The gg →HH amplitude induced by bottom quarks at two-loop level

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Outline

✓ Motivation

- HH production in gluon fusion
- Beyond LO status
- QCD & EW
- Mass uncertainty
- Bottom quark effect
- ✓ Methods
 - Differential Equations and Asymptotic Expansion
 - Boundary Conditions
- ✓Example

✓ Conclusion

Motivation: HH production in gluon fusion

✓ Understand the scalar sector of the Standard Model (SM)

- The Higgs self-coupling is important for understanding the mechanism of electroweak symmetry breaking and testing the electroweak theory.
- The most promising such measurement is the production of a pair of Higgs bosons.
- One central objective of the LHC at CERN is to refine the experimental bounds on the trilinear self-coupling of the Higgs boson ($\lambda = \frac{m_H^2}{2n^2} \approx 0.13$).

 κ factor ($\lambda_{HHH}/\lambda_{HHH}^{SM}$): $-1.2 < \kappa_{\lambda} < 7.2$ [ATLAS collaboration '24]

 $-1.4 < \kappa_{\lambda} < 6.4$ [CMS collaboration '24]

 Gluon fusion dominates Higgs pair production at the LHC due to the large gluon luminosity. (key process at HL-LHC)



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Motivation: beyond LO status

- ✓ Beyond LO: no fully analytic results
- 1. Numerical evaluation
- 2. Expansion in various kinematic limits

NLO QCD:

- Large-m_t [Dawson et al. '98; Grigo, et al. '13]
- Threshold expansion [Gröber et al. '18]
- High-energy expansion [Davies et al. '18, '19]
- Small- p_T expansion [Bonciani et al. '18]
- Small-t expansion [Davies et al. '23]
- Numeric [Borowka et al. '16; Baglio et al. '20] NNLO QCD:
 - Large-m_t [Davies et al. '19; Grigo et al. '15]
- Small-t expansion [Davies et al. '23] N³LO QCD:
 - Large- m_t limit [Chen, Li, Shao, Wang '19]

EW calculations:

- Yukawa-top corrections in high-energy expansion and large- m_t limit [Davies et al. '22; Mühlleitner et al. '22]
- Full top-induced EW corrections in large- m_t expansion [Davies et al. '23]
- Higgs self-coupling corrections with SecDec [Borowka et al. '19]
- Full EW corrections with AMFlow [Bi, Huang, Huang, Ma, Yu '23]
- EW corrections mediated by light quarks [Bonetti et al. '25]

Motivation: QCD & EW



$$\sigma^{\text{NLO}} = 27.80^{+13.8\%}_{-12.8\%} \text{ fb} \pm 0.3\% \text{ (stat.)} \pm 0.1\% \text{ (i}$$

$$\sigma^{LO} = 16.72^{+28\%}_{-21\%} \text{ fb.} \quad (\sqrt{s} = 13 \text{ TeV})$$

About 40% larger than LO

NLO EW [Bi, Huang, Huang, Ma, Yu '23] ($\sqrt{s} = 14$ TeV)



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Motivation: mass uncertainty [Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher '20]



Motivation: bottom quark effect

✓ Percent level corrections

LO cross sections under different cuts at the 13 TeV LHC

M ^{max} HH	σ_t (fb)	$\sigma_{t,b}$ (fb)	$rac{\sigma_{t,b}-\sigma_t}{\sigma_t}$
300	0.229(3)	0.242(6)	5.7%
350	1.38(0)	1.41(3)	2.2%
400	4.33(2)	4.37(7)	0.92%

✓ Enhanced by logarithms

• At one-loop level the bottom quark contribution to single Higgs production is suppressed by $m_b^2/m_H^2 \approx 0.0014$ but enhanced by $\ln^2(m_b^2/m_H^2) \approx 43.05$ relative to the top quark contribution

✓ Results can be used to other physical processes induced by light quarks

• $gg \rightarrow ZZ$, $gg \rightarrow ZH$ (expand one mass)

Notations



♦ Hierarchy of scales:



Master Integrals

- ◆ Integral families:
 - I. One-loop: 2 families, 11 MIs
 - II. Two-loop planar: 10 families, 177 MIs
 - III. Two-loop non-planar: 3 families



Differential Equations (DEs) and Asymptotic Expansion

DEs:

$$\partial_k I_i(x, z, \kappa, \epsilon) = A_{ij}^k(x, z, \kappa, \epsilon) I_j(x, z, \kappa, \epsilon), \quad k \in \{x, z, \kappa\}$$
 In the limit of $\kappa \ll 1$

Asymptotic Expansion: [Beneke, Smirnov '98; Smirnov '02]

Considering the following ansatz as the solutions: [Melnikov, Tancredi, Wever '16] [Davies, Mishima, Steinhauser, Wellmann, '17, '18]

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

- 1. n_{max} can goes to infinity, high order terms can be obtained without solving x, z-DEs.
- 2. *n* may take half-integer values such as in the non-planar case of $gg \rightarrow Hg$. [Melnikov, Tancredi, Wever '16]
- 3. $c_{0,0,0}^{i}$ corresponds to the integrals with massless internal quarks. [Gehrmann, Manteuffel, Tancredi, Weihs '14]

Methods: overview



Methods



$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

1. Insert into κ -DEs

$$\partial_{\kappa} I_i(x, z, \kappa, \epsilon) = A_{ij}^{\kappa}(x, z, \kappa, \epsilon) I_j(x, z, \kappa, \epsilon)$$

- 2. Require the coefficient of $\kappa^{n-j\epsilon} \log^k \kappa$ are independent, leaving a linear system
- 3. Solve this system, arriving at independent $c_{n,j,k}^{i}$, with their number equal to that of MIs
- 4. Expand $c_{n,j,k}^i$ in powers of ϵ

$$c_{n,j,k}^{i}(x,z,\epsilon) = \sum \epsilon^{r} c_{n,j,k}^{i,(r)}(x,z)$$

Methods

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

$$c_{n,j,k}^{i}(x,z,\epsilon) = \sum \epsilon^{r} c_{n,j,k}^{i,(r)}(x,z)$$



5. Insert into *x*, *z*-DEs

$$\partial_{x(z)} I_i(x, z, \kappa, \epsilon) = A_{ij}^{x(z)}(x, z, \kappa, \epsilon) I_j(x, z, \kappa, \epsilon)$$

Require the coefficient of ε^rκ^{n-jε} log^k κ are independent, leaving a system of differential equations (coupled or decoupled)

7. Solve this system and express the results in terms of Multiple polylogarithms (MPLs)

$$G(a_1, ..., a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, ..., a_n; t)$$



Example: top-sector in PL5, 4 in 60 MIs



PL5[1, 1, 1, 1, 1, 1, 1, 0, 0]

- ✓ Solving κ -DEs leaves 60 independent $c_{n,j,k'}^i$ 4 in top-sector:
 - $c_{0,-1,0}^{57}(x,z), c_{0,0,0}^{57}(x,z), c_{0,0,0}^{58}(x,z), c_{0,0,0}^{60}(x,z)$
- \checkmark Expanding them in powers of ϵ :

$$c_{0,-1,0}^{57}(x,z) = \sum_{i=-4}^{0} c_{0}^{(i)}(x,z)\epsilon^{i} + O(\epsilon), \quad c_{0,0,0}^{57}(x,z) = \sum_{i=-4}^{0} c_{1}^{(i)}(x,z)\epsilon^{i} + O(\epsilon),$$
$$c_{0,0,0}^{58}(x,z) = \sum_{i=-2}^{0} c_{2}^{(i)}(x,z)\epsilon^{i} + O(\epsilon), \quad c_{0,0,0}^{60}(x,z) = \sum_{i=-4}^{0} c_{3}^{(i)}(x,z)\epsilon^{i} + O(\epsilon).$$

- ✓ x, z-DEs for $c_{0,-1,0}^{57}$ are decoupled.
- ✓ x, z-DEs for $c_{0,0,0}^{57}$, $c_{0,0,0}^{58}$, $c_{0,0,0}^{60}$ form a coupled system.

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

Example: top-sector in PL5, 4 in 60 MIs



PL5[1, 1, 1, 1, 1, 1, 1, 0, 0]

- ✓ Solving κ -DEs leaves 60 independent $c_{n,j,k'}^i$ 4 in top-sector:
- ✓ Expanding them in powers of ϵ :
- ✓ x, z-DEs for $c_{0,-1,0}^{57}$ are decoupled.

$$\frac{\partial c_0^{(-4)}(x,z)}{\partial z} = \frac{-1}{z} c_0^{(-4)}(x,z) ,$$
$$\frac{\partial c_0^{(-4)}(x,z)}{\partial x} = \frac{2(1-x)}{x(1+x)} c_0^{(-4)}(x,z) .$$

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

$$c_{0,-1,0}^{57}(x,z) = \sum_{i=-4}^{0} c_0^{(i)}(x,z)\epsilon^i + O(\epsilon)$$

Example: top-sector in PL5, 4 in 60 MIs



- ✓ Expanding them in powers of ϵ :
- ✓ x, z-DEs for $c_{0,-1,0}^{57}$ are decoupled.

✓ x, z-DEs for $c_{0,0,0}^{57}$, $c_{0,0,0}^{58}$, $c_{0,0,0}^{60}$ form a coupled system.

$$\begin{split} \frac{\partial c_1^{(-4)}(x,z)}{\partial z} &= \frac{\left(x^3 z - x^4 + xz - 1\right) c_1^{(-4)}(x,z)}{\left(x^2 + 1\right) \left(x - z\right) \left(xz - 1\right)} - \frac{x \left(x^2 - 2xz + 1\right) c_3^{(-4)}(x,z)}{\left(x^2 + 1\right) \left(x - z\right) \left(xz - 1\right)} \,, \\ \frac{\partial c_1^{(-4)}(x,z)}{\partial x} &= \frac{\left(x - 1\right) \left(2x \left(x^2 + 1\right) z^2 - 2 \left(x^2 + 1\right)^2 z + x(x - 1)^2\right) c_1^{(-4)}(x,z)}{x(x + 1) \left(x^2 + 1\right) \left(x - z\right) \left(xz - 1\right)} \,, \\ + \frac{\left(1 - x^2\right) z c_3^{(-4)}(x,z)}{\left(x^2 + 1\right) \left(x - z\right) \left(xz - 1\right)} \,, \\ \frac{\partial c_3^{(-4)}(x,z)}{\partial z} &= \frac{\left(xz - x^2 - 1\right) \left(x(xz - 2) + z\right) c_3^{(-4)}(x,z)}{\left(x^2 + 1\right) z(x - z) \left(xz - 1\right)} \,+ \frac{\left(x^4 - \left(x^2 + 1\right) xz + 1\right) c_1^{(-4)}(x,z)}{\left(x^2 + 1\right) z(x - z) \left(xz - 1\right)} \,, \\ \frac{\partial c_3^{(-4)}(x,z)}{\partial x} &= \frac{\left(1 - x\right) \left(-2 \left(x^3 + x\right) + \left(x^2 + 1\right) \left(x - 1\right)^2 z - x(x - 1)^2 z^2\right) c_3^{(-4)}(x,z)}{x(x + 1) \left(x^2 + 1\right) \left(x - z\right) \left(xz - 1\right)} \,, \\ + \frac{\left(x^4 - x^3 z + xz - 1\right) c_1^{(-4)}(x,z)}{x(x^2 + 1) \left(x - z\right) \left(xz - 1\right)} \,. \end{split}$$



PL5[1, 1, 1, 1, 1, 1, 1, 0, 0]

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^2 \sum_{k=0}^2 c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$

$$\begin{split} c_{0,0,0}^{57}(x,z) &= \sum_{i=-4}^{0} c_{1}^{(i)}(x,z) \epsilon^{i} + O(\epsilon) \,, \\ c_{0,0,0}^{58}(x,z) &= \sum_{i=-2}^{0} c_{2}^{(i)}(x,z) \epsilon^{i} + O(\epsilon) \,, \\ c_{0,0,0}^{60}(x,z) &= \sum_{i=-4}^{0} c_{3}^{(i)}(x,z) \epsilon^{i} + O(\epsilon) \,. \end{split}$$

Example: boundary conditions function of MPLs \checkmark After solving x, z-DEs, we have PL5[1, 1, 1, 1, 1, 1, 1, 0, 0] $c_{j}^{(i)}(x,z) = f_{j}^{(i)}(x,z) + c_{j}^{(i)}$ $c_{0,-1,0}^{57}(x,z) = \sum_{i=1}^{0} c_{0}^{(i)}(x,z)\epsilon^{i} + O(\epsilon) ,$ $c_{0,0,0}^{58}(x,z) = \sum_{i=-2}^{0} c_2^{(i)}(x,z)\epsilon^i + O(\epsilon) \,,$ constant ✓ Analytical form of $c_{0,0,0}^{57}$, $c_{0,0,0}^{58}$, $c_{0,0,0}^{60}$ are easy to fit (no dependence on mass) $c_{0,0,0}^{57}(x,z) = \sum_{i=-4}^{5} c_1^{(i)}(x,z)\epsilon^i + O(\epsilon),$ $\checkmark \kappa = 10^{-25}, x = \frac{1}{6}, z = 2$: $c_{0,0,0}^{60}(x,z) = \sum_{i=-4}^{0} c_3^{(i)}(x,z)\epsilon^i + O(\epsilon) \,.$ $= (4366.89695 \dots + 891.86882 \dots i) + O(\epsilon)$ **AMFLOW** $\cong \frac{0.007...\left(-1+c_0^{(-4)}\right)}{c_0^4} + \frac{0.007...c_0^{(-3)}}{c_0^3} + \frac{-0.003...(\pi^2 - 2c_0^{(-2)})}{c_0^3} + \frac{-0.003...(14\,\zeta(3) + i\,\pi^3 - 2c_0^{(-1)})}{c_0^3} + \frac{-0.003...(\pi^2 - 2c_0^{(-2)})}{c_0^3} + \frac{-0.003...(\pi^2 - 2c_0^{(-2)})}{c_0^3}$ PL5[1, 1, 1, 1, 1, 1, 1, 0, 0] + $(4366.67787 \dots + 892.01038 \dots i + 0.00749 \dots c_0^{(0)}) + O(\epsilon)$ GiNaC $c_0^{(-4)} = 1$, $c_0^{(-3)} = 0$, $c_0^{(-2)} = \frac{\pi^2}{2}$, $c_0^{(-1)} = 7\zeta(3) + \frac{i\pi}{2}$, $c_0^{(0)} = 29.2227310 \dots - i18.881865 \dots = \frac{3\pi^4}{10} - 5i\pi\zeta(3)$ 2025粒子物理标准模型及新物理精细计算研讨会

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Crosschecks

- ✓ One loop results are consistent with the results of *package*-X [Patel '16]
- $\checkmark c_{0,0,0}^{i}(x, z, \kappa)$ are consistent with the massless internal case
- \checkmark Set values to x, z, κ for different groups and compare with numerical results
- ✓ Choose another basis of MIs and check their differential equations

Conclusion

- \checkmark We computed the two-loop planar master integrals for gg \rightarrow HH with an expansion for bottom quark mass
- \checkmark Results are expressed in Multiple polylogarithms with dimensionless variables x, z
- ✓ Analytical form of boundary conditions are obtained by numerical approach
- \checkmark High order terms can be obtained by differential equations for κ , which are easy to solve
- \checkmark We believe that the non-planar Feynman integrals could also be solved in the same way

On-going work: non-planar MIs

Family	PL5	NPL1	NPL2	NPL3
File size	30 MB	2.5 GB	10.1 GB	18.7 GB

Thank you for your attention!