Exclusive quarkonium production in Z decays

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2025粒子物理精细计算研讨会@ 河北大学, 2025.3.29

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1. Background

- **Quarkonium is a bound state of a heavy quark Q and heavy antiquark $\bar{Q}$.**

- **There are many important scales in a heavy quarkonium:**

$$M \gg Mv \gg Mv^2$$

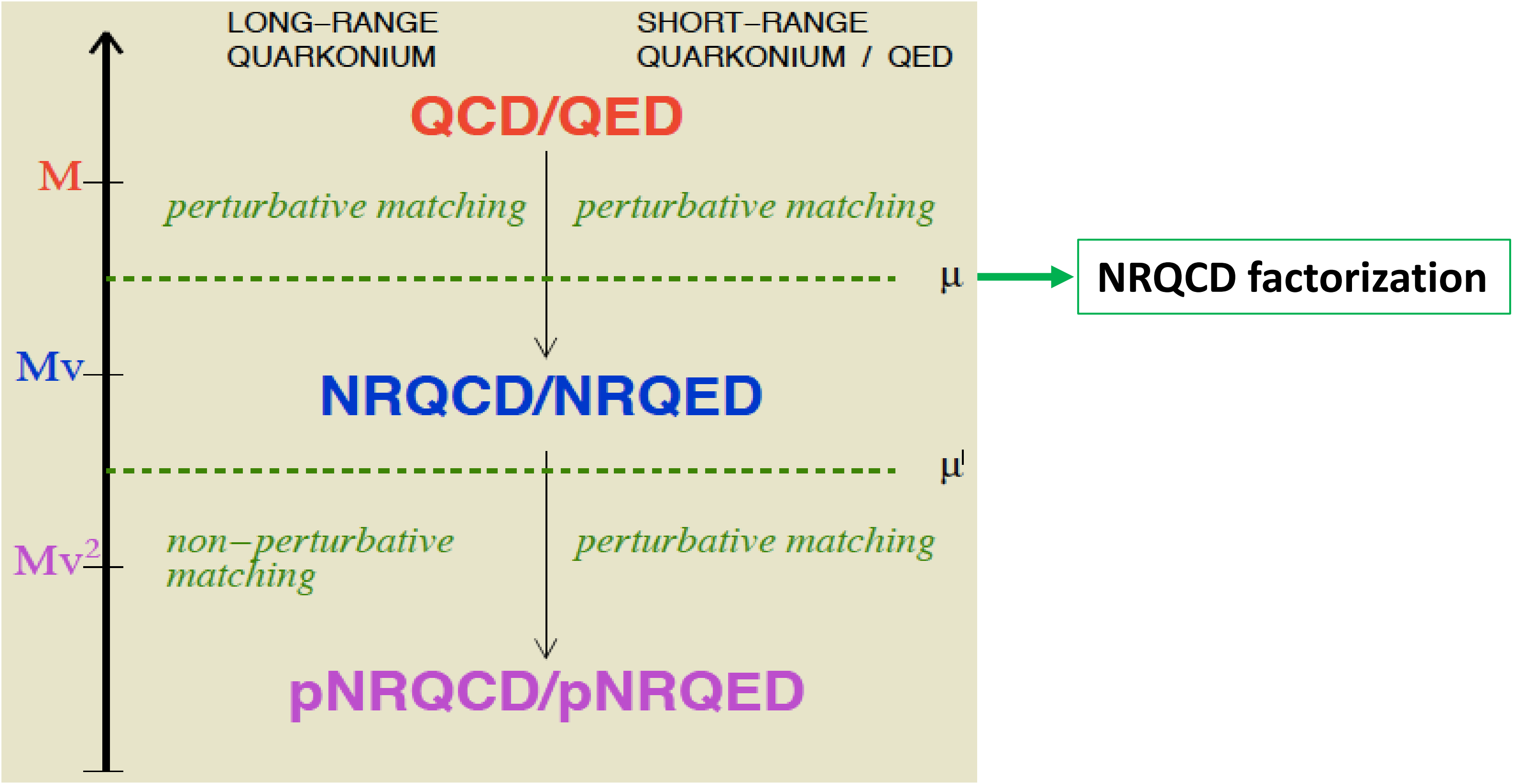
heavy quark mass ← M

typical heavy –quark momentum ← Mv

typical heavy-quark kinetic energy and binding energy ← Mv^2

$v^2 \approx 0.3$ for charmonium, $v^2 \approx 0.1$ for bottomonium.

- **Effective field theories provide a convenient way to separate scales**



➢ The features of exclusive quarkonium production processes

- They have clean experimental signature

  Can be easily distinguished from the background.

- Their production mechanism is simple

  Can be calculated to a higher accuracy.

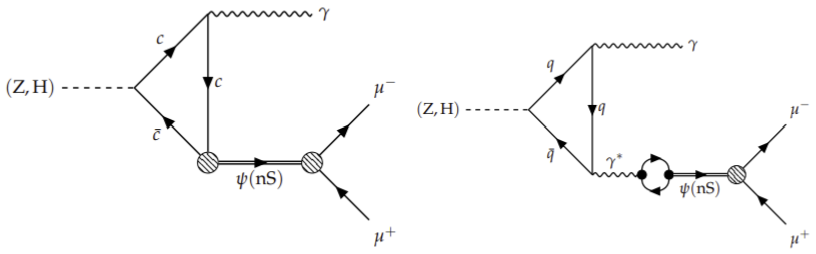
➢ A large number of Z bosons can be produced at colliders

Platforms	LHC	FCC-ee	CEPC	ILC
Number of Z bosons	$5 \times 10^9/yr$	5×10^{12}	4×10^{12}	$\sim 10^9$

2. Z-boson decays to quarkonium plus photon

➢ **Experimental searches**

- ATLAS Collaboration, PRL 114, 121801(2015); PLB 786, 134(2018); EPJC 83,781(2023).
- CMS Collaboration, EPJC 79,94(2019); arXiv:2411.15000.



These searches could calibrate the experimental techniques for measuring the $H \to V + \gamma$ decays.

Process	Experimental upper limit	Theoretical prediction
$Br(Z \to J/\psi + \gamma)$	6.0×10^{-7} (CMS2024)	1.1×10^{-7}
$Br(Z \to \psi(2S) + \gamma)$	1.3×10^{-6}(CMS2024)	4.8×10^{-8}
$Br(Z \to \Upsilon + \gamma)$	1.1×10^{-6}(ATLAS2023)	5.4×10^{-8}

➢ Theoretical studies

- NRQCD, $\mathcal{O}(\alpha_s^0 v^0)$, [B. Guberina et al, NPB 174, 317 (1980)].
- NRQCD,LCDA, $\mathcal{O}(\alpha_s^0 v^0)$, [A.V.Luchinsky et al, arXiv:1706.04091].
- LCDA, $\mathcal{O}(\alpha_s^1 v^0)$, [X.P. Wang and D.S.Yang, JHEP 06,121(2014)].

$Z \to V + \gamma$ (vector quarkonium production)

- LCDA, $[\mathcal{O}(\alpha_s^1 v^0) + \mathcal{O}(\alpha_s^0 v^2)]$, [T.C.Huang et al, PRD 92,014007(2015)].
- LCDA, $[\mathcal{O}(\alpha_s^1 v^0) + \mathcal{O}(\alpha_s^0 v^2)] + \mathrm{NLL}$, [Y.Grossman et al, JHEP 04,101(2015)
  G.T. Bodwin et al, PRD 97,016009(2018)].
- NRQCD+ LCDA, $\mathcal{O}(\alpha_s^2) + \mathrm{NLL}$, [W.L. Sang et al, PRD 106,094023(2022);
  W.L. Sang et al, PRD 108,014021(2023)].
- $Zb\bar{b}$ anomalous coupling, [B.Yan and P.Sun et al, PLB 829,137076(2022)].

The high-order corrections have been calculated up to $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(v^2)$.
The joint (QCD and relativistic) $\mathcal{O}(\alpha_s v^2)$ corrections?

NRQCD factorization

$$\mathcal{M}_{Z\to H+\gamma} = \left(c_0 + c_2\langle v^2\rangle_H\right)\sqrt{2m_H}\langle H|\psi^\dagger\mathcal{K}\chi|0\rangle,$$

(Up to order v^2)

$\mathcal{K} = 1$ for η_c (η_b)

$\mathcal{K} = \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$ for J/ψ (Υ)

$$c_i = c_i^{(0)} + c_i^{(1)}\frac{\alpha_s}{\pi} + \cdots$$

Short-distance coefficients (SDCs), IR safe

Can be calculated through perturbation theory

$$\langle v^2\rangle_H = \frac{\langle \boldsymbol{q}^2\rangle_H}{m_Q^2} = \frac{\langle H|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\mathcal{K}\chi|0\rangle}{m_Q^2\langle H|\psi^\dagger\mathcal{K}\chi|0\rangle},$$

Long-distance matrix elements (LDMEs),

Can be extracted from experimental data

We calculate the SDCs c_0 and c_2 up to $\mathcal{O}(\alpha_s)$.

➢ Extracting SDCs through matching

Applying the NRQCD factorization formulism to an on-shell $(Q\bar{Q})$ pair:

$$\mathcal{M}_{Z\to(Q\bar{Q})[n]+\gamma} = (c_0 + c_2 v^2)\langle (Q\bar{Q})[n]|\psi^\dagger \mathcal{K}\chi|0\rangle,$$

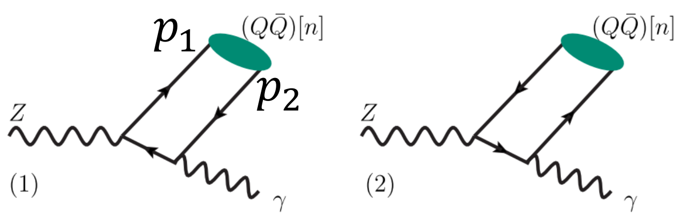
Calculated by Feynman diagrams

Calculated perturbatively based on the definition

The SDCs c_0 and c_2 up to $\mathcal{O}(\alpha_s)$ can be extracted.

➢ Calculating the amplitude for the on-shell $(Q\bar{Q})$ pair

Tree level Feynman diagrams



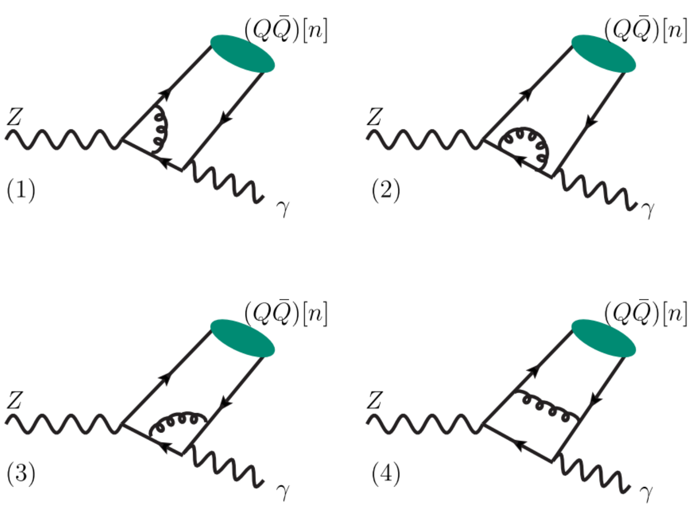
The relations of momenta:

$$p_1 = p + q \quad p_2 = p - q$$

$$p_1^2 = p_2^2 = m_Q^2, \qquad p^2 = E^2, \qquad p \cdot q = 0,$$

$$q^2 = m_Q^2 - E^2 = -m_Q^2 v^2.$$

One-loop level Feynman diagrams (part)



The projector for the $(Q\bar{Q})$ pair :

$$\Pi_1 = \frac{1}{2\sqrt{2}E(E+m_Q)}(\not{p}_2 - m_Q)\gamma_5(\not{p} + E)(\not{p}_1 + m_Q),$$

$$\Pi_3 = \frac{1}{2\sqrt{2}E(E+m_Q)}(\not{p}_2 - m_Q)\not{\epsilon}^*(\not{p} + E)(\not{p}_1 + m_Q),$$

$$\Lambda_1 = \frac{\mathbf{1}}{\sqrt{3}},$$

➢ Expanding the amplitude in v

$$\mathcal{M}_{Z\to(Q\bar{Q})[n]+\gamma} = \mathcal{M}|_{q=0} + q^\alpha \frac{\partial \mathcal{M}}{\partial q^\alpha}\bigg|_{q=0} + \frac{1}{2!} q^\alpha q^\beta \frac{\partial^2 \mathcal{M}}{\partial q^\alpha \partial q^\beta}\bigg|_{q=0} + \cdots,$$

Averaging over the spatial direction of q:

$$\int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} q^\alpha = 0, \qquad \int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} q^\alpha q^\beta = \frac{\mathbf{q}^2}{D-1}\Pi^{\alpha\beta}, \qquad \Pi_{\alpha\beta} \equiv -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{p^2}.$$

$$\mathcal{M}_{Z\to(Q\bar{Q})[n]+\gamma} = \mathcal{M}|_{q=0} + \frac{1}{2!}\frac{\mathbf{q}^2 \Pi_{\alpha\beta}}{(D-1)}\frac{\partial^2 \mathcal{M}}{\partial q^\alpha \partial q^\beta}\bigg|_{q=0}. \qquad \mathbf{q}^2 = m_Q^2 v^2$$

➢ γ_5 scheme in dimensional regularization （Larin scheme）

$$\gamma_\mu \gamma_5 = i\frac{1}{6}\epsilon_{\mu\rho\sigma\tau}\gamma^\rho\gamma^\sigma\gamma^\tau.$$

$$Z_5 = 1 - \frac{\alpha_s}{\pi}C_F.$$

Finite renormalization should be introduced to the axial vector vertex.

➢ Calculating the LDMEs for the on-shell $(Q\bar{Q})$ pair

The LDMEs for $(Q\bar{Q})$ pair up to $\mathcal{O}(\alpha_s v^2)$:

$$\langle (Q\bar{Q})[n]|\psi^\dagger \mathcal{K}\chi|0\rangle_{\overline{\rm MS}} = \left\{1 + \frac{2\alpha_s C_F}{3\pi}\left(\frac{\mu_R^2}{\mu_\Lambda^2}\right)^\epsilon \left[\frac{1}{\epsilon_{\rm IR}} + \ln(4\pi) - \gamma_E\right]\frac{\boldsymbol{q}^2}{m_Q^2}\right\} \times \langle (Q\bar{Q})[n]|\psi^\dagger \mathcal{K}\chi|0\rangle^{(0)},$$

The LO LDMEs for $(Q\bar{Q})$ pair:

$$|\langle (Q\bar{Q})[^1S_0]|\psi^\dagger \mathcal{K}\chi|0\rangle^{(0)}|^2 = 2N_c(2E)^2,$$
$$|\langle (Q\bar{Q})[^3S_1]|\psi^\dagger \mathcal{K}\chi|0\rangle^{(0)}|^2 = 2N_c(D-1)(2E)^2.$$

Input parameters

$$m_c = 1.4 \pm 0.2\ \text{GeV}, \quad m_b = 4.6 \pm 0.1\ \text{GeV}, \quad m_Z = 91.1876\ \text{GeV},$$

$$\sin^2\theta_W = 0.231, \quad \alpha = 1/128,$$

$$\alpha_s(\mu_R) = \frac{4\pi}{\beta_0 L}\left(1 - \frac{\beta_1 \ln L}{\beta_0^2 L}\right), \quad \Lambda_{\text{QCD}}^{n_f=5} = 0.226\ \text{GeV}, \quad \Lambda_{\text{QCD}}^{n_f=4} = 0.327\ \text{GeV}.$$

Input for LDMEs:

$$\langle \mathcal{O}_1 \rangle_{\eta_c} = 0.437\ \text{GeV}^3, \quad \langle \boldsymbol{q}^2 \rangle_{\eta_c} = 0.442\ \text{GeV}^2,$$

$$\langle \mathcal{O}_1 \rangle_{J/\psi} = 0.440\ \text{GeV}^3, \quad \langle \boldsymbol{q}^2 \rangle_{J/\psi} = 0.441\ \text{GeV}^2.$$

$$\langle \mathcal{O}_1 \rangle_{\Upsilon} = 3.069\ \text{GeV}^3, \quad \langle \boldsymbol{q}^2 \rangle_{\Upsilon} = -0.193\ \text{GeV}^2.$$

$$\langle \mathcal{O}_1 \rangle_{\eta_b} \approx \langle \mathcal{O}_1 \rangle_{\Upsilon} \quad \langle \boldsymbol{q}^2 \rangle_{\eta_b} \approx \langle \boldsymbol{q}^2 \rangle_{\Upsilon}.$$

Extracted from the exclusive decays of quarkonia.

[G.T. Bodwin et al, PRD 77,094017(2008); H. S. Chung et al, PLB 697,48(2011)].

Numerical results

- Decay width of $Z \to \eta_c + \gamma$ (eV)

[G.Y. Wang, X.C. Zheng, X.G. Wu et al, PRD 109,074004(2024)].

$\alpha_s(\mu_R)$	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s v^2)$	Total
$\alpha_s(2m_c)=0.263$	40.32	8.37	−7.59	−7.54	33.57
$\alpha_s(m_Z/2)=0.132$	40.32	4.20	−7.59	−3.78	33.15
$\alpha_s(m_Z)=0.118$	40.32	3.76	−7.59	−3.38	33.11

- Decay width of $Z \to J/\psi + \gamma$ (eV)

$\alpha_s(\mu_R)$	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s v^2)$	Total
$\alpha_s(2m_c)=0.263$	275.58	15.00	−10.26	−23.58	256.75
$\alpha_s(m_z/2)=0.132$	275.58	7.53	−10.26	−11.83	261.02
$\alpha_s(m_z)=0.118$	275.58	6.73	−10.26	−10.58	261.48

The $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s v^2)$ corrections have important contributions.

Numerical results

- Decay width of $Z \to \eta_b + \gamma$ (eV)

[G.Y. Wang, X.C. Zheng, X.G. Wu et al, PRD 109,074004(2024)].

$\alpha_s(\mu_R)$	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s v^2)$	Total
$\alpha_s(2m_b) = 0.178$	69.32	−0.61	0.53	0.22	69.46
$\alpha_s(m_z/2) = 0.132$	69.32	−0.46	0.53	0.16	69.56
$\alpha_s(m_z) = 0.118$	69.32	−0.41	0.53	0.14	69.59

- Decay width of $Z \to \Upsilon + \gamma$ (eV)

$\alpha_s(\mu_R)$	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s v^2)$	Total
$\alpha_s(2m_b) = 0.178$	146.24	−15.50	0.20	0.19	131.13
$\alpha_s(m_z/2) = 0.132$	146.24	−11.50	0.20	0.14	135.09
$\alpha_s(m_z) = 0.118$	146.24	−10.28	0.20	0.13	136.29

➢ Numerical results

[G.Y. Wang, X.C. Zheng, X.G. Wu et al, PRD 109,074004(2024)].

- Comparison of the contributions (eV) from different orders:

Decays	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s v^2)$	$\mathcal{O}(\alpha_s^2)$	Total
$Z \to \eta_c + \gamma$	29.1	2.4	−5.5	−2.4	−4.4	19.2
$Z \to J/\psi + \gamma$	197.7	1.9	−7.3	−7.5	−21.8	163.1
$Z \to \eta_b + \gamma$	65.9	−0.7	0.5	0.1	−1.5	64.4
$Z \to \Upsilon + \gamma$	139.2	−10.8	0.2	0.1	−3.6	125.0

$\mathcal{O}(\alpha_s^2)$ corrections are taken from [W.L.Sang, D.S.Yang et al, 2023]

Numerical results

- Branching fractions from theoretical calculation

$$\mathrm{Br}(Z \to \eta_c + \gamma) = 1.33^{+0.56}_{-0.54} \times 10^{-8},$$

$$\mathrm{Br}(Z \to J/\psi + \gamma) = 1.05^{+0.24}_{-0.20} \times 10^{-7},$$

$$\mathrm{Br}(Z \to \eta_b + \gamma) = 2.79^{+0.20}_{-0.19} \times 10^{-8},$$

$$\mathrm{Br}(Z \to \Upsilon + \gamma) = 5.41^{+0.39}_{-0.39} \times 10^{-8}.$$

- Experimental upper limits

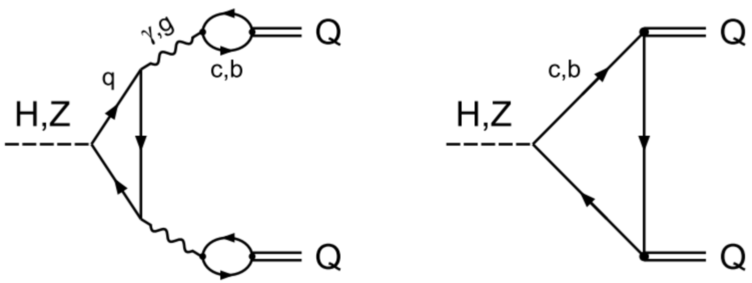
$$\mathrm{Br}(Z \to J/\psi + \gamma) < 6.0 \times 10^{-7}$$ [CMS,2024]

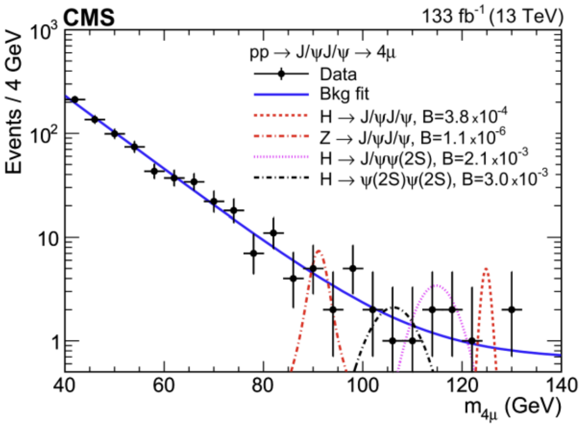
$$\mathrm{Br}(Z \to \Upsilon + \gamma) < 1.1 \times 10^{-6}$$ [ATLAS,2023]

3. Z-boson decays to double J/ψ

➢ Experimental searches

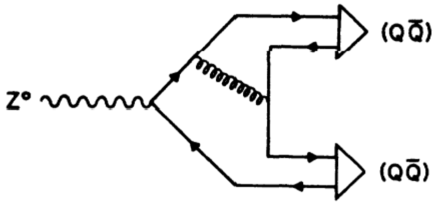
- CMS Collaboration, PLB 797, 134811 (2019);
  PLB 842, 137534 (2023);





$$Br(Z \to J/\psi + J/\psi) < 1.4 \times 10^{-6}$$

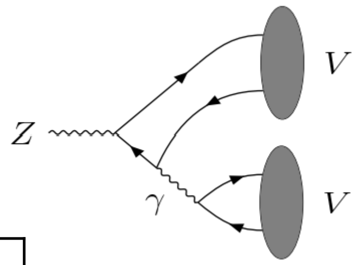
Theoretical studies



- LO QCD, [L. Bergstrom et al, PRD 41, 3513 (1990)].
- LO QCD, [A.K. Likhoded et al, Mod. Phys. Lett. A 33, 1850078 (2018)].

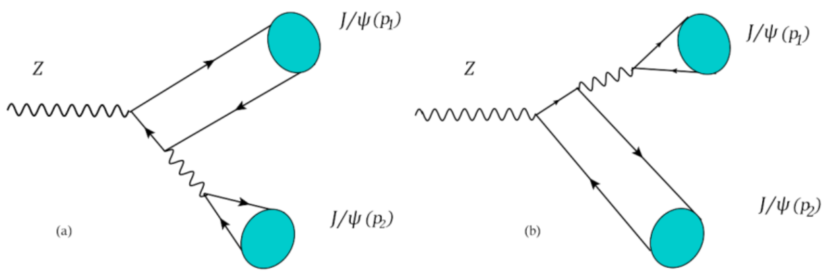
Including the photon fragmentation contribution

- LO QCD+QED Fragmentation, [D.N. Gao et al, CPC 47,043106 (2018)].
- QCD+QED Frag (NLO), [C. Li et al, JHEP 10,120 (2023);
  X. Luo et al, arXiv:2209.08802].



Theoretical branching fractions	Reference
2.3×10^{-14}	A.K. Likhoded(2018)
1.1×10^{-10}	D.N. Gao (2018)
1.1×10^{-10}	C. Li(2023)

➢ Motivation



The dominant contribution comes from the photon fragmentation.

- **The perturbative expansion of the amplitude of $\gamma^* \to J/\psi$ does not converge**

Γ(keV) \ V	LO	NLO	NNLO	N^{3}LO Direct ($m_M = 0$)	N^{3}LO Direct ($m_M \neq 0$)	N^{3}LO Total	PDG
Υ	1.6529	$1.1095^{+0.0888}_{-0.2922}$	$0.9750^{+0.0642}_{-0.0942}$	$0.1948^{+1.5900}_{-0.1948}$	$0.1763^{+1.9577}_{-0.1763}$	$0.1764^{+1.9560}_{-0.1764}$	1.340 ± 0.018
J/ψ	4.8392	$2.6999^{+0.4925}_{-1.0391}$	$1.3138^{+0.7094}_{-1.1444}$	$3.2219^{+123.4838}_{-3.2219}$			5.53 ± 0.10

F. Feng and Y. Jia et al, arXiv:2207.14259

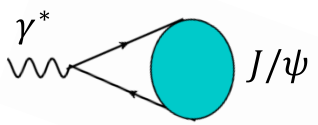
- **Large Logs $\ln(m_Z^2/m_c^2)$ appear in the amplitude of $Z \to J/\psi + \gamma^*$**

➢ Fragmentation amplitude

$$i\mathcal{M}^{\text{fr}}_{Z\to J/\psi+J/\psi} = i\mathcal{M}^{\mu}_{Z\to J/\psi(P_1)+\gamma^*(P_2)} \frac{-i}{m^2_{J/\psi}} i\mathcal{M}_{\gamma^*\to J/\psi,\mu} + (1 \leftrightarrow 2),$$

- **Extracting** $\mathcal{M}_{\gamma^*\to J/\psi,\mu}$ **from experimental data**

$$i\mathcal{M}^{\mu}_{\gamma^*\to J/\psi} = -ie\langle J/\psi|J^{\mu}(x=0)|0\rangle = -ieg_{J/\psi\gamma}\epsilon^{*\mu}_{J/\psi},$$



$$\Gamma(J/\psi \to e^+e^-) = \frac{4\pi\alpha^2(m_{J/\psi})g^2_{J/\psi\gamma}}{3m^3_{J/\psi}}.$$

$Z \to J/\psi + J/\psi$

- **Calculating $\mathcal{M}^{\mu}_{Z\to J/\psi(P_1)+\gamma^*(P_2)}$ through light-cone approach**

$$i\mathcal{M}_{Z\to J/\psi+\gamma,\mu} = i\mathcal{A}\,\epsilon_{\xi\mu\nu\rho}\epsilon_Z^{\xi}\epsilon_{J/\psi}^{*\nu}p_\gamma^{\rho},$$

$$i\mathcal{A} = -\frac{ee_c g_Z g_A^c m_{J/\psi}}{m_Z^2} f_{J/\psi}^{\parallel}\int_0^1 dx\, T_H(x,\mu)\phi_{J/\psi}^{\parallel}(x,\mu). \qquad \mu = m_Z$$

Hard-scattering kernel ($T_H(x,\mu)$)

LCDA ($\phi_{J/\psi}^{\parallel}$)

$$T_H(x,\mu) = T_H^{(0)}(x,\mu) + \frac{\alpha_s(\mu)}{4\pi}T_H^{(1)}(x,\mu),$$

X.P. Wang and D.S. Yang et al, JHEP 06,121(2014)

$$T_H^{(0)}(x,\mu) = \frac{1}{x(1-x)},$$

$$T_H^{(1)}(x,\mu) = C_F\frac{1}{x(1-x)}\Big\{\big[3+2x\ln(1-x) + 2(1-x)\ln x\big]\left(\ln\frac{m_Z^2}{\mu^2} - i\pi\right) + x\ln^2(1-x) + (1-x)\ln^2 x - (1-x)\ln(1-x) - x\ln x - 9\Big\}.$$

Calculating the LCDA based on the NRQCD factorization

$$\phi^{\parallel}_{J/\psi}(x,\mu_0) = \phi^{\parallel(0)}_{J/\psi}(x,\mu_0) + \langle v^2\rangle_{J/\psi}\phi^{\parallel(v^2)}_{J/\psi}(x,\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi}\phi^{\parallel(1)}_{J/\psi}(x,\mu_0) + \mathcal{O}(\alpha_s^2, \alpha_s v^2, v^4).$$

$\mu_0 = m_c$

$$\phi^{\parallel(0)}_{J/\psi}(x,\mu_0) = \delta(x - \tfrac{1}{2}).$$

$$\phi^{\parallel(v^2)}_{J/\psi}(x,\mu_0) = \frac{\delta^{(2)}(x-\frac{1}{2})}{24},$$

W. Wang, J. Xu and D.S. Yang et al, JHEP 12,012(2017).

$$\phi^{\parallel(1)}_{J/\psi}(x,\mu_0) = C_F\theta(1-2x)\left\{\left[\left(4x + \frac{8x}{1-2x}\right)\left(\log\frac{\mu_0^2}{m_c^2(1-2x)^2} - 1\right)\right]_+ - [8x]_+ + \left[\frac{16x(1-x)}{(1-2x)^2}\right]_{++}\right\} + (x \leftrightarrow 1-x),$$

X.P. Wang and D.S. Yang et al, JHEP 06,121(2014)

J.P. Ma and Z.G. Si, PLB 419,426(2007)

Solving the ERBL equation

$\phi^{\parallel}_{J/\psi}(x,\mu_0)$ ERBL evolution $\phi^{\parallel}_{J/\psi}(x,\mu)$

$\mu_0 = m_c$ → $\mu = m_Z$

The large logs of m_Z^2/m_c^2 are resumed through the ERBL evolution.

$$\frac{\partial}{\partial \ln \mu^2}\phi^{\parallel}_{J/\psi}(x,\mu) = \int_0^1 dy\, V_{\parallel}[x,y;\alpha_s(\mu)]\,\phi^{\parallel}_{J/\psi}(y,\mu),$$

$$\phi^{\parallel}_{J/\psi}(x,\mu) = \sum_{n=0}^{\infty} a_n^{\parallel}(\mu)\, x(1-x)\, C_n^{(3/2)}(2x-1),$$

Gegenbauer expansion of the LCDA

$$T_H(x,\mu) = \sum_{n=0}^{\infty} N_n b_n(\mu) C_n^{(3/2)}(2x-1).$$

$$\int_0^1 dx\, T_H(x,\mu)\phi^{\parallel}_{J/\psi}(x,\mu) = \sum_{n=0}^{\infty} b_n(\mu)a_n(\mu) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} b_n(\mu)U_{nk}(\mu,\mu_0)a_k(\mu_0).$$

The sum over *n* is divergent!

Abel summation:

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} b_n(\mu)U_{nk}(\mu,\mu_0)a_k(\mu_0) = \lim_{z\to 1}\sum_{n=0}^{\infty}\sum_{k=0}^{n} b_n(\mu)z^n U_{nk}(\mu,\mu_0)a_k(\mu_0).$$

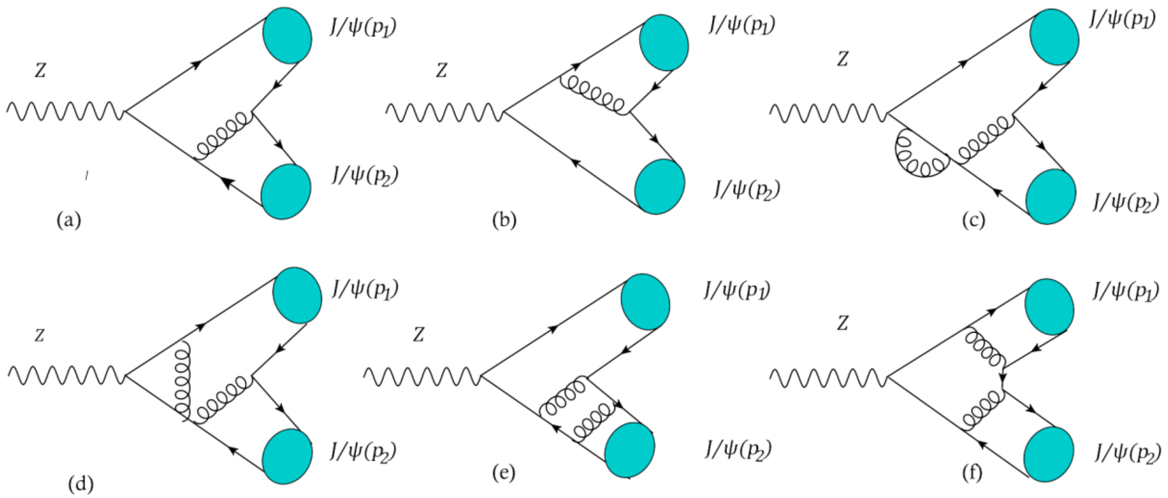
Padé approximant:

$$f(z) = \sum_{k=0}^{\infty} c_k z^k,$$

$$R_{[m/n]}(z) = \frac{P_m(z)}{Q_n(z)} = \frac{a_0 + a_1 z + \cdots + a_m z^m}{1 + b_1 z + \cdots + b_m z^n},$$

$$f(z) - R_{[m/n]}(z) = \mathcal{O}(z^{m+n+1}).$$

➢ Non-fragmentation amplitude



NRQCD factorization:

$$\mathcal{M}^{\rm nfr}_{Z\to J/\psi+J/\psi} = 2m_{J/\psi}\left(c_0 + c_2\langle v^2\rangle_{J/\psi}\right)\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\chi|0\rangle^2,$$

➢ Numerical results

[G.Y. Wang, X.G. Wu , X.C. Zheng et al, EPJC 84,544(2024)].

- **The decay width of $Z \to J/\psi + J/\psi$ $(10^{-12}$GeV)**

	Fragmentation	Non-fragmentation	Interference	Total
Γ	154.0	2.4	60.2	216.6

- **The branching fraction of $Z \to J/\psi + J/\psi$**

Br(this work)	Br(C. Li et al,2023)	Br(CMS,2023)
$8.66^{+1.48}_{-0.69} \times 10^{-11}$	$1.110^{+0.334+0.054}_{-0.241-0.001} \times 10^{-10}$	$< 1.4 \times 10^{-6}$

Summary

- The **exclusive quarkonium production** in **Z-boson decays** provides a good platform for studying the strong interaction and Z boson properties;
- The $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s v^2)$ **corrections** are important in the precise prediction of the decay widths of $Z \to \mathcal{Q} + \gamma$;
- The accuracy of the theoretical prediction for the decay width of $Z \to J/\psi + J/\psi$ are improved (**light-cone approach, $M_{\gamma^* \to J/\psi}$**) compared to the fixed-order calculations in the literature .

Thank you!