基于 QCD 因子化方案 $B \to \pi\pi$ 中色八重态的贡献

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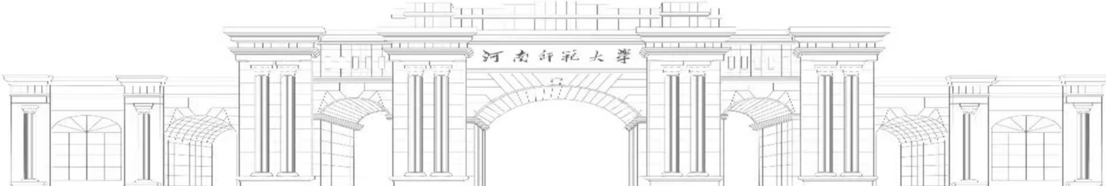

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CKM 矩阵

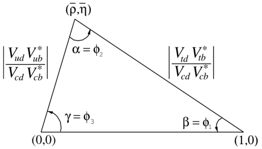

figure1: $(\bar\rho,\bar\eta)$ 复平面上的幺正三角形

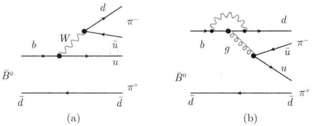

figure2: $\overline{B}^0 \to \pi^+\pi^-$ 过程的费曼图

(a) 树图; (b)QCD 企鹅图

CKM 混合矩阵 $\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

三个相角的约束条件: $\alpha+\beta+\gamma = 180°$

$$V_{ub} = |V_{ub}|e^{-i\gamma} \qquad V_{td} = |V_{td}|e^{-i\beta}$$
$$V_{ub}V_{ud}^* = |V_{ub}V_{ud}^*|e^{-i\gamma} \qquad V_{tb}V_{td}^* = |V_{tb}V_{td}^*|e^{+i\beta}$$

$$\begin{aligned}\mathcal{A} &= V_{ub}V_{ud}^*T - V_{tb}V_{td}^*P \\ &= |V_{ub}V_{ud}^*|e^{-i\gamma}T - |V_{tb}V_{td}^*|e^{+i\beta}P \\ &= |V_{ub}V_{ud}^*|e^{-i\gamma}T(1+\frac{|V_{tb}V_{td}^*|}{|V_{ub}V_{ud}^*|}e^{-i\alpha}\frac{P}{T})\end{aligned}$$

$B\to\pi\pi$ 衰变道可以提取 CKM 相角 α

动机

理论预测的 $\overline{B}^0 \to \pi^0\pi^0$ 的分支比与实验值相比小很多 （$\pi\pi$ puzzle）

Experimental data[1]

mode	PDG	Belle	BaBar	Belle II
$10^6 \times \mathcal{B}(B^- \to \pi^-\pi^0)$	5.31±0.26	5.86±0.46	5.02±0.54	5.10±0.40
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)$	1.55±0.17	1.31±0.27	1.83±0.25	1.38±0.35
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^+\pi^-)$	5.43±0.26	5.04±0.28	5.5 ±0.5	5.83±0.28
$\mathcal{A}_{CP}(B^- \to \pi^-\pi^0)$	−0.01 ±0.04	0.025±0.044	0.03 ±0.08	−0.081±0.055
$\mathcal{C}_{CP}(\overline{B}^0 \to \pi^0\pi^0)$	−0.25 ±0.20	−0.14 ±0.37	−0.43 ±0.26	0.14 ±0.47
$\mathcal{C}_{CP}(\overline{B}^0 \to \pi^+\pi^-)$	−0.314±0.030	−0.33 ±0.07	−0.25 ±0.08	
$\mathcal{S}_{CP}(\overline{B}^0 \to \pi^+\pi^-)$	−0.67 ±0.03	−0.64 ±0.09	−0.68 ±0.10	

Theoretical results

mode	QCDF		PQCD		
	+NLO[2]	+NNLO[3]	+NLO[4]	+NLO[5]	+NLOG[5]
$10^6 \times \mathcal{B}(B^- \to \pi^-\pi^0)$	5.1	5.82±1.42	$4.27^{+1.85}_{-1.47}$	3.35±1.10	4.45±1.43
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)$	0.7	0.63±0.65	$0.23^{+0.19}_{-0.15}$	0.29±0.11	0.61±0.21
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^+\pi^-)$	5.2	5.70±1.35	$7.67^{+3.47}_{-2.64}$	6.19±2.12	5.39±1.88

[1] Phys. Rev. D 110, 030001 (2024).
[2] M. Beneke, M. Neubert, Nucl. Phys. B 675, 333 (2003). [3] M. Beneke, T. Huber, X. Li, Nucl. Phys. B 832, 109 (2010).
[4] Y. Zhang, X. Liu, Y. Fan, S. Cheng, Z. Xiao, Phys. Rev. D 90, 014029 (2014). [5] X. Liu, H. Li, Z. Xiao, Phys. Rev. D 91, 114019 (2015).

$B \to \pi\pi$ 的振幅表达式 A_{ij}

$$\mathcal{A}_{+-} = \mathcal{A}_{\pi\pi}\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*[a_4 + a_{10} + (a_6 + a_8)R]\}$$

$$\mathcal{A}_{-0} = \frac{\mathcal{A}_{\pi\pi}}{\sqrt{2}}\{V_{ub}V_{ud}^*(a_1 + a_2) - V_{tb}V_{td}^*\frac{3}{2}(-a_7 + a_8 R + a_9 + a_{10})\}$$

$$\mathcal{A}_{00} = -\mathcal{A}_{\pi\pi}\{V_{ub}V_{ud}^* a_2 + V_{tb}V_{td}^*[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}(a_7 - a_9) + (a_6 - \frac{1}{2}a_8)R]\}$$

其中，$\mathcal{A}_{\pi\pi} = i\frac{G_F}{2}(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi$，$R = \frac{2m_\pi^2}{\bar{m}_b(\bar{m}_u+\bar{m}_d)}$

NLO 系数 $a_i = C_i^{\text{NLO}} + \frac{1}{N}C_j^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_j^{\text{LO}}V_i$ 当 i 为奇数时，j=i+1；i 为偶数时，j=i-1。

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N}C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2^{\text{LO}}V_1$$

$$a_2 = C_2^{\text{NLO}} + \frac{1}{N}C_1^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1^{\text{LO}}V_2$$

色八重态贡献

根据颜色 SU(3) 群生成元的代数关系 $T^a_{i,j}T^a_{k,l} = -\frac{1}{2N}\delta_{i,j}\delta_{k,l} + \frac{1}{2}\delta_{i,l}\delta_{k,j}$

一个四夸克算符可以表示为颜色单态和颜色八重态算符的和，Γ 代表任意 Dirac 流结构：

$$(\bar{q}_{1,\alpha}\Gamma_1 q_{2,\beta})(\bar{q}_{3,\beta}\Gamma_2 q_{4,\alpha}) = \frac{1}{N}(\bar{q}_{1,\alpha}\Gamma_1 q_{2,\alpha})(\bar{q}_{3,\beta}\Gamma_2 q_{4,\beta}) + 2(\bar{q}_1\Gamma_1 T^a q_2)(\bar{q}_3\Gamma_2 T^a q_4)$$

$$\begin{aligned}
&C_1\langle\pi^0\pi^0|O_1|\overline{B}^0\rangle\\
=&C_1\langle\pi^0\pi^0|(\overline{u}_\alpha b_\alpha)_{V-A}(\overline{d}_\beta u_\beta)_{V-A}|\overline{B}^0\rangle\\
=&C_1\langle\pi^0\pi^0|(\overline{u}_\alpha u_\beta)_{V-A}(\overline{d}_\beta b_\alpha)_{V-A}|\overline{B}^0\rangle\\
=&\frac{C_1}{N}\langle\pi^0\pi^0|(\overline{u}_\alpha u_\alpha)_{V-A}(\overline{d}_\beta b_\beta)_{V-A}|\overline{B}^0\rangle\\
+&2C_1\langle\pi^0\pi^0|(\overline{u}T^a u)_{V-A}(\overline{d}T^a b)_{V-A}|\overline{B}^0\rangle
\end{aligned}$$

参考 PQCD 的方法 [1,2]

[1] S. Lü and M. Z. Yang, Phys. Rev. D 107, 013004 (2023). [2] R. X. Wang and M. Z. Yang, Phys. Rev. D 108, 013003 (2023).

色八重态贡献 (continue.)

$$\langle \pi^0\pi^0|(\bar{u}_\alpha u_\alpha)_{V-A}(\bar{d}_\beta b_\beta)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi$$

类比色单态的强子矩阵元参数化色八重态矩阵元贡献：

$$\langle \pi^0\pi^0|(\bar{u}T^a u)_{V-A}(\bar{d}T^a b)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi X_{LL}$$

同理，

$$\langle \pi^0\pi^0|(\bar{d}T^a d)_{V+A}(\bar{d}T^a b)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi X_{RL}$$
$$-2\langle \pi^0\pi^0|(\bar{d}T^a d)_{S+P}(\bar{d}T^a b)_{S-P}|\overline{B}^0\rangle = +i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi R X_{SP}$$

假设 $X_{LL} = X_{RL} = X_{SP} = X = |X|e^{i\delta}$，则色八重态的振幅为 $\mathcal{C}_{ij} = 2X\mathcal{A}_{ij}$ $(a \to C)$

待定参数：$F_0^{B\pi}$、$|X|$、δ χ^2 函数：$\chi^2(\theta) = \sum_i \frac{(y_i - \mu(x_i;\theta))^2}{\sigma_i^2}$

待定参数

the fit results

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
PDG			
$\chi^2/n_{\rm dof}$	$120.5/4$	$170.1/4$	$205.2/4$
$\lvert X\rvert$	0.31 ± 0.02	0.40 ± 0.02	0.42 ± 0.02
δ	$(-61.5 \pm 5.5)^\circ$	$(75.6 \pm 4.3)^\circ$	$(83.5 \pm 4.0)^\circ$
$F_0^{B\pi}$	0.218 ± 0.004	0.218 ± 0.005	0.220 ± 0.005
$\rho_{\lvert X\rvert,F_0}$	-0.58	-0.51	-0.47
Belle			
$\chi^2/n_{\rm dof}$	$22.4/4$	$28.2/4$	$31.0/4$
$\lvert X\rvert$	0.28 ± 0.03	0.36 ± 0.04	0.37 ± 0.04
δ	$(-42.7 \pm 13.0)^\circ$	$(59.3 \pm 10.3)^\circ$	$(67.1 \pm 9.4)^\circ$
$F_0^{B\pi}$	0.220 ± 0.006	0.217 ± 0.006	0.216 ± 0.005
$\rho_{\lvert X\rvert,F_0}$	-0.50	-0.47	-0.38
BaBar			
$\chi^2/n_{\rm dof}$	$10.8/4$	$15.9/4$	$21.6/4$
$\lvert X\rvert$	0.33 ± 0.03	0.36 ± 0.03	0.37 ± 0.04
δ	$(-70.6 \pm 9.0)^\circ$	$(-79.7 \pm 8.8)^\circ$	$(-87.4 \pm 8.5)^\circ$
$F_0^{B\pi}$	0.216 ± 0.009	0.217 ± 0.009	0.221 ± 0.009
$\rho_{\lvert X\rvert,F_0}$	-0.63	-0.60	-0.54

[arXiv:2502.12461]

- 随能标 μ 变化
- 在 $\mu = m_b/2$ 时 χ^2 最小
- PDG 的最小 χ^2 更大
- $\lvert X\rvert \sim 0.3/0.4$
- $F_0^{B\pi} \sim 0.22/0.23$
- $\rho_{\lvert X\rvert,F_0} < 0$

格点计算结果 $F_0^{B\pi}$=0.183(92)[1]
光锥求和规则结果 $F_0^{B\pi}$=0.19(5)[2]

[1] JLQCD Collaboration, Phys. Rev. D106, 054502 (2022). [2] B. Cui, Y. Huang, Y. Shen, C. Wang, Y. Wang, JHEP 03, 140 (2023).

待定参数 (continue.)

our result: $|X| \sim 0.3/0.4$

PQCD: $|X| \sim 0.25$, $F_0^{B\pi}$=0.27[1]

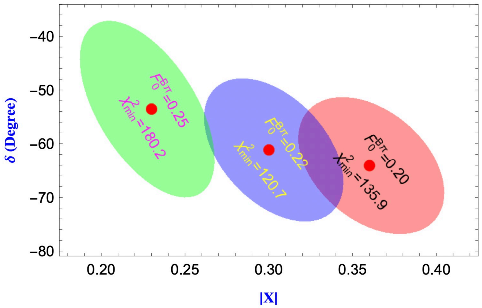

FIG. 1: The distribution of the fit parameter X obtained from the PGD data with different form factor $F_0^{B\pi}$ at the scale $\mu = m_b/2$, where the dots correspond to the optimal values of X, and the ellipses correspond to the errors of X.

[1] S. Lü and M. Z. Yang, Phys. Rev. D 107, 013004 (2023).

分支比

Branching ratios

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
				PDG
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.35^{+0.55}_{-0.52}$	$5.29^{+0.55}_{-0.52}$	$5.20^{+0.54}_{-0.51}$	5.31 ± 0.26
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.62^{+0.29}_{-0.26}$	$1.64^{+0.28}_{-0.25}$	$1.70^{+0.28}_{-0.25}$	1.55 ± 0.17
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.35^{+0.42}_{-0.41}$	$5.38^{+0.35}_{-0.34}$	$5.42^{+0.31}_{-0.30}$	5.43 ± 0.26
				Belle
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.88^{+0.95}_{-0.93}$	$5.89^{+0.98}_{-0.94}$	$5.82^{+0.97}_{-0.92}$	5.86 ± 0.46
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.35^{+0.43}_{-0.36}$	$1.34^{+0.43}_{-0.35}$	$1.39^{+0.41}_{-0.35}$	1.31 ± 0.27
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.02^{+0.62}_{-0.55}$	$5.02^{+0.51}_{-0.48}$	$5.04^{+0.40}_{-0.39}$	5.04 ± 0.28
				BaBar
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.04^{+1.07}_{-0.94}$	$5.01^{+1.09}_{-0.96}$	$4.98^{+1.09}_{-0.98}$	5.02 ± 0.54
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.85^{+0.49}_{-0.41}$	$1.82^{+0.49}_{-0.41}$	$1.81^{+0.48}_{-0.41}$	1.83 ± 0.25
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.46^{+0.77}_{-0.74}$	$5.52^{+0.69}_{-0.64}$	$5.55^{+0.61}_{-0.57}$	5.5 ± 0.5
				Belle II
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.11^{+0.62}_{-0.60}$	$5.10^{+0.61}_{-0.60}$	$5.10^{+0.61}_{-0.60}$	5.10 ± 0.40
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.42^{+0.44}_{-0.37}$	$1.40^{+0.44}_{-0.37}$	$1.39^{+0.45}_{-0.37}$	1.38 ± 0.35
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.82^{+0.43}_{-0.42}$	$5.82^{+0.42}_{-0.40}$	$5.83^{+0.38}_{-0.36}$	5.83 ± 0.28

$$\mathcal{A}_{+-} \sim a_1$$
$$\mathcal{A}_{00} \sim a_2$$
$$\mathcal{A}_{-0} \sim a_1 + a_2$$

拟合得到的分支比在误差范围内都与实验值符合很好。

CP 破坏

PDG

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0013^{+0.0003}_{-0.0002}$	-0.0024 ± 0.0001	0.0009 ± 0.0003	-0.01 ± 0.04
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$-0.504^{+0.043}_{-0.042}$	$0.418^{+0.035}_{-0.034}$	$0.314^{+0.024}_{-0.022}$	-0.25 ± 0.20
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$0.049^{+0.059}_{-0.054}$	$-0.053^{+0.045}_{-0.041}$	$-0.108^{+0.029}_{-0.027}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	$-0.023^{+0.002}_{-0.001}$	-0.012 ± 0.001	-0.025 ± 0.001	-0.314 ± 0.030
$\mathcal{S}_{CP}(\pi^+\pi^-)$	$-0.522^{+0.001}_{-0.002}$	-0.443 ± 0.001	-0.365 ± 0.002	-0.67 ± 0.03

Belle

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0018^{+0.0005}_{-0.0004}$	-0.0024 ± 0.0002	0.0001 ± 0.0005	0.025 ± 0.044
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$-0.459^{+0.113}_{-0.100}$	$0.396^{+0.082}_{-0.081}$	$0.307^{+0.061}_{-0.054}$	-0.14 ± 0.37
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$0.218^{+0.144}_{-0.129}$	$0.095^{+0.117}_{-0.104}$	$-0.002^{+0.076}_{-0.068}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	$-0.020^{+0.004}_{-0.003}$	$-0.010^{+0.002}_{-0.001}$	$-0.022^{+0.004}_{-0.003}$	-0.33 ± 0.07
$\mathcal{S}_{CP}(\pi^+\pi^-)$	$-0.526^{+0.003}_{-0.004}$	$-0.441^{+0.002}_{-0.001}$	$-0.359^{+0.004}_{-0.003}$	-0.64 ± 0.09

- 拟合值 $|\mathcal{A}_{CP}(\pi^-\pi^0)| < 1\%$
- $\mathcal{C}_{CP}(\pi^0\pi^0)$ 对色八重态贡献敏感
- $|\mathcal{A}_{CP}(\pi^-\pi^0)| < |\mathcal{C}_{CP}(\pi^-\pi^+)| < |\mathcal{C}_{CP}(\pi^0\pi^0)|$

振幅关系 $\mathcal{A}_{+-} \sim a_1 \qquad \mathcal{A}_{00} \sim a_2$

$$\mathcal{A}_{-0} = \frac{\mathcal{A}_{\pi\pi}}{\sqrt{2}}\{V_{ub}V_{ud}^*(a_1 + a_2) - V_{tb}V_{td}^*\frac{3}{2}(-a_7 + a_8 R + a_9 + a_{10})\}$$

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$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0018^{+0.0005}_{-0.0004}$	-0.0024 ± 0.0002	0.0001 ± 0.0005	0.025 ± 0.044
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$$\mathcal{A}_{-0} = \frac{\mathcal{A}_{\pi\pi}}{\sqrt{2}}\{V_{ub}V_{ud}^*(a_1+a_2) - V_{tb}V_{td}^*\frac{3}{2}(-a_7+a_8R+a_9+a_{10})\}$$

总结

- 我们将色八重态矩阵元的贡献考虑在内，基于 QCD 因子化方案在领头阶近似下重新研究了 $B \to \pi\pi$ 衰变。类比色单态矩阵元对色八重态矩阵元进行参数化，采用最小 χ^2 方法进行拟合。
- 色八重态贡献相对于色单态贡献较小，但是不能被忽略。色八重态矩阵元的引入有力加强了 $\overline{B}^0 \to \pi^0\pi^0$ 的分支比，并且 $B \to \pi\pi$ 过程所有分支比在误差范围内都与实验值符合很好。
- 拟合得到的形状因子 $F_0^{B\pi}$ ≈0.22。$F_0^{B\pi}$ 与 X 之间有紧密的关联性。
- 拟合得到的 CP 破坏结果与当前实验值的不一致还需要更多理论和实验的努力！

谢谢大家，欢迎批评指正！

威尔森系数

TABLE IV: Wilson coefficients C_i with the naive dimensional regularization scheme.

μ	$m_b/2$		m_b		$2\,m_b$	
	LO	NLO	LO	NLO	LO	NLO
C_1	1.168	1.128	1.110	1.076	1.070	1.041
C_2	−0.338	−0.269	−0.237	−0.173	−0.160	−0.100
C_3	0.019	0.020	0.012	0.014	0.007	0.009
C_4	−0.046	−0.048	−0.032	−0.034	−0.022	−0.024
C_5	0.010	0.010	0.008	0.008	0.006	0.006
C_6	−0.057	−0.060	−0.037	−0.039	−0.023	−0.025
C_7/α_{em}	−0.103	−0.012	−0.096	0.004	−0.080	0.027
C_8/α_{em}	0.023	0.080	0.014	0.052	0.009	0.034
C_9/α_{em}	−0.095	−1.372	−0.090	−1.297	−0.076	−1.234
$C_{10}/\alpha_{\mathrm{em}}$	−0.025	0.360	−0.018	0.249	−0.013	0.166

待定参数

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
PDG			
$\chi^2/n_{\rm dof}$	$120.5/4$	$170.1/4$	$205.2/4$
$\lvert X\rvert$	0.31 ± 0.02	0.40 ± 0.02	0.42 ± 0.02
δ	$(-61.5 \pm 5.5)^\circ$	$(75.6 \pm 4.3)^\circ$	$(83.5 \pm 4.0)^\circ$
$F_0^{B\pi}$	0.218 ± 0.004	0.218 ± 0.005	0.220 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	-0.44	0.47	0.41
$\rho_{\lvert X\rvert,F_0}$	-0.58	-0.51	-0.47
ρ_{δ,F_0}	0.14	0.08	0.22
Belle			
$\chi^2/n_{\rm dof}$	$22.4/4$	$28.2/4$	$31.0/4$
$\lvert X\rvert$	0.28 ± 0.03	0.36 ± 0.04	0.37 ± 0.04
δ	$(-42.7 \pm 13.0)^\circ$	$(59.3 \pm 10.3)^\circ$	$(67.1 \pm 9.4)^\circ$
$F_0^{B\pi}$	0.220 ± 0.006	0.217 ± 0.006	0.216 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	-0.63	0.74	0.68
$\rho_{\lvert X\rvert,F_0}$	-0.50	-0.47	-0.38
ρ_{δ,F_0}	0.49	-0.28	-0.05

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
BaBar			
$\chi^2/n_{\rm dof}$	$10.8/4$	$15.9/4$	$21.6/4$
$\lvert X\rvert$	0.33 ± 0.03	0.36 ± 0.03	0.37 ± 0.04
δ	$(-70.6 \pm 9.0)^\circ$	$(-79.7 \pm 8.8)^\circ$	$(-87.4 \pm 8.5)^\circ$
$F_0^{B\pi}$	0.216 ± 0.009	0.217 ± 0.009	0.221 ± 0.009
$\rho_{\lvert X\rvert,\delta}$	-0.27	-0.27	-0.27
$\rho_{\lvert X\rvert,F_0}$	-0.63	-0.60	-0.54
ρ_{δ,F_0}	0.15	-0.05	-0.22
Belle II			
$\chi^2/n_{\rm dof}$	$2.8/2$	$2.5/2$	$2.4/2$
$\lvert X\rvert$	0.36 ± 0.05	0.36 ± 0.05	0.37 ± 0.05
δ	$(72.2 \pm 6.9)^\circ$	$(78.1 \pm 7.0)^\circ$	$(83.9 \pm 7.0)^\circ$
$F_0^{B\pi}$	0.224 ± 0.006	0.225 ± 0.005	0.227 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	0.69	0.67	0.66
$\rho_{\lvert X\rvert,F_0}$	-0.57	-0.49	-0.38
ρ_{δ,F_0}	-0.48	-0.28	-0.05

- 随能标 μ 变化
- PDG 的 χ^2 更大
- $\lvert X\rvert \sim 0.3/0.4$
- $F_0^{B\pi} \sim 0.22/0.23$
- $\rho_{\lvert X\rvert,F_0} < 0$

CP 破坏 (continue.)

BaBar

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
$\mathcal{A}_{CP}(\pi^-\pi^0)$	-0.0009 ± 0.0004	-0.0024 ± 0.0002	-0.0049 ± 0.0005	0.03 ± 0.08
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$-0.493^{+0.055}_{-0.051}$	$-0.404^{+0.044}_{-0.043}$	$-0.308^{+0.033}_{-0.034}$	-0.43 ± 0.26
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$-0.035^{+0.087}_{-0.076}$	$-0.092^{+0.068}_{-0.060}$	$-0.123^{+0.050}_{-0.044}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	-0.024 ± 0.002	0.005 ± 0.001	0.020 ± 0.002	-0.25 ± 0.08
$\mathcal{S}_{CP}(\pi^+\pi^-)$	-0.519 ± 0.003	-0.444 ± 0.001	-0.367 ± 0.003	-0.68 ± 0.10

Belle II

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0052^{+0.0003}_{-0.0004}$	-0.0026 ± 0.0002	$0.0006^{+0.0006}_{-0.0005}$	-0.081 ± 0.055
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$0.572^{+0.079}_{-0.065}$	$0.469^{+0.070}_{-0.055}$	$0.358^{+0.057}_{-0.044}$	0.14 ± 0.47
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$0.011^{+0.124}_{-0.097}$	$-0.057^{+0.095}_{-0.074}$	$-0.097^{+0.070}_{-0.054}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	0.012 ± 0.003	-0.011 ± 0.001	-0.022 ± 0.003	
$\mathcal{S}_{CP}(\pi^+\pi^-)$	-0.522 ± 0.002	-0.444 ± 0.001	$-0.365^{+0.002}_{-0.003}$	

PQCD 结果

TABLE I. Branching ratios and direct CP violation ($\delta_8 = \delta_8^{SP}$, $l^2 = \frac{m_b^2}{4}$, $m_c = 1.3$ GeV), where NLO is the hard contribution up to next-to-leading order in QCD, "$+\xi_{B\pi}$" contribution of NLO + the contribution of the soft transition form factor $\xi_{B\pi}$, "$+T_8$" contribution of NLO + color-octet matrix element, "$+\xi_{\pi\pi}$" contribution of NLO + contribution of soft production form factor of $\pi\pi$, "$+\xi_{B\pi} + T_8 + \xi_{\pi\pi}$" total contribution of NLO + $\xi_{B\pi} + T_8 + \xi_{\pi\pi}$, for which the first uncertainty comes from the constraint of experimental data, the second is the quadratic combination of uncertainties from the variation of input parameters in B and pion wave functions. The last column is the experimental data from PDG [8].

Mode	NLO	$+\xi_{B\pi}$	$+\xi_{\pi\pi}$	$+T_8$	$+\xi_{B\pi} + \xi_{\pi\pi} + T_8$	Data [8]
$\mathrm{B}(B^0 \to \pi^+\pi^-) \times 10^{-6}$	4.95	7.48	3.32	4.37	$5.14 \pm 0.61^{+0.34}_{-0.37}$	5.12 ± 0.19
$\mathrm{B}(B^+ \to \pi^+\pi^0) \times 10^{-6}$	3.27	4.40	3.27	4.23	$5.72 \pm 0.44^{+0.29}_{-0.37}$	5.5 ± 0.4
$\mathrm{B}(B^0 \to \pi^0\pi^0) \times 10^{-6}$	0.13	0.14	0.22	0.67	$1.50 \pm 0.24^{+0.18}_{-0.19}$	1.59 ± 0.26
$A_{CP}(B^0 \to \pi^+\pi^-)$	0.17	0.11	0.44	0.22	$0.33 \pm 0.04^{+0.04}_{-0.03}$	0.32 ± 0.04
$A_{CP}(B^+ \to \pi^+\pi^0)$	−0.0007	−0.0007	−0.0007	0.0053	$0.0054 \pm 0.0004^{+0.0001}_{-0.0001}$	0.03 ± 0.04
$A_{CP}(B^0 \to \pi^0\pi^0)$	0.27	0.48	−0.16	0.53	$0.23 \pm 0.07^{+0.07}_{-0.05}$	0.33 ± 0.22

QCD 因子化方案

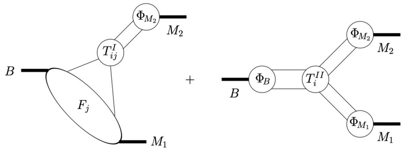
- 1999 年 M. Beneke 等人首次提出一种计算强子矩阵元的方法——QCD 因子化方法 [1]。
- 在 QCD 因子化方案下，非微扰效应包含在普适的介子光锥分布振幅和形状因子中。
- 在重夸克近似下，$B \to M_1 M_2$ 过程的强子矩阵元公式表示为

（1）当 M_1 和 M_2 都是轻介子

$$\begin{aligned}\langle M_1 M_2|O_i|B\rangle &= \sum_j F_j^{B\to M_1}\int_0^1 dx\ T_{ij}^I(x)\Phi_{M_2}(x) + (M_1 \leftrightarrow M_2)\\ &+\int_0^1 d\xi\int_0^1 dx\int_0^1 dy\ T_i^{II}(\xi,x,y)\Phi_B(\xi)\Phi_{M_1}(x)\Phi_{M_2}(y)\end{aligned}$$

（2）当 M_1 是重介子，M_2 是轻介子

$$\langle M_1 M_2|O_i|B\rangle = \sum_j F_j^{B\to M_1}\int_0^1 dx\ T_{ij}^I(x)\Phi_{M_2}(x)$$



- QCD 因子化方案在 B 介子两体非轻弱衰变已经得到广泛应用 [1−9]，但也存在一些问题。

[1]Phys. Rev. Lett. 83, 1914 (1999). [2]Nucl. Phys. B 591, 313 (2000). [3]Nucl. Phys. B 606, 245 (2001). [4]Phys. Lett. B 488, 46 (2000). [5]Phys. Lett. B 509, 263 (2001). [6]Phys. Rev. D 64, 014036 (2001). [7]Nucl. Phys. B 675, 333 (2003). [8]Nucl. Phys. B 832, 109 (2010). [9]Phys. Rev. D 90, 054019 (2014).

QCDF 方案下 $B \to \pi\pi$ 的振幅表达式 (continue.)

NLO 系数 $a_i = C_i^{\text{NLO}} + \frac{1}{N} C_j^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N} C_j^{\text{LO}} V_i$ 当 i 为奇数时，j=i+1；i 为偶数时，j=i-1。

顶角修正的贡献

$$V_i = \begin{cases} 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 dx\; g(x)\; \Phi_M(x) & \text{if } i = 1,2,3,4,9,10 \\ -[12\ln\frac{m_b}{\mu} - 6\int_0^1 dx\; g(\bar{x})\; \Phi_M(x)] & \text{if } i = 5,7 \\ -6 & \text{if } i = 6,8 \end{cases}$$

$$g(x) = 3(\tfrac{1-2x}{1-x}\ln x - i\pi) + [2\text{Li}_2(x) - \ln^2 x + \tfrac{2\ln x}{1-x} - (3+2i\pi)\ln x - (x \leftrightarrow \bar{x})]$$

对于 $B \to \pi\pi$，twist-2 光锥分布振幅 $\Phi_\pi(x) = 6x(1-x)[1 + \sum_{n=1}^{\infty} a_n^\pi C_n^{(3/2)}(2x-1)]$ 。

故 $\int_0^1 dx\; g(x)\; \Phi_\pi(x)$ 可以表示为 $-\frac{1}{2} - i\,3\pi - \frac{21}{20}a_2^\pi - \frac{12}{35}a_4^\pi$

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CKM 矩阵

- 1963 年，Cabibbo 引入混合角 θ_c，认为轻子是以味道本征态参加弱相互作用，夸克要先混合再参加弱相互作用，混合用混合角 θ_c 来描写。

- 1970 年，Glashow, Iliopoulos, Maiani 从对称性出发提出 GIM 机制，预言存在第四种味道“charm”。

弱作用本征态 $$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$ 质量本征态

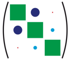
- 1973 年，Kobayashi 和 Maskawa 将 Cabibbo 两代夸克混合和 GIM 机制的思想推广到三代夸克，提出了描写三个下夸克混合的 3×3CKM 混合矩阵。

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

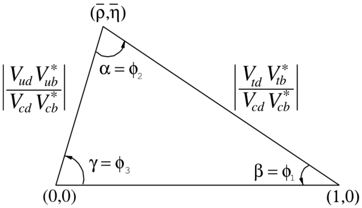
CKM 矩阵

采用 Wolfenstein 参数化方案，在领头阶近似下 CKM 矩阵可以展开为

$$\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4).$$



由 CKM 矩阵的幺正性，可以得到 $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*$ =0，对应复平面上一个幺正三角形。



$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

人们通过对 $B \to \pi\pi, \rho\pi$ 和 $\rho\rho$ 衰变道的研究来测量相角 α。