

天球全息中的带质量振幅

2404.05137, 2503.15174

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Celestial Holography

Flat holography:

- 4D bulk, Minkowski $\Rightarrow$ 2D Celestial CFT (CCFT), celestial sphere
- Massless: Mellin transforms of plane waves.
- Massive: Fourier transforms of bulk-to-boundary propagators.

起源, 2010-2016 年: Barnich, Cachazo, Strominger, Pasterski, Sundrum...

- 渐近对称: 构造保持解不变的变换 $\rightarrow$ 守恒荷 $\rightarrow$ 代数 [*BMS algebra* 1962]
global $SL(2, \mathbb{C})$ 推广到: extended *BMS algebra* $\supset$ Virasoro algebra [0909.2617...]
- 软定理 $\sim$ CCFT Ward identities of *BMS* [1308.0589, 1312.2229...]
- AdS/CFT : Minkowski $\rightarrow$ 维度约化 $AdS_3 \rightarrow CFT_2$ [1609.00732]

偏振幅的讲义: 2310.12922, 2203.13022, 2111.11392, 2107.02075, 1703.05448

例子: Soft graviton theorem

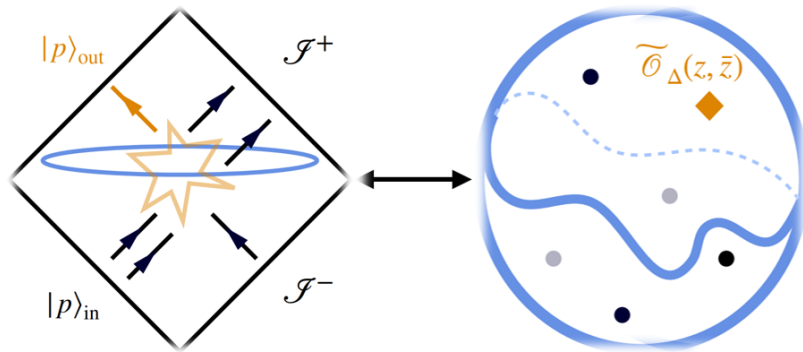
[low 1958] [Weinberg 1965]: universal soft factor $\Rightarrow$ underlying symmetry

$$\mathcal{M}_{\mu\nu}(q, p'_1, \dots, p'_m, p_1, \dots, p_n) \stackrel{q \rightarrow 0}{=} \frac{\kappa}{2} \left[\sum_{k=1}^m \frac{p'_{k\mu} p'_{k\nu}}{p'_k \cdot q} - \sum_{k=1}^n \frac{p_{k\mu} p_{k\nu}}{p_k \cdot q} \right] \mathcal{M}(p'_1, \dots, p'_m, p_1, \dots, p_n) \quad (1)$$

等价于 CCFT 中的 Ward identity, Supertranslation current $P(z, \bar{z})$

$$\begin{aligned} & \left\langle P(z, \bar{z}) \mathcal{O}_{h_1, \bar{h}_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{h_n, \bar{h}_n}(z_n, \bar{z}_n) \right\rangle \\ &= \sum_{k=1}^n \frac{1}{z - z_k} \left\langle \mathcal{O}_{h_1, \bar{h}_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{h_k + \frac{1}{2}, \bar{h}_k + \frac{1}{2}}(z_k, \bar{z}_k) \dots \mathcal{O}_{h_n, \bar{h}_n}(z_n, \bar{z}_n) \right\rangle \end{aligned} \quad (2)$$

Celestial amplitudes [1701.00049,1705.01027]



渐近态 $\rightarrow$ 共形波包; 闵氏时空 $\rightarrow$ foliation $\rightarrow$ AdS/CFT: bulk-boundary propagator
 bulk 中的动量 $\rightarrow$ 天球中的坐标和共形量纲 $\Delta = \frac{d}{2} + i\lambda$, $\lambda \in \mathbb{R}$: putative CCFT

Conformal primary wave function

- massless limit:

$$\Phi_{\Delta}^m(X; \vec{w}) \xrightarrow{m \rightarrow 0} \left(\frac{m}{2}\right)^{-\Delta} \frac{\pi \Gamma(\Delta - 1)}{\Gamma(\Delta)} \phi_{\Delta}(X^{\mu}; \vec{w}) + \left(\frac{m}{2}\right)^{-\tilde{\Delta}} \frac{\pi \Gamma(\tilde{\Delta} - 1)}{\Gamma(\tilde{\Delta})} \widetilde{\phi_{\tilde{\Delta}}}(X^{\mu}; \vec{w}) \quad (3)$$

- shadow 变换 [1205.1941]: 对于 $\phi_{\Delta=h+\bar{h}}(z, \bar{z})$: 共形量纲 $(h, \bar{h})$, 自旋 $J = h - \bar{h}$

$$\tilde{\phi}_{\tilde{\Delta}}(z, \bar{z}) = \widetilde{\phi_{\Delta}(y, \bar{y})} = \frac{(-1)^{2(h-\bar{h})} \Gamma(2-2h)}{(\pi \Gamma(2\bar{h}-1))} \int \frac{d^2 y}{(z-y)^{2-2h} (\bar{z}-\bar{y})^{2-2\bar{h}}} \phi_{\Delta}(y, \bar{y}) \quad (4)$$

有 shadow field $\tilde{\phi}_{\tilde{\Delta}}(z, \bar{z})$: $(1-h, 1-\bar{h}), \tilde{J} = -J, \tilde{\Delta} = 2 - \Delta$.

3pt massive-massless-massless:

• 2312.08597

$$\begin{aligned}
 \langle \Phi_{\Delta_1}^{m_1}(\vec{w}_1) \phi_{\Delta_2}(\vec{w}_2) \phi_{\Delta_3}(\vec{w}_3) \rangle &= i\lambda \int d^4X \Phi_{\Delta_1}^{m_1(-)}(X^\mu; \vec{w}_1) \prod_{i=2}^3 \phi_{\Delta_i}^{(+)}(X^\mu; \vec{w}_i) \\
 &= \frac{i\lambda(2\pi)^4}{16} \left(\frac{m_1}{2}\right)^{\Delta_2+\Delta_3-4} \frac{\Gamma(\frac{\Delta_1+\Delta_2-\Delta_3}{2})\Gamma(\frac{\Delta_1-\Delta_2+\Delta_3}{2})/\Gamma(\Delta_1)}{|\vec{w}_{12}|^{\Delta_1+\Delta_2-\Delta_3} |\vec{w}_{13}|^{\Delta_1+\Delta_3-\Delta_2} |\vec{w}_{23}|^{\Delta_2+\Delta_3-\Delta_1}}.
 \end{aligned} \tag{5}$$

• 我们需要其 Shadowed amplitude:

$$\begin{aligned}
 \langle \Phi_{\Delta_1}^{m_1}(\vec{w}_1) \widetilde{\phi_{\tilde{\Delta}_2}}(\vec{w}_2) \phi_{\Delta_3}(\vec{w}_3) \rangle &= k_{h,\bar{h}} \int \frac{d^2\vec{w}'_2}{|\vec{w}'_2 - \vec{w}_2|^{4-2\tilde{\Delta}_2}} \langle \Phi_{\Delta_1}^{m_1}(\vec{w}_1) \phi_{\tilde{\Delta}_2}(\vec{w}'_2) \phi_{\Delta_3}(\vec{w}_3) \rangle \\
 &= \frac{i\lambda(2\pi)^4 m_1^{\tilde{\Delta}_2+\Delta_3-4}}{2^{\tilde{\Delta}_2+\Delta_3}} \frac{\frac{\Gamma(\frac{\Delta_1+\Delta_2-\Delta_3}{2})\Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2})}{\Gamma(\Delta_1)} \frac{\Gamma(-1+\frac{\Delta_1+\Delta_2+\Delta_3}{2})}{\Gamma(1+\frac{\Delta_3-\Delta_1-\Delta_2}{2})}}{|\vec{w}_{12}|^{\Delta_1+\Delta_2-\Delta_3} |\vec{w}_{13}|^{\Delta_1+\Delta_3-\Delta_2} |\vec{w}_{23}|^{\Delta_2+\Delta_3-\Delta_1}}.
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 \langle \Phi_{\Delta_1}^{m_1}(\vec{w}_1) \Phi_{\Delta_2}^{m_2}(\vec{w}_2) \phi_{\Delta_3}(\vec{w}_3) \rangle &= i\lambda \int d^4X \Phi_{\Delta_1}^{m_1(-)}(X; \vec{w}_1) \Phi_{\Delta_2}^{m_2(+)}(X; \vec{w}_2) \phi_{\Delta_3}^{(+)}(X; \vec{w}_3) \\
 &= i\lambda (2\pi)^4 \int \left(\prod_{i=1}^2 \frac{dy_i d^2z_i}{y_i^3} \left(\frac{y_i}{y_i^2 + |\vec{z}_i - \vec{w}_i|^2} \right)^{\Delta_i} \right) \int \frac{d\omega_3}{\omega_3} \omega_3^{\Delta_3} \delta^4(-m_1 \hat{p}_1 + m_2 \hat{p}_2 + \omega_3 \hat{q}_3) \\
 &= i\lambda (2\pi)^4 \frac{(m_1^2 - m_2^2)^{\Delta_3-1}}{2^{\Delta_3} m_1^{2-\Delta_1} m_2^{\Delta_1+\Delta_3}} \frac{\pi \Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2})}{\Gamma(\Delta_2) \Gamma(\Delta_3) \Gamma(\frac{\Delta_1-\Delta_2-\Delta_3}{2} + 1)} \\
 &\times \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n}{n!} \left(\frac{m_2}{m_1} \right)^{\Delta_1+\Delta_2+\Delta_3-2+2n} \Gamma(1-\Delta_2-n) \Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3}{2} - 1 + n\right) \right. \\
 &\times \left. \Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_3}{2} + n\right) {}_2F_1\left(\Delta_3 - \Delta_1, \frac{\Delta_1+\Delta_2+\Delta_3}{2} - 1 + n; \Delta_3; 1 - \frac{m_2^2}{m_1^2}\right) + (\Delta_2 \rightarrow \tilde{\Delta}_2) \right\} \quad (7)
 \end{aligned}$$

Massless limit of Amp

Leading order

$$\left(\frac{m_2}{2}\right)^{\Delta_2} \frac{\Gamma(\Delta_2)}{\pi\Gamma(\Delta_2-1)} \langle \Phi_{\Delta_1}^{m_1} \Phi_{\Delta_2}^{m_2} \phi_{\Delta_3} \rangle \xrightarrow[m_2 \rightarrow 0]{\Delta_2 \rightarrow 1} \langle \Phi_{\Delta_1}^{m_1} \phi_{\Delta_2=1} \phi_{\Delta_3} \rangle \quad (8)$$

$$\left(\frac{m_2}{2}\right)^{\tilde{\Delta}_2} \frac{\Gamma(\tilde{\Delta}_2)}{\pi\Gamma(\tilde{\Delta}_2-1)} \langle \Phi_{\Delta_1}^{m_1} \Phi_{\Delta_2}^{m_2} \phi_{\Delta_3} \rangle \xrightarrow[m_2 \rightarrow 0]{\tilde{\Delta}_2 \rightarrow 1} \langle \Phi_{\Delta_1}^{m_1} \widetilde{\phi_{\tilde{\Delta}_2=1}} \phi_{\Delta_3} \rangle. \quad (9)$$

Subleading order

$$\left(\frac{m_2}{2}\right)^{\Delta_2} \frac{\Gamma(\Delta_2)}{\pi\Gamma(\Delta_2-1)} \langle \Phi_{\Delta_1}^{m_1} \Phi_{\Delta_2}^{m_2} \phi_{\Delta_3} \rangle \xrightarrow[m_2 \rightarrow 0]{\Delta_2 \rightarrow 1-n} \langle \Phi_{\Delta_1}^{m_1} \phi_{\Delta_2=1-n} \phi_{\Delta_3} \rangle \quad (10)$$

$$\left(\frac{m_2}{2}\right)^{\tilde{\Delta}_2} \frac{\Gamma(\tilde{\Delta}_2)}{\pi\Gamma(\tilde{\Delta}_2-1)} \langle \Phi_{\Delta_1}^{m_1} \Phi_{\Delta_2}^{m_2} \phi_{\Delta_3} \rangle \xrightarrow[m_2 \rightarrow 0]{\tilde{\Delta}_2 \rightarrow 0, \Delta_2 \rightarrow 2} \langle \Phi_{\Delta_1}^{m_1} \widetilde{\phi_{\tilde{\Delta}_2=1-n}} \phi_{\Delta_3} \rangle. \quad (11)$$

Generalized conformal primaries $\Delta \in 2 - \mathbb{Z}_{\geq 0}$ [2402.09256]: analytic constraints $w_{1+\infty}$ algebra [2105.14346]: current algebra of BMS

2d integrable QFT to 0d CCFT

[2209.02776] Sinh-Gordon: no bound state. Rapidity parameter θ : $p^\mu = m (\cosh \theta, \sinh \theta)$

- Exact 2d S-matrix $\rightarrow$ Fourier transform $\rightarrow$ Celestial dual

$$S(\theta) = \frac{\sinh \theta - i \sin \alpha}{\sinh \theta + i \sin \alpha} = 1 + b^2 S^{(1)}(\theta) + b^4 S^{(2)}(\theta) + \dots, \quad \alpha = \frac{\pi b^2}{8\pi + b^2}$$
$$\mathcal{A}(\omega) = \int_{-\infty}^{\infty} d\theta e^{i\omega\theta} S(\theta) = 2\pi \left(\delta(\omega) + b^2 f_1(\omega) + b^4 f_2(\omega) + \dots \right). \quad (12)$$

- Unitarity and crossing symmetry $\rightarrow$ Celestial bootstrap

$$S(\theta) = S(i\pi - \theta), \quad |S(\theta)|^2 = S(\theta)S(-\theta) = 1, \quad 0 < \text{Im } \theta < \pi. \quad (13)$$

$$f_n^+(\omega) = \frac{1}{(1 + e^{-\pi\omega})} \left[-e^{-\pi\omega} i \text{Res} \left[e^{i\omega\theta} S^{(n)}(\theta) \right]_{\theta=0} - \sum_{j=1}^{n-1} \int_{-\infty}^{\infty} d\omega' f_{n-j}^+(\omega + \omega') f_j^-(\omega') \right]. \quad (14)$$

Bullough-Dodd model: particle as the bound state itself

[Arinshtein, Fateev, Zamolodchikov 1979]

$$S(\theta) = \frac{\tanh \frac{1}{2}(\theta + \frac{2i\pi}{3}) \tanh \frac{1}{2}(\theta - \frac{2i\pi}{3bQ}) \tanh \frac{1}{2}(\theta - \frac{2i\pi b}{3Q})}{\tanh \frac{1}{2}(\theta - \frac{2i\pi}{3}) \tanh \frac{1}{2}(\theta + \frac{2i\pi}{3Qb}) \tanh \frac{1}{2}(\theta + \frac{2i\pi b}{3Q})}, \quad Q = b^{-1} + b. \quad (15)$$

Linear space of dimension-seven:

$$f_n^+(\omega) = \frac{1}{e^{2\pi\omega} - 1} \sum_{k=0}^5 P_{k;n-1}^+(\omega) e^{\frac{k\pi\omega}{3}}, \quad f_n^-(\omega) = \frac{1}{e^{2\pi\omega} - 1} \sum_{k=1}^6 P_{k;n-1}^-(\omega) e^{\frac{k\pi\omega}{3}}, \quad (16)$$

$$f_n^+(\omega) = \frac{1}{(1 + e^{-\pi\omega})} \left[-e^{-\pi\omega} i \operatorname{Res} \left[e^{i\omega\theta} S^{(n)}(\theta) \right]_{\theta=0} \right. \\ \left. + i \sum_{k=1}^2 \operatorname{Res} \left[e^{-i\omega\theta} S^{(n)}(\theta) \right]_{\theta=-i\frac{k\pi\omega}{3}} - \sum_{j=1}^{n-1} \int_{-\infty}^{\infty} d\omega' f_{n-j}^+(\omega + \omega') f_j^-(\omega') \right]. \quad (17)$$

Convolution integral: 11 kinds of integrals

$$\int d\omega' f_{n-j}^+(\omega + \omega') f_j^-(\omega') \propto \sum_{q=0}^5 \sum_{k=1}^{11} e^{\frac{q\pi\omega}{3}} \int d\omega' \frac{P_{q;n-2}(\omega') e^{\frac{k\pi\omega'}{3}}}{(-1 + e^{2\pi(\omega+\omega')})(-1 + e^{2\pi\omega'})} \quad (18)$$

$$I(\omega, k, \alpha) = \int_{-\infty}^{\infty} d\omega' \frac{e^{\frac{k\alpha\pi}{3}\omega'}}{(-1 + e^{2\pi(\omega+\omega')})(-1 + e^{2\pi\omega'})}, \quad k = 1, 2, \dots, 11,$$
$$I^{[m]}(\omega, k, \alpha) = \left(\frac{3}{k\pi}\right)^m \frac{\partial^m I(\omega, k, \alpha)}{\partial^m \alpha} = \int_{-\infty}^{\infty} d\omega' \frac{\omega'^m e^{\frac{k\alpha\pi}{3}\omega'}}{(-1 + e^{2\pi(\omega+\omega')})(-1 + e^{2\pi\omega'})}, \quad (19)$$

Hamonic number:

$$\begin{aligned} I(\omega, k, \alpha) &= \frac{1}{2\pi} \frac{-1 + (e^{-2\pi\omega})^{k\alpha/6-1}}{-1 + e^{2\pi\omega}} (H_{-k\alpha/6} - H_{k\alpha/6-1}) \\ &= \frac{1}{2} \frac{-1 + (e^{-2\pi\omega})^{k\alpha/6-1}}{-1 + e^{2\pi\omega}} \cot\left(\frac{k\alpha\pi}{6}\right). \end{aligned} \quad (20)$$

Workable bootstrap

Require: $\{S^{(1)}(\theta), S^{(2)}(\theta), \dots, S^{(N)}(\theta)\}, f_1^+(\omega), f_1^-(\omega)$

1: $n \leftarrow 2$

2: **while** $n \leq N$ **do**

3: compute residue: $\text{Res} [e^{i\omega\theta} S^{(n)}(\theta)]_{\theta=0}, \text{Res} [e^{-i\omega\theta} S^{(n)}(\theta)]_{\theta=-i\frac{\pi\omega}{3}, -i\frac{2\pi\omega}{3}}$

4: compute $\sum_{j=1}^{n-1} \int_{-\infty}^{\infty} d\omega' f_{n-j}^+(\omega + \omega') f_j^-(\omega')$ using $l^{[m]}(\omega, k, \alpha) = (\frac{3}{k\pi})^m \frac{\partial^m l(\omega, k, \alpha)}{\partial^m \alpha}$

5: get $f_n^+(\omega), f_n^-(\omega)$ from the bootstrap formula

6: $n \leftarrow n + 1$

7: **end while**

- 基本代数运算：比直接上（下）半平面留数的无穷级数求和快。
- Revisit Sinh-Gordon model: Bullough-Dodd model 的由 $k = 0, 3, 6$ 组成的子空间。

$$f_n^+(\omega) = \frac{1}{e^{2\pi\omega} - 1} \sum_{k=0}^1 P_{k;n-1}^+(\omega) e^{k\pi\omega}, \quad f_n^-(\omega) = \frac{1}{e^{2\pi\omega} - 1} \sum_{k=1}^2 P_{k;n-1}^-(\omega) e^{k\pi\omega}, \quad (21)$$

谢谢!