Analytic decay width of the Higgs boson to massive bottom quarks at $\mathcal{O}\left(\alpha_s^3\right)$

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Motivation

Higgs is important in the Standard Model.

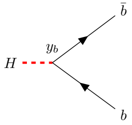
The dominant decay mode of the Higgs boson is $H \to b\bar{b}$ [P.D.G 2024].

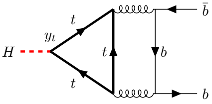
The process $H \to b\bar{b}$ has been observed in LHC [CMS 2018, ATLAS 2018].

The accuracy of y_b will be improved at future electron colliders. [CEPC group 2024].

Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	2.1%
$H \to ZZ$	2.62×10^{-2}	$\pm1.5\%$
$H \to W^+W^-$	2.14×10^{-1}	$\pm1.5\%$
$H \to \tau^+\tau^-$	6.27×10^{-2}	$\pm1.6\%$
$H \to b\bar{b}$	5.82×10^{-1}	$^{+1.2\%}_{-1.3\%}$
$H \to c\bar{c}$	2.89×10^{-2}	$^{+5.5\%}_{-2.0\%}$
$H \to Z\gamma$	1.53×10^{-3}	$\pm5.8\%$
$H \to \mu^+\mu^-$	2.18×10^{-4}	$\pm1.7\%$

Motivation





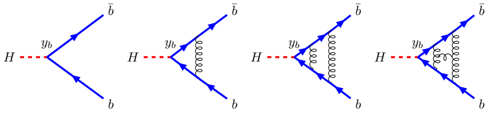
Inclusive decay width up to $\mathcal{O}(y_b^2\alpha_s^4)$ with massless bottom quarks [Gorishnii, Kataev, Larin, Surguladze 1990, Chetyrkin 1997, Baikov, Chetyrkin, Kuhn 2006]

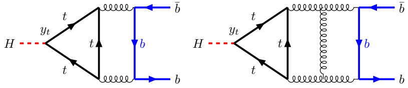
Differential decay width at $\mathcal{O}(y_b^2\alpha_s^2)$ with massive bottom quarks [Bernreuther, Chen, Si 2018, Behring, Bizoń 2020, Somogyi, Tramontano 2020]

Large m_t limit, differential decay width at $\mathcal{O}(\alpha_s^3)$ [Mondini, Schubert, Williams 2020, Chen, Jakubčík, Marcoli, Stagnitto 2023]

Large m_t limit, analytic calculations at $\mathcal{O}(\alpha_s^3)$ [This talk]

$H \to b\bar{b}$ with bottom quark Yukawa coupling and top quark Yukawa coupling up to $\mathcal{O}(\alpha_s^3)$.



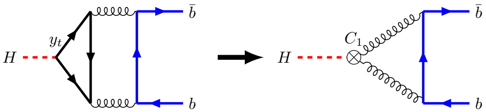


The decay width can be decomposed to

$$\Gamma_{H\to b\bar{b}} = \Gamma^{y_b y_b}_{H\to b\bar{b}} + \Gamma^{y_b y_t}_{H\to b\bar{b}} + \Gamma^{y_t y_t}_{H\to b\bar{b}}. \tag{1}$$

Effective theory

In the effective theory, the top quark can be integrated in the large top quark mass limit, $m_t \to \infty$.



And

$$\text{top quark loop}\,(y_t) \to C_1 \mathcal{O}_1, \quad \alpha_s^{(6)} \to \alpha_s^{(5)} \tag{2}$$

$$\mathcal{O}_1 = H G_{a,\mu\nu} G^{a,\mu\nu} \tag{3}$$

This approximation works exceedingly well.

Effective theory

Higgs boson decay to bottom quarks can be written as

$$\mathcal{L}_{\text{eff}} = -\frac{H}{v}\left(C_1\mathcal{O}_1^R + C_2\mathcal{O}_2^R\right) + \mathcal{L}_{\text{QCD}}\,,$$
$$\mathcal{O}_1 = (G^0_{a,\mu\nu})^2, \quad \mathcal{O}_2 = m_b^0\bar{b}^0 b^0. \tag{4}$$

$$\begin{aligned} C_1 &= -\left(\frac{\alpha_s}{\pi}\right)\frac{1}{12} - \left(\frac{\alpha_s}{\pi}\right)^2\frac{11}{48} \\ &\quad - \left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{2777}{3456} + \frac{19}{192}L_t - n_f\left(\frac{67}{1152} - \frac{1}{36}L_t\right)\right] + \mathcal{O}(\alpha_s^4), \\ C_2 &= 1 + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{5}{18} - \frac{1}{3}L_t\right] \\ &\quad + \left(\frac{\alpha_s}{\pi}\right)^3\left[-\frac{841}{1296} + \frac{5}{3}\zeta(3) - \frac{79}{36}L_t - \frac{11}{12}L_t^2 + n_f\left(\frac{53}{216} + \frac{1}{18}L_t^2\right)\right] + \mathcal{O}(\alpha_s^4), \end{aligned} \tag{5}$$

$L_t = \log(\mu^2/m_t^2)$ in the on-shell scheme, n_f is the number of quark flavors.

Effective theory

$$\mathcal{L}_{\text{eff}} = -\frac{H}{v}\left(C_1 \mathcal{O}_1^R + C_2 \mathcal{O}_2^R\right) + \mathcal{L}_{\text{QCD}}\,,$$
$$\mathcal{O}_1 = (G^0_{a,\mu\nu})^2, \quad \mathcal{O}_2 = m_b^0 \bar{b}^0 b^0. \tag{6}$$

$H \to b\bar{b}$ can be decomposed into three parts

$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}. \tag{7}$$

up to $\mathcal{O}(\alpha_s^3)$,

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right],$$
$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$
$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right], \tag{8}$$

Optical Theorem

The optical theorem,

$$\Gamma_{Hb\bar{b}} = \frac{\mathrm{Im}(\Sigma)}{m_H}, \tag{9}$$

where Σ represents the forward scattering amplitudes of the process $H \to b\bar{b} \to H$.

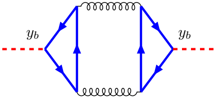
The complicated multi-body phase space integration can be avoided.

$\Gamma^{C_2C_2}_{H\to b\bar{b}}\ \left(\Gamma^{y_by_b}_{H\to b\bar{b}}\right)$

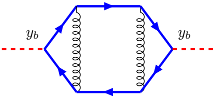
The analytical result of $\Delta^{C_2C_2}_{2,b\bar{b}}$ have been calculated in the massive m_b. [Wang, Wang, Zhang 2023]

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right]$$

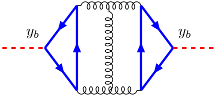
$\Delta^{C_2C_2}_{3,b\bar{b}}$ have been calculated in the massless m_b long time ago. [Gorishnii, Kataev, Larin, Surguladze 1990, Chetyrkin 1997, Baikov, Chetyrkin, Kuhn 2006]



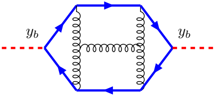
$\Delta^{C_2C_2}_{2,b\bar{b}}$



$\Delta^{C_2C_2}_{2,b\bar{b}}$



$\Delta^{C_2C_2}_{3,b\bar{b}}$



$\Delta^{C_2C_2}_{3,b\bar{b}}$

$$\Gamma_{H\to b\bar{b}}^{C_2C_2} = C_2C_2 \left[\Delta_{0,b\bar{b}}^{C_2C_2} + \left(\frac{\alpha_s}{\pi}\right)\Delta_{1,b\bar{b}}^{C_2C_2} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta_{2,b\bar{b}}^{C_2C_2} + \left(\frac{\alpha_s}{\pi}\right)^3 \Delta_{3,b\bar{b}}^{C_2C_2} + \mathcal{O}(\alpha_s^4)\right]$$

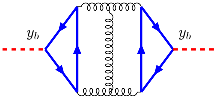
At $\mathcal{O}(\alpha_s^2)$, the finite bottom quark mass effect is quite small, since $\overline{m}_b^2/m_H^2 \approx 5\times 10^{-4}$.

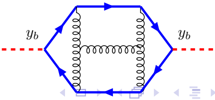
Therefore we neglect m_b in propagators for the $\mathcal{O}(\alpha_s^3)$ corrections to $\Delta_{3,b\bar{b}}^{C_2C_2}$

$$\Delta_{3,b\bar{b}}^{C_2C_2}(\mu = m_H,\, m_b \to 0) = \Delta_{0,b\bar{b}}^{C_2C_2}\left(\frac{1945\zeta(5)}{36} - \frac{5\pi^4}{108} - \frac{80095\zeta(3)}{216} - \frac{10225\pi^2}{324} + \frac{34873057}{46656}\right) \quad (10)$$

Only the bottom mass in Yukawa coupling are kept,

$$\Delta_{0,b\bar{b}}^{C_2C_2} = \frac{3\overline{m}_b^2 m_H}{8\pi v^2}, \quad \mu = m_H \quad (11)$$



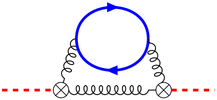


$\Gamma^{C_1C_2}_{H\to b\bar{b}}$ $\left(\Gamma^{y_b y_t}_{H\to b\bar{b}}\right)$ and $\Gamma^{C_1C_1}_{H\to b\bar{b}}$ $\left(\Gamma^{y_t y_t}_{H\to b\bar{b}}\right)$

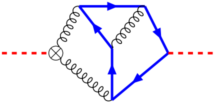
However, the results of $\Gamma^{C_1C_2}_{H\to b\bar{b}}$ and $\Gamma^{C_1C_1}_{H\to b\bar{b}}$ cannot be Taylor expanded in m_b^2 because of the logarithmic dependence even in leading power (keep the m_b in Yukawa coupling).

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$
$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right], \qquad (12)$$

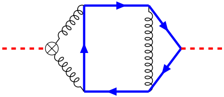
The bottom mass need to be kept in the calculations.



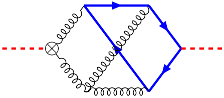
$\Delta^{C_1C_1}_{1,b\bar{b}}$



$\Delta^{C_1C_2}_{2,b\bar{b}}$



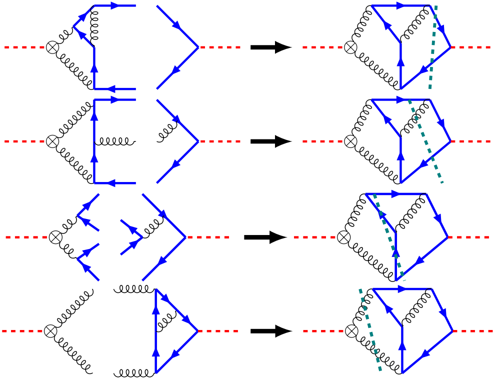
$\Delta^{C_1C_2}_{2,b\bar{b}}$



$\Delta^{C_1C_2}_{2,b\bar{b}}$

$\Delta_{2,b\bar{b}}^{C_1 C_2}$

The imaginary part comes from cut diagrams. For example,



$H \to b\bar{b}$, $H \to b\bar{b}g$, $H \to b\bar{b}b\bar{b}$ and $H \to gg$.

The last cut diagram needs to be subtracted.

Calculation framework

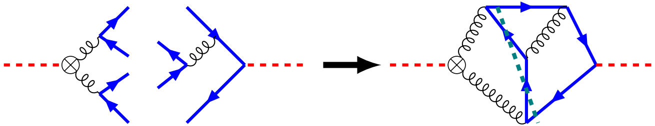
Focus on the $\Delta_{2,b\bar{b}}^{C_1C_2}$,

1. Generating diagrams and amplitudes.
2. IBP reduction with `Kira` [Klappert, Lange, Maierhöfer, Usovitsch 2021].
3. Master integrals calculations.

The contributions from only two bottom quark final states ($b\bar{b}$, $b\bar{b}g$, $b\bar{b}q\bar{q}$, $b\bar{b}gg$) are defined as $\tilde{\Delta}_{2,b\bar{b}}^{C_1C_2}$, and that from four bottom quark final states are $\Delta_{2,b\bar{b}b\bar{b}}^{C_1C_2}$.

$$\Delta_{2,b\bar{b}}^{C_1C_2} \equiv \tilde{\Delta}_{2,b\bar{b}}^{C_1C_2} + \Delta_{2,b\bar{b}b\bar{b}}^{C_1C_2} \tag{13}$$

These two parts are computed separately.



Master integrals calculations in $\tilde{\Delta}^{C_1 C_2}_{2,b\bar{b}}$ (two bottom quark final states)

Canonical differential equation [Henn 2013]. To rationalize the square root $r = \sqrt{z(z-4)}$,

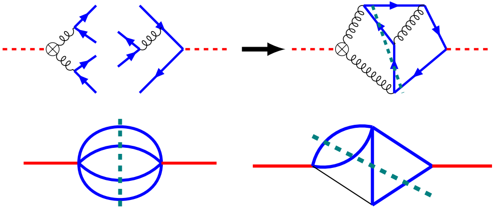
$$z = \frac{m_H^2}{m_b^2} = -\frac{(w-1)^2}{w}. \tag{14}$$

The solutions of canonical differential equations are multiple polylogarithms (GPLs).

$$\begin{aligned}
\tilde{\Delta}^{C_1 C_2}_{2,b\bar{b}} \; &= \; \frac{m_H m_b \overline{m_b}(\mu)}{v^2 \pi} \frac{(-w)}{(w-1)^2} \times \\
&\Bigg(\Big[(w+1)\left(5w^2 - 2w + 101\right) + i\pi \left(101 w^3 + 99 w^2 + 93 w + 91\right) \Big] \frac{(w+1) G(-1,0,w)}{3(w-1)^2 w} \\
&+ \Big[12(w-1)(w+1)^3 - i\pi \left(11 w^4 + 20 w^3 + 34 w^2 + 20 w + 11\right) \Big] \frac{8 G(1,-1,w)}{3(w-1)^2 w} \\
&+ \Big[(w-1)(w+1) - i\pi \left(w^2 + 1\right) \Big] \frac{32 (w+1)^2 G(-1,1,w)}{(w-1)^2 w} \Bigg) + \cdots
\end{aligned} \tag{15}$$

Master integrals calculations in $\Delta^{C_1 C_2}_{2,b\bar{b}b\bar{b}}$

The master integrals contributing to $\Delta^{C_1 C_2}_{2,b\bar{b}b\bar{b}}$ contain elliptic integrals and cannot be written as GPLs



They have been calculated in [Lee, Onishchenko 2019] for the process $e^+e^- \to Q\bar{Q}Q\bar{Q}$.

After choosing a regular basis, only the $\mathcal{O}(\epsilon^0)$ parts of the MIs are needed.

Complete elliptic integrals or one-fold integrals of them. For example,

$$F_1^{4b}(z) = (z-16)f(z) = \frac{16\pi(z-16)}{z}\left[\mathrm{K}(1-k_-)\mathrm{K}(k_+) - \mathrm{K}(k_-)\mathrm{K}(1-k_+)\right],$$
$$F_4^{4b}(z) = \frac{\beta s F_1^{4b}(z)}{s-4} - \int_{16}^{z} dz_1 \frac{4\beta_1(z_1+2)F_1^{4b}(z_1)}{(z_1-16)(z_1-4)^2} \qquad (16)$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right].$$
$$\Delta^{C_1C_2}_{2,b\bar{b}} \equiv \tilde{\Delta}^{C_1C_2}_{2,b\bar{b}} + \Delta^{C_1C_2}_{2,b\bar{b}b\bar{b}} \tag{17}$$

$z = m_H^2/m_b^2$, $\beta = \sqrt{1-4/z}$ is the velocity of bottom quarks.

$$\Delta^{C_1C_2}_{2,b\bar{b}b\bar{b}} = \frac{m_H m_b \overline{m_b}(\mu)}{v^2}\frac{1}{6912\pi^2 z^2}\Big[-3\left(279z^2-1284z+1024\right)F_1^{4b}(z)$$
$$+6\left(77z^2-764z+1376\right)F_2^{4b}(z)+24(z-4)(71z-172)F_3^{4b}(z)$$
$$+16\left(49z^2-266z+256\right)\beta F_4^{4b}(z)-24z^2F_5^{4b}(z)+48(5z-18)z\beta F_6^{4b}(z)$$
$$-192\left(z^2-3z-7\right)F_7^{4b}(z)-48(5z-18)z\beta F_8^{4b}(z)-96\left(z^2-4z+2\right)F_9^{4b}(z)\Big] \tag{18}$$

$$F_1^{4b}(z) = (z-16)f(z) = \frac{16\pi(z-16)}{z}\left[\mathrm{K}(1-k_-)\mathrm{K}(k_+)-\mathrm{K}(k_-)\mathrm{K}(1-k_+)\right],$$
$$F_4^{4b}(z) = \frac{\beta s F_1^{4b}(z)}{s-4} - \int_{16}^{z} dz_1 \frac{4\beta_1(z_1+2)F_1^{4b}(z_1)}{(z_1-16)(z_1-4)^2} \tag{19}$$

Asymptotic expansion

$\Delta^{C_1 C_2}_{1,b\bar{b}}|_{z\to\infty}$ is induced by soft quarks, which is differs from Sudakov double logarithm.

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right]$$
$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right], \quad (20)$$

$z = m_H^2/{m_b}^2$

$$\Delta^{C_1C_2}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H m_b \overline{m_b}(\mu)}{\pi v^2} C_A C_F\left[-\frac{1}{8}\log^2(z) - \frac{3}{4}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{\pi^2}{8} - \frac{19}{8}\right.$$
$$\left. + \frac{1}{2}\frac{\log^2(z)}{z} + 2\frac{\log(z)}{z} + \frac{9}{2z}\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{\pi^2}{2z} + \frac{15}{2z}\right] + \mathcal{O}(z^{-2}),$$
$$\Delta^{C_1C_1}_{1,b\bar{b}}|_{z\to\infty} = \frac{m_H^3}{\pi v^2} C_A C_F\left[\frac{1}{6}\log(z) - \frac{7}{12} + \frac{3}{z}\right] + \mathcal{O}(z^{-2}). \quad (21)$$

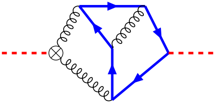
$$\Gamma^{C_1 C_2}_{H\to b\bar{b}} = C_1 C_2 \left[\left(\frac{\alpha_s}{\pi}\right) \Delta^{C_1 C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{C_1 C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3) \right] \tag{22}$$

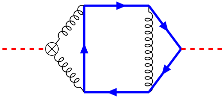
$$\Delta^{C_1 C_2}_{2,b\bar{b}} \equiv \tilde{\Delta}^{C_1 C_2}_{2,b\bar{b}} + \Delta^{C_1 C_2}_{2,b\bar{b}b\bar{b}} \tag{23}$$

$z = m_H^2 / {m_b}^2$

$$\Delta^{C_1 C_2}_{2,b\bar{b}} = -\frac{m_H m_b \overline{m_b}(\mu)}{192\pi v^2} C_A C_F \left(C_A - C_F\right) \log^4(z) + \cdots \tag{24}$$

This color structure distinguishes it from the Sudakov double logarithms and shares the same features as the results for the quark-gluon splitting function [Vogt 2010], $Hb\bar{b}$ form factor [Liu, Penin 2017], and off-diagonal "gluon" thrust [Moult, Stewart, Vita, Zhu 2020, Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang 2020].





Asymptotic expansion of $H \to b\bar{b}$ in $\overline{\text{MS}}$ scheme,

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \mathcal{O}(\alpha_s^4)\right],$$

$$\Gamma^{C_1C_2}_{H\to b\bar{b}} = C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \mathcal{O}(\alpha_s^3)\right],$$

$$\Gamma^{C_1C_1}_{H\to b\bar{b}} = C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \mathcal{O}(\alpha_s^2)\right],$$

$$\Gamma_{H\to b\bar{b}} = \frac{3m_H\overline{m_b}^2}{8v^2\pi}\Bigg\{1 + \left(\frac{\alpha_s}{\pi}\right)\frac{17}{3} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{1}{9}\log^2(\bar{z}) - \frac{2}{3}\log(x) - \frac{97\zeta(3)}{6} - \frac{17\pi^2}{12} + \frac{9235}{144}\right]$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\Bigg[\frac{5}{648}\log^4(\bar{z}) + \frac{59}{324}\log^3(\bar{z}) - \frac{31\pi^2}{324}\log^2(\bar{z}) + \frac{989}{648}\log^2(\bar{z}) + \frac{32\zeta(3)}{27}\log(\bar{z})$$
$$-\frac{41\pi^2}{324}\log(\bar{z}) + \frac{137}{216}\log(\bar{z}) - \frac{23}{18}\log^2(x) - \frac{49}{6}\log(x) + \frac{1945\zeta(5)}{36} - \frac{13\pi^4}{3240} - \frac{81239\zeta(3)}{216}$$
$$-\frac{22291\pi^2}{648} + \frac{37434709}{46656}\Bigg] + \mathcal{O}(\bar{z}^{-1})\Bigg\} + \frac{m_H^3}{v^2\pi}\left(\frac{\alpha_s}{\pi}\right)^3\left[\frac{\log(\bar{z})}{216} - \frac{7}{432} + \mathcal{O}(\bar{z}^{-1})\right] + \mathcal{O}(x) + \mathcal{O}(\alpha_s^4)$$

with $\bar{z} = m_H^2/\overline{m_b}^2$, $x = m_H^2/m_t^2$ and $\mu = m_H$. The large logarithmic terms provide significant corrections to the decay width.

$\overline{z} = m_H^2/\overline{m_b}^2$ and $x = m_H^2/m_t^2$ Apply the same method to different cut,

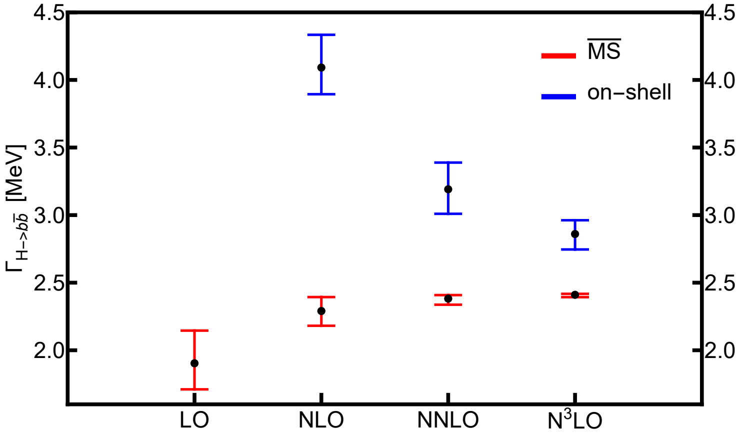
$$\Gamma_{H\to gg} = \frac{m_H \overline{m_b}^2}{v^2\pi}\Bigg\{ \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{1}{24}\log^2(\overline{z}) + \frac{\pi^2}{24} + \frac{1}{6}\right]$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[-\frac{5}{1728}\log^4(\overline{z}) - \frac{59}{864}\log^3(\overline{z}) + \frac{31\pi^2}{864}\log^2(\overline{z}) - \frac{989}{1728}\log^2(\overline{z})\right.$$
$$- \frac{4\zeta(3)}{9}\log(\overline{z}) + \frac{41\pi^2}{864}\log(\overline{z}) - \frac{137}{576}\log(\overline{z}) - \frac{137\pi^4}{8640} - \frac{29\zeta(3)}{36}$$
$$\left. + \frac{1277\pi^2}{1728} + \frac{17275}{3456}\right]\Bigg\} + \frac{m_H^3}{v^2\pi}\Bigg\{ \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{72} + \left(\frac{\alpha_s}{\pi}\right)^3 \left[-\frac{1}{216}\log(\overline{z}) + \frac{229}{864}\right]\Bigg\}$$

$$\Gamma_{H\to \text{hadron}} = \frac{3 m_H \overline{m_b}^2}{8v^2\pi}\Bigg\{1 + \left(\frac{\alpha_s}{\pi}\right)\frac{17}{3} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{2}{3}\log(x) - \frac{97\zeta(3)}{6} - \frac{47\pi^2}{36} + \frac{9299}{144}\right]$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[-\frac{23}{18}\log^2(x) - \frac{49}{6}\log(x) + \frac{1945\zeta(5)}{36} - \frac{5\pi^4}{108} - \frac{81703\zeta(3)}{216}\right.$$
$$\left. - \frac{10507\pi^2}{324} + \frac{38056609}{46656}\right]\Bigg\} + \frac{m_H^3}{v^2\pi}\Bigg\{ \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{72} + \left(\frac{\alpha_s}{\pi}\right)^3 \frac{215}{864}\Bigg\} \tag{25}$$

Large logarithmic are canceled in hadronic state, the result is consistent with [Chetyrkin, Steinhauser 1997, Davies, Steinhauser, Wellmann 2017]

Numerical result

$\overline{m_b}\,(m_H/2) = 2.95631$ GeV, $\overline{m_b}\,(m_H) = 2.78425$ GeV, $\overline{m_b}\,(2m_H) = 2.63908$ GeV



The $\mathcal{O}(\alpha_s^3)$ correction increases the NNLO decay rate by 1%.

$\Gamma^{\mathrm{N^3LO\ \ QCD}}_{H\to b\bar{b}}\,(\overline{\mathrm{MS}}) = 2.410^{+0.007}_{-0.017}$ MeV.

Conclusion

We have calculated the analytic result of the dominant decay channel of the Higgs boson, $H \to b\bar{b}$, at $\mathcal{O}(\alpha_s^3)$.

The $\mathcal{O}(\alpha_s^3)$ correction increases the NNLO decay rate by 1% due to the large logarithms

The coefficient of the double logarithm at $\mathcal{O}(\alpha_s^3)$ is proportional to $C_A - C_F$, which is a typical color structure in the next-to-leading power resummation with soft quarks.

Our analytic results provide a useful reference to check the resummation formula in future.