Footprints of vector-like quark models

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  - UV completion
  - Higgs physics

1. Introduction of vector-like quark

What is vector-like quark (VLQ)?

standard model (SM) gauge group $SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$

fermion representation under $SU_L(2)\otimes U_Y(1)$

~~chiral fermion~~ ← Higgs constraints

vector-like fermion → quantum anomaly free; mass term

representation under $SU_C(3)$

mix with SM fermion via renormalizable interactions

vector-like lepton

Leptons	N	E	$\begin{pmatrix} N \\ E^- \end{pmatrix}$	$\begin{pmatrix} E^- \\ E^{--} \end{pmatrix}$	$\begin{pmatrix} E^+ \\ N \\ E^- \end{pmatrix}$	$\begin{pmatrix} N \\ E^- \\ E^{--} \end{pmatrix}$
Notation			Δ_1	Δ_3	Σ_0	Σ_1
$SU(2)_L \otimes U(1)_Y$	1_0	1_{-1}	$2_{-\frac{1}{2}}$	$2_{-\frac{3}{2}}$	3_0	3_{-1}
Spinor	Dirac or Majorana	Dirac	Dirac	Dirac	Dirac or Majorana	Dirac

F. del. Aguilar *et al* (2008)

VLQ

Multiplet	U	D	$\begin{pmatrix} U \\ D \end{pmatrix}$	$\begin{pmatrix} X \\ U \end{pmatrix}$	$\begin{pmatrix} D \\ Y \end{pmatrix}$	$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$
$SU(2)_L$	**1**	**1**	**2**	**2**	**2**	**3**	**3**
$U(1)_Y$	$2/3$	$-1/3$	$1/6$	$7/6$	$-5/6$	$2/3$	$-1/3$

J. M. Alves *et al* (2023)

Why we need VLQ?

G. C. Branco, L. Lavoura, and J. P. Silva, CP Violation, Int. Ser. Monogr. Phys. 103, 1-536 (1999). Chap 24 "MODELS WITH VECTOR-LIKE QUARKS"

Theoretical aspect

- grand unified theory

$E_6: 27 = (3,2)_{1/6} + (\bar{3},1)_{-2/3} + (\bar{3},1)_{1/3} + (1,2)_{-1/2} + (1,1)_1$

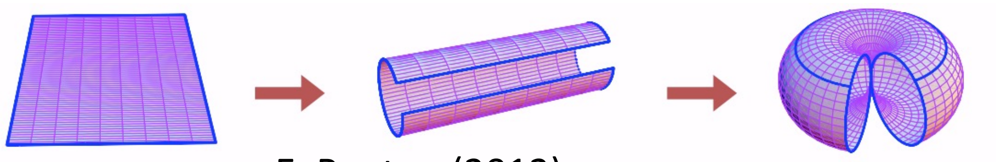
$+(3,1)_{-1/3} + (\bar{3},1)_{1/3} + (1,2)_{1/2} + (1,2)_{-1/2} + (1,1)_0 + (1,1)_0$

- composite Higgs model

simplest little Higgs model $SU_C(3)\otimes SU_L(3)\otimes U_Y(1)$

$Q_3 = (t_L, b_L, iT_L), t_R, b_R, T_R$ M. Schmaltz (2004)

- extra dimension



E. Ponton (2012)

Figure 1: Torus compactification: the identification of opposite sides leads to a smooth manifold. Fermions propagating on the torus lead to a vector-like spectrum at low energies.

- ...

Phenomenological aspect

- CP violation
- CKM unitarity
- B physics anomalies
- Muon g-2 anomaly
- ...

label	X	T (also U)	B (also D)	Y
electric charge	$\frac{5}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$

Theoretical status

- VLQ models: minimal and non-minimal
- vacuum stability

VLQ A. Arsenault, K. Y. Cingiloglu and M. Frank (2023)

HSM+VLQ M. L. Xiao and J. H. Yu (2014)

2HDM+VLQ K. Y. Cingiloglu and M. Frank (2024)

- effective field theory

matching dim-6 SMEFT and VLQ models C. Y. Chen, S. Dawson and E. Furlan (2017)

matching dim-8 ZZZ operators and VLQ models J. Ellis, H. J. He, R. Q. Xiao, S. P. Zeng and J. Zheng (2025)

VLQ representations from dim-5 operators J. C. Criado and M. Perez-Victoria (2020)

J. J. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer, and M. P. Victoria (2013)

Phenomenological and experimental status

- direct search — ATLAS, CMS: $T \to bW, tZ, th$ → 1.5TeV (minimal VLQ models)
  R. Benbrik, M. Boukidi, M. Ech-chaouy, S. Moretti, K. Salime, and Q. S. Yan (2024)

pair:

$T\bar{T}$
- X. Qin and C. Wang (2023). CLIC
- G. S. Lv, X. M. Cui, Y. Q. Li and Y. B. Liu (2022). muon collider

$B\bar{B}$ J. Z. Han, Y. B. Liu, and S. Y. Xu (2024). CLIC

TT X. M. Cui, Y. Q. Li, and Y. B. Liu (2022). HL-LHC

single:

T
- H. Li, J. Chao, and G. Zhang (2023)
- B. Yang, S. Wang, X. Sima, and L. Shang (2023). CLIC
- L. Han, L. F. Du and Y. B. Liu (2022). CLIC
- B. Yang, X. Sima, S. Wang and L. Shang (2022). pp
- X. Qin, L. F. Du and J. F. Shen (2022). CLIC
- X. Y. Tian, L. F. Du and Y. B. Liu (2021). pp
- B. Yang, M. Wang, H. Bi and L. Shang (2021). pp

X
- Y. J. Zhang, Y. T. Zhu, L. Han, Y. P. Bi, and T. G. Liu (2024). FCC-eh
- Y. J. Zhang, J. L. Chang, and T. G. Liu (2024). CLIC
- Y. B. Liu, B. Hu, and C. Z. Li (2024). LHC
- J. Z. Han, S. Xu, W. J. Mao, and H. Q. Song (2023). HL-LHC
- L. Shang and K. Sun (2023). pp

B
- B. Yang, Z. Li, X. Jia, S. Moretti, and L. Shang (2024). LHC
- Q. G. Zeng, Y. S. Pan, and J. Zhang (2023). LHeC
- J. Z. Han, S. Xu, H. Q. Song and Y. J. Wang (2022). CLIC
- L. Shang, C. Chen, S. Wang and B. Yang (2022). ep
- J. Z. Han, Y. B. Liu, L. Xing and S. Xu (2022). LHC
- L. Han, J. F. Shen and Y. B. Liu (2022). CLIC
- J. Z. Han, J. Yang, S. Xu and H. K. Wang (2022). HL-LHC
- J. Z. Han, J. Yang, S. Xu and H. K. Wang (2022). CLIC
- X. Gong, C. X. Yue, H. M. Yu, and D. Li (2020). LHeC
- L. Han and J. F. Shen (2021). CLIC
- X. Qin and J. F. Shen (2021). CLIC

Y
- L. Shang, Y. Yan, S. Moretti, and B. Yang (2024). pp
- V. Cetinkaya, A. Ozansoy, V. Ari, O. M. Ozsimsek and O. Cakir (2021). HL-LHC

exotic:

$T \to bW^+ \to b\ell^+\ell^+qq'$ H. Zhou and N. Liu (2020).

$T \to ta(a \to \gamma\gamma)$ D. Wang, L. Wu, and M. Zhang (2021).

$T \to bH^+, tA, tH$ A. Arhrib, R. Benbrik, M. Boukidi, B. Manaut, and S. Moretti (2025).

leptoquark $\to T\mu$ S. P. He (2022).

$\bar{t}\sigma^{\mu\nu}G_{\mu\nu}T$ A. Belyaev, R. S. Chivukula, B. Fuks, E. H. Simmons and X. Wang (2021)

Higgs physics

F. Nortier, G. Rigo and P. Sesma (2025)
S. P. He (2021)
S. P. He (2020)
K. Kumar, R. Vega-Morales and F. Yu (2012)

electroweak precision observables

H. Abouabid, A. Arhrib, R. Benbrik, M. Boukidi and J. E. Falaki (2024)
F. Albergaria, L. Lavoura and J. C. Romao (2023)
S. P. He (2023)
H. Cai and G. Cacciapaglia (2022)
J. Cao, L. Meng, L. Shang, S. Wang, and B. Yang (2022)

top physics $Zt\bar{t}, Wtb, ht\bar{t}$

CMS (2014) $V_{tb} \geq 0.92$ → $s_L = \sqrt{1 - V_{tb}^2} \leq 0.4$

flavour physics

J. M. Alves, G. C. Branco, A. L. Cherchiglia, C. C. Nishi, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo, and J. I. Silva-Marcos (2024)

CP violation

F. Albergaria, G. C. Branco, J. F. Bastos and J. I. Silva-Marcos (2023)
A. L. Cherchiglia and C. C. Nishi (2020)

2. Our studies in vector-like quark models

- Higgs physics
  - Shi-Ping He, Di-Higgs production as a probe of flavor changing neutral Yukawa couplings, Chin. Phys. C 45, 073108 (2021), arXiv:2011.11949.
  - Shi-Ping He, Higgs boson to γZ decay as a probe of flavor-changing neutral Yukawa couplings, Phys. Rev. D 102, 075035 (2020), arXiv:2004.12155.

- STU parameters → Shi-Ping He, Leptoquark and vector-like quark extended model for simultaneous explanation of W boson mass and muon g-2 anomalies, Chin. Phys. C 47 (2023), 043102, arXiv:2205.02088.

- muon g-2
  - Shi-Ping He, Complete one-loop analytic and expansion formulae for the muon magnetic dipole moment, arXiv:2308.07133.
  - Shi-Ping He, Scalar leptoquark and vector-like quark extended models as the explanation of the muon g-2 anomaly: bottom partner chiral enhancement case, Chin. Phys. C 47, 073101 (2023), arXiv:2211.08062.
  - Shi-Ping He, Leptoquark and vectorlike quark extended models as the explanation of the muon g-2 anomaly, Phys. Rev. D 105 (2022), 035017, [Phys. Rev. D 106 (2022), 039901 (erratum)], arXiv:2112.13490.

Shi-Ping He, Higgs boson to γZ decay as a probe of flavor-changing neutral Yukawa couplings, Phys. Rev. D 102, 075035 (2020), arXiv:2004.12155.

minimal VLQ model with a pair of singlet $T_{L,R}$

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_T^{Yukawa} + \mathcal{L}_T^{gauge}$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \frac{v + h + i\chi}{\sqrt{2}} \end{pmatrix}, \tilde{\Phi} = i\sigma_2 \Phi^*$$

$$\mathcal{L}_T^{Yukawa} = -\Gamma_T^i \bar{Q}_L^i \tilde{\Phi} T_R - M_T \bar{T}_L T_R + h.c.$$

$$\mathcal{L}_T^{gauge} = \bar{T}_L i\, D_\mu \gamma^\mu T_L + \bar{T}_R i\, D_\mu \gamma^\mu T_R, D_\mu \equiv \partial_\mu - i g' Y_T B_\mu$$

gauge eigenstates

$$\mathcal{L}_{mass} \to (\bar{t}_L \quad \bar{T}_L) \begin{pmatrix} \Gamma_t^{33}/\sqrt{2}v & \Gamma_T^3/\sqrt{2} \\ 0 & M_T \end{pmatrix} \begin{pmatrix} t_R \\ T_R \end{pmatrix} + h.c.$$

$$\theta_L \sim \frac{\Gamma_T^3 v}{\sqrt{2} M_T}$$

$$\begin{pmatrix} t_L \\ T_L \end{pmatrix} \to \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \begin{pmatrix} t_L \\ T_L \end{pmatrix}, \begin{pmatrix} t_R \\ T_R \end{pmatrix} \to \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} t_R \\ T_R \end{pmatrix}$$

$$\left(\tan\theta_R = \frac{m_t}{m_T} \tan\theta_L, \qquad M_T^2 = m_T^2 c_L^2 + m_t^2 s_L^2 \right)$$

free parameters

m_T, θ_L

$$\mathcal{L}_{Yukawa} \supseteq -m_t \bar{t}t - m_T \bar{T}T - \frac{m_t}{v} c_L^2 h \bar{t}t - \frac{m_T}{v} s_L^2 h \bar{T}T$$
$$- \frac{m_t}{v} s_L c_L h(\bar{T}_L t_R + \bar{t}_R T_L) - \frac{m_T}{v} s_L c_L h(\bar{t}_L T_R + \bar{T}_R t_L)$$

mass eigenstates

the simplified model based on singlet $T_{L,R}$

$$\mathcal{L} \supseteq -eA_\mu \sum_{f=t,T} Q_f \bar{f}\gamma^\mu f + eZ_\mu[\bar{t}\gamma^\mu(g_L^t\omega_- + g_R^t\omega_+)t$$
$$+\bar{T}\gamma^\mu(g_L^T\omega_- + g_R^T\omega_+)T + \bar{t}\gamma^\mu\left(g_L^{tT}\omega_- + g_R^{tT}\omega_+\right)T + \bar{T}\gamma^\mu\left(g_L^{tT}\omega_- + g_R^{tT}\omega_+\right)t]$$
$$+\frac{gc_L}{\sqrt{2}}\left(W_\mu^+\bar{t}_L\gamma^\mu b_L + W_\mu^-\bar{b}_L\gamma^\mu t_L\right) + \frac{gs_L}{\sqrt{2}}\left(W_\mu^+\bar{T}_L\gamma^\mu b_L + W_\mu^-\bar{b}_L\gamma^\mu T_L\right)$$
$$-m_t\bar{t}t - m_T\bar{T}T - \frac{m_t}{v}h\bar{t}(\kappa_t + i\gamma^5\tilde{\kappa}_t)t + h\bar{T}(y_T + i\gamma^5\tilde{y}_T)T$$
$$+h\bar{t}\left(y_L^{tT}\omega_- + y_R^{tT}\omega_+\right)T + h\bar{T}\left((y_R^{tT})^*\omega_- + (y_L^{tT})^*\omega_+\right)t$$

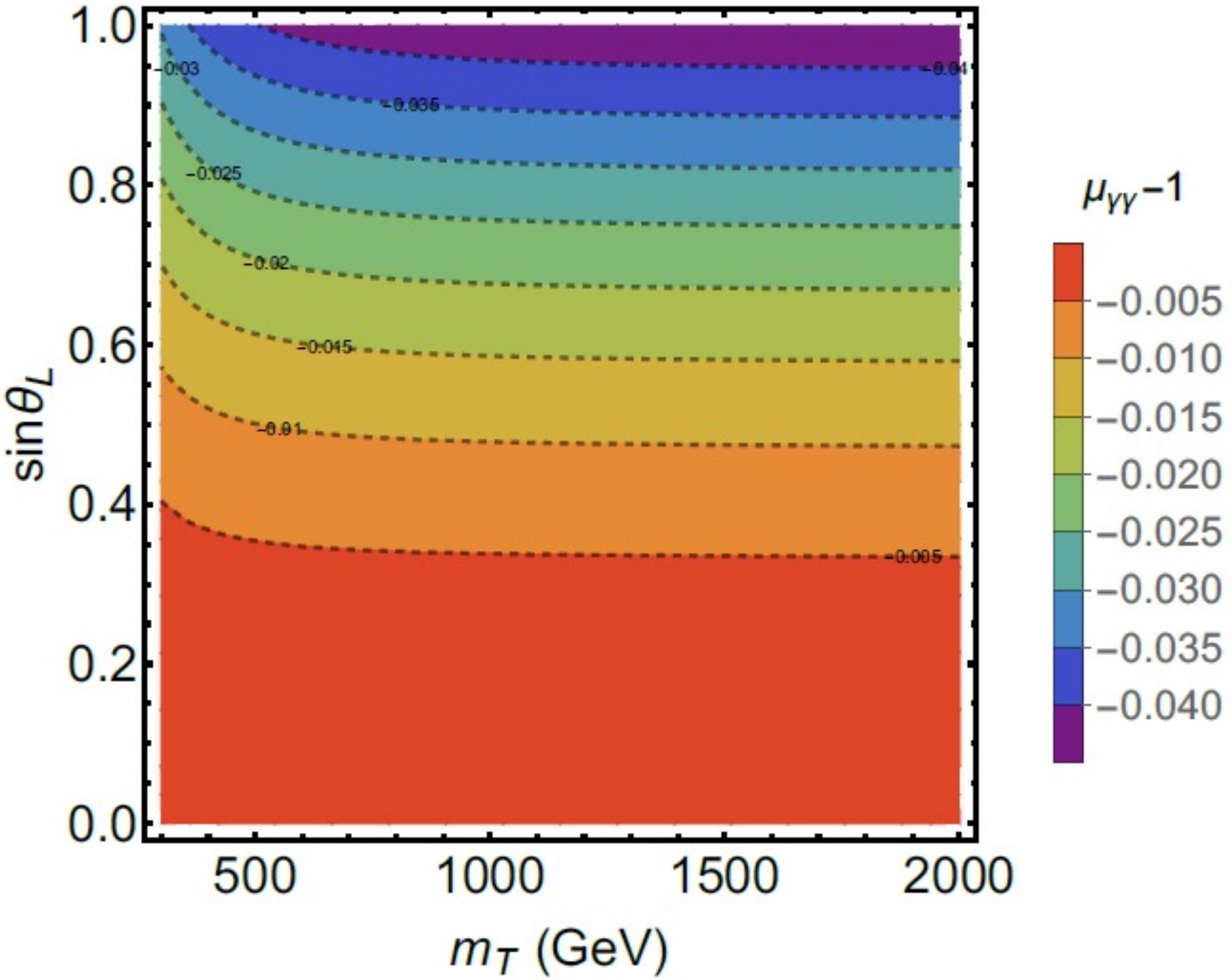
$$g_L^t = \frac{1}{s_W c_W}\left(\frac{1}{2}c_L^2 - \frac{2}{3}s_W^2\right),$$
$$g_L^T = \frac{1}{s_W c_W}\left(\frac{1}{2}s_L^2 - \frac{2}{3}s_W^2\right),$$
$$g_R^t = -\frac{2s_W}{3c_W}, \qquad g_R^T = -\frac{2s_W}{3c_W},$$
$$g_L^{tT} = \frac{s_L c_L}{2s_W c_W}, \qquad g_R^{tT} = 0$$

flavor-changing neutral Yukawa (FCNY)

free parameters:
- real parameters → m_T, θ_L; κ_t, y_T; $\tilde{\kappa}_t, \tilde{y}_T$
- complex parameters → y_L^{tT}, y_R^{tT}

$$\kappa_t = c_L^2, \qquad y_T = -\frac{m_T}{v}s_L^2$$
$$\tilde{\kappa}_t = 0, \qquad \tilde{y}_T = 0$$

constraints from Higgs signal strength in $gg \to h \to \gamma\gamma$



$$\mu_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)\Gamma(h \to \gamma\gamma)}{\sigma^{SM}(gg \to h)\Gamma^{SM}(h \to \gamma\gamma)}$$

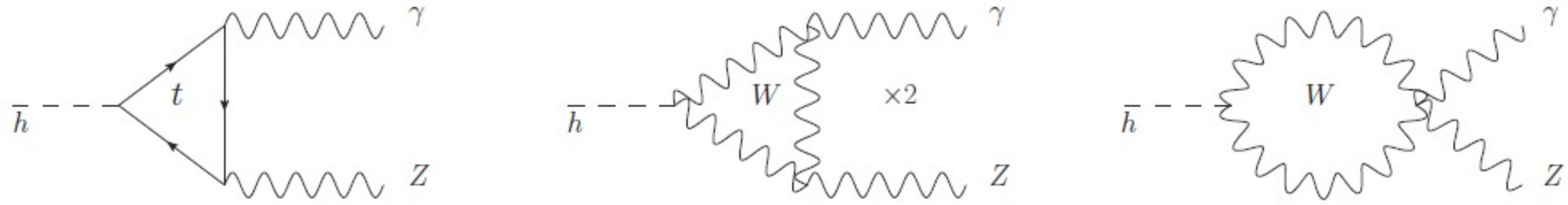
$$\mu_{\gamma\gamma} - 1 \sim -\theta_L^2 \cdot \frac{m_h^2}{4m_t^2}$$

suppression by mixing angle and heavy quark mass

loose m_T & s_L

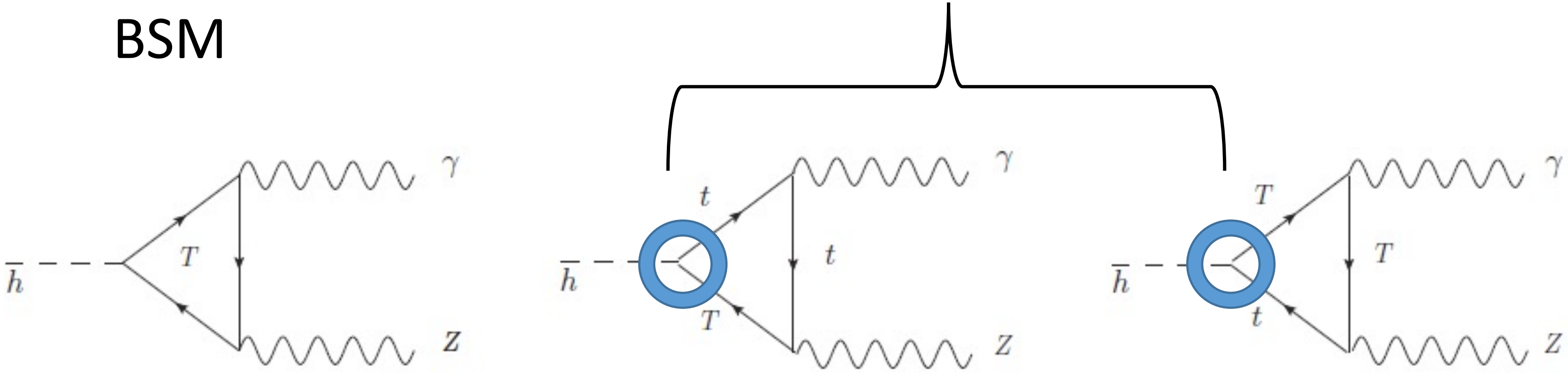
constraints from $h \to \gamma Z$ decay

SM



BSM

FCNY interaction htT



the amplitude

$$i\mathcal{M} = i\varepsilon_\mu(p_1)\varepsilon_\nu(p_2)\left[\left(p_2^\mu p_1^\nu - p_1\cdot p_2 g^{\mu\nu}\right)\mathcal{A} + \varepsilon^{\mu\nu p_1 p_2}\mathcal{B}\right] \;(\varepsilon^{\mu\nu p_1 p_2} \equiv \varepsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma})$$

$$\mathcal{A} \equiv \frac{e^2}{8\pi^2 v}(\mathcal{A}_W + \mathcal{A}_t + \mathcal{A}_T + \mathcal{A}_{tT}), \qquad \mathcal{B} \equiv \frac{e^2}{8\pi^2 v}\mathcal{B}_{tT}$$

the partial decay width

$$\Gamma(h\to\gamma Z) = \frac{G_F \alpha^2 m_h^3}{64\sqrt{2}\pi^3}\left(1-\frac{m_Z^2}{m_h^2}\right)^3\left[|\mathcal{A}_W + \mathcal{A}_t + \mathcal{A}_T + \mathcal{A}_{tT}|^2 + |\mathcal{B}_{tT}|^2\right]$$

BSM $\xrightarrow{\kappa_t = 1,\, \tilde{\kappa}_t = 0,\, y_T = \tilde{y}_T = 0,\, y_{L,R}^{tT} = 0,\, s_L = 0}$ SM

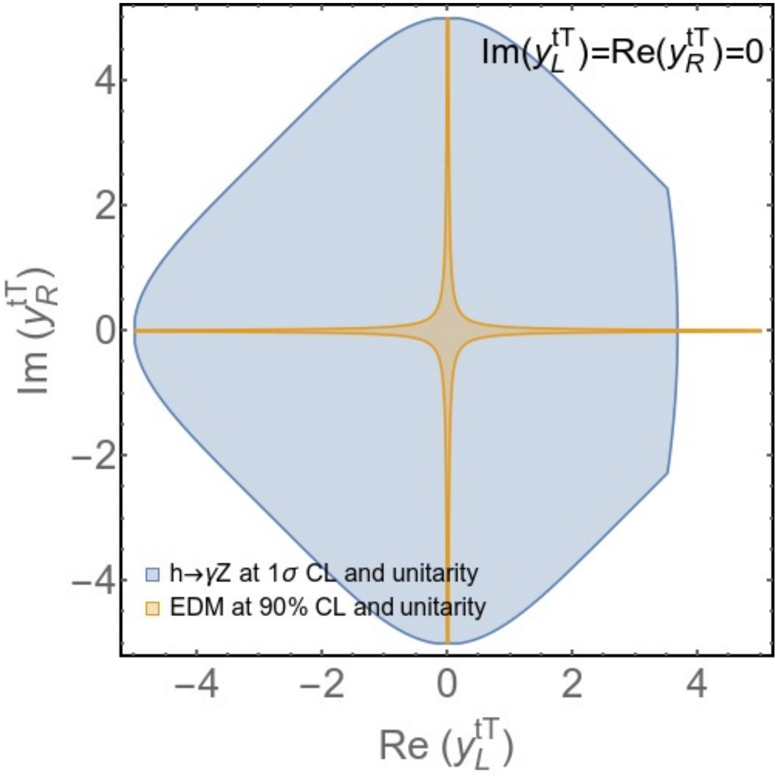
HL-LHC uncertainty: 19.1% at 1σ CL

M. Cepeda *et al*, CERN Yellow Rep. Monogr. 7 (2019) 221-584

$$\left|\frac{\Gamma(h\to\gamma Z)}{\Gamma^{SM}(h\to\gamma Z)} - 1\right| \leq 19.1\% \quad \Rightarrow \quad \left|\frac{|\mathcal{A}|^2 + |\mathcal{B}|^2}{|\mathcal{A}^{SM}|^2} - 1\right| \leq 19.1\%$$

constraints combined with top EDM

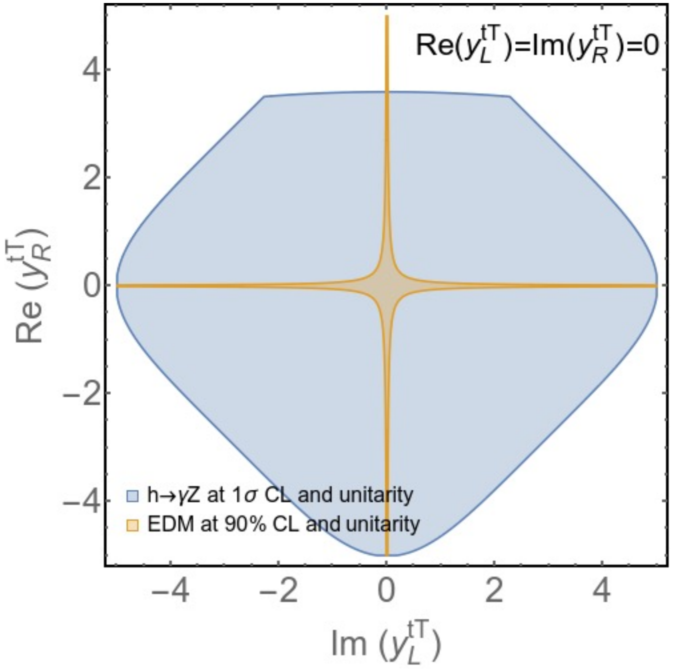
$m_T = 400\text{GeV}, s_L = 0.2$ fixed

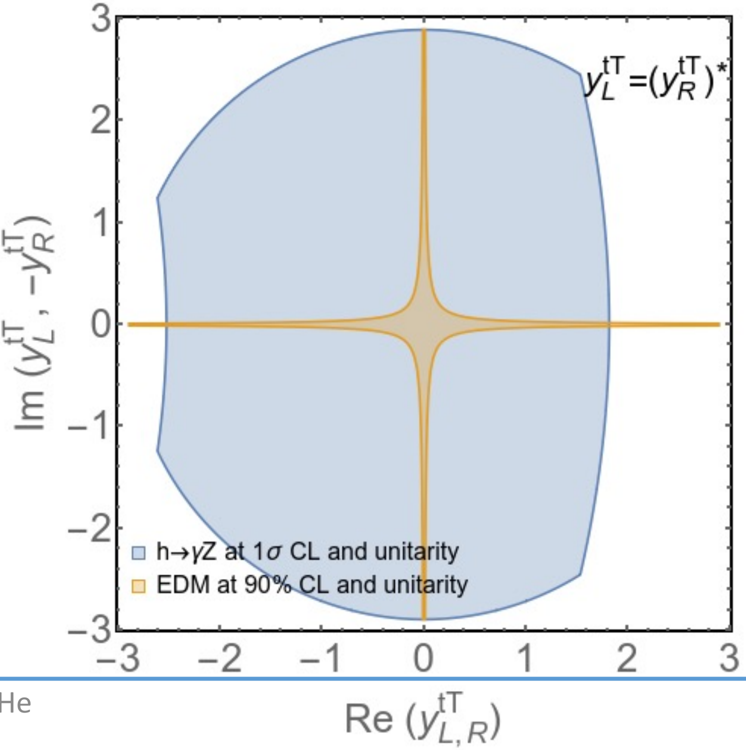


$$\mathcal{A}_{tT} \sim [m_T \mathrm{Re}(y_R^{tT}) - (3 + 2\ln r_{tT}^2) m_t \mathrm{Re}(y_L^{tT})]$$

$$\mathcal{B}_{tT} \sim [(1 + \ln r_{tT}^2) m_t \mathrm{Im}(y_L^{tT}) + m_T \mathrm{Im}(y_R^{tT})]$$

$$r_{tT} \equiv \frac{m_t}{m_T}$$





$$d_t^{EDM} \sim [y_R^{tT}(y_L^{tT})^* - y_L^{tT}(y_R^{tT})^*]$$

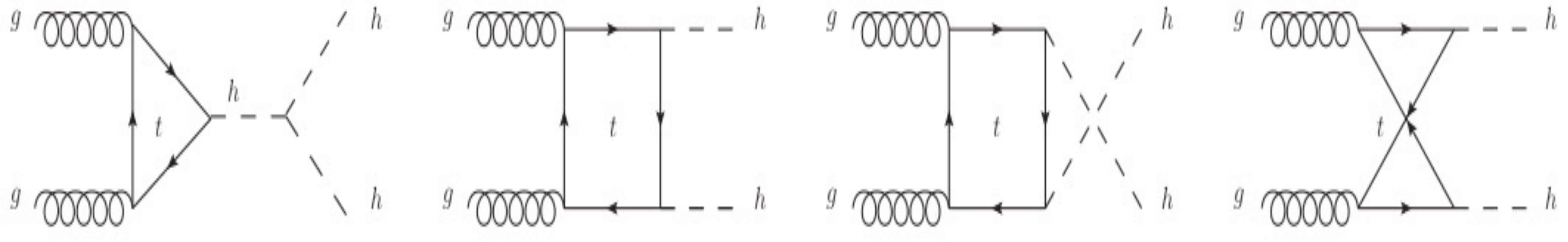
$$= 2i[\mathrm{Re}(y_L^{tT})\mathrm{Im}(y_R^{tT}) - \mathrm{Re}(y_R^{tT})\mathrm{Im}(y_L^{tT})]$$

constraints from di-Higgs production

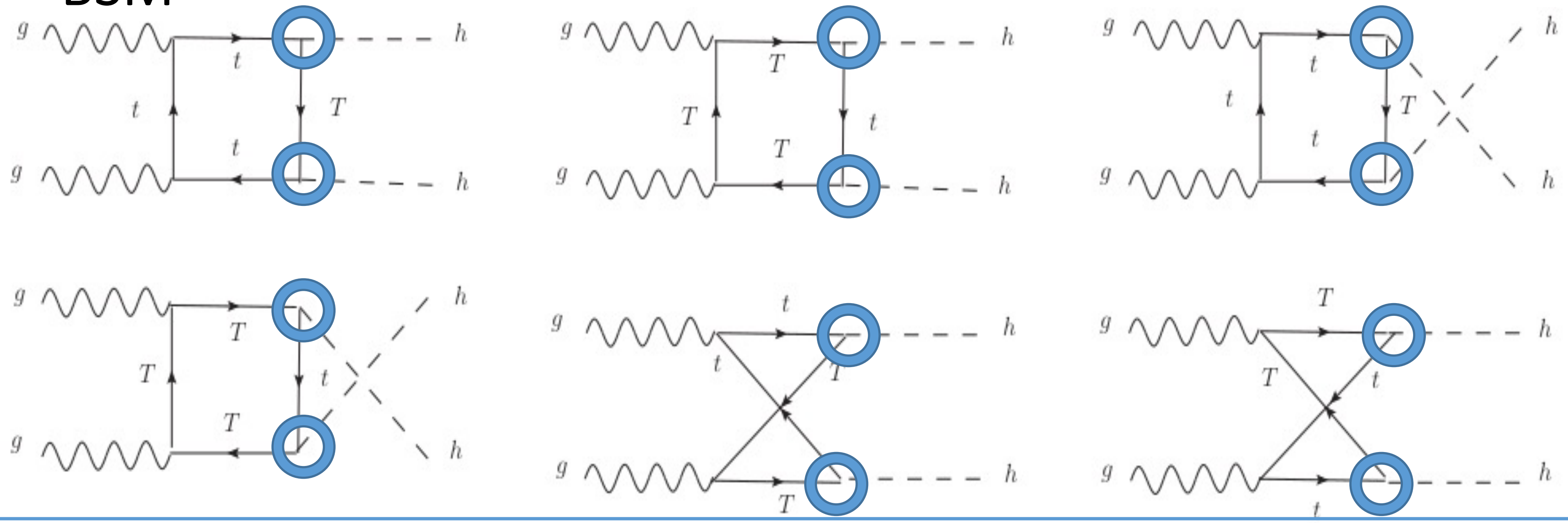
Shi-Ping He, Di-Higgs production as a probe of flavor changing neutral Yukawa couplings, Chin. Phys. C 45, 073108 (2021), arXiv:2011.11949.

$$\frac{\lambda_{hhh}}{\lambda_{hhh}^{SM}} \equiv 1 + \delta_{hhh}$$

SM



BSM



the amplitude of $gg \to hh$

$$f_{A,B,C} \equiv f^{t}_{A,B,C} + f^{T}_{A,B,C} + f^{tT}_{A,B,C}$$

$$i\mathcal{M} = -i\frac{g_s^2 \hat{s}}{16\pi^2 v^2}\varepsilon_\mu^{a,r1}(k_1)\varepsilon_\nu^{a,r2}(k_2)[\mathcal{A}^{\mu\nu} f_A + \mathcal{B}^{\mu\nu} f_B + \mathcal{C}^{\mu\nu} f_C]$$

$$\tilde{\kappa}_t = 0, \qquad \tilde{y}_T = 0$$

$$\mathcal{A}^{\mu\nu} \equiv g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2}, \qquad \mathcal{C}^{\mu\nu} \equiv \frac{\varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}}{k_1 \cdot k_2},$$

$$\mathcal{B}^{\mu\nu} \equiv g^{\mu\nu} + \frac{m_h^2 k_2^\mu k_1^\nu}{p_T^2 (k_1 \cdot k_2)} - \frac{2(k_1 \cdot p_1) k_2^\mu p_1^\nu}{p_T^2 (k_1 \cdot k_2)} - \frac{2(k_2 \cdot p_1) p_1^\mu k_1^\nu}{p_T^2 (k_1 \cdot k_2)} + \frac{2 p_1^\mu p_1^\nu}{p_T^2}, p_T^2 \equiv \frac{\hat{t}\,\hat{u} - m_h^4}{\hat{s}}$$

f_A

$$f_A^t = \kappa_t f_A^{t,\triangle} + \kappa_t^2 f_A^{t,\boxdot 1}, f_A^T = \left(-\frac{v y_T}{m_T}\right) f_A^{T,\triangle} + \left(\frac{v y_T}{m_T}\right)^2 f_A^{T,\boxdot 1},$$

$$f_A^{tT} = \left(\left|y_L^{tT}\right|^2 + \left|y_R^{tT}\right|^2\right) f_A^{tT,\boxdot 1} + [y_L^{tT}(y_R^{tT})^* + y_R^{tT}(y_L^{tT})^*] f_A^{tT,\boxdot 2}$$

f_B

$$f_B^t = \kappa_t^2 f_B^{t,\boxdot 1}, f_B^T = \left(\frac{v y_T}{m_T}\right)^2 f_B^{T,\boxdot 1},$$

$$f_B^{tT} = \left(\left|y_L^{tT}\right|^2 + \left|y_R^{tT}\right|^2\right) f_B^{tT,\boxdot 1} + [y_L^{tT}(y_R^{tT})^* + y_R^{tT}(y_L^{tT})^*] f_B^{tT,\boxdot 2}$$

$f_C \longrightarrow$

$$f_C^t = 0, f_C^T = 0, f_C^{tT} = -i[y_L^{tT}(y_R^{tT})^* - y_R^{tT}(y_L^{tT})^*] f_C^{tT,\boxdot}$$

estimation and analysis

μ_{hh} can be parametrized as

$$\mu_{hh} = 1 + A_1 + A_0^{hhh}\delta_{hhh} + A_1^{hhh}\delta_{hhh}^2 + (A_2 + A_2^{hhh}\delta_{hhh})(|y_L^{tT}|^2 + |y_R^{tT}|^2)$$
$$+ (A_3 + A_3^{hhh}\delta_{hhh})[y_L^{tT}(y_R^{tT})^* + y_R^{tT}(y_L^{tT})^*] + A_4(|y_L^{tT}|^2 + |y_R^{tT}|^2)^2 + A_5[y_L^{tT}(y_R^{tT})^* + y_R^{tT}(y_L^{tT})^*]^2$$
$$+ A_6(|y_L^{tT}|^2 + |y_R^{tT}|^2)[y_L^{tT}(y_R^{tT})^* + y_R^{tT}(y_L^{tT})^*] - A_7[y_L^{tT}(y_R^{tT})^* - y_R^{tT}(y_L^{tT})^*]^2$$

11 cross section values from **MadGraph**

parameters need to be fixed

$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_0^{hhh}, A_1^{hhh}, A_2^{hhh}, A_3^{hhh}$

$\sqrt{s}$ (TeV)	$(m_T/\text{GeV}, s_L)$	A_1	A_2	A_3	A_4	A_5	A_6	A_7
14	(400, 0.2)	−0.1919	0.3717	1.672	0.07449	1.114	0.3166	3.071
	(800, 0.1)	−0.04939	0.1279	1.087	0.01943	0.378	0.0711	0.9907
$\sqrt{s}$ (TeV)	$(m_T/\text{GeV}, s_L)$	A_0^{hhh}	A_1^{hhh}	A_2^{hhh}	A_3^{hhh}			
14	(400, 0.2)	−0.6958	0.2754	−0.1343	−0.9956			
	(800, 0.1)	−0.7814	0.281	−0.04494	−0.5731			

2.2 STU parameters

Shi-Ping He, Leptoquark and vector-like quark extended model for simultaneous explanation of W boson mass and muon g-2 anomalies, Chin. Phys. C 47 (2023), 043102, arXiv:2205.02088.

triplet scalar leptoquark + triplet VLQ

S_3: $(\bar{3}, 3, 1/3)$ $(X, T, B)_{L,R}$: $(3,3,2/3)$

$$S_3 \equiv \begin{pmatrix} S_3^{1/3} & \sqrt{2} S_3^{4/3} \\ \sqrt{2} S_3^{-2/3} & -S_3^{1/3} \end{pmatrix} \qquad \Psi_{L,R} \equiv \begin{pmatrix} T_{L,R} & \sqrt{2} X_{L,R} \\ \sqrt{2} B_{L,R} & -T_{L,R} \end{pmatrix}$$

VLQ gauge interactions

$\mathrm{Tr}(\bar{\Psi} i \not{D} \Psi)/2$

charged current interactions:

$$\mathcal{L}_{XTB}^{gauge} \supset \frac{g}{\sqrt{2}} W_\mu^+ \Big\{ \bar{t}\gamma^\mu [(c_L^t c_L^b + \sqrt{2} s_L^t s_L^b)\omega_- + \sqrt{2} s_R^t s_R^b \omega_+] b + \bar{t}\gamma^\mu [(c_L^t s_L^b - \sqrt{2} s_L^t c_L^b)\omega_- - \sqrt{2} s_R^t c_R^b \omega_+] B$$
$$+ \bar{T}\gamma^\mu [(s_L^t c_L^b - \sqrt{2} c_L^t s_L^b)\omega_- - \sqrt{2} c_R^t s_R^b \omega_+] b + \bar{T}\gamma^\mu [(s_L^t s_L^b + \sqrt{2} c_L^t c_L^b)\omega_- + \sqrt{2} c_R^t c_R^b \omega_+] B \Big\}$$
$$+ g W_\mu^+ [\overline{X_L}\gamma^\mu (s_L^t t_L - c_L^t T_L) + \overline{X_R}\gamma^\mu (s_R^t t_R - c_R^t T_R)] + \text{h.c..} \tag{15}$$

neutral current interactions:

$$\mathcal{L}_{XTB}^{gauge} \supset \frac{g}{2c_W} Z_\mu \Big\{ \bar{t}\gamma^\mu [((c_L^t)^2 - \frac{4}{3} s_W^2)\omega_- - \frac{4}{3} s_W^2 \omega_+] t + \bar{T}\gamma^\mu [((s_L^t)^2 - \frac{4}{3} s_W^2)\omega_- - \frac{4}{3} s_W^2 \omega_+] T$$
$$+ s_L^t c_L^t (\overline{t_L}\gamma^\mu T_L + \overline{T_L}\gamma^\mu t_L)] + \bar{b}\gamma^\mu [(-1 - (s_L^b)^2 + \frac{2}{3} s_W^2)\omega_- + (-2(s_R^b)^2 + \frac{2}{3} s_W^2)\omega_+] b$$
$$+ \bar{B}\gamma^\mu [(-1 - (c_L^b)^2 + \frac{2}{3} s_W^2)\omega_- + (-2(c_R^b)^2 + \frac{2}{3} s_W^2)\omega_+] B + s_L^b c_L^b (\overline{b_L}\gamma^\mu B_L + \overline{B_L}\gamma^\mu b_L)$$
$$+ 2 s_R^b c_R^b (\overline{b_R}\gamma^\mu B_R + \overline{B_R}\gamma^\mu b_R) + 2(1 - \frac{5}{3} s_W^2)(\overline{X_L}\gamma^\mu X_L + \overline{X_R}\gamma^\mu X_R) \Big\}. \tag{16}$$

contributions of self energy

$\bar{f}_i\gamma_\mu(g^{ij}_{V_1} + g^{ij}_{A_1}\gamma^5)f_j V_1^\mu + \bar{f}_i\gamma_\mu(g^{ij}_{V_2} + g^{ij}_{A_2}\gamma^5)f_j V_2^\mu$

L. Lavoura and J. P. Silva (1993)

$$\Pi_{V_1V_2}(0) = \frac{N_C}{16\pi^2}\{(g^{ij}_{V_1}g^{ij}_{V_2} + g^{ij}_{A_1}g^{ij}_{A_2})[2(m_i^2 + m_j^2)\Delta_\epsilon - 2(m_i^2\log\frac{m_i^2}{\mu^2} + m_j^2\log\frac{m_j^2}{\mu^2}) + \theta_+(m_i^2, m_j^2)]$$
$$+ (g^{ij}_{V_1}g^{ij}_{V_2} - g^{ij}_{A_1}g^{ij}_{A_2})[-4m_im_j\Delta_\epsilon + 2m_im_j\log\frac{m_i^2m_j^2}{\mu^4} + \theta_-(m_i^2, m_j^2)]\}. \quad \text{(C1)}$$

$$\Pi'_{V_1V_2}(0) \equiv \frac{d\Pi_{V_1V_2}(p^2)}{dp^2}|_{p^2=0} = \frac{N_C}{4\pi^2}\{(g^{ij}_{V_1}g^{ij}_{V_2} + g^{ij}_{A_1}g^{ij}_{A_2})[-\frac{1}{3}\Delta_\epsilon + \frac{1}{6} + \frac{1}{6}\log\frac{m_i^2m_j^2}{\mu^4} - \frac{1}{2}\chi_+(m_i^2, m_j^2)]$$
$$+ (g^{ij}_{V_1}g^{ij}_{V_2} - g^{ij}_{A_1}g^{ij}_{A_2})[-\frac{m_i^2 + m_j^2}{12m_im_j} - \frac{1}{2}\chi_-(m_i^2, m_j^2)]\}. \quad \text{(C2)}$$

assumptions

$$S = \frac{N_c}{2\pi}\Big\{\sum_\alpha\sum_i[(|V^L_{\alpha i}|^2 + |V^R_{\alpha i}|^2)\,\psi_+(y_\alpha, y_i) + 2\,\mathrm{Re}(V^L_{\alpha i}V^{R*}_{\alpha i})\,\psi_-(y_\alpha, y_i)]$$
$$- \sum_{\beta<\alpha}[(|\mathbf{U}^L_{\alpha\beta}|^2 + |\mathbf{U}^R_{\alpha\beta}|^2)\,\chi_+(y_\alpha, y_\beta) + 2\,\mathrm{Re}(\mathbf{U}^L_{\alpha\beta}\mathbf{U}^{R*}_{\alpha\beta})\,\chi_-(y_\alpha, y_\beta)]$$
$$- \sum_{j<i}[(|\mathbf{D}^L_{ij}|^2 + |\mathbf{D}^R_{ij}|^2)\,\chi_+(y_i, y_j) + 2\,\mathrm{Re}(\mathbf{D}^L_{ij}\mathbf{D}^{R*}_{ij})\,\chi_-(y_i, y_j)]\Big\}.$$

$$T = \frac{N_c}{16\,\pi\,\sin^2\theta_W\cos^2\theta_W}\Big\{\sum_\alpha\sum_i[(|V^L_{\alpha i}|^2 + |V^R_{\alpha i}|^2)\,\theta_+(y_\alpha, y_i) + 2\,\mathrm{Re}(V^L_{\alpha i}V^{R*}_{\alpha i})\,\theta_-(y_\alpha, y_i)]$$
$$- \sum_{\beta<\alpha}[(|\mathbf{U}^L_{\alpha\beta}|^2 + |\mathbf{U}^R_{\alpha\beta}|^2)\,\theta_+(y_\alpha, y_\beta) + 2\,\mathrm{Re}(\mathbf{U}^L_{\alpha\beta}\mathbf{U}^{R*}_{\alpha\beta})\,\theta_-(y_\alpha, y_\beta)]$$
$$- \sum_{j<i}[(|\mathbf{D}^L_{ij}|^2 + |\mathbf{D}^R_{ij}|^2)\,\theta_+(y_i, y_j) + 2\,\mathrm{Re}(\mathbf{D}^L_{ij}\mathbf{D}^{R*}_{ij})\,\theta_-(y_i, y_j)]\Big\}.$$

$$U = -\frac{N_c}{2\pi}\Big\{\sum_\alpha\sum_i[(|V^L_{\alpha i}|^2 + |V^R_{\alpha i}|^2)\,\chi_+(y_\alpha, y_i) + 2\,\mathrm{Re}(V^L_{\alpha i}V^{R*}_{\alpha i})\,\chi_-(y_\alpha, y_i)]$$
$$- \sum_{\beta<\alpha}[(|\mathbf{U}^L_{\alpha\beta}|^2 + |\mathbf{U}^R_{\alpha\beta}|^2)\,\chi_+(y_\alpha, y_\beta) + 2\,\mathrm{Re}(\mathbf{U}^L_{\alpha\beta}\mathbf{U}^{R*}_{\alpha\beta})\,\chi_-(y_\alpha, y_\beta)]$$
$$- \sum_{j<i}[(|\mathbf{D}^L_{ij}|^2 + |\mathbf{D}^R_{ij}|^2)\,\chi_+(y_i, y_j) + 2\,\mathrm{Re}(\mathbf{D}^L_{ij}\mathbf{D}^{R*}_{ij})\,\chi_-(y_i, y_j)]\Big\}.$$

V^α: W boson coupling matrix
U^α: Z boson coupling matrix with up quark
D^α: Z boson coupling matrix with down quark

assumption 1:

$$\mathbf{U}^\alpha \equiv V^\alpha (V^\alpha)^\dagger$$
$$\mathbf{D}^\alpha \equiv (V^\alpha)^\dagger V^\alpha$$

$$(\mathbf{U}^\alpha)^2 = \mathbf{U}^\alpha \,,$$
$$(\mathbf{D}^\alpha)^2 = \mathbf{D}^\alpha \,,$$
$$\mathbf{D}^L M_d \mathbf{D}^R = (V^L)^\dagger M_u V^R$$
$$\mathbf{U}^L M_u \mathbf{U}^R = V^L M_d (V^R)^\dagger$$

only valid for the singlet and doublet
not for the triplet

$$U^X_{L/R} = V^{Xt}_{L/R}(V^{Xt}_{L/R})^\dagger, \qquad U^t_{L/R} = V^{tb}_{L/R}(V^{tb}_{L/R})^\dagger - (V^{Xt}_{L/R})^\dagger V^{Xt}_{L/R}, \qquad U^b_{L/R} = (V^{tb}_{L/R})^\dagger V^{tb}_{L/R}.$$

$$(U^X_{L/R})^2 = U^X_{L/R} + 2, (U^t_{L/R})^2 = U^t_{L/R}, (U^b_{L/R})^2 = U^b_{L/R} + 2\begin{bmatrix} sin^2\theta^b_{L/R} & -sin\theta^b_{L/R}cos\theta^b_{L/R} \\ -sin\theta^b_{L/R}cos\theta^b_{L/R} & cos^2\theta^b_{L/R} \end{bmatrix}$$ for (X, T, B) triplet

assumptio

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) - i\frac{g}{\cos\theta_w} Z_\mu (T^3 - \sin^2\theta_w Q) - ieA_\mu Q$$

$W^+_\mu \bar{\psi} T_+ \gamma^\mu \psi$ $\qquad$ $Z_\mu \bar{\psi} T_3 \gamma^\mu \psi$

$[T_i, T_j] = i\varepsilon_{ijk} T_k$

$T_\pm = T_1 \pm iT_2$

$T_+T_- = T^2 - T_3^2 + T_3$? T_3

$V^\alpha (V^\alpha)^\dagger \Rightarrow U^\alpha$

assumption 2:

$$\frac{\Pi_{VV}(m_V^2) - \Pi_{VV}(0)}{m_V^2} \approx \Pi'_{VV}(0)$$

imply the heavy quark approximation

Paper	Earliest time	Representation	Results	Comments
L. Lavoura and J. P. Silva Phys. Rev. D 47 (1993), 2046	1993	$T, B, \begin{pmatrix}T\\B\end{pmatrix}$	$S \sim \psi_+, \psi_-, \chi_+, \chi_-$ $T \sim \theta_+, \theta_-$ $U \sim \chi_+, \chi_-$	
Haiying Cai arXiv:1210.5200 (JHEP)	2012	$T, \begin{pmatrix}X\\T\end{pmatrix}, \begin{pmatrix}T\\B\end{pmatrix}, \begin{pmatrix}X\\T\\B\end{pmatrix}$	$T \sim \theta_+, \theta_-$	based on Lavoura's results valid
Chen, Dawson, and Furlan arXiv:1703.06134 (PRD)	2017	$T, B, \begin{pmatrix}X\\T\end{pmatrix}, \begin{pmatrix}T\\B\end{pmatrix}, \begin{pmatrix}B\\Y\end{pmatrix}, \begin{pmatrix}X\\T\\B\end{pmatrix}, \begin{pmatrix}T\\B\\Y\end{pmatrix}$	$S \sim (\theta_{L,R}^{t,b})^2$ $T \sim (\theta_{L,R}^{t,b})^2$	based on Lavoura's results some are valid
Junjie Cao *et al.* arXiv:2204.09477 (PRD) ↕ discussions	12 July 2022 (arXiv v3)	$T, B, \begin{pmatrix}X\\T\end{pmatrix}, \begin{pmatrix}T\\B\end{pmatrix}, \begin{pmatrix}B\\Y\end{pmatrix}, \begin{pmatrix}X\\T\\B\end{pmatrix}, \begin{pmatrix}T\\B\\Y\end{pmatrix}$	$S \sim (\theta_{L,R}^{t,b})^2$ $T \sim (\theta_{L,R}^{t,b})^2$ $U \sim (\theta_{L,R}^{t,b})^2$	first correct expansion for S, U parameters of triplet
Shi-Ping He arXiv:2205.02088 (CPC) ↕ discussions	24 July 2022 (arXiv v2)	$\begin{pmatrix}X\\T\\B\end{pmatrix}$	$S \sim \chi_+, \chi_-, \mathrm{non}\chi_\pm$ $T \sim \theta_+, \theta_-$ $U \sim \chi_+, \chi_-, \mathrm{non}\chi_\pm$	find the problems (arXiv v1) claim the reason (arXiv v2)
Haiying Cai and G. Cacciapaglia arXiv:2208.04290 (JHEP)	8 Aug 2022 (arXiv v1)	$SU(4)/Sp(4)$ CHM bidoublet	S, T, U	(X, T, B) triplet
Albergaria, Lavoura, and Romao arXiv:2212.06509 (JHEP)	13 Dec 2022 (arXiv v1)	$T, B, \begin{pmatrix}X\\T\end{pmatrix}, \begin{pmatrix}T\\B\end{pmatrix}, \begin{pmatrix}B\\Y\end{pmatrix}, \begin{pmatrix}X\\T\\B\end{pmatrix}, \begin{pmatrix}T\\B\\Y\end{pmatrix}$	S, T, U $\sim B$ functions V, W, X	more general analysis

results of STU parameters from triplet VLQ

C. Y. Chen, S. Dawson and E. Furlan, Vectorlike fermions and Higgs effective field theory revisited, Phys. Rev. D 96, 015006 (2017), arXiv:1703.06134.

L. Lavoura and J. P. Silva, The Oblique corrections from vector - like singlet and doublet quarks, Phys. Rev. D 47, 2046-2057 (1993).

original results

$$r_F \equiv \frac{M_F^2}{m_t^2}.$$

$$(XTB)\ \text{triplet}:\ \Delta T^{XTB} \sim \frac{N_c m_t^2}{8\pi s_W^2 M_W^2}(s_L^t)^2\left[3\log(r_T)-5\right] \qquad \Delta S^{XTB} \sim -\frac{N_c}{18\pi}(s_L^t)^2\left[7+4\log(r_b)-6\log(r_T)\right]+\mathcal{O}\left((s_L^t)^4, \frac{1}{r_T}\right)$$

H. Cai and G. Cacciapaglia, Partial compositeness under precision scrutiny, JHEP 12, 104 (2022), arXiv:2208.04290.

Shi-Ping He, Leptoquark and vector-like quark extended model for simultaneous explanation of W boson mass and muon g-2 anomalies, Chin. Phys. C 47 (2023), 043102, arXiv:2205.02088.

J. Cao, L. Meng, L. Shang, S. Wang and B. Yang, Interpreting the W-mass anomaly in vectorlike quark models, Phys. Rev. D 106, 055042 (2022), arXiv:2204.09477.

$$\Delta S^{XTB} \approx \frac{N_C (s_L^t)^2}{18\pi}\left(-12\log\frac{m_T}{m_t}-16\log\frac{m_t}{m_b}+29\right),$$

$$\Delta T^{XTB} \approx \frac{N_C m_t^2 (s_L^t)^2}{8\pi s_W^2 m_W^2}\left(6\log\frac{m_T}{m_t}-5\right), \qquad \Delta U^{XTB} \approx \frac{N_C (s_L^t)^2}{18\pi}\left(24\log\frac{m_t}{m_b}-5\right).$$

results in 2022

H. Abouabid, A. Arhrib, R. Benbrik, M. Boukidi, and J. El Falaki, The oblique parameters in the 2HDM with vector-like quarks: confronting M_W CDF-II anomaly, J. Phys. G 51, 075001 (2024), arXiv:2302.07149.

more papers after

F. Albergaria, L. Lavoura and J. C. Romao, Oblique corrections from triplet quarks, JHEP 03, 031 (2023), arXiv:2212.06509.

2.3 $(g-2)_\mu$ contributions

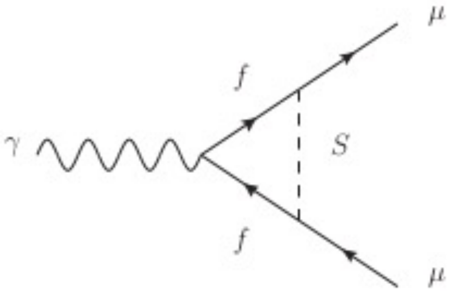
Shi-Ping He, Complete one-loop analytic and expansion formulae for the muon magnetic dipole moment, arXiv:2308.07133.

Shi-Ping He, Scalar leptoquark and vector-like quark extended models as the explanation of the muon g-2 anomaly: bottom partner chiral enhancement case, Chin. Phys. C 47, 073101 (2023), arXiv:2211.08062.

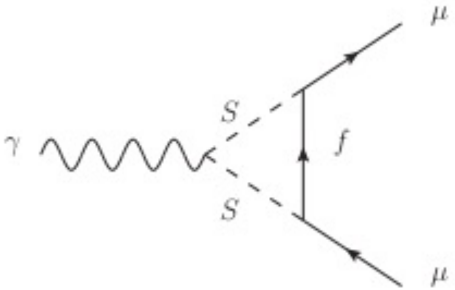
Shi-Ping He, Leptoquark and vectorlike quark extended models as the explanation of the muon g-2 anomaly, Phys. Rev. D 105 (2022), 035017, [Phys. Rev. D 106 (2022), 039901 (erratum)], arXiv:2112.13490.

new physics contributions at one-loop (four basic diagrams)

scalar mediator $\bar{\mu}(y_S + i\gamma^5 y_P) f S$ or $\bar{\mu}(y_L \omega_- + y_R \omega_+) f S$

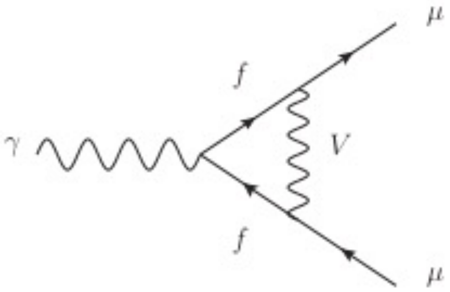


$ff\gamma$ vertex

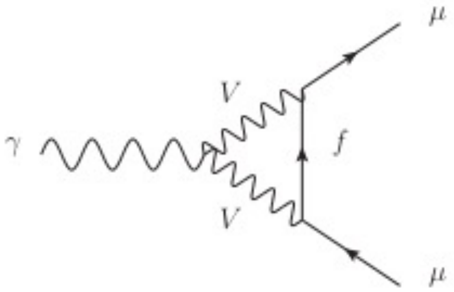


$SS\gamma$ vertex

vector mediator $\bar{\mu}\gamma^\mu(g_V + \gamma^5 g_A) f V_\mu$ or $\bar{\mu}\gamma^\mu(g_L \omega_- + g_R \omega_+) f V_\mu$



$ff\gamma$ vertex



$VV\gamma$ vertex

effective operators

$$O_{eB} = \overline{L_L^i}\sigma^{\mu\nu} e_R^j\, H\, B_{\mu\nu}$$

$$O_{eW} = \overline{L_L^i}\sigma^{\mu\nu}\sigma^a e_R^j\, H\, W_{\mu\nu}^a \;\Rightarrow\; -\frac{e a_\mu}{4 m_\mu}\bar{\mu}\sigma^{\mu\nu}\mu\, F_{\mu\nu}$$

$SU_L(2) \otimes U_Y(1)$ invariant

scalar leptoquark + VLQ

scalar leptoquark

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation	label	$F(3B+L)$
$(\bar{3}, 3, 1/3)$	S_3	-2
$(3, 2, 7/6)$	R_2	0
$(3, 2, 1/6)$	$\widetilde{R}_2$	0
$(\bar{3}, 1, 4/3)$	$\widetilde{S}_1$	-2
$(\bar{3}, 1, 1/3)$	S_1	-2
$(\bar{3}, 1, -2/3)$	$\bar{S}_1$	-2

I. Dorsner, S. Fajfer, A. Greljo, J. F. Kamenik and N. Kosnik (2016)

VLQ

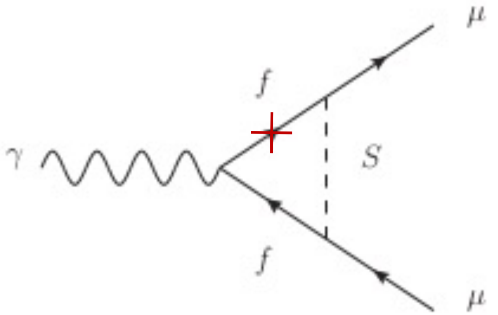
$6\times7 = 42$ combinations

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation	label
$(3, 1, 2/3)$	$T_{L,R}$
$(3, 1, -1/3)$	$B_{L,R}$
$(3, 2, 7/6)$	$(X, T)_{L,R}$
$(3, 2, 1/6)$	$(T, B)_{L,R}$
$(3, 2, -5/6)$	$(B, Y)_{L,R}$
$(3, 3, 2/3)$	$(X, T, B)_{L,R}$
$(3, 3, -1/3)$	$(T, B, Y)_{L,R}$

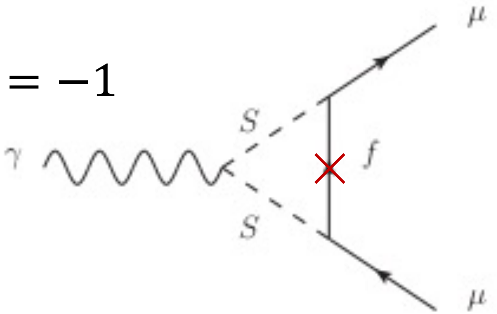
J. J. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer and M. P. Victoria (2013)

$$\mathcal{L}_{F=0} \supset \bar{\mu}(y_L^{S_A\mu q_A}\omega_- + y_R^{S_A\mu q_A}\omega_+)q_A S_A + \text{h.c.}$$

$$\mathcal{L}_{F=2} \supset \bar{\mu}(y_L^{S_B\mu q_B}\omega_- + y_R^{S_B\mu q_B}\omega_+)q_B^C S_B + \text{h.c.}$$



$Q_f + Q_S = -1$



chiral enhancement from VLQ mass

$ff\gamma$ vertex mediated $\propto Q_f$

$SS\gamma$ vertex mediated $\propto Q_S$

Model	chiral enhancement
R_2	m_t/m_μ
S_1	m_t/m_μ

minimal scalar LQ models

17 leptoquark and VLQ extended models

$R_2 + B_{L,R}/(B,Y)_{L,R}$	m_t/m_μ
$S_1 + B_{L,R}/(B,Y)_{L,R}$	m_t/m_μ
$R_2 + T_{L,R}/(X,T)_{L,R}/(T,B)_{L,R}/(T,B,Y)_{L,R}$	$m_t/m_\mu, m_T/m_\mu$
$S_1 + T_{L,R}/(X,T)_{L,R}/(T,B)_{L,R}/(X,T,B)_{L,R}/(T,B,Y)_{L,R}$	$m_t/m_\mu, m_T/m_\mu$
$R_2 + (X,T,B)_{L,R}$	$m_t/m_\mu, m_T/m_\mu, m_b/m_\mu, m_B/m_\mu$
$S_3 + (X,T,B)_{L,R}$	$m_t/m_\mu, m_T/m_\mu, m_b/m_\mu, m_B/m_\mu$

up-type quark chiral enhancements

(up-type LQ + VLQ)

Shi-Ping He, Phys. Rev. D 105 (2022), 035017, [Phys. Rev. D 106 (2022), 039901 (erratum)], arXiv:2112.13490

$\widetilde{\mathbf{R}}_\mathbf{2} + (\mathbf{B},\mathbf{Y})_{\mathbf{L},\mathbf{R}}$	$m_b/m_\mu, m_B/m_\mu$
$\widetilde{\mathbf{S}}_\mathbf{1} + (\mathbf{B},\mathbf{Y})_{\mathbf{L},\mathbf{R}}$	$m_b/m_\mu, m_B/m_\mu$

down-type quark chiral enhancements

(down-type LQ + VLQ)

Shi-Ping He, Chin. Phys. C 47 (2023), 073101, arXiv:2211.08062

VLQ phenomenology

new production and decay channels of VLQ in two example models

Model	Scenario	LQ decay	VLQ decay	new signatures
$\widetilde{R}_2 + (B,Y)_{L,R}$	$m_{\rm LQ} > m_B$	$\widetilde{R}_2^{2/3} \to \mu^+ b, \mu^+ B$	$B \to bZ, bh$	$\widetilde{R}_2^{2/3} \to \mu j_b Z, \mu j_b h$
		$\widetilde{R}_2^{-1/3} \to \mu^+ Y, \nu_L b$	$Y \to bW^-$	$\widetilde{R}_2^{-1/3} \to \mu j_b W$
	$m_{\rm LQ} < m_B$	$\widetilde{R}_2^{2/3} \to \mu^+ b$	$B \to bZ, bh, \mu^- \widetilde{R}_2^{2/3}$	$B \to \mu^+ \mu^- j_b$
		$\widetilde{R}_2^{-1/3} \to \nu_L b$	$Y \to bW^-, \mu^- \widetilde{R}_2^{-1/3}$	$Y \to \mu \not{E}_T j_b$
$\widetilde{S}_1 + (B,Y)_{L,R}$	$m_{\rm LQ} > m_B$	$\widetilde{S}_1 \to \mu^+ \bar{b}, \mu^+ \bar{B}, \nu_L \bar{Y}$	$B \to bZ, bh$	$\widetilde{S}_1 \to \mu j_b Z,\ \mu j_b h, \not{E}_T j_b W$
			$Y \to bW^-$	
	$m_{\rm LQ} < m_B$	$\widetilde{S}_1 \to \mu^+ \bar{b}$	$B \to bZ, bh, \mu^+ (\widetilde{S}_1)^*$	$B \to \mu^+ \mu^- j_b$
			$Y \to bW^-, \nu_L (\widetilde{S}_1)^*$	$Y \to \mu \not{E}_T j_b$

3. Outlook

Next steps

- **Theoretical aspect**
  - UV completion

    gauge group and representations

    mixing with SM quark
  - …

- **Phenomenological aspect**
  - CP violation
  - Higgs physics $gg \to Z\,h$

    analytic amplitude and expansion
  - …

Summary and conclusions

- Introduction to VLQ

- Our studies in vector-like quark models

  - Higgs physics $h\bar{t}T/h\bar{T}t$

  $h \to \gamma Z/gg \to hh$ in singlet $T_{L,R}$ extended models

  - STU parameters

  triplet scalar leptoquark and triplet $(X, T, B)_{L,R}$ model

  - $(g-2)_\mu$ contributions

  scalar leptoquark and VLQ models (17 out of 42)

- Outlook

  - UV completion: representations

  - Higgs physics: $gg \to Z\ h$

Thank you!

Modifications to SM couplings

J. J. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer, and M. P. Victoria (2013)

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\bar{t}\gamma^\mu \left(V_{tb}^L P_L + V_{tb}^R P_R\right) b W_\mu^+ + \text{H.c.}\,,$$

$$\mathcal{L}_Z = -\frac{g}{2c_W}\bar{t}\gamma^\mu \left(X_{tt}^L P_L + X_{tt}^R P_R - 2Q_t s_W^2\right) t Z_\mu - \frac{g}{2c_W}\bar{b}\gamma^\mu \left(-X_{bb}^L P_L - X_{bb}^R P_R - 2Q_b s_W^2\right) b Z_\mu\,,$$

$$\mathcal{L}_H = -\frac{g m_t}{2M_W} Y_{tt}\bar{t}tH - \frac{g m_b}{2M_W} Y_{bb}\bar{b}bH\,,$$

	V_{tb}^L	V_{tb}^R
(T)	c_L	0
(B)	c_L	0
$(X\,T)$	c_L	0
$(T\,B)$	$c_L^u c_L^d + s_L^u s_L^d e^{i(\phi_u-\phi_d)}$	$s_R^u s_R^d e^{i(\phi_u-\phi_d)}$
$(B\,Y)$	c_L	0
$(X\,T\,B)$	$c_L^u c_L^d + \sqrt{2} s_L^u s_L^d$	$\sqrt{2} s_R^u s_R^d$
$(T\,B\,Y)$	$c_L^u c_L^d + \sqrt{2} s_L^u s_L^d$	$\sqrt{2} s_R^u s_R^d$

Table 2: Light-light couplings to the W boson.

	X_{tt}^L	X_{tt}^R	X_{bb}^L	X_{bb}^R
(T)	c_L^2	0	1	0
(B)	1	0	c_L^2	0
$(X\,T)$	$c_L^2 - s_L^2$	$-s_R^2$	1	0
$(T\,B)$	1	$(s_R^u)^2$	1	$(s_R^d)^2$
$(B\,Y)$	1	0	$c_L^2 - s_L^2$	$-s_L^2$
$(X\,T\,B)$	$(c_L^u)^2$	0	$1 + (s_L^d)^2$	$2(s_R^d)^2$
$(T\,B\,Y)$	$1 + (s_L^u)^2$	$2(s_R^u)^2$	$(c_L^d)^2$	0

Table 3: Light-light couplings to the Z boson.

	Y_{tt}	Y_{bb}
(T)	c_L^2	1
(B)	1	c_L^2
$(X\,T)$	c_R^2	1
$(T\,B)$	$(c_R^u)^2$	$(c_R^d)^2$
$(B\,Y)$	1	c_R^2
$(X\,T\,B)$	$(c_L^u)^2$	$(c_L^d)^2$
$(T\,B\,Y)$	$(c_L^u)^2$	$(c_L^d)^2$

Table 4: Light-light couplings to the Higgs boson.

Higgs Yukawa interactions for the triplet $(X,T,B)_{L,R}$

$(X,T,B)_{L,R}$: (3,3,2/3)

ϕ: (1,2,1/2)
Q_L: (3,2,1/6)
u_R: (3,1,2/3)
d_R: (3,1,−1/3)

$$-M_T\overline{(X,T,B)_L}\begin{pmatrix} X \\ T \\ B \end{pmatrix}_R - y_{ij}^u \overline{Q_L}^i u_R^j \widetilde{\phi} - y_{ij}^d \overline{Q_L}^i d_R^j \phi - y_{iT}\overline{(Q_L)}^i \Psi_R \widetilde{\phi} + \text{h.c.}.$$

third generation quark and heavy quark mixing

$$\tan\theta_R^t = \frac{m_t}{m_T}\tan\theta_L^t, \qquad \tan\theta_R^b = \frac{m_b}{m_B}\tan\theta_L^b, \qquad \sin 2\theta_L^b = \frac{\sqrt{2}(m_T^2 - m_t^2)}{m_B^2 - m_b^2}\sin 2\theta_L^t$$

$$M_T^2 = m_T^2(c_L^t)^2 + m_t^2(s_L^t)^2 = m_B^2(c_L^b)^2 + m_b^2(s_L^b)^2$$

$$\mathcal{L}_H^{Yukawa} \supset -\frac{m_t}{v}(c_L^t)^2 h\bar{t}t - \frac{m_T}{v}(s_L^t)^2 h\bar{T}T - \frac{m_b}{v}(c_L^b)^2 h\bar{b}b - \frac{m_B}{v}(s_L^b)^2 h\bar{B}B - \frac{m_T}{v}s_L^t c_L^t h(\bar{t}_L T_R + \bar{T}_R t_L)$$
$$- \frac{m_t}{v}s_L^t c_L^t h(\bar{T}_L t_R + \bar{t}_R T_L) - \frac{m_B}{v}s_L^b c_L^b h(\bar{b}_L B_R + \bar{B}_R b_L) - \frac{m_b}{v}s_L^b c_L^b h(\bar{B}_L b_R + \bar{b}_R B_L). \qquad (6)$$

Electroweak precision measurement constraints

STU parameters

$$\frac{\alpha S}{4s_W^2c_W^2} \equiv \frac{\Pi_{ZZ}(m_Z^2)-\Pi_{ZZ}(0)}{m_Z^2} - \frac{c_W^2-s_W^2}{s_Wc_W}\Pi'_{\gamma Z}(0) - \Pi'_{\gamma\gamma}(0) = \Pi'_{ZZ}(0) - \frac{c_W^2-s_W^2}{s_Wc_W}\Pi'_{\gamma Z}(0) - \Pi'_{\gamma\gamma}(0)$$

$$\alpha T \equiv \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}$$

$$\frac{\alpha U}{4s_W^2} \equiv \frac{\Pi_{WW}(m_W^2)-\Pi_{WW}(0)}{m_W^2} - c_W^2\frac{\Pi_{ZZ}(m_Z^2)-\Pi_{ZZ}(0)}{m_Z^2} - 2s_Wc_W\Pi'_{\gamma Z}(0) - s_W^2\Pi'_{\gamma\gamma}(0)$$
$$= \Pi'_{WW}(0) - c_W^2\Pi'_{ZZ}(0) - 2s_Wc_W\Pi'_{\gamma Z}(0) - s_W^2\Pi'_{\gamma\gamma}(0).$$

relation with W mass $$\frac{\Delta m_W^2}{m_W^2} = \frac{2\Delta m_W}{m_W} = \frac{\alpha}{c_W^2-s_W^2}\left(-\frac{1}{2}\Delta S + c_W^2\Delta T + \frac{c_W^2-s_W^2}{4s_W^2}\Delta U\right)$$

assumption: oblique corrections are dominated in the muon decay

large T correction

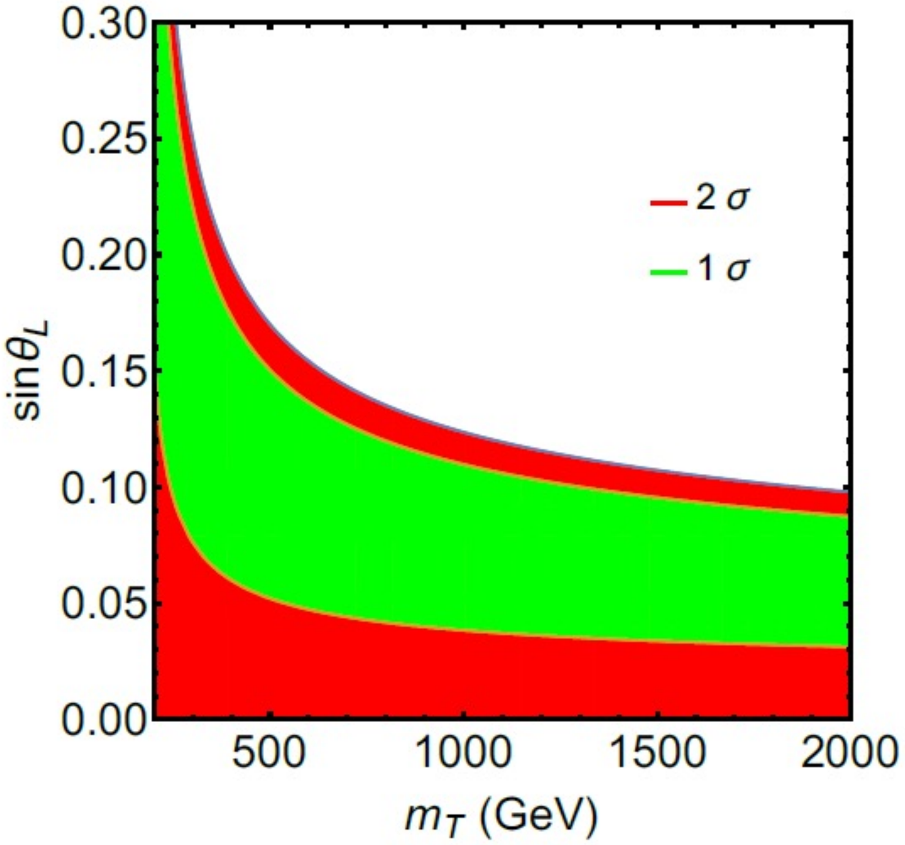
singlet $T_{L,R}$

$$\Delta S \equiv S - S^{SM}$$
$$= -\frac{N_t^C s_L^2}{18\pi}[-2\ln r_{tT} + c_L^2 \frac{5 - 22r_{tT}^2 + 5r_{tT}^4}{(1 - r_{tT}^2)^2} + c_L^2 \frac{6(1 + r_{tT}^2)(1 - 4r_{tT}^2 + r_{tT}^4)}{(1 - r_{tT}^2)^3}\ln r_{tT}]$$

$$\Delta T \equiv T - T^{SM} = \frac{N_t^C m_t^2 s_L^2}{16\pi s_W^2 m_W^2}\left[-1 - c_L^2 + \frac{s_L^2}{r_{tT}^2} - \frac{4c_L^2}{1 - r_{tT}^2}\ln r_{tT}\right](r_{tT} = \frac{m_t}{m_T})$$

$$\Delta\chi^2 \equiv \sum_{i,j=1,2} (O_i - O_i^{exp})(\sigma^2)_{ij}^{-1}\left(O_j - O_j^{exp}\right), O_i \in \{\Delta S, \Delta T\}, (\sigma^2)_{ij} = \sigma_i \rho_{ij} \sigma_j$$

$$\Delta S^{exp} = 0.02, \sigma_{\Delta S} = 0.07, \Delta T^{exp} = 0.06, \sigma_{\Delta T} = 0.06,$$
$$\rho = \begin{bmatrix} 1 & 0.92 \\ 0.92 & 1 \end{bmatrix}, \sigma^2 = \begin{bmatrix} \sigma_{\Delta S} & 0 \\ 0 & \sigma_{\Delta T} \end{bmatrix} \rho \begin{bmatrix} \sigma_{\Delta S} & 0 \\ 0 & \sigma_{\Delta T} \end{bmatrix}$$



triplet $(X,T,B)_{L,R}$

T参数的贡献

$$\Delta T^{XTB} = \frac{N_C}{16\pi s_W^2 m_W^2}\Big\{2[(s_L^t)^2 + (s_R^t)^2]\theta_+(m_X^2, m_t^2) + 4 s_L^t s_R^t \theta_-(m_X^2, m_t^2)$$
$$+ 2[(c_L^t)^2 + (c_R^t)^2]\theta_+(m_X^2, m_T^2) + 4 c_L^t c_R^t \theta_-(m_X^2, m_T^2)$$
$$+ [(c_L^t c_L^b + \sqrt{2} s_L^t s_L^b)^2 + 2(s_R^t s_R^b)^2 - 1]\theta_+(m_t^2, m_b^2) + 2\sqrt{2} s_R^t s_R^b (c_L^t c_L^b + \sqrt{2} s_L^t s_L^b)\theta_-(m_t^2, m_b^2)$$
$$+ [(c_L^t s_L^b - \sqrt{2} s_L^t c_L^b)^2 + 2(s_R^t c_R^b)^2]\theta_+(m_t^2, m_B^2) - 2\sqrt{2} s_R^t c_R^b (c_L^t s_L^b - \sqrt{2} s_L^t c_L^b)\theta_-(m_t^2, m_B^2)$$
$$+ [(s_L^t c_L^b - \sqrt{2} c_L^t s_L^b)^2 + 2(c_R^t s_R^b)^2]\theta_+(m_T^2, m_b^2) - 2\sqrt{2} c_R^t s_R^b (s_L^t c_L^b - \sqrt{2} c_L^t s_L^b)\theta_-(m_T^2, m_b^2)$$
$$+ [(s_L^t s_L^b + \sqrt{2} c_L^t c_L^b)^2 + 2(c_R^t c_R^b)^2]\theta_+(m_T^2, m_B^2) + 2\sqrt{2} c_R^t c_R^b (s_L^t s_L^b + \sqrt{2} c_L^t c_L^b)\theta_-(m_T^2, m_B^2)$$
$$- (s_L^t c_L^t)^2 \chi_+(m_t^2, m_T^2) - [(s_L^b c_L^b)^2 + 4(s_R^b c_R^b)^2]\theta_+(m_b^2, m_B^2) - 4(s_L^b c_L^b)(s_R^b c_R^b)\theta_-(m_b^2, m_B^2)\Big\}.$$

$$\Delta S^{XTB} = S^{XTB}_{non-\chi\pm} + \frac{N_C}{2\pi}\Big\{-\psi_+(m_t^2, m_b^2) - (s_L^t c_L^t)^2 \chi_+(m_t^2, m_T^2)$$
$$- [(s_L^b c_L^b)^2 + 4(s_R^b c_R^b)^2]\chi_+(m_b^2, m_B^2) - 4(s_L^b c_L^b)(s_R^b c_R^b)\chi_-(m_b^2, m_B^2)\Big\}.$$

S参数的贡献

$$\frac{\alpha S^{XTB}_{non-\chi\pm}}{4 s_W^2 c_W^2}$$
$$= \frac{N_C g^2}{96\pi^2 c_W^2}\Big\{[U_L^X U_L^X + U_R^X U_R^X - 2Q_X(U_L^X + U_R^X)](-\Delta_\epsilon + \log\frac{m_X^2}{\mu^2}) + \frac{U_L^X U_L^X + U_R^X U_R^X}{2} - U_L^X U_R^X$$
$$+ \mathrm{Tr}[\big(U_L^t U_L^t + U_R^t U_R^t - 2Q_t(U_L^t + U_R^t)\big).(-\Delta_\epsilon + \log\frac{M_u^2}{\mu^2})] + \frac{\mathrm{Tr}[U_L^t U_L^t + U_R^t U_R^t]}{2} - \mathrm{Tr}[U_L^t.M_u.U_R^t.M_u^{-1}]$$
$$+ \mathrm{Tr}[\big(U_L^b U_L^b + U_R^b U_R^b + 2Q_b(U_L^b + U_R^b)\big).(-\Delta_\epsilon + \log\frac{M_d^2}{\mu^2})] + \frac{\mathrm{Tr}[U_L^b U_L^b + U_R^b U_R^b]}{2} - \mathrm{Tr}[U_L^b.M_d.U_R^b.M_d^{-1}]\Big\}$$
$$= \frac{N_C g^2}{32\pi^2 c_W^2}\Big\{\frac{2}{3} - \frac{1}{3}\cos(2\theta_L^b)\cos(2\theta_R^b) - \frac{(m_b^2 + m_B^2)\sin(2\theta_L^b)\sin(2\theta_R^b)}{6 m_b m_B} - \frac{16}{9}[(s_L^t)^2 \log\frac{m_X^2}{m_t^2} + (c_L^t)^2 \log\frac{m_X^2}{m_T^2}]$$
$$- \frac{5}{3}[(s_L^t)^2 \log\frac{m_t^2}{m_b m_B} + (c_L^t)^2 \log\frac{m_T^2}{m_b m_B}] - \frac{1}{9}\log\frac{m_t^2 m_T^2}{m_b^2 m_B^2} + \frac{7\cos(2\theta_L^b) + 8\cos(2\theta_R^b)}{18}\log\frac{m_B^2}{m_b^2}\Big\}.$$

T参数的验证

$$\alpha T^{XTB}_{non-\theta_\pm} = \frac{N_C g^2}{32\pi^2 m_W^2}\Big\{[V_L^{Xt}(V_L^{Xt})^\dagger - U_L^X U_L^X + V_R^{Xt}(V_R^{Xt})^\dagger - U_R^X U_R^X] m_X^2(\Delta_\epsilon - \log\frac{m_X^2}{\mu^2})$$

$$+ [V_L^{Xt}.M_u.(V_R^{Xt})^\dagger - U_L^X m_X U_R^X + V_R^{Xt}.M_u.(V_L^{Xt})^\dagger - U_R^X m_X U_L^X] m_X(-\Delta_\epsilon + \log\frac{m_X^2}{\mu^2})$$

$$+ \mathrm{Tr}[\big(V_L^{tb}(V_L^{tb})^\dagger + (V_L^{Xt})^\dagger V_L^{Xt} - U_L^t U_L^t + V_R^{tb}(V_R^{tb})^\dagger + (V_R^{Xt})^\dagger V_R^{Xt} - U_R^t U_R^t\big).M_u^2.(\Delta_\epsilon - \log\frac{M_u^2}{\mu^2})]$$

$$+ \mathrm{Tr}[\big(V_L^{tb}.M_d.(V_R^{tb})^\dagger + (V_L^{Xt})^\dagger m_X V_R^{Xt} - U_L^t.M_u.U_R^t + V_R^{tb}.M_d.(V_L^{tb})^\dagger + (V_R^{Xt})^\dagger m_X V_L^{Xt} - U_R^t.M_u.U_L^t\big).$$

$$M_u.(-\Delta_\epsilon + \log\frac{M_u^2}{\mu^2})] + \mathrm{Tr}[\big((V_L^{tb})^\dagger V_L^{tb} - U_L^b U_L^b + (V_R^{tb})^\dagger V_R^{tb} - U_R^b U_R^b\big).M_d^2.(\Delta_\epsilon - \log\frac{M_d^2}{\mu^2})]$$

$$+ \mathrm{Tr}[\big((V_L^{tb})^\dagger.M_u.V_R^{tb} - U_L^b.M_d.U_R^b + (V_R^{tb})^\dagger.M_u.V_L^{tb} - U_R^b.M_d.U_L^b\big).M_d.(-\Delta_\epsilon + \log\frac{M_d^2}{\mu^2})]\Big\}$$

$$= \frac{N_C g^2}{16\pi^2 m_W^2}\Big\{m_t(\Delta_\epsilon - \log\frac{m_t^2}{\mu^2})[2m_t\big((s_L^t)^2 + (s_R^t)^2\big) + \sqrt{2}c_L^t s_R^t(m_B c_R^b s_L^b - m_b c_L^b s_R^b) - 4m_X s_L^t s_R^t]$$

$$+ m_T(\Delta_\epsilon - \log\frac{m_T^2}{\mu^2})[2m_T\big((c_L^t)^2 + (c_R^t)^2\big) - \sqrt{2}s_L^t c_R^t(m_B c_R^b s_L^b - m_b c_L^b s_R^b) - 4m_X c_L^t c_R^t]$$

$$+ m_b(\Delta_\epsilon - \log\frac{m_b^2}{\mu^2})[m_b\big(-(s_L^b)^2 + (s_R^b)^2\big) + \sqrt{2}c_L^b s_R^b(m_T c_R^t s_L^t - m_t c_L^t s_R^t)]$$

$$+ m_B(\Delta_\epsilon - \log\frac{m_B^2}{\mu^2})[m_B\big(-(c_L^b)^2 + (c_R^b)^2\big) - \sqrt{2}c_R^b s_L^b(m_T c_R^t s_L^t - m_t c_L^t s_R^t)]\Big\} = 0, \quad \text{(C9)}$$

U参数的贡献

$$\Delta U^{XTB} = \Delta U^{XTB}_{\chi\pm} + U^{XTB}_{non-\chi\pm}.$$

$$\Delta U^{XTB}_{\chi\pm} = -\frac{N_C}{2\pi}\Big\{2[(s^t_L)^2 + (s^t_R)^2]\chi_+(m_X^2, m_t^2) + 4s^t_L s^t_R \chi_-(m_X^2, m_t^2)$$
$$+ 2[(c^t_L)^2 + (c^t_R)^2]\chi_+(m_X^2, m_T^2) + 4c^t_L c^t_R \chi_-(m_X^2, m_T^2)$$
$$+ [(c^t_L c^b_L + \sqrt{2}s^t_L s^b_L)^2 + 2(s^t_R s^b_R)^2 - 1]\chi_+(m_t^2, m_b^2) + 2\sqrt{2}s^t_R s^b_R(c^t_L c^b_L + \sqrt{2}s^t_L s^b_L)\chi_-(m_t^2, m_b^2)$$
$$+ [(c^t_L s^b_L - \sqrt{2}s^t_L c^b_L)^2 + 2(s^t_R c^b_R)^2]\chi_+(m_t^2, m_B^2) - 2\sqrt{2}s^t_R c^b_R(c^t_L s^b_L - \sqrt{2}s^t_L c^b_L)\chi_-(m_t^2, m_B^2)$$
$$+ [(s^t_L c^b_L - \sqrt{2}c^t_L s^b_L)^2 + 2(c^t_R s^b_R)^2]\chi_+(m_T^2, m_b^2) - 2\sqrt{2}c^t_R s^b_R(s^t_L c^b_L - \sqrt{2}c^t_L s^b_L)\chi_-(m_T^2, m_b^2)$$
$$+ [(s^t_L s^b_L + \sqrt{2}c^t_L c^b_L)^2 + 2(c^t_R c^b_R)^2]\chi_+(m_T^2, m_B^2) + 2\sqrt{2}c^t_R c^b_R(s^t_L s^b_L + \sqrt{2}c^t_L c^b_L)\chi_-(m_T^2, m_B^2)$$
$$- (s^t_L c^t_L)^2\chi_+(m_t^2, m_T^2) - [(s^b_L c^b_L)^2 + 4(s^b_R c^b_R)^2]\chi_+(m_b^2, m_B^2) - 4(s^b_L c^b_L)(s^b_R c^b_R)\chi_-(m_b^2, m_B^2)\Big\}.$$

$$
\begin{aligned}
&\frac{\alpha U^{XTB}_{non-\chi_\pm}}{4s_W^2}\\
&= \frac{N_C g^2}{96\pi^2}\Big\{[V_L^{Xt}(V_L^{Xt})^\dagger - U_L^X U_L^X + V_R^{Xt}(V_R^{Xt})^\dagger - U_R^X U_R^X](-\Delta_\epsilon + \log\frac{m_X^2}{\mu^2})\\
&+ \mathrm{Tr}[\big(V_L^{tb}(V_L^{tb})^\dagger + (V_L^{Xt})^\dagger V_L^{Xt} - U_L^t U_L^t + V_R^{tb}(V_R^{tb})^\dagger + (V_R^{Xt})^\dagger V_R^{Xt} - U_R^t U_R^t\big).(-\Delta_\epsilon + \log\frac{M_u^2}{\mu^2})]\\
&+ \mathrm{Tr}[\big((V_L^{tb})^\dagger V_L^{tb} - U_L^b U_L^b + (V_R^{tb})^\dagger V_R^{tb} - U_R^b U_R^b\big).(-\Delta_\epsilon + \log\frac{M_d^2}{\mu^2})] + V_L^{Xt}(V_L^{Xt})^\dagger - \frac{1}{2}U_L^X U_L^X\\
&+ V_R^{Xt}(V_R^{Xt})^\dagger - \frac{1}{2}U_R^X U_R^X + \mathrm{Tr}[V_L^{tb}(V_L^{tb})^\dagger - \frac{1}{2}U_L^t U_L^t - \frac{1}{2}U_L^b U_L^b + V_R^{tb}(V_R^{tb})^\dagger - \frac{1}{2}U_R^t U_R^t - \frac{1}{2}U_R^b U_R^b]\\
&- \frac{1}{2}[V_L^{Xt}.M_u^{-1}.(V_R^{Xt})^\dagger m_X + \frac{V_L^{Xt}.M_u.(V_R^{Xt})^\dagger}{m_X} - U_L^X U_R^X + V_R^{Xt}.M_u^{-1}.(V_L^{Xt})^\dagger m_X + \frac{V_R^{Xt}.M_u.(V_L^{Xt})^\dagger}{m_X} - U_R^X U_L^X]\\
&- \frac{1}{2}\mathrm{Tr}[V_L^{tb}.M_d^{-1}.(V_R^{tb})^\dagger.M_u - U_L^b.M_d^{-1}.U_R^b.M_d + V_R^{tb}.M_d^{-1}.(V_L^{tb})^\dagger.M_u - U_R^b.M_d^{-1}.U_L^b.M_d]\\
&- \frac{1}{2}\mathrm{Tr}[V_L^{tb}.M_d.(V_R^{tb})^\dagger.M_u^{-1} - U_L^t.M_u^{-1}.U_R^t.M_u + V_R^{tb}.M_d.(V_L^{tb})^\dagger.M_u^{-1} - U_R^t.M_u^{-1}.U_L^t.M_u]\Big\}\\
&= -\frac{N_C g^2}{32\pi^2}\Big\{\frac{1}{3} - \frac{1}{3}\cos(2\theta_L^b)\cos(2\theta_R^b) - \frac{(m_b^2+m_B^2)\sin(2\theta_L^b)\sin(2\theta_R^b)}{6m_b m_B} - \frac{4}{3}[(s_L^t)^2 + (s_R^t)^2]\log\frac{m_t^2}{m_X^2}\\
&- \frac{4}{3}[(c_L^t)^2 + (c_R^t)^2]\log\frac{m_T^2}{m_X^2} + \frac{2}{3}[(s_L^b)^2 + (s_R^b)^2]\log\frac{m_b^2}{m_X^2} + \frac{2}{3}[(c_L^b)^2 + (c_R^b)^2]\log\frac{m_B^2}{m_X^2}\Big\}. \qquad \text{(C17)}
\end{aligned}
$$

Higgs to di-photon and γZ analysis

di-photon signal strength constraints

$$\mu_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)\Gamma(h \to \gamma\gamma)}{\sigma^{SM}(gg \to h)\Gamma^{SM}(h \to \gamma\gamma)}$$

$$= \frac{\Gamma(h \to gg)\Gamma(h \to \gamma\gamma)}{\Gamma^{SM}(h \to gg)\Gamma^{SM}(h \to \gamma\gamma)}$$

$$= \left| c_L^2 + s_L^2 \frac{F_f(\tau_T)}{F_f(\tau_t)} \right|^2 \frac{\left| Q_t^C Q_t^2 [c_L^2 F_f(\tau_t) + s_L^2 F_f(\tau_T)] + F_W(\tau_W) \right|^2}{\left| Q_t^C Q_t^2 F_f(\tau_t) + F_W(\tau_W) \right|^2}$$

$$F_f(\tau_f) \equiv -2\tau_f\left[1 + \left(1 - \tau_f\right) f\left(\tau_f\right)\right], \qquad F_W(\tau_W) \equiv 2 + 3\tau_W + 3\tau_W(2 - \tau_W) f(\tau_W)$$

$$f(\tau) \equiv \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right), & \text{for } \tau \geq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2, & \text{for } \tau < 1 \end{cases}$$

amplitude estimation and analysis

SM	Top	W	W/Top
$h \to \gamma\gamma$	-1.84	8.32	-4.5
$h \to \gamma Z$	-0.65	12.03	-18.5

$$\mathcal{A}_{tT} \sim [m_T \mathrm{Re}(y_R^{tT}) - (3 + 2\ln r_{tT}^2) m_t \mathrm{Re}(y_L^{tT})]$$

- $\mathcal{A}_{tT} > 0$, enhance $\Gamma(h \to \gamma Z)$
- $\mathcal{A}_{tT} < 0$, suppress $\Gamma(h \to \gamma Z)$

$$\mathcal{B}_{tT} \sim [(1 + \ln r_{tT}^2) m_t \mathrm{Im}(y_L^{tT}) + m_T \mathrm{Im}(y_R^{tT})]$$

→ always enhance $\Gamma(h \to \gamma Z)$

Di-Higgs analysis

the partonic cross section

$$\widehat{\sigma}_{LO}(gg \to hh; \hat{s}) = \frac{{\alpha_S}^2 {G_F}^2 \sqrt{\hat{s}(\hat{s} - 4m_h^2)}}{128(4\pi)^3} \int_{-1}^{1} d\cos\theta \left(|f_A|^2 + |f_B|^2 + |f_C|^2\right)$$

$$= \frac{{\alpha_S}^2 {G_F}^2}{64(4\pi)^3} \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} d\hat{t} \left(|f_A|^2 + |f_B|^2 + |f_C|^2\right)$$

$$\hat{t}_{\min} = -\frac{1}{4}\left(\sqrt{\hat{s}} + \sqrt{\hat{s} - 4m_h^2}\right)^2, \hat{t}_{\max} = -\frac{1}{4}\left(\sqrt{\hat{s}} - \sqrt{\hat{s} - 4m_h^2}\right)^2$$

the hadron level cross section

$$\sigma_{LO}(pp \to hh) = \int_{4m_h^2/s}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f(x, {\mu_F}^2) f\left(\frac{\tau}{x}, {\mu_F}^2\right) \widehat{\sigma}_{LO}(gg \to hh; \hat{s} = \tau\, s)$$

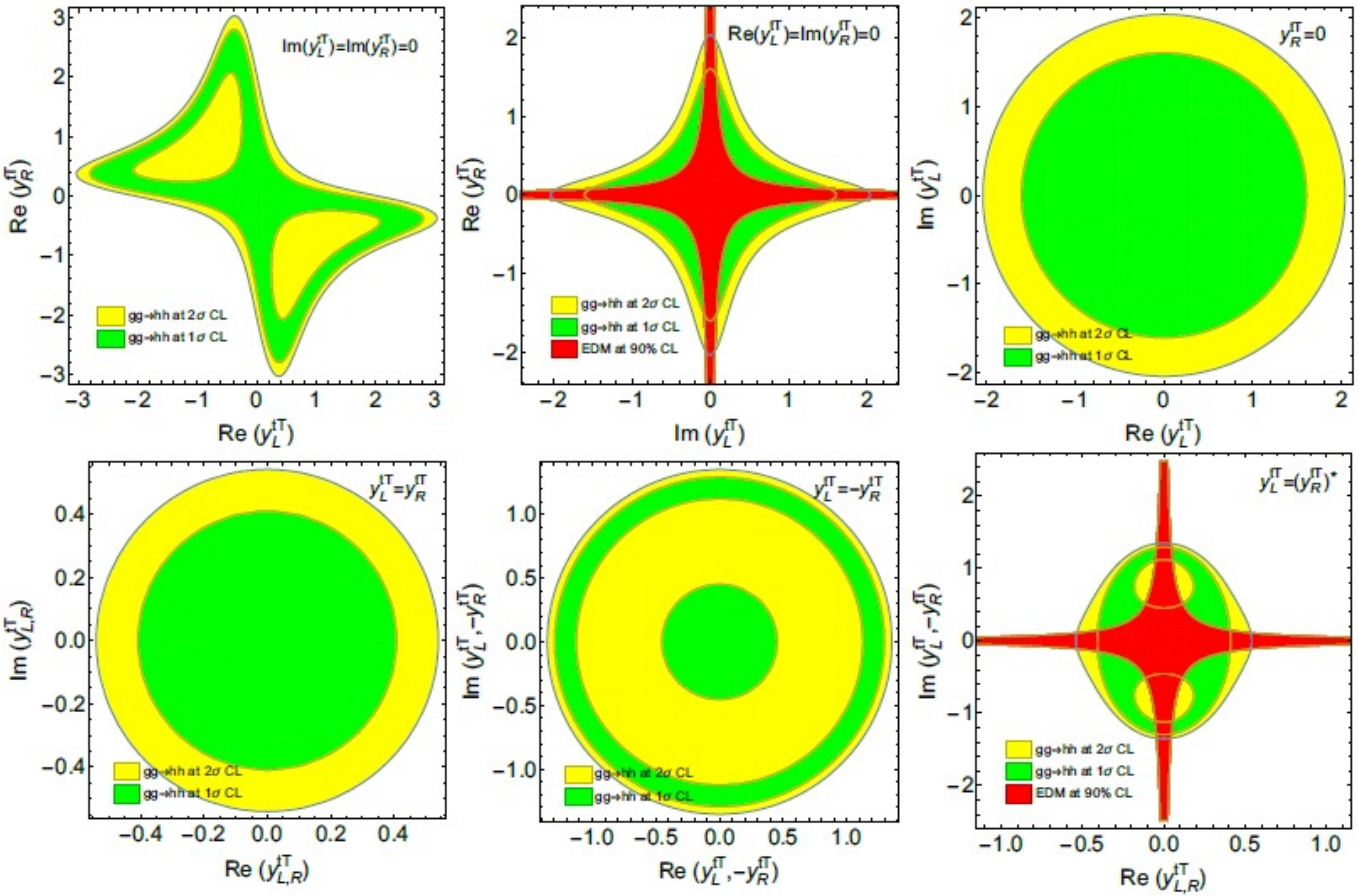
$$\mu_{hh} \equiv \frac{\sigma_{LO}(pp \to hh)}{\sigma_{LO}^{SM}(pp \to hh)}$$

G. Aad *et al* (ATLAS), Phys. Lett. B. 800 (2020), 135103
A. M. Sirunyan *et al* (CMS), Phys. Rev. Lett 122 (2019) 12, 121803

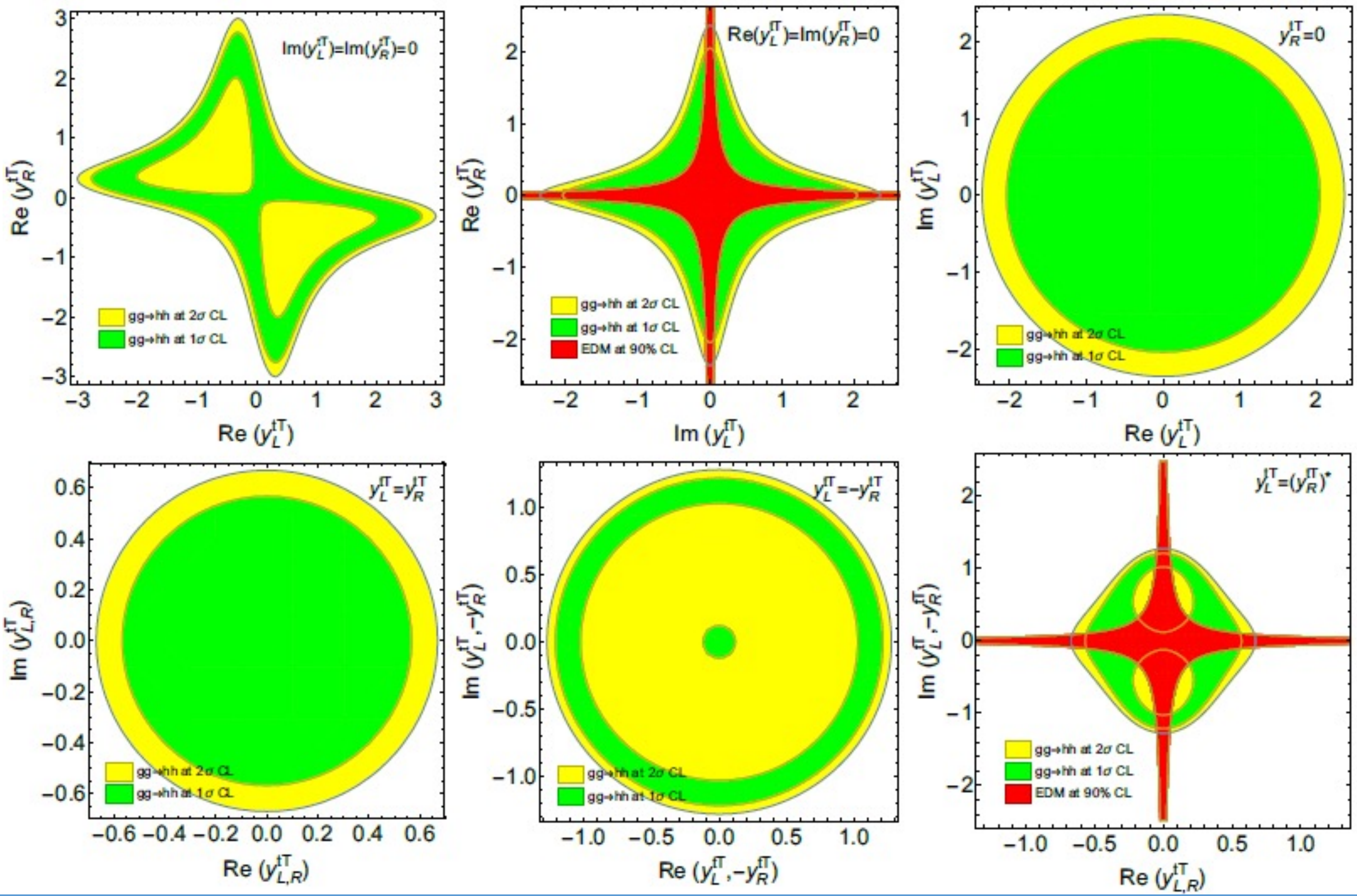
Current limit: $|\mu_{hh}| < 6.9$ at 95% CL

HL-LHC uncertainty: $\mu_{hh} = 1.00^{+0.41}_{-0.39}$ at 1σ CL
M. Cepeda *et al*, CERN Yellow Rep. Monogr. 7 (2019) 221-584

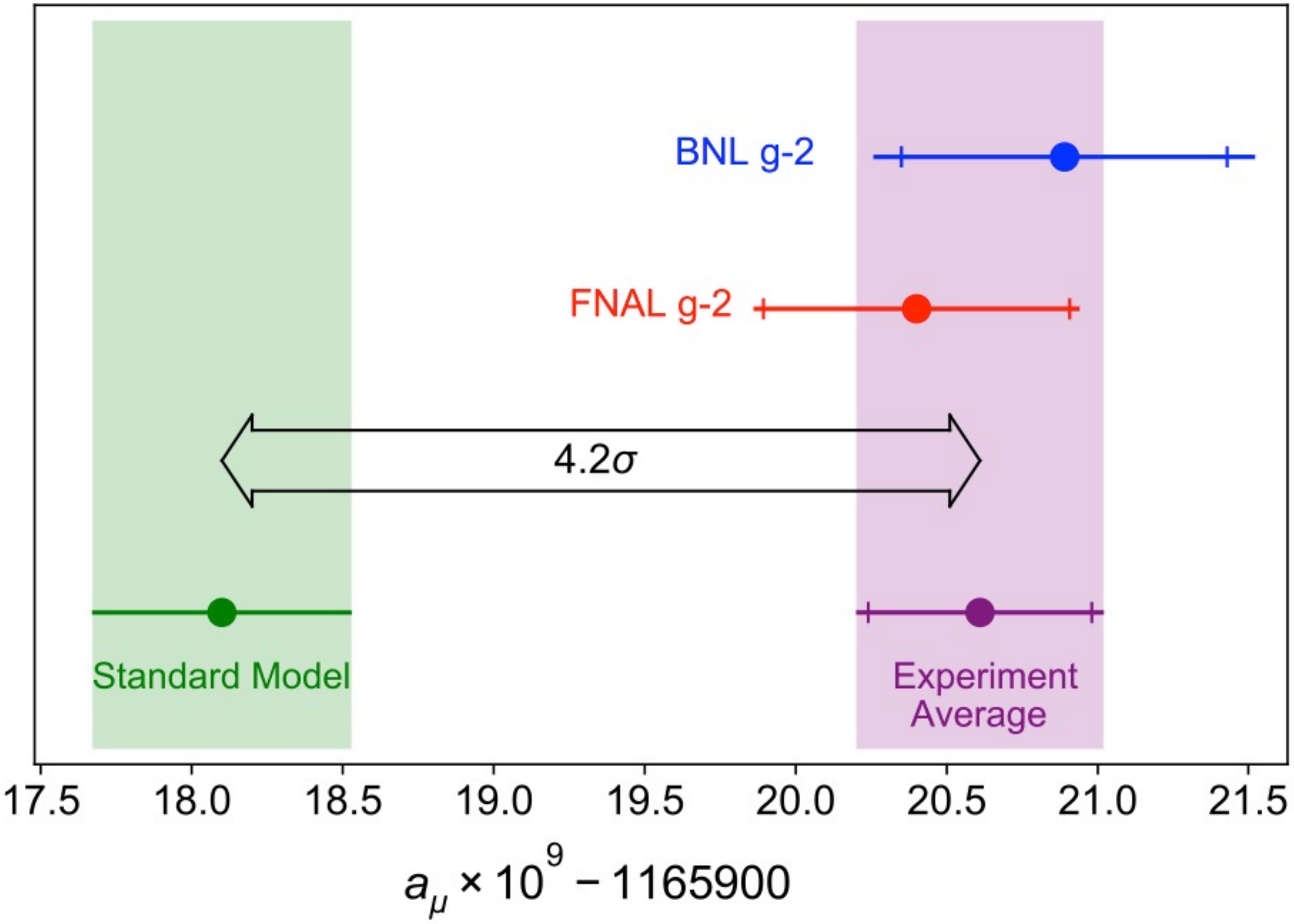
$m_T = 800\text{GeV}, s_L = 0.1$ & $\delta_{hhh} = 0$



$m_T = 800\text{GeV}, s_L = 0.1\ \&\ \delta_{hhh} = 0.5$



Muon g-2 anomaly



FNAL (2021)

$a_\mu(\mathrm{BNL}) = 116592080(63)\times10^{-11}$

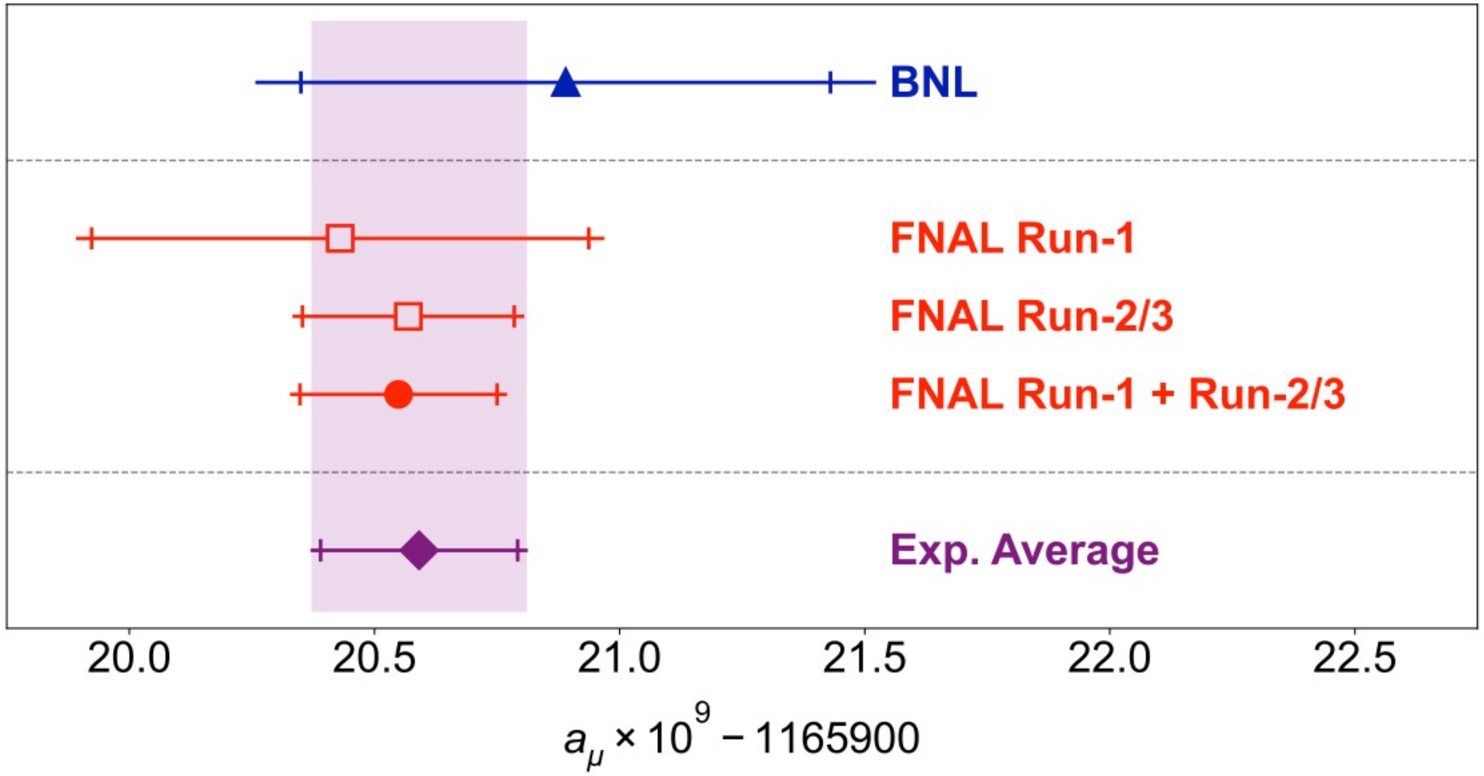
$a_\mu(\mathrm{FNAL}) = 116592040(54)\times10^{-11}$

$a_\mu(\mathrm{Exp}) = 116592061(41)\times10^{-11}$

FNAL muon g-2 experiment Run-1 data
(collected in 2018)

$a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = (251 \pm 59)\times10^{-11}$

muon g-2 or $(g-2)_\mu$ anomaly



FNAL (2023)

$a_\mu(\text{FNAL}) = 116592057(25)\times10^{-11}$

$a_\mu(\text{Exp}) = 116592059(22)\times10^{-11}$

FNAL muon g-2 experiment Run-2/3 data (collected in 2019/2020)

5.1σ?