東南大學
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Precision Gravitational Dynamics from Feynman Integrals

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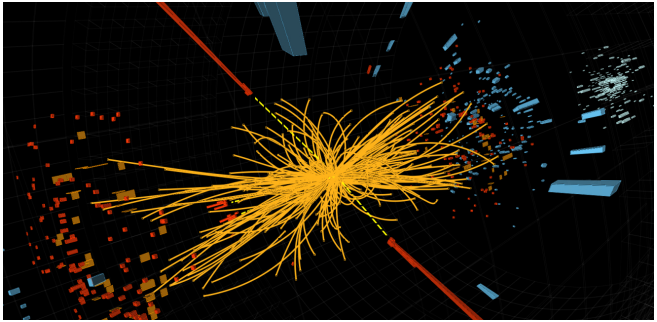
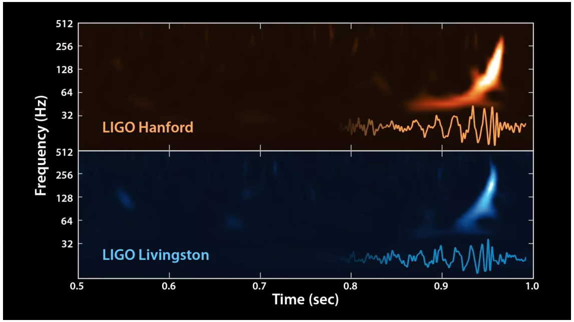
2025粒子物理标准模型及新物理精细计算研讨会

保定 2025.04.30

Precision era of fundamental physics

止於至善



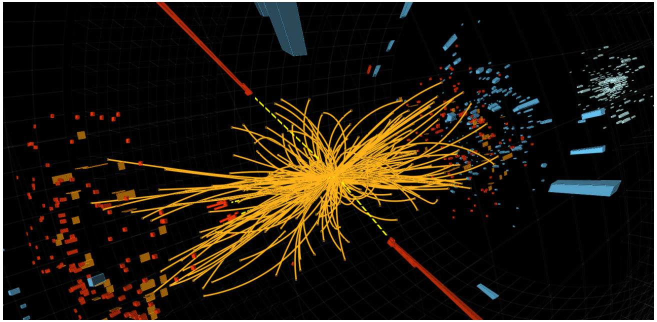


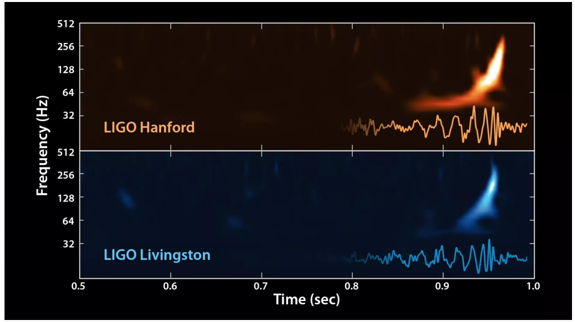
Two historic breakthroughs in science:

- Higgs bosons at the LHC (2012)
- Gravitational waves by the LIGO (2016)
- High-energy and gravitational physics entered a precision era!

Opened a new window on the Universe!

Precision era of fundamental physics





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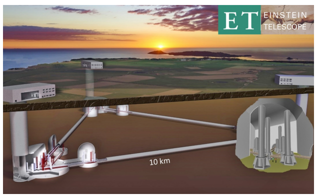
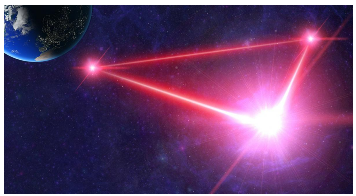
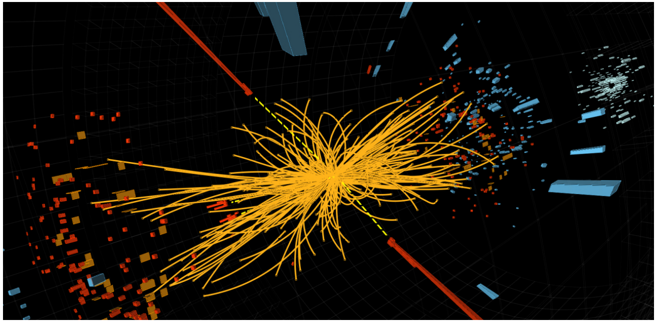
Opened a new window on the Universe! discovery potential = precise theoretical predictions!

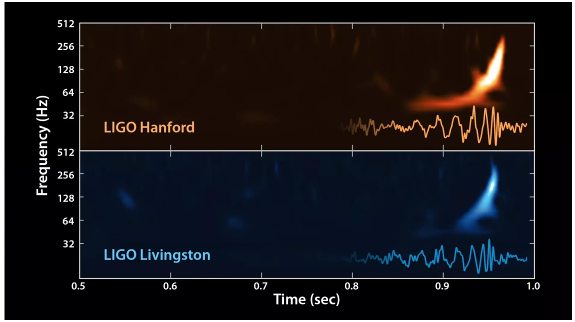
LIGO VIRGO KAGRA





Precision era of fundamental physics

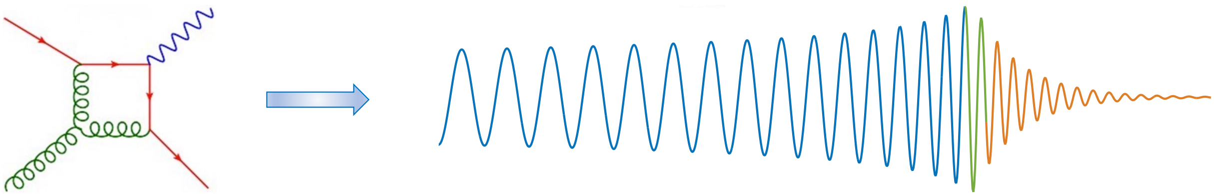




Two historic breakthroughs in science:

- Higgs bosons from the LHC (2012)
- Gravitational waves from the LIGO (2016)
- High-energy and gravitational physics entered a precision era!

Modern techniques from particle physics are playing a crucial role in precision GW physics!



Gravitational waves from binary coalescences

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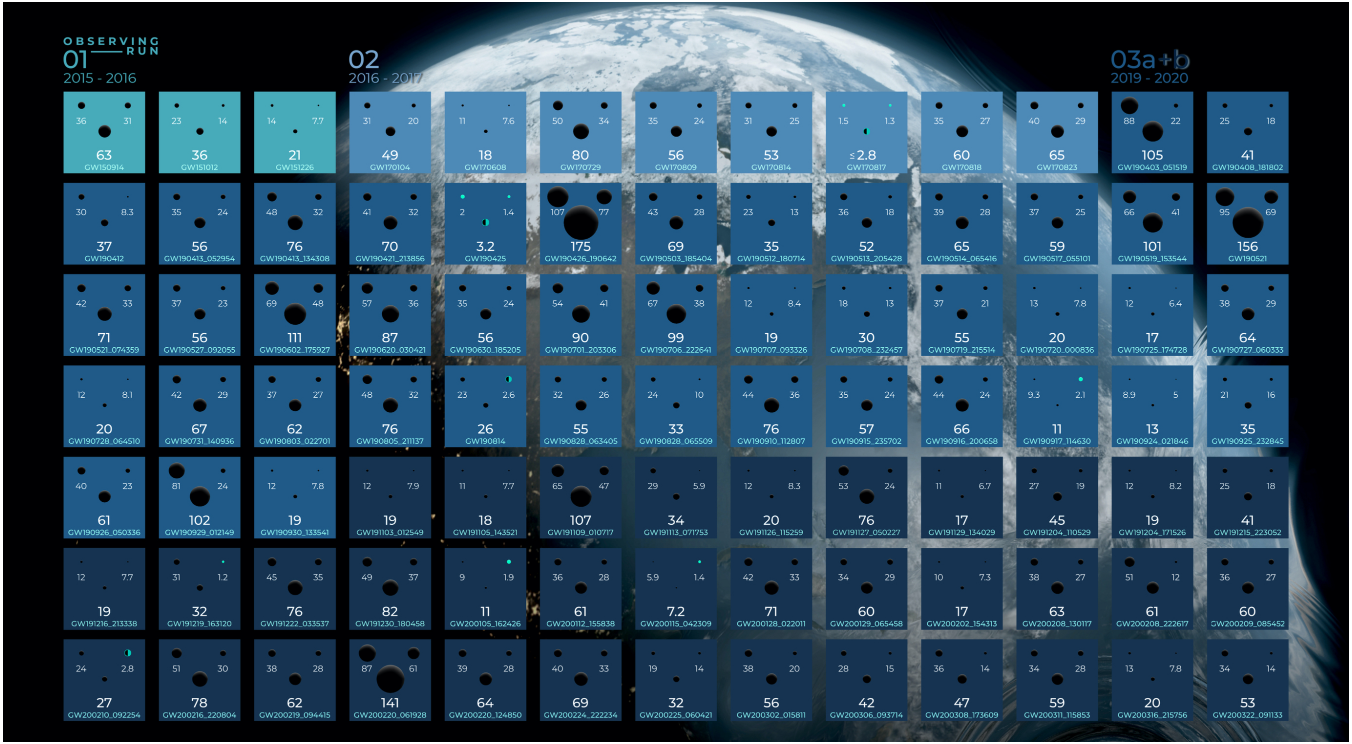

Credit: Carl Knox (OzGrav, Swinburne)

GWTC-3: 90 GW events – the majority are binary black holes (BH), but also several binary neutron stars (NS) and mixed NS-BHs.

Gravitational waves from binary coalescences

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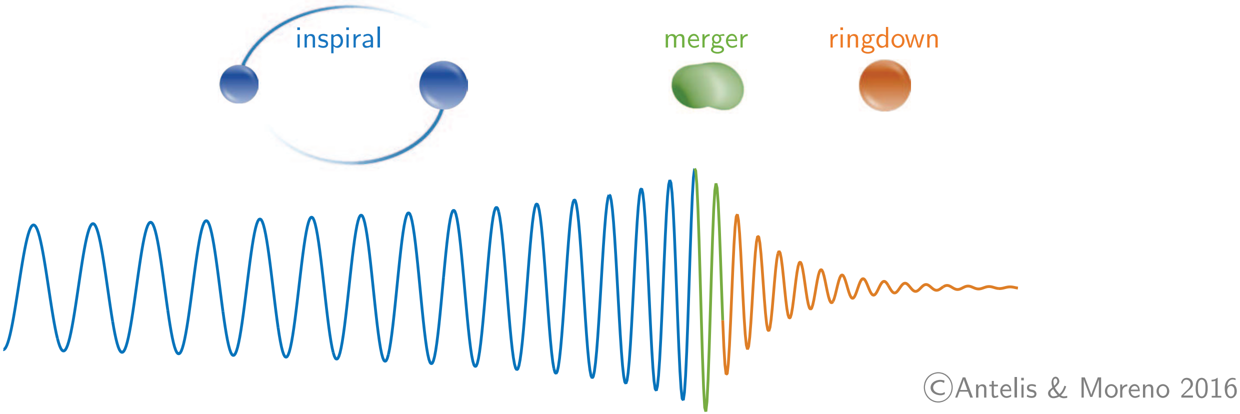

Merger: Numerical Relativity

Ringdown: black hole perturbation theory

Inspiral: the interaction between two bodies is weak

$$v^2 \sim \frac{GM}{r} \ll 1$$

- Numerical Relativity: accurately, but computationally expensive
- Analytic methods: corrections in v or G are perturbatively calculable

Post-Newtonian/post-Minkowskian expansion

  - Particle theory technology, QFT methodology, shown great power!

Effective Field Theory

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- Gravitational binary system

$$S_{\rm WL} = \sum_{i=1,2}\left[-\frac{m_i}{2}\int dt\, g_{\mu\nu}\dot{x}_i^\mu \dot{x}_i^\nu + \cdots\right] \qquad S_{\rm GR} = \frac{-1}{16\pi G}\int d^4x\,\sqrt{-g}\,R + \cdots$$

- Effective action for gravitational binary systems

Goldberger-Rothstein hep-th/0409156

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$e^{iS_{\rm eff}[x_a(\tau)]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm WL}+iS_{\rm GR}}$$

- Post-Minkowskian expand in powers of G

$$L_{\rm eff} = L_0 + GL_1 + G^2L_2 + \cdots \qquad L_0 = -\sum_i \frac{m_i}{2}\eta_{\mu\nu}\dot{x}_i^\mu \dot{x}_i^\nu$$

- The equations of motion for trajectories:

Kälin-Porto 2006.01184

$$m_i \ddot{x}_i^\mu = -\eta^{\mu\nu}\sum_{n=1}^{\infty} G^n \left(\frac{\partial L_n}{\partial x_i^\nu} - \frac{d}{d\tau_i}\frac{\partial L_n}{\partial \dot{x}_i^\nu}\right) \qquad x_i^\mu = b_i^\mu + u_i^\mu \tau_i + \delta x_i^\mu(\tau_i) + \cdots$$

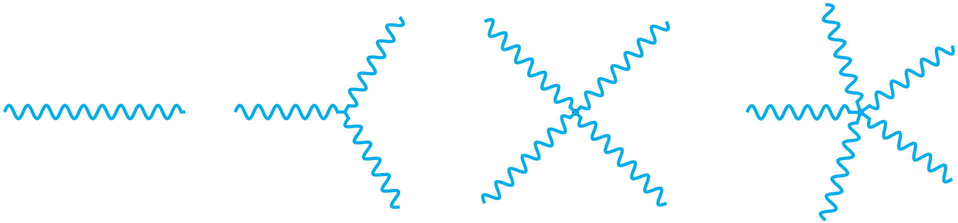
- Physical observables:

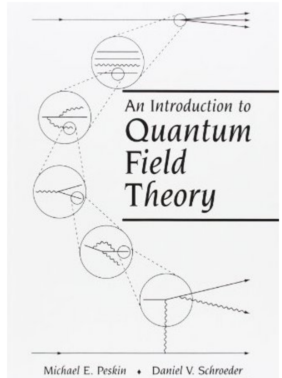
$$\Delta p_i^\mu = p_i^\mu(+\infty) - p_i^\mu(-\infty) = -\eta^{\mu\nu}\sum_{n=1}^{\infty} G^n \int_{-\infty}^{\infty} d\tau_i \left(\frac{\partial L_n}{\partial x_i^\nu}\right)$$

Effective Field Theory

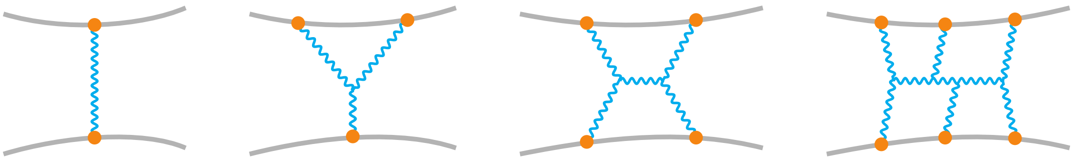
- Worldlines as classical sources in path integral:

- Hilbert-Einstein: $\mathcal{L}_{\text{HE}} = \mathcal{L}_{hh} + \mathcal{L}_{hhh} + \mathcal{L}_{hhhh} + \cdots$





- Classical physics: we use the saddle-point approximation in path integrals.



- Enjoy the advantages of quantum field theory methods and classical physics

powerful and systematic & purely classical at all steps (simplicity)

Effective Field Theory

- Observables at $\mathcal{O}(G^N)$ Kälin-**ZL**-Porto PRL2020 Dlapa-Kälin-**ZL**-Porto PRL2022 JHEP2024

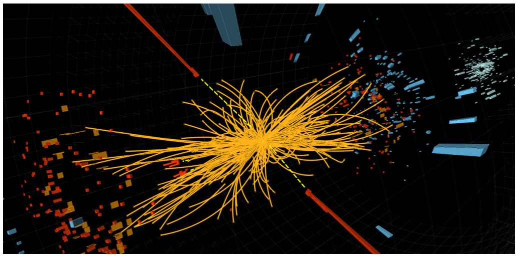
$$\Delta p_i^\mu \sim \int d^D q \, \frac{e^{iq\cdot b}\,\delta(q\cdot u_1)\delta(q\cdot u_2)}{|q^2|^\sharp} \int \left(\prod_{i=1}^{N-1} d^D\ell_i \, \frac{\delta(\ell_j\cdot u_a)}{(\ell_i\cdot u_b - i0)^{\nu_i}} \right) \frac{\mathcal{N}^\mu(q,u_a)}{D_1 D_2 D_3 \cdots}$$

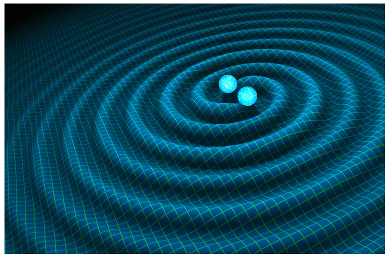
  - Graviton propagators: **Classical Feynman Integrals**

$$\frac{1}{D_i} \longrightarrow \frac{1}{(\ell^0 \pm i0)^2 - \vec{\ell}^{\,2}} \quad \text{or} \quad \frac{1}{\ell^2 + i0}$$

  - Cut: always one delta function $\delta(\ell_i\cdot u_a)$ for each loop
  - Kinematics: $q \cdot u_a = 0,\ u_a^2 = 1,\ u_1 \cdot u_2 = \gamma \implies$ single scale γ to all orders

- Multi-loop technology from particle physics can be used to solve gravitational problems!



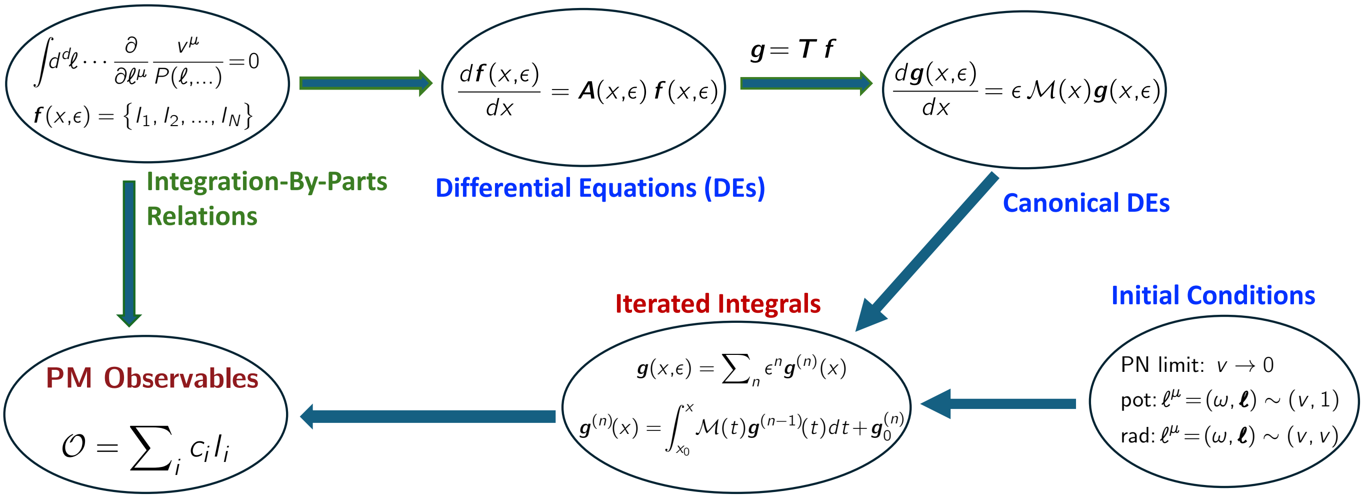


LHC theory toolbox

- Observables at $\mathcal{O}(G^N)$ Kälin-**ZL**-Porto PRL 2020 Dlapa-Kälin-**ZL**-Porto PRL 2022 Dlapa-Kälin-**ZL**-Neef-Porto PRL 2023

$$\Delta p_i^\mu \sim \int d^D q \, \frac{e^{i q\cdot b}\,\delta(q\cdot u_1)\delta(q\cdot u_2)}{|q^2|^\sharp} \int \left(\prod_{i=1}^{N-1} d^D \ell_i \, \frac{\delta(\ell_j\cdot u_a)}{(\ell_i\cdot u_b - i0)^{\nu_i}} \right) \frac{\mathcal{N}^\mu(q,u_a)}{D_1 D_2 D_3 \cdots}$$

- Perturbative QFT toolbox: Dlapa-Kälin-**ZL**-Porto JHEP 2024



- Post-Minkowskian physics can be bootstrapped from Post-Newtonian data using DEs!

NLO: 2PM

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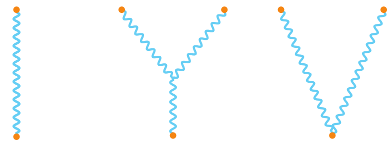
Spin interactions $g_{\mu\nu}e_a^\mu e_b^\nu = \eta_{ab}$

$$-\frac{1}{2}\left(\omega_\mu^{ab}S_{ab}v^\mu + \frac{1}{m}R_{\beta\rho\mu\nu}e_a^\alpha e_b^\beta e_c^\mu e_d^\nu S^{ab}S^{cd}v^\rho v_\alpha - \frac{C_{\rm ES}}{m}E_{\mu\nu}e_a^\mu e_b^\nu S^{ac}S_c{}^b + \cdots\right)$$

- EFT provides a systematic way to include spin effects.
- Needed one-loop integrals are simple

$$\int d^D\ell\, \frac{\delta(\ell\cdot u_1)}{[(\pm\ell\cdot u_2)]^{a_1}}\, \frac{1}{[\ell^2]^{a_2}\,[(\ell-q)^2]^{a_3}}$$



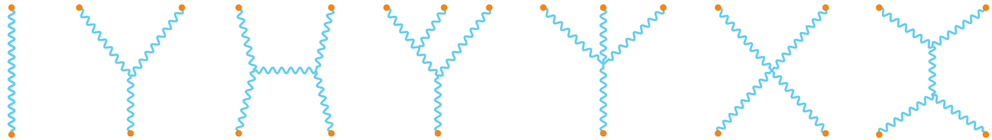
- Nontrivial to simplify complicated tensor expressions

Observables: **ZL**-Porto-Yang JHEP 2021

$$\Delta p_1^\mu = \frac{\nu G^2 M^3}{|b|^3}\left[3D_1\epsilon_{\alpha\rho\beta\sigma}\hat{b}^\mu\hat{b}^\alpha u_1^\beta u_2^\sigma a_1^\rho + \cdots + \frac{D_{20}}{|b|}u_1^\mu(a_1\cdot a_2) + \cdots + \frac{D_{14}}{|b|}u_2^\mu a_1^2 + \cdots\right]$$

$$s_\mu = m a_\mu \equiv \frac{1}{2m}\epsilon_{\mu\nu\alpha\beta}p^\nu S^{\alpha\beta}$$

NNLO: 3PM



- $\mathcal{O}(G^3)$: two-loop integrals Kälin-**ZL**-Porto PRL 2020 PRD 2020

$$\int \frac{d^D\ell_1 d^D\ell_2\, \delta(\ell_1\cdot u_1)\, \delta(\ell_2\cdot u_2)}{[\ell_1\cdot u_2]^{a_1}\, [\pm\ell_2\cdot u_1]^{a_2}} \frac{1}{[\ell_1^2]^{a_3}\, [\ell_2^2]^{a_4}\, [(\ell_1+\ell_2-q)^2]^{a_5}\, [(\ell_1-q)^2]^{a_6}\, [(\ell_2-q)^2]^{a_7}}$$

- The reduction and evaluation of integrals can be performed in standard techniques.

$$\partial_x \vec{f}(x,\epsilon) = \epsilon\left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}\right)\vec{f}(x,\epsilon)$$

- Conservative dynamics at $\mathcal{O}(G^3)$: Kälin-**ZL**-Porto PRL 2020

$$\Delta p_1^\mu = \frac{G^3 b^\mu}{|b^2|^2}\left(\frac{8m_1^2m_2^2(4\gamma^4-12\gamma^2-3)}{(\gamma^2-1)}\log\left(\gamma-\sqrt{\gamma^2-1}\right) - \frac{2m_1m_2(m_1^2+m_2^2)(16\gamma^6-32\gamma^4+16\gamma^2-1)}{(\gamma^2-1)^{5/2}} - \frac{4m_1^2m_2^2\gamma(20\gamma^6-90\gamma^4+120\gamma^2-53)}{3(\gamma^2-1)^{5/2}}\right) + \frac{3\pi}{2}\frac{(2\gamma^2-1)\,(5\gamma^2-1)}{(\gamma^2-1)^2}\frac{G^3m_1m_2(m_1+m_2)}{|b^2|^{3/2}}\Big((m_1+\gamma m_2)u_2^\mu - (m_2+\gamma m_1)u_1^\mu\Big)$$

- We provided the first confirmation for the result from a scattering amplitude clacualtion.

Bern-Cheung-Roiban-Shen-Solon-Zeng 2019

- We computed quadrupolar and octupole tidal corrections at $\mathcal{O}(G^3)$. Kälin-**ZL**-Porto PRD 2020

NNNLO: 4PM

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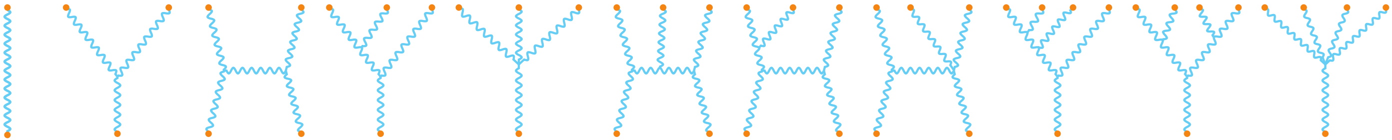

$\mathcal{O}(G^4)$: three-loop integrals

Dlapa-Kälin-**ZL**-Porto PRL 2022 PLB 2022 Dlapa-Kälin-**ZL**-Neef-Porto PRL 2023

$$\int d^D\ell_1 d^D\ell_2 d^D\ell_3 \frac{\delta(\ell_1\cdot u_1)\,\delta(\ell_2\cdot u_2)\,\delta(\ell_3\cdot u_2)}{[\ell_1\cdot u_2]^{\alpha_1}\,[\ell_2\cdot u_1]^{\alpha_2}\,[\ell_3\cdot u_1]^{\alpha_3}} \frac{D_8^{-\nu_8} D_9^{-\nu_9}}{D_1^{\nu_1} D_2^{\nu_2}\cdots D_7^{\nu_7}}$$

$$\{\ell_1^2, \ell_2^2, (\ell_1-q)^2, (\ell_2-q)^2, (\ell_3-q)^2, \ell_3^2, (\ell_1-\ell_2)^2, (\ell_2-\ell_3)^2, (\ell_3-\ell_1)^2\}$$

IBP reduction:

conservative: $\mathcal{O}(10^2)$ master integrals full: $\mathcal{O}(10^3)$ master integrals

Differential Equations

Dlapa-Kälin-**ZL**-Porto PRL 2022 PLB 2022 Dlapa-Henn-Wagner 2211.16357

$$\frac{d\vec{f}(x,\epsilon)}{dx} = \epsilon\,\mathcal{M}(x)\,\vec{f}(x,\epsilon)$$

- The majority can be solved in terms of multiple polylogarithms.
- Elliptic integrals appear in post-Minkwskian gravity for the first time.

NNNLO: 4PM

以於至善

The full impulse at $\mathcal{O}(G^4)$: Dlapa-Kälin-**ZL**-Porto PRL 2022 JHEP 2023 Dlapa-Kälin-**ZL**-Neef-Porto PRL 2023

$$\Delta p_1^\mu\big|_{\text{NNNLO}} = \frac{G^4}{|b|^4}\left(c_b \frac{b^\mu}{|b|} + c_1 \frac{\gamma u_2^\mu - u_1^\mu}{\gamma^2-1} + c_2 \frac{\gamma u_1^\mu - u_2^\mu}{\gamma^2-1} \right)$$

$$\begin{aligned}
\frac{c_b}{\pi} = &-\frac{3h_{34}m_2m_1(m_1^3+m_2^3)}{64v_\infty^5} + \frac{m_1^2m_{12}m_2^2}{4}\Bigg[\frac{3h_6\mathrm{K}^2(w_2)}{4v_\infty^3} - \frac{3h_8\mathrm{K}(w_2)\mathrm{E}(w_2)}{4v_\infty^3} + \frac{21h_5w_3\mathrm{E}^2(w_2)}{8v_\infty^3} - \frac{\pi^2h_{16}v_\infty}{4(\gamma+1)} + \frac{3\gamma h_{10}(\mathrm{Li}_2(w_2)-4\mathrm{Li}_2(\sqrt{w_2}))}{w_3v_\infty^2} \\
&+\log(v_\infty)\left(\frac{h_{32}}{2v_\infty^3} - \frac{3h_{14}\log(\frac{w_3}{2})}{v_\infty} - \frac{3\gamma h_{22}\log(w_1)}{2v_\infty^4}\right)\Bigg] + m_2^2m_1^3\Bigg[\frac{h_{52}}{48v_\infty^6} - \frac{h_{63}}{768\gamma^9w_3v_\infty^5} - \frac{3v_\infty(h_{40}\mathrm{Li}_2(w_2)+2w_3h_{33}\mathrm{Li}_2(-w_2))}{64w_3} \\
&+\frac{3h_{14}\log(\frac{w_3}{2})\log(w_3)}{4v_\infty} + \frac{\gamma h_{39}\log(w_1)}{8w_3^3v_\infty^2} + \frac{3\gamma h_{22}\log(w_3)\log(w_1) - h_{35}\log(\frac{w_3}{2})}{8v_\infty^4} + \frac{h_{56}\log(2) - h_{57}\log(w_3) + 2\gamma h_{55}\log(\gamma)}{32v_\infty^5} - \frac{\gamma h_{51}\log(w_1)}{16v_\infty^7}\Bigg] \\
&+m_1^2m_2^3\Bigg[\frac{h_{58}}{192\gamma^7v_\infty^5} + \frac{h_{53}}{48v_\infty^6} + \frac{\gamma h_{49}\log(w_1)}{16v_\infty^6} - \frac{2\gamma h_{50}\log(w_1) + 3\gamma^2h_{13}\log^2(w_1)}{32v_\infty^7} - \frac{h_{41}\log(\frac{w_3}{2})}{8v_\infty^4} + \frac{3\gamma\log(w_1)(5h_{26}\log(2)+8h_{12}\log(w_3))}{8v_\infty^4} \\
&-\frac{h_{36}\log(w_3)}{4v_\infty^3} + \frac{\gamma h_{30}\log(\gamma)}{2v_\infty^3} + \frac{h_{37}\log(2)}{8v_\infty^3} + \frac{3(h_{17}w_3\mathrm{Li}_2(w_2) - 2h_{23}\mathrm{Li}_2(-w_2) + h_{15}\log^2(w_3) - h_9\log^2(2))}{8v_\infty} - \frac{3h_7\log(2)\log(w_3)}{v_\infty}\Bigg]
\end{aligned}$$

$$c_1 = m_1m_2^2\left(\frac{2h_{46}m_{12s}}{v_\infty^6} + \frac{9\pi^2h_1m_{12}^2}{32v_\infty^2}\right) + m_1^2m_2^3\left(\frac{4\gamma h_{47}}{3v_\infty^6} - \frac{8\gamma h_2\log(w_1)}{v_\infty^6} + \frac{16h_{25}\log(w_1)}{v_\infty^3} - \frac{8h_3}{3v_\infty^5}\right)$$

$$\begin{aligned}
c_2 = &-m_1^4m_2\left(\frac{9\pi^2h_1}{32v_\infty^2} + \frac{2h_{46}}{v_\infty^6}\right) + m_2^2m_1^3\Bigg[+\frac{h_{60}}{705600\gamma^8v_\infty^5} - \frac{4\gamma h_{48}}{3v_\infty^6} + \frac{3h_{38}(\mathrm{Li}_2(w_2)-4\mathrm{Li}_2(\sqrt{w_2})) - \gamma h_{21}(\mathrm{Li}_2(-w_1^2)+2\log(\gamma)\log(w_1))}{16v_\infty^4} \\
&+\frac{3\gamma h_{31}(2\mathrm{Li}_2(-w_1)+\log(w_1)\log(w_3))}{8v_\infty^4} + \frac{h_{62}\log(w_1)}{6720\gamma^9v_\infty^6} + \frac{32\gamma^2h_{44}\log^2(w_1)}{v_\infty^7} + \frac{8\gamma(2h_4\log(2) - h_{27}\log(w_1))\log(w_1)}{v_\infty^4} - \frac{32h_{29}\log(w_1)}{3v_\infty^3} + \frac{\pi^2h_{42}}{192v_\infty^4}\Bigg] \\
&+m_2^3m_1^2\Bigg[\frac{h_{59}}{1440\gamma^7v_\infty^5} - \frac{h_{19}(\mathrm{Li}_2(-w_1^2)+2\log(\gamma)\log(w_1))}{8v_\infty^4} + \frac{h_{43}(\mathrm{Li}_2(w_2)-4\mathrm{Li}_2(\sqrt{w_2}))}{32v_\infty^4} - \frac{h_{20}(2\mathrm{Li}_2(-w_1)+\log(w_1)\log(w_3))}{4v_\infty^4} \\
&-\frac{h_{61}\log(w_1)}{480\gamma^8v_\infty^6} - \frac{16\gamma^2h_{11}\log^2(w_1)}{v_\infty^4} - \frac{32\gamma h_{45}\log^2(w_1)}{v_\infty^7} + \frac{16\gamma h_{28}\log(w_1)}{5v_\infty^3} - \frac{32h_{24}\log(2)\log(w_1)}{v_\infty^4} - \frac{\pi^2h_{18}}{48v_\infty^4} - \frac{2h_{54}}{45v_\infty^6}\Bigg]
\end{aligned}$$

with $\gamma \equiv u_1\cdot u_2$, $v_\infty = \sqrt{\gamma^2-1}$, $w_1 = \gamma - v_\infty$, $w_2 = \frac{\gamma-1}{\gamma+1}$, $w_3 = \gamma+1$, h_i = polynomial in γ.

$$\mathrm{Li}_2(z) \equiv \int_0^z dx\,\frac{\log(1-x)}{-x}$$

$$K(z) \equiv \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-zx^2)}}$$

$$E(k) \equiv \int_0^1 dx\,\frac{\sqrt{1-zx^2}}{\sqrt{1-x^2}}$$

NNNLO: 4PM

止於至善

The full impulse at $\mathcal{O}(G^4)$: Dlapa-Kälin-**ZL**-Porto JHEP 2023 Dlapa-Kälin-**ZL**-Neef-Porto PRL 2023

$$\Delta p_1^\mu\big|_{\text{NNNLO}} = \frac{G^4}{|b|^4}\left(c_b \frac{b^\mu}{|b|} + c_1 \frac{\gamma u_2^\mu - u_1^\mu}{\gamma^2 - 1} + c_2 \frac{\gamma u_1^\mu - u_2^\mu}{\gamma^2 - 1}\right)$$

- We obtained the full dynamics of binary inspirals at $\mathcal{O}(G^4)$ for the first time.
- Conservative part agrees perfectly with previous derivations.

  Bern-Parra-Martinez-Roiban-Ruf-Shen-Solon-Zeng 2022 Dlapa-Kälin-**ZL**-Porto PRL 2022
- Perfect agreement with the state-of-the-art PN computations

  Cho-Dandapat-Gopakumar 2021 Cho 2022 Bini-Geralico 2021 2022 Bini-Damour 2022
- Later, two new calculations confirmed our results.

  Damgaard-Hansen-Planté-Vanhove 2023 (exponentiation of amplitudes)

  Jakobsen-Mogull-Plefka-Sauer-Xu 2023 (worldline formalism)

Analytic vs Numerical Relativity

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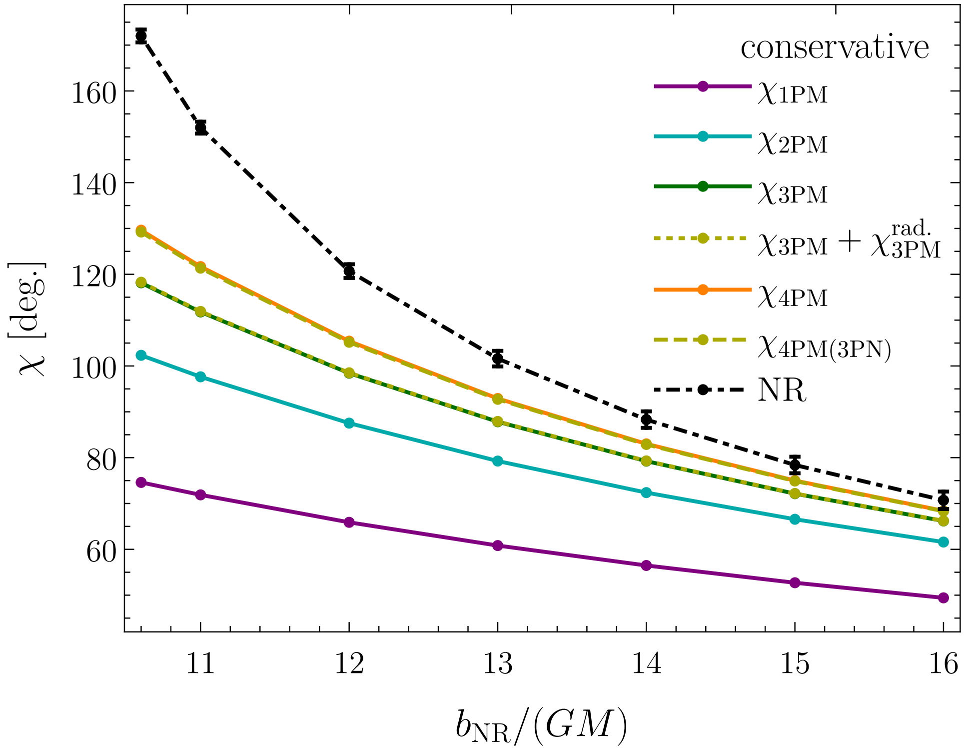


Khalil-Buonanno-Steinhoff-Vines 2204.05047

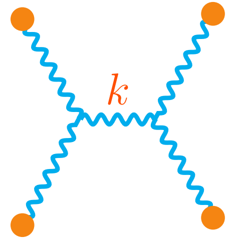
NNNLO: local-in-time part

- The full result does not describe generic elliptic-like motion due to nonlocal-in-time effects.

  Damour-Jaranowski-Schäfer 2014 Galley-Leibovich-Porto-Ross 2015 Cho-Kälin-Porto 2021

- Nonlocal-in-time radial action:

$$\mathcal{S}_r^{(\mathrm{nloc})} = -\frac{GE}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$



- The 4PM integrand can be built from 3PM diagrams. $\omega \equiv k\cdot u_{\mathrm{com}}$

$$\int d^D\ell_1 d^D\ell_2 \frac{\delta(\ell_1\cdot u_1)\delta(\ell_2\cdot u_2)}{[\ell_1\cdot u_2][\ell_2\cdot u_1]}\frac{\log(\omega^2)}{[\ell_1^2][\ell_2^2][(\ell_1+\ell_2-q)^2][(\ell_1-q)^2][(\ell_2-q)^2]}$$

- We managed to compute the integrals and obtained nonlocalin-time contribution:

$$\frac{\nu}{(\gamma^2-1)^2}\Bigg[h_1 + \frac{\pi^2 h_2}{\sqrt{\gamma^2-1}} + h_3\log\frac{\gamma+1}{2} + \frac{h_4\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_5\log\frac{\gamma-1}{8} + h_6\log^2\frac{\gamma+1}{2} + \frac{h_8\log(2)\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}}$$
$$+h_7\,\mathrm{arccosh}(\gamma)^2 + h_9\log\frac{\gamma-1}{8}\log\frac{\gamma+1}{2} + \frac{h_{10}\log\frac{\gamma^2-1}{16}\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_{11}\mathrm{Li}_2\frac{\gamma-1}{\gamma+1} + h_{12}\frac{\mathrm{arccosh}^2(\gamma)+4\mathrm{Li}_2(\sqrt{\gamma^2-1}-\gamma)}{\sqrt{\gamma^2-1}}\Bigg]$$

Coefficients h_i: exact-ν (iterated elliptic integrals) and SF-expanded (30SF) forms

Dlapa-Kälin-**ZL**-Porto PRL 2024

- Using 6PN results in the literature, we constructed an improved bound Hamiltonian.

Conclusion & Outlook

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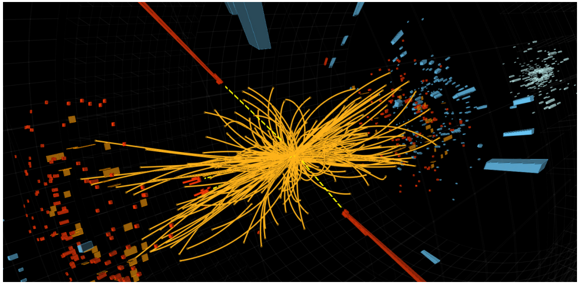
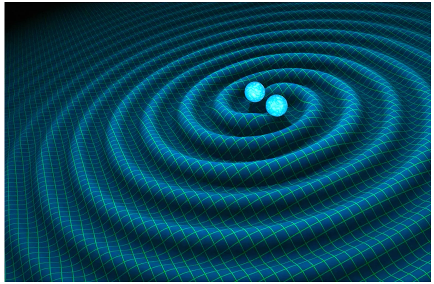
Modern techniques from Quantum Field Theory have already proven useful to improve theoretical predictions for gravitational-wave observables.





We have developed an efficient framework and made breakthroughs to NNNLO.

- Conservative spin & tidal effects at NLO JHEP 06 (2021) 012 PRD 102 (2020) 124025
- Conservative dynamics at NNLO PRL 125 (2020) 261103
- Conservative dynamics at NNNLO PLB 822 (2021) 136698 PRL 128 (2022) 161104 PRL 130 (2023) 101401
- Local-in-time & nonlocal-in-time separation PRL 132 (2024) 221401
- Novel techniques to evaluate loop integrals in gravity JHEP 07 (2023) 181 JHEP 08 (2023) 109

Outlook: precision frontier

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GW science is just starting! discovery potential = precise theoretical predictions!

0PN 1PN 2PN 3PN 4PN 5PN 6PN ⋯

$$\text{1PM}\quad \left(\tfrac{GM}{r}\right)\times\left(1+v^2+v^4+v^6+v^8+v^{10}+v^{12}+\cdots\right)=\left(\tfrac{GM}{r}\right)\times\left(1\right)$$

$$\text{2PM}\quad \left(\tfrac{GM}{r}\right)^2\times\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right)=\left(\tfrac{GM}{r}\right)^2\times\left(1\right)$$

$$\text{3PM}\quad \left(\tfrac{GM}{r}\right)^3\times\left(1+v^2+v^4+v^6+v^8+\cdots\right)=\left(\tfrac{GM}{r}\right)^3\times\left(1+\nu\right)$$

$$\text{4PM}\quad \left(\tfrac{GM}{r}\right)^4\times\left(1+v^2+v^4+v^6+\cdots\right)=\left(\tfrac{GM}{r}\right)^4\times\left(1+\nu\right)$$

$$\text{5PM}\quad \left(\tfrac{GM}{r}\right)^5\times\left(1+v^2+v^4+\cdots\right)=\left(\tfrac{GM}{r}\right)^5\times\left(1+\nu+\nu^2\right)$$

$$\text{6PM}\quad \left(\tfrac{GM}{r}\right)^6\times\left(1+v^2+\cdots\right)=\left(\tfrac{GM}{r}\right)^6\times\left(1+\nu+\nu^2\right)$$

$$\text{7PM}\quad \left(\tfrac{GM}{r}\right)^7\times\left(1+\cdots\right)=\left(\tfrac{GM}{r}\right)^7\times\left(1+\nu+\nu^2+\nu^3\right)$$

0SF 1SF 2SF 3SF ⋯

- NNNNLO (5PM)
  - Nonlocal conservative dynamics: ongoing   Dlapa-Kälin-**ZL**-Porto PRL 2024
  - 1SF: Driesse-Jakobsen-Mogull-Plefka-Sauer-Usovitsch 2024   D-J-Klemm-M-Nega-P-S-U 2024

Outlook: precision frontier

GW science is just starting! discovery potential = precise theoretical predictions!

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	$\cdots$

$$\text{1PM}\quad \left(\tfrac{GM}{r}\right)\times\left(1+v^2+v^4+v^6+v^8+v^{10}+v^{12}+\cdots\right)=\left(\tfrac{GM}{r}\right)\times\left(1\right)$$

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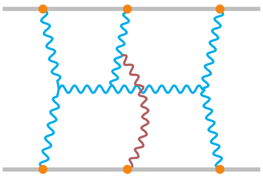

0SF 1SF 2SF 3SF $\cdots$

- NNNNLO (5PM)
  - Nonlocal conservative dynamics: ongoing — Dlapa-Kälin-**ZL**-Porto PRL 2024
  - 1SF: Driesse-Jakobsen-Mogull-Plefka-Sauer-Usovitsch 2024 — D-J-Klemm-M-Nega-P-S-U 2024
  - 2SF: nonplanar diagram & more complicated functions

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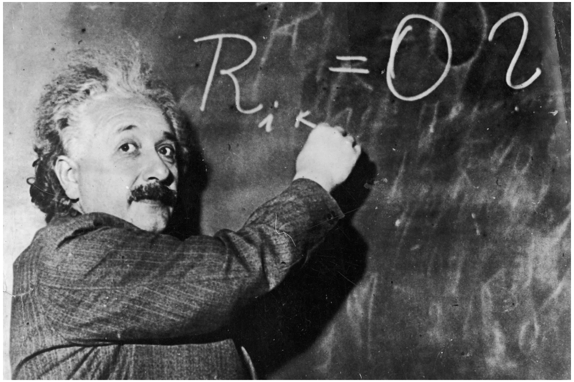
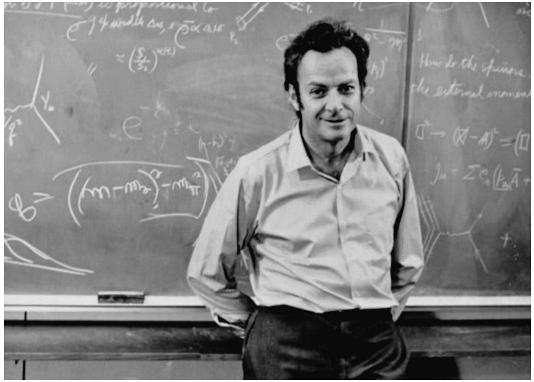
Feynman integrals solve Einstein's equations!





谢谢!

EFT: closed-time-path integral

止於至善

The in-in effective action is obtained by performing a closed-time-path integral

$$e^{i\mathcal{S}_{\rm eff}[x_{a,1},x_{a,2}]} = \int \mathcal{D}h_1\mathcal{D}h_2\, e^{i(S_{\rm GR}[h_1]-S_{\rm GR}[h_2]+S_{\rm WL}[h_1,x_{a,1}]-S_{\rm WL}[h_2,x_{a,2}])}$$

It is convenient to use the Keldysh basis — Galley PRL 110 (2013) 174301

$$h^-_{\mu\nu} = \frac{1}{2}(h_{1\mu\nu}+h_{2\mu\nu}) \qquad x^\alpha_{a,+} = \frac{1}{2}(x^\alpha_{a,1}+x^\alpha_{a,2})$$
$$h^+_{\mu\nu} = h_{1\mu\nu}-h_{2\mu\nu} \qquad x^\alpha_{a,-} = x^\alpha_{a,1}-x^\alpha_{a,2}$$

for which the matrix of (classical) propagators for gravitions becomes

$$i\begin{pmatrix} 0 & -\Delta_{\rm adv}(x-y) \\ -\Delta_{\rm ret}(x-y) & 0 \end{pmatrix}$$

The worldline equations of motion: — Kälin-Neef-Porto JHEP 01 (2023) 140

$$m_i\ddot{x}^\mu_i(\tau) = -\eta^{\mu\nu}\frac{\delta\mathcal{S}_{\text{eff, int}}[x_{a,\pm}]}{\delta x^\nu_{i,-}(\tau)}\bigg|_{\rm PL} \qquad \Delta p^\mu_i = -\eta^{\mu\nu}\int_{-\infty}^{\infty}d\tau\frac{\delta\mathcal{S}_{\text{eff, int}}[x_{a,\pm}]}{\delta x^\nu_{i,-}(\tau)}\bigg|_{\rm PL}$$

Physical Limit (PL): $x_{a,-}\to 0,\ x_{a,+}\to x_a$.

Closed-time path integrals

EQUILIBRIUM AND NONEQUILIBRIUM FORMALISMS MADE UNIFIED

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