

Classical simulation

Electron in laser field Zhao et al, 1303.3273

Ultrarelativistic quark-nucleus scattering Li et al, 2002.09757

Scattering and gluon emission in a color field

Jet in Glasma field Li, Lappi, Zhao, 2107.02225
Li et al, 2305.12490

… Ongoing work by Avramescu et al

Quantum simulation

Nuclear inelastic scattering Du et al, 2006.01369

Strategy to Jet quenching parameter Barata, Salgado, 2104.04661

Medium-induced QCD jet

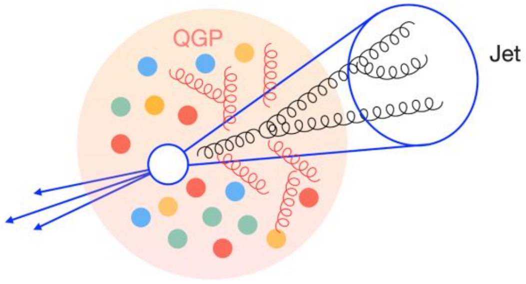
… Barata et al, 2208.06750, 2307.01792
Yao, 2205.07902
Wu et al, 2404.00819
WQ et al, 2411.09762

QCD Lagrangian

We start with the QCD lagrangian, with an external field

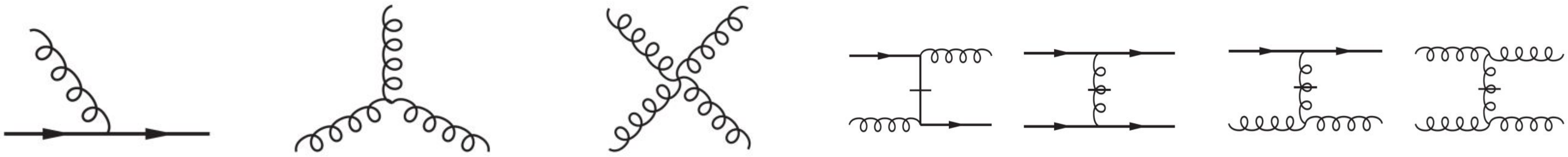
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m_q)\Psi$$

$$D^\mu \equiv \partial_\mu + ig(A^\mu + \mathcal{A}^\mu)$$

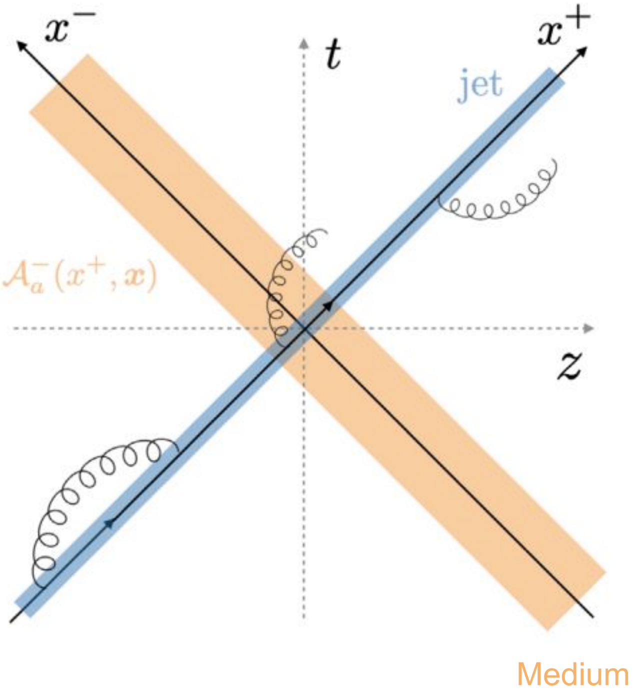


The light-front Hamiltonian is obtained by the canonical light-front quantization via the standard Legendre transformation,

For review: Brodsky, Pauli, Pinsky (1997)



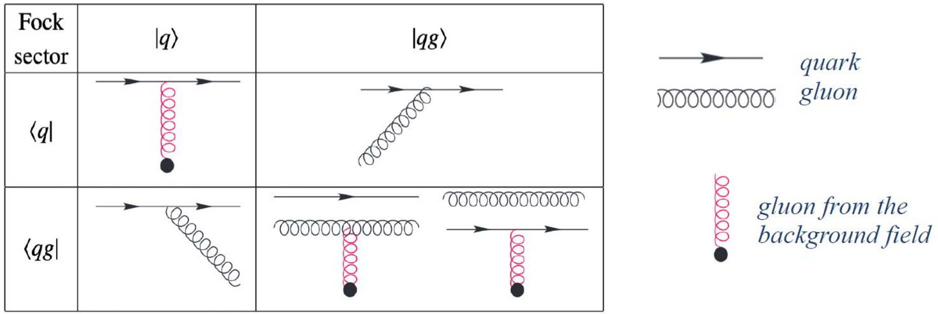
Physical setup



High-energy quark jet moving close to the light cone scattering on a dense nucleus medium

For example, light-front Hamiltonian in $|q\rangle + |qg\rangle$ Fock space

$$P^-(x^+) = P^-_{\mathrm{KE}} + V(x^+) = P^-_{\mathrm{KE}} + \left\{V_{qg} + V_{\mathcal{A}}(x^+)\right\}$$



Barata, Du, Li, WQ, Salgado, 2208.06750 & 2307.01792

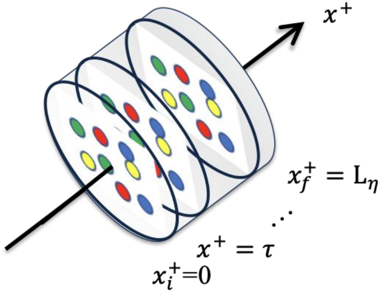
Medium and Evolution

Classical stochastic background field (to reduce problem complexity)

$$\langle\langle \rho_a(x^+,\boldsymbol{x})\rho_b(y^+,\boldsymbol{y})\rangle\rangle = g^2\mu^2\delta_{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta(x^+-y^+)$$

$$(m_g^2-\nabla_\perp^2)\mathcal{A}_a^-(x^+,\boldsymbol{x}) = \rho_a(x^+,\boldsymbol{x}) \qquad Q_s^2 \equiv \frac{C_F g^4\mu^2 L_\eta}{2\pi}$$

saturation scales



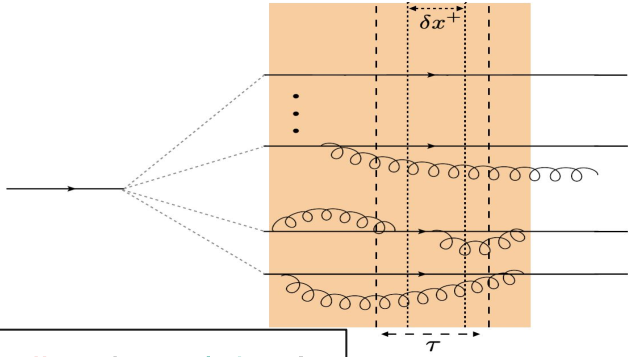
McLerran and Venugopalan, 9309289 (1993)

Jet probe evolution, decomposed as sequence of unitary operators

$$|\psi_{L_\eta}\rangle = U(L_\eta;0)\,|\psi_0\rangle \equiv \mathcal{T}_+ e^{-i\int_0^{L_\eta} \mathrm{d}x^+\, P^-(x^+)}\,|\psi_0\rangle$$

$$U(L_\eta;0) = \prod_{k=1}^{N_t} U(x_k^+;x_{k-1}^+)$$

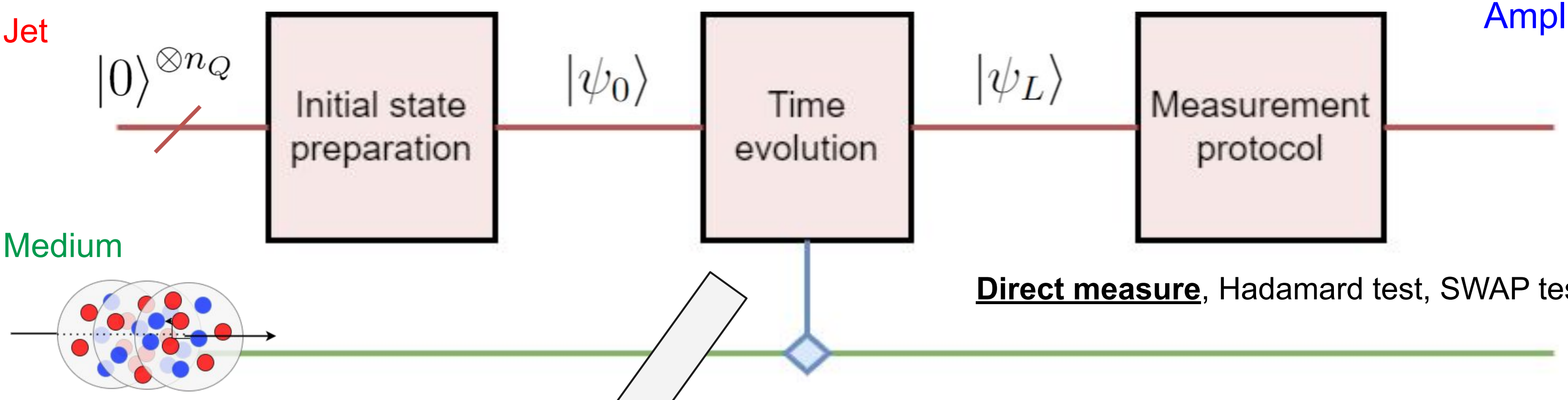
non-perturbative



Universal framework to simulate (3+1)-d QCD jet probe evolution in medium in real-time!

Quantum simulation algorithm

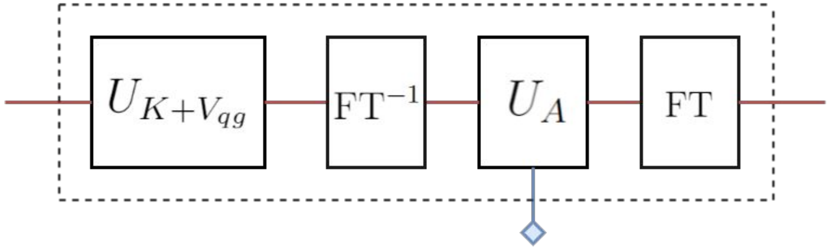
Wiesner, 9603028 (1996); Zalka, 9603026 (1996)



Amplitude level

Direct measure, Hadamard test, SWAP test, etc

many **time steps**



$$U_A(x^+, \delta x^+)\,|\boldsymbol{r}\rangle = \exp(-iV_{\mathcal{A}}\delta x^+)\,|\boldsymbol{r}\rangle$$
$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iP^-_{\mathrm{KE}}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_{V_{qg}}(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{qg}\delta x^+)\,|\boldsymbol{p}\rangle$$

FT allows efficient/sparse simulation in the respective basis

Sparse H => fewer pauli strings => less quantum gates

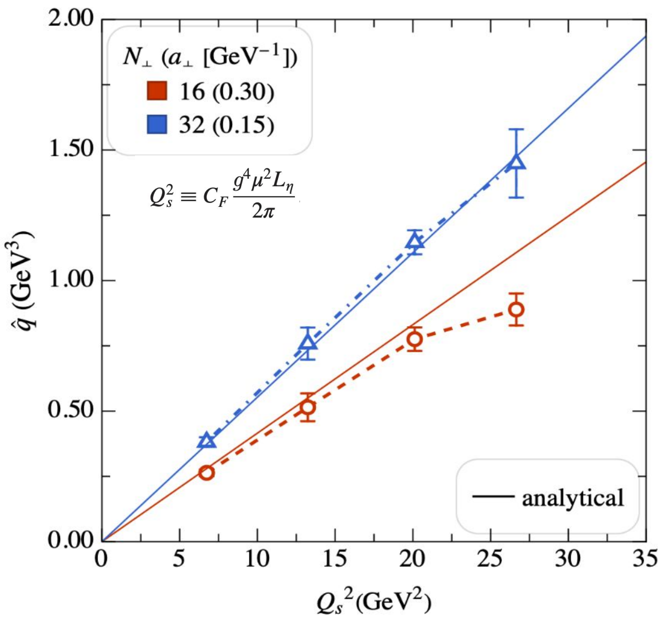
Extracting quenching parameter

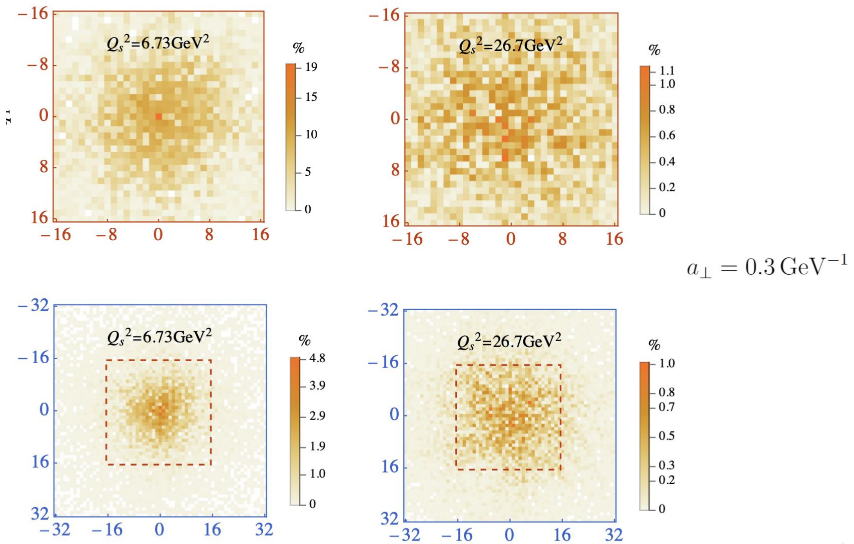
First quenching parameter calculation on QC

Barata, Du, Li, WQ, Salgado, 2208.06750

Jet quenching parameter: $\hat{q} = \frac{\Delta \langle p_\perp^2(x^+)\rangle}{\Delta x^+} = \langle \boldsymbol{p}_\perp^2 \rangle / L_\eta$

Eikonal analytical: $\sim Q_s^2 / L_\eta$





$p_\perp = [-\pi/a_\perp, \pi/a_\perp]$

$(p_x, p_y) = (0, 0) \qquad L_\eta = 50\,\mathrm{GeV}^{-1} \approx 10\,\mathrm{fm}$

Similarly done for momentum broadening at finite p+

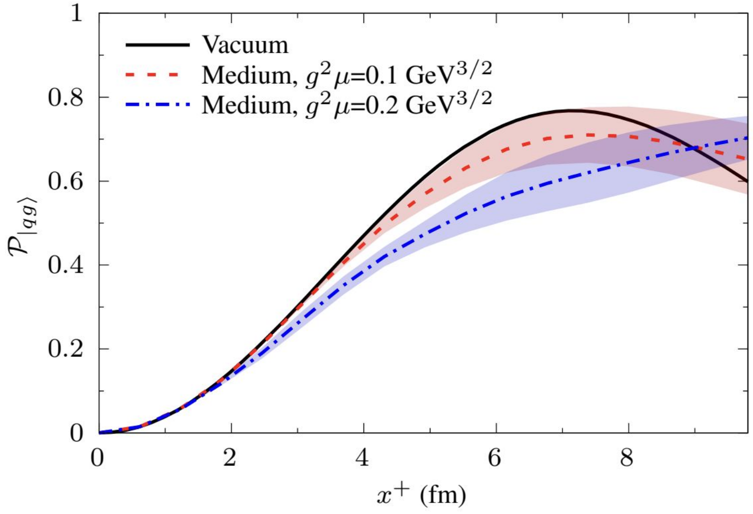
simulator, 10-12 qubits

Qiskit

Gluon production and entropy growth

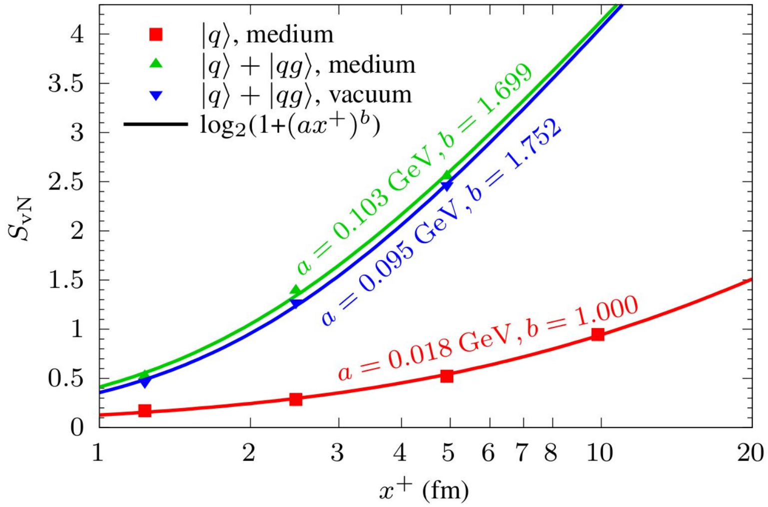
Barata, Du, Li, WQ, Salgado, 2307.01792

Gluon production in mediums
(error from stochastic medium)



Entropy expansion linear in Fock |q> and power-law in Fock |q> + |qg> with radiation

Verified in leading |q> with classical calc, Barata et al, 2305.10476



$$S_{\rm vN}(x^+) = -\mathrm{Tr}[\rho(x^+)\log\rho(x^+)]$$

simulator, 9 qubits

Qiskit

Towards efficient simulation of many more particles

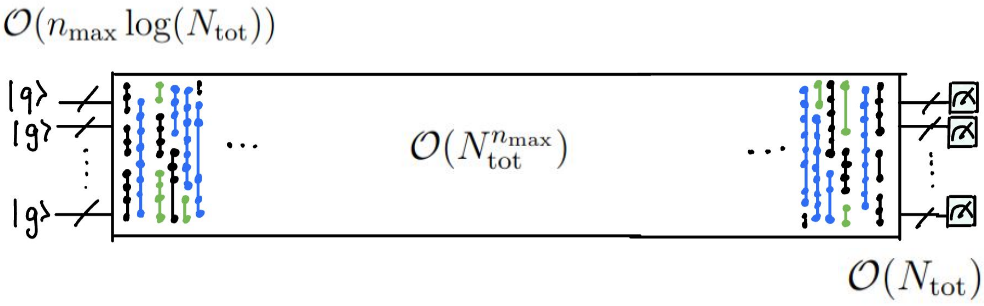
Go far beyond classical computations. But current setup is still expensive with increasing particle.

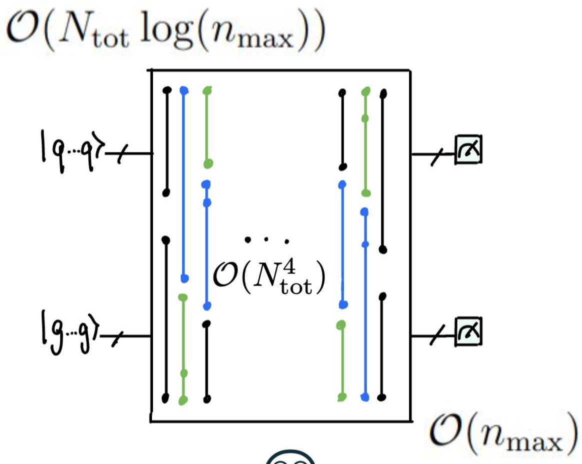
Our new solution: **Direct encoding** on the particle operators

Aspuru-Guzik et al, 0604193

- No need to evaluate Hamiltonian matrix to Pauli operators (instantly)
- Shallow + sparse quantum circuits
- Particle exchange symmetry automatically satisfied

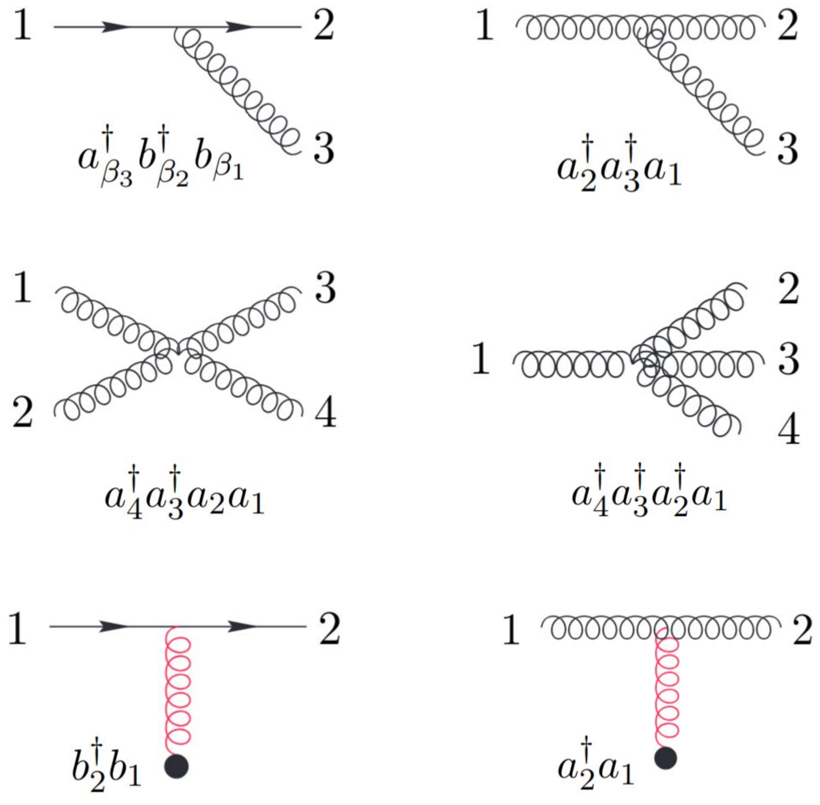
total modes N_{tot}
gluon occupancy $n_{\max}$





Polynomial scaling

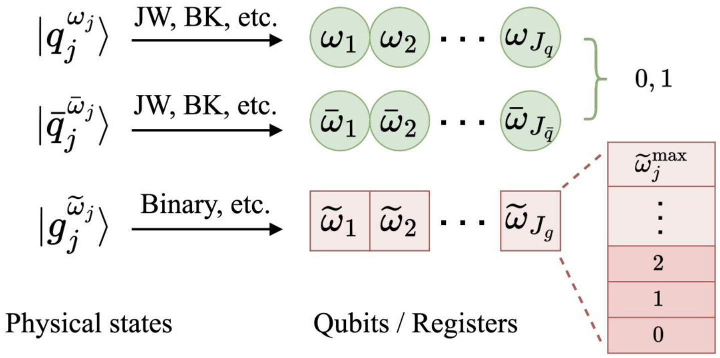
Qubit encoding of quantized operators, *“Direct encoding”*



- QCD vertices encoded into bosonic (a) and fermonic (b) creation/annihilation operators

- Operator encoding:

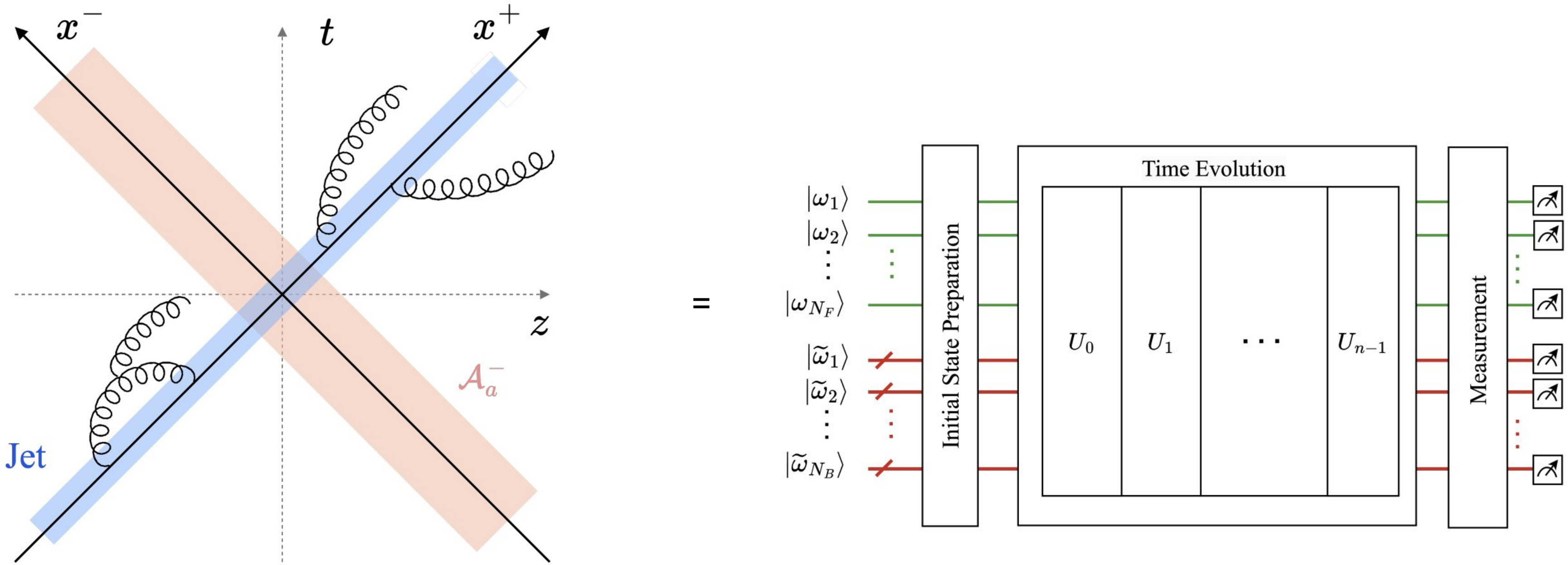
Aspuru-Guzik et al, 0604193
Sawaya et al, 1909.12847



- Vertex coefficients are instantly computed

Efficient quantum simulation of quark/gluon jet

WQ, Li, Kreshchuk, Salgado, 2411.09762



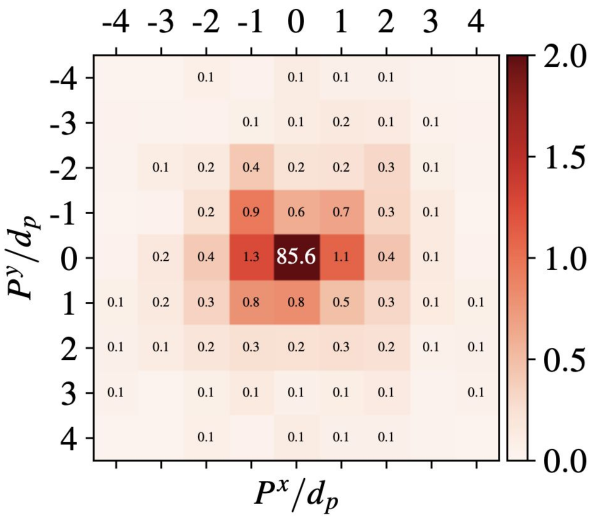
Unified, **Efficient**, **Scalable** procedure for **3+1** jet evolution

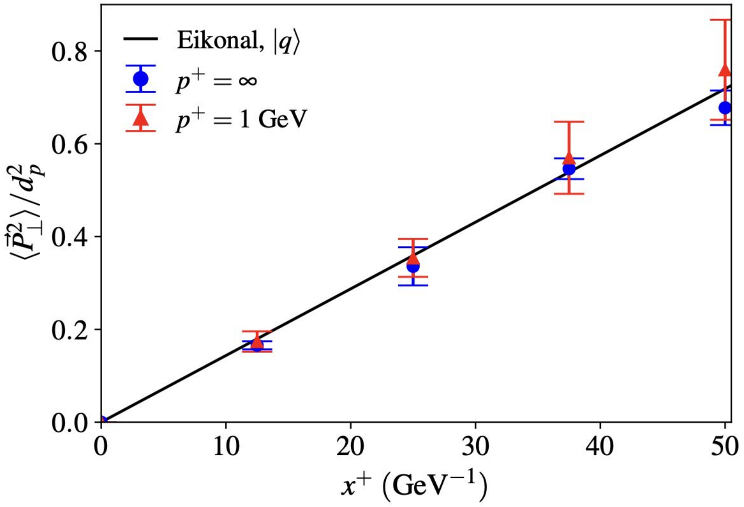
Quark jet results

Transverse Momenta operator

$$\vec{P}_{\perp}^{2} = \Big(\sum_{\beta} b_{\beta}^{\dagger} b_{\beta} \vec{p}_{\perp} + \sum_{\beta} a_{\beta}^{\dagger} a_{\beta} \vec{p}_{\perp} \Big)^{2}$$

WQ, Li, Kreshchuk, Salgado, 2411.09762



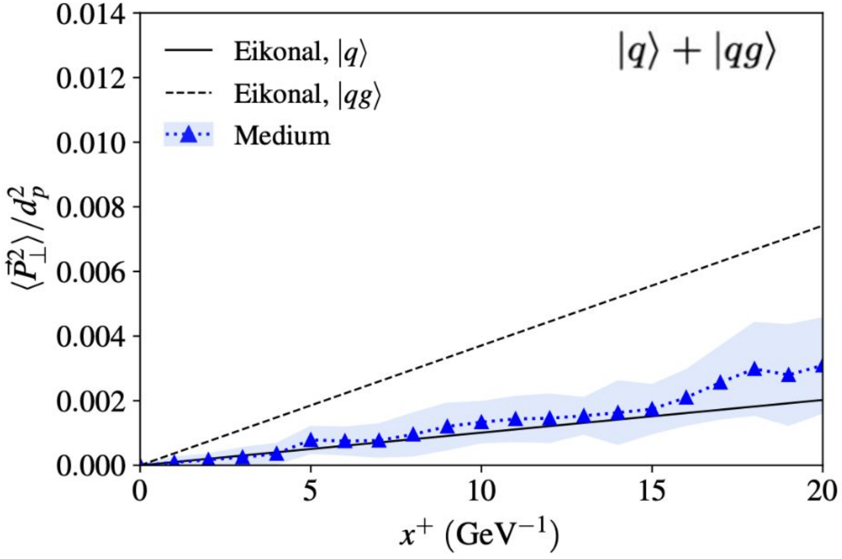


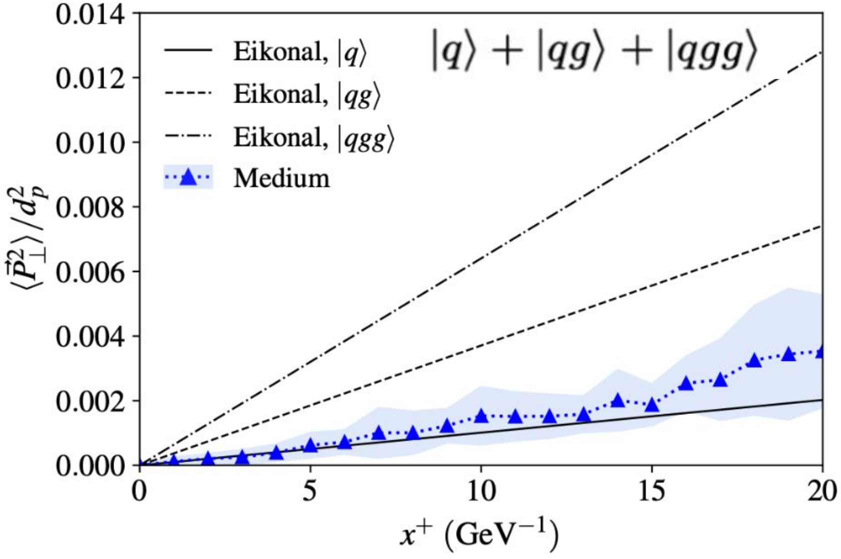
Fock space: $|q\rangle$

Quark jet results

Transverse Momenta operator $$\vec{P}_\perp^2 = \left(\sum_\beta b_\beta^\dagger b_\beta \vec{p}_\perp + \sum_\beta a_\beta^\dagger a_\beta \vec{p}_\perp \right)^2$$

WQ, Li, Kreshchuk, Salgado, 2411.09762

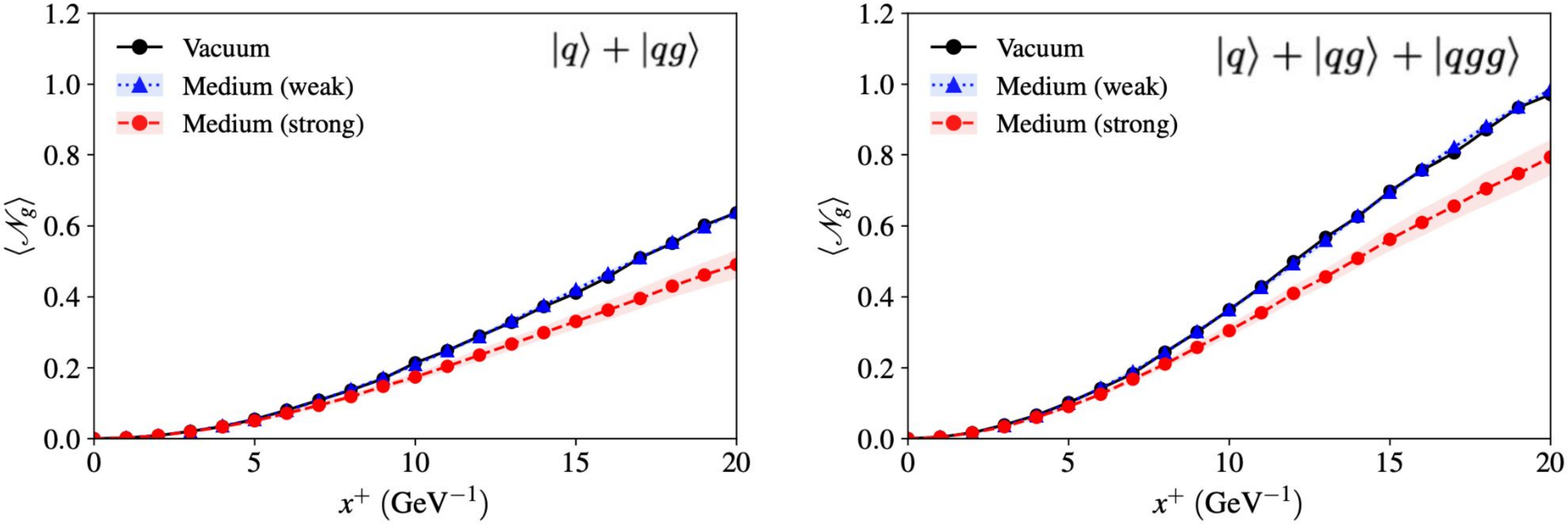




Quark jet results

Gluon number operator $$\mathcal{N}_g \equiv \sum_\beta a_\beta^\dagger a_\beta$$

WQ, Li, Kreshchuk, Salgado, 2411.09762

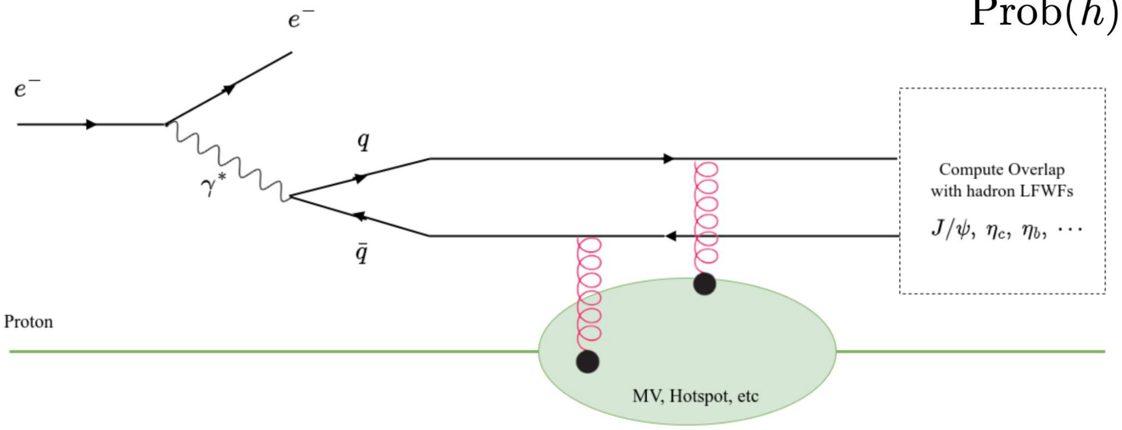


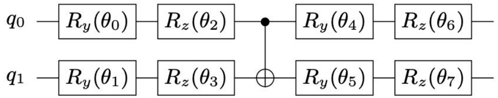
Qiskit

Future jet work

Meson production for qq pair to hadrons, connecting time evolution with hadron states

$$\text{Prob}(h) = |\langle\psi(x^+)|\psi_h\rangle|^2$$





WQ et al, 2112.01927

Quantum simulation to obtain incoming final state

$$\langle\psi(x^+)|$$

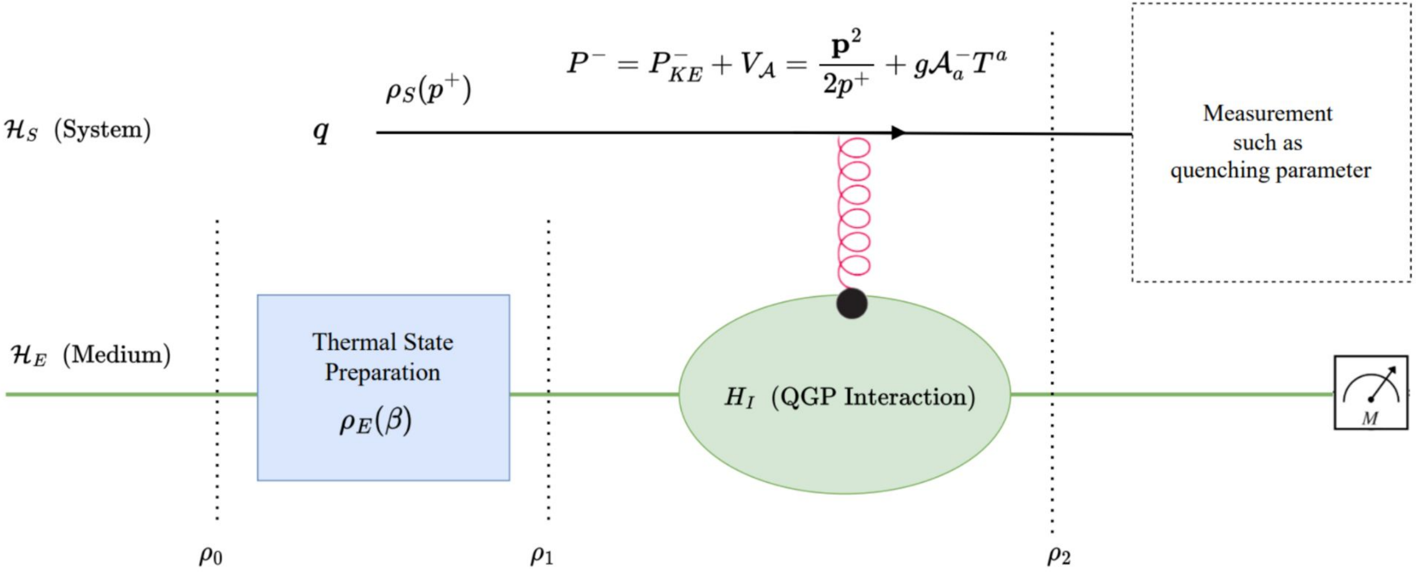
Variational quantum eigensolver to prepare target meson states

$$|\psi_h\rangle$$

Quantum Antenna
(ongoing with Joao, Meijian, Carlos)

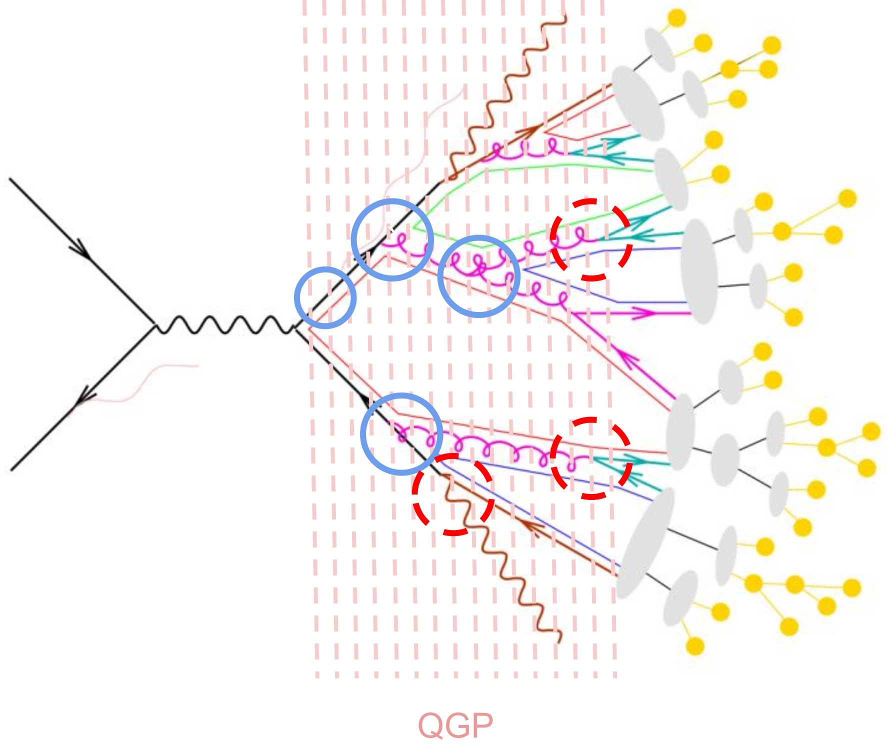
Future jet work

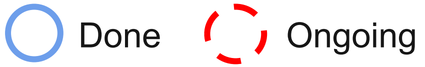
Prepare a more realistic QGP background on the circuit directly, surpassing the classical MV model.



Quantum simulation of jet in HIC - Ultimate path

Full description requires much more but we will get there, together with hardware





Theoretical lab to simulate jet physics!

- Medium property
- Jet as fully quantum object
- Extract useful observable

5. Quantum simulation for thermalization

[from 1+1 fermionic/QED/bosonic to speedup in heavy quark thermal]

Du and WQ, Phys.Rev.D, 2312.16294
WQ and Wu, JHEP, 2404.07912
Ikeda, Kang, Kharzeev, WQ, Zhao, Phys.Rev.D, 2407.21496
Cuntin, WQ, Wu (to appear)
…

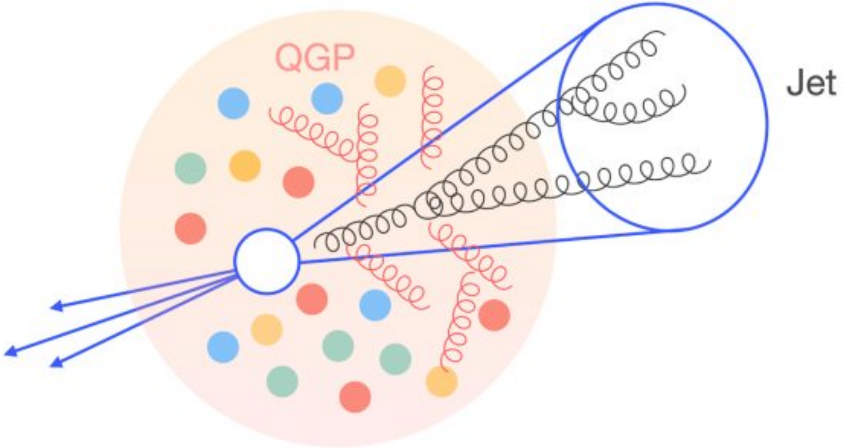
Quantum field theory at finite temperature

My original goal is understand the medium for QGP…

State preparation itself is super challenging (harder than evolution) involving non-trivial adiabatic process.

So, I started to understand thermal quantum field theory. Essentially:

- Prepare thermal (Gibbs) state
- Measure observables of interests
  such as energy density, pressure, distribution etc



Classical strategies

For small system, one can exactly diagonalize the Hamiltonian to compute the thermal average.

$$\langle \hat{O} \rangle_\beta = \frac{\mathrm{Tr}\left[e^{-\beta \hat{H}} \hat{O}\right]}{\mathrm{Tr}\left[e^{-\beta \hat{H}}\right]}$$

For large systems with high degrees of freedom, it is more favorable to formulate the problem as a path-integral and use Monte Carlo (MC) methods, such as

$$Z = \mathrm{Tr}[e^{-\beta H}] = \lim_{N \to \infty} \sum_{\{x_\tau\}} \langle x_0 | e^{-\frac{\beta}{N} H} | x_1 \rangle \langle x_1 | ... | x_N \rangle \langle x_N | e^{-\frac{\beta}{N} H} | x_0 \rangle$$

For MC to work, any subpath needs be real and non-negative (Importance sampling) => difficult to deal with non-zero chemical potentials, topological terms, or real-time dynamics… “sign” problem

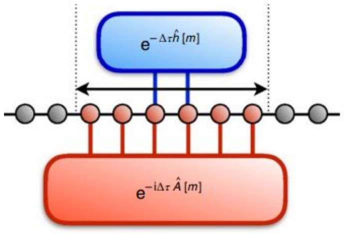
Czajka et al,
2112.03944 & 2210.03062

Honda et al, 2105.03276

Here, we use quantum strategies to circumvent it

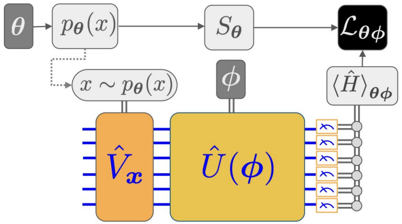
Quantum strategies

Because of the non-unitary nature, medium preparation is typically hard on quantum circuit. Still, there are many proposals…



Quantum imaginary time evolution (QITE)

Motta et al, 1901.07653
Davoudi et al, 2208.13112



Variational Quantum Thermalizer

Verdon et al, 1910.02071

“Maxwell demon”, quantum fridge — Ball and Cohen, 2212.06730

Thermofield Double States — Cottrell et al, 1811.11528

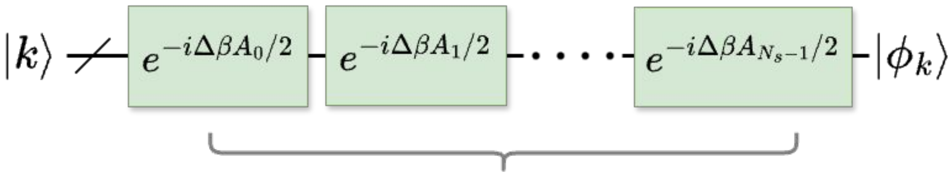
Open quantum system approaches — de Jong et al, 2010.03571 & 2106.08394
Cleve and Wang, 1612.09512
Kamakari et al, 2104.07823

…

Quantum imaginary time evolution

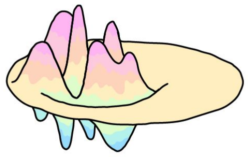
Key idea: trotterize the non-unitary evolution by replacing imaginary time with real time evolution

$$|\psi(\beta/2)\rangle = e^{-\beta\hat{H}/2}|\psi_0\rangle \approx \prod_{i=1}^{N_s} e^{-\Delta\beta\hat{H}/2}|\psi_0\rangle \qquad |\psi_{i+1}\rangle = e^{-\Delta\beta\hat{H}/2}|\psi_i\rangle = e^{-i\Delta\beta\hat{A}_i/2}|\psi_i\rangle\sqrt{c_i(\Delta\beta)} + \mathcal{O}(\Delta\beta^2)$$

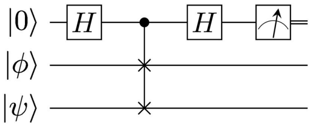


$N_s = \beta/\Delta\beta$ trotter steps using quantum gates

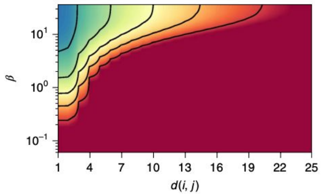
Practical advantage:



Exact, free from barren plateaus



No ancilla qubits



Correlation saturation

Recent QITE applied to field theories

Davoudi et al, 2208.13112 (Lattice gauge theory)
Czajka et al, 2112.03944 (Chemical potentials)
Pedersen et al, 2311.11616 (Schwinger topological)
WQ and Wu, 2404.07912 (Fermion systems)

QITE + Thermal sampling

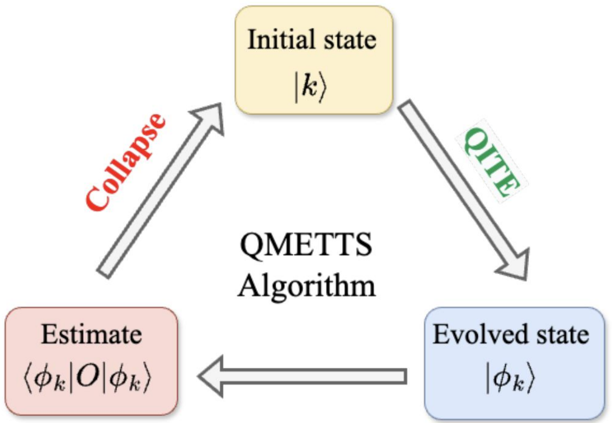
QITE + QMETTS (quantum minimally entangled typical thermal state) algorithm to compute thermal observable.

Stoudenmire and White, 1002.1305

$$\langle \hat{O} \rangle_\beta = \sum_{k \in \mathcal{S}} \frac{P_k}{Z} \langle \phi_k | \hat{O} | \phi_k \rangle \qquad |\phi_k\rangle = P_k^{-1/2} e^{-\beta \hat{H}/2} |k\rangle \qquad P_k = \langle k | e^{-\beta \hat{H}} | k \rangle \qquad Z = \sum_{k \in \mathcal{S}} P_k$$

Super efficient for large systems because state collapsing picks out dominant contribution $N_S \ll |\mathcal{S}|$

$$\langle \hat{O} \rangle_\beta \approx \frac{1}{N_S} \sum_{k=1}^{N_S} \langle \hat{O}_k \rangle_\beta \qquad |\langle k' | \phi_k \rangle|^2 \sim P_{k'}/Z$$



Fermionic QFT in 1+1 dimensions

WQ and Wu, 2404.07912

We study the Majorana Fermions in 1+1 dimensions and use Majorana representation:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}(i\partial\!\!\!/ - m)\psi - \mathcal{H}_I(\psi)$$

In the discretized theory:

$$\hat{H} = \frac{1}{2}\sum_n \bar{\psi}_n[-\frac{i}{2a}\gamma^1(\psi_{n+1} - \psi_{n-1}) + m\psi_n] - \frac{r}{4a}\sum_n \bar{\psi}_n(\psi_{n+1} - 2\psi_n + \psi_{n-1}) + \hat{H}_I$$

kinetic mass Wilson terms interaction

Map to qubits using Jordan-Wigner encoding: 1 qubit per 1 spatial

$$\psi_n = \begin{pmatrix} \frac{a_n + a_n^\dagger}{\sqrt{2}} \\ \frac{a_n^\dagger - a_n}{i\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} a_n \\ a_n^\dagger \end{pmatrix} \equiv R\begin{pmatrix} a_n \\ a_n^\dagger \end{pmatrix} \qquad a_n^\dagger = \frac{\sigma_n^X - i\sigma_n^Y}{2}\prod_{i=0}^{n-1}\sigma_i^Z$$

Wilson (1974); Jordan et al, 1404.7115

Fermion fields at finite temperature

WQ and Wu, 2404.07912

We study the N-fermion systems in thermal equilibrium to approximate the thermal field at $\beta = 1/T$

$$\langle \hat{O} \rangle_\beta \equiv Z_\beta^{-1} \mathrm{Tr}[e^{-\beta \hat{H}} \hat{O}] \qquad\qquad Z_\beta \equiv \mathrm{Tr}[e^{-\beta \hat{H}}]$$

From the spectral function, the free-theory thermal distribution is:

$$f_p = \frac{1}{2E_p} \sum_n [\gamma^0 u_p]_{\alpha'} [\bar{u}_p \gamma^0]_\alpha e^{ipan} \langle \bar{\psi}_n^{\alpha'} \psi_0^\alpha \rangle_\beta \qquad f_p = \sqrt{\frac{N}{2E_p}} \langle \hat{a}_p^\dagger [\bar{u}_p \gamma^0 \psi_0] \rangle_\beta = \langle \hat{a}_p^\dagger \hat{a}_p \rangle_\beta \qquad f_p = \frac{1}{e^{\beta E_p} + 1}$$

Energy density $\hat{\epsilon} \equiv T^{00}$ and pressure $\hat{P} \equiv T^{11}$ are

$$\hat{P}_n = -\frac{i}{4a^2} [a_n (a_{n+1} - a_{n-1}) + a_n^\dagger (a_{n+1}^\dagger - a_{n-1}^\dagger)]$$

$$\hat{\epsilon}_n - \hat{P}_n = \frac{m}{a} \left(a_n^\dagger a_n - \frac{1}{2} \right) - \frac{r}{4a^2} [a_n^\dagger (a_{n+1} - 2a_n + a_{n-1}) - a_n (a_{n+1}^\dagger - 2a_n^\dagger + a_{n-1}^\dagger)]$$

position space

$$\langle \hat{P}_n \rangle_\beta = \frac{1}{L} \sum_p \frac{p_a^2}{E_p} (\langle \hat{a}_p^\dagger \hat{a}_p \rangle - 1/2)$$

$$\langle \hat{\epsilon}_n \rangle_\beta = \frac{1}{L} \sum_p E_p (\langle \hat{a}_p^\dagger \hat{a}_p \rangle - 1/2)$$

momentum space

$$\bar{P} = \int \frac{dp}{2\pi} \frac{p^2}{\sqrt{p^2 + m^2}} \bar{f}_p$$

$$\bar{\epsilon} = \int \frac{dp}{2\pi} \sqrt{p^2 + m^2} \bar{f}_p$$

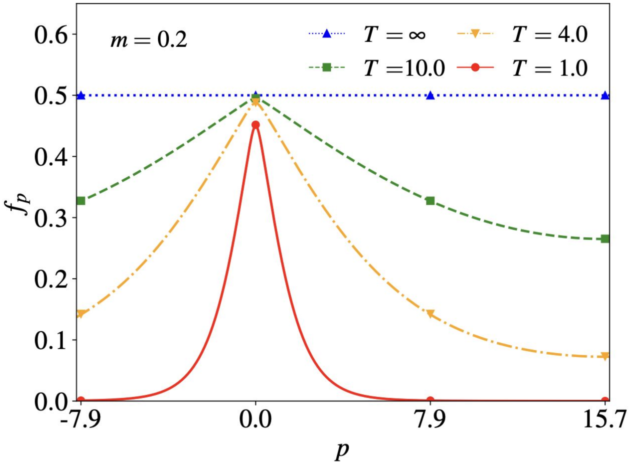
continuum

Free field thermal distribution

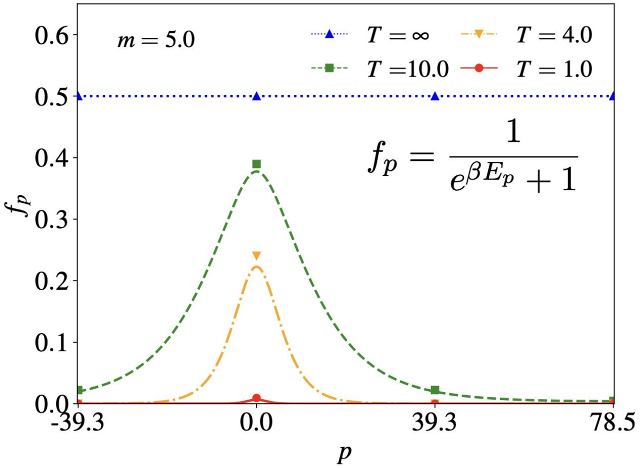
WQ and Wu, 2404.07912

We evolve the 4-qubit fermionic fields using QITE algorithm for the free fields. Results agree with analytical distribution of Fermi-Dirac.

$$E_p = \sqrt{\left(m + \frac{2r}{a}\sin^2\frac{ap}{2}\right)^2 + p_a^2}, \qquad p_a = \frac{1}{a}\sin(pa), \qquad \Delta\beta = 0.001$$



(a) $T \gg m$ case



(b) $m \gg T$ case

Interacting field thermal distribution

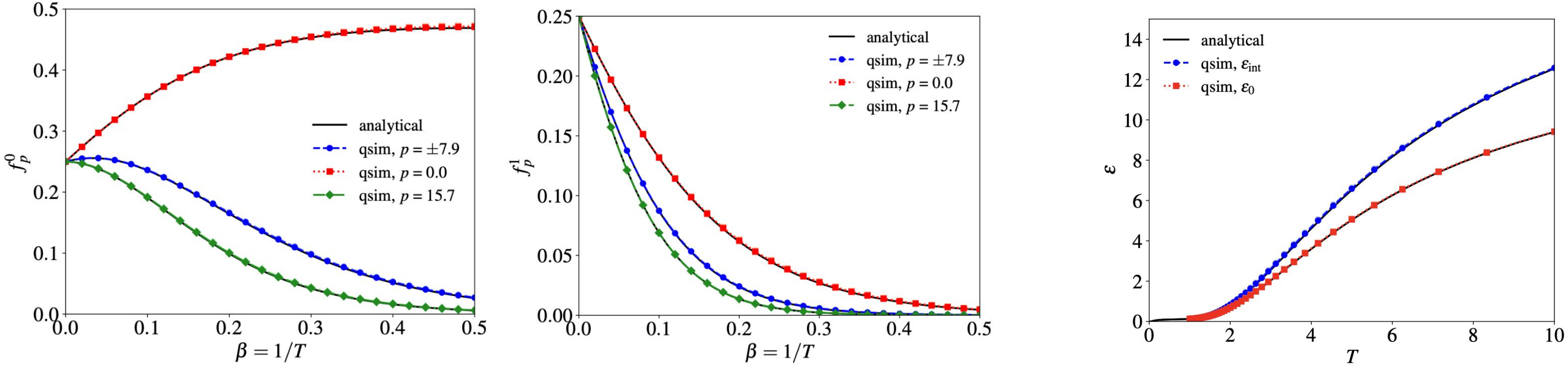
WQ and Wu, 2404.07912

For simplicity, we consider another external Majorana field ψ_M that is massive and *homogeneous in space*:

$$L = \int dx \left[\frac{1}{2}\bar{\psi}(i\partial\!\!\!/ - m_0)\psi - \frac{g}{4}(\bar{\psi}\psi)(\bar{\psi}_B\psi_B)\right] + \frac{1}{2}\bar{\psi}_B(i\gamma^0\partial_t - M)\psi_B$$

Mass renormalization is solved analytically and two quasiparticles emerge

$$\frac{Z^0_\beta}{Z_\beta}\epsilon^0 + \frac{Z^1_\beta}{Z_\beta}\epsilon^1$$



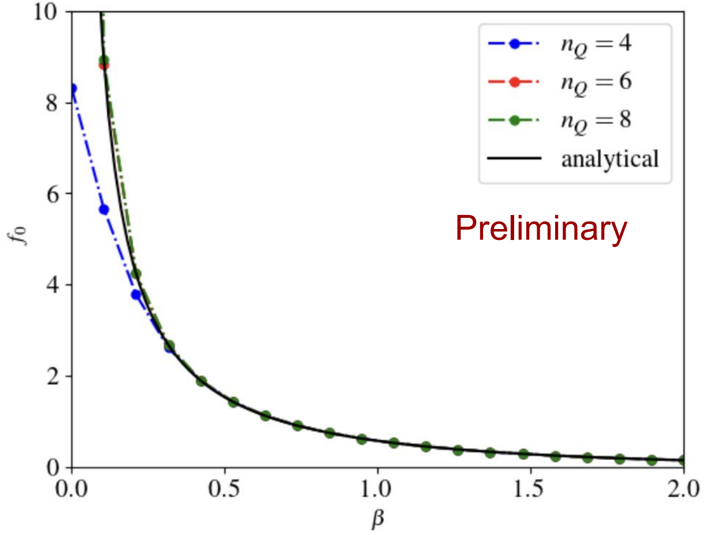
(a) $T \gg \bar{m}$ case with $\bar{m} = 0.2, M = 10.0$

Extending to bosons in scalar field theory

Similarly, we study Bose Einstein distribution (1 site = (0+1) scalar FT)

Cuntin, WQ, Wu, 2411.19601 (ICHEP 2023)
Cuntin, WQ, Wu (work in progress)

The ultimate goal is studying non-equilibrium effects toward thermalization with initial conditions in 1+1



$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \phi \partial^\mu \phi - m\phi^2\right] - \frac{\lambda}{4!}\phi^4,$$

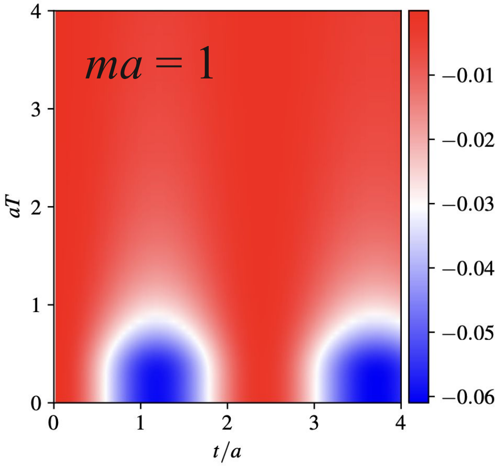
$$\varphi_\alpha = \Delta_\varphi\left(\alpha - \frac{N_\varphi - 1}{2}\right) \quad , \quad \Delta_\varphi = \sqrt{\frac{2\pi}{N_\varphi \bar{m}}},$$

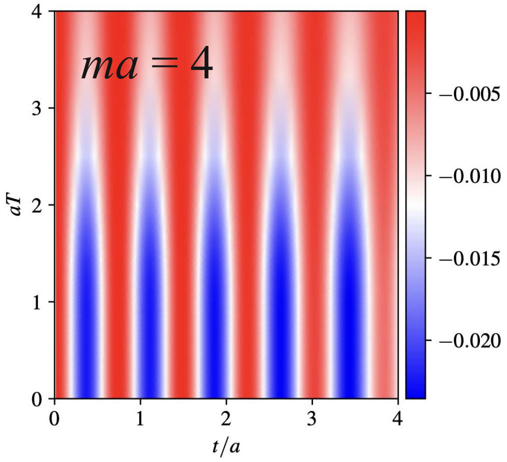
Extending to QED2

Schwinger model (1+1 QED) with quenched evolution (imbalance of chirality) at finite temperature to study dynamics of charge operators => explain Chiral Magnetic Effect (CME)

CME in 1+1, Kharzeev, Yee, 1012.6026
Ikeda, et al, 2407.21496

CME in 1+1 = generation of electric current





Thermal damping controlled by the mass

Real & **Imag** evolution simultaneously!

$$\langle O(t)\rangle_\beta = \frac{\mathrm{Tr}\left(e^{-\beta H} O(t)\right)}{\mathrm{Tr}\left(e^{-\beta H}\right)}$$

Vector current

$$J(x) \equiv \bar{\psi}(x)\gamma^1\psi(x)$$

Heavy quark thermalization (macroscopic, application of quantum search)

Distinguished separation of scales

Two dominant factors:

- Sudden change of momentum from radiation
- Slow change of momentum from environment

Brownian motion can be described by SDE:

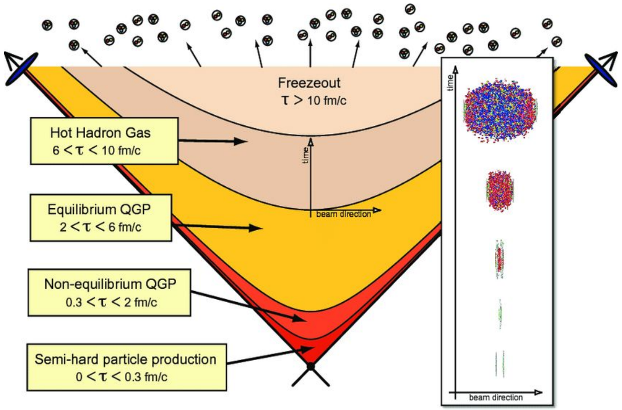
- Drag term: Energy loss
- Diffusion term: Momentum broadening

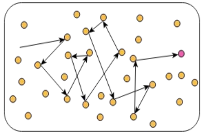
Full SDE (Langevin) equation

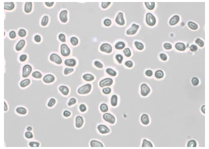
$$dx_i = \frac{p_i}{E(\vec{p})}dt, \quad i = x, y, z,$$

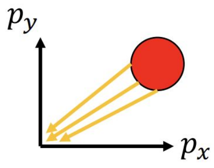
$$dp_i = -A(\vec{x}, \vec{p}, t)p_i dt + \sigma_{ij}(\vec{x}, \vec{p}, t)dW_j,$$

Drag coefficient   Stochastic Wiener process

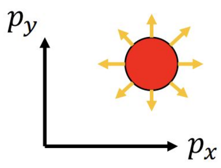








Drag: Energy loss



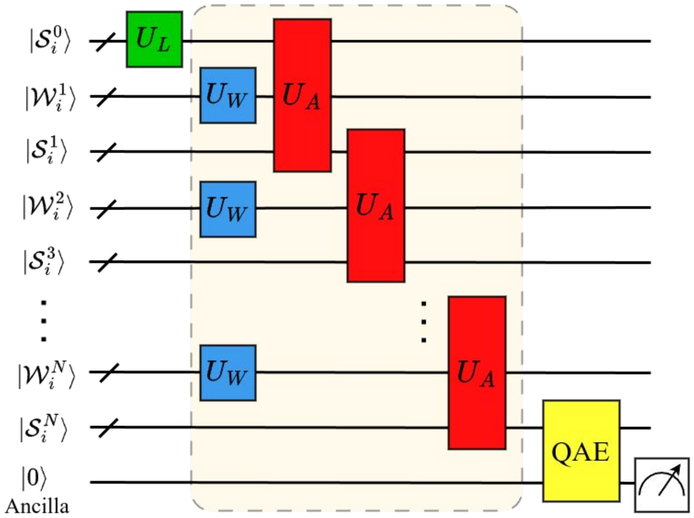
Diffusion: Momentum broadening

Accelerated Quantum Circuit MC (aQCMC)

Rebentrost et al, 1805.00109
Stamatopoulos et al, 905.02666v5
Du and WQ, arXiv:2312.16294

The idea: use Grover-like operator to accelerate the extraction of **expectation** at final step (“maturity”) by a square root over classical Monte Carlo methods

$$\mathbb{E}\left[F(\vec{p},T)\right] = \frac{1}{N}\sum_{i=1}^{N} F(\vec{p},T)$$



$$dq_i = -q_i d\tilde{t} + d\tilde{W}_i$$

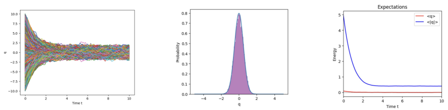
- Distribution loading gate $\quad q_i = p_i/M \quad d\tilde{t} = Adt$
- Stochastic Wiener gate $\quad d\tilde{W}_i \sim \mathcal{N}(0, 2Td\tilde{t}/(M\chi_i^2))$
- Quantum evolution gate $\quad A = \sigma_{ii}^2\chi_i^2/(2MT)$
- Quantum Amplitude Estimation (QAE)

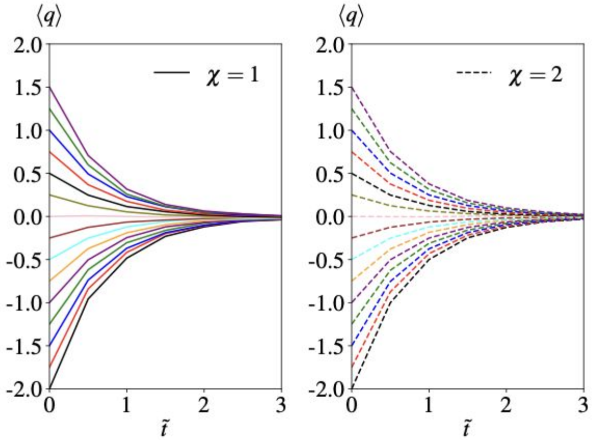
$\epsilon = \mathcal{O}(1/N_q)$ quadratically faster than MC

Simulation results (1D)

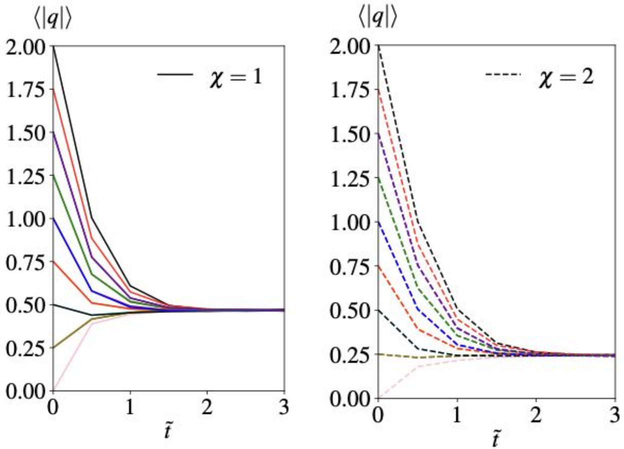
Du and WQ, arXiv:2312.16294

Large time thermalization, aQCMC **without** QAE = Classical MC

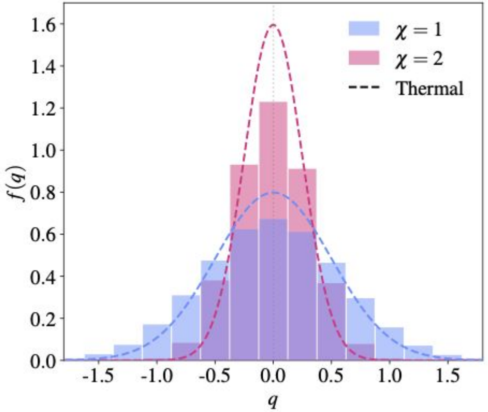




Momentum



Abs Momentum



Thermal Distribution

$$f^{\mathrm{eq}}(\vec{q}) \propto \exp\left[-\frac{q_x^2}{\tilde{\sigma}_x^2} - \frac{q_y^2}{\tilde{\sigma}_y^2} - \frac{q_z^2}{\tilde{\sigma}_z^2}\right]$$

$$M = 1.5\,\mathrm{GeV}$$

$$T \simeq 300\text{ - }500\,\mathrm{MeV}$$

$$\tilde{\sigma}_i^2 d\tilde{t} = 2T d\tilde{t}/(M\chi_i^2) \simeq d\tilde{t}/(2\chi_i^2)$$

n = 4, 20 qubits, dq = 0.25, dt = 0.5

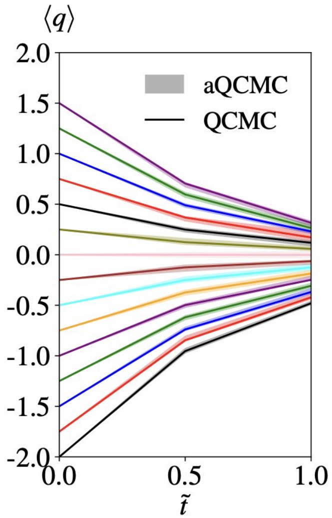
Qiskit

Simulation results (1D)

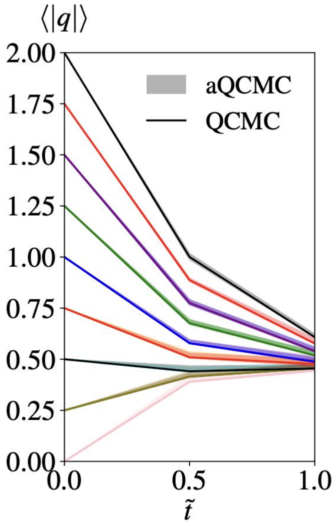
Du and WQ, arXiv:2312.16294

Early time thermalization, aQCMC **with** QAE

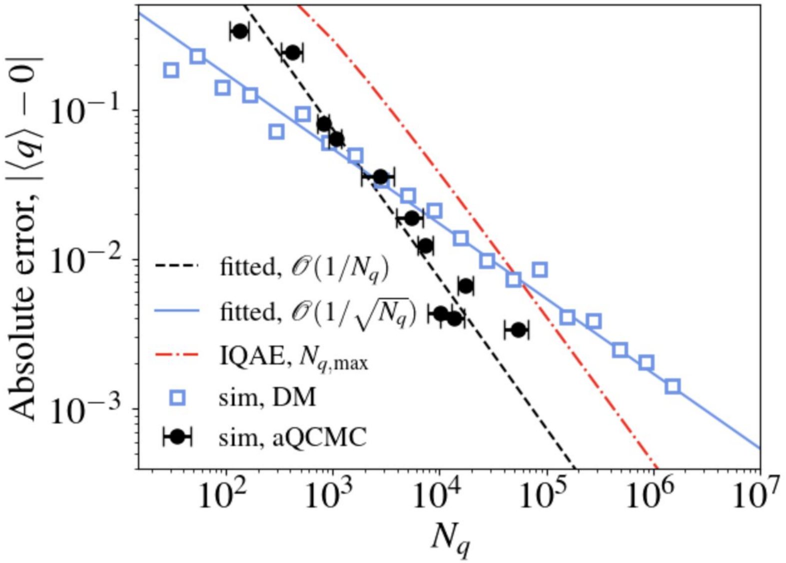
Quantum advantage to quadratically reduce sampling shots



Momentum



Abs Momentum



$$M = 1.5\,\mathrm{GeV}$$

$$T \simeq 300 \text{ - } 500\,\mathrm{MeV}$$

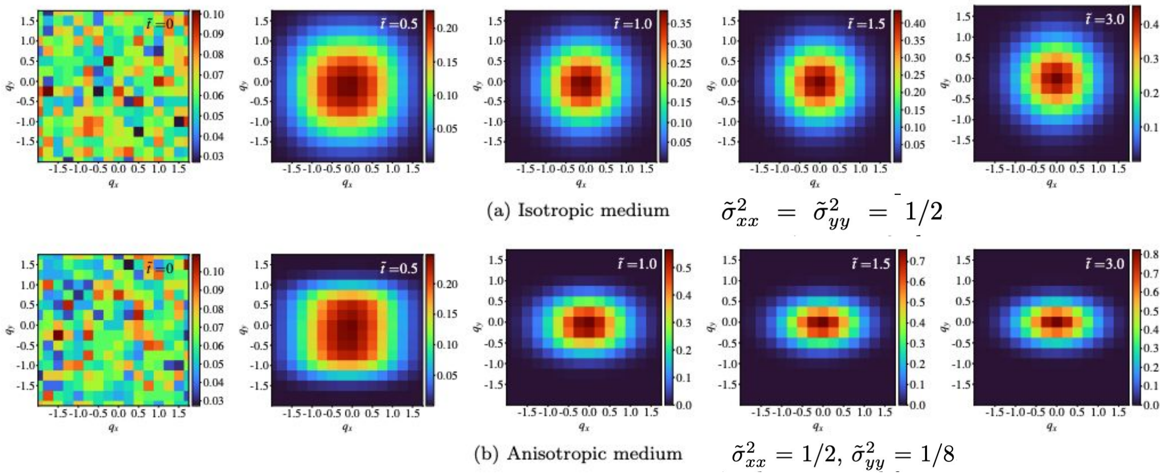
$$\tilde{\sigma}_i^2 d\tilde{t} = 2T d\tilde{t}/(M\chi_i^2) \simeq d\tilde{t}/(2\chi_i^2).$$

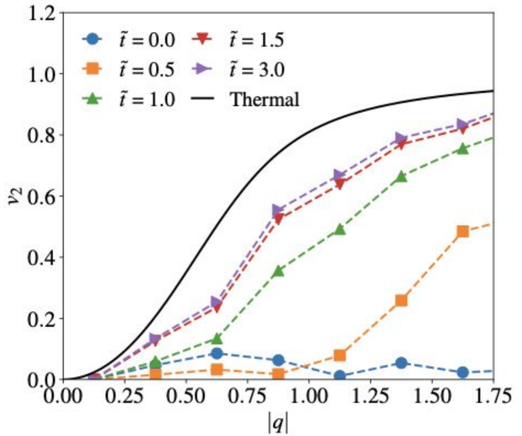
n = 4, 20 qubits, dq = 0.25, dt = 0.5

Qiskit

Simulation results (2D)

Du and WQ, arXiv:2312.16294





Non-equilibrium initial conditions towards thermalization with medium profiles

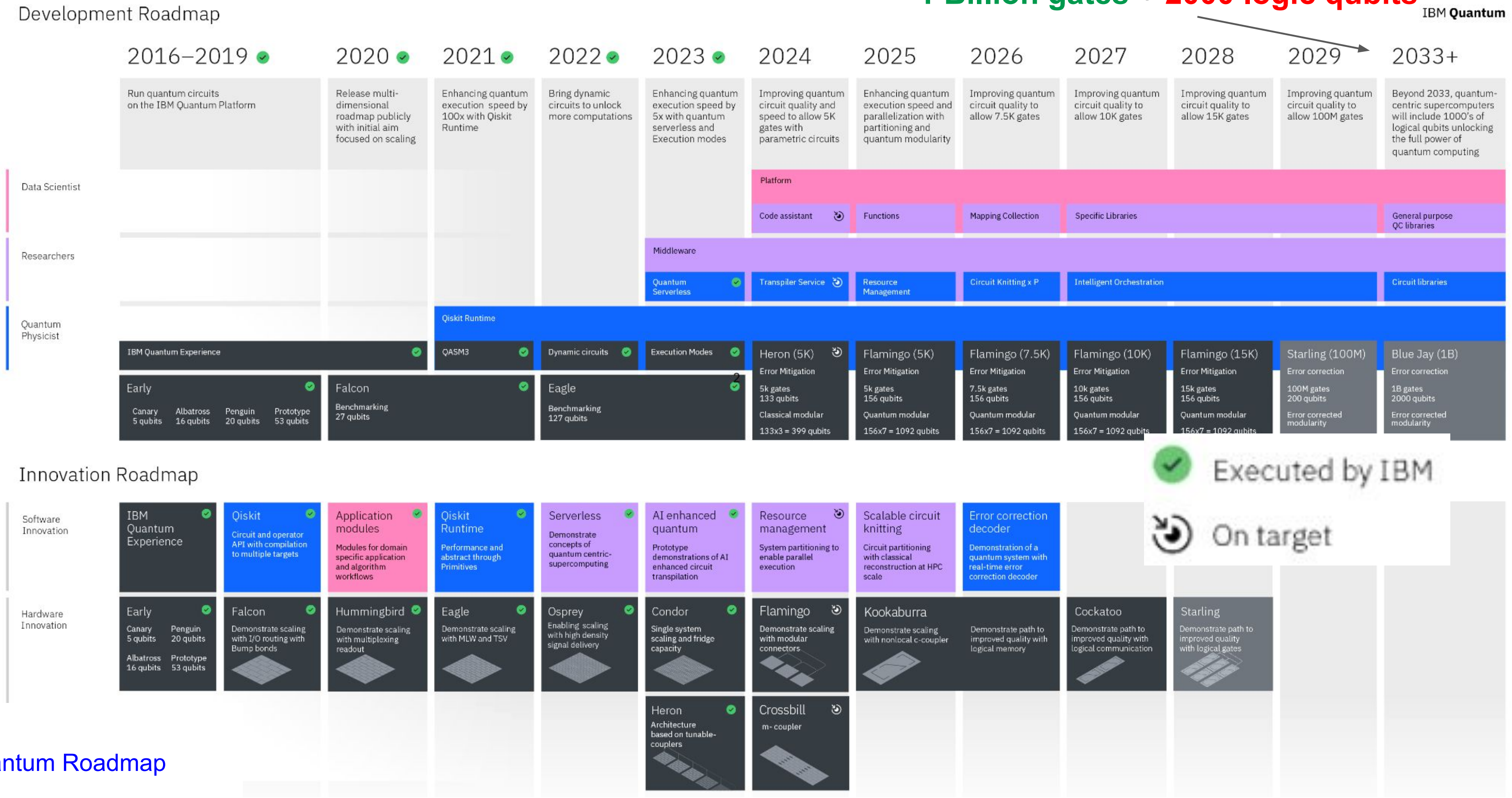
Elliptic flow “v2” build up

$$v_2 = \frac{\int f(q, \cos(\phi), t)\cos(2\phi)\mathrm{d}\phi}{\int f(q, \cos(\phi), t)\mathrm{d}\phi} \overset{\text{thermal}}{=} \frac{I_1\left(\frac{1}{2q^2}\left|\frac{1}{\tilde{\sigma}_x^2} - \frac{1}{\tilde{\sigma}_y^2}\right|\right)}{I_0\left(\frac{1}{2q^2}\left|\frac{1}{\tilde{\sigma}_x^2} - \frac{1}{\tilde{\sigma}_y^2}\right|\right)}$$

6. The future of quantum computing

Quantum technology is growing at fast pace

1 Billion gates + **2000 logic qubits**



IBM Quantum Roadmap

Quantum technology is growing at fast pace

Our focus is to unlock the full potential of quantum computing by developing a large-scale computer capable of complex, error-corrected computations. We're guided by a roadmap featuring six milestones that will lead us toward top-quality quantum computing hardware and software for meaningful applications.

MILESTONE 1 — 2019

BEYOND CLASSICAL

Physical Qubits: 54
Logical Qubit Error Rate: -

MILESTONE 2 — 2023

QUANTUM ERROR CORRECTION

Physical Qubits: 10^2
Logical Qubit Error Rate: 10^{-2}

MILESTONE 3

BUILDING A LONG-LIVED LOGICAL QUBIT

Physical Qubits: 10^3
Logical Qubit Error Rate: 10^{-6}

MILESTONE 4

CREATING A LOGICAL GATE

Physical Qubits: 10^4
Logical Qubit Error Rate: 10^{-6}

MILESTONE 5

ENGINEERING SCALE UP

Physical Qubits: 10^5
Logical Qubit Error Rate: 10^{-6}

MILESTONE 6

LARGE ERROR-CORRECTED QUANTUM COMPUTER

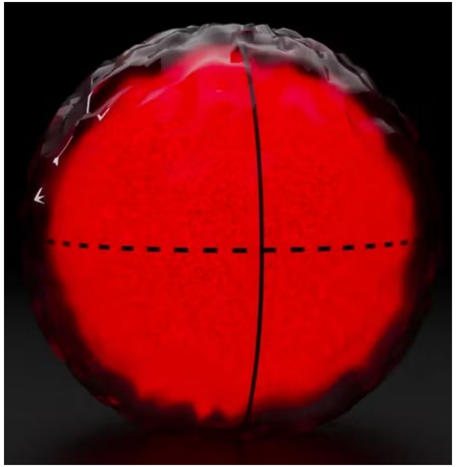
Physical Qubits: 10^6
Logical Qubit Error Rate: 10^{-13}

1000 qubits, eror rate: 10^-6

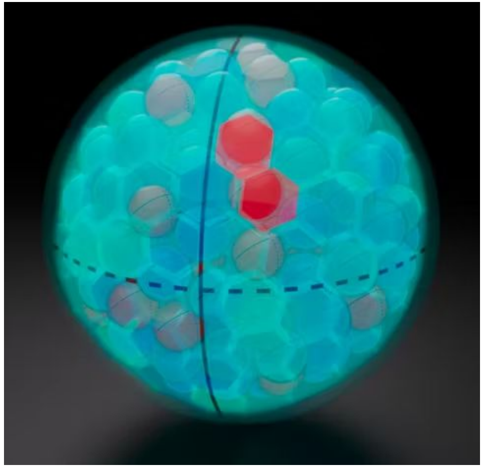
Error correction, 100000 qubits, 10^-13 error

Google Quantum AI Roadmap

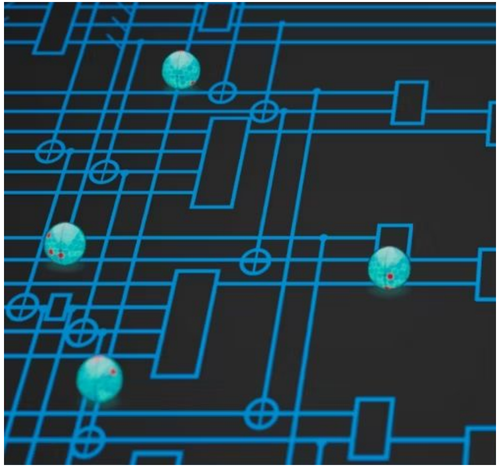
Quantum technology is growing at fast pace



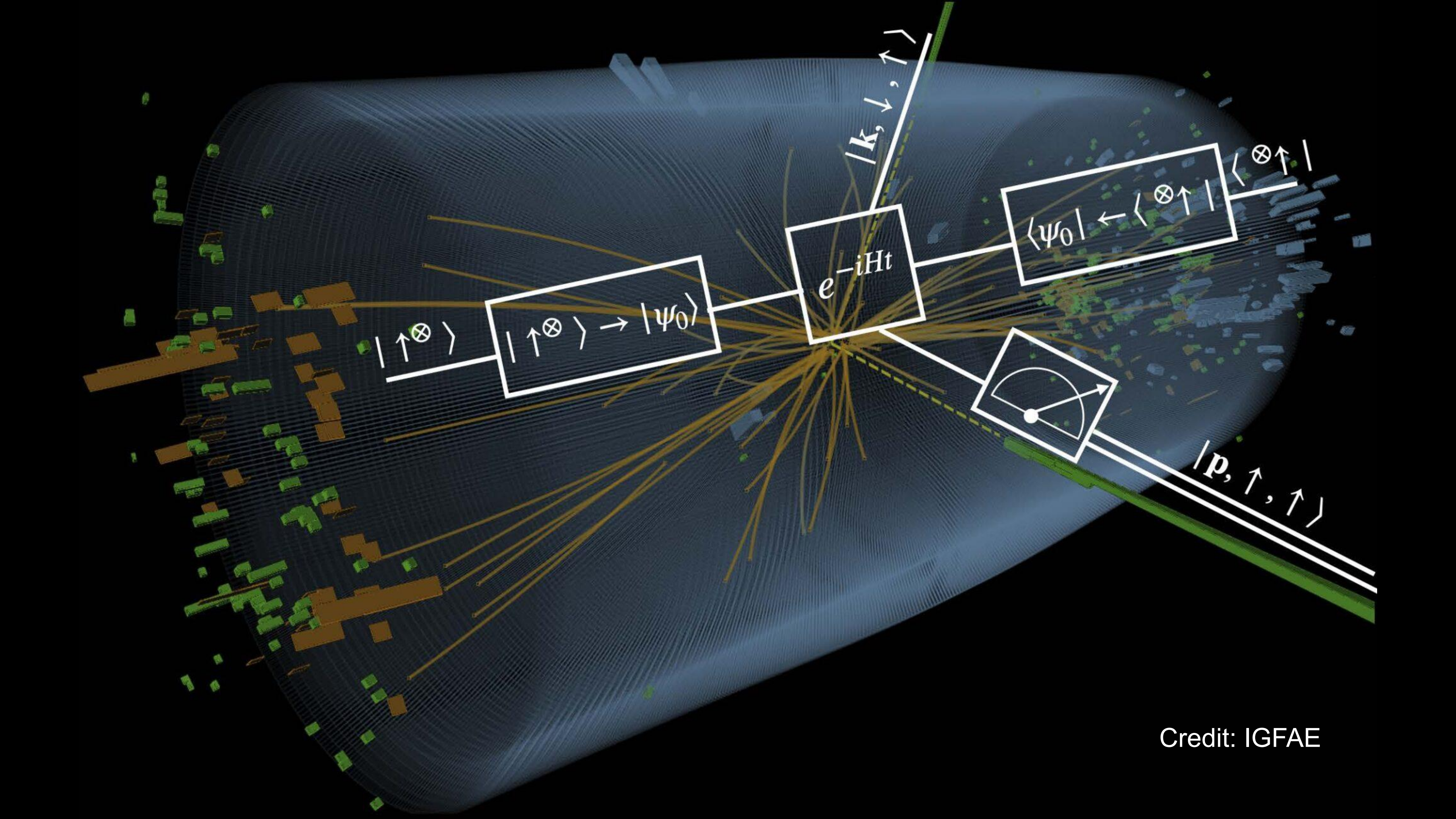
Noisy physical qubits

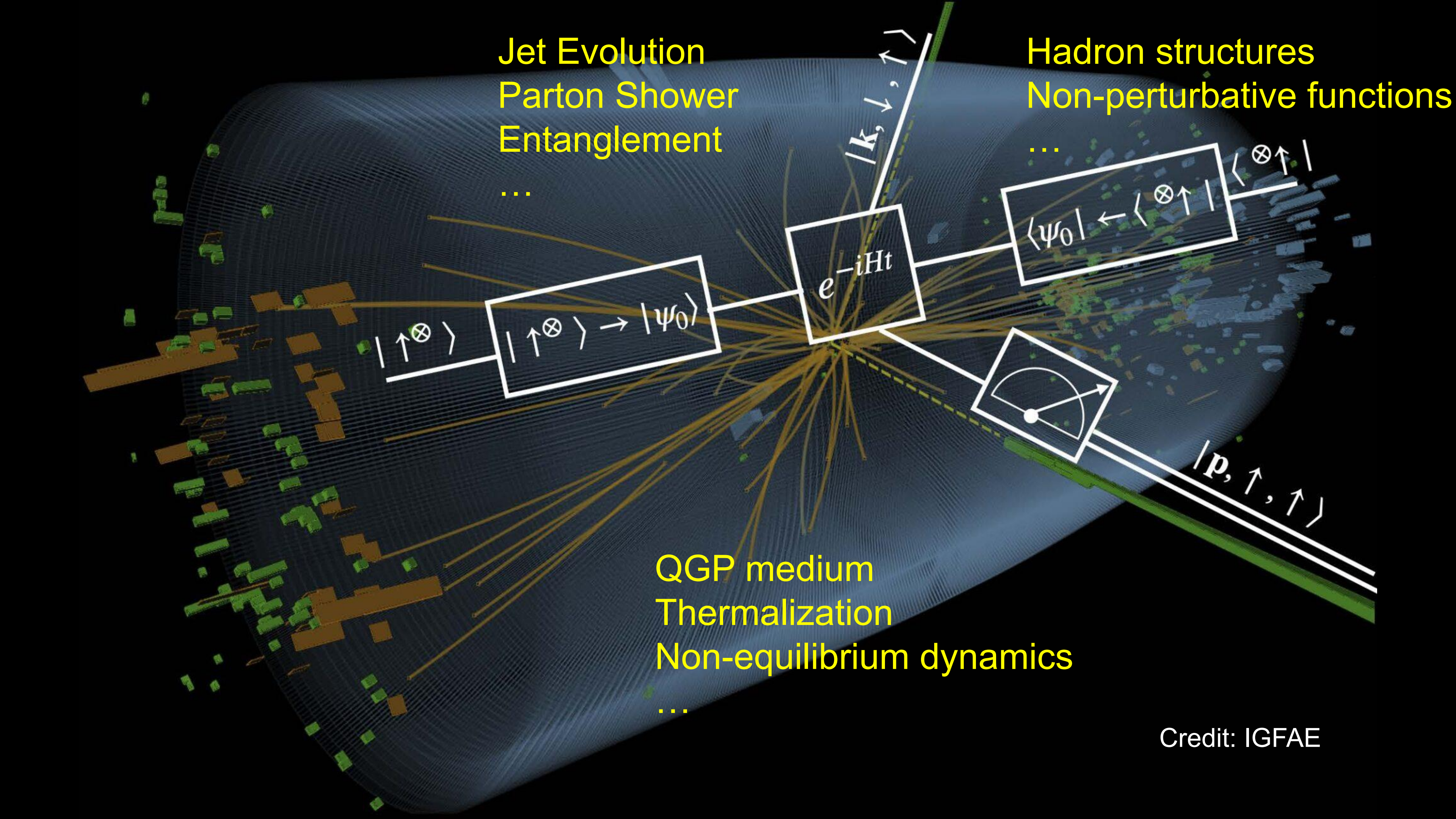


Reliable logical qubits



Quantum supercomputers


$|\uparrow^{\otimes}\rangle$
$|\uparrow^{\otimes}\rangle \to |\psi_0\rangle$
e^{-iHt}
$|\mathbf{k}, \downarrow, \uparrow\rangle$
$\langle\psi_0| \leftarrow \langle {}^{\otimes}\uparrow|$
$\langle {}^{\otimes}\uparrow|$
$|\mathbf{p}, \uparrow, \uparrow\rangle$
Credit: IGFAE

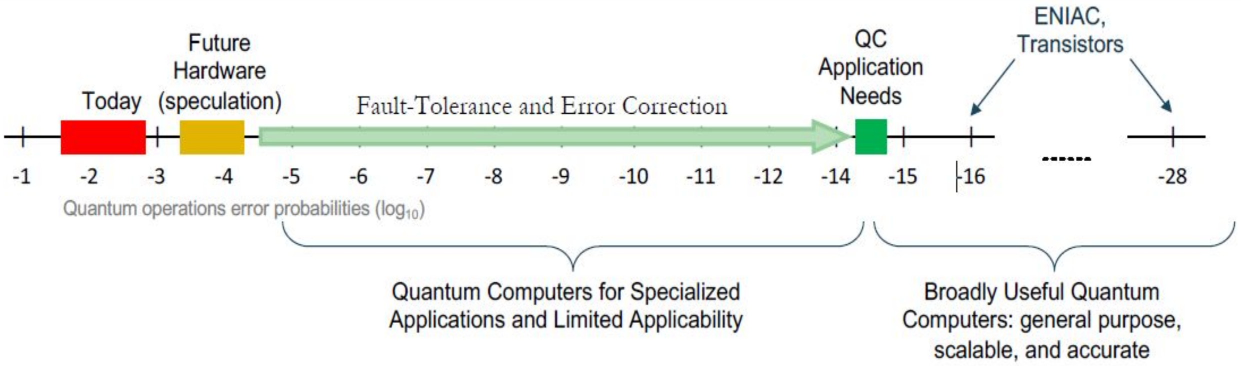


Credit: IGFAE

Lessons

- Quantum computing technology is available today and developing fast.
- Lots of quantum computing applications in experiment and theory for physics.
- Quantum simulation is promising to solve hadron structures, QCD jet evolution, and thermalization.
  (a conceptually clean, algorithmic proven path)
- We may be in reach of fault-tolerant quantum computing sooner than we expect.

Thanks for your attention!



IGFAE INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS 25 1999 2024

Acknowledgements

USC
XUNTA DE GALICIA
CENTROS
Cofinanciado pola Unión Europea
Fondos Europeos
erc
European Research Council
Established by the European Commission
EXCELENCIA MARÍA DE MAEZTU 2024-2029
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