



Exploring high energy nuclear physics by quantum computing

邢宏喜

QuNu Collaboration

2106.03865, 2205.12767, 2207.13258,
2301.04179, 2406.05683, 2411.18869

中国科学院大学高能核物理课题组前沿讲座

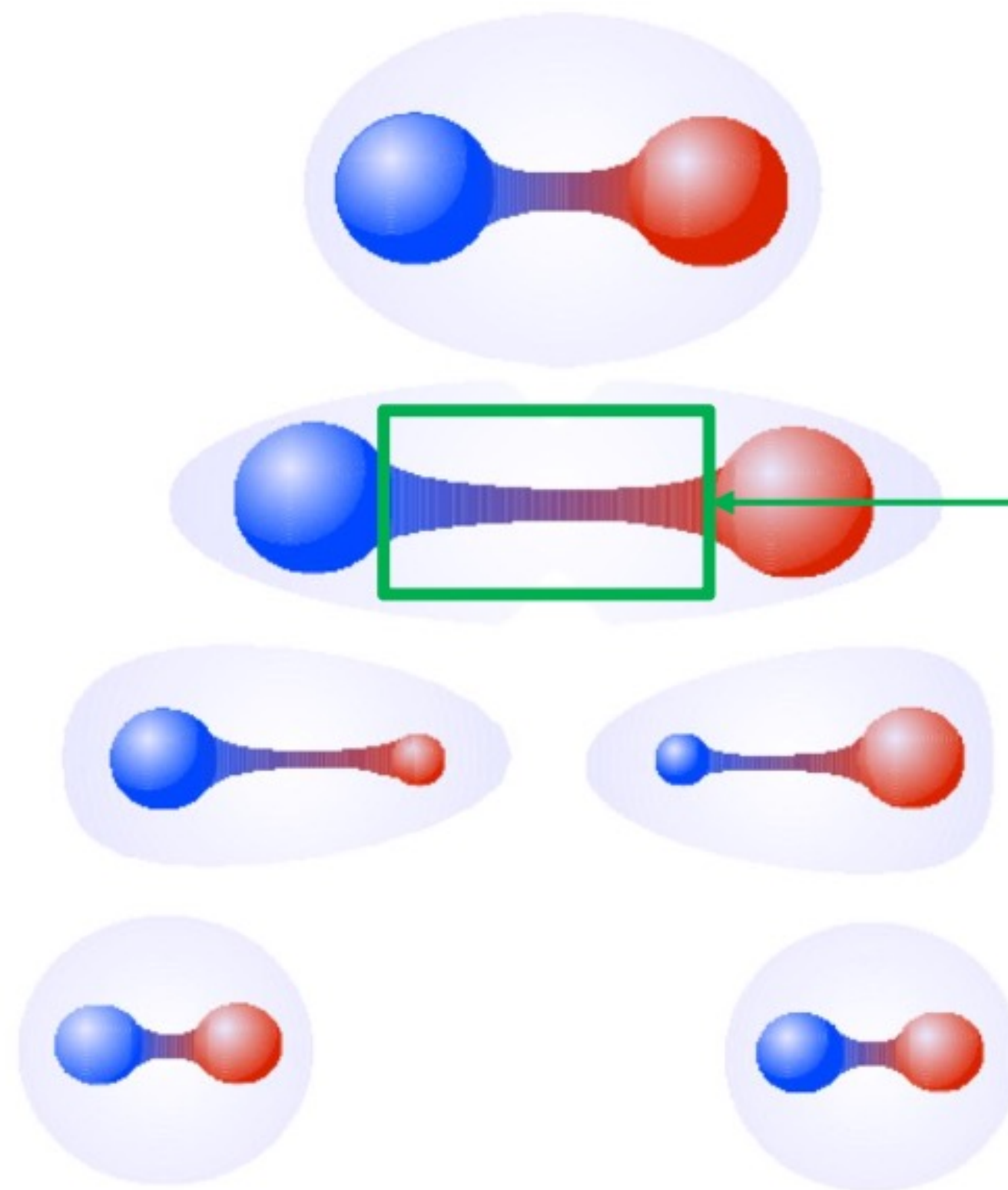
2024.12.22

Outline

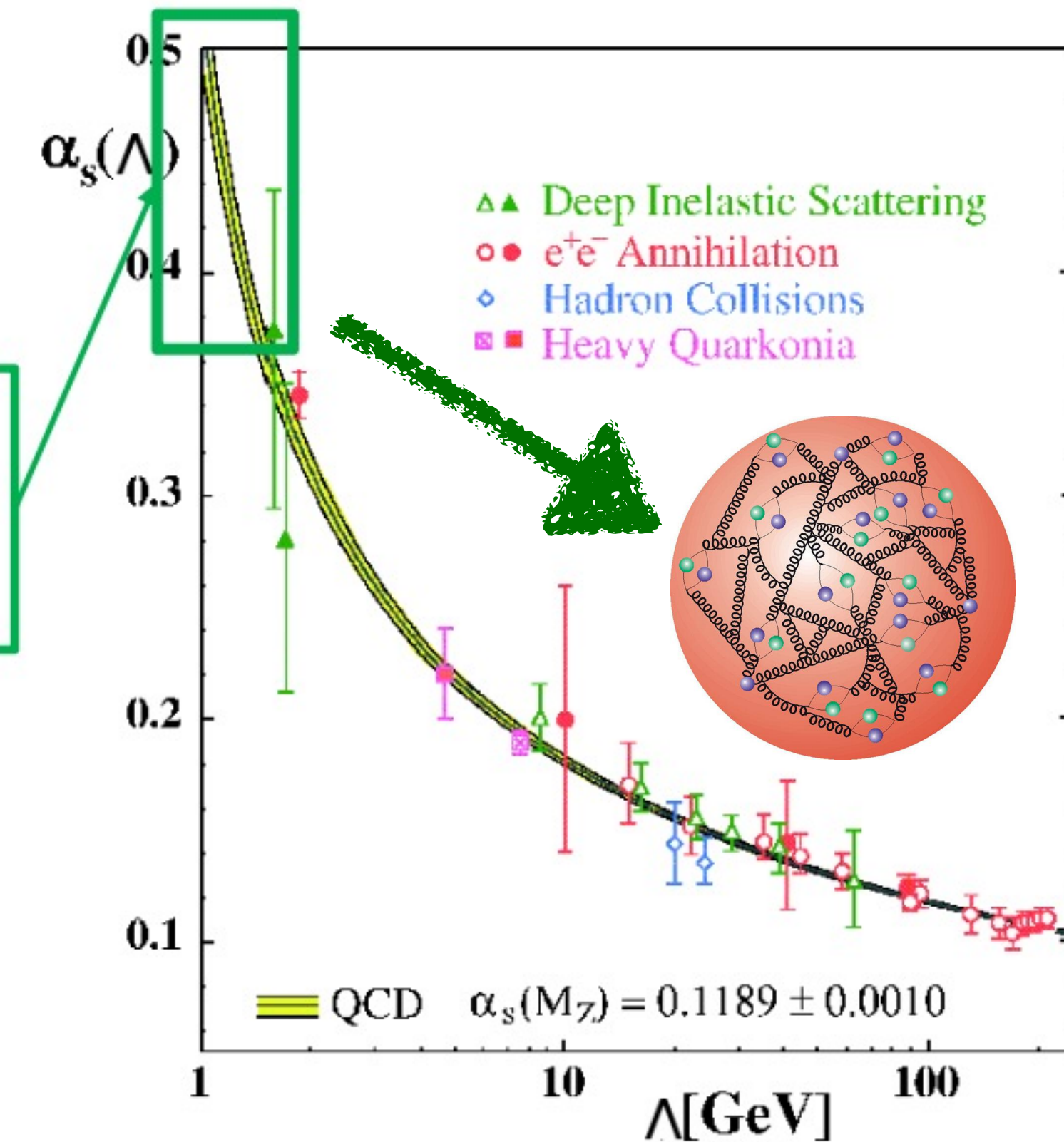
- ◆ Introduction
- ◆ Simulate hadronic structure from quantum computing
 - ➡ parton distribution in hadron
 - ➡ partonic scatterings
 - ➡ hadronization
- ◆ Chiral condensate from quantum computing
- ◆ Summary and outlook

Two scientific pillars in high energy nuclear physics

◆ QCD confinement: nucleon/nucleus partonic structure

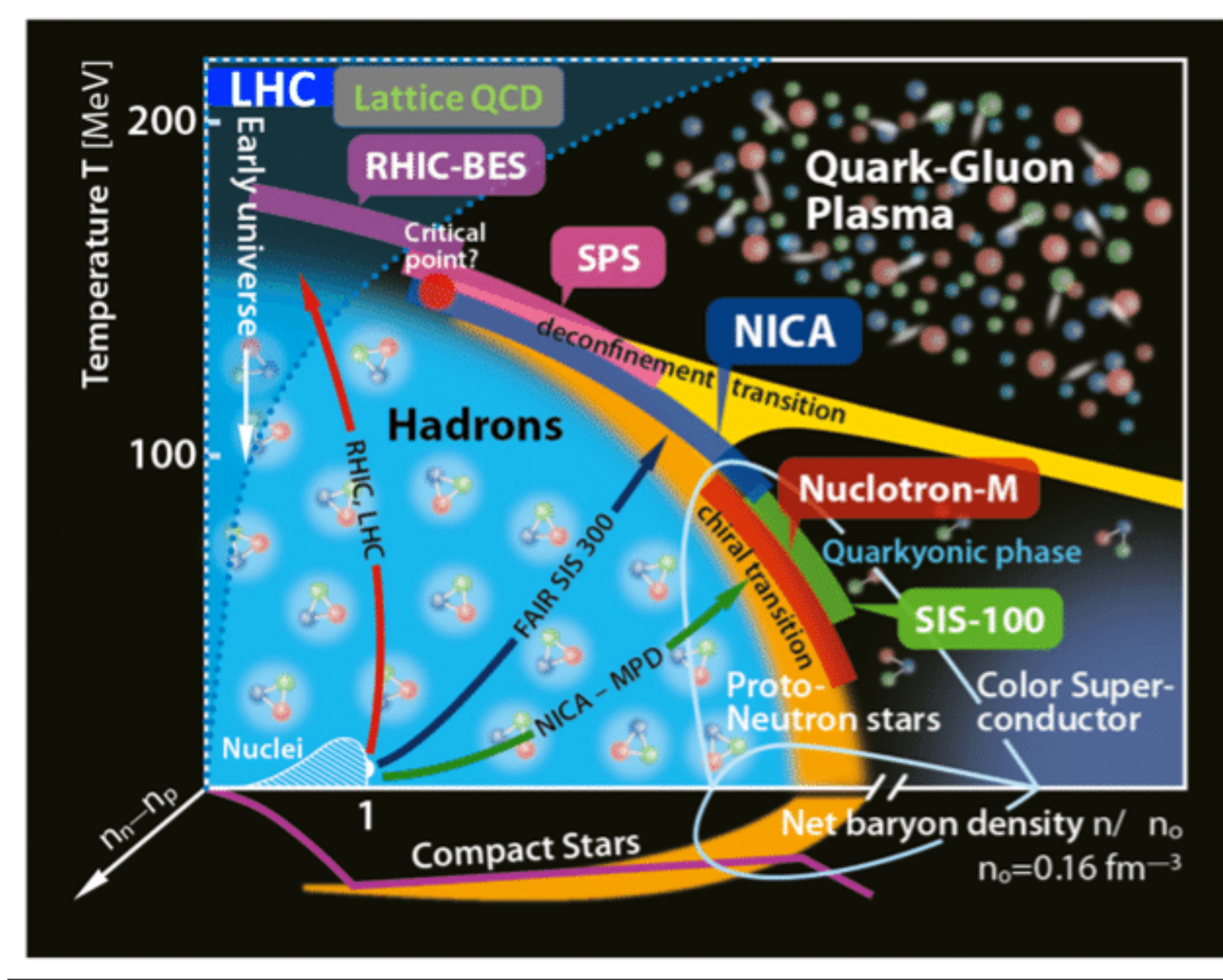


Long-distance physics of QCD, which is Non-perturbative



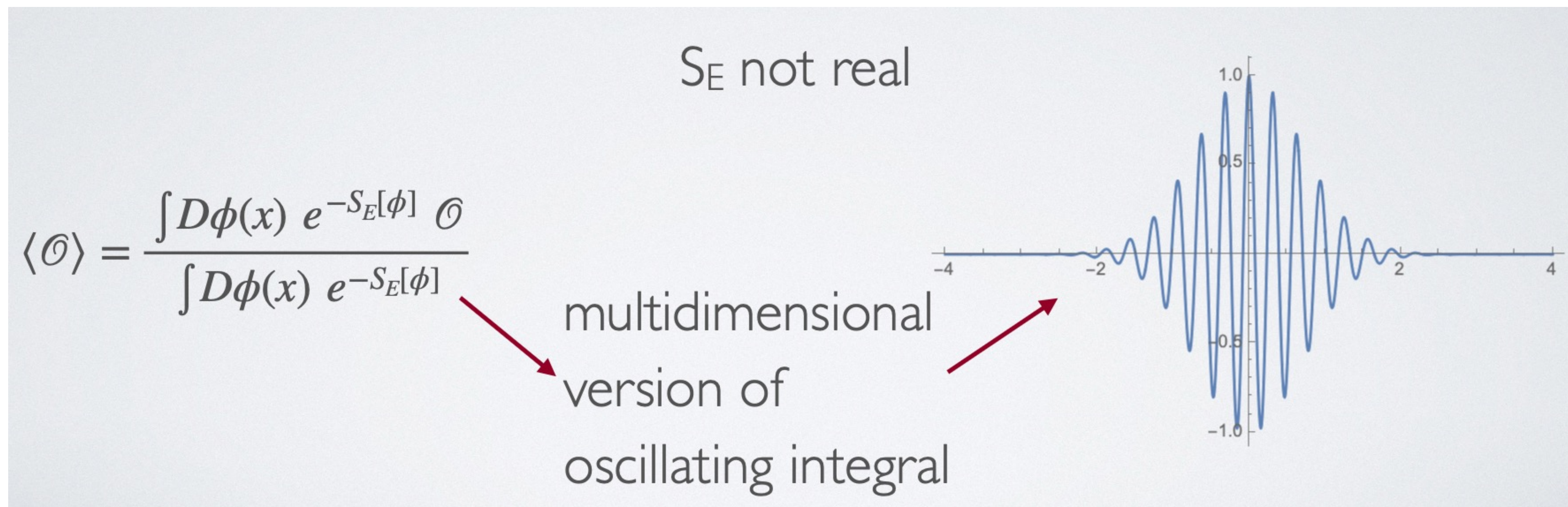
Two scientific pillars in high energy nuclear physics

◆ QCD confinement: QCD phase diagram



Main reasons make classical computations hard

- ◆ Complicated initial and final state, i.e. proton, heavy ions, hadrons, etc.
- ◆ Notorious sign problem for simulating real time dynamics and finite density system using classical Monte-Carlo calculations



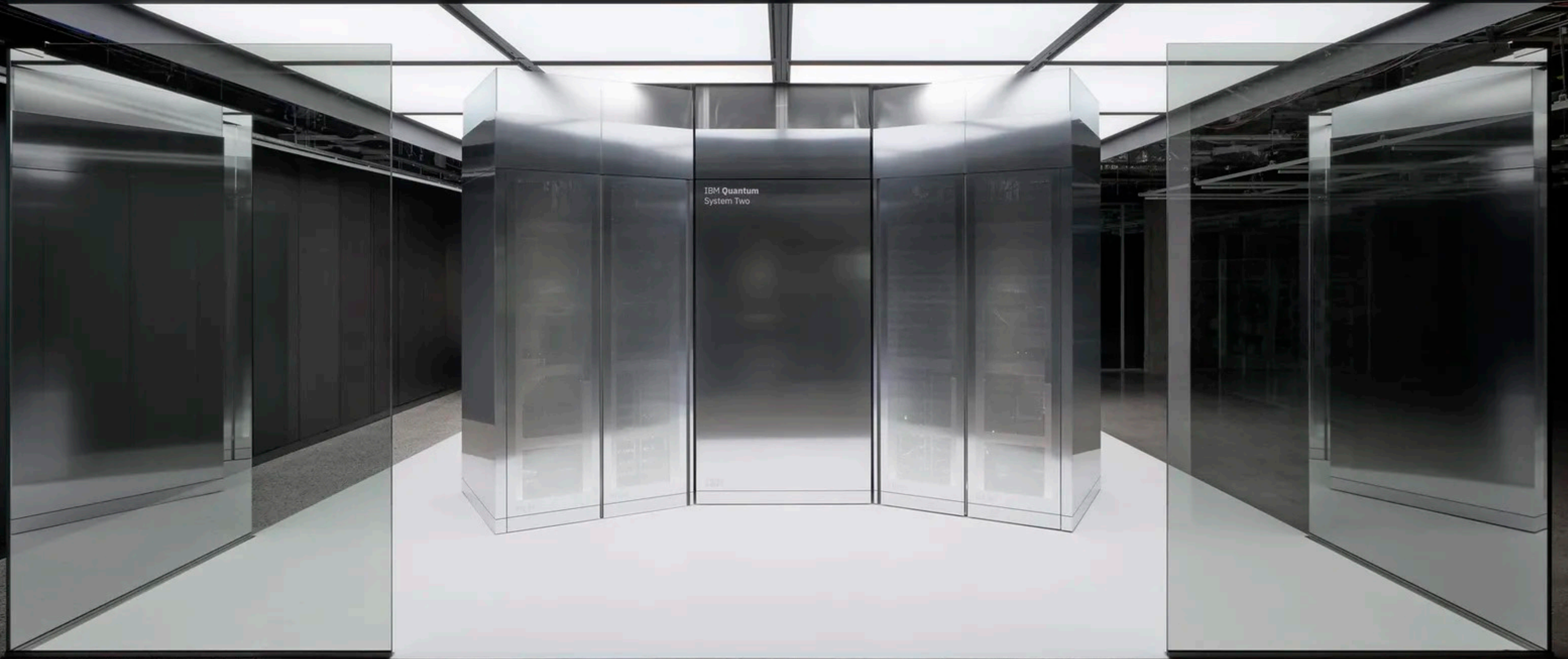
Main reasons make classical computations hard

- ◆ Complicated initial and final state, i.e. proton, heavy ions, hadrons, etc.
- ◆ Notorious sign problem for simulating real time dynamics and finite density system using classical Monte-Carlo calculations

$$\langle \phi(t)\phi(t') \rangle = \frac{\int D\phi(x) e^{iS[\phi]} \phi(t)\phi(t')}{\int D\phi(x) e^{iS[\phi]}}$$

← multidimensional oscillating integral

IBM Can we simulate high energy physics from first principles?



Quantum computing

◆ A bit history

The Computer as a Physical System: A Microscopic Quantum Mechanical Hamiltonian Model of Computers as Represented by Turing Machines

Paul Benioff^{1,2}

Received June 11, 1979; revised August 9, 1979

In this paper a microscopic quantum mechanical model of computers as represented by Turing machines is constructed. It is shown that for each number N and Turing machine Q there exists a Hamiltonian H_N^Q and a class of appropriate initial states such that if $\Psi_Q^N(0)$ is such an initial state, then $\Psi_Q^N(t) = \exp(-iH_N^Q t) \Psi_Q^N(0)$ correctly describes at times t_3, t_0, \dots, t_{3N} model states that correspond to the completion of the first, second, ..., N th computation step of Q . The model parameters can be adjusted so that for an arbitrary time interval Δ around t_3, t_0, \dots, t_{3N} , the “machine” part of $\Psi_Q^N(t)$ is stationary.

KEY WORDS: Computer as a physical system; microscopic Hamiltonian models of computers; Schrödinger equation description of Turing machines; Coleman model approximation; closed conservative system; quantum spin lattices.



P. Benioff, 1979

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don’t know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there’s no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain.



R. Feynman, 1981

Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor
AT&T Bell Labs
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA

Abstract

A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting with David Deutsch, have developed models for quantum mechanical computers and have investigated their computational properties. This paper gives Las Vegas algorithms for finding discrete logarithms and factoring integers on a quantum computer that take a number of steps which is polynomial in the input size, e.g., the number of digits of the integer to be factored. These two problems are generally considered hard on a classical computer and have been used as the basis of several proposed cryptosystems. (We thus give the first examples of quantum cryptanalysis.)

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

The next part of this paper discusses how quantum computation relates to classical complexity classes. We will thus first give a brief intuitive discussion of complexity classes for those readers who do not have this background. There are generally two resources which limit the ability of computers to solve large problems: time and space (i.e., memory). The field of analysis of algorithms considers the asymptotic demands that algorithms make for these resources as a function of the problem size. Theoretical computer scientists generally classify algorithms as efficient when the number of steps of the algorithms grows as



P. Shor, 1994

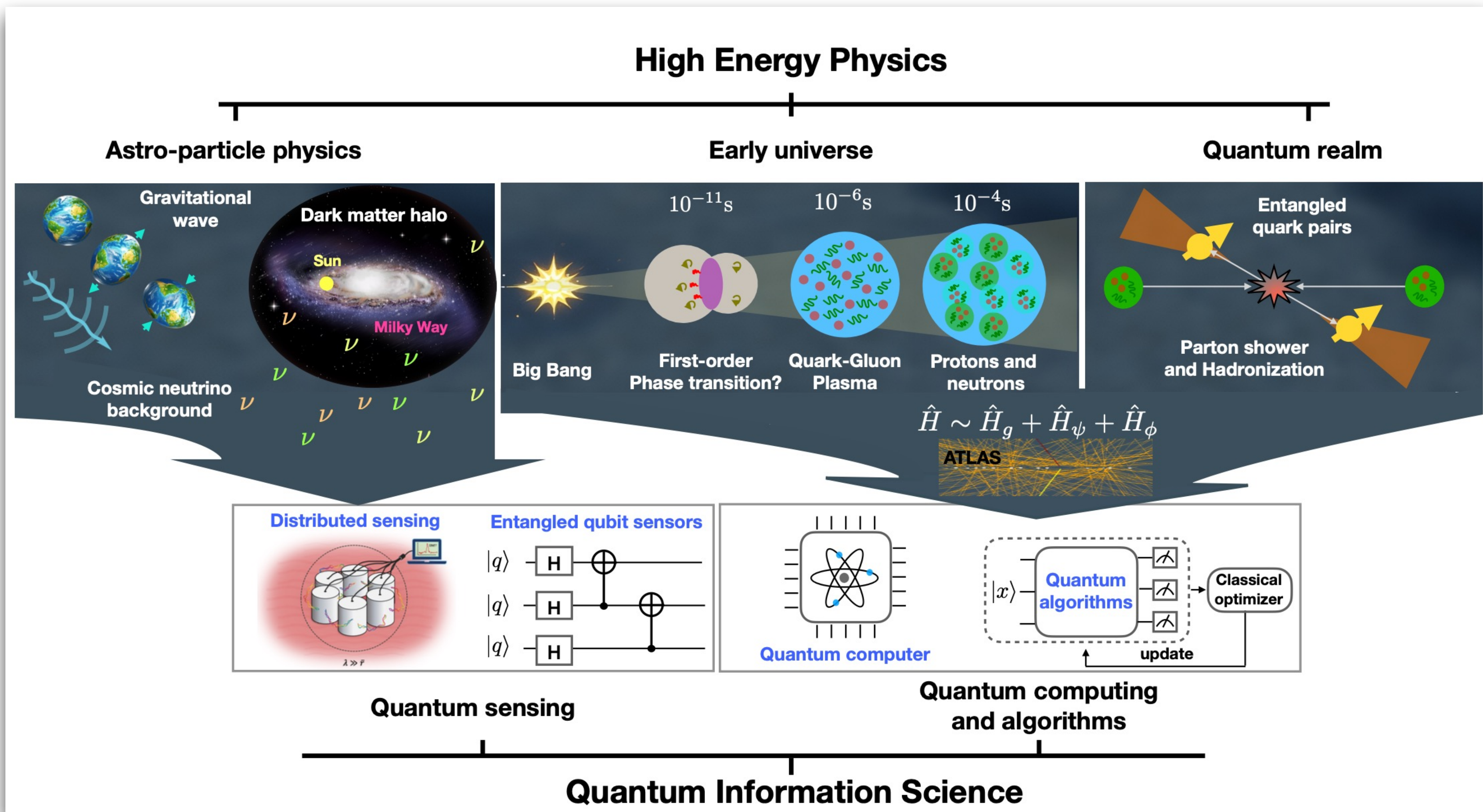


IBM Q System One (2019), the first circuit-based commercial quantum computer

“... and if you want to make a simulation of nature, you’d better make it quantum mechanical, ...”

—Feynman

Quantum simulation for nuclear and high energy physics



Increasing interest in HEP and NP using quantum computing

Solving a Higgs optimization problem with quantum annealing for machine learning

Alex Mott, Joshua Job, Jean-Roch Vlimant, Daniel Lidar & Maria Spiropulu 

Nature **550**, 375–379 (2017) | [Cite this article](#)

9683 Accesses | **53** Citations | **180** Altmetric | [Metrics](#)

Abstract

The discovery of Higgs-boson decays in a background of standard-model processes was assisted by machine learning methods^{1,2}. The classifiers used to separate signals such as these from background are trained using highly unerring but not completely perfect simulations of the physical processes involved, often resulting in incorrect labelling of background processes or signals (label noise) and systematic errors. Here we use quantum^{3,4,5,6} and classical^{7,8} annealing (probabilistic techniques for approximating the global maximum or minimum of a given function) to solve a Higgs-signal-versus-background machine learning optimization problem, mapped to a problem of finding the ground state of a corresponding Ising spin model. We build a set of weak classifiers based on the kinematic observables of the Higgs decay photons, which we then use to construct a

Quantum Algorithm for High Energy Physics Simulations

Benjamin Nachman, Davide Provasoli, Wibe A. de Jong, and Christian W. Bauer
Phys. Rev. Lett. **126**, 062001 – Published 10 February 2021

ArticleReferencesCiting Articles (6)Supplemental MaterialPDFHTMLExport Ci

>

ABSTRACT

Simulating quantum field theories is a flagship application of quantum computing. However, calculating experimentally relevant high energy scattering amplitudes entirely on a quantum computer is prohibitively difficult. It is well known that such high energy scattering processes can be factored into pieces that can be computed using well established perturbative techniques, and pieces which currently have to be simulated using classical Markov chain algorithms. These classical Markov chain simulation approaches work well to capture many of the salient features, but cannot capture all quantum effects. To exploit quantum resources in the most efficient way, we introduce a new paradigm for quantum algorithms in field theories. This approach uses quantum computers only for those parts of the problem which are not computable using existing techniques. In particular, we develop a polynomial time quantum final state shower that accurately models the effects of intermediate spin states similar to those present in high energy electroweak showers with a global evolution variable. The algorithm is explicitly demonstrated for a simplified quantum field theory on a quantum computer.

Featured in Physics

Editors' Suggestion

Access by Si

Cloud Quantum Computing of an Atomic Nucleus

E. F. Dumitrescu, A. J. McCaskey, G. Hagen, G. R. Jansen, T. D. Morris, T. Papenbrock, R. C. Pooser, D. J. Dean, and P. Lougovski
Phys. Rev. Lett. **120**, 210501 – Published 23 May 2018

 See Viewpoint: [Cloud Quantum Computing Tackles Simple Nucleus](#)

ArticleReferencesCiting Articles (127)PDFHTMLExport Citation

>

ABSTRACT

We report a quantum simulation of the deuteron binding energy on quantum processors accessed via cloud servers. We use a Hamiltonian from pionless effective field theory at leading order. We design a low-depth version of the unitary coupled-cluster ansatz, use the variational quantum eigensolver algorithm, and compute the binding energy to within a few percent. Our work is the first step towards scalable nuclear structure computations on a quantum processor via the cloud, and it sheds light on how to map scientific computing applications onto nascent quantum devices.

Letter

Open Access

Access by South

Quantum simulation of open quantum systems in heavy-ion collisions

Wibe A. de Jong, Mekena Metcalf, James Mulligan, Mateusz Płoskoń, Felix Ringer, and Xiaojun Yao
Phys. Rev. D **104**, L051501 – Published 7 September 2021

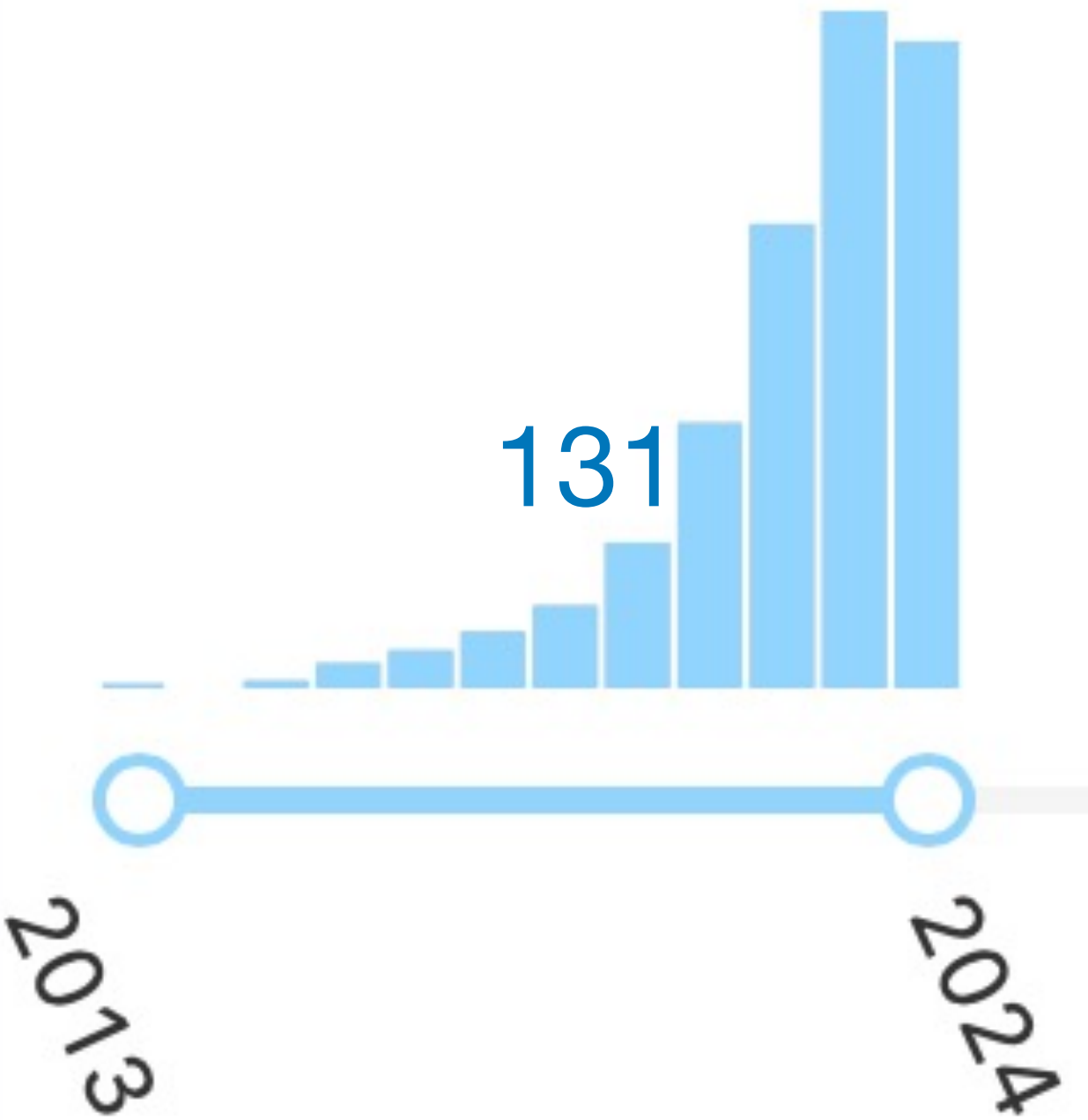
ArticleReferencesNo Citing ArticlesSupplemental MaterialPDFHTMLExport Citati

>

ABSTRACT

We present a framework to simulate the dynamics of hard probes such as heavy quarks or jets in a hot, strongly coupled quark-gluon plasma (QGP) on a quantum computer. Hard probes in the QGP can be treated as open quantum systems governed in the Markovian limit by the Lindblad equation. However, due to large computational costs, most current phenomenological calculations of hard probes evolving in the QGP use semiclassical approximations of the quantum evolution. Quantum computation can mitigate these costs and offers the potential for a fully quantum treatment with exponential speed-up over classical techniques. We report a simplified demonstration of our framework on IBM Q quantum devices and apply the random identity insertion method to account for CNOT depolarization noise, in addition to measurement error mitigation. Our work demonstrates the feasibility of simulating open quantum systems on current and near-term quantum devices, which is of broad relevance to applications in nuclear physics, quantum information, and other fields.

Date of paper



Inspire:

find t quantum computing and date>2015

Community-wide efforts

QUANTUM COMPUTING FOR THEORETICAL NUCLEAR PHYSICS


A White Paper prepared for the U.S. Department of
Energy, Office of Science, Office of Nuclear Physics




Opportunities for Nuclear Physics & Quantum Information Science

13 Mar 2019

CERN

 **QUANTUM
TECHNOLOGY
INITIATIVE**



Quantum support vector machines for Higgs boson classification

arXiv > quant-ph > arXiv:2209.14839

Search...
Help | Advanced

Quantum Physics

[Submitted on 29 Sep 2022]

Report of the Snowmass 2021 Theory Frontier Topical Group on Quantum Information Science

Simon Catterall, Roni Harnik, Veronika E. Hubeny, Christian W. Bauer, Asher Berlin, Zohreh Davoudi, Thomas Faulkner, Thomas Hartman, Matthew Headrick, Yonatan F. Kahn, Henry Lamm, Yannick Meurice, Surjeet Rajendran, Mukund Rangamani, Brian Swingle

arXiv > quant-ph > arXiv:2307.03236

Search...
Help | Advanced

Quantum Physics

[Submitted on 6 Jul 2023]

Quantum Computing for High-Energy Physics: State of the Art and Challenges. Summary of the QC4HEP Working Group

Alberto Di Meglio, Karl Jansen, Ivano Tavernelli, Constantia Alexandrou, Srinivasan Arunachalam, Christian W. Bauer, Kerstin Borras, Stefano Carrazza, Arianna Crippa, Vincent Croft, Roland de Putter, Andrea Delgado, Vedran Dunjko, Daniel J. Egger, Elias Fernandez-Combarro, Elina Fuchs, Lena Funcke, Daniel Gonzalez-Cuadra, Michele Grossi, Jad C. Halimeh, Zoe Holmes, Stefan Kuhn,

arXiv > nucl-ex > arXiv:2303.00113

Search...
Help | Advanced

Nuclear Experiment

[Submitted on 28 Feb 2023]

Quantum Information Science and Technology for Nuclear Physics. Input into U.S. Long-Range Planning, 2023

Douglas Beck, Joseph Carlson, Zohreh Davoudi, Joseph Formaggio, Sofia Quaglioni, Martin Savage, Joao Barata, Tanmoy Bhattacharya, Michael Bishof, Ian Cloet, Andrea Delgado, Michael DeMarco, Caleb Fink, Adrien Florio, Marianne Francois, Dorota Grabowska, Shannon Hoogerheide, Mengyao Huang, Kazuki Ikeda, Marc Illa, Kyungseon Joo, Dmitri Kharzeev, Karol Kowalski, Wai Kin Lai, Kyle Leach, Ben Loer, Ian Low, Joshua Martin, David Moore, Thomas

Different approaches in quantum simulation

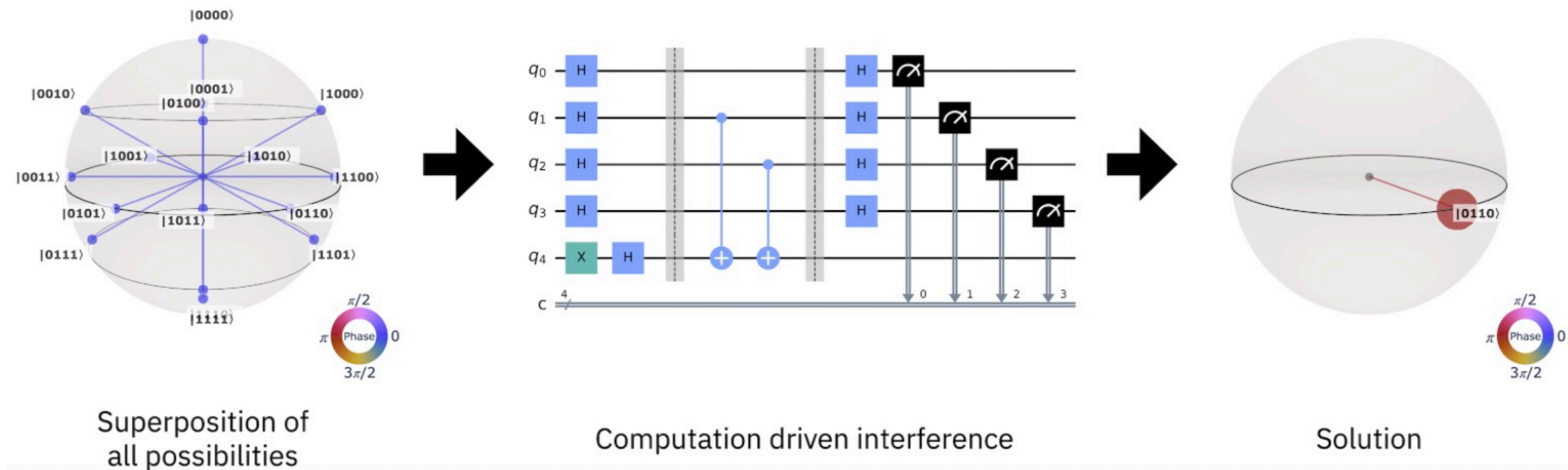
	Analog	Hybrid	Digital
Degreed of freedom	Bosons, fermions, qubits, qudits, etc.	Bosons, fermions, qubits, qudits, etc.	Qubits
Time evolution	Continuous	Digitized (gate based)	Digitized (gate based)
Hardware agnostic	No	No	Yes (hence universal)
Simulation challenge	Hamiltonian engineering	Gate decomposition	Gate decomposition
Theoretical error	Imperfect effective Hamiltonian	Imperfect digitalization	Imperfect digitalization
Error correction	Not known	Possible	Yes

Hamiltonian vs. Lagrangian formulation of LGTs

	Path integral (Lagrangian)	Hamiltonian
Degrees of freedom	Fields and their derivatives	Fields and their conjugate variables
Spacetime signature	Often Euclidean	Minkowski
Starting point	$\mathcal{L}[\varphi, \partial\varphi]$	$\hat{H}[\hat{\varphi}, \hat{\pi}]$
Hilbert space	Not explicitly constructed/relevant	Built out of $\hat{H}^n \text{vac.}\rangle^*$ * $ \text{vac.}\rangle = \text{empty state}\rangle$
Expectation values	$\frac{1}{Z} \int \mathcal{D}\varphi e^{-S} O$	$\langle \psi \hat{O} \psi \rangle$
Dynamical quantities	Sometimes accessible with indirect methods, e.g., Luescher method.	In principle accessible: $\langle \psi e^{i\hat{H}t} \hat{O} e^{-i\hat{H}t} \psi \rangle$
Computational methods	Monte Carlo, etc.	Classical Hamiltonian methods like exact diag., tensor networks/quantum simulation
Computational challenge	Sign and signal-to-noise problem for real-time quantities and finite-density systems.	Exponential scaling of the Hilbert space with the number of DOF.

Quantum computing

<https://qiskit.org/>



♦ Building blocks of quantum computing

- Qubit: takes infinitely many different values $|\psi\rangle := \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

- Quantum gate: unitary operators (X, Y, Z, CNOT)

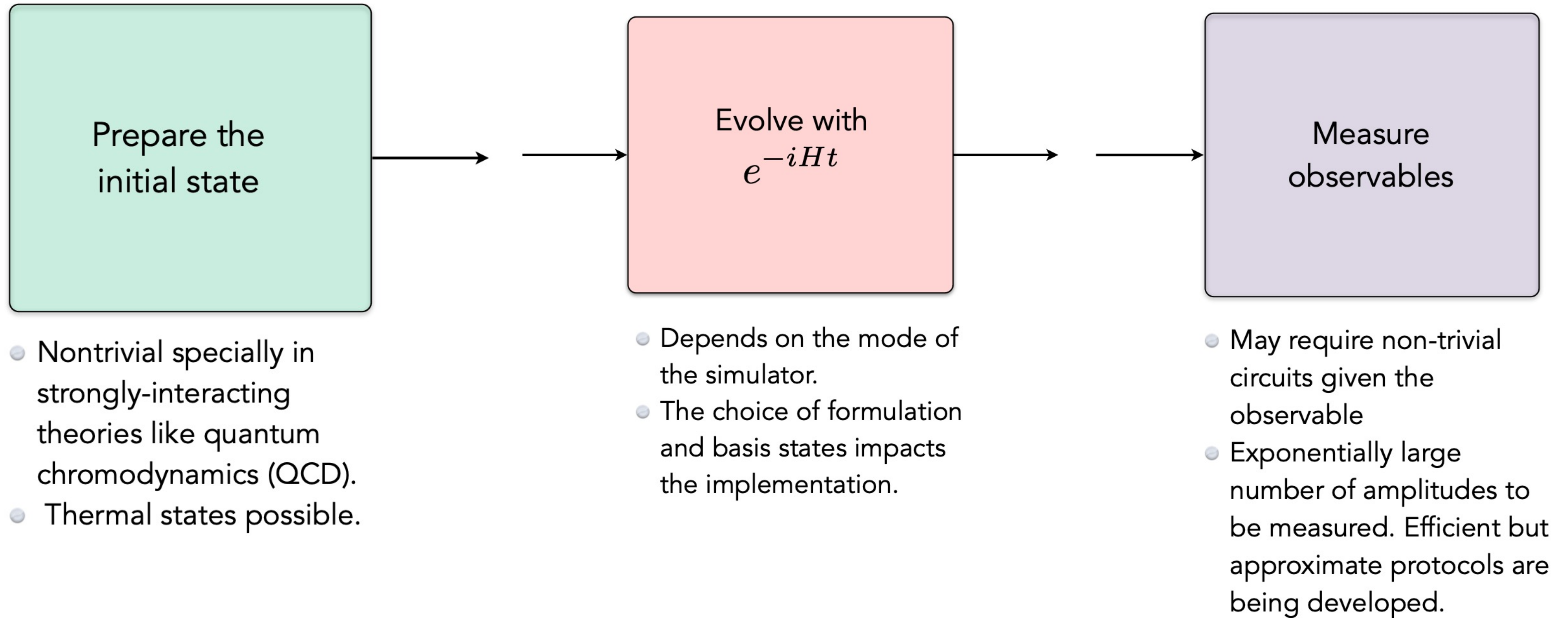
$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{X} \beta|0\rangle + \alpha|1\rangle$$

$$|0\rangle \xrightarrow{H} \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

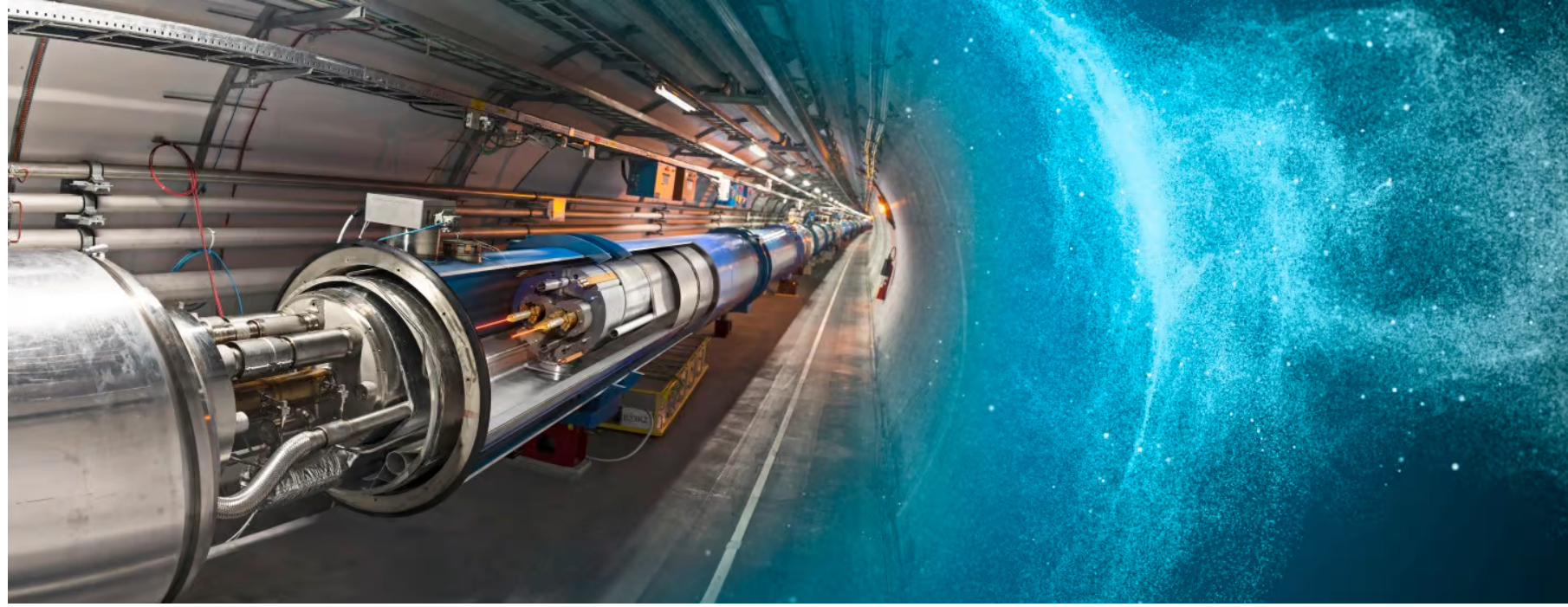
$$\begin{matrix} |x\rangle \\ |y\rangle \end{matrix} \xrightarrow{\text{CNOT}} \begin{matrix} |x\rangle \\ |y \oplus x\rangle \end{matrix}$$

- Measurements: Hermitian

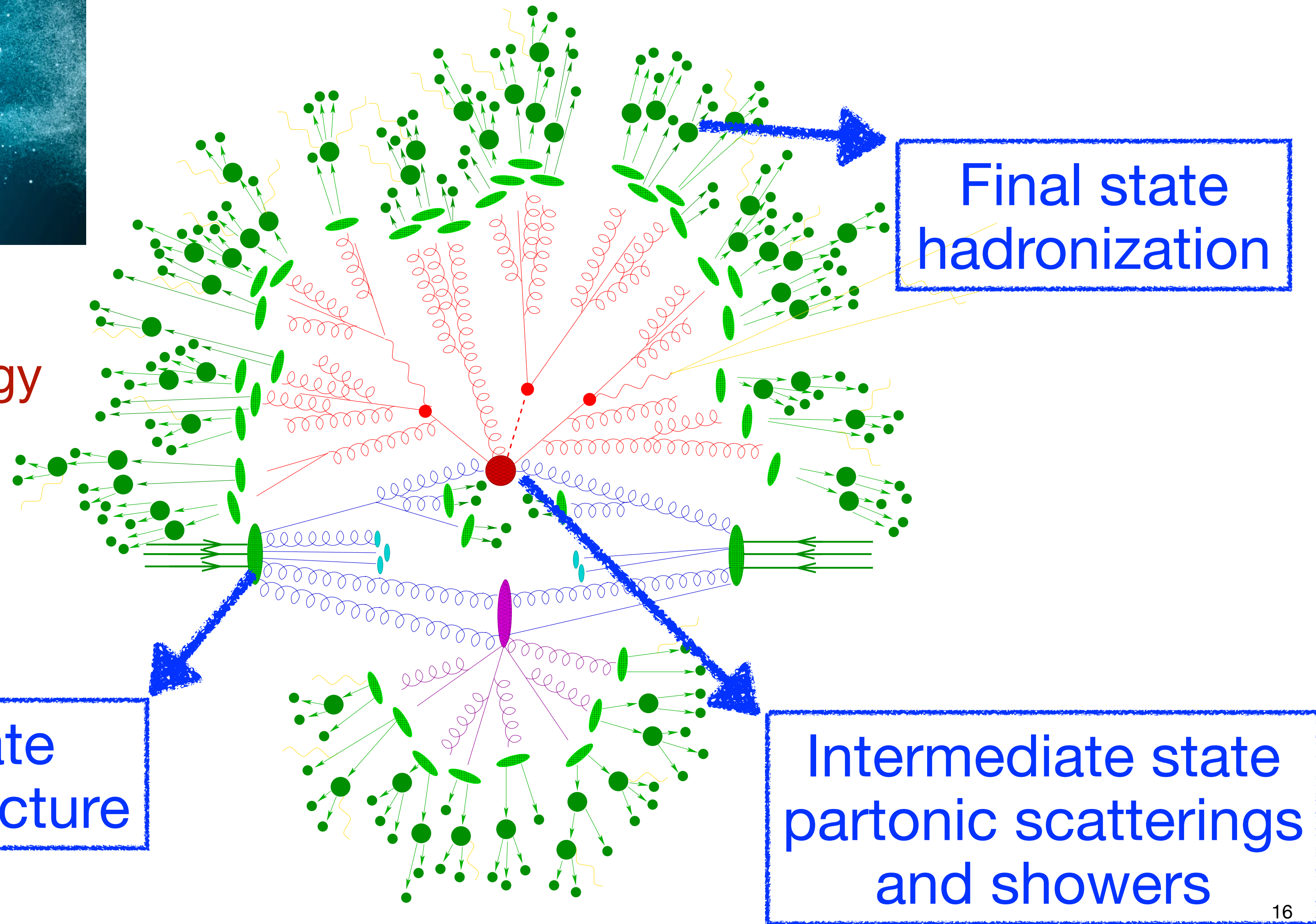
What we usually do on quantum machine?



High energy hadron/nucleus collisions



LHC \sim TeV
the highest collision energy
in the world!



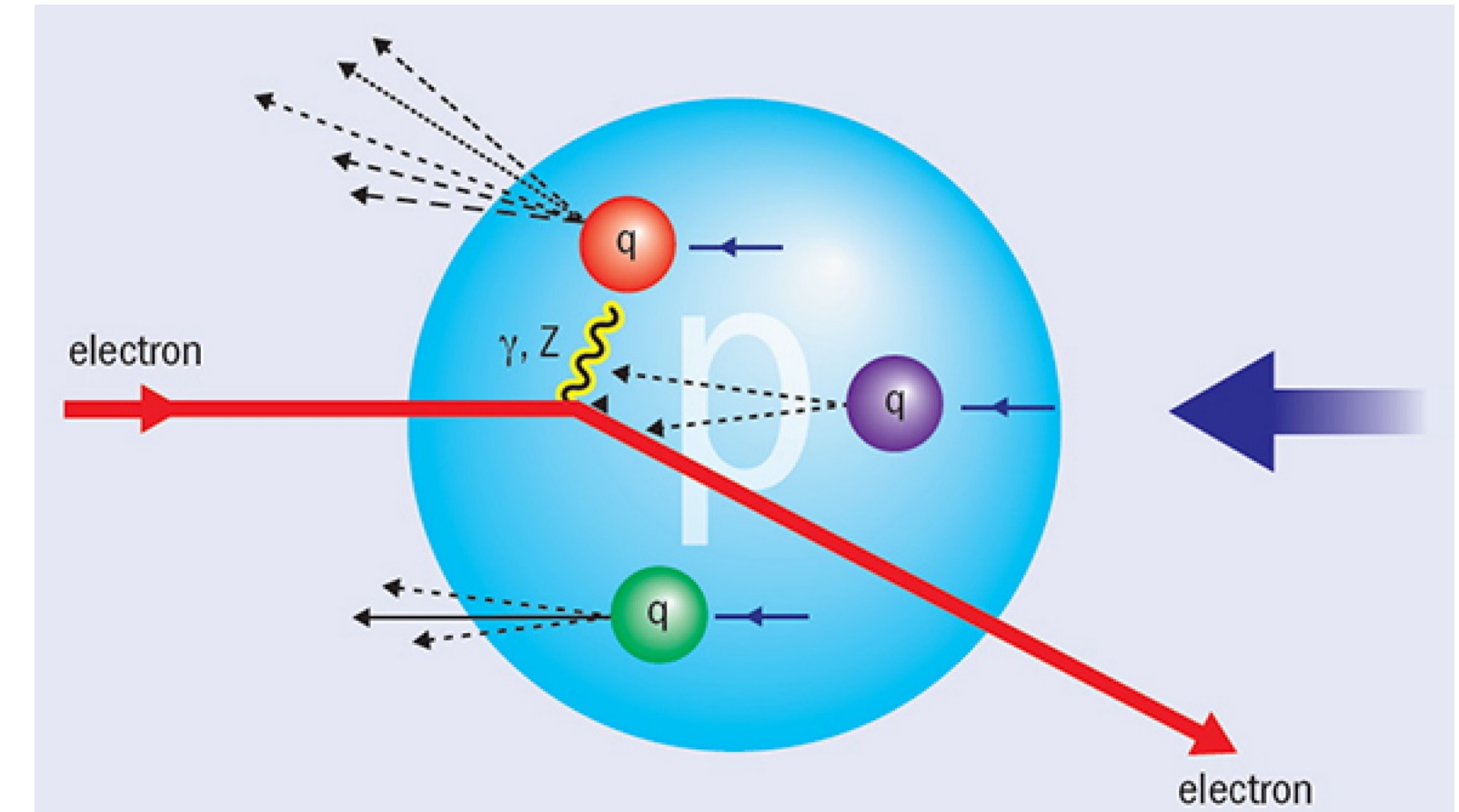
First principle calculation on lattice

◆ Electron-proton collisions

$$| \langle X(T) | U(T, -T) | ep (-T) \rangle |^2$$

◆ Key steps

- Prepare initial states from the distance past $(-T)$
- Evolve these states from the distance past to time T , $U(T, -T) \rightarrow e^{-iH(\psi)T}$
- Perform measurement in final state

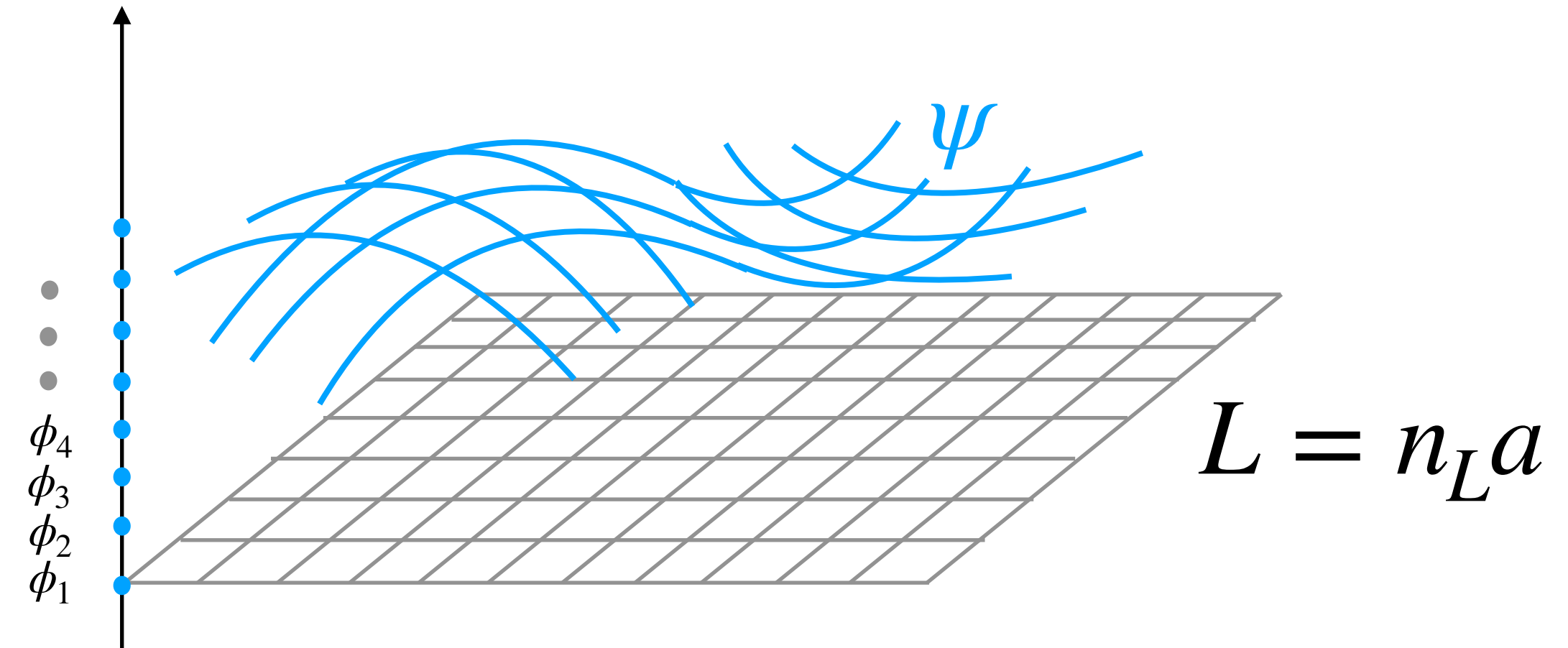
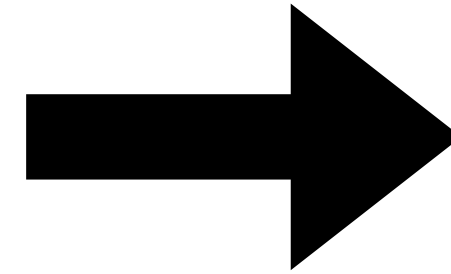


However, the Hilbert space in quantum field theory is infinite ...

First principle calculation on lattice

- ◆ Digitize field ϕ at discrete points x

$$|\langle X(T) | U(T, -T) | ep(-T) \rangle|^2$$



- Hilbert space dimension: $n_H = (n_\phi)^{n_L^d}$

n_ϕ : # of digitized field values

n_L : # of lattice points per dimension

d : # of dimensions

- Energy range can be described by lattice

$$(n_L a)^{-1} \lesssim E \lesssim a^{-1}$$

Full energy range of LHC: $100\text{MeV} \lesssim E \lesssim 13\text{TeV}$

$$n_L^D \sim 10^{15}$$

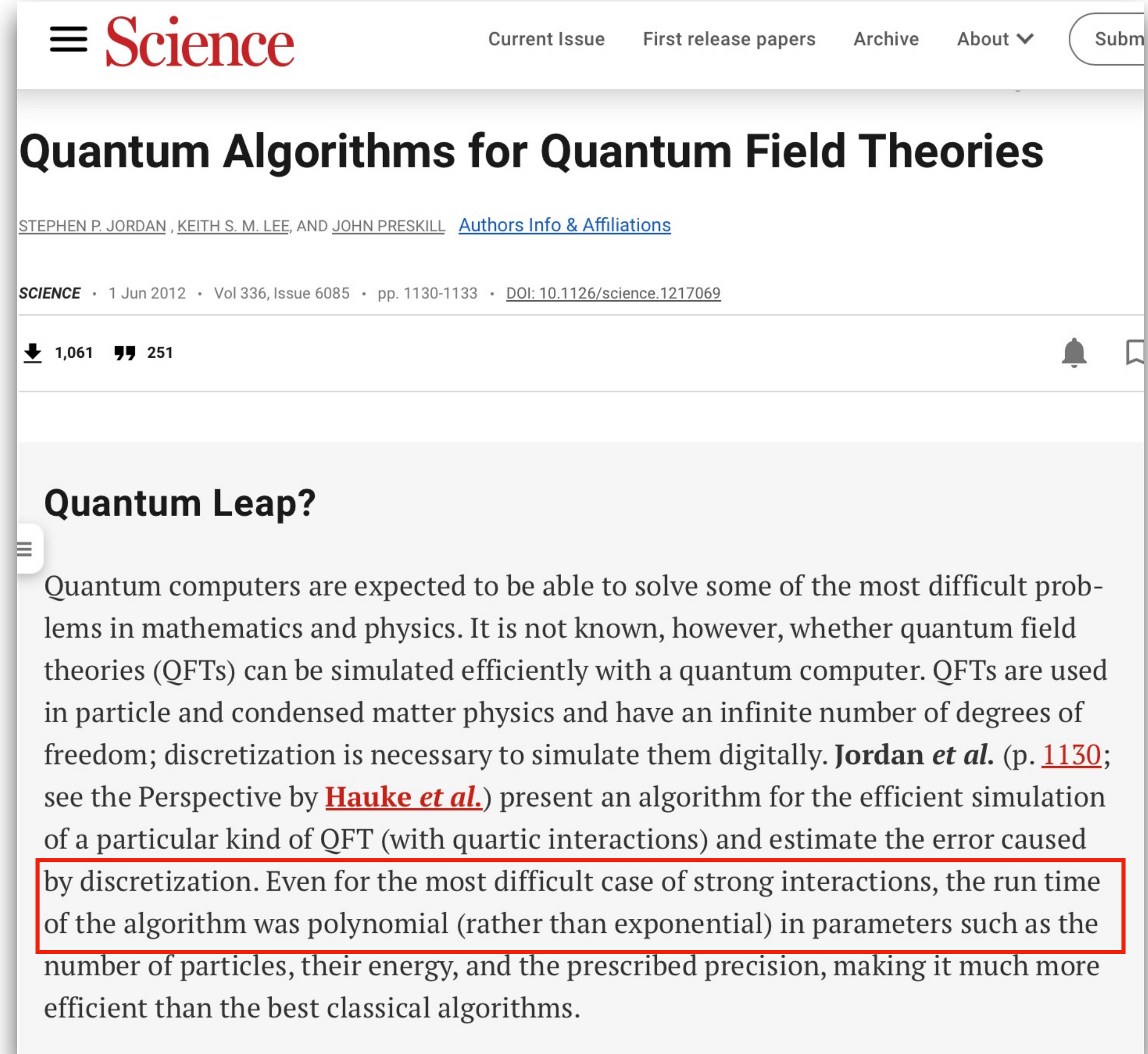
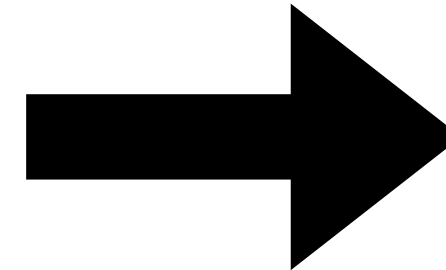
Assume 5 bit digitization: $n_\phi = 2^5 = 32$

Dimension of Hilbert space: $n_H = 32^{10^{15}} \sim \infty$

First principle calculation on lattice

- ◆ Digitize field ϕ at discrete points x

$$|\langle X(T) | U(T, -T) | ep(-T) \rangle|^2$$



- Hilbert space dimension: $n_H = (n_\phi)^{n_L^d}$

Quantum computing: encoding in qubits

$$n_q = \ln_2 n_H = n_L^D \ln_2 n_\phi$$

$$\text{For LHC: } n_q = 5 \times 10^{15}$$

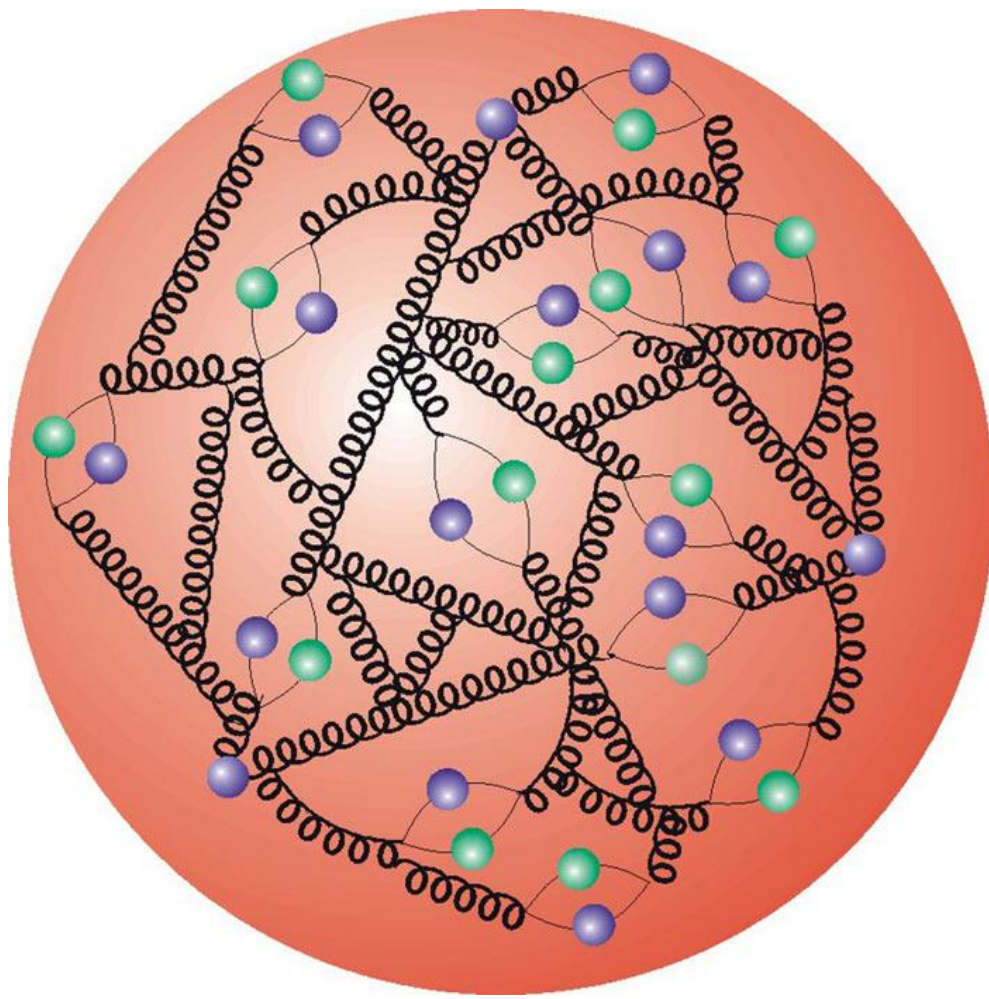
Quantum computing run time was polynomial in # of particles
Way beyond NISQ era in quantum computing

Quantum simulation using effective field theory

- For the hadron

$$100\text{MeV} \lesssim E \lesssim 1\text{GeV} \quad n_L^D \sim 10^3$$

$$\text{\# of qubits: } n_q = 5 \times 10^3$$



Promising in NISQ era in quantum computing!

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Editoria

Open Access

Simulating Collider Physics on Quantum Computers Using Effective Field Theories

Christian W. Bauer, Benjamin Nachman, and Marat Freytsis
Phys. Rev. Lett. **127**, 212001 – Published 18 November 2021

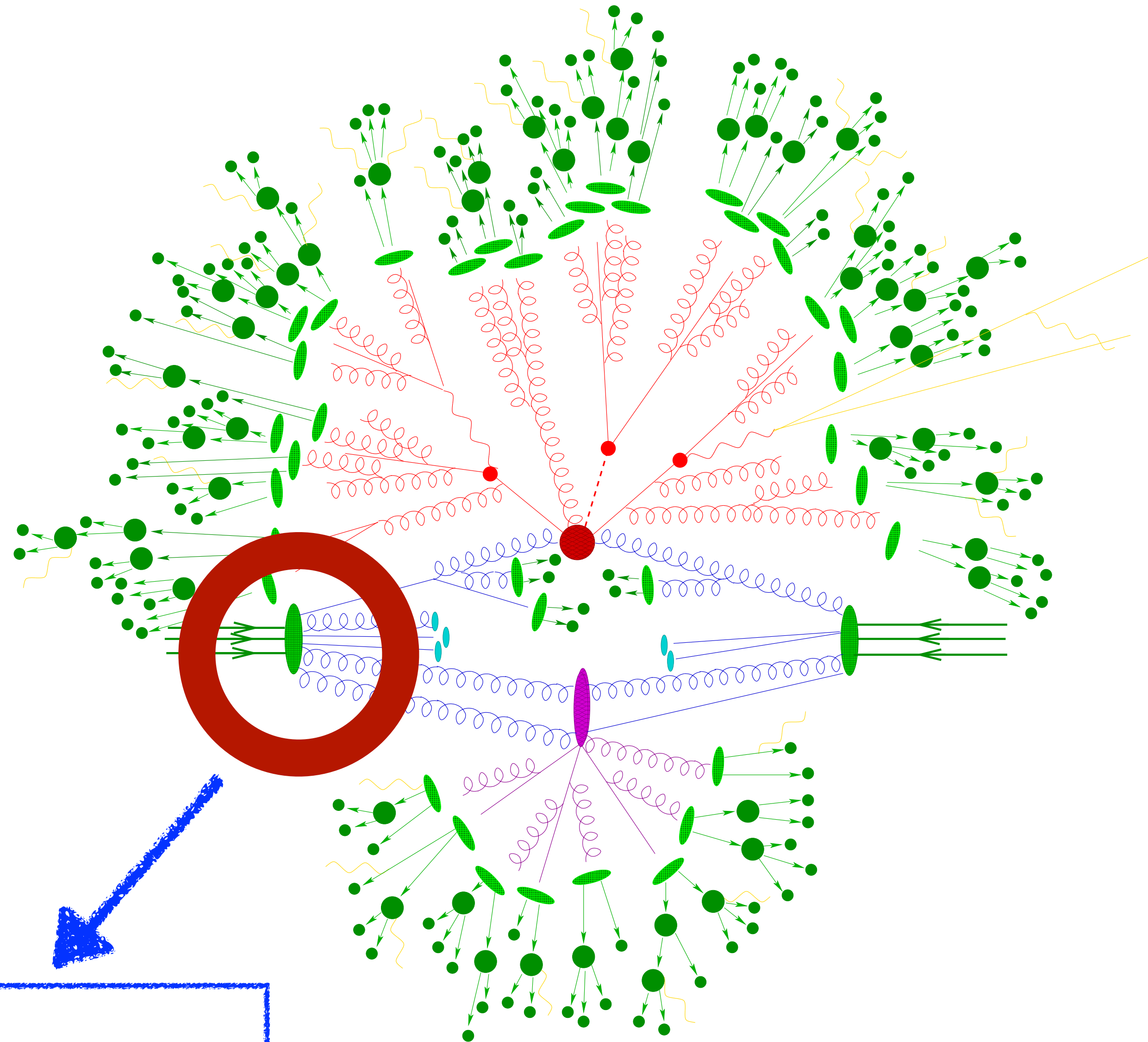
Article References Citing Articles (35) Supplemental Material PDF HTML Export Citation

>

ABSTRACT

Simulating the full dynamics of a quantum field theory over a wide range of energies requires exceptionally large quantum computing resources. Yet for many observables in particle physics, perturbative techniques are sufficient to accurately model all but a constrained range of energies within the validity of the theory. We demonstrate that effective field theories (EFTs) provide an efficient mechanism to separate the high energy dynamics that is easily calculated by traditional perturbation theory from the dynamics at low energy and show how quantum algorithms can be used to simulate the dynamics of the low energy EFT from first principles. As an explicit example we calculate the expectation values of vacuum-to-vacuum and vacuum-to-one-particle transitions in the presence of a time-ordered product of two Wilson lines in scalar field theory, an object closely related to those arising in EFTs of the standard model of particle physics. Calculations are performed using simulations of a quantum computer as well as measurements using the IBMQ Manhattan machine.

1



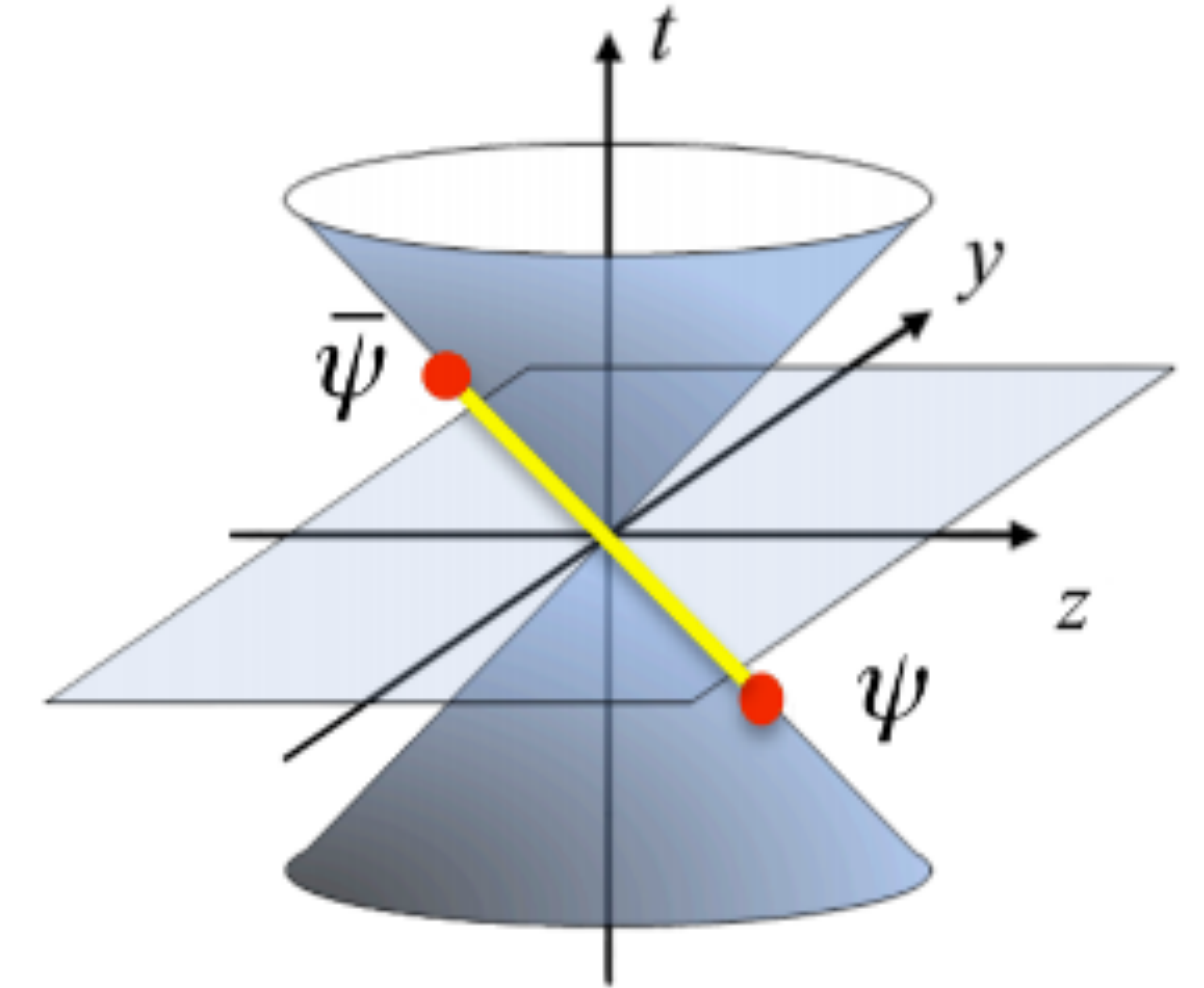
Initial state

parton distribution function f

Simulate hadron partonic structure on quantum computer

- ◆ Nucleon structure - 1D parton distribution function

$$f_{q/p}(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \underbrace{\langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathcal{W}(0, y^-) \psi(y^-) | p \rangle}_{\text{real time correlation function}}$$
$$y^- = (t - y_3)/\sqrt{2}$$



- ◆ Lattice calculation: moments, LaMET ...
- ◆ QC can naturally simulate real-time dynamics.
- ◆ We are far from QCD Quantum Supremacy, start from a toy model for proof of concept study

Simulate hadron partonic structure on quantum computer

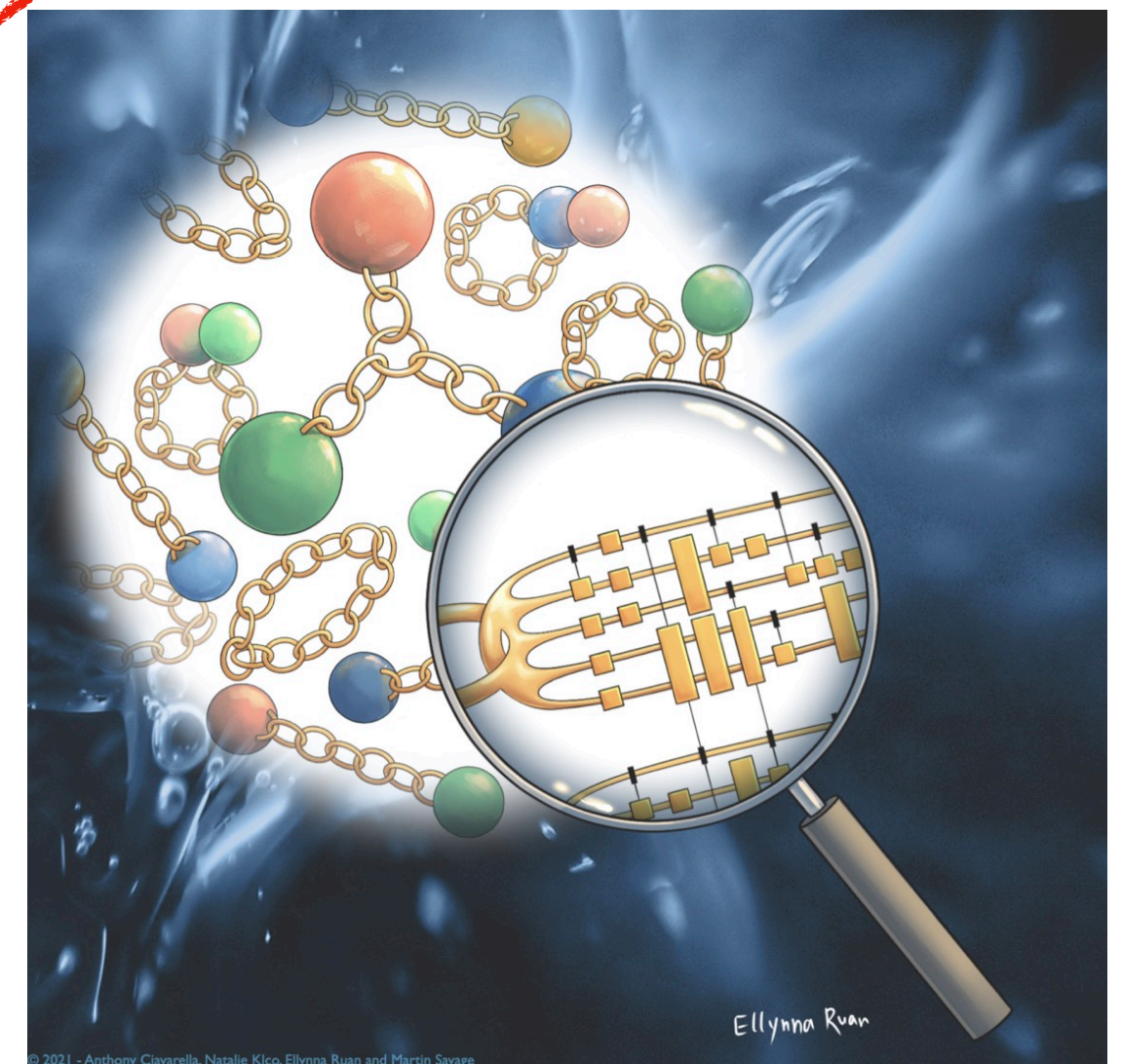
- ◆ A toy model - 1+1D NJL (Gross, Neveu, 1974), no gauge field

$$\mathcal{L} = \bar{\psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha) \psi_\alpha + g(\bar{\psi}_\alpha \psi_\alpha)^2$$

$$f(x) = \int dz^- e^{-ixM_h z^-} \langle h | \bar{\psi}(z^-) \gamma^+ \psi(0) | h \rangle = \int dz^- e^{-ixM_h z^-} \langle h | e^{iH_z} \bar{\psi}(0, -z) e^{-iH_z} \gamma^+ \psi(0) | h \rangle$$

- ◆ Challenges in quantum computing

- Jordan-Wigner: map QFT to qubits+gates system
- VQE: prepare the external hadronic state $|h\rangle$
- Evaluate the real-time dynamical correlation function
- Measurement of final observable



Simulate hadron partonic structure on quantum computer

◆ Quantum field to qubits+gates $\mathcal{L} = \bar{\psi}(i\partial - m)\psi + g(\bar{\psi}\psi)^2$

- Discretization: staggered fermion, put different fermion components, flavors on different sites

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_{2n} \\ \phi_{2n+1} \end{pmatrix}$$

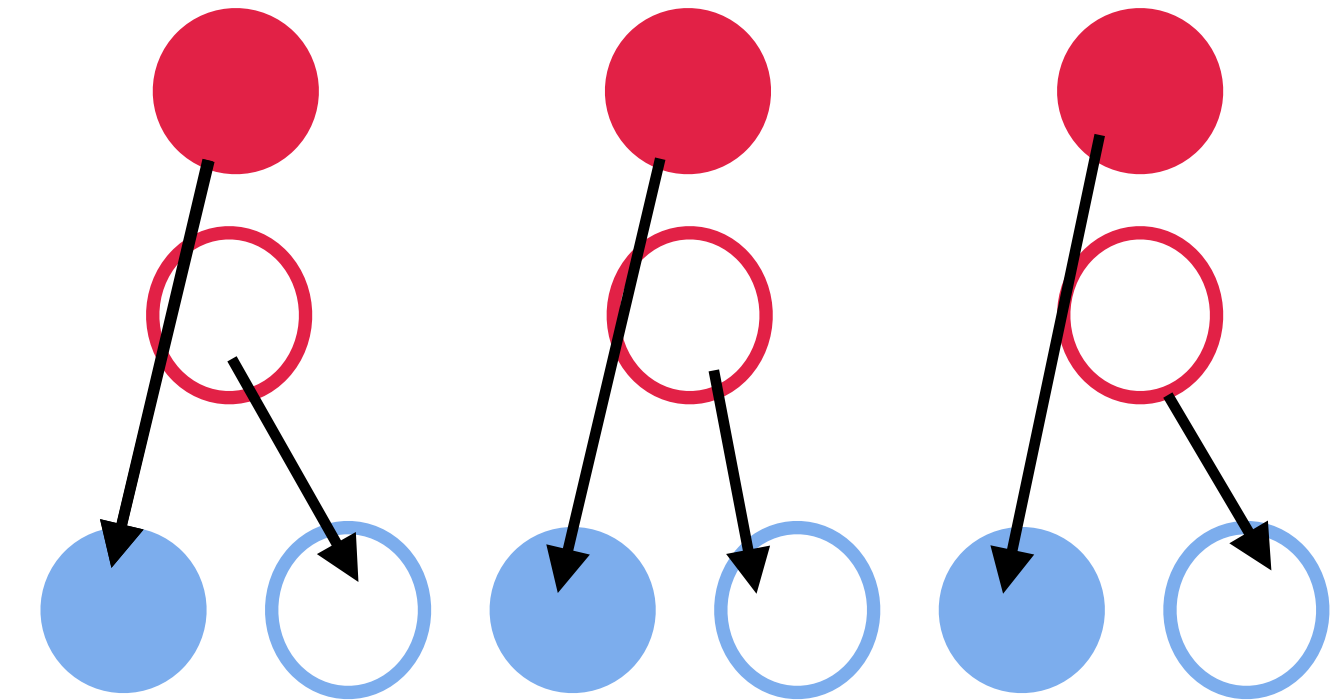
- Jordan-Wigner transformation

$$\phi_n = \prod_{i < n} Z_i (X + iY)_n$$

- Discretized PDF:

$$f(x) \rightarrow \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iH_z} \phi_{-2z+i}^\dagger e^{-iH_z} \phi_j | h \rangle$$

$$H = H_1 + H_2 + H_3 + H_4 \quad H_1 = \sum_{n=\text{even}} \frac{1}{4} [X_n Y_{n+1} - Y_n X_{n+1}]$$



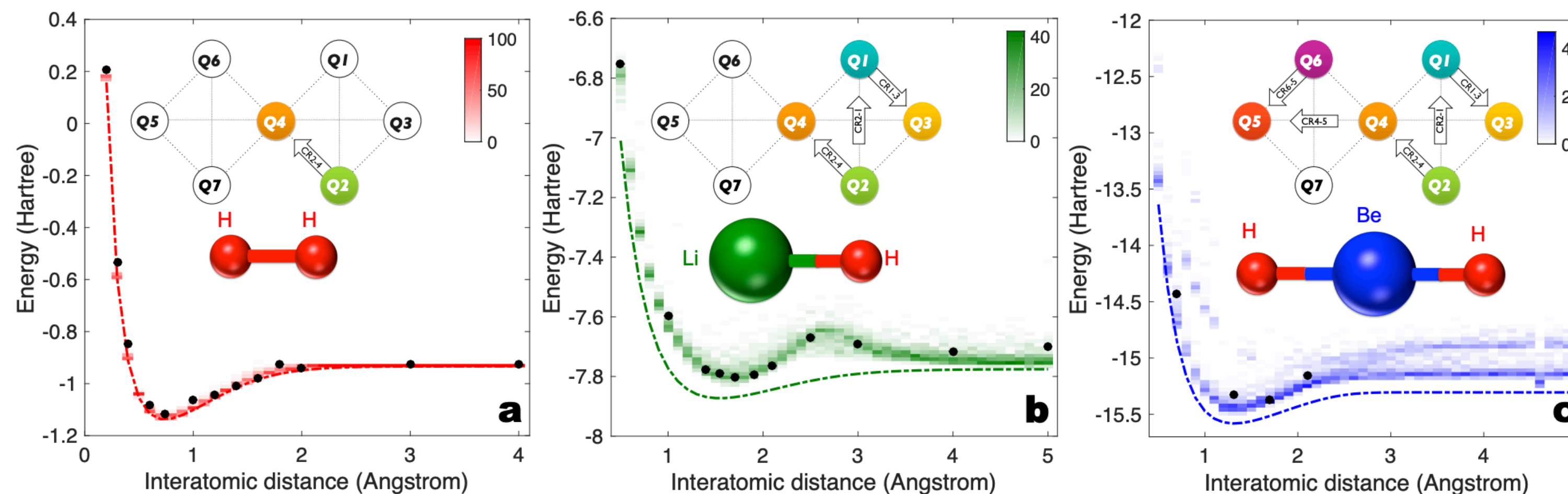
Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

- Hadron states are the eigenstates of the Hamiltonian with certain quantum numbers.
- Prepare the state by variational quantum eigensolver (VQE) 2103.08505 + ...
- VQE is a hybrid method involves both classical and quantum computers

Potential energy surfaces

Nature 549, 242 (2017)



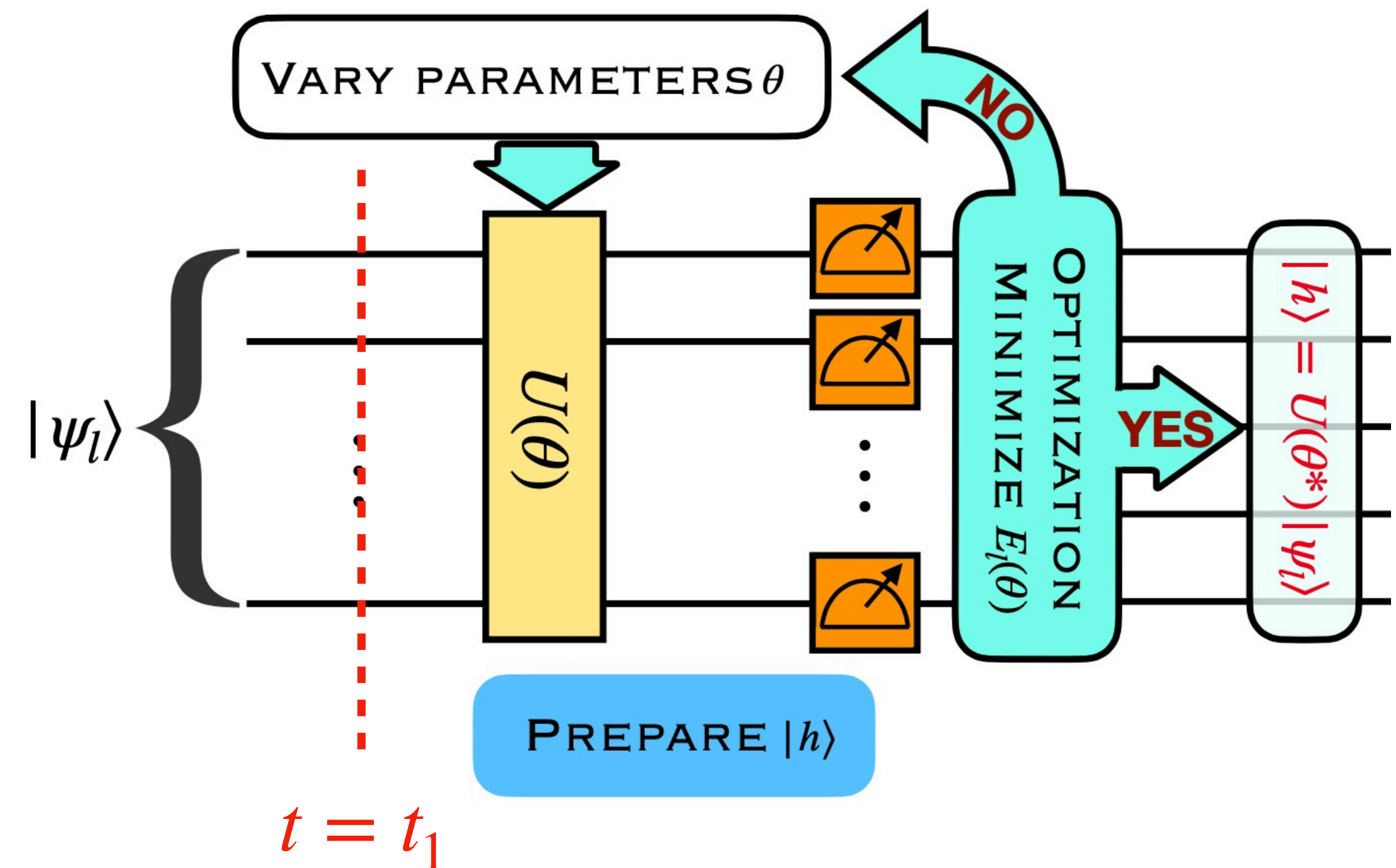
show its power in quantum chemistry

Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_l\rangle$



Simulate hadron partonic structure on quantum computer

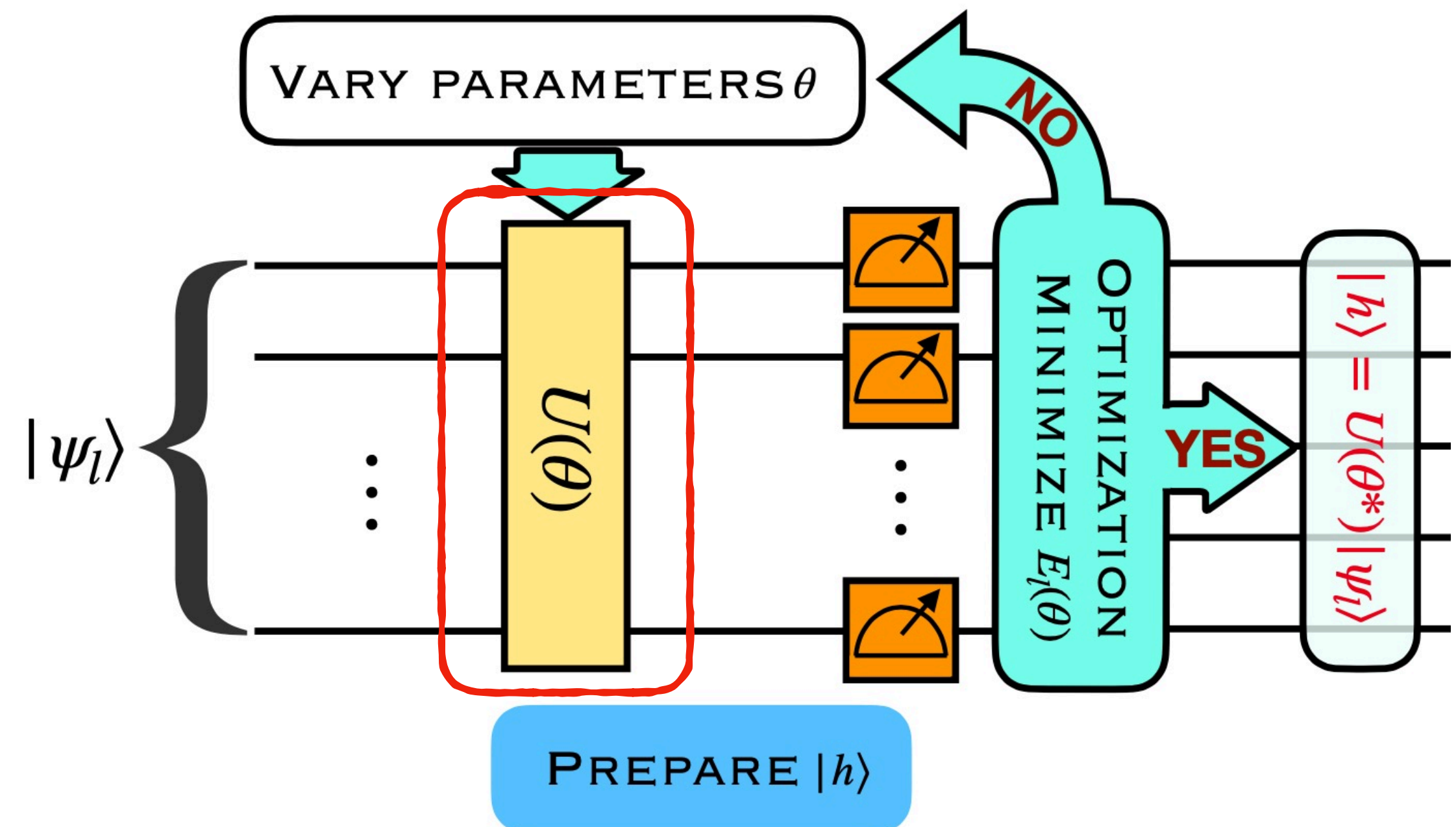
◆ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_l\rangle$

2. Divide $H = H_1 + H_2 + H_3 + H_4$

$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i \theta_{ij} H_j)$$



Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

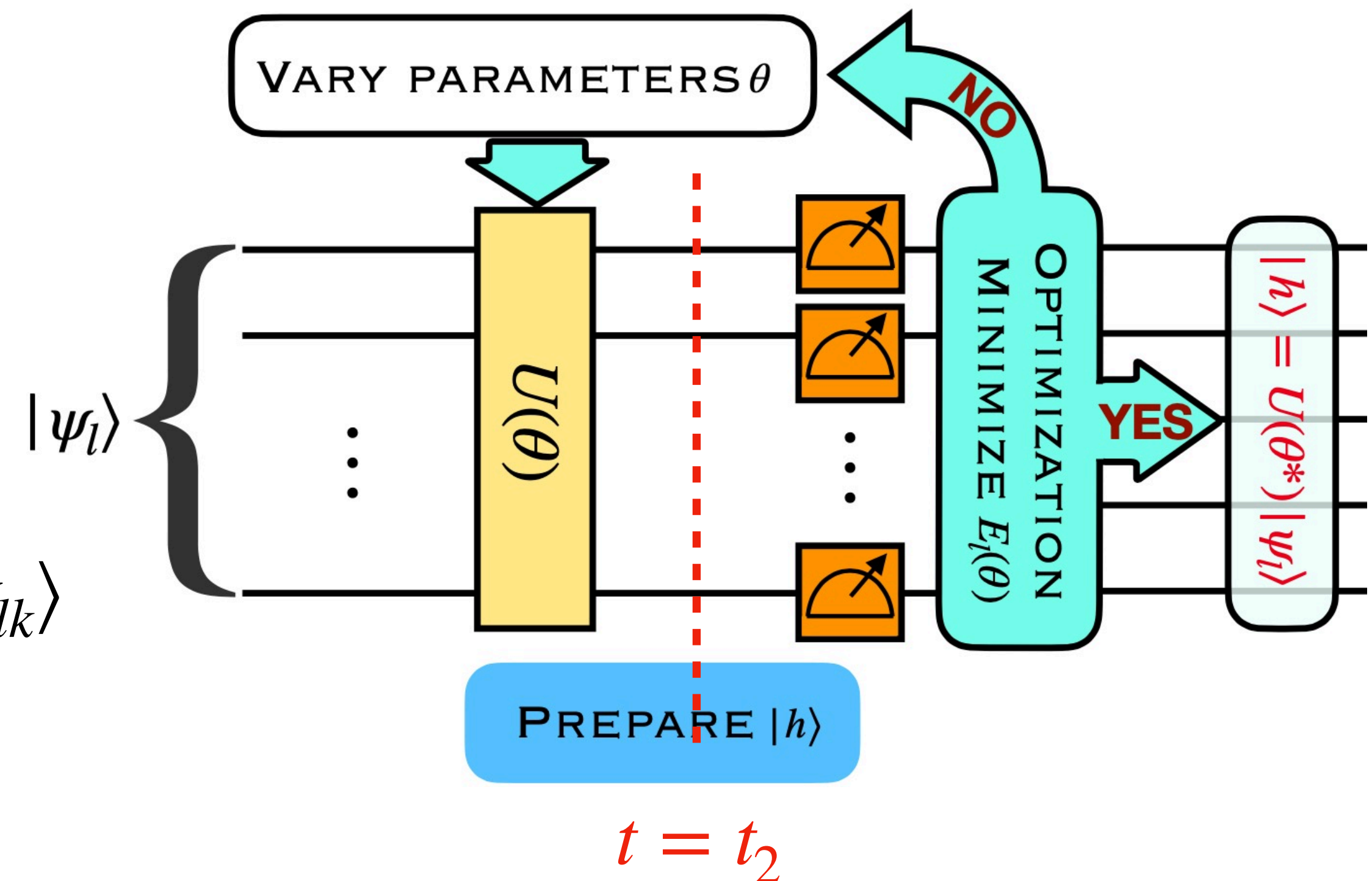
Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_{lk}\rangle$

2. Divide $H = H_1 + H_2 + H_3 + H_4$

$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i \theta_{ij} H_j)$$

3. Generate the trial state: $|\psi_{lk}(\theta)\rangle = U(\theta) |\psi_{lk}\rangle$



Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_{lk}\rangle$

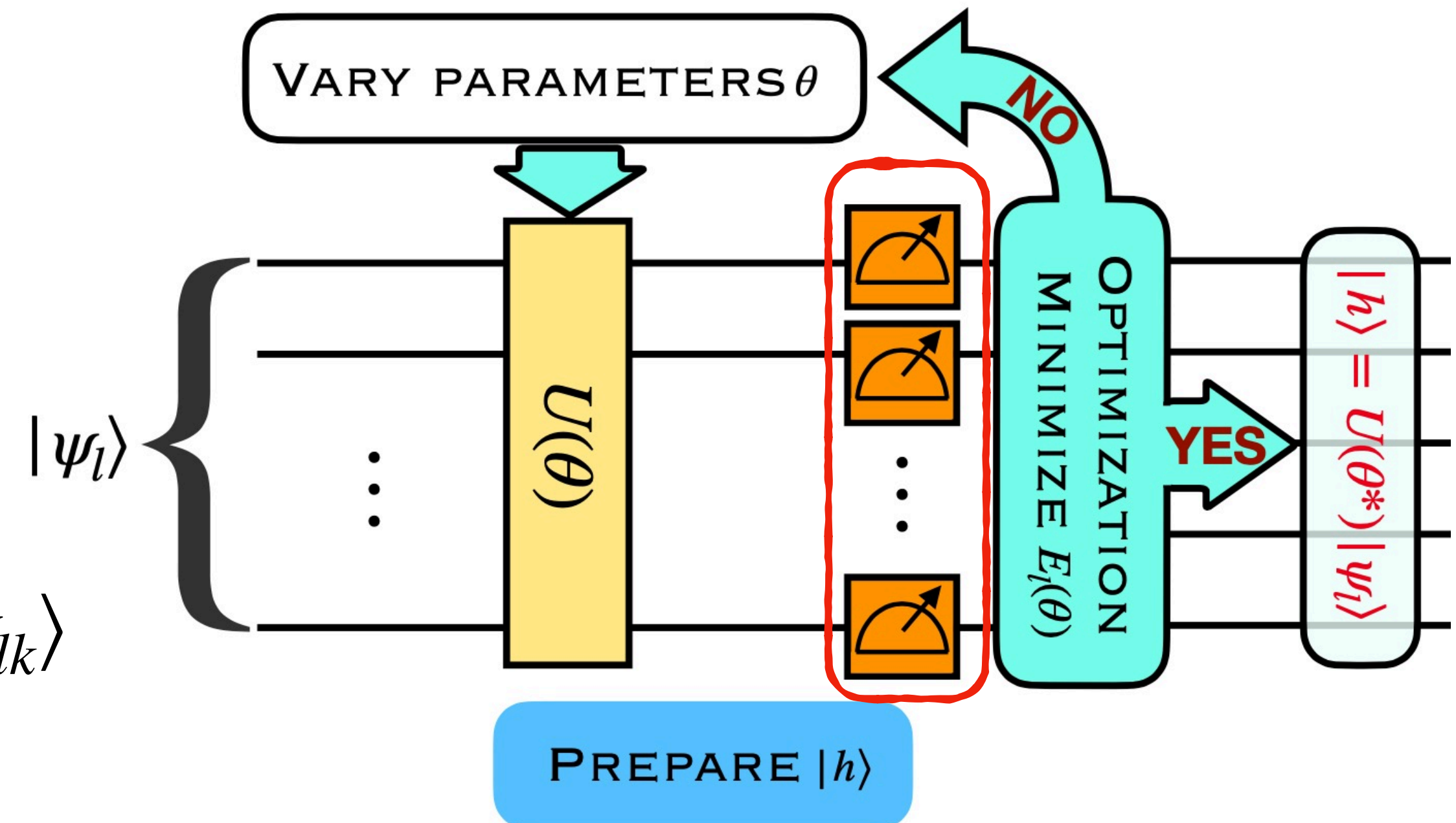
2. Divide $H = H_1 + H_2 + H_3 + H_4$

$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i \theta_{ij} H_j)$$

3. Generate the trial state: $|\psi_{lk}(\theta)\rangle = U(\theta) |\psi_{lk}\rangle$

4. Measure the loss function:

$$E_l(\theta) = \sum_{i=1}^k w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$$



Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_{lk}\rangle$

2. Divide $H = H_1 + H_2 + H_3 + H_4$

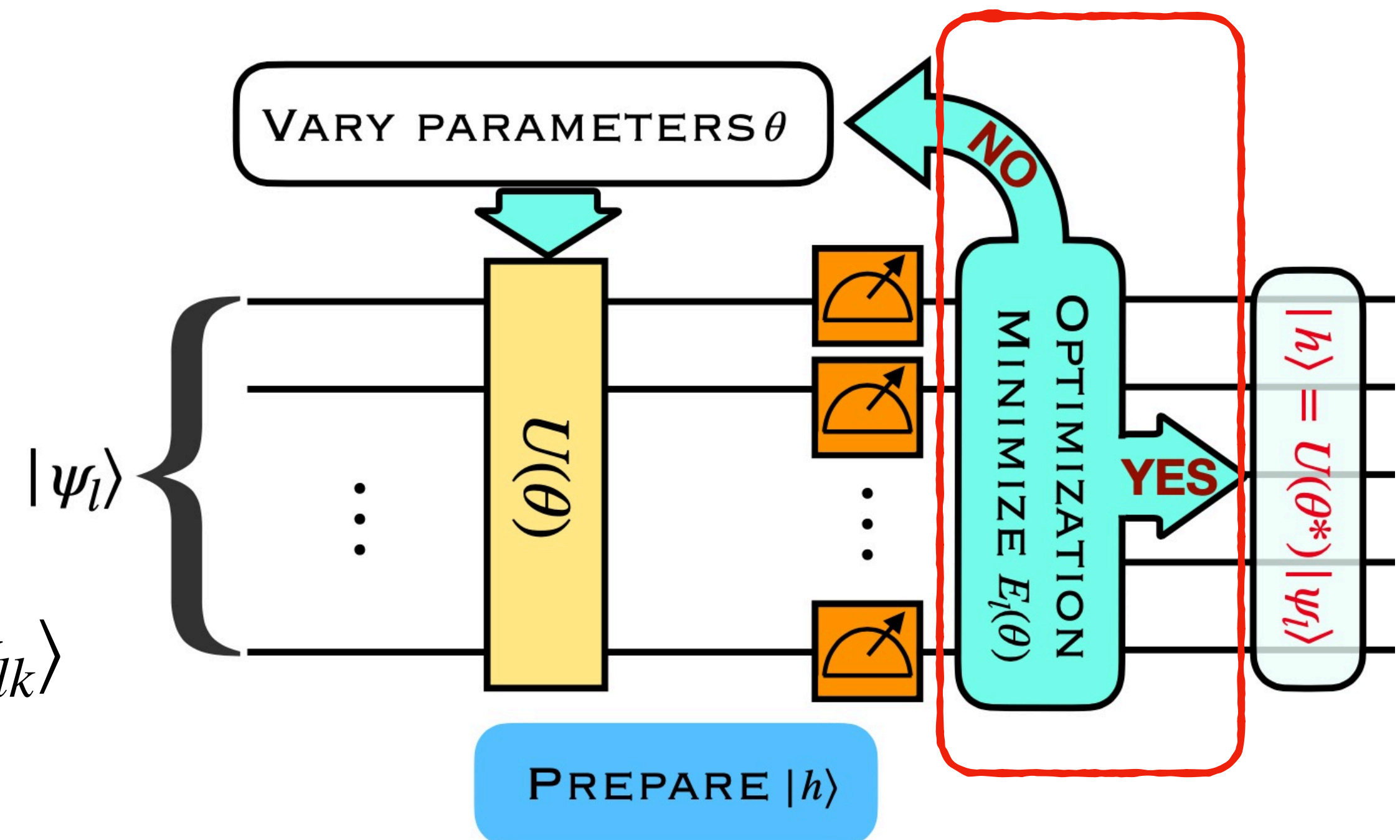
$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i \theta_{ij} H_j)$$

3. Generate the trial state: $|\psi_{lk}(\theta)\rangle = U(\theta) |\psi_{lk}\rangle$

4. Measure the loss function:

$$E_l(\theta) = \sum_{i=1}^k w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$$

5. Optimize the parameters θ^* on classical machine



Simulate hadron partonic structure on quantum computer

◆ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

1. For a giving quantum number l and first k excited states, construct a trial hadronic state $|\psi_{lk}\rangle$

2. Divide $H = H_1 + H_2 + H_3 + H_4$

$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i \theta_{ij} H_j)$$

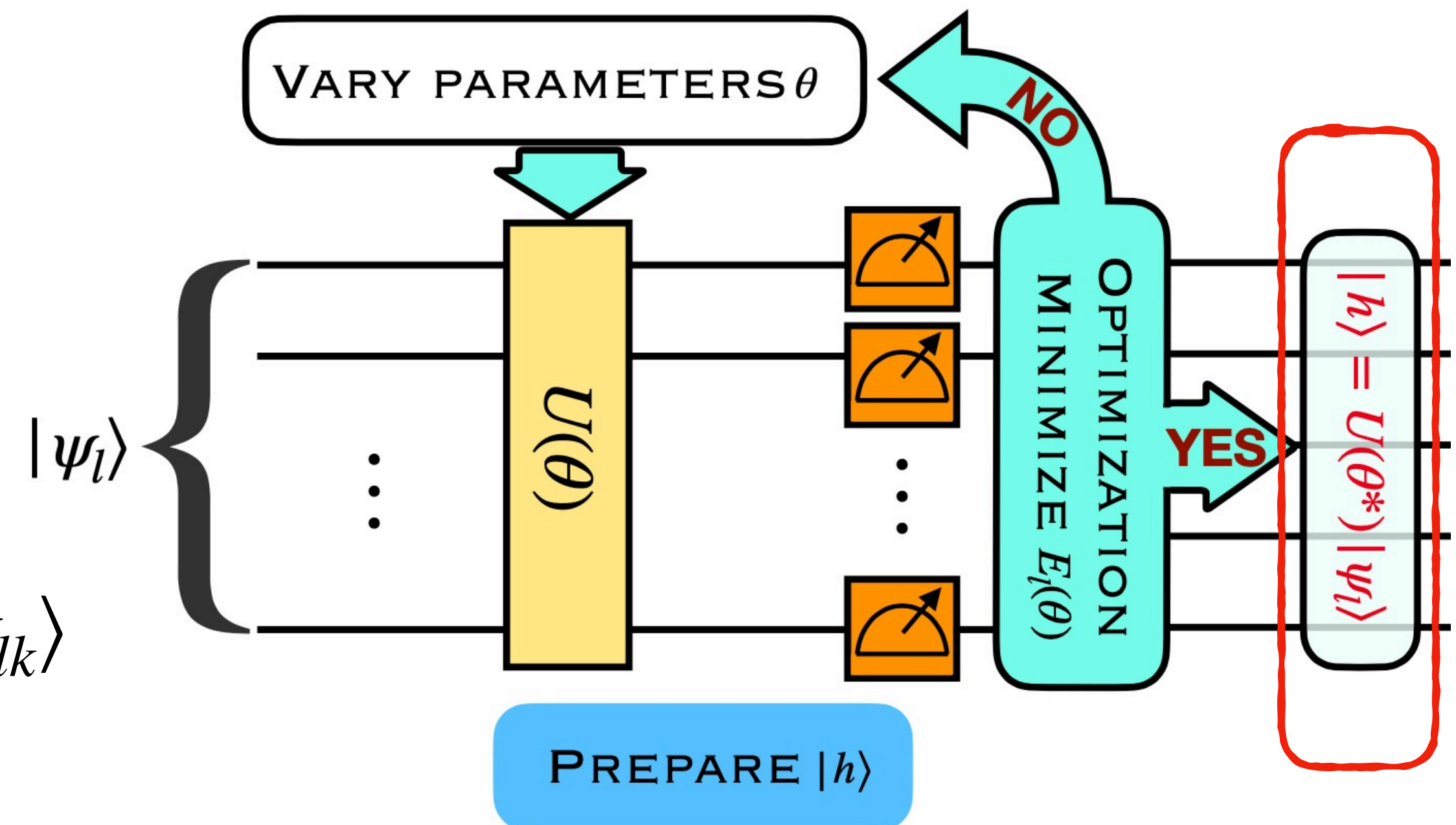
3. Generate the trial state: $|\psi_{lk}(\theta)\rangle = U(\theta) |\psi_{lk}\rangle$

4. Measure the loss function:

$$E_l(\theta) = \sum_{i=1}^k w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$$

5. Optimize the parameters θ^* on classical machine

6. Generate the hadron state $|h\rangle = U(\theta^*) |\psi_{lk}\rangle$



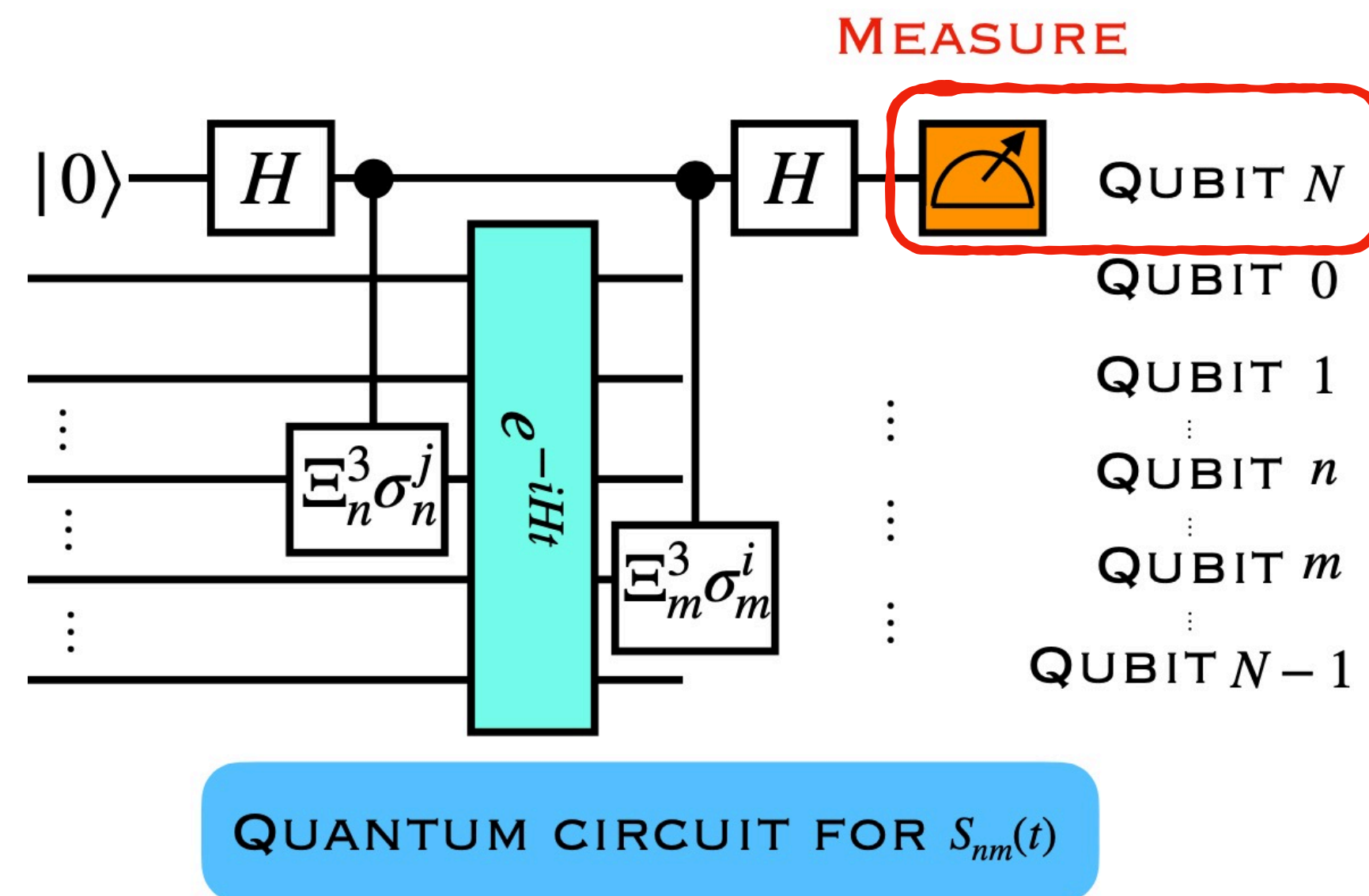
Simulate hadron partonic structure on quantum computer

- ◆ Evaluate the real-time dynamical correlation function

$$S_{mn}(t) = \langle h | e^{iHt} \Xi_m^3 \sigma_m^i e^{-iHt} \Xi_n^3 \sigma_n^j | h \rangle$$

PDFs can be written as a sum of such correlation functions

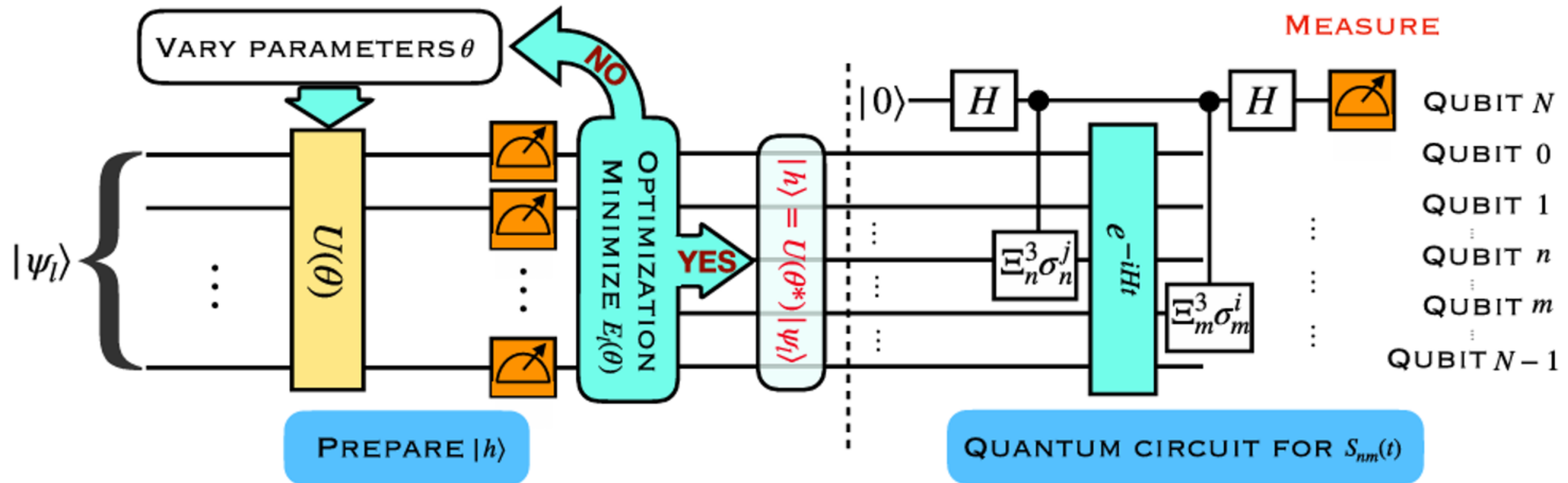
- ◆ Measure the observable with one auxiliary qubit



Measure the ancillary qubit on X (Y) basis to get the real (imaginary) part of $S_{mn}(t)$

Simulate hadron partonic structure on quantum computer

♦ Quantum circuits for PDFs



Hadron state preparation

Measure correlation functions

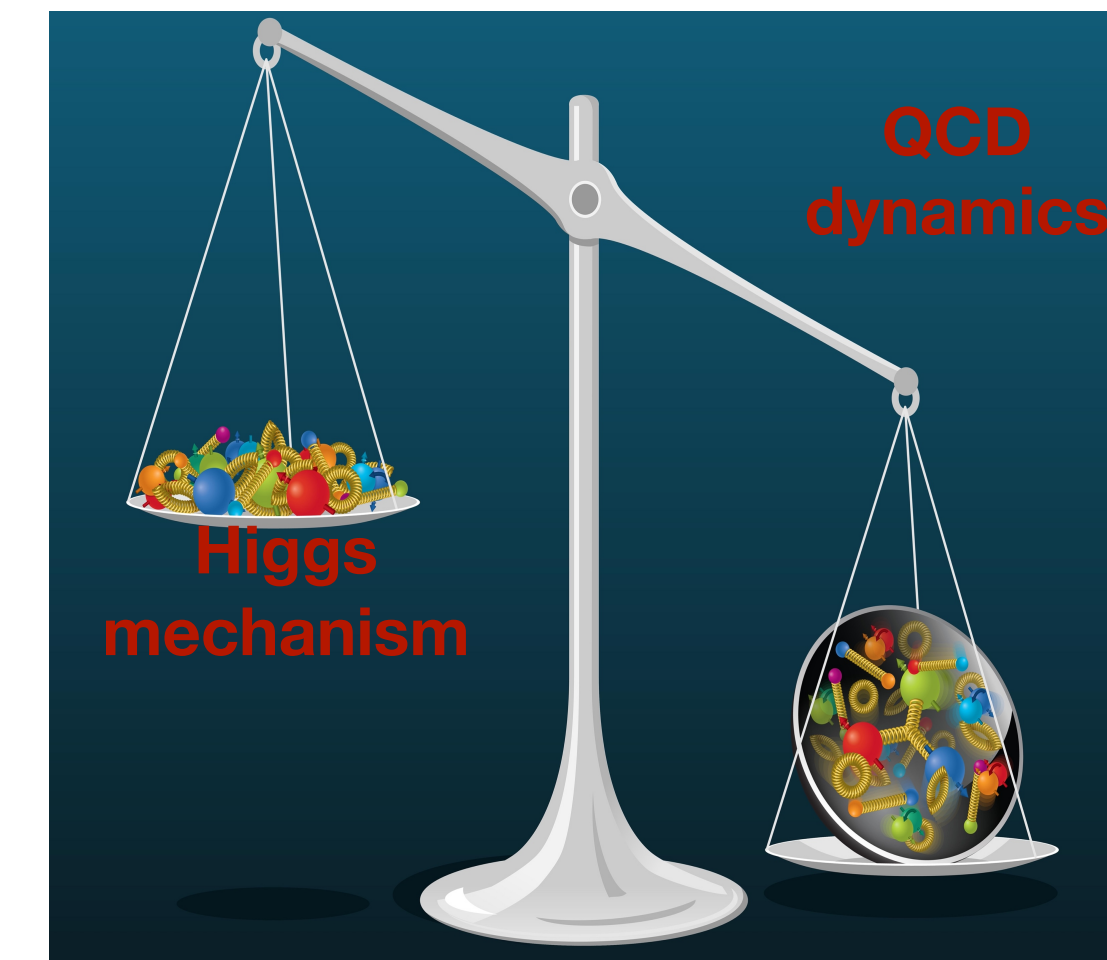
Numerical results from quantum computing

◆ Measurement of hadron mass $M_h = \langle h | H | h \rangle - \langle \Omega | H | \Omega \rangle$

g	0.2	0.4	0.6	0.8	1.0
$M_{h,\text{QCA}}$	1.002	1.810	2.674	3.534	4.352
$M_{h,\text{NUM}}$	1.001	1.801	2.659	3.509	4.342

$N = 12$

$ma = 0.2$



- Considering the current limitations of using real quantum devices, the results are generated using a classical simulation of the quantum circuit
- Measure the mass of the lowest-lying ud -like hadron in NJL model with 2 flavors, QAOA has good accuracy
- For small quark mass, the dominant contribution comes from the interaction rather than the quark masses
- For $ma = 0.8$, the quark masses are dominant

Numerical results from quantum computing

◆ quark PDF of the lowest-lying zero-charge hadron

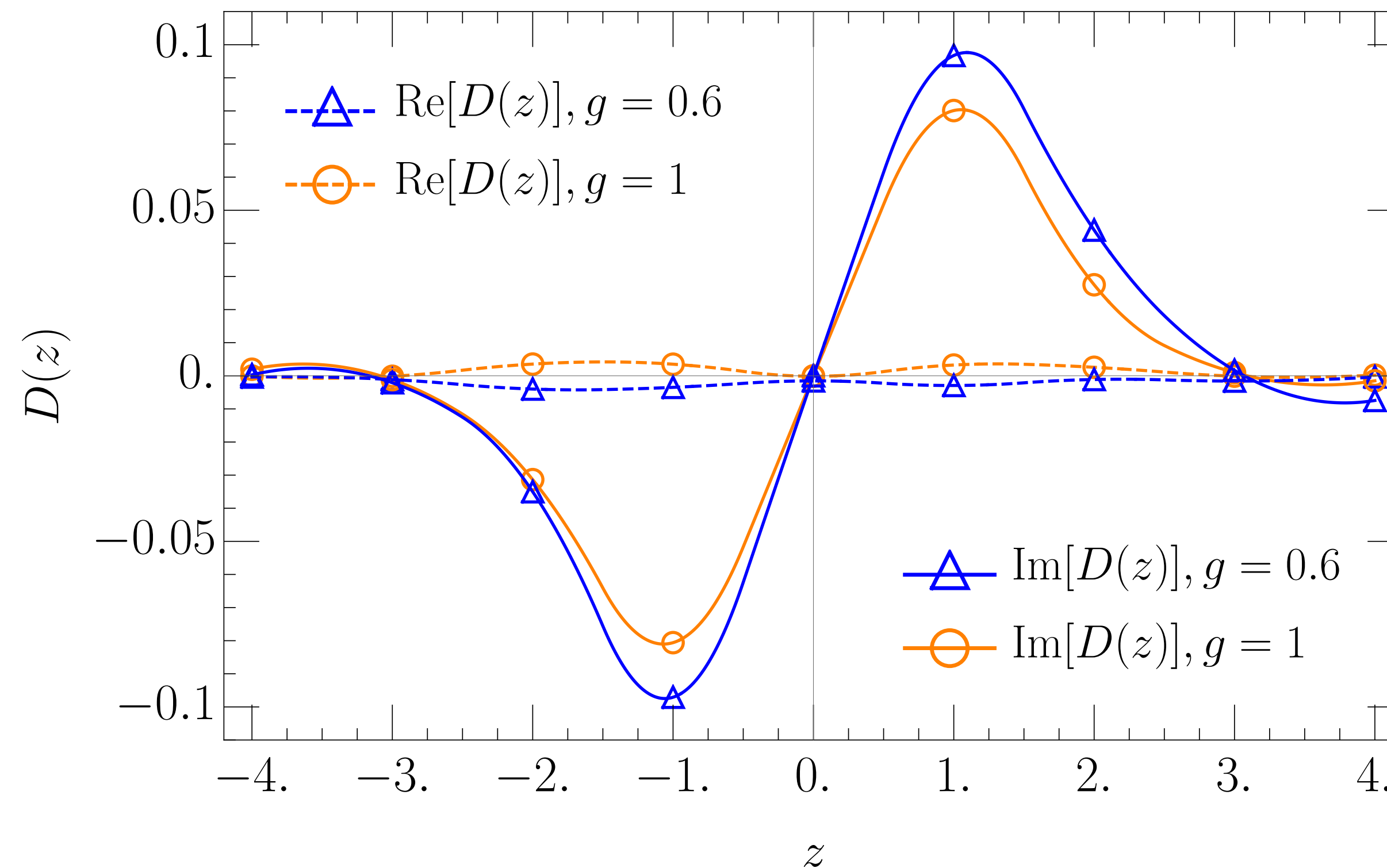
- quark PDF in position space

$$ma = 0.8 \quad N = 18 \quad n_f = 1$$

- The real part is consistent with 0

$$f_q(x) = f_{\bar{q}}(x) = -f_q(-x)$$

- The bound state behavior

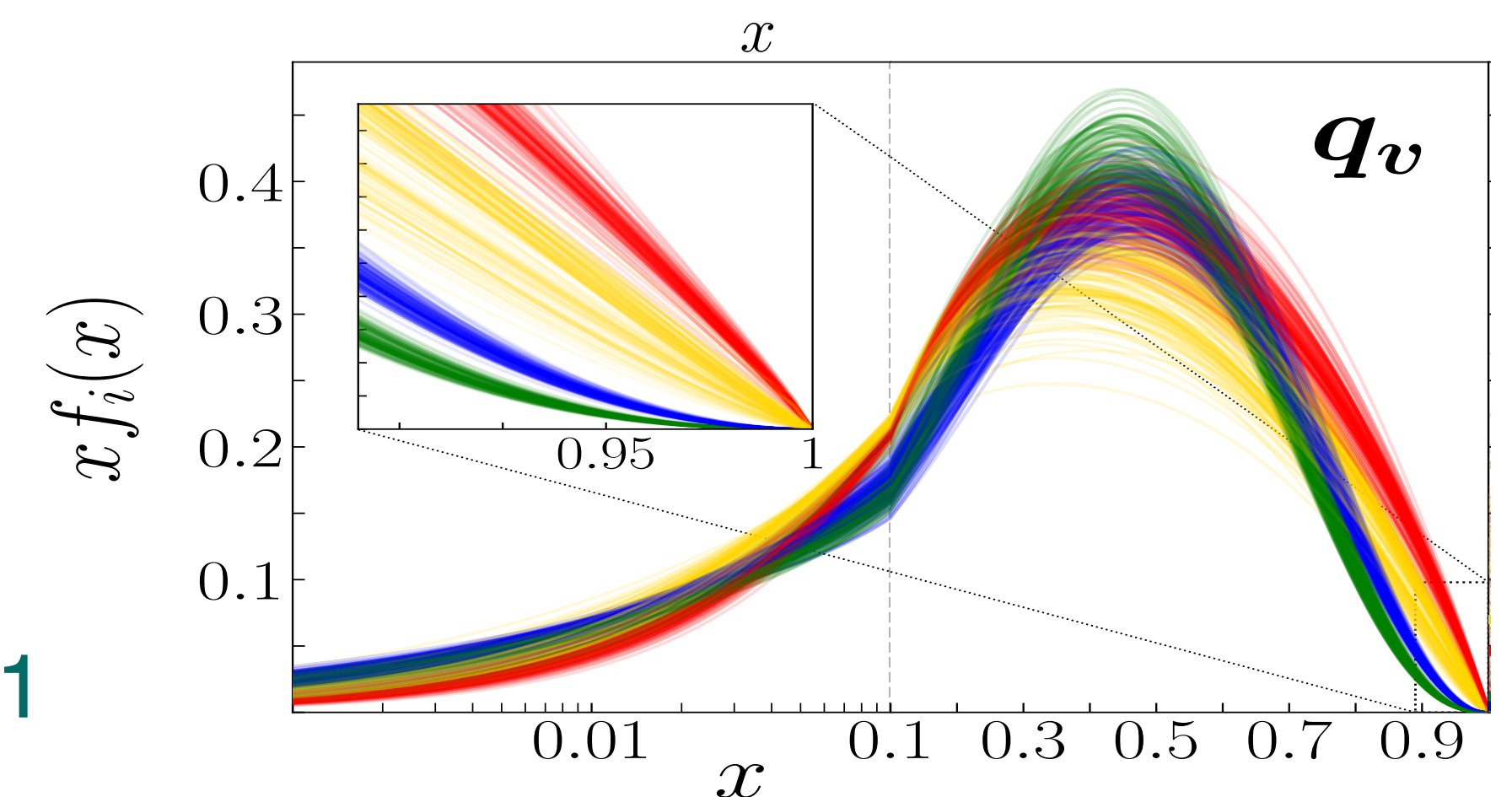
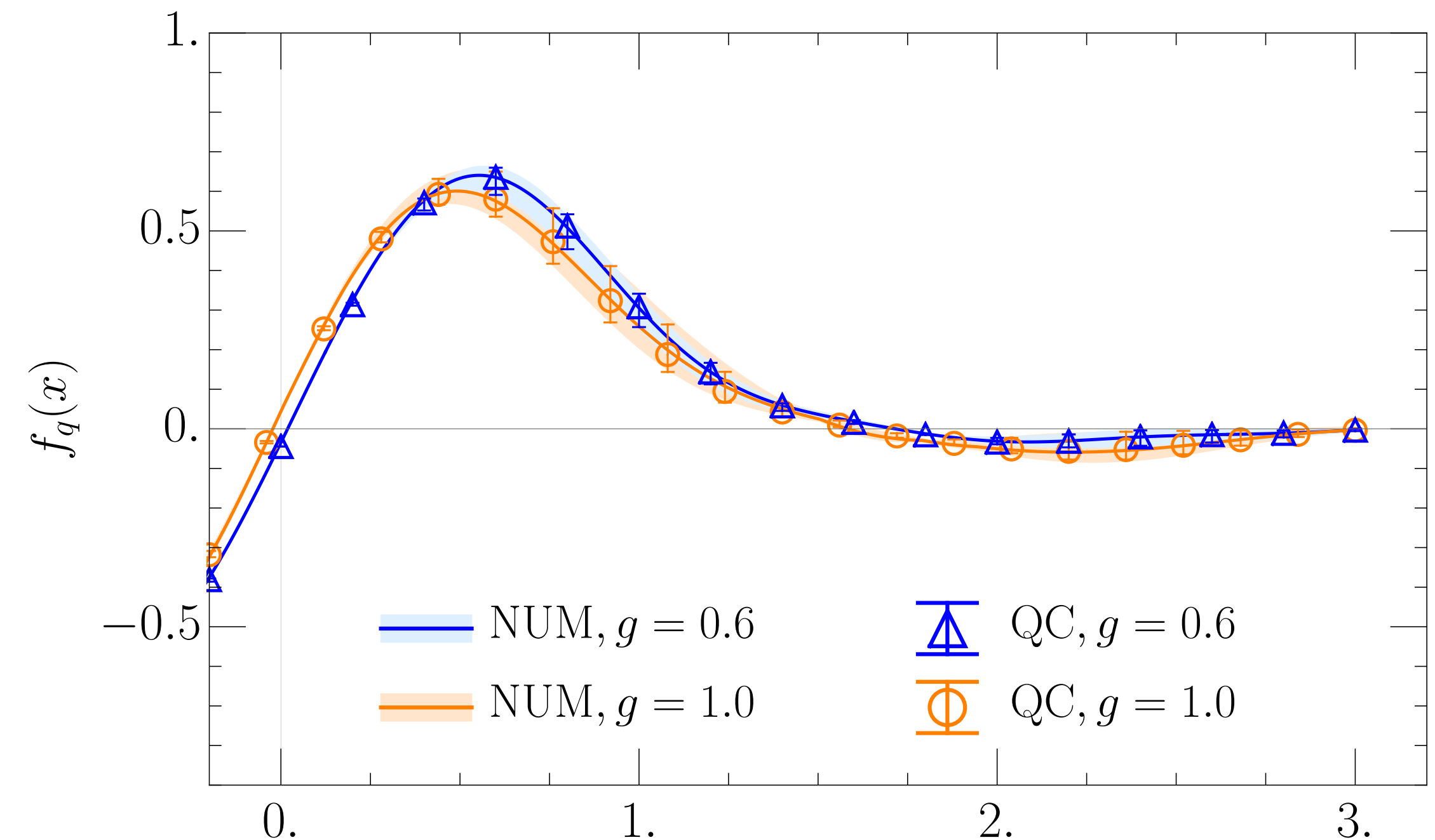


Numerical results from quantum computing

Li et al (QuNu), PRD (letter, 2022)

◆ quark PDF of the lowest-lying zero-charge hadron

- Good agreement between quantum computing and numerical diagonalization
- The non-vanishing contributions in the $x > 1$ are partly due to the finite volume effect
- We observe the expected peak around $x = 0.5$ and qualitative agreement with pion PDFs

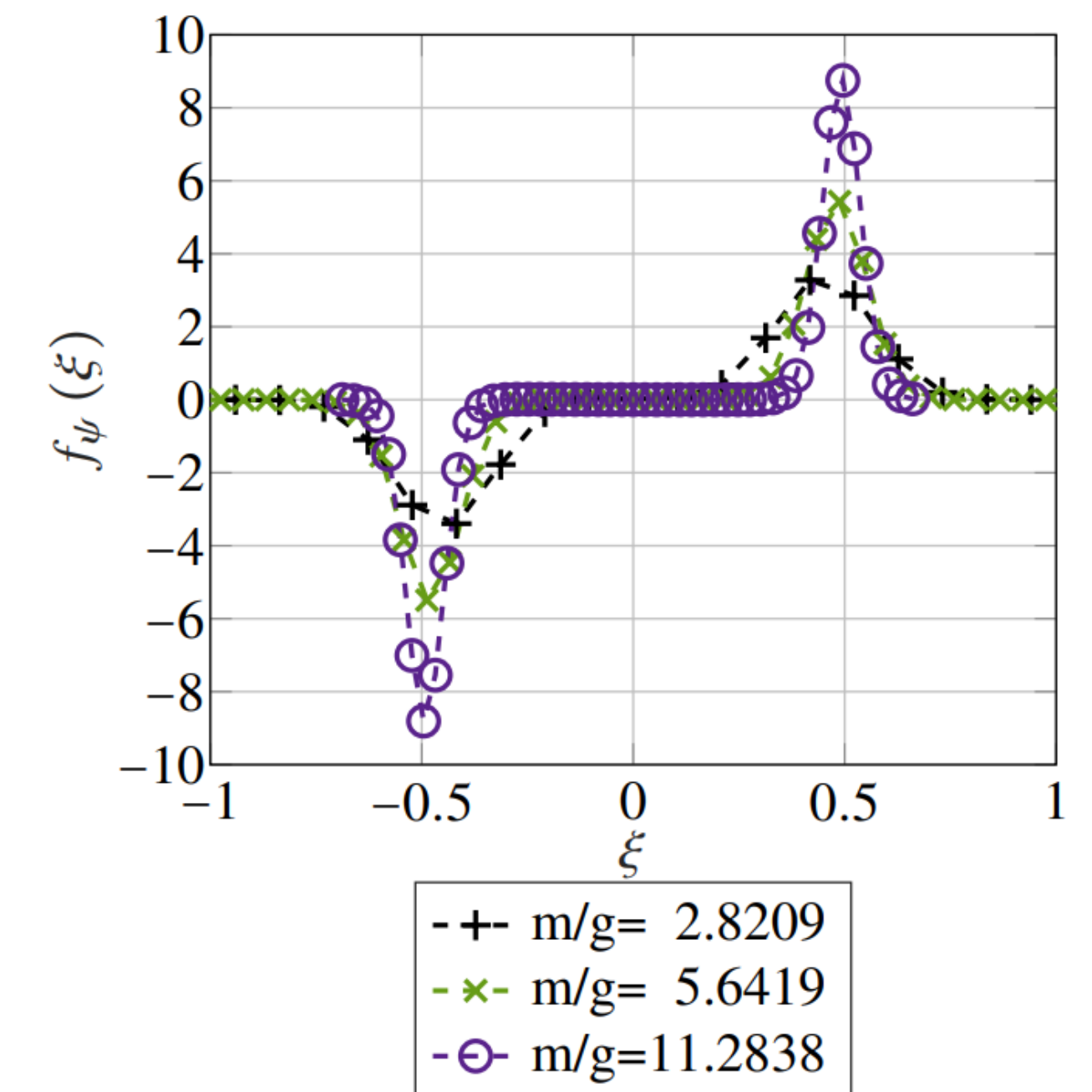
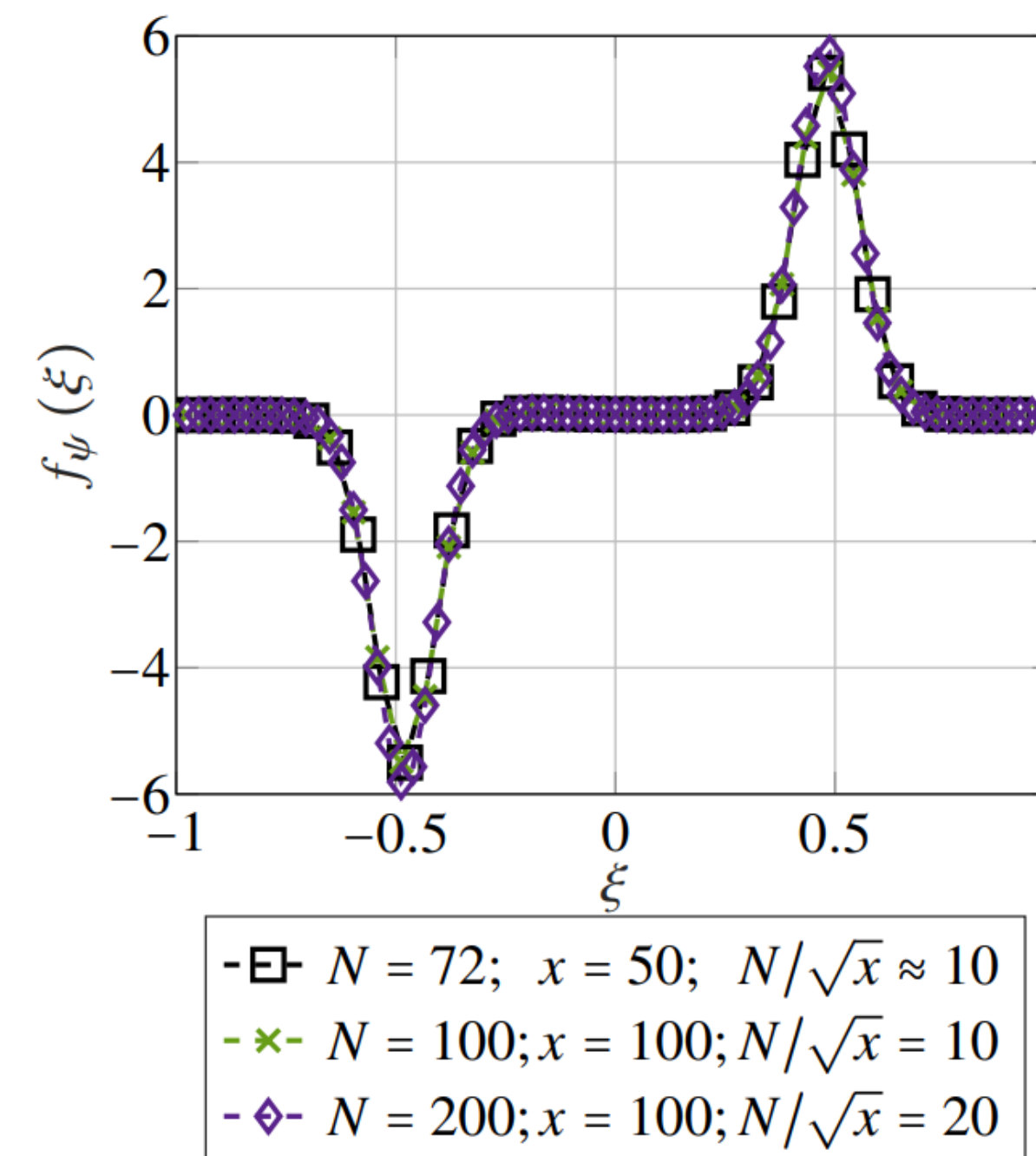
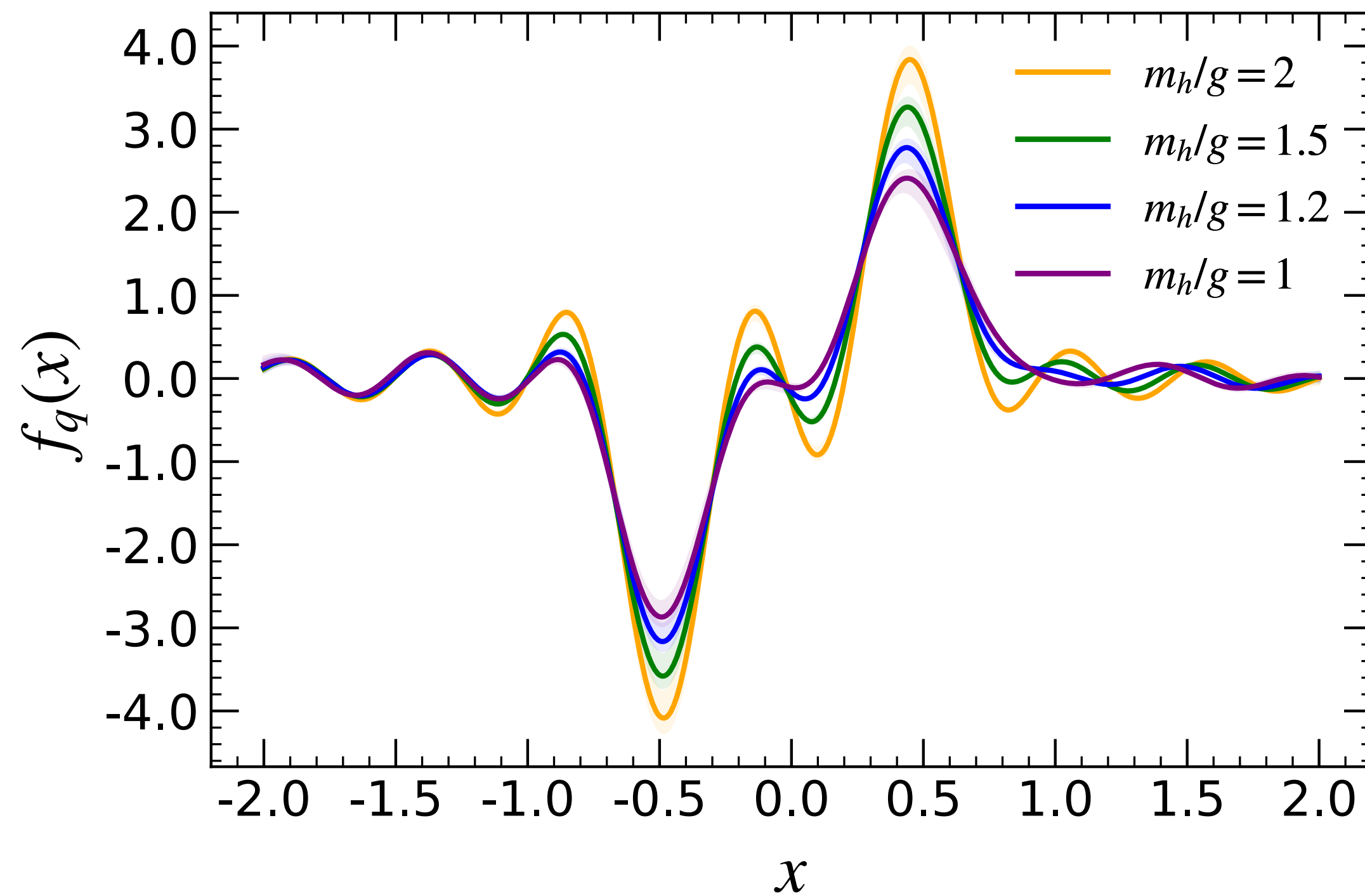


JAM Collaboration, PRL, 2021

1+1D QED - Schwinger model

Li et al (QuNu), in preparation

◆ QED PDF of the lowest-lying zero-charge bound state

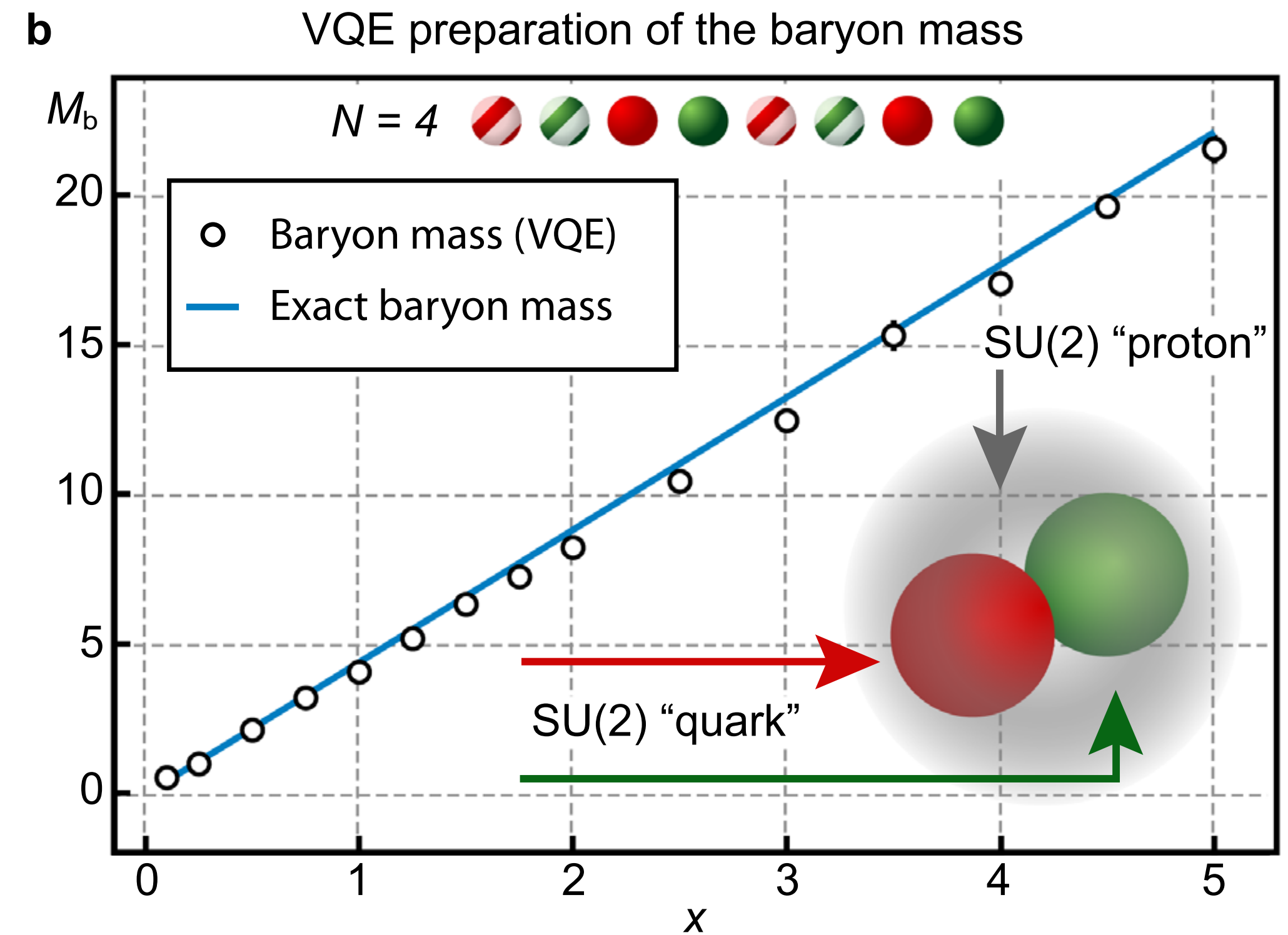
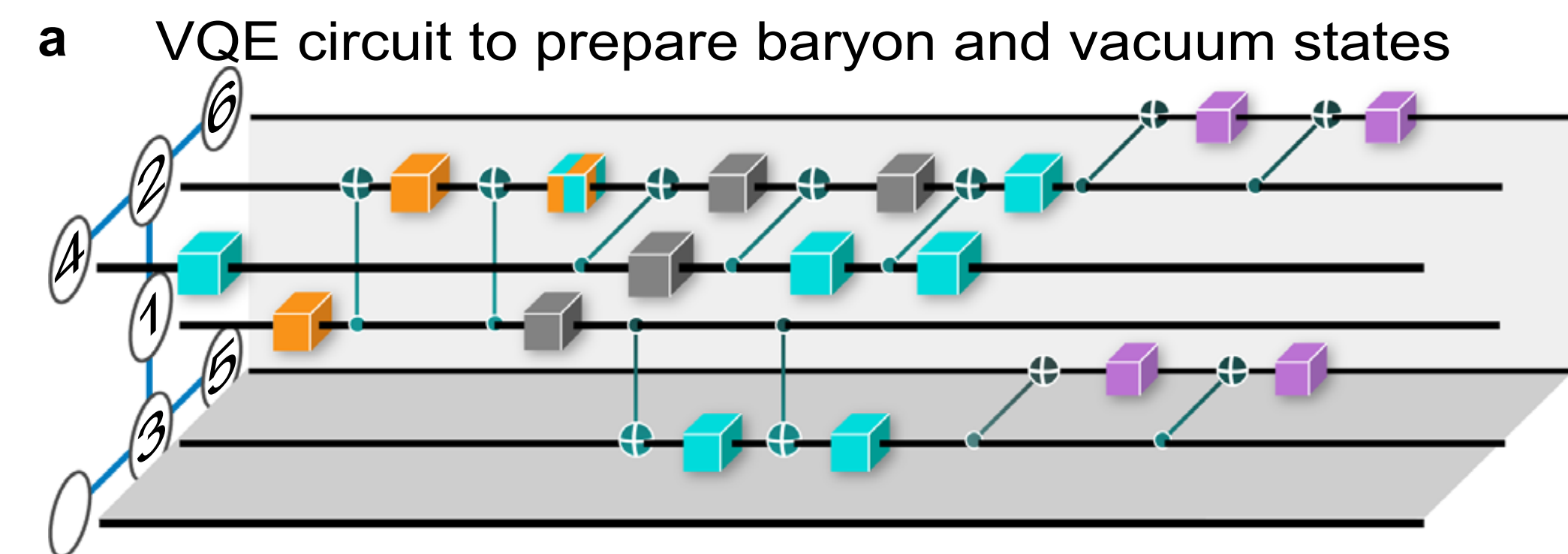
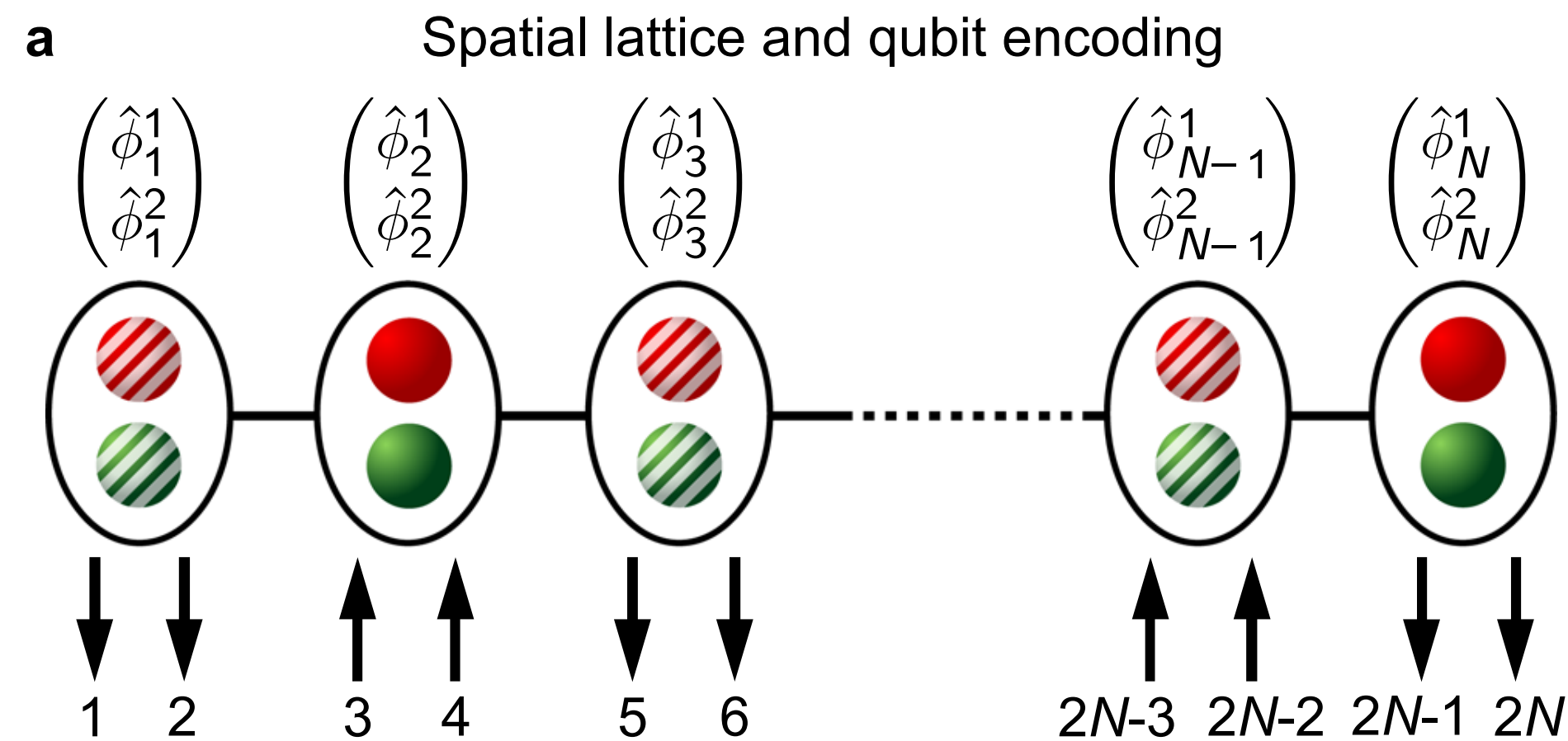


Tensor network: Banuls et al, 2409.16996

Simulate SU(2) hadron on quantum computer

- Global fitting with quantum circuit at initial scale Atas et al, Nature Commun. 2021

SU(2) Hamiltonian:
$$\hat{H}_l = \frac{1}{2a_l} \sum_{n=1}^{N-1} \left(\hat{\phi}_n^\dagger \hat{U}_n \hat{\phi}_{n+1} + \text{H.C.} \right) + m \sum_{n=1}^N (-1)^n \hat{\phi}_n^\dagger \hat{\phi}_n + \frac{a_l g^2}{2} \sum_{n=1}^{N-1} \hat{L}_n^2$$

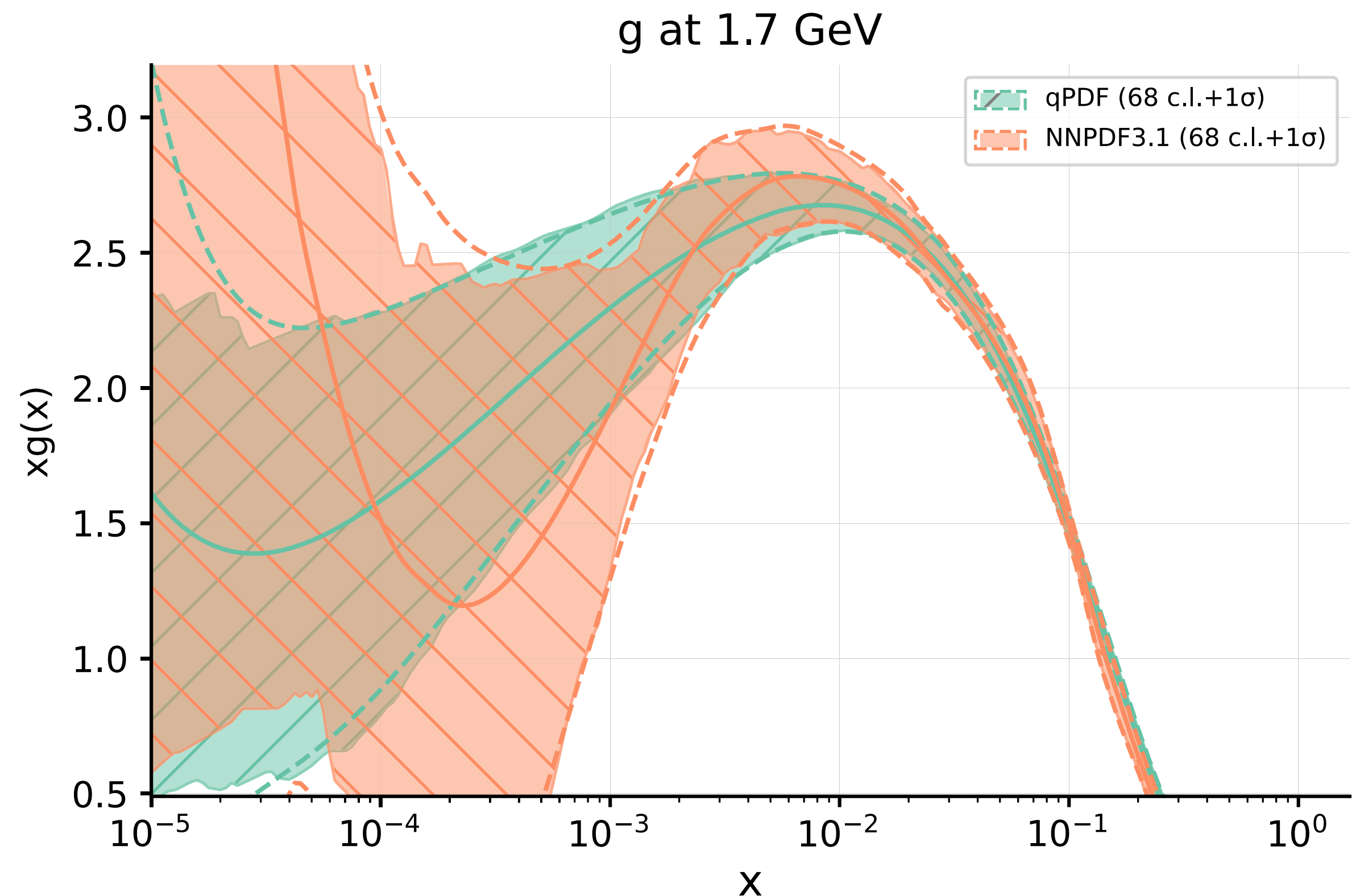
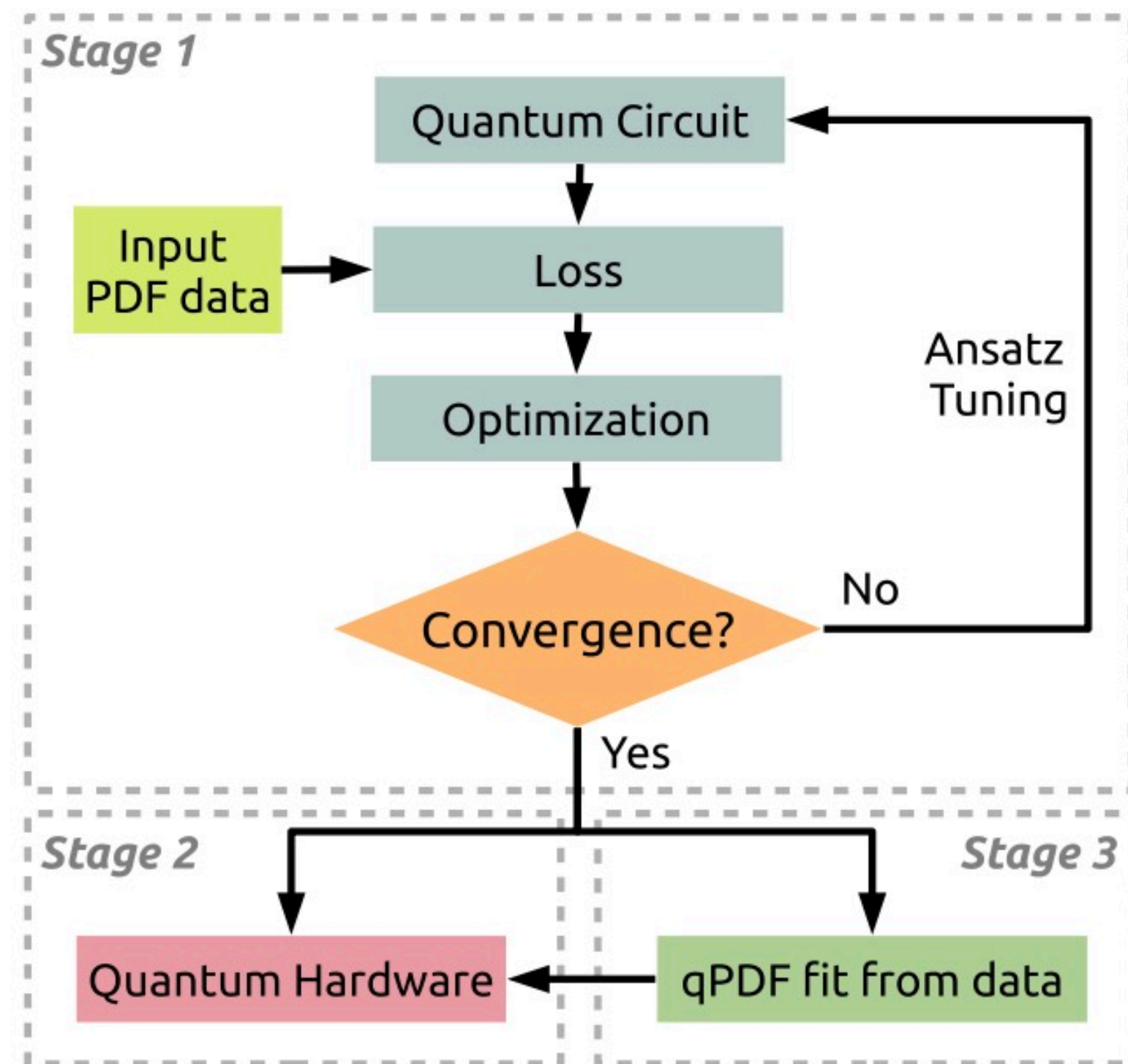
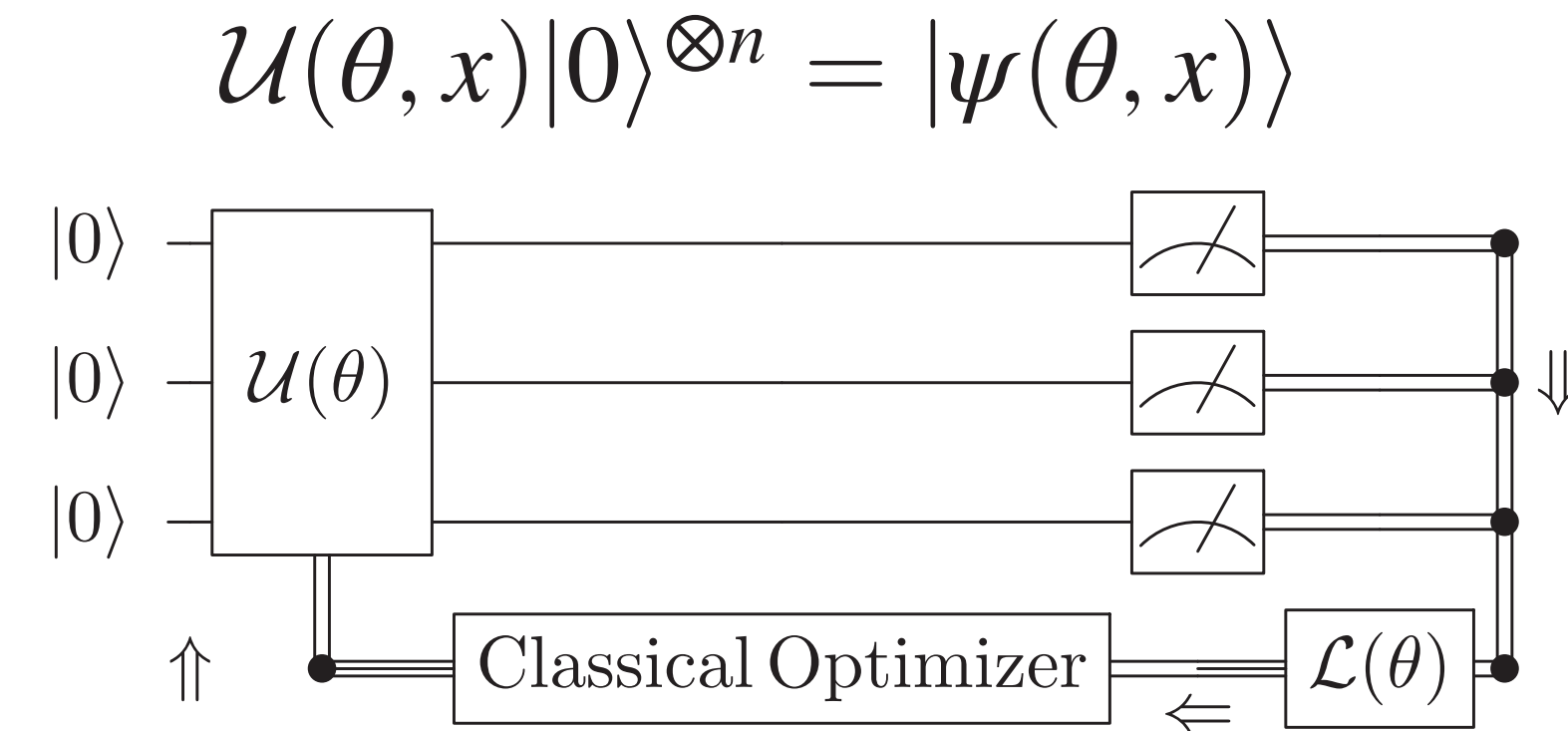


Alternative approach

- Global fitting with quantum circuit at initial scale

quantum parametrization:
$$\text{qPDF}_i(x, Q_0, \theta) = \frac{1 - z_i(\theta, x)}{1 + z_i(\theta, x)}$$

variational quantum circuit:
$$z_i(\theta, x) = \langle \psi(\theta, x) | Z_i | \psi(\theta, x) \rangle$$



Alternative approach

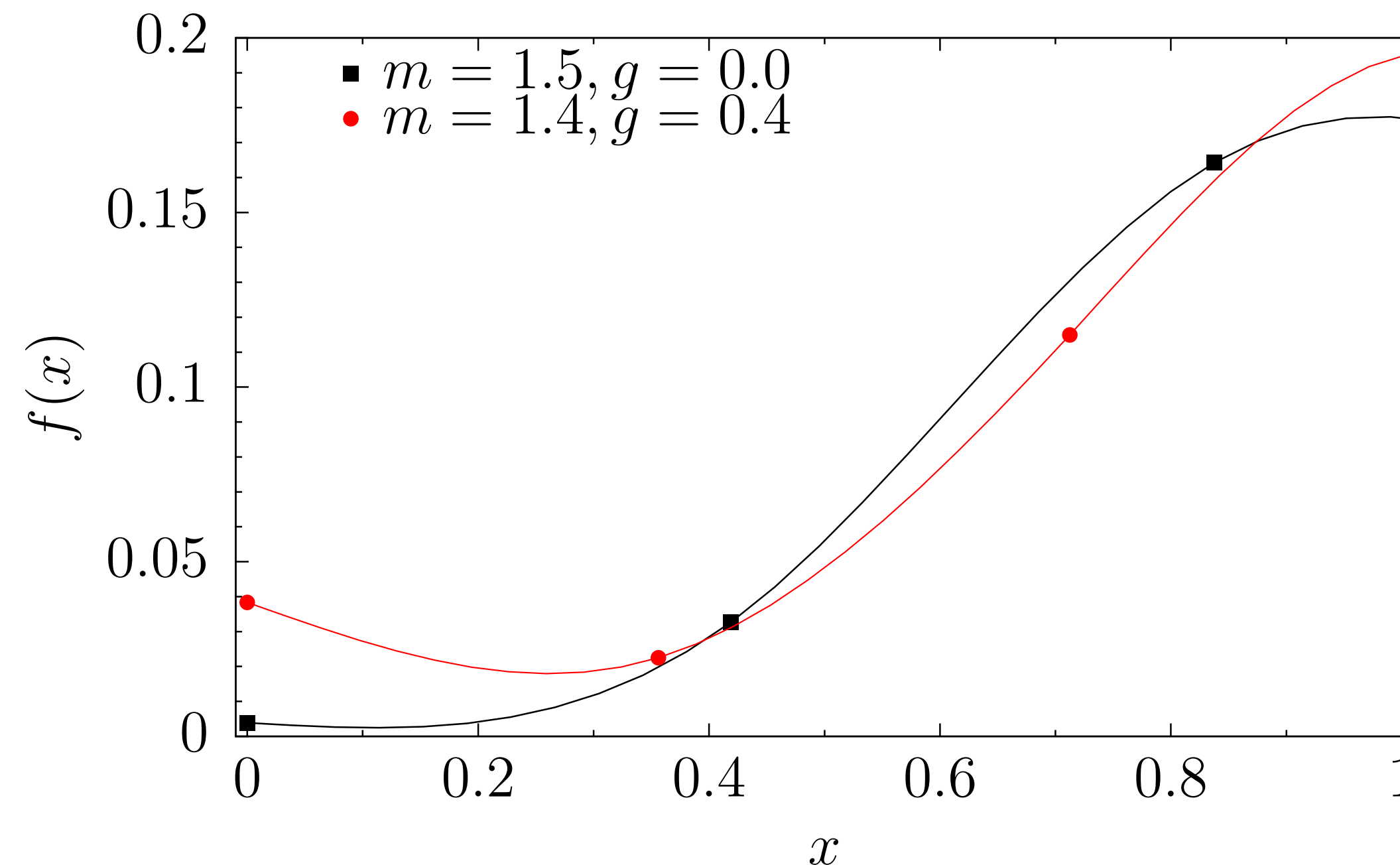
- Global fitting based hadronic tensor

NuQS, PRR 2020

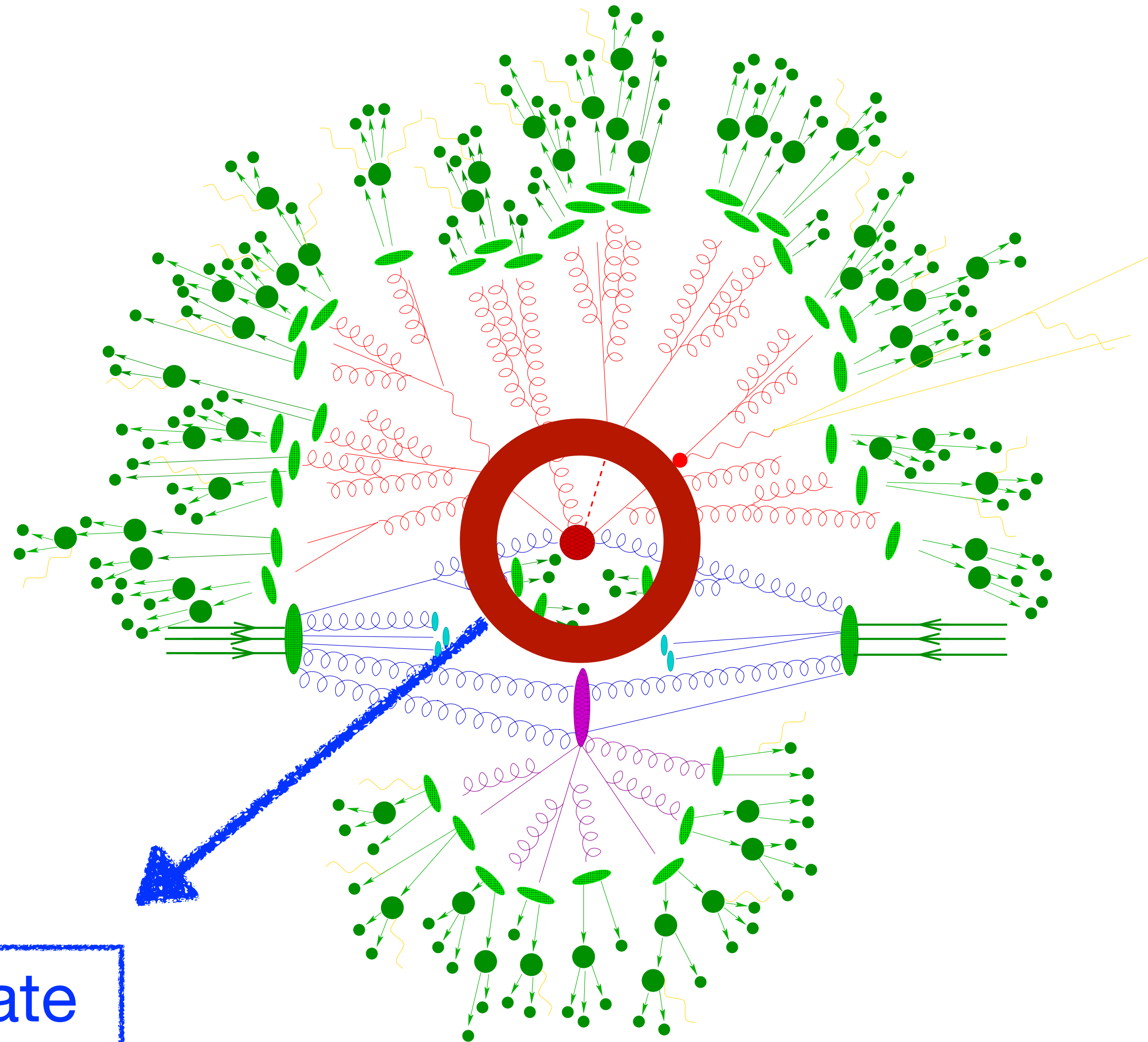
Hadronic tensor:
$$W^{\mu\nu}(q) = \text{Re} \int d^d x e^{iqx} \langle P | T \{ J^\mu(x) J^\nu(0) \} | P \rangle$$

Collinear factorization:
$$W^{\mu\nu} = \sum_{i,j} f_i \otimes P_{i \rightarrow j} \otimes \hat{W}^{\mu\nu}$$

- A test from exact diagonalization of Hamiltonian in Thirring model



2

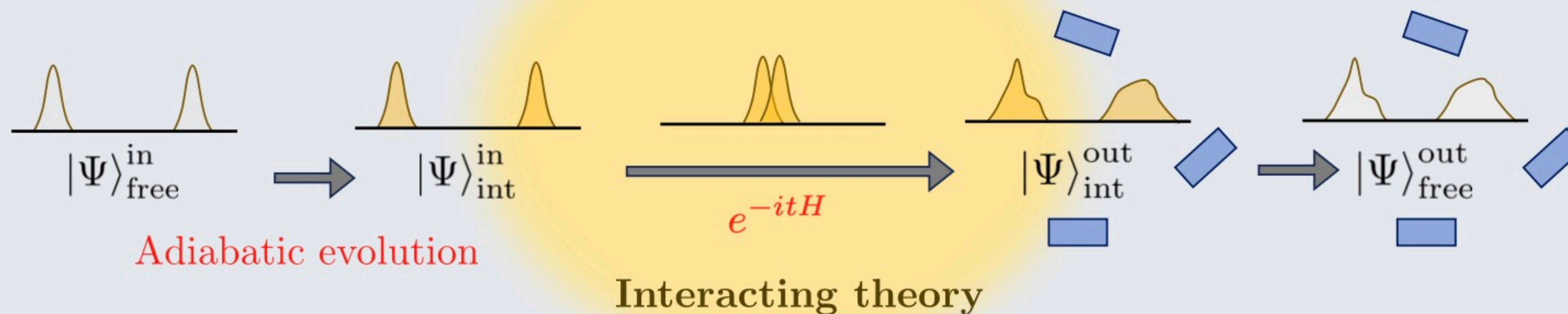


Intermediate state
partonic scatterings

Quantum computing for scattering amplitude

◆ Computing scattering amplitudes for strongly-coupled QFT

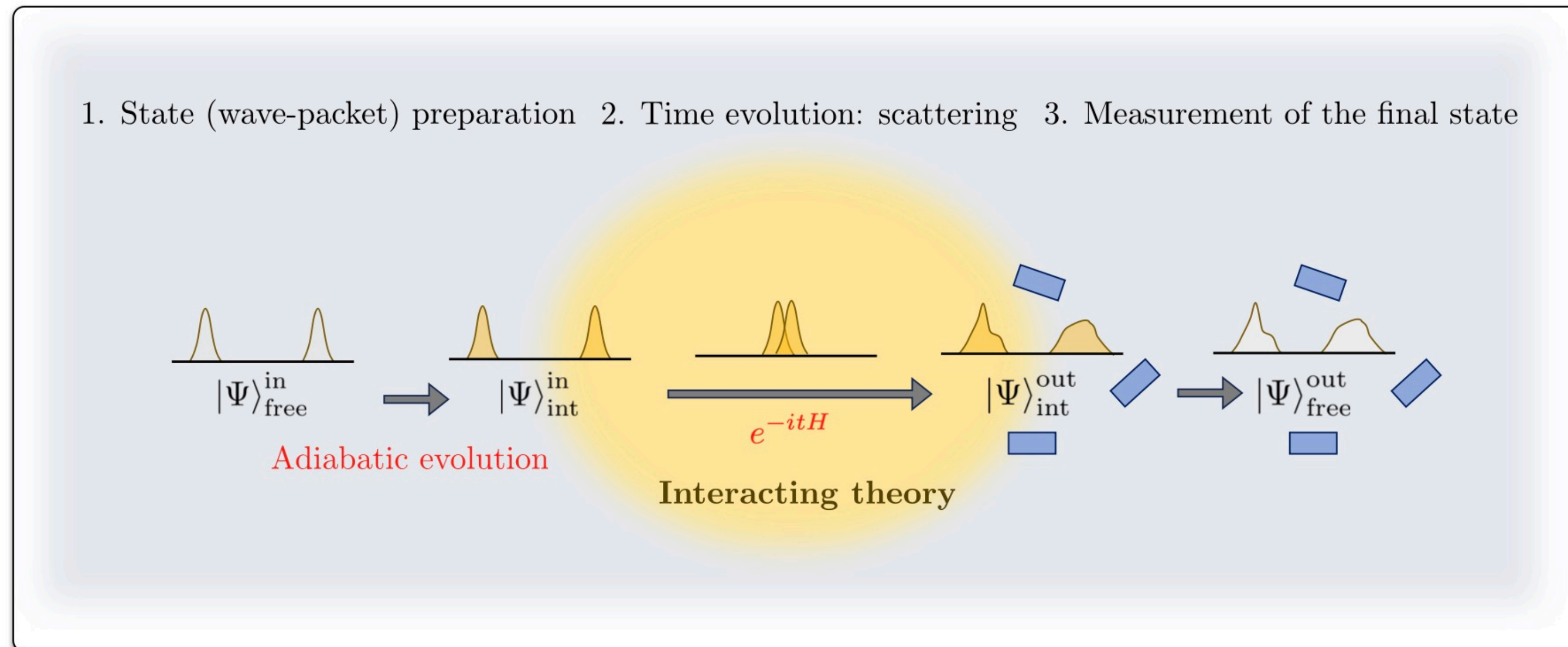
1. State (wave-packet) preparation 2. Time evolution: scattering 3. Measurement of the final state



Jordan, Lee, Preskill, Science 336, 1130–1133 (2012)

Quantum computing for scattering amplitude

◆ Computing scattering amplitudes for strongly-coupled QFT



1. Incoming particles are widely separated wave packets

$$L \gg d_{ij} \gg 1/|p_i| \rightarrow \text{requires large lattice}$$

2. Adiabatically turn on coupling, interactions happen

Long time span of evolution, broadening of wave packet

3. Adiabatically turned off coupling, measure final states

Quantum computing for scattering amplitudes

◆ A new proposal - LSZ reduction formula

Li et al (QuNu), PRD 2024

- Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \rightarrow m^2 \\ k_j^2 \rightarrow m^2}} G(\{p_i\}, \{k_j\}) \left(\prod_{r=1}^{n_{\text{out}}} K^{-1}(p_r) \right) \left(\prod_{s=1}^{n_{\text{in}}} K^{-1}(k_s) \right)$$

- connected n-point function in momentum space

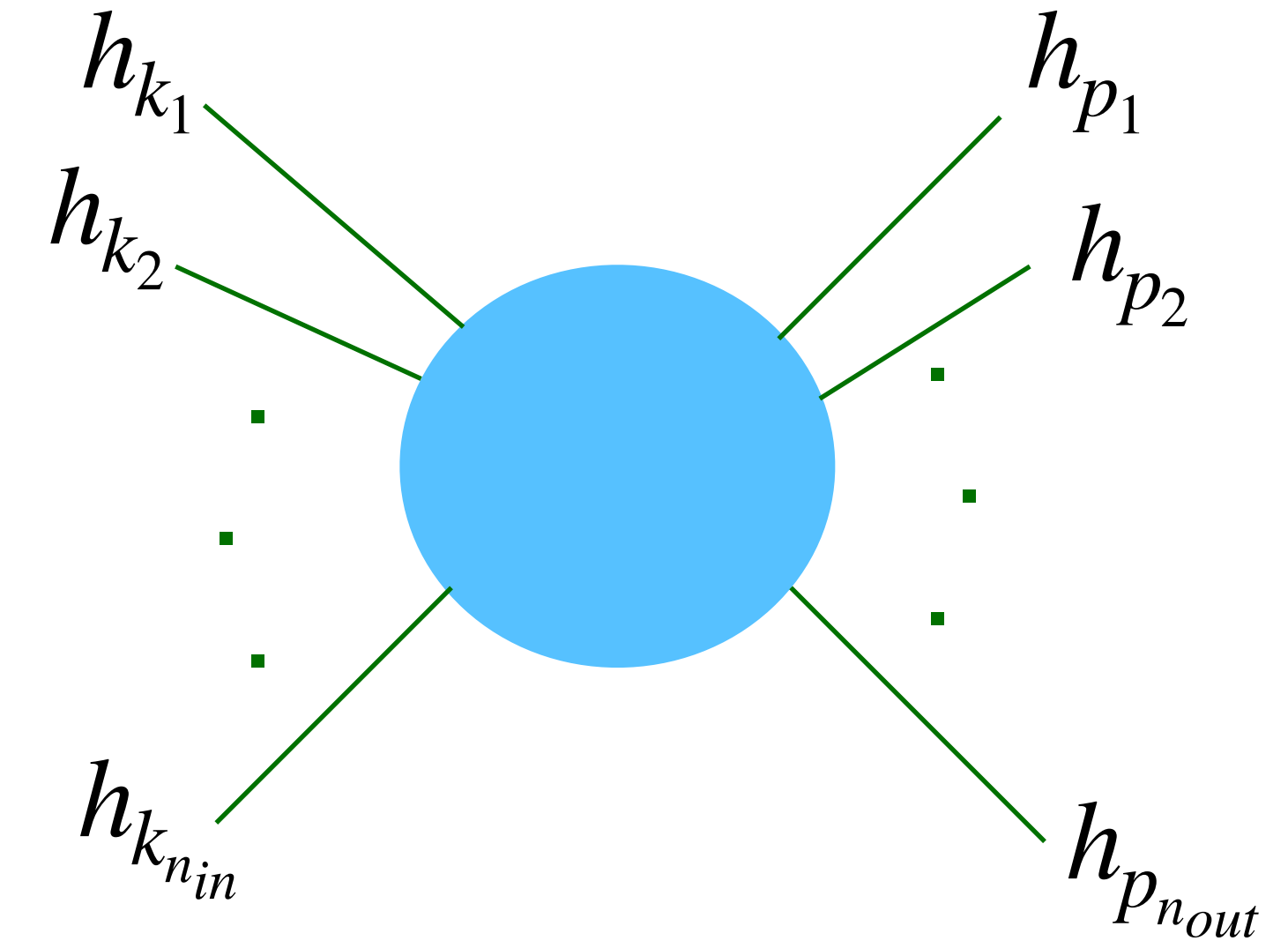
$$G(\{p_i\}, \{k_j\}) = \left(\prod_{i=1}^{n_{\text{out}}} \int d^4x_i e^{ip_i \cdot x_i} \right) \left(\prod_{j=1}^{n_{\text{in}}-1} \int d^4y_j e^{-ik_j \cdot y_j} \right) \\ \times \langle \Omega | T \{ \phi(x_1) \cdots \phi(x_{n_{\text{out}}}) \phi^\dagger(y_1) \cdots \phi^\dagger(y_{n_{\text{in}}-1}) \phi^\dagger(0) \} | \Omega \rangle_{\text{con}}$$

- two-point function in momentum space (propagator)

$$K(p) = \int d^4x e^{ip \cdot x} \langle \Omega | T \{ \phi(x) \phi^\dagger(0) \} | \Omega \rangle$$

- field normalization

$$R = |\langle \Omega | \phi(0) | h(\mathbf{p} = 0) \rangle|^2$$



Quantum computing for scattering amplitudes

♦ A new proposal - LSZ reduction formula

- Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \rightarrow m^2 \\ k_j^2 \rightarrow m^2}} G(\{p_i\}, \{k_j\}) \left(\prod_{r=1}^{n_{\text{out}}} K^{-1}(p_r) \right) \left(\prod_{s=1}^{n_{\text{in}}} K^{-1}(k_s) \right)$$

- connected n-point function in momentum space

$$G(\{p_i\}, \{k_j\}) = \left(\prod_{i=1}^{n_{\text{out}}} \int d^4x_i e^{ip_i \cdot x_i} \right) \left(\prod_{j=1}^{n_{\text{in}}-1} \int d^4y_j e^{-ik_j \cdot y_j} \right) \\ \times \langle \Omega | T \{ \phi(x_1) \cdots \phi(x_{n_{\text{out}}}) \phi^\dagger(y_1) \cdots \phi^\dagger(y_{n_{\text{in}}-1}) \phi^\dagger(0) \} | \Omega \rangle_{\text{con}}$$

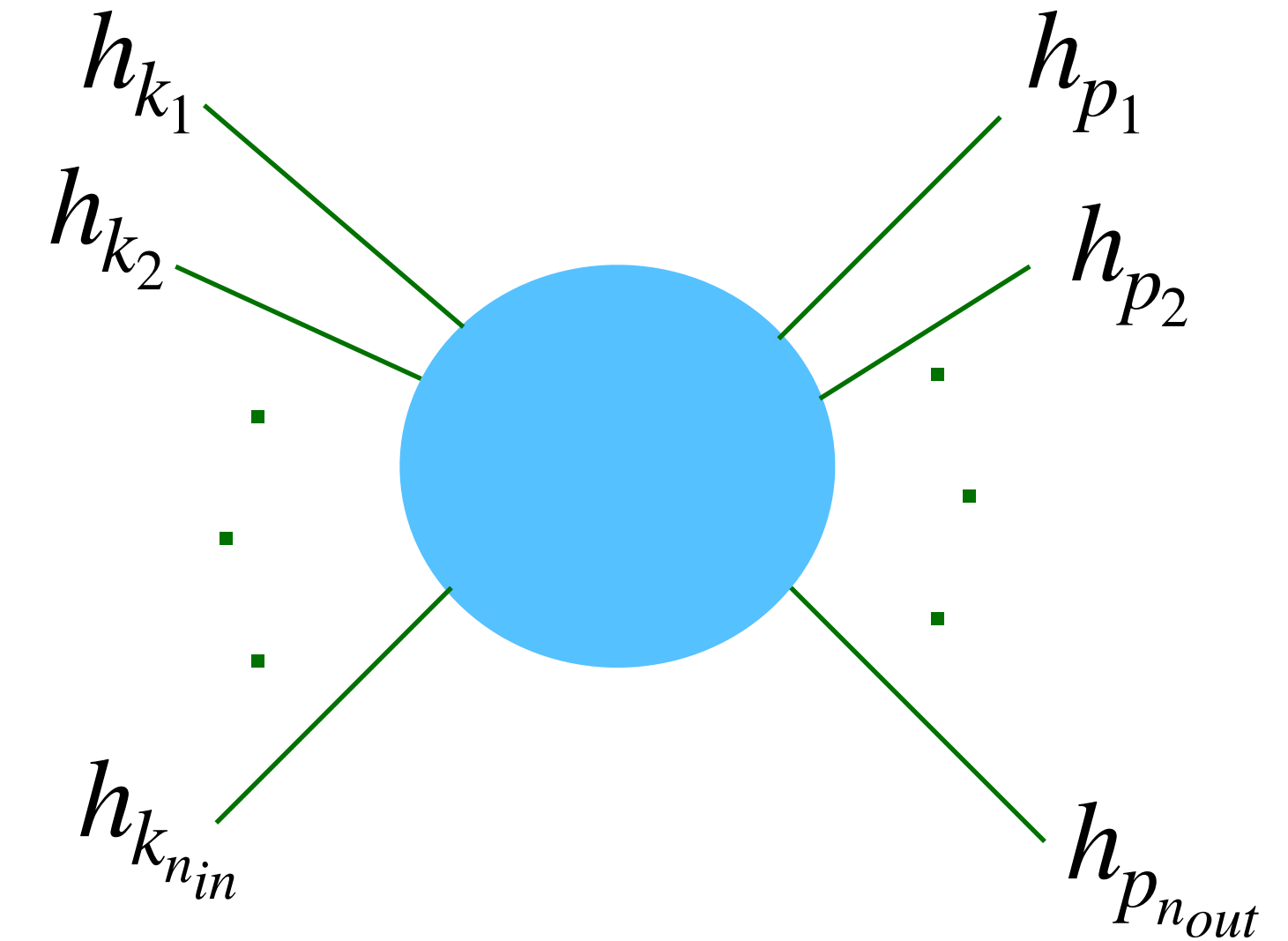
- two-point function in momentum space (propagator)

$$K(p) = \int d^4x e^{ip \cdot x} \langle \Omega | T \{ \phi(x) \phi^\dagger(0) \} | \Omega \rangle$$

- field normalization

$$R = |\langle \Omega | \phi(0) | h(\mathbf{p} = 0) \rangle|^2$$

QAOA for $|\Omega\rangle$ and $|h\rangle$



pole singularities cancel on mass-shell, giving finite scattering amplitude

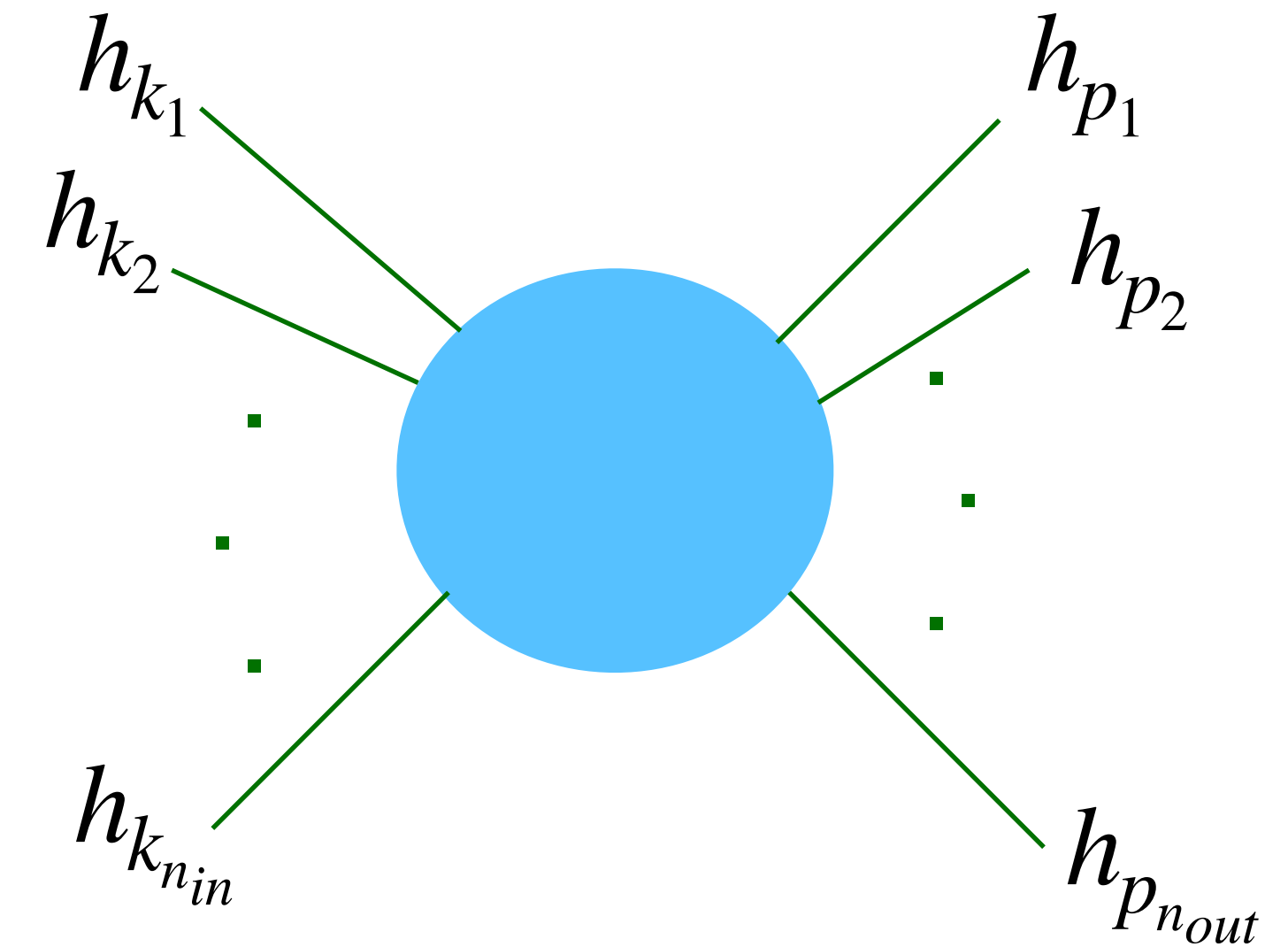
Quantum computing for scattering amplitudes

♦ A new proposal - LSZ reduction formula

- Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \rightarrow m^2 \\ k_j^2 \rightarrow m^2}} G(\{p_i\}, \{k_j\}) \left(\prod_{r=1}^{n_{\text{out}}} K^{-1}(p_r) \right) \left(\prod_{s=1}^{n_{\text{in}}} K^{-1}(k_s) \right)$$

- No preparation of incoming wave packets, smaller lattice is allowed.
- No adiabatic turn on and turn off of coupling constants, no associated extra time evolution
- Bound-states are allowed as incoming and outgoing particles
- Complexity scales exponentially in particle number n , ideal for exclusive scattering process, e.g. $2 \rightarrow 2$ scattering. JLP formalism scales polynomially with n .



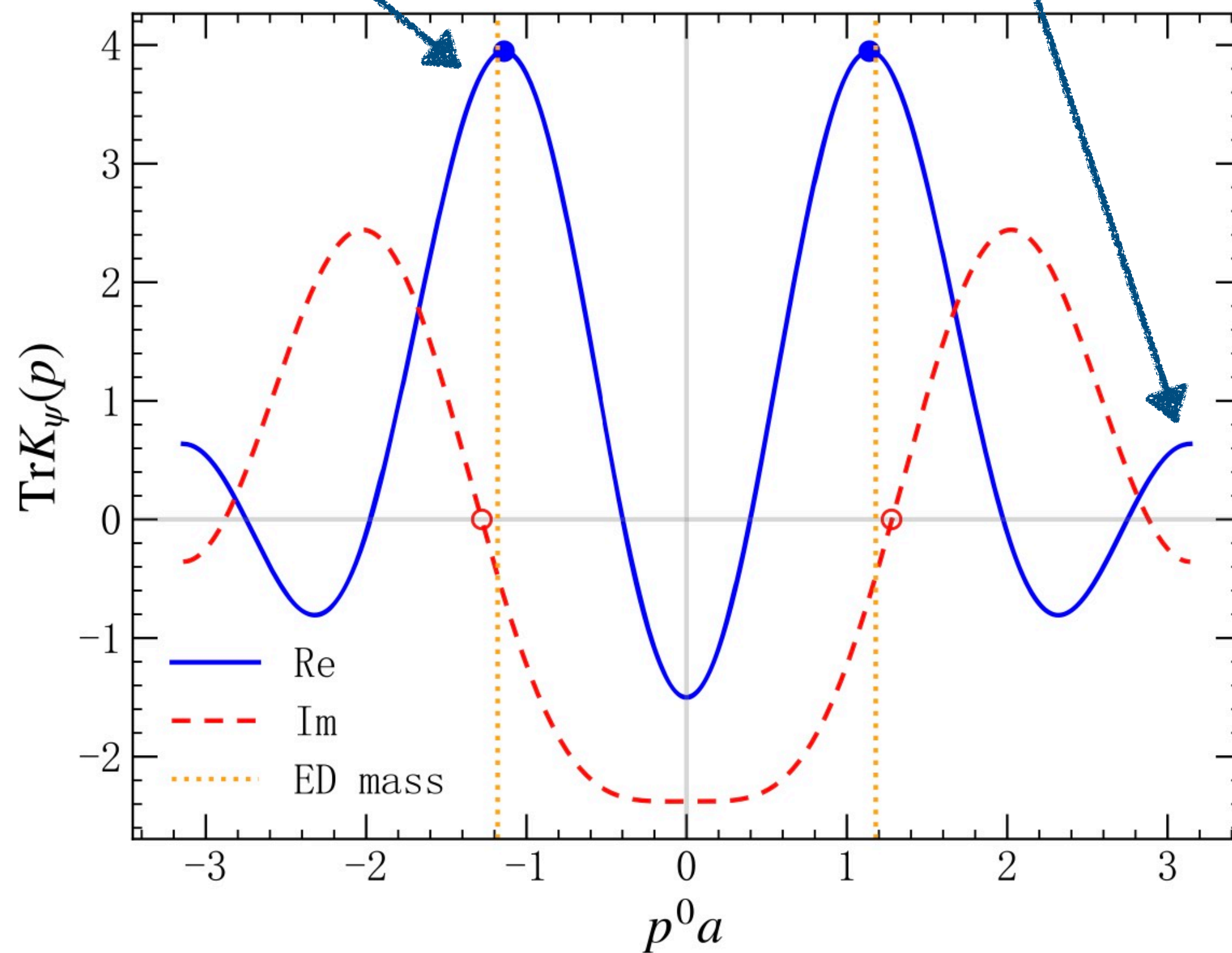
Quantum computing for scattering amplitudes

◆ LSZ reduction formula - 1+1 NJL

- Fermion propagator $K_\psi(p) = \int d^2x e^{ip \cdot x} \langle \Omega | T \{ \psi(x) \bar{\psi}(0) \} | \Omega \rangle$

Lowest lying quark state

Lowest lying bound state
(2q+qbar)

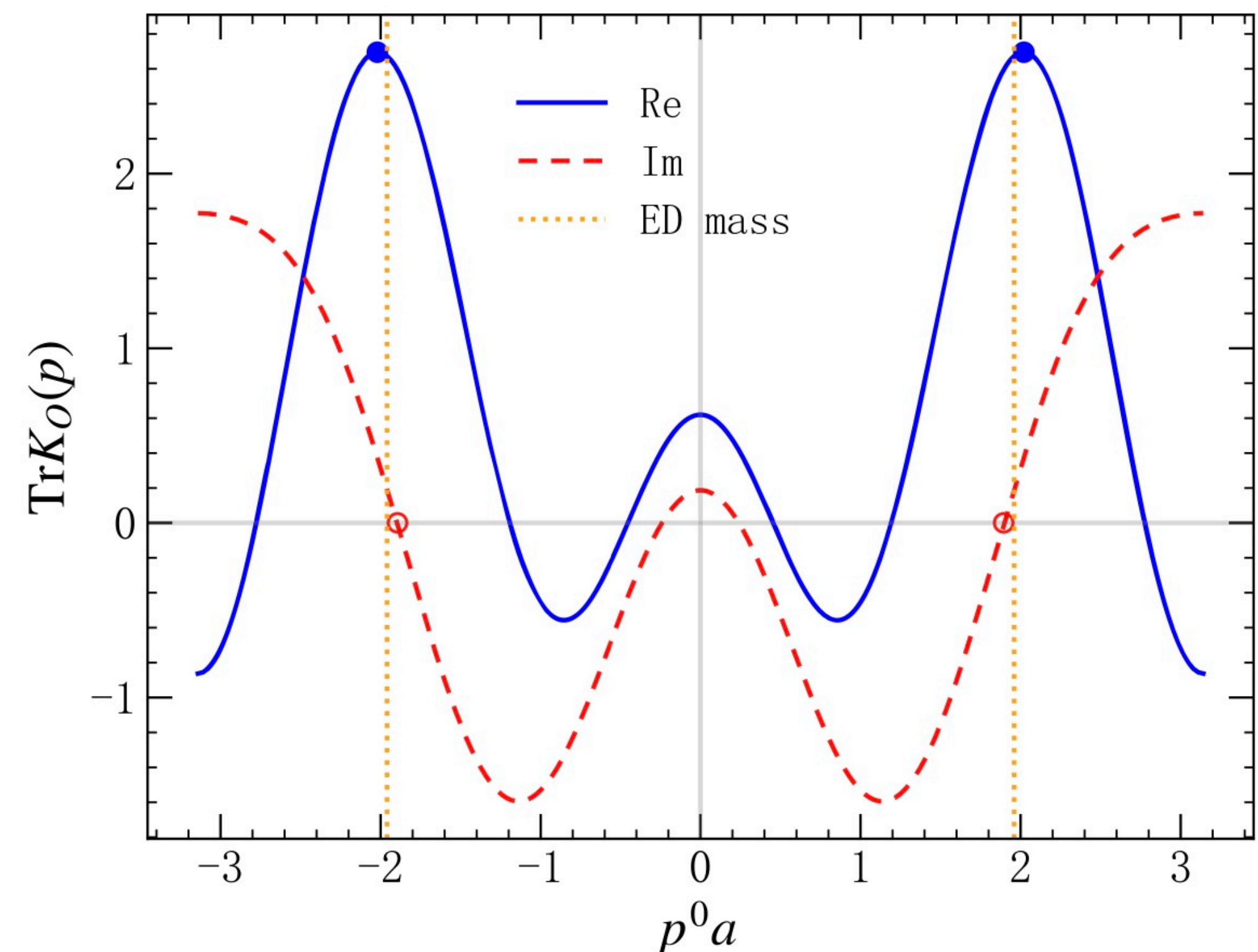


Li et al (QuNu), PRD 2024

- propagator of composite operator

$$K_O(p) = \int d^2x e^{ip \cdot x} \langle \Omega | T \{ O(x) O(0) \} | \Omega \rangle_{\text{con}}$$

$$O(x) = \bar{\psi}(x) \psi(x)$$



Quantum computing for scattering amplitudes

◆ LSZ reduction formula - 1+1 NJL

- Four point correlation function

Our quantum algorithm succeeds in recovering the expected pole structure, which is crucial to the implementation of LSZ formula.

Lowest lying quark state

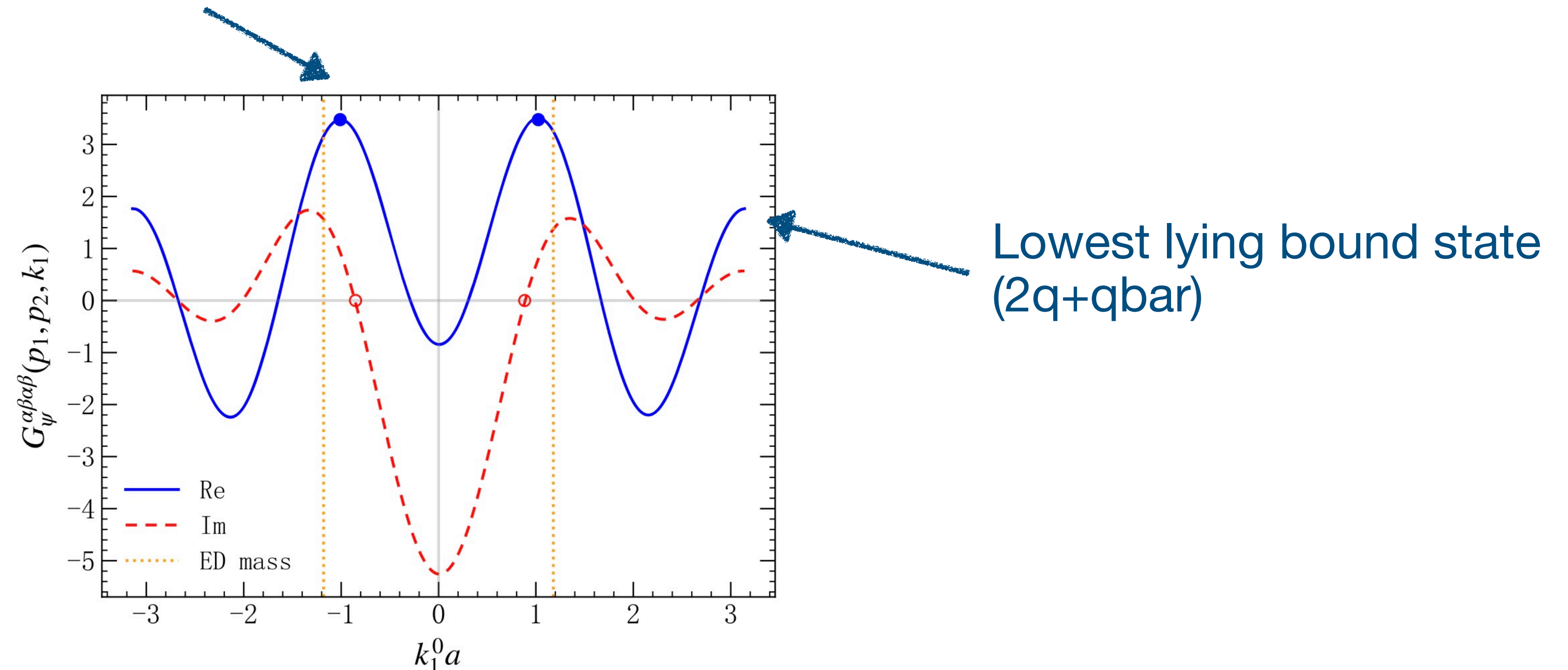


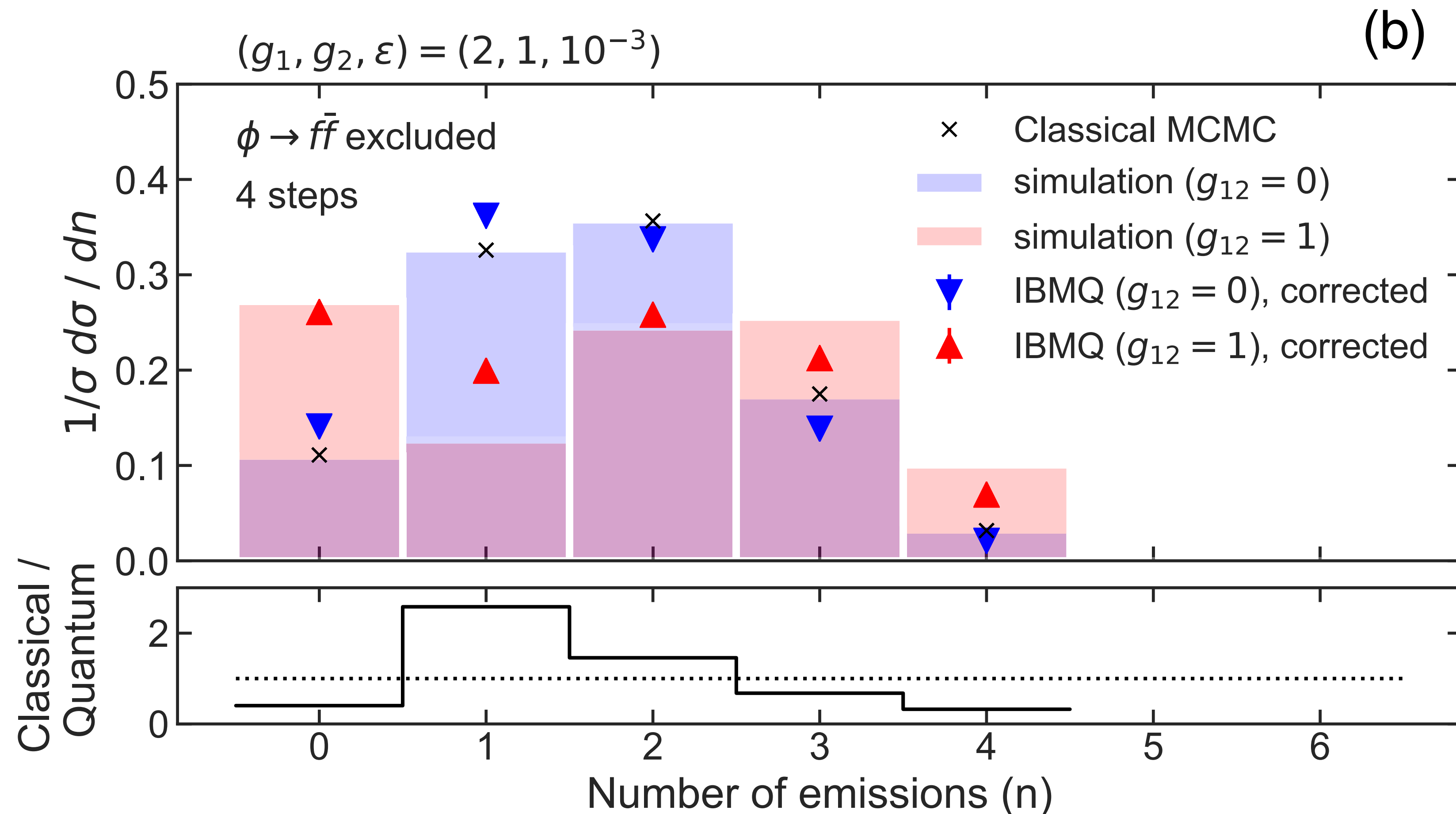
FIG. 2. Real part (solid line) and imaginary part (dashed line) of $G_{\psi}^{\alpha\beta\alpha\beta}(p_1, p_2, k_1)$ in the one-flavor Gross-Neveu model as a function of $k_1^0 a$, with $k_1 = (k_1^0, 0)$, $p_1 = (0, 0)$, $p_2 = (k_1^0, \pi/a)$,

Quantum computing for scattering amplitudes

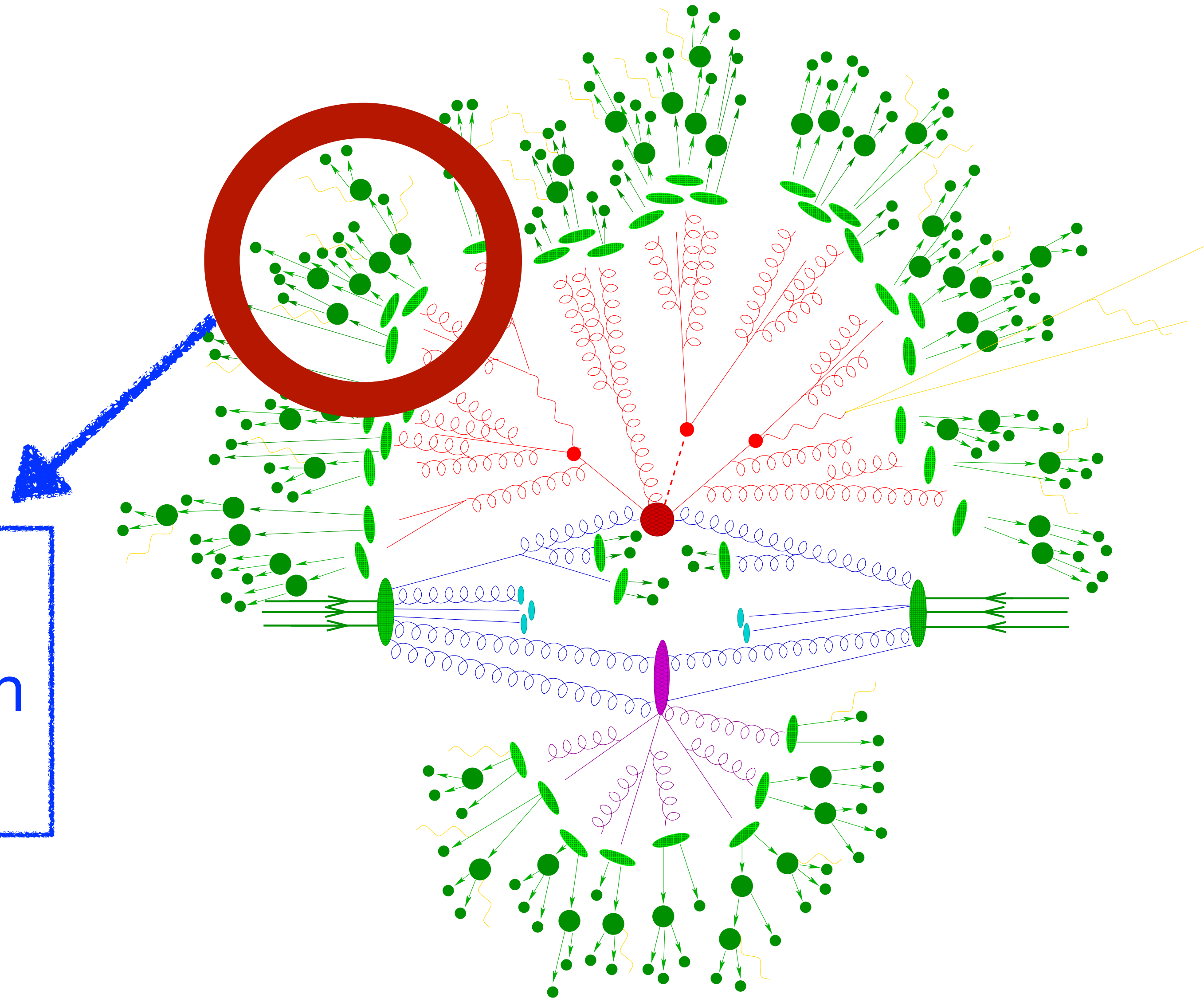
◆ Simulate the quantum interference effect in parton shower

$$\mathcal{L} = \bar{f}_1(i\not{\partial} + m_1)f_1 + \bar{f}_2(i\not{\partial} + m_2)f_2 + (\partial_\mu\phi)^2 \\ + g_1\bar{f}_1f_1\phi + g_2\bar{f}_2f_2\phi + g_{12}[\bar{f}_1f_2 + \bar{f}_2f_1]\phi$$

Nachman et al, PRL 2021



3



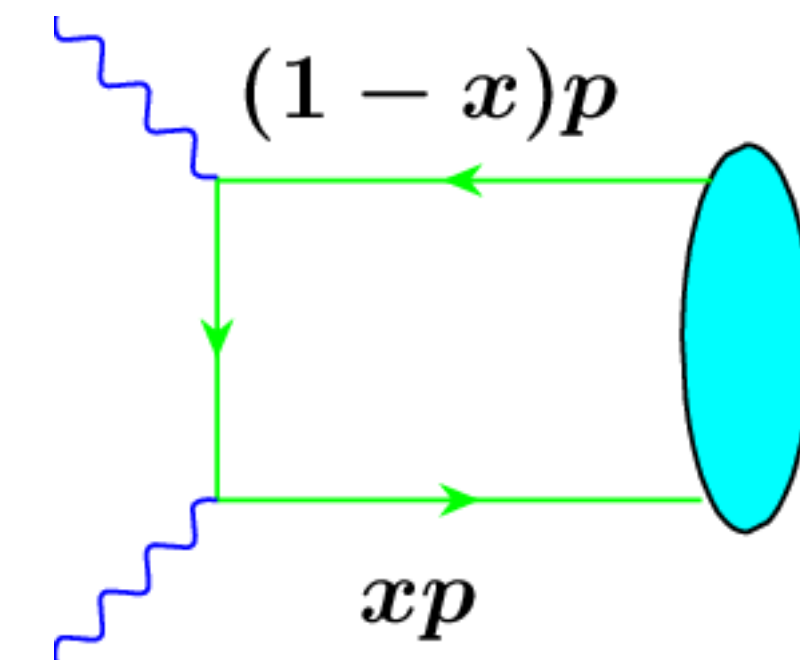
Final state
hadron fragmentation
function $D_{q \rightarrow h}$

Quantum computing for exclusive hadronization

- ◆ LCDA - light cone distribution amplitude, describes the formation/decay of a hadron
- ◆ LCDA is an essential ingredient in exclusive high-energy QCD processes, e.g. form factor in the process $\gamma^*\gamma \rightarrow \pi^0$

$$F(Q^2) = f_\pi \int_0^1 dx T_H(x, Q^2; \mu) \phi_\pi(x; \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2/Q^2)$$

$$\phi(x) = \frac{1}{f} \int dz e^{-i(x-1)n \cdot Pz} \langle \Omega | \bar{\psi}(zn) \gamma^+ \psi(0) | h(P) \rangle$$

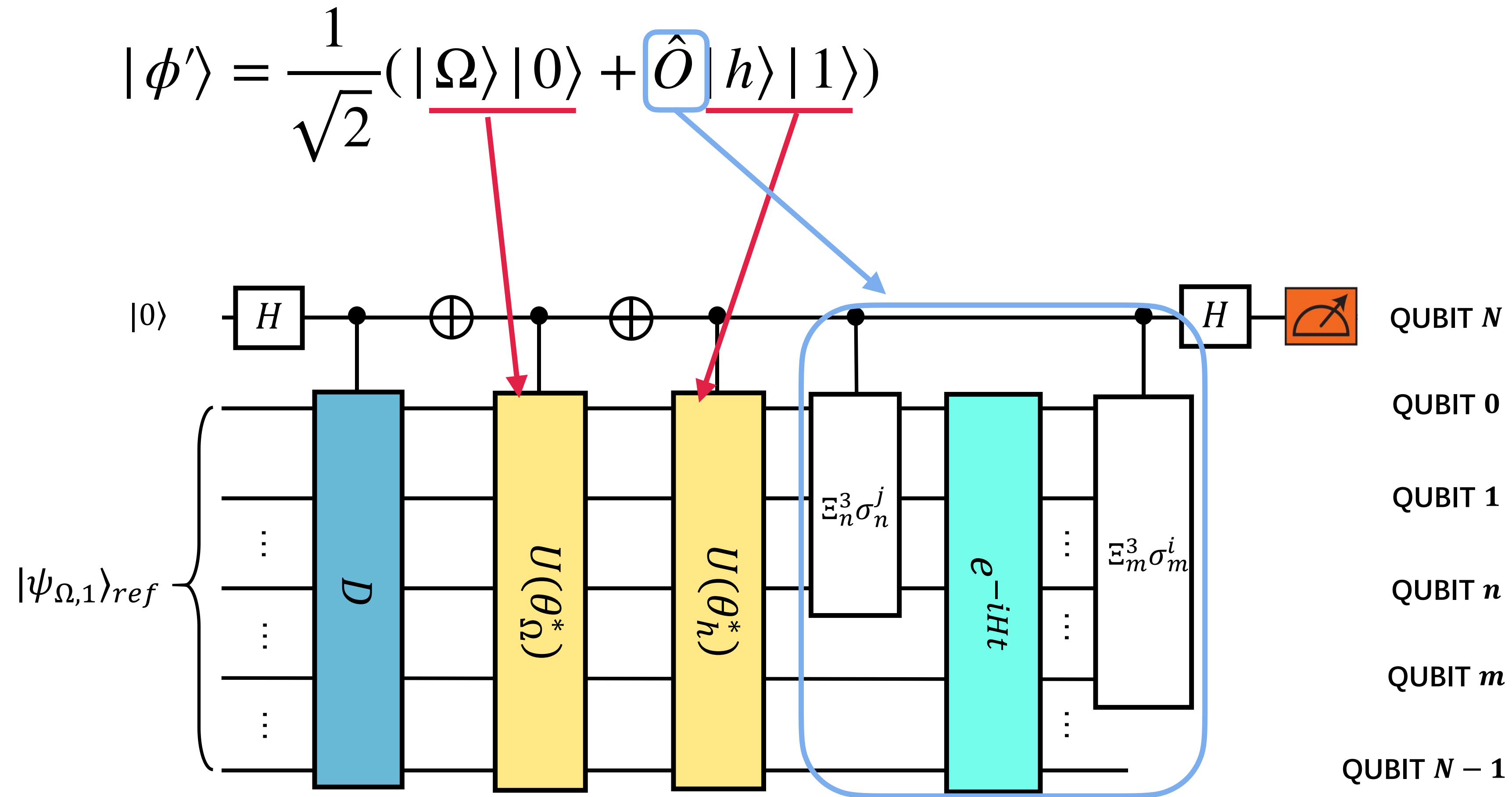


- ◆ The current knowledge on LCDA is limited, mainly on models and lattice calculations
- ◆ First try using quantum computing

Quantum computing for exclusive hadronization

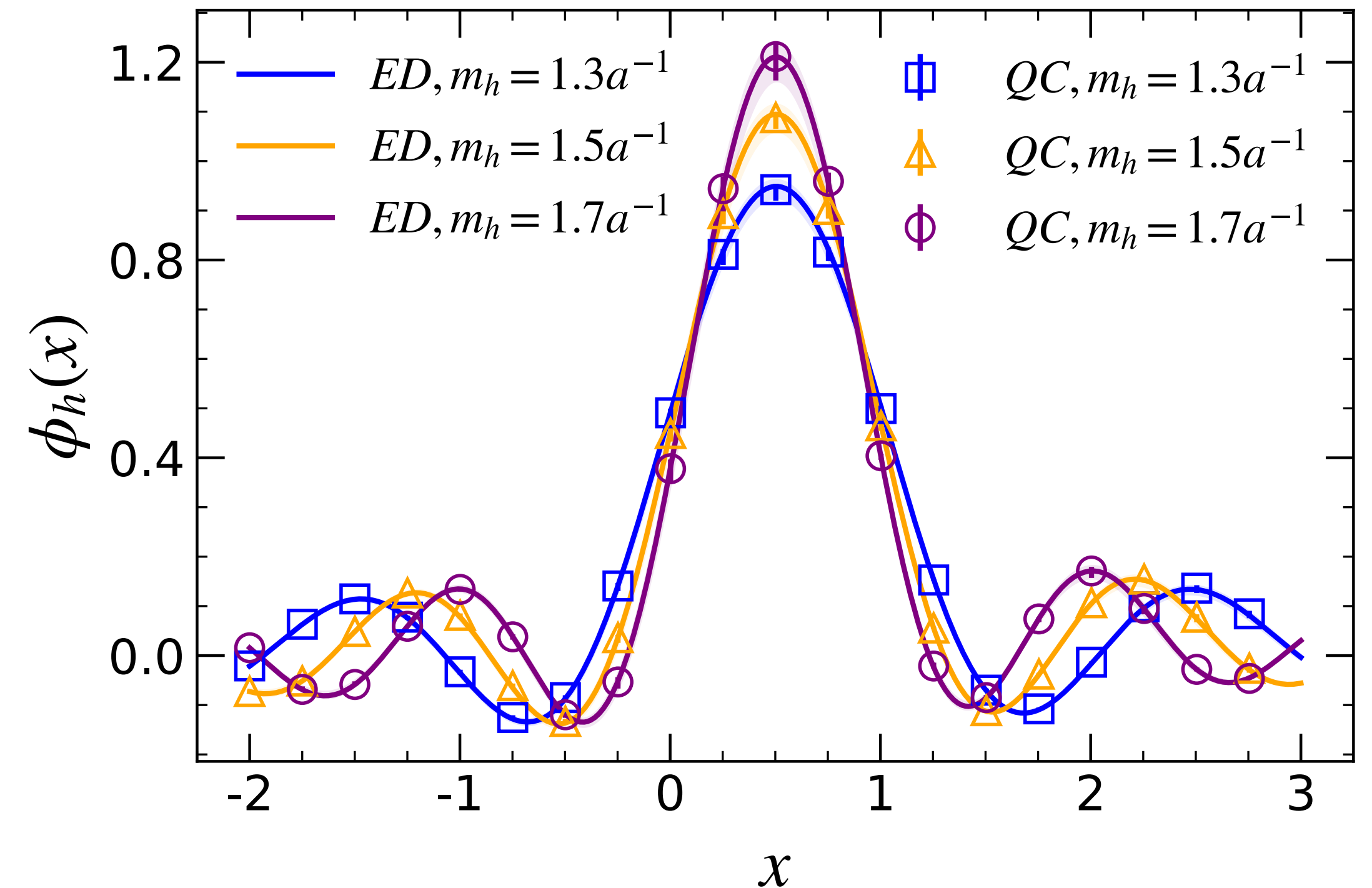
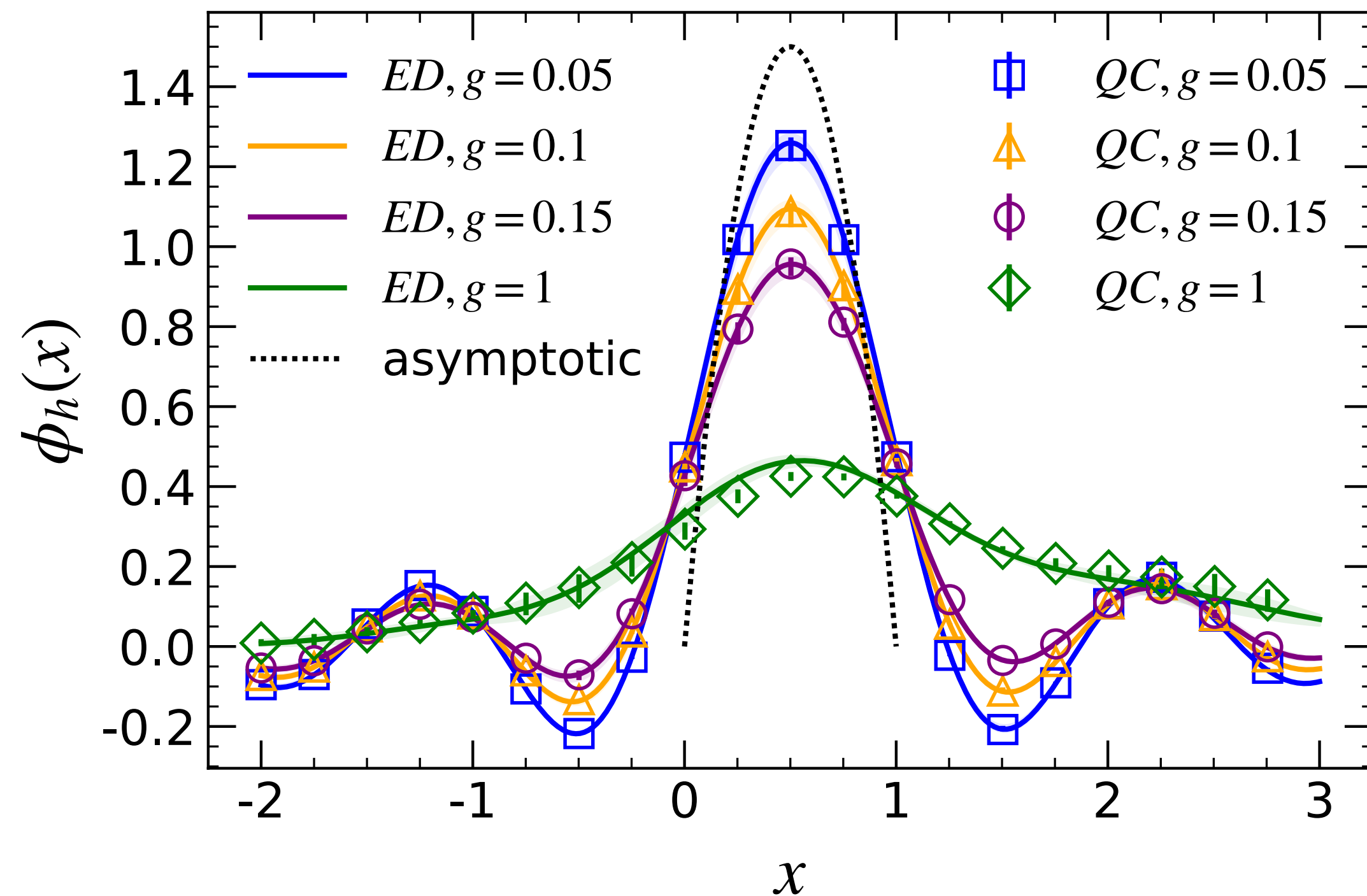
◆ Quantum circuit

Li et al (QuNu), SCPMA (2023)



Quantum computing for exclusive hadronization

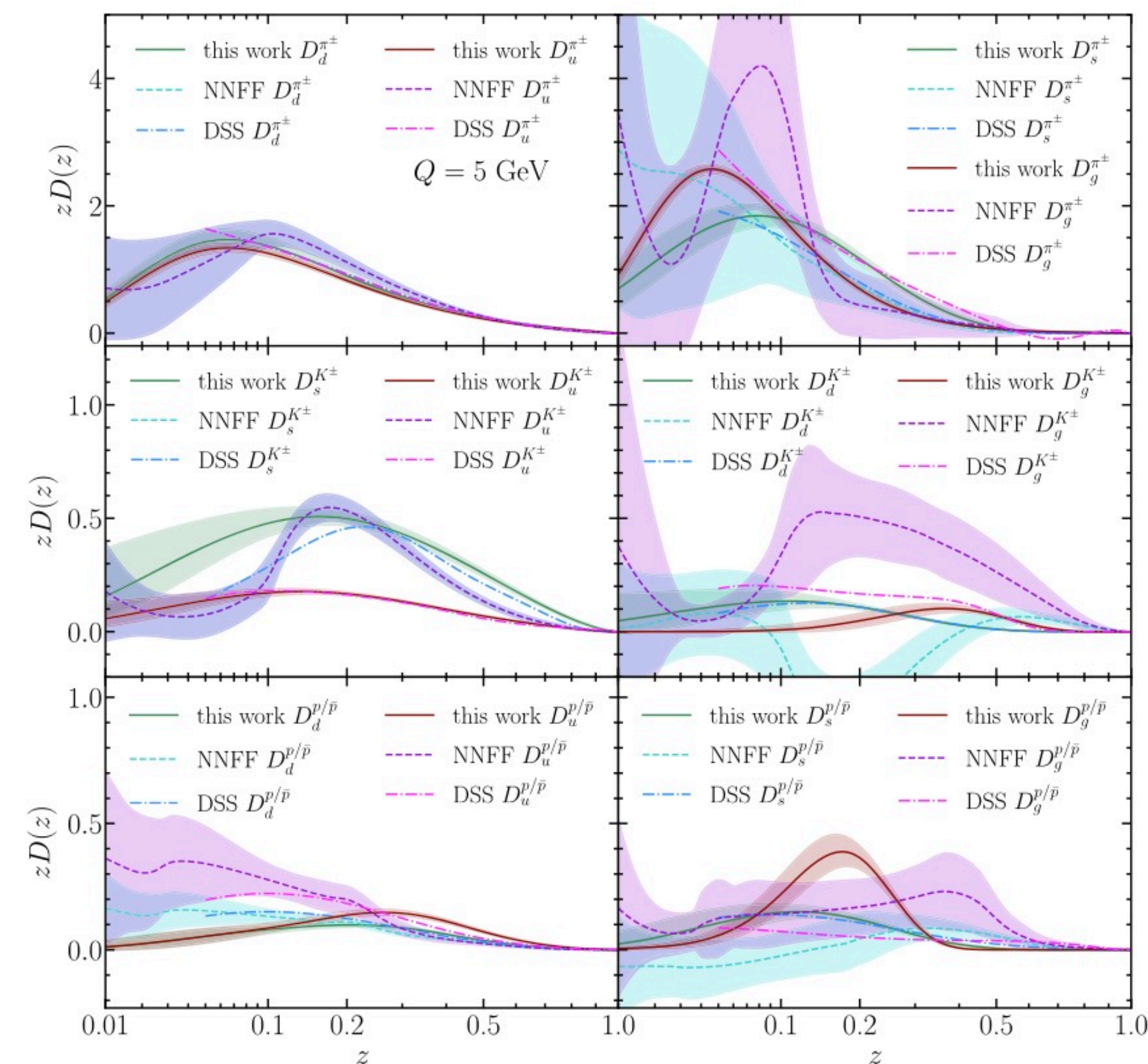
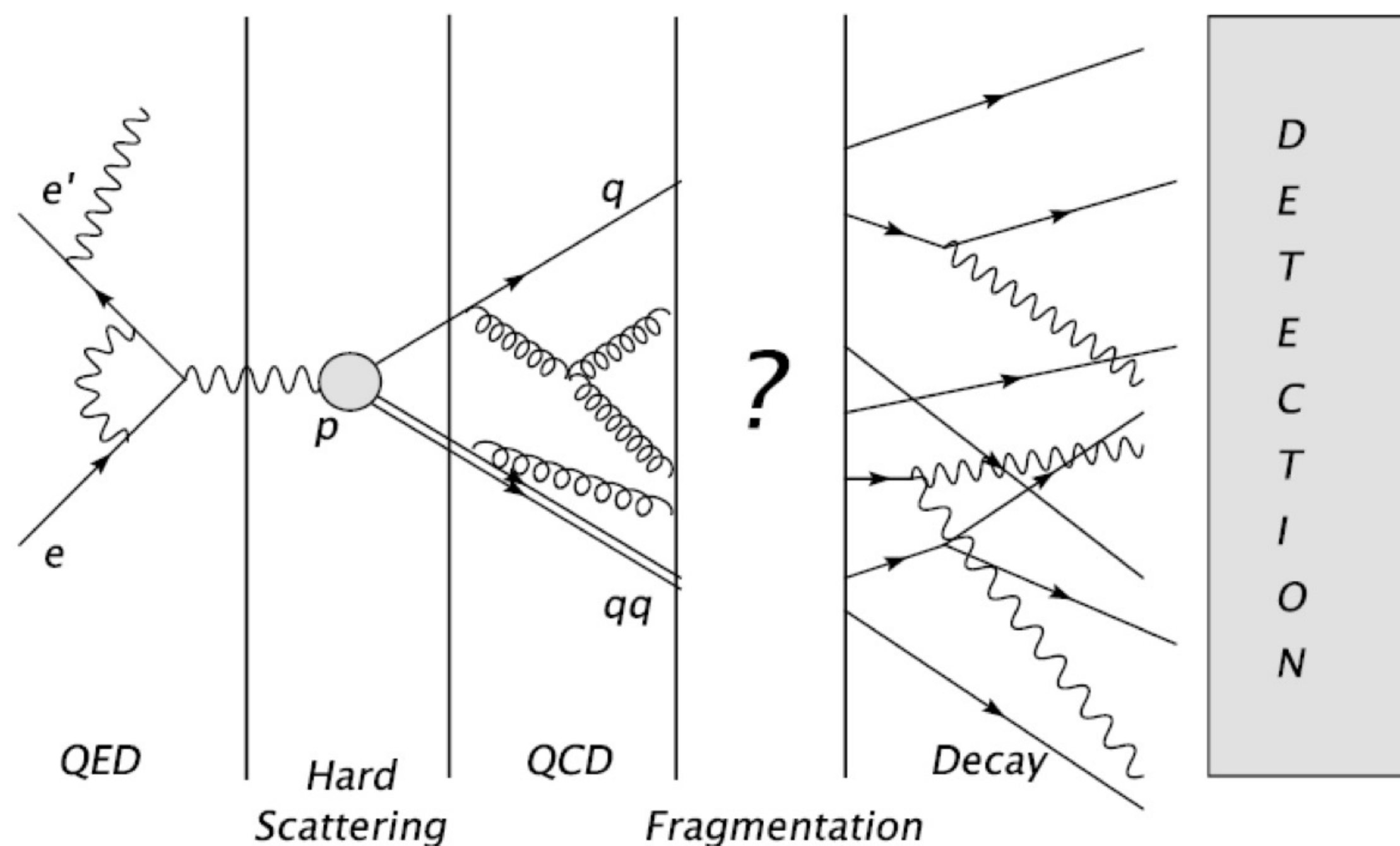
◆ Numerical results



- peak gets narrower with decreasing coupling constant or increasing hadron mass
- Converges to asymptotic result in weak coupling limit

Quantum computing for inclusive hadronization

- ◆ Global fitting - the only reliable way to extract hadron fragmentation functions

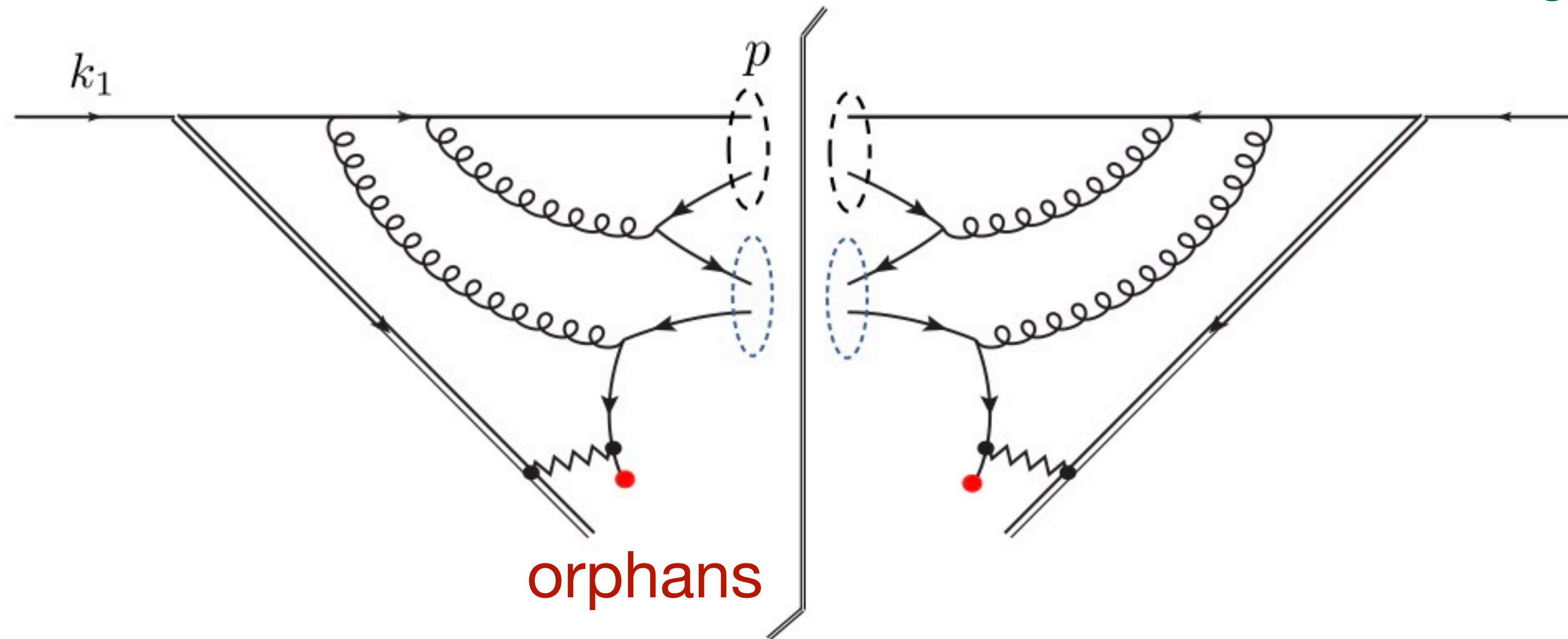


Gao, Liu, Shen, **HX**, Zhao, PRL, 2024

Gao, Liu, Shen, **HX**, Zhao, arXiv: 2407.04422, PRD Editor's suggestion

Challenges in lattice QCD for FFs

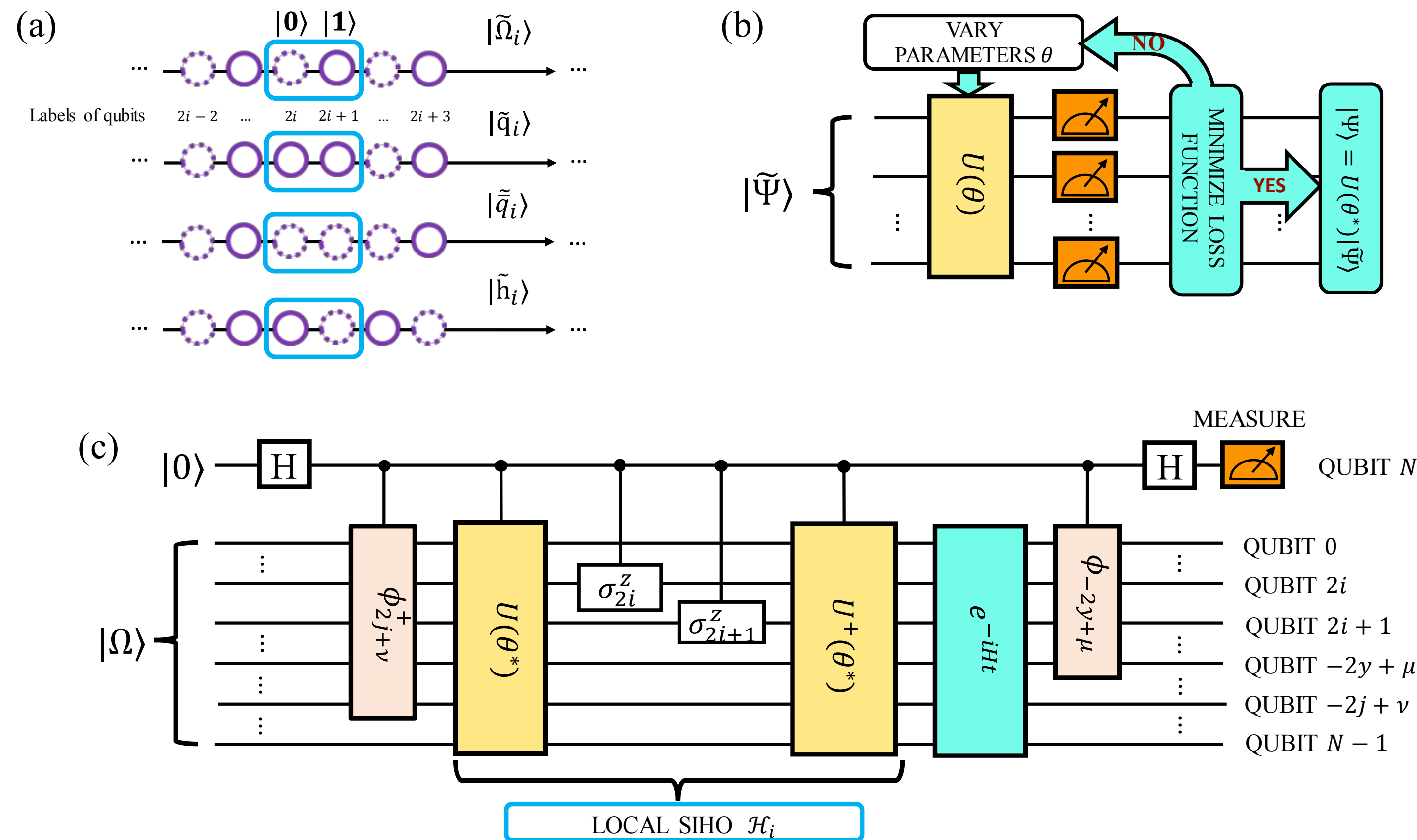
Collins, Rogers, PRD 2024



$$D_q^h(z) = z^{d-3} \int \frac{dy^-}{4\pi} e^{-iy^- p^+/z} \text{Tr} \left\{ \langle \Omega | \psi(y^-) \sum_X |h, X\rangle \langle h, X| \bar{\psi}(0) | \Omega \rangle \gamma^+ \right\}$$

1. Real-time dynamical quantity -> sign problem
2. Unidentified X -> exponentially increasing complexity

Quantum circuit for FFs



- VQE

$$U|I_i^{\Omega}\rangle = |\Omega_i\rangle$$

$$U|I_i^q\rangle = |q_i\rangle$$

$$U|I_i^h\rangle = |h_i\rangle$$

...

- Semi-inclusive hadronic operator

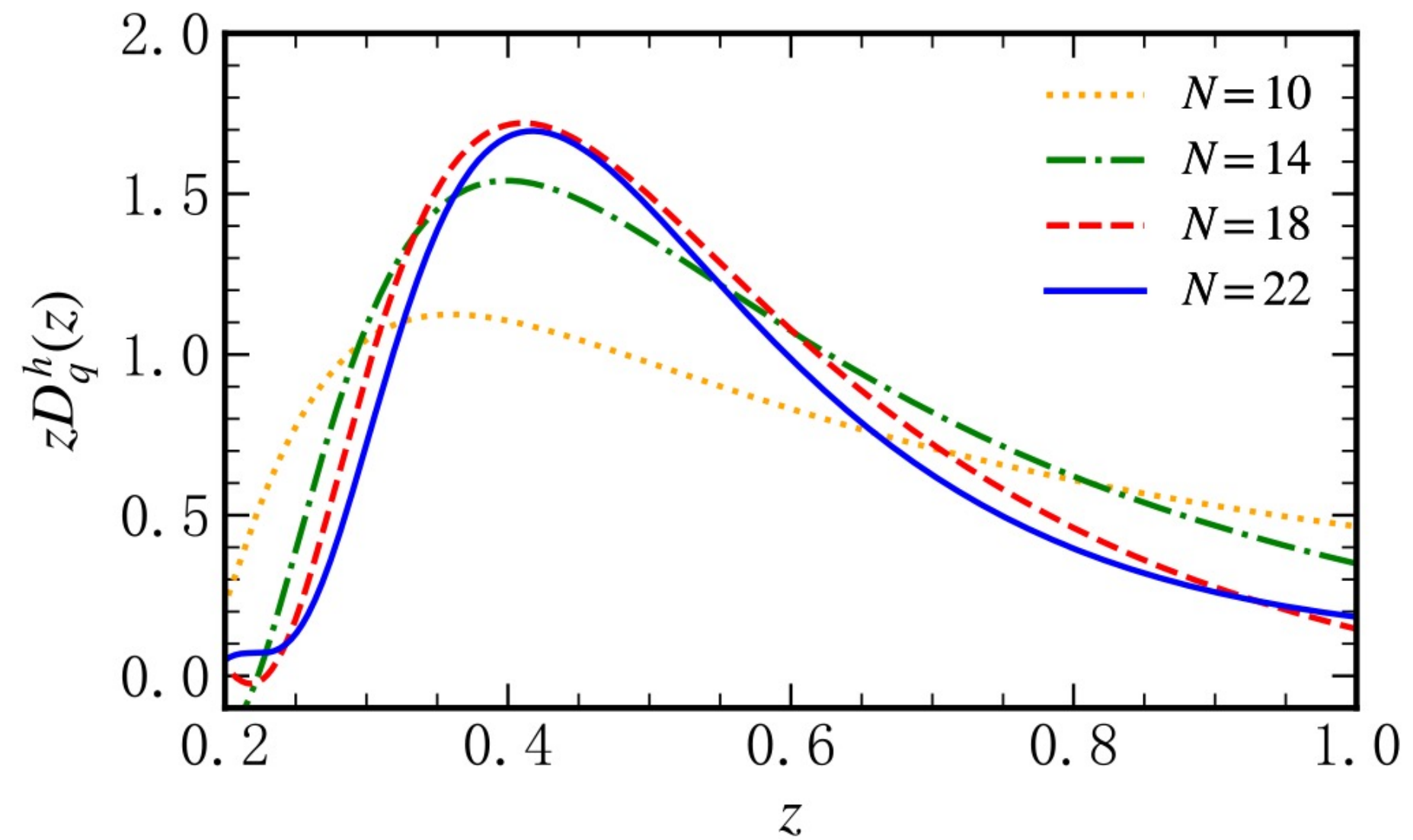
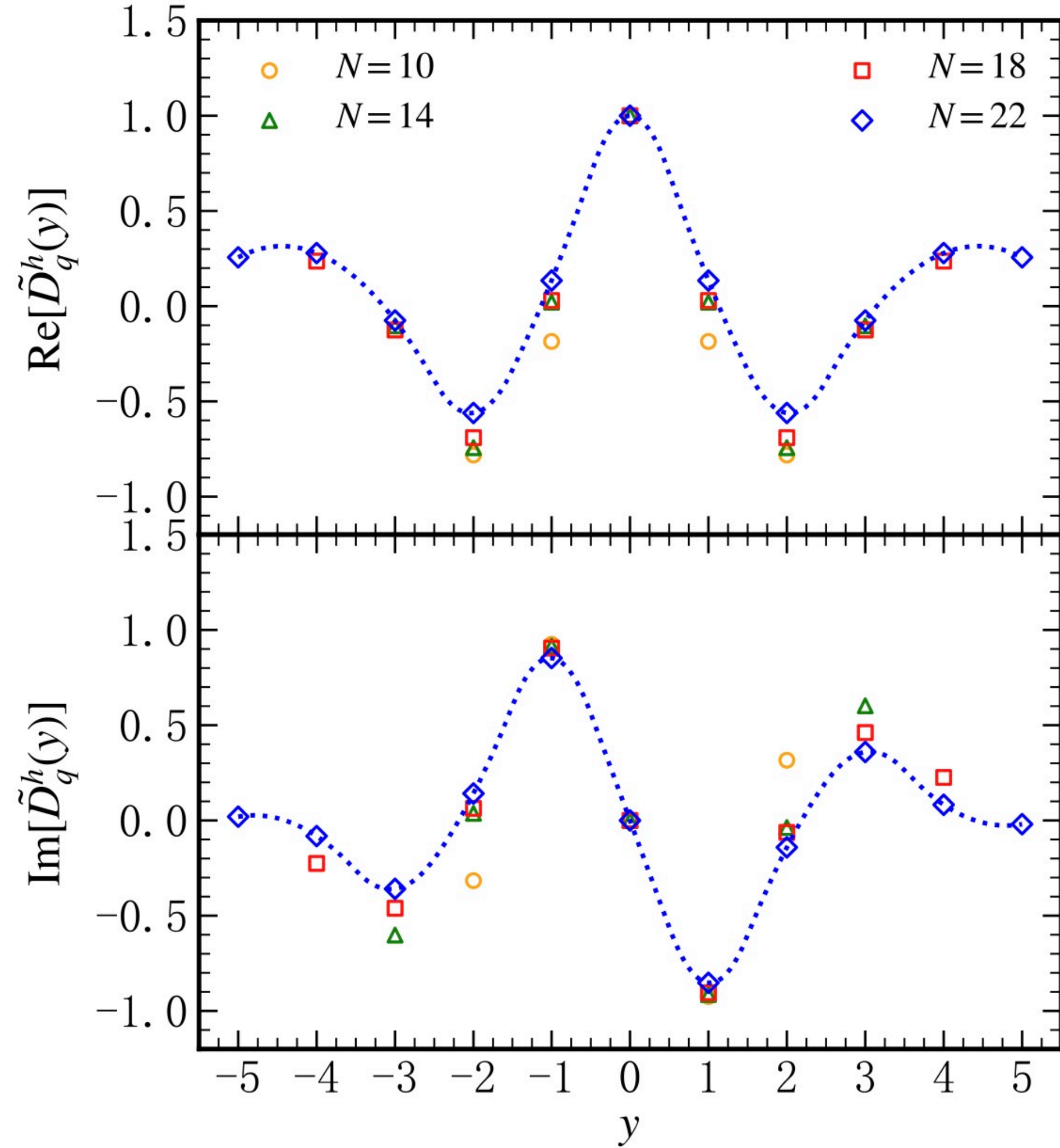
$$|h_i, X_{\{j \neq i\}}\rangle = U |\tilde{h}_i, \tilde{X}_{\{j \neq i\}}\rangle$$

$$\sum_a |I_i^a\rangle \langle I_i^a| = \text{Id}_i$$

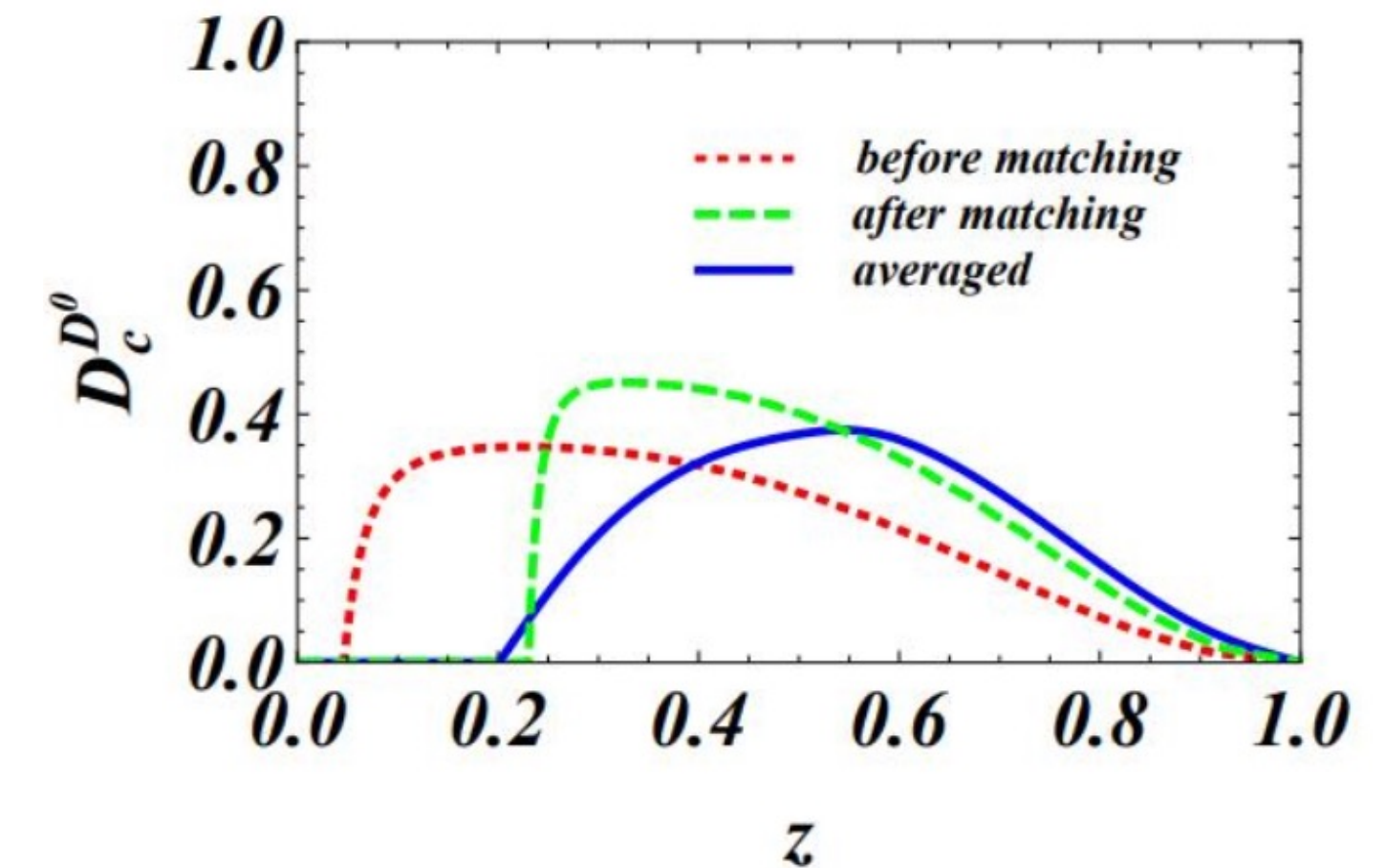
$$\begin{aligned} \mathcal{H}_i &= U \text{Tr}_{\{j \neq i\}} |\tilde{h}_i, \tilde{X}_{\{j \neq i\}}\rangle \langle \tilde{h}_i, \tilde{X}_{\{j \neq i\}}| U^\dagger \\ &= U |I_i^h\rangle \langle I_i^h| \otimes \text{Id}_{\{j \neq i\}} U^\dagger, \end{aligned}$$

FFs from quantum simulation

Li, **HX**, Zhang, arXiv:2406.05683



D.-J. Yang Phys. Rev. D (2020).

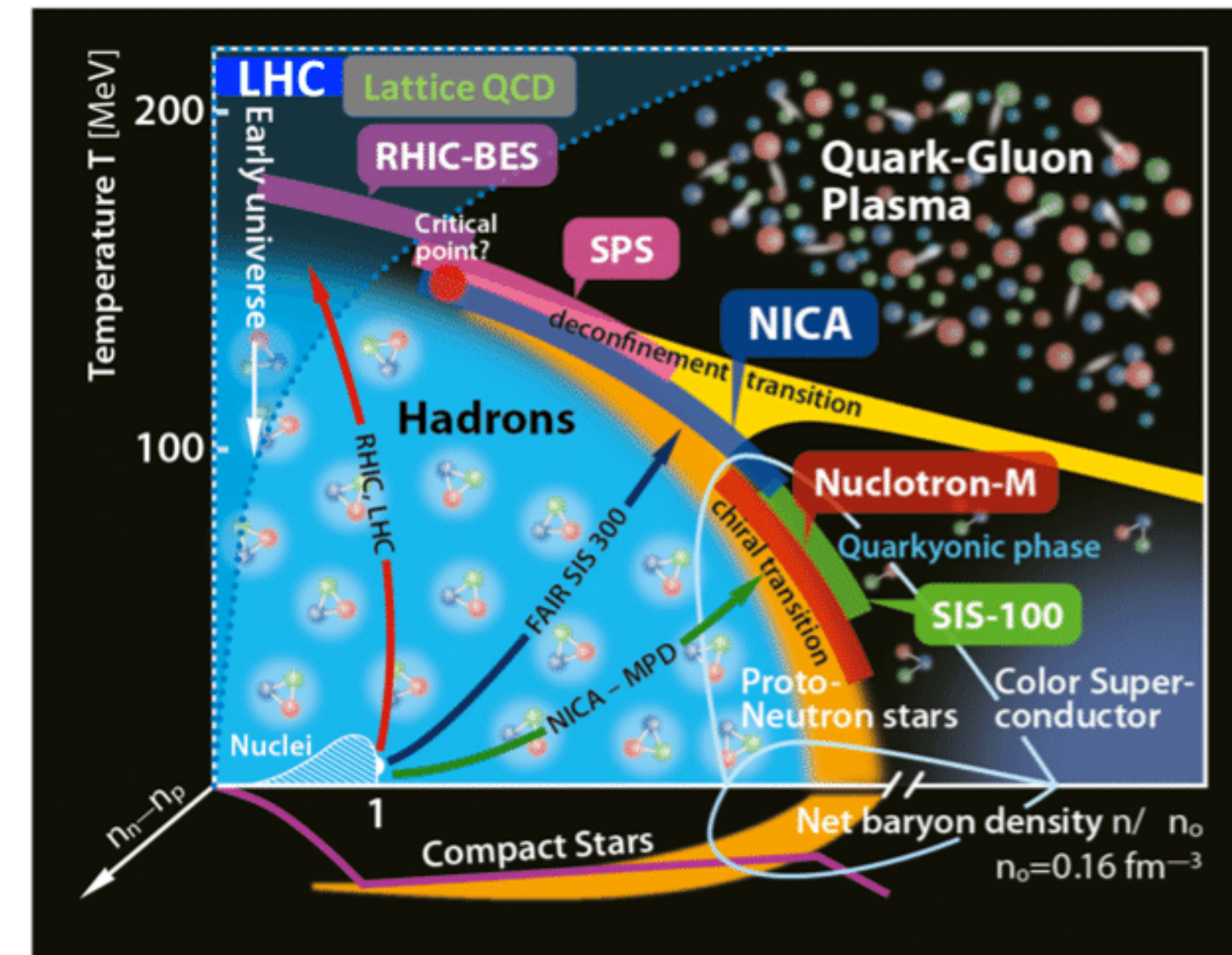


- Converges with the increase of qubit number N
- consistent with analytical calculations

Chiral condensate in SU(2)

◆ Spontaneous chiral symmetry breaking

- One of the key features of QCD
- Origin of mass
- Chiral magnetic effect, chiral vortical effect...
- Non-perturbative, high baryon chemical potential
- Challenging for traditional methods
- Chiral condensate: $\sigma = \langle \bar{\psi}\psi \rangle$



Chiral condensate in SU(2)

Li, **HX**, Zhang, arXiv:2411.18869

- 1+1D SU(2) model: simplest non-Abelian model

$$H = -i\bar{\psi}\gamma^1(\partial_1 + igA_1^a t^a)\psi + m\bar{\psi}\psi + \mu\psi^\dagger\psi + \frac{1}{2} \sum_a (L^a)^2$$

- Discretization: Staggered fermion

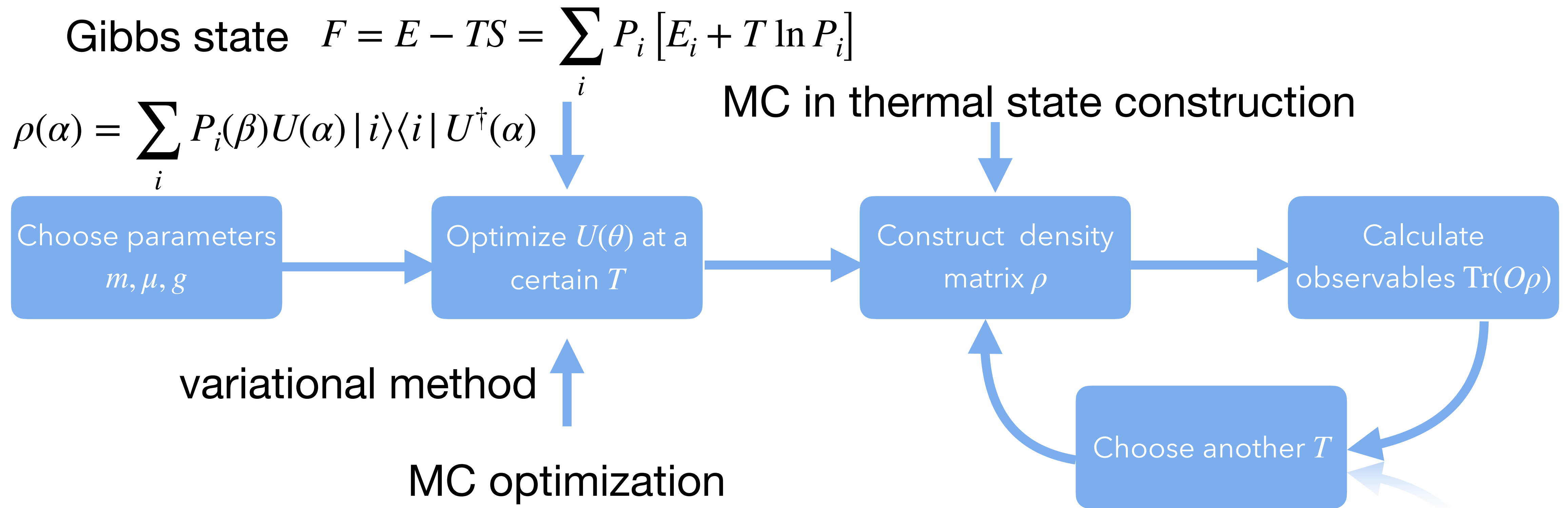
$$\psi_1(x) \rightarrow \phi_{2n}, \quad \psi_2(x) \rightarrow \phi_{2n+1}$$

$$H = \frac{1}{2\Delta} \sum_{n=0}^{N-2} (\phi_n^\dagger U_n \phi_{n+1} + H.C.) + m \sum_{n=0}^{N-1} (-1)^{n+1} \phi_n^\dagger \phi_n + \mu \sum_{n=0}^{N-1} \phi_n^\dagger \phi_n + \frac{\Delta g^2}{2} \sum_{n=0}^{N-2} \mathbf{L}_n^2$$

- Taking advantage of Gauss's law: $\mathbf{L}_n^a - \mathbf{R}_{n-1}^a = \mathbf{Q}_{n-1}^a = \phi_n^\dagger t^a \phi_n \rightarrow \mathbf{L}_n^a = \sum_{i < n} \mathbf{Q}_i^a$

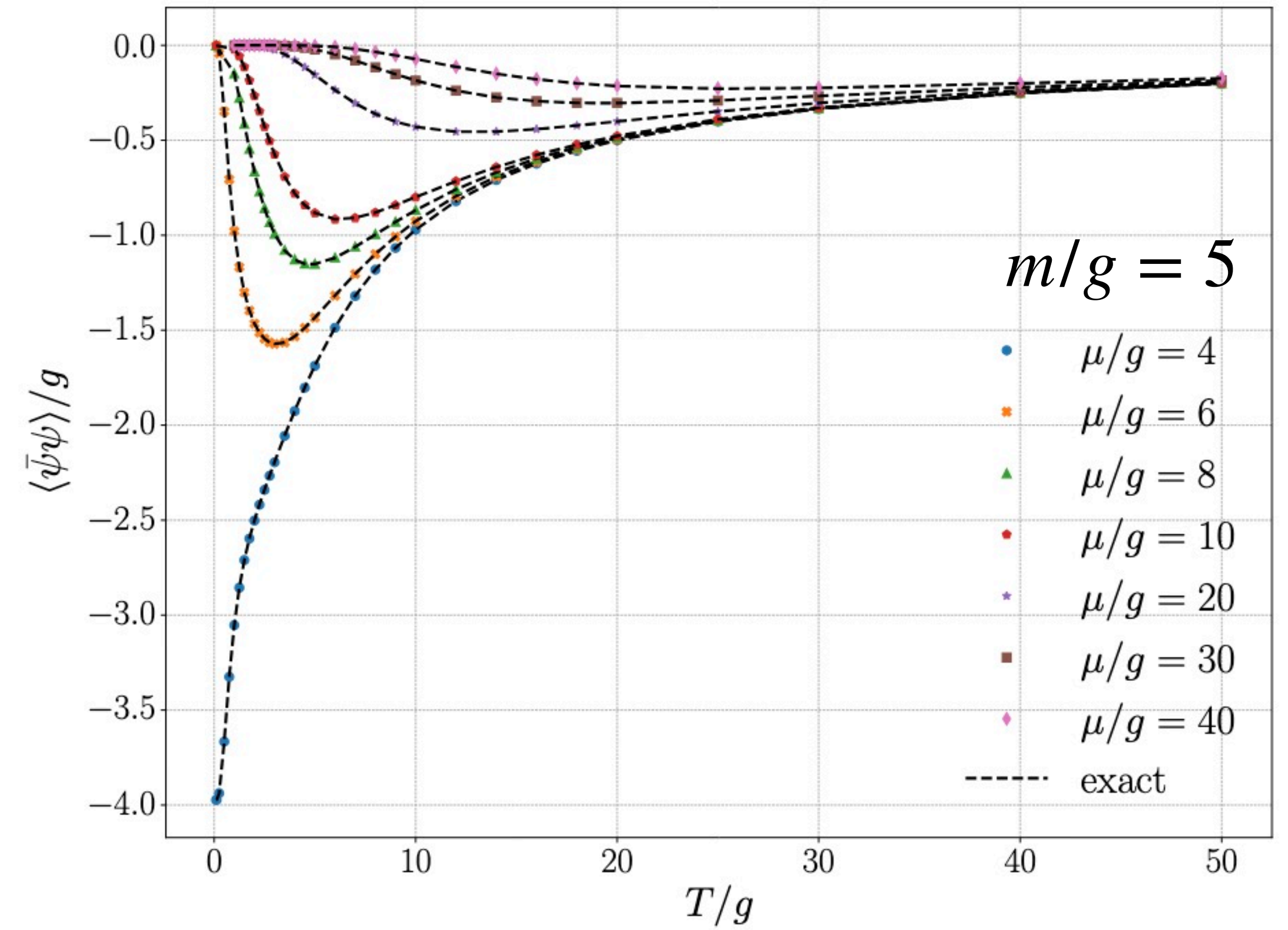
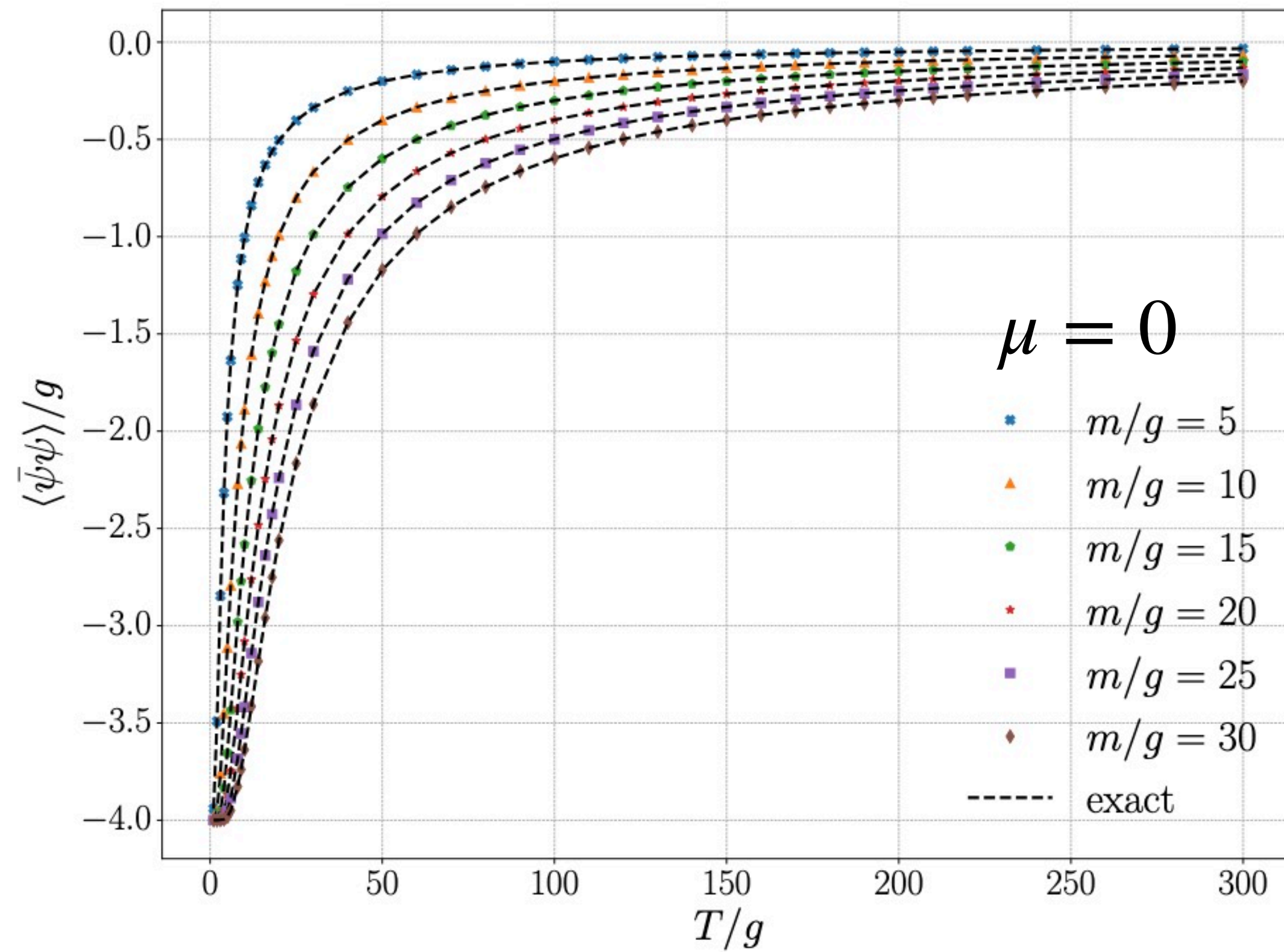
$$H = \frac{1}{2} \sum_{n=0}^{N-2} (\phi_n^\dagger \phi_{n+1} + H.C.) + \Delta m \sum_{n=0}^{N-1} (-1)^{n+1} \phi_n^\dagger \phi_n + \Delta \mu \sum_{n=0}^{N-1} \phi_n^\dagger \phi_n + \frac{\Delta^2 g^2}{2} \sum_{n=0}^{N-2} \left(\sum_{k \leq n} \mathbf{Q}_k \right)$$

algorithm workflow



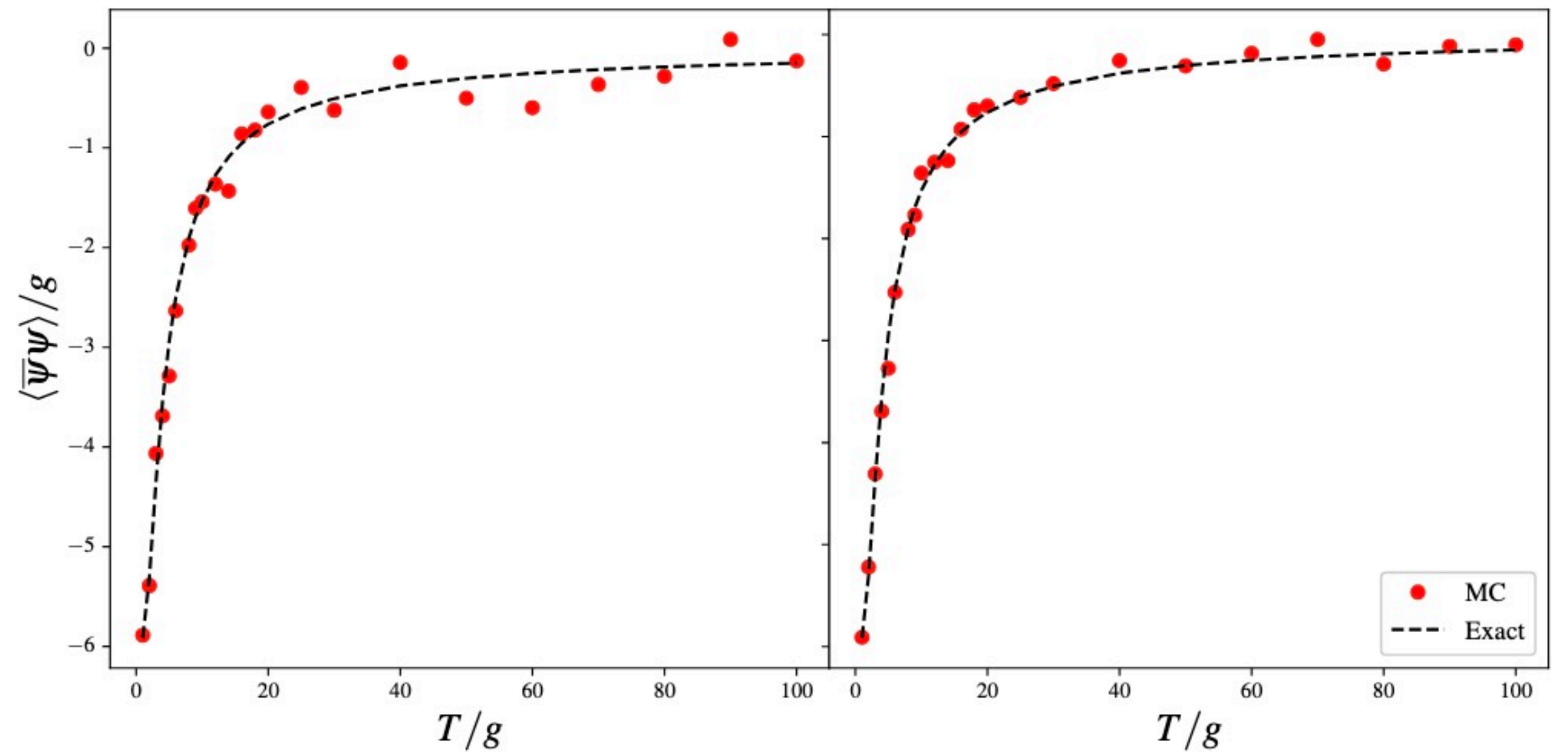
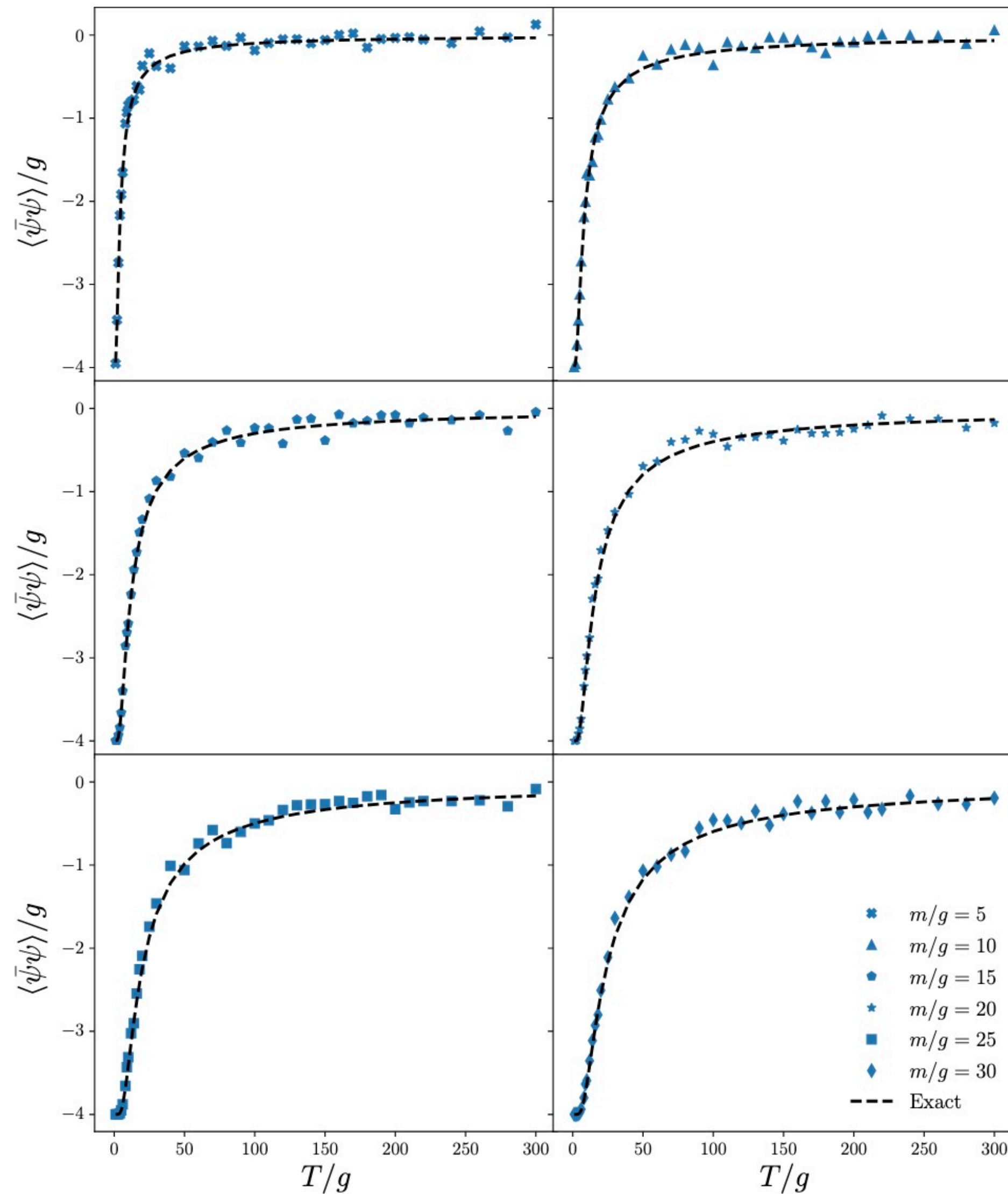
- The variational method is only used once for all different temperatures.
- Many part of the calculation is analytical.

Results: Full Gibbs state



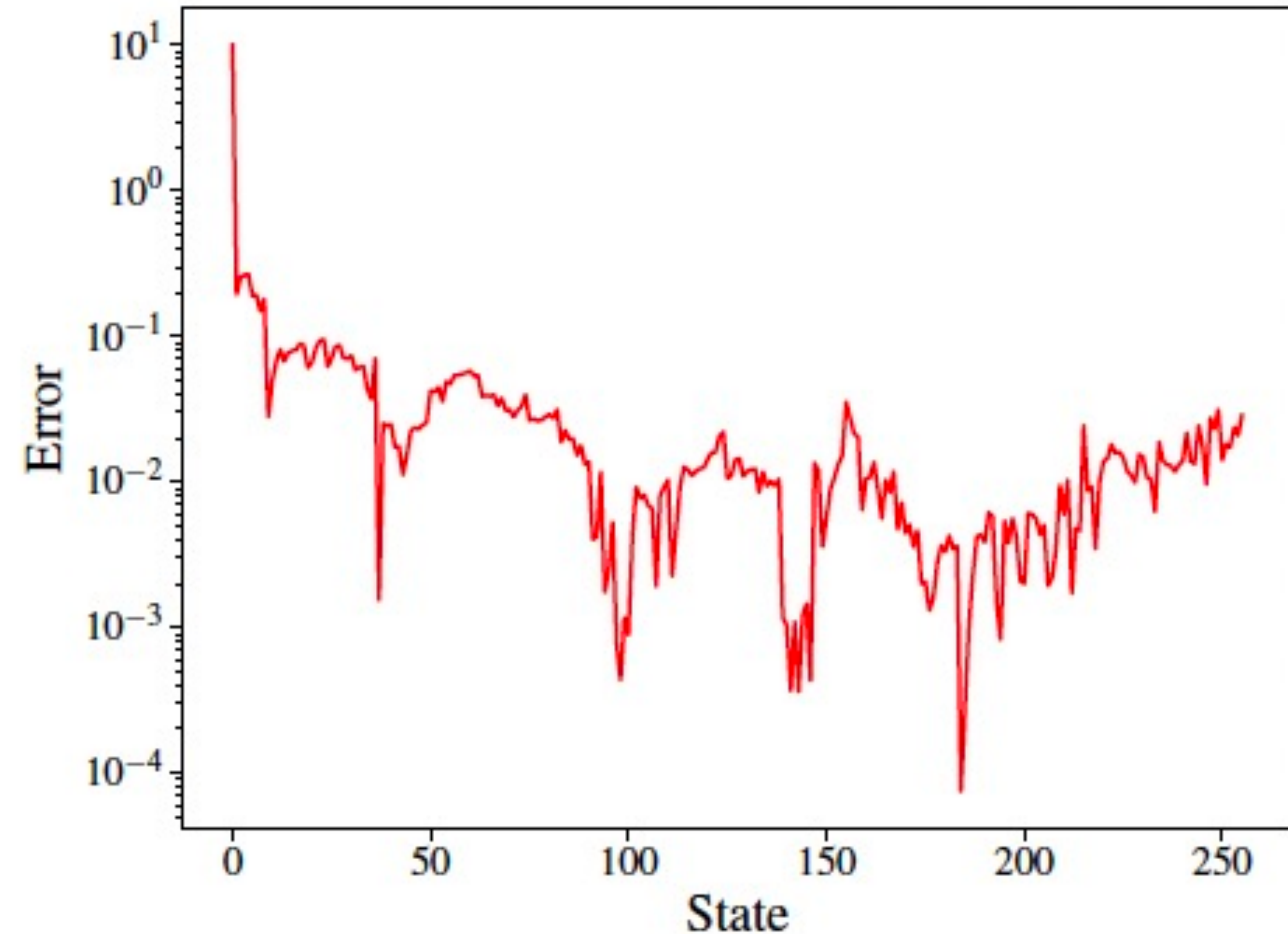
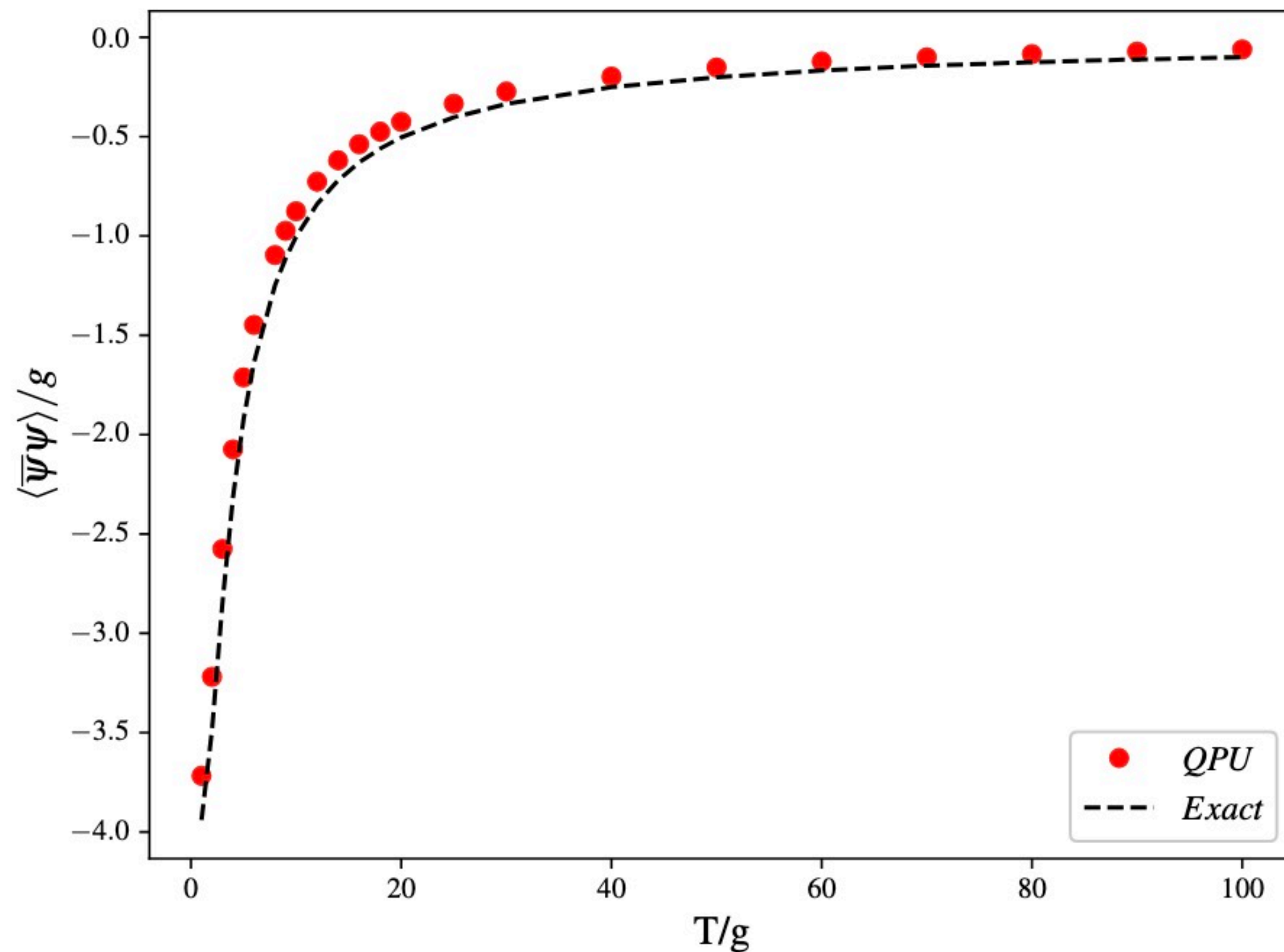
- The VQE method produces the Gibbs state very accurately.

Results: Monte-Carlo



- 12 qubits, 1000 (left) and 2000 (right) states.
- Required number of sampling increases only as power law of number of qubits.
- 2000 vs. 4096 is already effective.
- 8 qubits, 1000 states for each sampling

Results: simulation on real IBM quantum machine



- 8 qubits, results from IBM's quantum hardware
- Our algorithm can achieve good precision on real QC
- Promising to apply to larger systems

Summary and outlook

- Systematic computing of hadronic scatterings
 1. Use NJL model as a proof of concept study
 2. Include both parton distribution function, scattering amplitude and fragmentation functions
- Quantum simulation for chiral condensate, many topics are not covered, such as jet quenching, quantum machine learning for data analysis ...
- The field is still at its infant age, many more need to be done
 1. Simulation of real QCD
 2. Extend to higher dimensions and spin dependent processes
 3. Consider noises

Thanks for your attention!

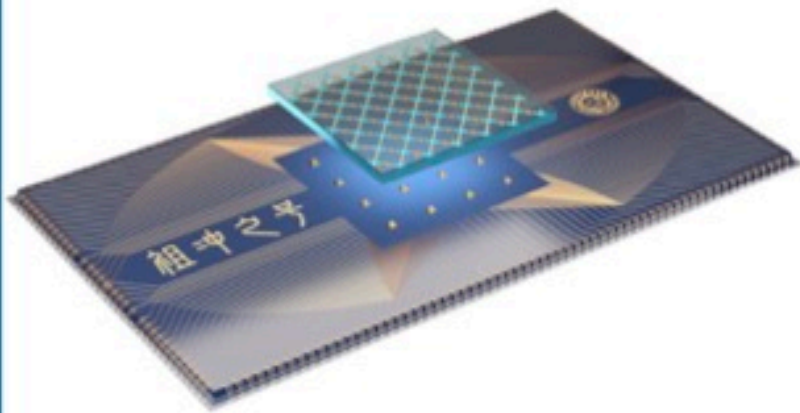


Now - Noisy Intermediate Scale Quantum (NISQ) era

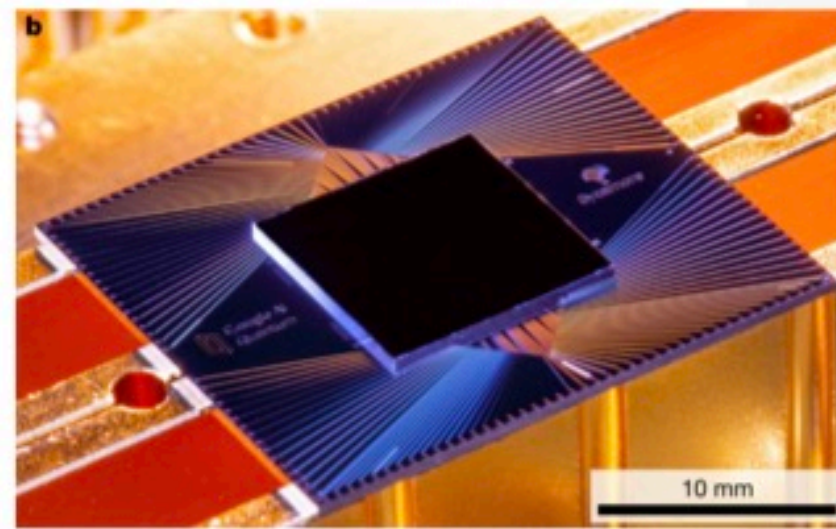
By YYLi

more than 50 well controlled qubits, not error-corrected yet

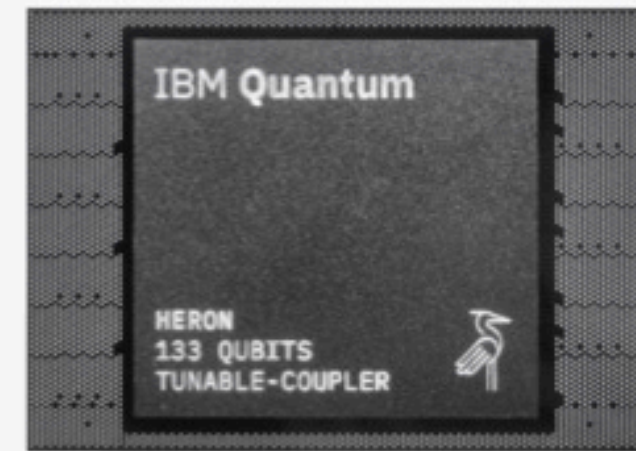
superconducting processor



176 qubits

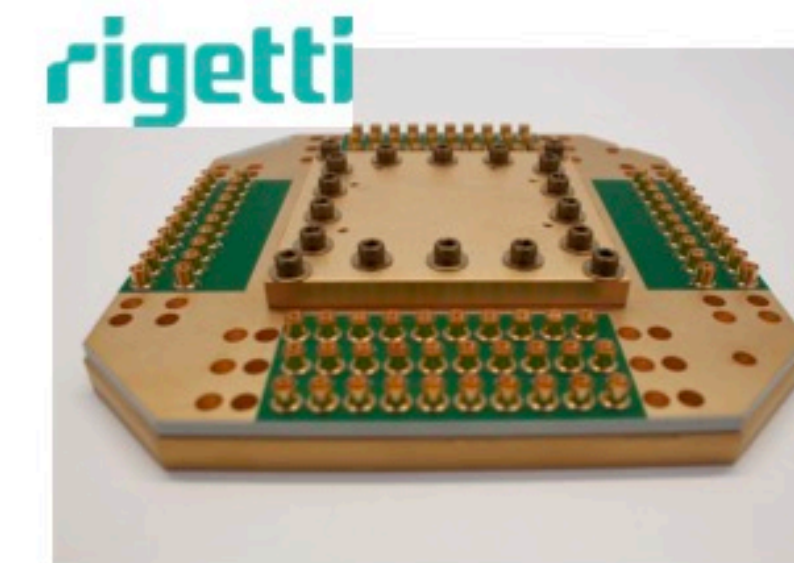


54 qubits



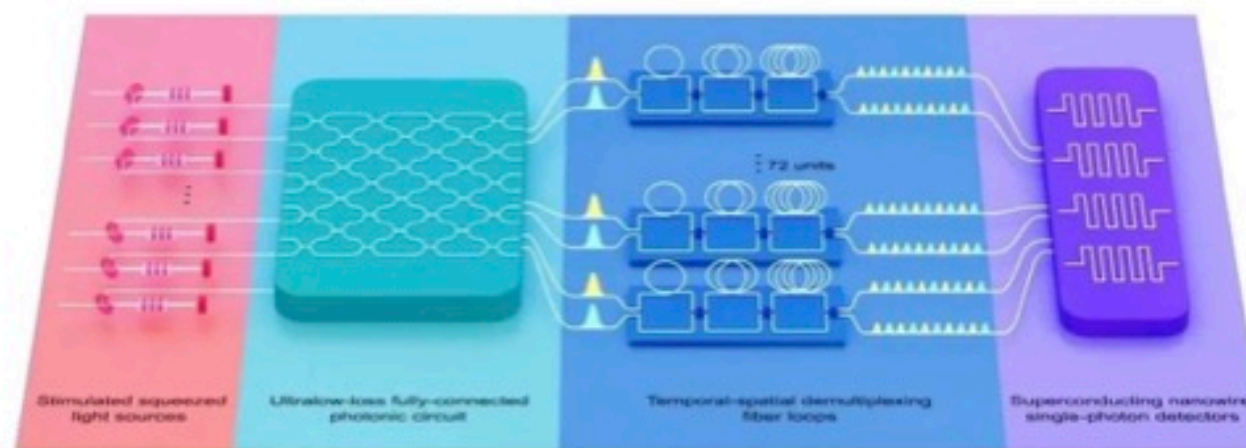
1121 qubits
access to 156 qubits

multi-chip quantum processor



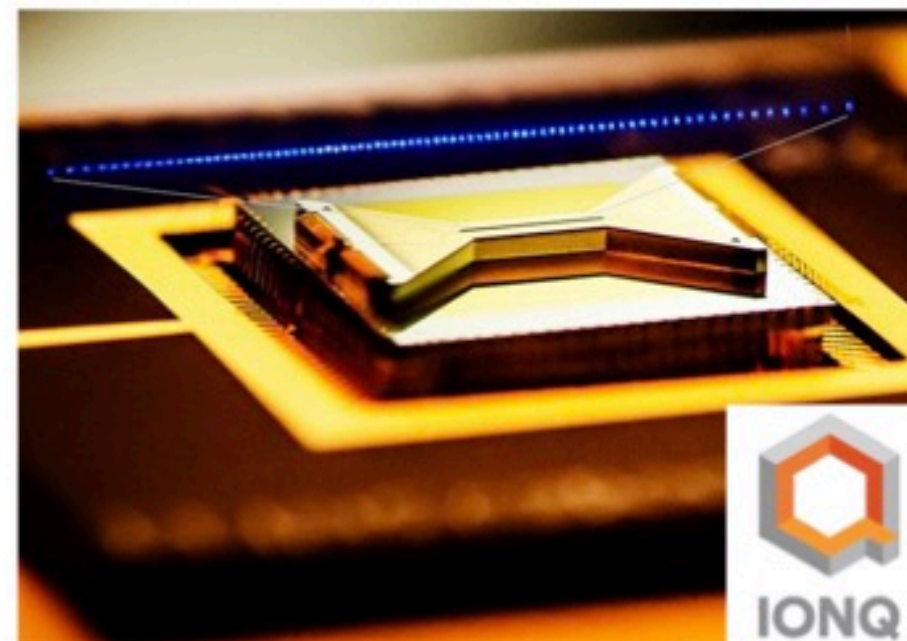
80 qubits

photon qubits



Jiuzhang - 255 qubits

trapped ion qubits



22 qubits

48 logical
qubits














Development Roadmap

IBM Quantum

	2016–2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2033+
	Run quantum circuits on the IBM Quantum Platform	Release multi-dimensional roadmap publicly with initial aim focused on scaling	Enhancing quantum execution speed by 100x with Qiskit Runtime	Bring dynamic circuits to unlock more computations	Enhancing quantum execution speed by 5x with quantum serverless and Execution modes	Improving quantum circuit quality and speed to allow 5K gates with parametric circuits	Enhancing quantum execution speed and parallelization with partitioning and quantum modularity	Improving quantum circuit quality to allow 7.5K gates	Improving quantum circuit quality to allow 10K gates	Improving quantum circuit quality to allow 15K gates	Improving quantum circuit quality to allow 100M gates	Beyond 2033, quantum-centric supercomputers will include 1000's of logical qubits unlocking the full power of quantum computing
Data Scientist						Platform						
						Code assistant	Functions	Mapping Collection	Specific Libraries			General purpose QC libraries
Researchers						Middleware						
						Quantum Serverless	Transpiler Service	Resource Management	Circuit Knitting x P	Intelligent Orchestration		Circuit libraries
Quantum Physicist				Qiskit Runtime								
	IBM Quantum Experience		QASM3	Dynamic circuits	Execution Modes	Heron (5K)	Flamingo (5K)	Flamingo (7.5K)	Flamingo (10K)	Flamingo (15K)	Starling (100M)	Blue Jay (1B)
	Early	Falcon		Eagle		Error Mitigation	Error Mitigation	Error Mitigation	Error Mitigation	Error Mitigation	Error correction	Error correction
	Canary 5 qubits Albatross 16 qubits Penguin 20 qubits Prototype 53 qubits	Benchmarking 27 qubits		Benchmarking 127 qubits		5k gates 133 qubits Classical modular 133x3 = 399 qubits	5k gates 156 qubits Quantum modular 156x7 = 1092 qubits	7.5k gates 156 qubits Quantum modular 156x7 = 1092 qubits	10k gates 156 qubits Quantum modular 156x7 = 1092 qubits	15k gates 156 qubits Quantum modular 156x7 = 1092 qubits	100M gates 200 qubits Error corrected modularity	1B gates 2000 qubits Error corrected modularity

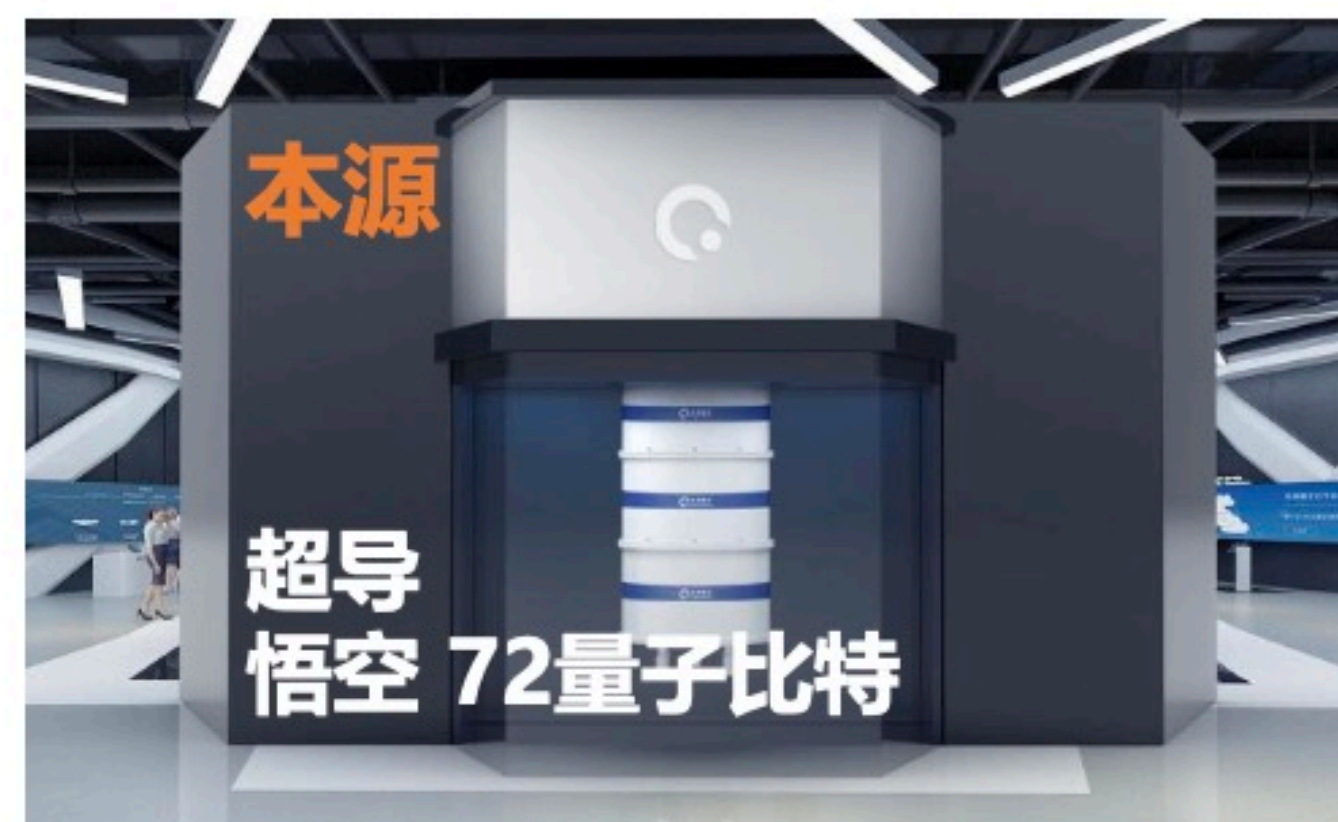
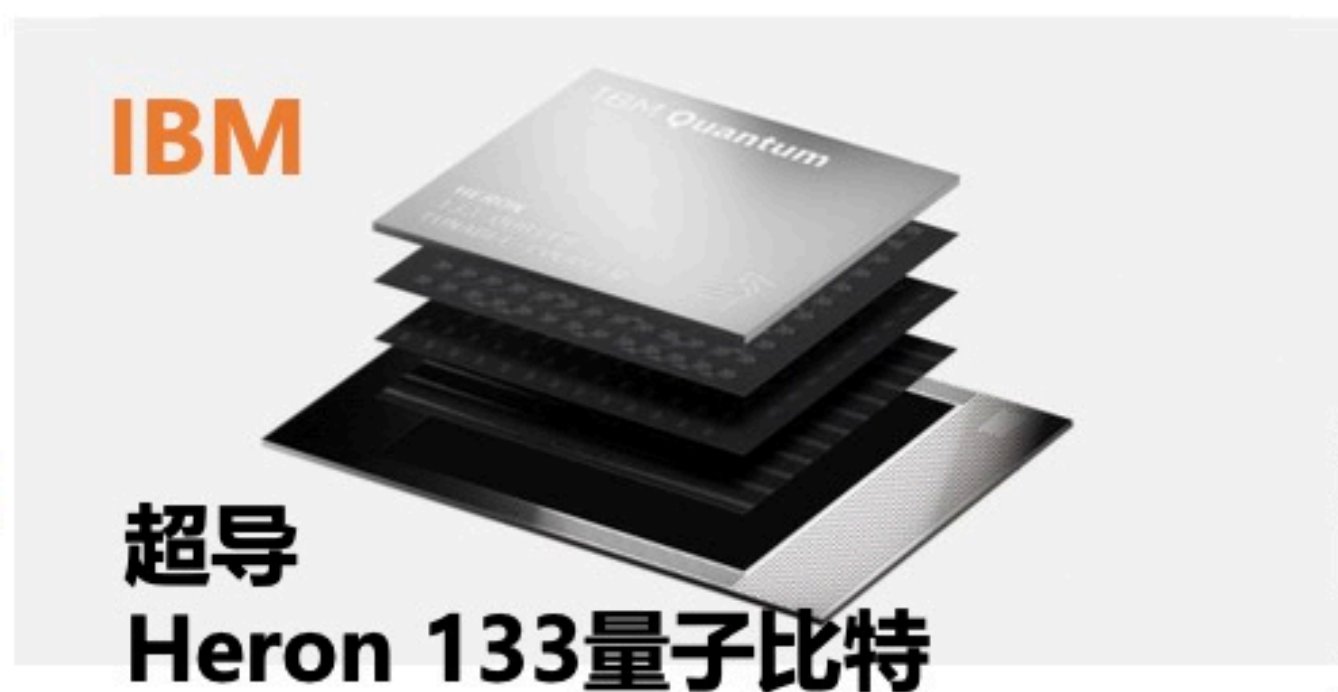
Innovation Roadmap

Software Innovation	IBM Quantum Experience ✔	Qiskit Circuit and operator API with compilation to multiple targets ✔	Application modules Modules for domain specific application and algorithm workflows ✔	Qiskit Runtime Performance and abstract through Primitives ✔	Serverless Demonstrate concepts of quantum centric-supercomputing ✔	AI enhanced quantum Prototype demonstrations of AI enhanced circuit transpilation ✔	Resource management System partitioning to enable parallel execution 🎯	Scalable circuit knitting Circuit partitioning with classical reconstruction at HPC scale ✔	Error correction decoder Demonstration of a quantum system with real-time error correction decoder ✔					
Hardware Innovation	Early Canary 5 qubits Penguin 20 qubits Albatross 16 qubits Prototype 53 qubits ✔	Falcon Demonstrate scaling with I/O routing with Bump bonds ✔ 	Hummingbird Demonstrate scaling with multiplexing readout ✔ 	Eagle Demonstrate scaling with MLW and TSV ✔ 	Osprey Enabling scaling with high density signal delivery ✔ 	Condor Single system scaling and fridge capacity ✔ 	Flamingo Demonstrate scaling with modular connectors 🎯 	Kookaburra Demonstrate scaling with nonlocal c-coupler ✔ 	Demonstrate path to improved quality with logical memory		Cockatoo Demonstrate path to improved quality with logical communication ✔ 	Starling Demonstrate path to improved quality with logical gates ✔ 		
						Heron Architecture based on tunable-couplers ✔ 	Crossbill m- coupler 🎯 							

✔ Executed by IBM

🎯 On target

中等规模带噪声的量子计算(NISQ)时代



- 现在是**中等规模带噪声的量子计算时代 (>50量子比特)**。
- 超导容错量子计算机要**约100万量子比特**。2030年左右可能达到？破加密可能要**约1亿量子比特**。

Postselection and noise extrapolation

$$\epsilon(\rho) = (1 - p)\rho + \frac{p}{3}(\sigma^x \rho \sigma^x + \sigma^y \rho \sigma^y + \sigma^z \rho \sigma^z)$$

- Physical states have fixed quantum numbers, such as particle number
- If these quantities changed, it must be due to noise.
- So results with inaccurate quantum numbers are excluded.
- Effectively, only even number of x and y flips are allowed.
- Any final measurement can be viewed as a function of noise probability

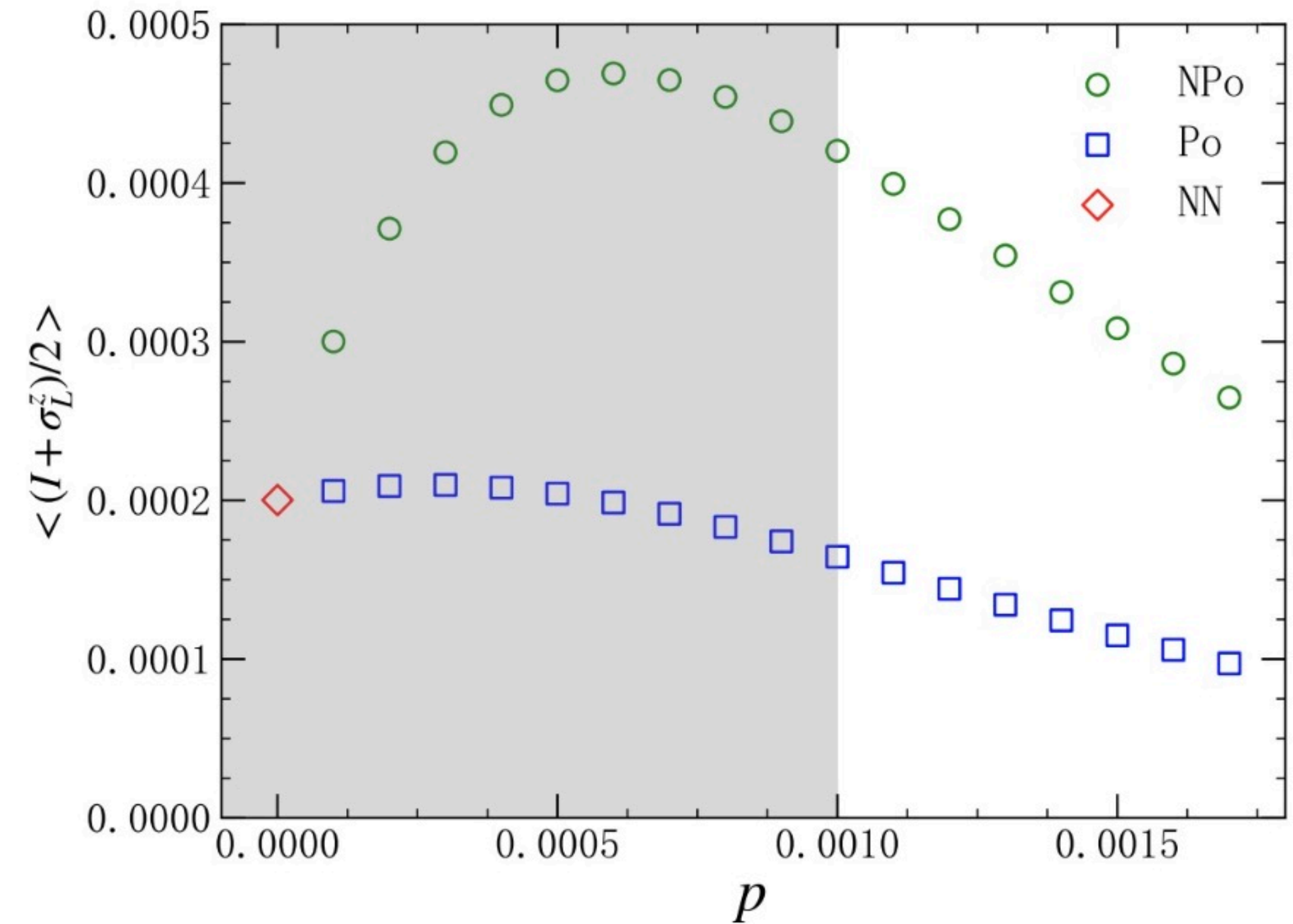
$$O = O(p)$$

- By measuring at different p and choosing a proper extrapolation method, theoretically one can get $O(0)$
- Richardson zero noise extrapolation of λ order:

$$O^\lambda = \sum_{j=0}^{\lambda} \gamma_j O(c_j p)$$

$$\gamma_j = \prod_{m \neq j} c_m (c_j - c_m)^{-1}$$

$$c_j = (1 + 0.1j) \times 10^{-3}, \quad j = 1, 2, \dots, 7$$



Eliminating gauge field

- Finite temperature: the Gibbs state

$$\rho(\beta) = \frac{1}{Z(\beta)} e^{-\beta H}, \quad Z(\beta) = \text{Tr}(e^{-\beta H})$$

$$\rho(\alpha) = \sum_i P_i(\beta) |\varphi_i\rangle\langle\varphi_i|$$

Analytically solvable

$$P_i = \frac{e^{-\beta E_i}}{\left(\sum_n e^{-\beta E_i}\right)}$$

Eigenstate of H
independent of β

- Variational method
 - Parametrization

$$\rho(\alpha) = \sum_i P_i(\beta) U(\alpha) |i\rangle\langle i| U^\dagger(\alpha)$$

Algorithm: Monte-Carlo

- Monte-Carlo in thermal state construction
 - Start from $|i\rangle$ such that $U|i\rangle$ is the ground state.
 - Randomly flip one qubit of $|i\rangle$ to get a new state $|j\rangle$
 - Calculate the energy expectation value $E_j \langle i | U^\dagger H U | i \rangle$
 - If $E_j < E_i$, accept the new state, otherwise, accept it with the probability $e^{-(E_j - E_i)/T}$
 - If the new state is rejected, the old state is added into the mixed state again.
 - Repeat until number of states reaches a predetermined limit.