重味夸克衰变的次次次领头阶微扰 QCD 修正

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粒子物理标准模型及新物理精细计算研讨会

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Based on on-going work with **Xiang Chen**, **Xin Guan** and **Yan-Qing Ma**

Why Study Top Quark in Particular

- Heaviest fundamental particle in SM

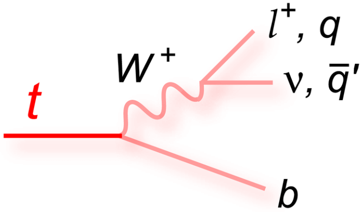
$$\boxed{m_t = 172.69 \pm 0.30\,\text{GeV}}$$

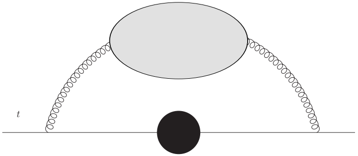
- Precision test of SM mechanism, and probs for possible BSM physics

- Decay exclusively to $b + W$ before hadronization:

$$\boxed{\Gamma_t = 1.4\,\text{GeV} \gg \Lambda_{\text{QCD}}}$$

- Convergence of the perturbative QCD series (e.g. renormalon issue)

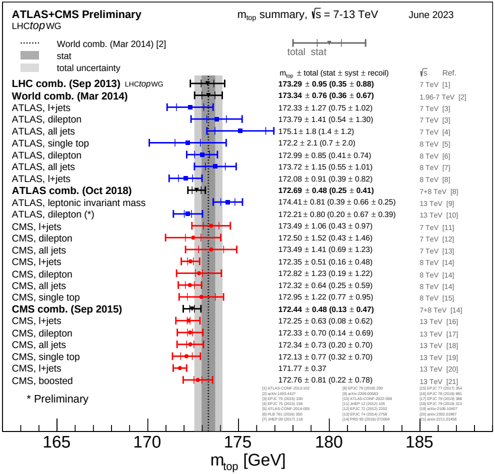




- **PDG average for m_t:**
  $172.69 \pm 0.30\,\mathbf{GeV}$

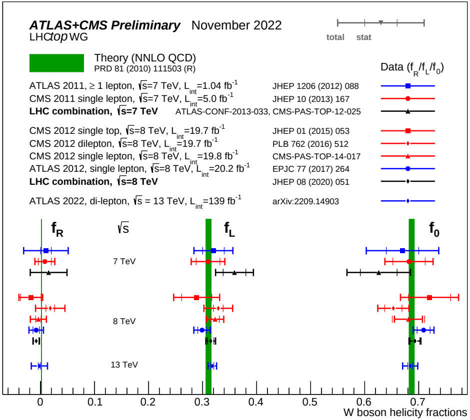
- **Current best measurement for Γ_t:**
  $1.36 \pm 0.02(\text{stat.})^{+0.14}_{-0.11}(\text{syst.})\,\text{GeV}$.

- **Experimental uncertainties anticipated at future colliders:**
  $20 \sim 26\,\text{MeV}$



The W-helicity fractions in Top Decay

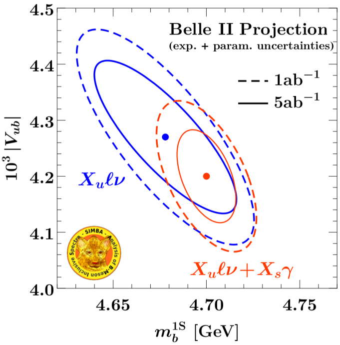
W from $t \to b + W^+ + X_{\text{QCD}}$ is polarized even if the t-quark is unpolarized



- The current best measurements: $f_0 = 0.684 \pm 0.005\,(\text{stat.}) \pm 0.014\,(\text{syst.})$, $f_\text{L} = 0.318 \pm 0.003\,(\text{stat.}) \pm 0.008\,(\text{syst.})$ and $f_\text{R} = -0.002 \pm 0.002\,(\text{stat.}) \pm 0.014\,(\text{syst.})$.
- Notoriously difficult to be predicted theoretically to high precision

The $B \to X_u \ell \bar{\nu}_\ell$ @ Belle II

- Measurement of $|V_{ub}|$ (and $|V_{cb}|$) at the unprecedented $\mathcal{O}(0.01)$ level
- Clarifying the long-standing discrepancy between the *inclusive* and *exclusive* measurements of $|V_{ub}|$
- Precision determination of the (global) non-perturbative parameters and *shape* functions for B-mesons
- $\cdots\cdots$ [→ See talks by Wang and Qin]



	Statistical	Systematic (reducible, irreducible)	Total expt.	Theory	Total
1 ab^{-1}	2.5	$(2.9, 1.6)$	4.1	$2.5 - 4.5$	$4.8 - 6.1$
5 ab^{-1}	1.1	$(1.3, 1.6)$	2.3	$2.5 - 4.5$	$3.4 - 5.1$
50 ab^{-1}	0.4	$(0.4, 1.6)$	1.7	$2.5 - 4.5$	$3.0 - 4.8$

Precise measurements of both the **inclusive and differential** observables for $B \to X_u \ell \bar{\nu}_\ell$ are crucial to achieve the above goals.

Much Theoretical Work Done So Far

Given the key role played by the top-quark both in SM precision test and searching for BSM, there have been vast amount of works done in literature regarding $t \to b + W^+ + X_{\text{QCD}}$.

- **The inclusive Γ_t**
  - Up to NNLO in QCD: [Jezabek etc 88; Czarnecki etc 90; Li etc 90; Czarnecki etc 98; Chetyrkin etc 99; Fischer etc 01; Blokland etc 04'05;......; Czarnecki etc 10; Meng etc 22; Chen etc 22]

    @**NNNLO** in QCD: [**LC**, Chen, Guan, Ma 23; Chen, Li, Li, Wang, Wang, Wu 23; Yan, Wu, Zhou, Li, Shan 24] [Datta, Rana, Ravindran, Sarkar 23 (only virtuals)]
  - NLO Electroweak: [Denner Sack 91; Eilam, Mendel Migneron Soni 91; Basso, Dittmaier, Huss, Oggero 15]

- **W-helicities $f_{\text{L,R,0}}$**
  - @NNLO in QCD: [Czarnecki, Korner, Piclum 10; Gao, Li, Zhu 12; Brucherseifer, Caola, Melnikov 13; Czarnecki, Groote Korner ,Piclum 18]

    @**NNNLO** in QCD: [**LC**, Chen, Guan, Ma 23]
  - NLO Electroweak: [Do, Groote, Korner, Mauser 02]

- **Differential results**
  - QCD:

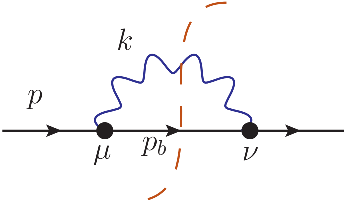
    @NLO [Fischer, Groote, Korner, Mauser 01; Brandenburg, Si, Uwer 02; Bernreuther, Gonzalez , Mellei 14; Kniehl, Nejad 21]
    @NNLO [Gao, Li, Zhu 12; Brucherseifer, Caola, Melnikov 13; Campbell, Neumann, Sullivan 20]
    @**NNNLO** in QCD: [**LC**, Chen, Guan, Ma 23]

Cut Diagrams for Heavy-to-light Decay Width

Γ_t in terms of the **semi-inclusive** $\mathcal{W}_{tb}^{\mu\nu}$

$$\Gamma_t = \frac{1}{2\,m_t} \int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1} 2E} \, \mathcal{W}_{tb}^{\mu\nu} \sum_{\lambda}^{L,R,0} \varepsilon_\mu^*(k,\lambda)\, \varepsilon_\nu(k,\lambda)\,,$$

$$\left| \mathcal{M}_{t\to b+W} \right|^2 \Rightarrow$$



$$\mathcal{W}_{tb}^{\mu\nu}(p,k) \;=\; W_1\, g^{\mu\nu} \;+\; W_2\, p^\mu p^\nu \;+\; W_3\, k^\mu k^\nu$$
$$+\; W_4 \left(p^\mu k^\nu \;+\; k^\mu p^\nu \right) \;+\; W_5\, i\epsilon^{\mu\nu\rho\sigma}\, p_\rho\, k_\sigma\,,$$

Selection Criteria: the cut diagrams of t-quark self-energy function with exactly one (cut) W propagator interacting with the *external* t-quark plus (up to 3) QCD loops

Results for the Inclusive Γ_t

The QCD effects on Γ_t in SM can be parameterized as

$$\Gamma_t = \Gamma_0 \left[\mathbf{c}_0 + \frac{\alpha_s}{\pi} \mathbf{c}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{c}_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \mathbf{c}_3 + \mathcal{O}(\alpha_s^4) \right],$$

with $\Gamma_0 \equiv \frac{G_F\, m_W^2\, m_t\, |V_{tb}|^2}{12\sqrt{2}}$.

At $\mu = m_t/2$, motivated by the QCD radiative energy $m_t - m_W - m_b$, our N3LO result reads: [LC, Chen, Guan, Ma 23]

$$\begin{aligned} \Gamma_t &= 1.48642 - 0.140877 - 0.023306 - 0.007240 \text{ GeV} \\ &= \mathbf{1.31500} \text{ GeV} \end{aligned}$$

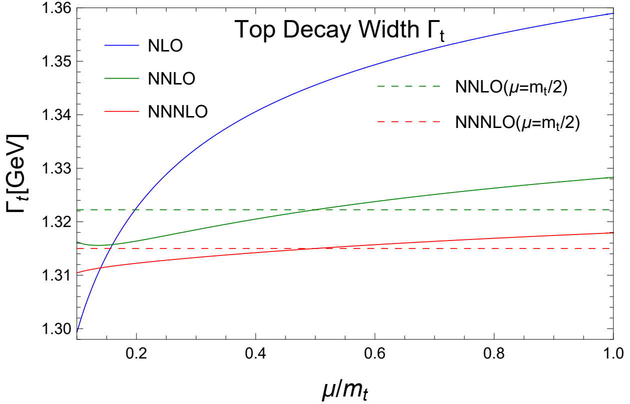
SM Inputs:

$$G_F = 1.166379 \times 10^{-5}\text{GeV}^{-2}, \quad m_t = 172.69\text{GeV},$$
$$m_W = 80.377\text{GeV}, \quad \alpha_s(m_t/2) \approx 0.1189.$$

- The leading-color part of Γ_t agrees with a parallel computation [Chen, Li, Li, Wang, Wang, Wu 23]
- Further improvement using (extended) PMC methods [Yan, Wu, Zhou, Li, Shan 24]

QCD Scale Uncertainty of Γ_t in OS scheme

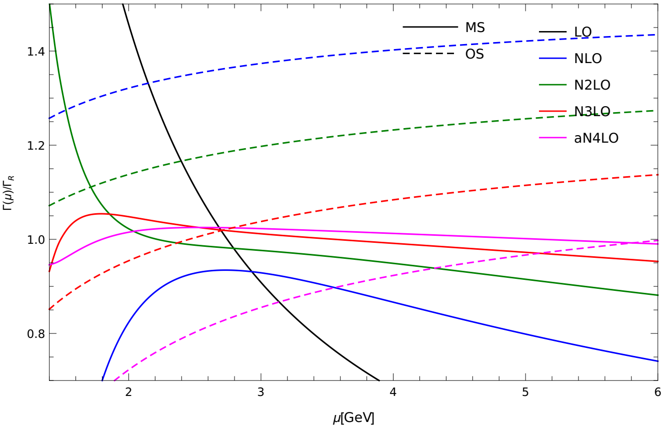
The scale dependence of the fixed-order results for Γ_t in $\mu/m_t \in [0.1, 1]$ (in OS scheme)



- NNLO scale-variation **never** cover the NNNLO result at any scales less than $\mu/m_t = 0.6$.
- Pure $\mathcal{O}(\alpha_s^3)$ correction decreases Γ_t by $\sim 0.8\%$ of the NNLO result at $\mu = m_t$ roughly 10 MeV(exceeding NNLO scale-hand)

Results for $\Gamma_{\rm sl}^{b\to u}$ in $\overline{\rm MS}$ scheme

$$\Gamma_{\rm sl}^{b\to u} \equiv \Gamma_0(m_b)\mathbf{C}_{\rm p} = \overline{\Gamma}_0 \left(1 + 0.3036075 + 0.1365820 + 0.06841766 + 0.034184\right)$$



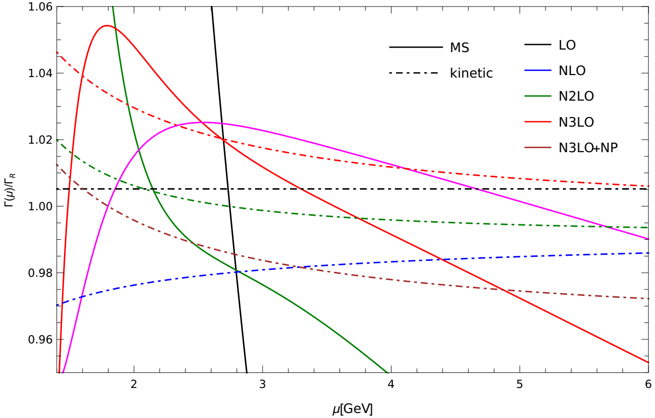
Every one more perturbative order higher in the $\overline{\rm MS}$ result, the term is reduced roughly by 1/2.

However, the scale uncertainty of $\bar{\Gamma}_{\rm sl}^{b\to u}$ at N3LO is still $[+6\%, -8\%]$ (!) for $\mu/\overline{m}_b \in [1/2, 2]$

Result for $\Gamma(B \to X_u \ell \bar{\nu}_\ell)$ in kinetic-mass scheme

In Heavy Quark Expansion theory

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0(m_b) \left[\mathbf{C}_\mathrm{p} \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2\, m_b^2} \right) - 2\, \frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^3 / m_b^3\right) \right].$$

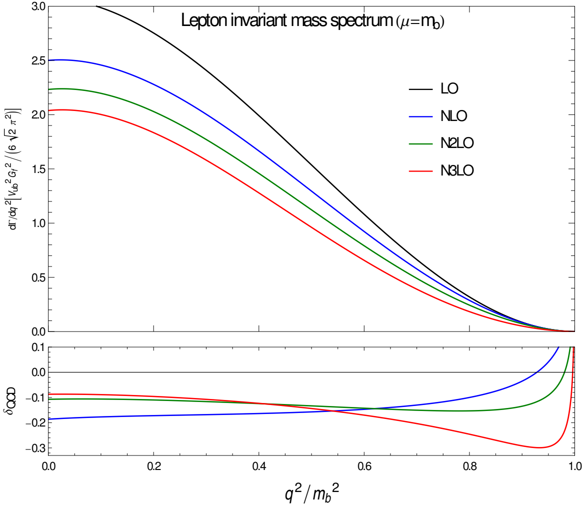


We obtain the **most accurate theoretical prediction for the inclusive** $B \to X_u \ell \bar{\nu}_\ell$ (kine-scheme):

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{|V_{ub}|^2}{|3.82 \times 10^{-3}|^2} \left(6.53 \pm 0.12 \pm 0.13 \pm 0.03\right) \times 10^{-16}\,\mathrm{GeV}.$$

Lepton-pair invariant-mass spectrum $d\Gamma_{sl}^{b\to u}/dq^2$

In the OS scheme, δ_{QCD} is large and does not converge quickly.



- $\mathcal{O}(\alpha_s^3)$ Leading-color **integrated** form agrees with [Chen, Li, Li, Wang, Wang, Wu 23] .
- $\mathcal{O}(\alpha_s^3)$ **integrated** fermionic part agrees with a recent evaluation [Fael, Usovitsch 23] .

Subtlety with Re-expanding non-inclusive Observables

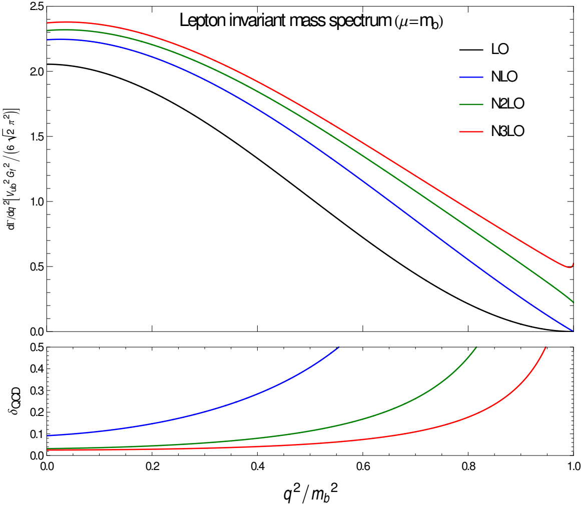
Rewrite $\Gamma_{\rm sl}^{b\to u} = \int_0^{m^2} f(q^2, m^2)\, \mathrm{d}q^2$ using an alter. $\bar{m}$ is **NOT** the same as changing mass-renormalization schemes in the usual sense.

- Consistency for the integral:

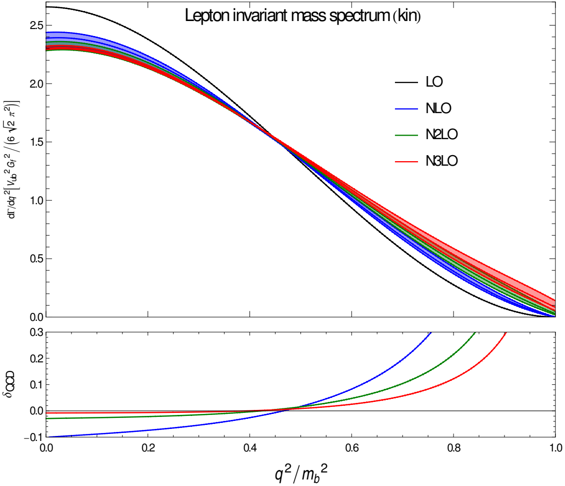
$$\int_0^{m^2} f(q^2, m^2)\, \mathrm{d}q^2 =$$

$$\int_0^{\bar{m}^2 + dm} f(q^2, \bar{m}^2 + dm)\, \mathrm{d}q^2$$

- q^2-range bounded by $\bar{m}(\mu)$ (phase-space expanded too)
- Not (necess.) 0 at **new** ends (non-0 high-order derivatives)
- Dirac-δ type boundary(!) term (histogramming in bins)

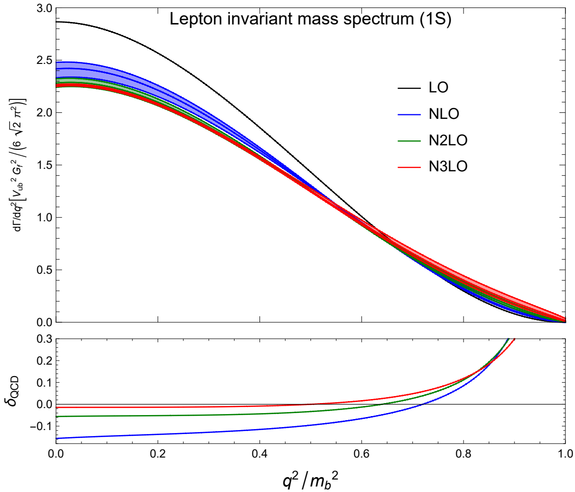


$\mathrm{d}\Gamma_{\mathrm{sl}}^{b\to u}/\mathrm{d}q^2$ in kinetic-mass scheme



- δ_{QCD} exhibits the usual expected (regular) behavior
- The particular (crossing) **pattern** helps to "understand" the puzzle (scale-uncertainty of the total inclusive result not reduced from N2LO to N3LO)
- The boundary-term is not included above (changing the last of 10-bin histograms by $3 \sim 5\%$)

$d\Gamma_{\rm sl}^{b\to u}/dq^2$ in 1S-scheme



- $\delta_{\rm QCD}$ exhibits the usual expected (regular) behavior
- Similar (crossing) **pattern** also helps to “understand” the puzzle (scale-uncertainty of the total inclusive result not reduced from N2LO to N3LO)
- The boundary-term is not included above (smaller than in the kinetic-mass case)

A Tentative Application to $\Gamma_{\rm sl}^{c\to q}$ (3+1)

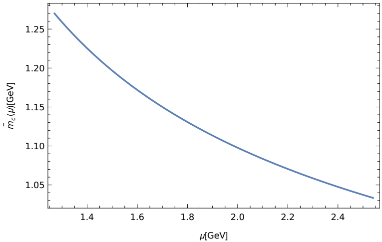
- Large QCD coupling at $\bar{m}_c$:

$$\alpha_s^{[4]}(\bar{m}_c) \approx 0.38 \approx 2\,\alpha_s^{[5]}(\bar{m}_b) \approx 4\,\alpha_s^{[6]}(m_t)$$

- Significant running-mass effect:

$$\bar{m}_c(\bar{m}_c) = 1.27\,\text{GeV};\ \bar{m}_c(\mu = 2\,\text{GeV}) = 1.10\text{GeV}$$

$$(\bar{m}_c + \Delta m)^5 \approx \bar{m}_c^5 \left(1 + 5\frac{\Delta m}{\bar{m}_c} + \cdots\right)$$



To get an idea of the sizes of the perturbative QCD correction factors:

$$\tilde{\Gamma}_0\, \bar{m}_c^5(2\,\text{GeV}) \Big(1 + 0.832 + 0.710 + 0.606\Big) = 5.01\,\tilde{\Gamma}_0$$

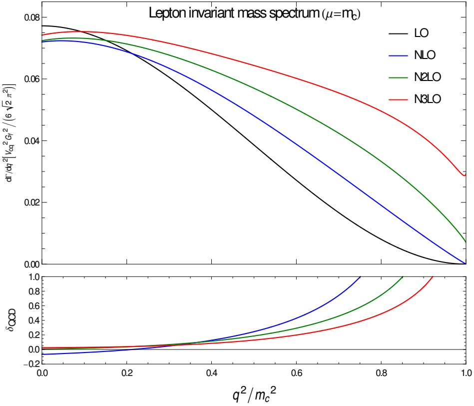
$$\tilde{\Gamma}_0\, \bar{m}_c^5(2 \times 1.27\,\text{GeV}) \Big(1 + 0.953 + 0.850 + 0.725\Big) = 4.16\,\tilde{\Gamma}_0$$

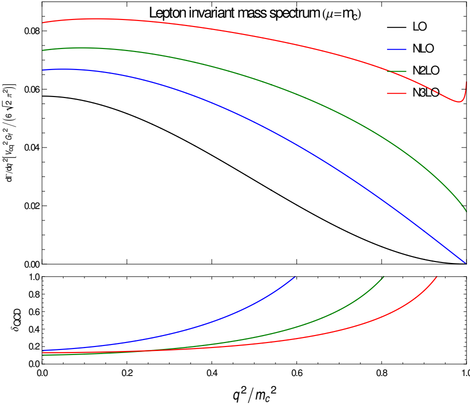
At the ref. scale $\mu = 2\,\text{GeV}$ (typically chosen for $\bar{m}_c$), the $\mathcal{O}(\alpha_s^3)$ correction could increase the perturbative NNLO correction by $\sim$24%.

The non-perturbative corrections (to higher twists) are important to obtain results for **semi-leptonic D-meson decays** comparable with the experiments. [→ See talks by Qin]

$d\Gamma_{sl}^{c\to q}/dq^2$ in $\overline{\text{MS}}$-scheme

Neither $\overline{\text{MS}}$ nor kinetic mass, unfortunately, works well for $d\Gamma_{sl}^{c\to q}/dq^2$



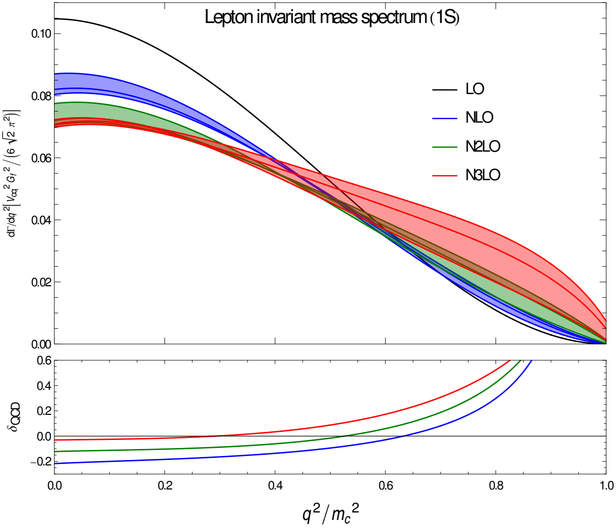


The left is for the result with kinetic-mass, slightly better than the right with $\overline{\text{MS}}$-mass, obtained using respectively standard parameter-settings employed in literature.

$\mathrm{d}\Gamma_{\mathrm{sl}}^{c\to q}/\mathrm{d}q^2$ in 1S-scheme

But, there is still **hope**!

[→ See talks by Qin for inclusive results]



(To estimate the uncertainty, the usual choice $\mu = m_{1S}/2$ is too low, and we used $[\overline{m}_c(\overline{m}_c), 2\,m_{1S}]$)

Summary and Outlook

- ☑ The first complete N3LO QCD corrections for t-decay width (and W-helicity fractions, W-energy distribution etc), the error of which meets the request by future colliders.

- ☑ Combining all available perturbative and non-perturbative information (relevant at %-level), an **accurate theoretical prediction for the inclusive** $B \to X_u \ell \bar{\nu}_\ell$:

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{|V_{ub}|^2}{|3.82 \times 10^{-3}|^2} \left(6.53 \pm 0.12 \pm 0.13\right) \times 10^{-16}\,\text{GeV}\,.$$

- ☑ The N3LO predictions for $\mathrm{d}\Gamma_{\mathrm{sl}}^{b\to u}/\mathrm{d}q^2$ and for $\mathrm{d}\Gamma_{\mathrm{sl}}^{c\to q}/\mathrm{d}q^2$ derived in the experimentally used kinetic and 1S scheme, useful to reveal certain puzzling behaviors.

- ☒ The method in use can be readily applied to the **triple distributions** (*"holy-grail"*) for $B \to X_u \ell \bar{\nu}_\ell$ @ Belle-II (in particular lepton-energy spectrum at N3LO)

Thank you for listening!

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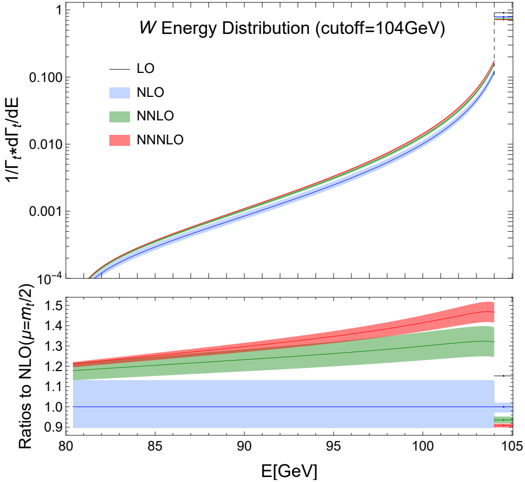
Backup Slides

Results for W-energy Distribution



- **In the bulk**: QCD corrections are positive and quite sizable: pure $\mathcal{O}(\alpha_s^3)$ correction modifies the lowest order by $7 \sim 14\%$ for $E \in [94, 104]$ GeV.
- **In the rightmost** 1 **GeV-bin**: QCD corrections up to $\mathcal{O}(\alpha_s^3)$ decrease the Born-level result.
- A change of x-axis $E_X = m_t - E_W \rightarrow$ distribution of the total energy of the **hadronic system**.