$B_{(s)}$ meson decays to even-parity charmed mesons

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Introduction

In the conventional quark model, these P-wave orbitally excited charmed mesons $D_0^*(2300)$, $D_{s0}^*(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_1(2420)$ and $D_1(2430)$ can be view as constituent quark-antiquark pairs .

- They are usually classified according to the quantum numbers ${}^{(2S+1)}L_J$.
- The scalar mesons $D_0^*(2300)$ and $D_{s0}^*(2317)$ correspond to ${}^3P_0$. While there exist two different axial-vector mesons, namely ${}^1P_1$ and ${}^3P_1$, which can undergo mixing when the two constituent quarks are different.

Introduction

- In the heavy quark limit, it is more convenient to use the L_J^j configurations with $j(L)$ being the total (orbital) angular momentum of the light quark to classify them.
- The scalar mesons $D_0^*(2300)$ and $D_{s0}^*(2317)$ belong to $P_0^{1/2}$, $D_1(2420)(D_{s1}(2460))$ and $D_1(2430)(D_{s1}(2536))$ correspond to $P_1^{1/2}$ and $P_1^{3/2}$, respectively.
- Beyond the heavy quark limit, there is a mixing between $P_1^{3/2}$ and $P_1^{1/2}$, denoted by $D_1^{3/2}$ and $D_1^{1/2}$, respectively.

$$
\begin{aligned}
|D_1(2420)\rangle &= |D_1^{1/2}\rangle \sin\theta_s + |D_1^{3/2}\rangle \cos\theta_s, \\
|D_1(2430)\rangle &= -|D_1^{3/2}\rangle \sin\theta_s + |D_1^{1/2}\rangle \cos\theta_s.
\end{aligned} \tag{1}
$$

Introduction

- While the states $D_1^{1/2}$ and $D_1^{3/2}$ are expected to be a mixture of states ${}^1P_1$ and ${}^3P_1$ denoted by ${}^1D_1$ and ${}^3D_1$, respectively,

$$
\begin{aligned}
|D_1^{3/2}\rangle &= \sqrt{\frac{2}{3}}|{}^1D_1\rangle + \sqrt{\frac{1}{3}}|{}^3D_1\rangle, \\
|D_1^{1/2}\rangle &= -\sqrt{\frac{1}{3}}|{}^1D_1\rangle + \sqrt{\frac{2}{3}}|{}^3D_1\rangle.
\end{aligned} \tag{2}
$$

- Combining Eq. (1) and Eq. (2), one can find that the physical states $D_1(2420)$ and $D_1(2430)$ can be written as

$$
\begin{aligned}
|D_1(2420)\rangle &= |{}^1D_1\rangle\cos\theta + |{}^3D_1\rangle\sin\theta, \\
|D_1(2430)\rangle &= -|{}^1D_1\rangle\sin\theta + |{}^3D_1\rangle\cos\theta,
\end{aligned} \tag{3}
$$

where $\theta = \theta_s + 35.3^\circ$, $\theta_s = 7^\circ$(the quark potential model).

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions descrbing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transtion form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirbel (BSW) transition form factors are more frequently used and defined by,

$$
\begin{aligned}
\langle D_0^*\left(P''\right)|A_\mu|B\left(P'\right)\rangle &= \left(P_\mu - \frac{m_B^2 - m_{D_0^*}^2}{q^2}\, q_\mu\right) F_1^{BD_0^*}\left(q^2\right) + \frac{m_B^2 - m_{D_0^*}^2}{q^2}\, q_\mu\, F_0^{BD_0^*}\left(q^2\right), \\
\left\langle {}^iD_1\left(P'', \varepsilon^{\mu *}\right)|V_\mu|B\left(P'\right)\right\rangle &= -i\left\{\left(m_B - m_{i_{D_1}}\right)\varepsilon_\mu^*\, V_1^{B^iD_1}\left(q^2\right) - \frac{\varepsilon^* \cdot P}{m_B - m_{i_{D_1}}}\, P_\mu\, V_2^{B^iD_1}\left(q^2\right)\right. \\
&\quad \left. - 2m_{i_{D_1}}\frac{\varepsilon^* \cdot P}{q^2}\, q_\mu\left[V_3^{B^iD_1}\left(q^2\right) - V_0^{B^iD_1}\left(q^2\right)\right]\right\}, \\
\left\langle {}^iD_1\left(P'', \varepsilon^{\mu *}\right)|A_\mu|B\left(P'\right)\right\rangle &= -\frac{1}{m_B - m_{i_{D_1}}}\,\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu}\, P^\alpha\, q^\beta\, A^{B^iD_1}\left(q^2\right),
\end{aligned}
\tag{4}
$$

where $P = P' + P''$, $q = P' - P''$, ϵ is the polarization vector. The four-momentum of the initial (final) meson is $P' = p_1' + p_2$ ($P'' = p_1'' + p_2$), with $p_1'^{(\prime\prime)}$ and p_2 being the momenta of the quark and antiquark inside the incoming (outgoing) meson.

The CLFQM general calculation procedure

- The form factors of the transitions $B \to D_0^*$ and $B \to {}^iD_1 (i = 1, 3)$ induced by the vector and aixal-vector currents are defined as

$$
\begin{aligned}
\langle D_0^* \left(P''\right) | A_\mu | B(P') \rangle &= i \left[u_+(q^2) P_\mu + u_-(q^2) q_\mu \right], \\
\left\langle {}^iD_1 \left(P'', \varepsilon\right) | A_\mu | B \left(P'\right) \right\rangle &= -q(q^2) \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} P^\alpha q^\beta, \\
\left\langle \ {}^iD_1 \left(P'', \varepsilon\right) | V_\mu | \ B \left(P'\right) \right\rangle &= i \left\{ l(q^2) \varepsilon_\mu^* + \varepsilon^* \cdot P \left[P_\mu c_+(q^2) + q_\mu c_-(q^2) \right] \right\}.
\end{aligned}
\tag{5}
$$

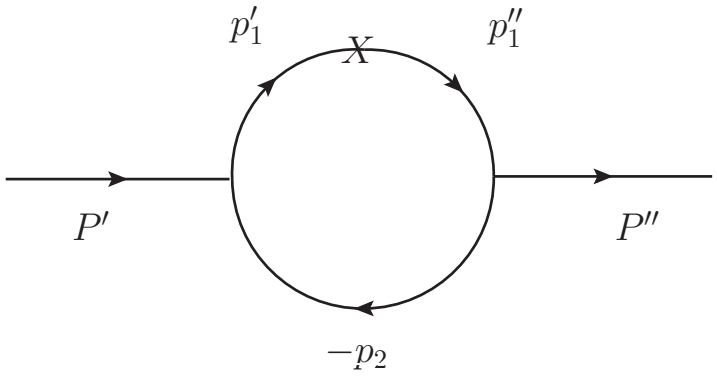
- This two kinds of form factors are related to each other via

$$
\begin{aligned}
F_1^{BD_0^*}(q^2) &= -u_+(q^2), F_0^{BD_0^*}(q^2) = -u_+(q^2) - \frac{q^2}{q \cdot P} u_-(q^2), \\
A^{B^iD_1}(q^2) &= -(m_B - m_{iD_1}) q(q^2), V_1^{B^iD_1}(q^2) = -\frac{l(q^2)}{m_B - m_{i_{D_1}}}, \\
V_2^{B^iD_1}(q^2) &= (m_B - m_{i_{D_1}}) c_+(q^2), V_3^{B^iD_1}(q^2) - V_0^{B^iD_1}(q^2) = \frac{q^2}{2m_{i_{D_1}}} c_-(q^2).
\end{aligned}
\tag{6}
$$

The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transition $B_{(s)} \to M$

$$\mathcal{B}_{\mu}^{B_{(s)}M} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h'_{B_{(s)}}(h''_M)}{N_1' N_1'' N_2} S_{\mu\nu}^{B_{(s)}M} \varepsilon^{*\nu}, \tag{7}$$



- $N_1'^{(\prime\prime)} = p_1'^{(\prime\prime)2} - m_1'^{(\prime\prime)2}, N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The CLFQM general calculation procedure

The trace $S_{\mu\nu}^{B_{(s)}M}$ can be obtained directly using Lorentz contraction,

$$
\begin{aligned}
S_{\mu}^{BD_0^*} &= Tr\left[\left(\not{p}_1'' + m_1''\right)\gamma_\mu\gamma_5\left(\not{p}_1' + m_1'\right)\gamma_5\left(-\not{p}_2 + m_2\right)\right], \\
S_{\mu\nu}^{B^1 D_1} &= \left(S_V^{B^1 D_1} - S_A^{B^1 D_1}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(-\frac{1}{W_{1\,D_1}''}\left(p_1'' - p_2\right)_\nu\right)\gamma_5\left(\not{p}_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not{p}_1' + m_1'\right)\gamma_5\left(-\not{p}_2 + m_2\right)\right], \\
S_{\mu\nu}^{B^3 D_1} &= \left(S_V^{B^3 D_1} - S_A^{B^3 D_1}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_{3\,D_1}''}\left(p_1'' - p_2\right)_\nu\right)\gamma_5\left(\not{p}_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not{p}_1' + m_1'\right)\gamma_5\left(-\not{p}_2 + m_2\right)\right]
\end{aligned}
\tag{8}
$$

The CLFQM general calculation procedure

The covariant vertex function $h'_{B_{(s)}}$,h'_M are defined as

$$
\begin{aligned}
h'_{B_{(s)}} &= (M'^2 - M_0'^2)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}'_0}\varphi', \\
h_{D_0^*} &= \sqrt{\frac{2}{3}}h'_{3\,D_1} = \left(M'^2 - M_0'^2\right)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}'_0}\frac{\widetilde{M_0'^2}}{2\sqrt{3}M'_0}\varphi'_p, \\
h'_{1\,D_1} &= \left(M'^2 - M'^2\right)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}'_0}\varphi'_p.
\end{aligned}
\tag{9}
$$

with

$$
M_0'^2 \quad = \quad \left(e'_1 + e_2\right)^2 = \frac{p_\perp'^2 + m_1'^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \quad \widetilde{M}'_0 = \sqrt{M_0'^2 - (m'_1 - m_2)^2}. \tag{10}
$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function $\varphi'_{(p)}$ depicts the light-front momentum distribution amplitude for the S(P)-wave mesons,

$$\begin{aligned} \varphi' &= \varphi'\left(x_2, p'_\perp\right) = 4\left(\frac{\pi}{\beta'^2}\right)^{\frac{3}{4}} \sqrt{\frac{dp'_z}{dx_2}} \exp\left(-\frac{p'^2_z + p'^2_\perp}{2\beta'^2}\right), \\ \varphi'_p &= \varphi'_p\left(x_2, p'_\perp\right) = \sqrt{\frac{2}{\beta'^2}}\varphi', \end{aligned} \tag{11}$$

- β' is a phenomenological parameter and can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_b = 4.8,\ m_c = 1.4,\ m_s = 0.37,$ $m_{u,d} = 0.25;$
- The masses of the initial and the final mesons(GeV): $m_{B_s} = 5.367, m_B = 5.279, m_{D_0^*} = 2.343, m_{D_{s0}^*} = 2.317, m_{D_{s1}} = 2.460, m_{D'_{s1}} = 2.536, m_{D_1} = 2.422, m_{D'_1} = 2.412;$
- The CKM matrix elements: $V_{cd} = 0.221 \pm 0.004,$ $V_{us} = 0.2243 \pm 0.0008,\ V_{cs} = 0.975 \pm 0.006,$ $V_{ud} = 0.97373 \pm 0.00031,\ V_{cb} = (40.8 \pm 1.4) \times 10^{-3}.$

Transition Form Factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta'_B = 0.555^{+0.048}_{-0.048}, \beta'_{B_s} = 0.628^{+0.035}_{-0.034}, \beta'_{D_s} = 0.497^{+0.032}_{-0.028},$$

$$\beta'_{D^*} = 0.409^{+0.021}_{-0.022},\ \beta'_{D_s^*} = 0.438^{+0.016}_{-0.027},\ \beta'_{D_0^*} = 0.373^{+0.063}_{-0.059},$$

$$\beta'_{D_{s0}^*} = 0.325^{+0.043}_{-0.043},\ \beta'_{{}^3D_{s1}} = 0.342^{+0.030}_{-0.034}, \beta'_{{}^1D'_{s1}} = 0.342^{+0.039}_{-0.039},$$

$$\beta'_{{}^3D_1} = 0.332^{+0.031}_{-0.034}, \beta'_{{}^1D'_1} = 0.329^{+0.038}_{-0.040}.$$

- Meson lifetimes(s): $\tau_{B_s} = (1.520 \pm 0.005) \times 10^{-12}, \tau_{B^\pm} = (1.638 \pm 0.004) \times 10^{-12}, \tau_{B^0} = (1.519 \pm 0.004) \times 10^{-12}.$

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F\left(q^2\right) = \frac{F(0)}{1 - aq^2/m^2 + bq^4/m^4}. \tag{12}$$

- $F(q^2)$ denotes different form factors $F_1(q^2), F_0(q^2), V(q^2)$, $A_0(q^2), A_1(q^2)$ and $A_2(q^2)$.

Transition Form Factors

表: The transition form factors $B_{(s)} \to D_0^*, D_{s0}^*, D_{s1}, D'_{s1}$ at $q^2 = 0$.

	$F_i(q^2=0)$	$F_i(q^2_{max})$	a	b
$F_1^{BD_0^*}$	$0.25^{+0.03+0.05}_{-0.02-0.05}$	$0.30^{+0.03+0.06}_{-0.03-0.07}$	$0.70^{+0.04+0.03}_{-0.05-0.11}$	$0.65^{+0.08+0.03}_{-0.07-0.07}$
$F_0^{BD_0^*}$	$0.25^{+0.03+0.05}_{-0.02-0.05}$	$0.22^{+0.02+0.04}_{-0.01-0.04}$	$-0.38^{+0.04+0.05}_{-0.04-0.02}$	$0.21^{+0.07+0.08}_{-0.07-0.08}$
$F_1^{B_sD_{s0}^*}$	$0.21^{+0.02+0.04}_{-0.01-0.04}$	$0.24^{+0.02+0.05}_{-0.01-0.05}$	$0.63^{+0.05+0.07}_{-0.06-0.12}$	$0.78^{+0.08+0.01}_{-0.09-0.04}$
$F_0^{B_sD_{s0}^*}$	$0.21^{+0.02+0.04}_{-0.01-0.04}$	$0.18^{+0.02+0.03}_{-0.01-0.03}$	$-0.43^{+0.01+0.01}_{-0.00-0.02}$	$0.28^{+0.03+0.01}_{-0.06-0.04}$
$A^{B_sD_{s1}}$	$0.20^{+0.01+0.02}_{-0.01-0.02}$	$0.18^{+0.02+0.03}_{-0.01-0.02}$	$-0.27^{+0.06+0.8}_{-0.07-0.09}$	$0.11^{+0.02+0.02}_{-0.02-0.03}$
$V_0^{B_sD_{s1}}$	$0.40^{+0.02+0.04}_{-0.02-0.04}$	$0.42^{+0.02+0.05}_{-0.02-0.05}$	$-0.17^{+0.02+0.04}_{-0.04-0.06}$	$-0.02^{+0.01+0.00}_{-0.00-0.01}$
$V_1^{B_sD_{s1}}$	$0.58^{+0.01+0.03}_{-0.02-0.04}$	$0.57^{+0.01+0.03}_{-0.02-0.04}$	$-0.05^{+0.01+0.01}_{-0.01-0.01}$	$0.02^{+0.00+0.01}_{-0.00-0.00}$
$V_2^{B_sD_{s1}}$	$-0.05^{+0.01+0.02}_{-0.00-0.01}$	$-0.05^{+0.01+0.03}_{-0.00-0.02}$	$0.56^{+0.06+0.22}_{-0.06-0.25}$	$2.50^{+0.25+1.67}_{-0.20-1.30}$
$A^{B_sD'_{s1}}$	$0.08^{+0.01+0.02}_{-0.01-0.02}$	$0.03^{+0.01+0.01}_{-0.02-0.03}$	$2.05^{+0.13+0.35}_{-0.10-0.35}$	$5.57^{+0.25+0.50}_{-0.20-0.41}$
$V_0^{B_sD'_{s1}}$	$-0.08^{+0.01+0.04}_{-0.01-0.04}$	$-0.05^{+0.02+0.05}_{-0.01-0.04}$	$1.24^{+0.05+0.23}_{-0.06-0.25}$	$0.74^{+0.02+0.21}_{-0.02-0.17}$
$V_1^{B_sD'_{s1}}$	$0.17^{+0.02+0.04}_{-0.03-0.03}$	$0.15^{+0.01+0.02}_{-0.03-0.03}$	$-0.52^{+0.06+0.06}_{-0.05-0.06}$	$0.36^{+0.01+0.03}_{-0.00-0.08}$
$V_2^{B_sD'_{s1}}$	$0.11^{+0.01+0.01}_{-0.02-0.02}$	$0.10^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.06+0.06}_{-0.07-0.07}$	$-0.07^{+0.03+0.01}_{-0.04-0.03}$

$F_0^{BD_0^*} = 0.24$ CLFQM; $F_0^{BD_0^*} = 0.18$ ISGW2; $F_0^{BD_0^*} = 0.95$ LCSR; $F_0^{B_sD_{s0}^*} = 0.20$ ISGW2; $F_0^{B_sD_{s0}^*} = 0.40$ QCDSR.

Transition Form Factors

表: The $B \to D_1, D_1'$ transition form factors at $q^2 = 0$.

	$F_i(q^2=0)$	$F_i(q^2_{max})$	a	b
A^{BD_1}	$0.21^{+0.02+0.02}_{-0.01-0.01}$	$0.20^{+0.02+0.02}_{-0.02-0.02}$	$-0.23^{+0.10+0.09}_{-0.09-0.09}$	$0.09^{+0.02+0.02}_{-0.02-0.02}$
$V_0^{BD_1}$	$0.42^{+0.03+0.04}_{-0.02-0.04}$	$0.44^{+0.04+0.05}_{-0.02-0.05}$	$0.16^{+0.02+0.03}_{-0.02-0.05}$	$-0.02^{+0.00+0.01}_{-0.00-0.01}$
$V_1^{BD_1}$	$0.57^{+0.03+0.03}_{-0.01-0.04}$	$0.56^{+0.03+0.03}_{-0.02-0.04}$	$-0.07^{+0.00+0.01}_{-0.01-0.00}$	$0.02^{+0.00+0.00}_{-0.00-0.00}$
$V_2^{BD_1}$	$-0.06^{+0.01+0.02}_{-0.01-0.02}$	$-0.06^{+0.01+0.04}_{-0.01-0.03}$	$0.57^{+0.06+0.17}_{-0.07-0.20}$	$2.13^{+0.21+1.30}_{-0.18-1.17}$
$A^{BD_1'}$	$0.07^{+0.02+0.02}_{-0.02-0.02}$	$0.02^{+0.03+0.01}_{-0.02-0.02}$	$-1.90^{+0.09+0.12}_{-0.10-0.14}$	$5.64^{+0.22+1.59}_{-0.18-1.29}$
$V_0^{BD_1'}$	$-0.08^{+0.02+0.04}_{-0.02-0.03}$	$-0.06^{+0.02+0.06}_{-0.02-0.03}$	$-0.11^{+0.01+0.06}_{-0.04-0.11}$	$3.67^{+0.05+0.51}_{-0.05-0.51}$
$V_1^{BD_1'}$	$0.14^{+0.04+0.03}_{-0.04-0.03}$	$0.13^{+0.03+0.03}_{-0.04-0.02}$	$-0.36^{+0.04+0.04}_{-0.06-0.06}$	$0.09^{+0.01+0.01}_{-0.01-0.01}$
$V_2^{BD_1'}$	$0.10^{+0.03+0.02}_{-0.02-0.02}$	$0.11^{+0.03+0.02}_{-0.04-0.04}$	$0.19^{+0.06+0.06}_{-0.09-0.09}$	$0.00^{+0.01+0.01}_{-0.00-0.01}$

Transition Form Factors

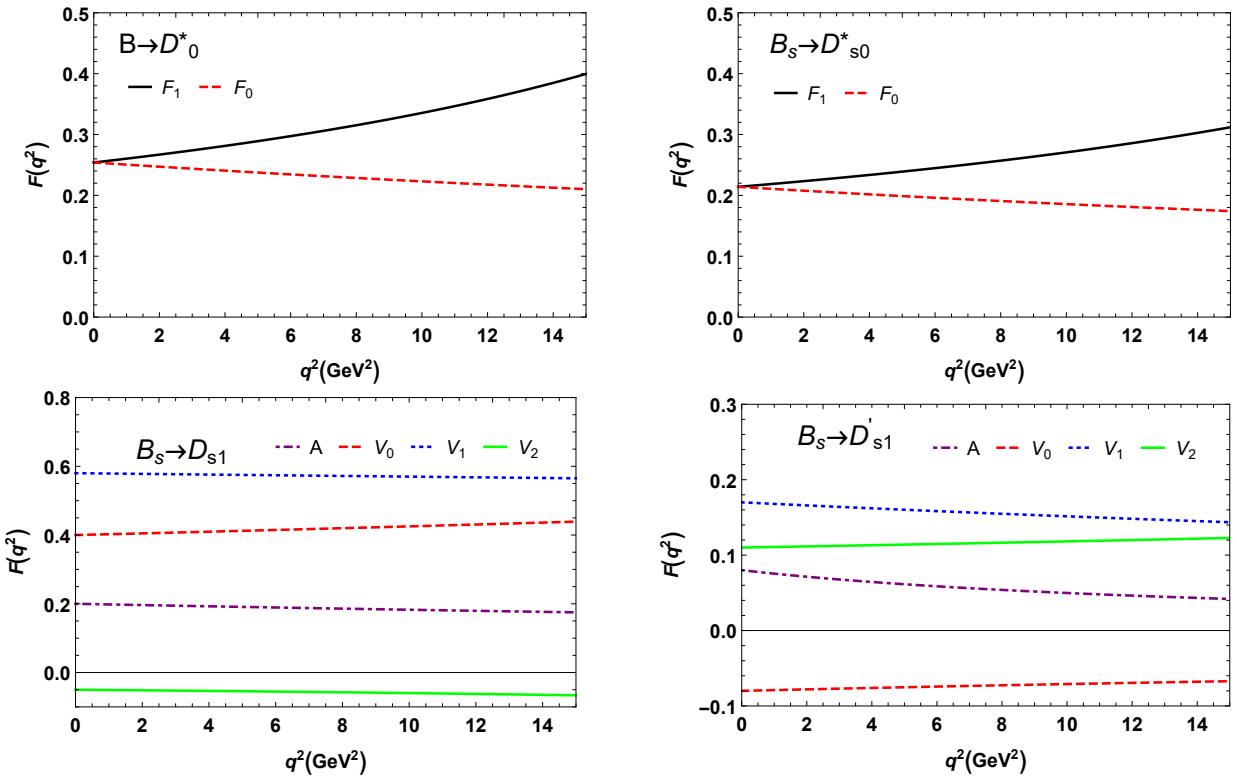
表: Form factors of the transitions $B_{(s)} \to D_{s1}^{3/2}, D_{s1}^{1/2}, D_1^{3/2}$ and $D_1^{1/2}$ at $q^2 = 0$ together with other theoretical results.

Transitions	References	$A(0)$	$V_0(0)$	$V_1(0)$	$V_2(0)$
$B_s \to D_{s1}^{3/2}$	This work	0.19	0.41	0.55	−0.07
	CLFQM[a]	0.24	0.49	0.57	−0.09
$B_s \to D_{s1}^{1/2}$	This work	0.10	−0.03	0.24	0.11
	CLFQM[a]	−0.17	0.13	−0.25	−0.17
$B \to D_1^{3/2}$	This work	0.20	0.43	0.55	−0.07
	CLFQM[1]	0.23	0.47	0.55	−0.09
	ISGW2	0.16	0.43	0.40	−0.12
	CLFQM[2]	0.25	0.52	0.58	−0.10
$B \to D_1^{1/2}$	This work	0.09	−0.02	0.21	0.09
	CLFQM[1]	−0.12	0.08	−0.19	−0.12
	ISGW2	−0.16	0.18	−0.19	−0.18
	CLFQM[2]	−0.13	0.11	−0.19	−0.14

In Refs.[1,2], due to the signs of the $D_{(s)1}^{1/2}$ mixing formula Eq. (2) between this work are opposite, the corresponding results are just contrary with our predictions.

Transition Form Factors

图: The q^2-dependence of the form factors of $B \to D_0^*, D_{s0}^*, D_{s1}, D'_{s1}$.



Non-leptonic decays

Non-leptonic decays

The decays rates of the nonleptonic weak decays $B_{(s)} \to D^*_{(s)0} M$ with M being a light pseudoscalar (vector) meson $P(V)$

$$\mathcal{B}r(B_{(s)} \to D^*_{(s)0} M) = \frac{\tau_{B_{(s)}}}{\hbar} \Gamma(B_{(s)} \to D^*_{(s)0} M), \tag{13}$$

with

$$\Gamma\left(B_{(s)} \to D_0^{*-} P(V)\right) = \frac{\left| G_F V_{cb} V_{uq}^* a_1 f_{P(V)} m_B^2 F_0^{BD_0^*}(m_{P(V)}^2) \right|^2}{32\pi m_B} \left(1 - r_{D_0^*}^2\right). \tag{14}$$

Non-leptonic decays

The decay widths for the decays $B \to D_1^{(\prime)} P$ and $B \to D_1^{(\prime)} V$

$$\Gamma\left(B \to D_1^{(\prime)} P\right) = \frac{\left|G_F V_{cb} V_{uq}^* a_1 f_P m_B^2 V_0^{BD_1^{(\prime)}}(m_P^2)\right|^2}{32\pi m_B}\left(1 - r_{D_1^{(\prime)}}^2\right), \tag{15}$$

$$\Gamma\left(B \to D_1^{(\prime)} V\right) = \frac{|\vec{p}|}{8\pi m_B^2}\left(|\mathcal{A}_L(B \to D_1^{(\prime)} V)|^2 + 2\left|\mathcal{A}_N\left(B \to D_1^{(\prime)} V\right)\right|^2 + 2\left|\mathcal{A}_T\left(B \to D_1^{(\prime)} V\right)\right|^2\right), \tag{16}$$

$$\begin{aligned} i\mathcal{A}_L(B \to D_1^{(\prime)} V) &= \frac{(-i)^3 G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^* a_1 f_V m_B^2 \frac{1}{2r_{D_1^{(\prime)}}} \\ &\times \left[(1 - r_V^2 - r_{D_1^{(\prime)}}^2)(1 - r_{D_1^{(\prime)}}) V_1^{BD_1^{(\prime)}}(m_V^2) - \frac{\lambda(1, r_V^2, r_{D_1^{(\prime)}}^2)}{1 - r_{D_1^{(\prime)}}} V_2^{BD_1^{(\prime)}}(m_V^2)\right], \\ i\mathcal{A}_N(B \to D_1^{(\prime)} V) &= \frac{(-i)^3 G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^* a_1 f_V m_B^2 r_V (1 - r_{D_1^{(\prime)}}) V_1^{BD_1^{(\prime)}}(m_V^2), \\ i\mathcal{A}_T(B \to D_1^{(\prime)} V) &= \frac{-iG_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^* a_1 f_V m_B^2 r_V \frac{\sqrt{\lambda(1, r_V^2, r_{D_1^{(\prime)}}^2)}}{1 - r_{D_1^{(\prime)}}} A^{BD_1^{(\prime)}}(m_V^2), \end{aligned} \tag{17}$$

Non-leptonic Decays

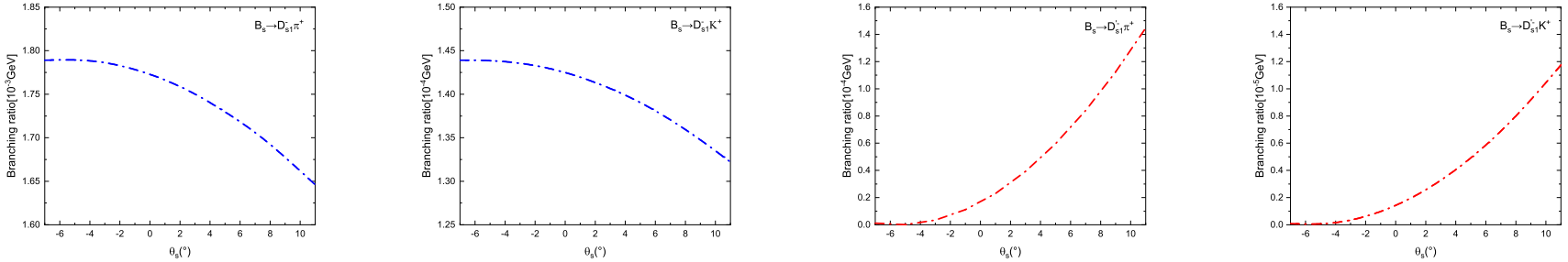
	$\mathcal{B}r(B_s \to D_{s0}^{*-}\pi^+)$	$\mathcal{B}r(B_s \to D_{s0}^{*-}K^+)$	$\mathcal{B}r(B_s \to D_{s0}^{*-}\rho^+)$	$\mathcal{B}r(B_s \to D_{s0}^{*-}K^{*+})$
LO	$5.06^{+0.83+2.09}_{-0.75-1.83}$	$0.40^{+0.07+0.17}_{-0.06-0.15}$	$12.87^{+2.11+5.31}_{-1.91-4.66}$	$0.74^{+0.12+0.30}_{-0.11-0.27}$
VC	$5.27^{+0.86+2.17}_{-0.78-1.91}$	$0.42^{+0.07+0.17}_{-0.06-0.15}$	$13.40^{+2.20+5.53}_{-1.99-4.86}$	$0.77^{+0.13+0.32}_{-0.11-0.28}$
HSSC	$4.96^{+0.81+2.05}_{-0.74-1.80}$	$0.38^{+0.06+0.16}_{-0.06-0.14}$	$12.59^{+2.06+5.20}_{-1.87-4.56}$	$0.72^{+0.12+0.30}_{-0.11-0.26}$
NLO	$5.17^{+0.85+2.13}_{-0.77-1.87}$	$0.40^{+0.07+0.16}_{-0.06-0.14}$	$13.12^{+2.15+5.42}_{-1.95-4.76}$	$0.75^{+0.12+0.31}_{-0.11-0.28}$
PQCD	$5.49^{+2.64+0.41+0.35}_{-1.68-0.27-0.35}$	$0.51^{+0.06+0.01+0.01}_{-0.04-0.01-0.01}$	$17.7^{+8.5+1.3+1.2}_{-5.3-0.8-1.1}$	$1.01^{+0.44+0.06+0.05}_{-0.31-0.06-0.07}$
LSCR	$5.2^{+2.5}_{-2.1}$	$0.4^{+0.2}_{-0.2}$	13^{+6}_{-5}	$0.8^{+0.4}_{-0.3}$
RQM	9	0.7	22	1.2
NRQM	10	0.9	27	16
ISGW2	3.3	–	8.3	–

Non-leptonic Decays

	LO	VC	HSSC	NLO	RQM	NRQM	ISGW2
$\mathcal{B}r(B_s \to D_{s1}^- D_s^+)$	$6.55^{+0.56+0.12+1.36}_{-0.55-0.18-1.29}$	$6.48^{+0.56+0.12+1.34}_{-0.55-0.17-1.28}$	$6.44^{+0.56+0.12+1.36}_{-0.55-0.17-1.28}$	$6.38^{+0.56+0.12+1.34}_{-0.54-0.17-1.27}$	3.0	–	–
$\mathcal{B}r(B_s \to D_{s1}^- D_s^{*+})$	$8.45^{+0.72+0.24+1.59}_{-0.70-0.32-1.53}$	$8.48^{+0.72+0.25+1.58}_{-0.70-0.33-1.52}$	$8.29^{+0.71+0.23+1.59}_{-0.69-0.31-1.52}$	$8.32^{+0.71+0.24+1.58}_{-0.70-0.32-1.52}$	5.9	–	–
$\mathcal{B}r(B_s \to D_{s1}^- \pi^+)$	$1.67^{+0.14+0.03+0.35}_{-0.14-0.04-0.33}$	$1.74^{+0.15+0.03+0.36}_{-0.15-0.05-0.34}$	$1.64^{+0.14+0.03+0.34}_{-0.14-0.04-0.33}$	$1.70^{+0.15+0.03+0.36}_{-0.15-0.01-0.34}$	1.9	1.5	0.52
$\mathcal{B}r(B_s \to D_{s1}^- \rho^+)$	$4.49^{+0.39+0.10+0.90}_{-0.38-0.14-0.86}$	$4.68^{+0.40+0.11+0.94}_{-0.39-0.15-0.90}$	$4.40^{+0.38+0.10+0.90}_{-0.38-0.13-0.86}$	$4.59^{+0.40+0.10+0.94}_{+0.39+0.14+0.89}$	4.9	3.6	1.3
$\mathcal{B}r(B_s \to D_{s1}^- K^+)$	$0.13^{+0.01+0.00+0.03}_{-0.01-0.00-0.03}$	$0.14^{+0.01+0.00+0.03}_{-0.01-0.00-0.03}$	$0.13^{+0.01+0.00+0.03}_{-0.01-0.00-0.03}$	$0.14^{+0.01+0.00+0.03}_{-0.01-0.00-0.03}$	0.14	0.12	–
$\mathcal{B}r(B_s \to D_{s1}^- K^{*+})$	$0.26^{+0.02+0.01+0.05}_{-0.02-0.01-0.05}$	$0.27^{+0.02+0.01+0.05}_{-0.02-0.01-0.05}$	$0.26^{+0.02+0.01+0.05}_{-0.02-0.01-0.05}$	$0.27^{+0.02+0.01+0.05}_{-0.02-0.01-0.05}$	0.26	0.20	–
$\mathcal{B}r(B_s \to D_{s1}^{'-} D_s^+)$	$0.27^{+0.10+0.04+0.29}_{-0.08-0.03-0.19}$	$0.26^{+0.10+0.04+0.29}_{-0.08-0.03-0.19}$	$0.32^{+0.11+0.04+0.31}_{-0.09-0.03-0.21}$	$0.31^{+0.11+0.04+0.31}_{-0.09-0.03-0.21}$	0.54	–	–
$\mathcal{B}r(B_s \to D_{s1}^{'-} D_s^{*+})$	$0.42^{+0.15+0.01+0.30}_{-0.12-0.00-0.18}$	$0.43^{+0.16+0.02+0.30}_{-0.13-0.00-0.18}$	$0.46^{+0.16+0.01+0.33}_{-0.13-0.01-0.21}$	$0.46^{+0.16+0.01+0.32}_{-0.13-0.01-0.21}$	1.5	–	–
$\mathcal{B}r(B_s \to D_{s1}^{'-} \pi^+)$	$0.068^{+0.025+0.010+0.074}_{-0.020-0.007-0.049}$	$0.071^{+0.026+0.011+0.077}_{-0.021-0.007-0.051}$	$0.081^{+0.028+0.011+0.080}_{-0.022-0.007-0.055}$	$0.084^{+0.029+0.011+0.083}_{-0.023-0.007-0.057}$	0.29	0.7	1.5
$\mathcal{B}r(B_s \to D_{s1}^{'-} \rho^+)$	$0.20^{+0.07+0.02+0.18}_{-0.06-0.00-0.12}$	$0.21^{+0.08+0.06+0.19}_{-0.06-0.01-0.13}$	$0.23^{+0.08+0.06+0.21}_{-0.06-0.04-0.14}$	$0.24^{+0.08+0.02+0.21}_{-0.07-0.01-0.14}$	0.83	1.9	3.8
$\mathcal{B}r(B_s \to D_{s1}^{'-} K^+)$	$0.006^{+0.002+0.008+0.006}_{-0.002-0.005-0.004}$	$0.006^{+0.002+0.009+0.006}_{-0.002-0.006-0.004}$	$0.007^{+0.002+0.009+0.007}_{-0.002-0.006-0.004}$	$0.007^{+0.002+0.009+0.007}_{-0.002-0.006-0.005}$	0.021	0.054	–
$\mathcal{B}r(B_s \to D_{s1}^{'-} K^{*+})$	$0.012^{+0.004+0.009+0.011}_{-0.004-0.004-0.007}$	$0.012^{+0.005+0.009+0.011}_{-0.004-0.004-0.007}$	$0.014^{+0.005+0.001+0.012}_{-0.004-0.005-0.008}$	$0.014^{+0.005+0.001+0.012}_{-0.004-0.005-0.008}$	0.044	0.1	–

Non-leptonic Decays

The dependences of the branching ratios of the decays $B_s \to D_{s1}^{(\prime)}\pi(K)$ on the mixing angle θ_s.



The branching ratios of the decays $B_s \to D'_{s1}\pi(K)$ are very sensitive to the mixing angle, while those of the decays $B_s \to D_{s1}\pi(K)$ show an insensitive dependence on θ_s. Furthermore, the changing trends for the branching ratios of these two kinds of decays are just opposite.

Semi-leptonic Decays

The differential widths of the decays $B \to D_0^* \ell \nu_\ell$,

$$\frac{d\Gamma(B \to D_0^* \ell \nu_\ell)}{dq^2} = (\frac{q^2 - m_\ell^2}{q^2})^2 \frac{\sqrt{\lambda(m_B^2, m_{D_0^*}^2, q^2)}\, G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3} \times \frac{1}{q^2}$$
$$\times \left\{ (m_\ell^2 + 2q^2)\lambda(m_B^2, m_{D_0^*}^2, q^2) F_1^2(q^2) + 3m_\ell^2 (m_B^2 - m_{D_0^*}^2)^2 F_0^2(q^2) \right\}, \tag{18}$$

where m_ℓ is the mass of the lepton ℓ and

$$\lambda(q^2) = \lambda(a^2, b^2, q^2) = (a^2 + b^2 - q^2)^2 - 4a^2 b^2,$$

Semi-leptonic Decays

The differential widths of the decays $B \to D_1^{(\prime)} \ell \nu_\ell$,

$$\frac{d\Gamma_L(B \to D_1^{(\prime)} \ell \nu_\ell)}{dq^2} = \left(\frac{q^2 - m_\ell^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2)}\, G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3 q^2} \left\{ 3 m_\ell^2 \lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2) V_0^2(q^2) \right.$$

$$\left. + (m_\ell^2 + 2q^2) \left| \frac{1}{2m_{D_1^{(\prime)}}} \left[(m_B^2 - m^2_{D_1^{(\prime)}} - q^2)(m_B - m_{D_1^{(\prime)}}) V_1(q^2) - \frac{\lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2)}{m_B - m_{D_1^{(\prime)}}} V_2(q^2) \right] \right|^2 \right\}, \quad (19)$$

$$\frac{d\Gamma_\pm(B \to D_1^{(\prime)} \ell \nu_\ell)}{dq^2} = \left(\frac{q^2 - m_\ell^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2)}\, G_F^2 |V_{cb}|^2}{384 m_B^3 \pi^3}$$

$$\times \left\{ (m_\ell^2 + 2q^2) \lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2) \left| \frac{A(q^2)}{m_B - m_{D_1^{(\prime)}}} \mp \frac{(m_B - m_{D_1^{(\prime)}}) V_1(q^2)}{\sqrt{\lambda(m_B^2, m^2_{D_1^{(\prime)}}, q^2)}} \right|^2 \right\}, \quad (20)$$

Semi-leptonic Decays

The combined transverse and total differential decay widths

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \tag{21}$$

For the existence of different polarizations, we difine the longitudinal polarization fraction

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-} \tag{22}$$

The forward-backward asymmetry is defined as

$$A_{FB} = \frac{\int_0^1 \frac{d\Gamma}{dcos\theta} dcos\theta - \int_{-1}^0 \frac{d\Gamma}{dcos\theta} dcos\theta}{\int_{-1}^1 \frac{d\Gamma}{dcos\theta} dcos\theta} = \frac{\int b_\theta(q^2) dq^2}{\Gamma_{B_{(s)}}}. \tag{23}$$

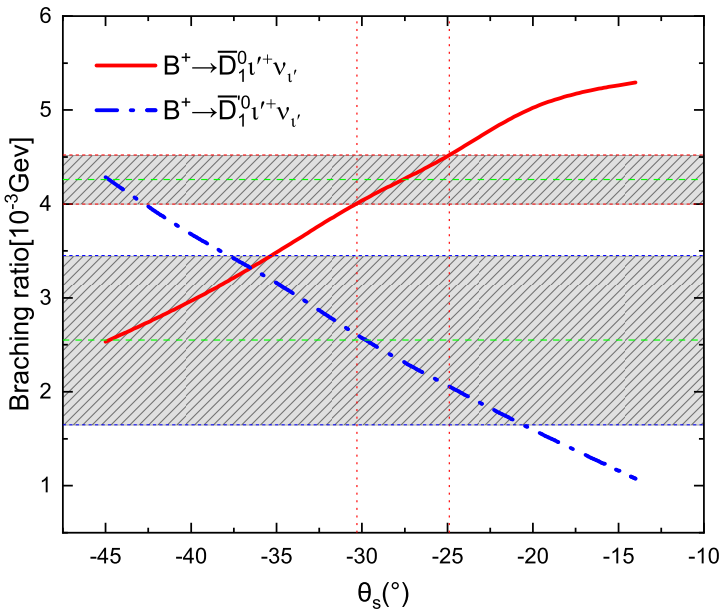
Semi-leptonic Decays

- The branching ratios of the semileptonic decays $B \to D_1^{(\prime)} \ell \nu_\ell$.

References	$B^+ \to \bar{D}_1^0 e^+ \nu_e$	$B^+ \to \bar{D}_1^0 \mu^+ \nu_\mu$	$B^+ \to \bar{D}_1^0 \tau^+ \nu_\tau$
This work	$6.21^{+0.94+0.43+1.00}_{-0.67-0.28-0.90}$	$6.16^{+0.93+0.42+0.99}_{-0.66-0.27-0.89}$	$0.57^{+0.08+0.03+0.07}_{-0.05-0.04-0.07}$
QCDSRs	$7.26^{+3.60+0.51}_{-2.87-0.49}$	$7.19^{+3.56+0.50}_{-2.84-0.48}$	$0.46^{+0.22+0.03}_{-0.17-0.03}$
PDG	4.26 ± 0.26	4.26 ± 0.26	–
LLSW	6.90 ± 0.47	6.90 ± 0.47	0.68 ± 0.06
HQEFT	5.3 ± 1.6	5.3 ± 1.6	–
References	$B^+ \to \bar{D}_1'^0 e^+ \nu_e$	$B^+ \to \bar{D}_1'^0 \mu^+ \nu_\mu$	$B^+ \to \bar{D}_1'^0 \tau^+ \nu_\tau$
This work	$0.15^{+0.15+0.07+0.08}_{-0.09-0.03-0.06}$	$0.15^{+0.15+0.07+0.08}_{-0.09-0.03-0.06}$	$0.02^{+0.01+0.00+0.00}_{-0.01-0.02-0.00}$
QCDSRs	$0.68^{+0.31+0.04}_{-0.24-0.05}$	$0.67^{+0.31+0.04}_{-0.23-0.05}$	$0.05^{+0.05+0.00}_{-0.02-0.00}$
LLSW	0.50 ± 0.40	0.50 ± 0.40	0.04 ± 0.03
HQEFT	0.51 ± 0.17	0.51 ± 0.17	–
PDG	2.55 ± 0.90	2.55 ± 0.90	–

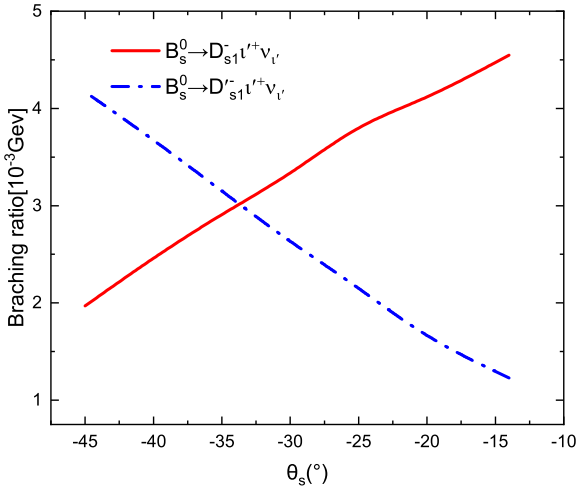
The first error arises from the decay widths of B, the second uncertainty is from the β' of the initial and final mesons.

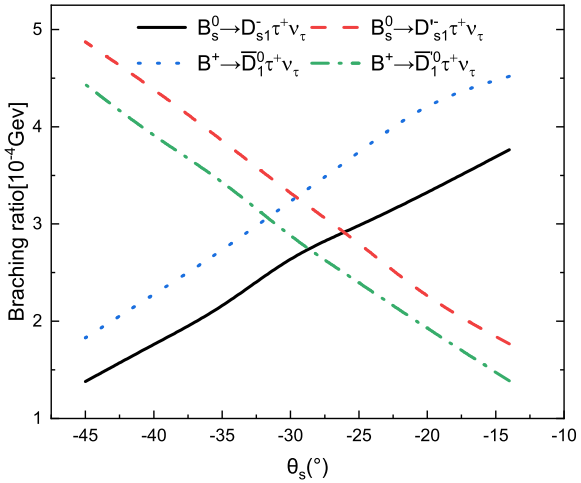
Semi-leptonic Decays



- The mixing angle θ_s dependencies of the branching ratios for the semileptonic decays $B^+ \to \bar{D}_1^0 \ell'^+ \nu_{\ell'}$ (the red solid line) and $B^+ \to \bar{D}_1'^0 \ell'^+ \nu_{\ell'}$ (the blue dash-dotted line).
- The upper (lower) shadow band and its horizontal image center line refers to the experimentally achievable range and the center value for the branching ratios of the decays $B^+ \to D_1^0 \ell' \nu_{\ell'} (B^+ \to \bar{D}_1'^0 \ell' \nu_{\ell'})$, respectively.
- Taking some negtive mixing angle θ_s values within a range from -30.3° to -24.9° can explain the data, which correspond to θ within the range $5^\circ \sim 10.4^\circ$.

Semi-leptonic Decays





The mixing angle θ_s dependencies of the branching ratios for the semileptonic decays $B_s^0 \to D_{s1}^{(\prime)-} \ell^+ \nu_\ell$ (left) and $B^+ \to \bar{D}_1^{(\prime)0} \tau^+ \nu_\tau$, $B_s^0 \to D_{s1}^{(\prime)-} \tau^+ \nu_\tau$ (right).

Semi-leptonic Decays

- The forward-backward asymmetry A_{FB}.

Channels		$B^+ \to \bar{D}_0^* e^+ \nu_e$	$B^+ \to \bar{D}_0^* \mu^+ \nu_\mu$	$B^+ \to \bar{D}_0^* \tau^+ \nu_\tau$
A_{FB}	This work	$(5.820^{+1.494+2.582}_{-0.904-2.108}) \times 10^{-7}$	$0.020^{+0.005+0.009}_{-0.003-0.007}$	$0.383^{+0.102+0.177}_{-0.063-0.143}$
Channels		$B_s^0 \to D_{s0}^{*-} e^+ \nu_e$	$B_s^0 \to D_{s0}^{*-} \mu^+ \nu_\mu$	$B_s^0 \to D_{s0}^{*-} \tau^+ \nu_\tau$
A_{FB}	This work	$(5.465^{+1.100+2.298}_{-0.515-1.898}) \times 10^{-7}$	$0.019^{+0.004+0.010}_{-0.002-0.007}$	$0.381^{+0.079+0.168}_{-0.038-0.138}$
	NCQM	8.22×10^{-7}	0.016	0.39
Channels		$B_s^0 \to D_{s1}^- e^+ \nu_e$	$B_s^0 \to D_{s1}^- \mu^+ \nu_\mu$	$B_s^0 \to D_{s1}^- \tau^+ \nu_\tau$
A_{FB}	This work	$-0.175^{+0.016+0.027+0.011}_{-0.014-0.016-0.017}$	$-0.174^{+0.016+0.026+0.010}_{-0.014-0.017-0.017}$	$-0.129^{+0.010+0.021+0.003}_{-0.015-0.017-0.016}$
	NCQM	-0.19	-0.18	0.10
Channels		$B_s^0 \to D_{s1}'^- e^+ \nu_e$	$B_s^0 \to D_{s1}'^- \mu^+ \nu_\mu$	$B_s^0 \to D_{s1}'^- \tau^+ \nu_\tau$
A_{FB}	This work	$-0.407^{+0.104+0.129+0.020}_{-0.101-0.185-0.029}$	$-0.405^{+0.104+0.129+0.020}_{-0.102-0.186-0.029}$	$-0.248^{+0.026+0.080+0.048}_{-0.064-0.027-0.007}$
	NCQM	-0.41	-0.40	-0.20

Semi-leptonic Decays

The longitudinal polarization fraction f_L. Region 1 $m_\ell^2 < q^2 < \frac{(m_{B_{(s)}} - m_{D^{(\prime)}_{(s1)}})^2 + m_\ell^2}{2}$ and Region 2 $\frac{(m_{B_{(s)}} - m_{D^{(\prime)}_{(s)1}})^2 + m_\ell^2}{2} < q^2 < (m_{B_{(s)}} - m_{D^{(\prime)}_{(s)1}})^2$.

Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B_s^0 \to D_{s1}^- e^+ \nu_e)$	0.81	0.50	0.71	$f_L(B_s^0 \to D_{s1}'^- e^+ \nu_e)$	0.42	0.12	0.31
$f_L(B_s^0 \to D_{s1}^- \mu^+ \nu_\mu)$	0.81	0.50	0.71	$f_L(B_s^0 \to D_{s1}'^- \mu^+ \nu_\mu)$	0.42	0.12	0.31
$f_L(B_s^0 \to D_{s1}^- \tau^+ \nu_\tau)$	0.66	0.48	0.56	$f_L(B_s^0 \to D_{s1}'^- \tau^+ \nu_\tau)$	0.23	0.24	0.24
$f_L(B^+ \to \bar{D}_1^0 e^+ \nu_e)$	0.81	0.50	0.72	$f_L(B^+ \to \bar{D}_1'^0 e^+ \nu_e)$	0.54	0.11	0.39
$f_L(B^+ \to \bar{D}_1^0 \mu^+ \nu_\mu)$	0.81	0.50	0.72	$f_L(B^+ \to \bar{D}_1'^0 \mu^+ \nu_\mu)$	0.54	0.11	0.39
$f_L(B^+ \to \bar{D}_1^0 \tau^+ \nu_\tau)$	0.67	0.49	0.56	$f_L(B^+ \to \bar{D}_1'^0 \tau^+ \nu_\tau)$	0.32	0.25	0.28

The longitudinal polarization fractions in small q^2 region are always larger than those in large q^2 region. The longitudinal polarization for the decays with $D_{(s)1}$ involved in the final states is dominant, while it is contrary for the decays with $D'_{(s)1}$ involved.

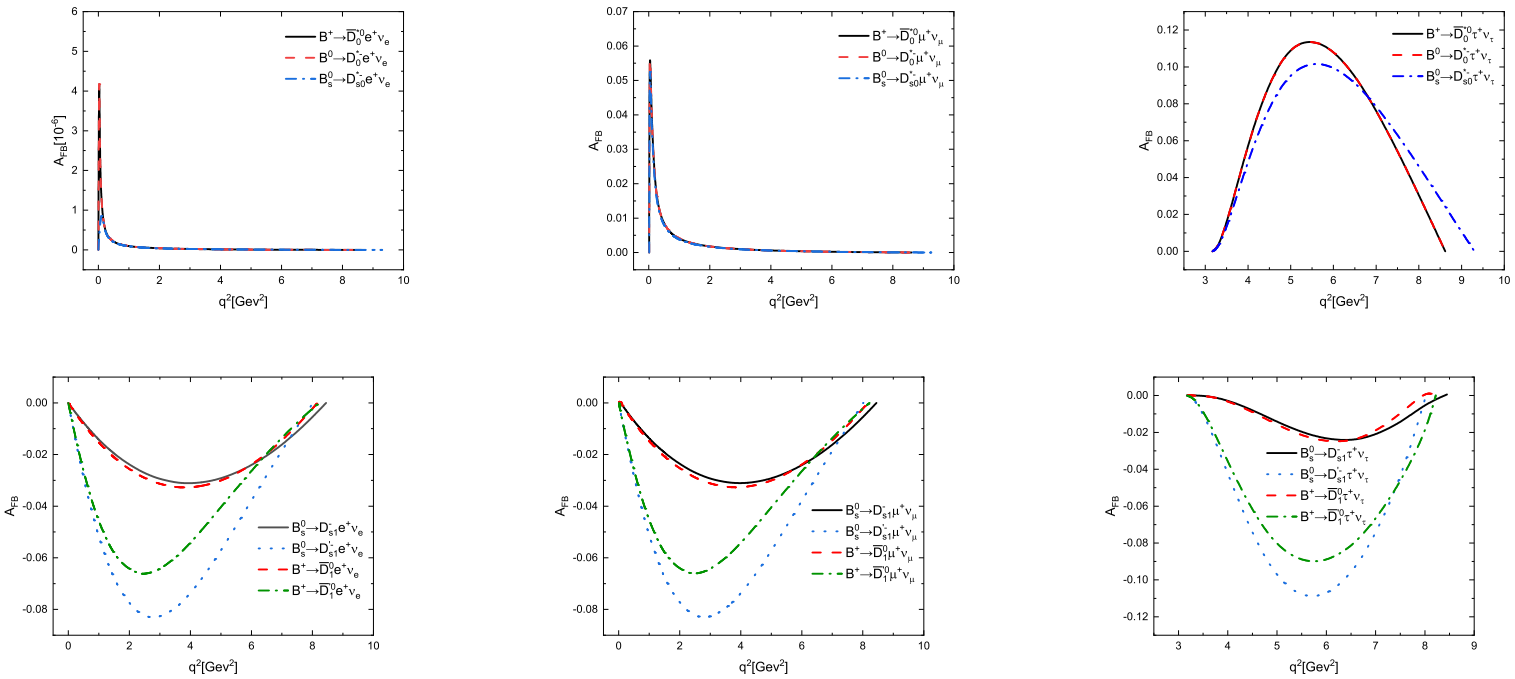
Summary

- Combining the helicity amplitudes with the form factors of the transitions $B_{(s)} \to D^{**}_{(s)}$ obtained from the CLFQM, the branching ratios of the corresponding $B_{(s)}$ weak decays are calculated.
- In order to provide more detailed physical picture for our considered decays, the longitudinal polarization fraction f_L and the forward-backward asymmetry A_{FB} are also calculated.
- Our predictions are helpful to clarify the different assumptions about the inner structures of these six charmed hadrons by comparing with the future data.

Thank you for your attention!

Backup

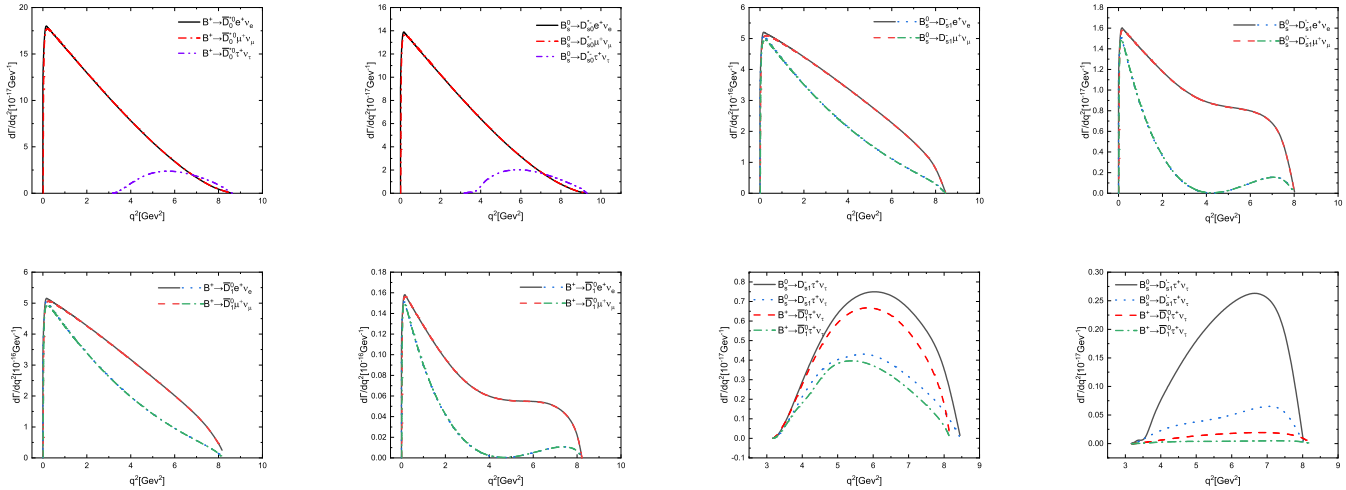
The q^2-dependence of the forward-backward asymmetry.



For the decays $B_{(s)} \to D_{(s)1}^{(\prime)}\ell'\nu_{\ell'}$, the values of the forward-backward asymmetries A_{FB}^{μ} and A_{FB}^{e} are almost equal to each

Backup

The q^2 dependences of differential decay rates $d\Gamma/dq^2$ and $d\Gamma^L/dq^2$.



It is interesting that taking some special q^2 values for the decays $B \to D_1' \ell' \nu_{\ell'}$ and $B_s \to D_{s1}' \ell' \nu_{\ell'}$, we find that the contribution from the longitudinal polarization almost disappears with only the transverse polarizations left.