基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

Asymptotic grand unification in SO(10) with one extra dimension

方高祥 (HIAS)

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Based on **GXF**, Z.W. Wang, Y.L. Zhou,
arXiv:2504.xxxxx

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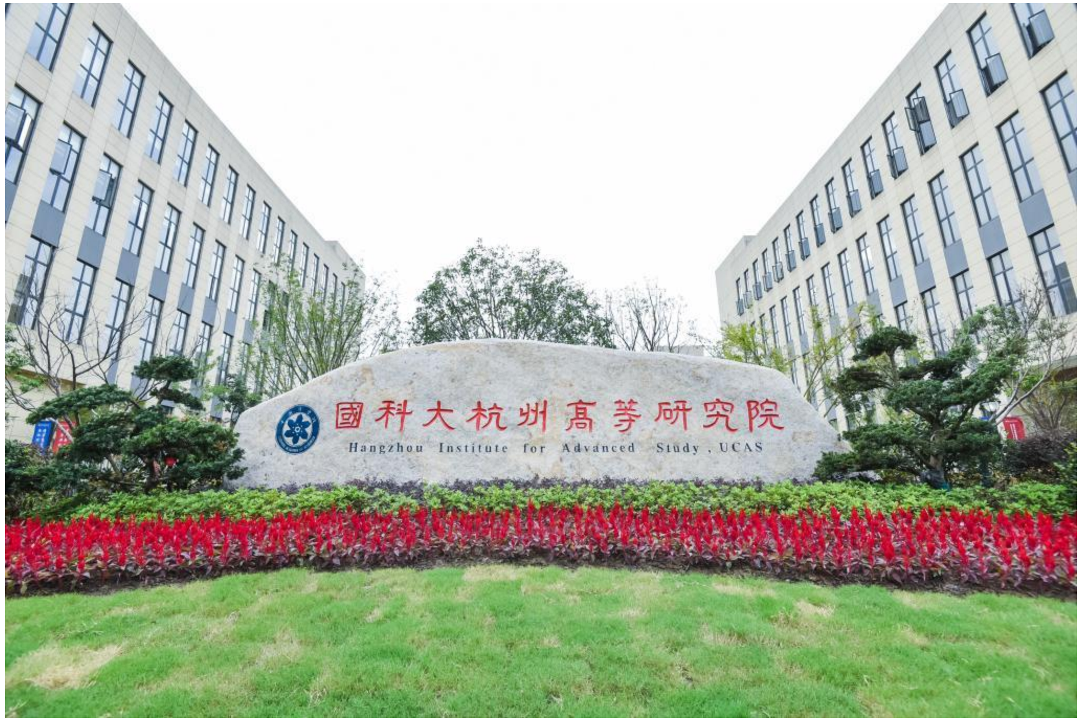

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Introduction

Electromagnetic force

Weak force

Strong force

Grand unified theory (GUT)

Unification of matter

Particles in GUT

$\cup$

Particles in SM

Unification of symmetries

$G_{\rm GUT} \supset G_{\rm SM}$

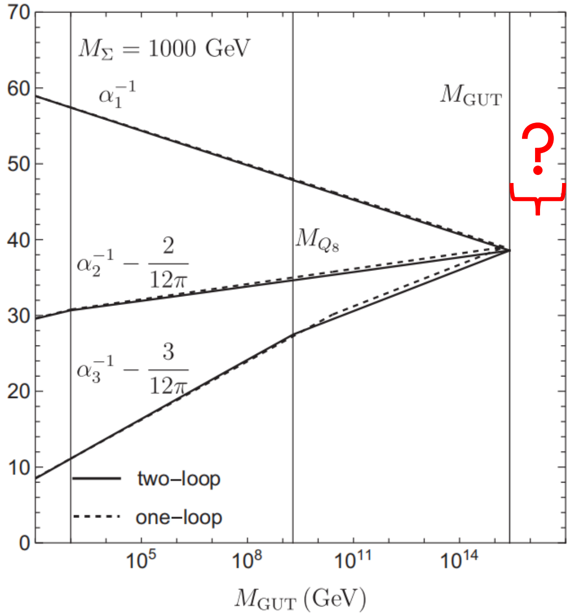
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Unification of gauge couplings

$g_{\rm GUT} = g_3 = g_2 = g_1$

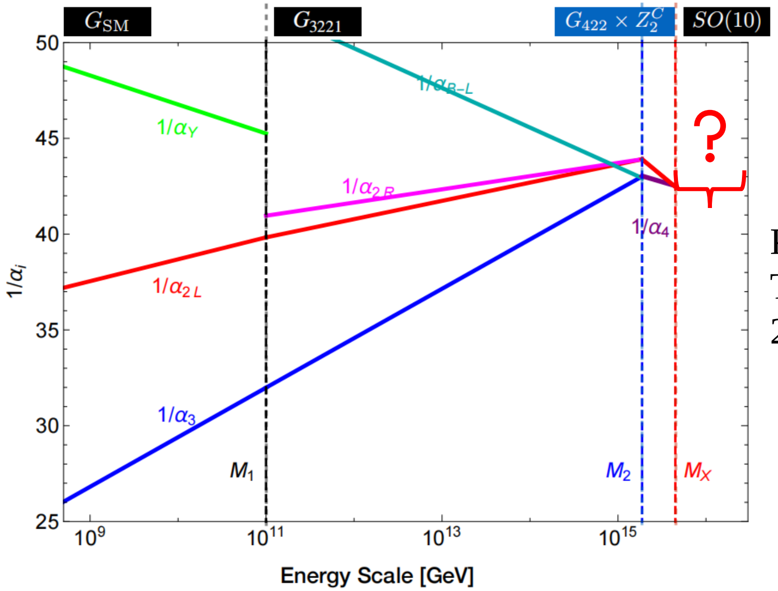
Running behavior between $M_{\rm GUT}$ and $M_{\rm Pl}$ is blank

$SU(5)$ GUT



Gao-Xiang Fang, Ye-Ling Zhou, 2406.06861

$SO(10)$ GUT



King, Pascoli, Turner, Zhou, 2106.15634

GUT scale: The scale where three gauge couplings are unified, denoted as $M_{\rm GUT}$

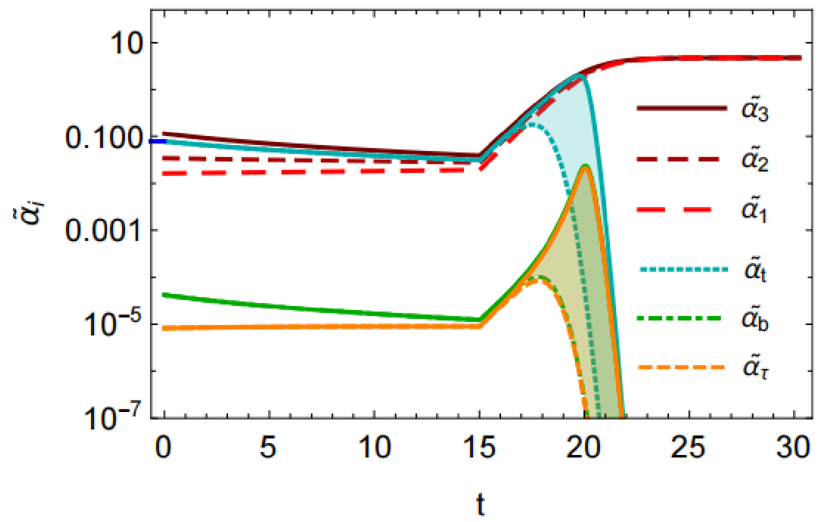
How about UV behavior of GUTs?

Introduction

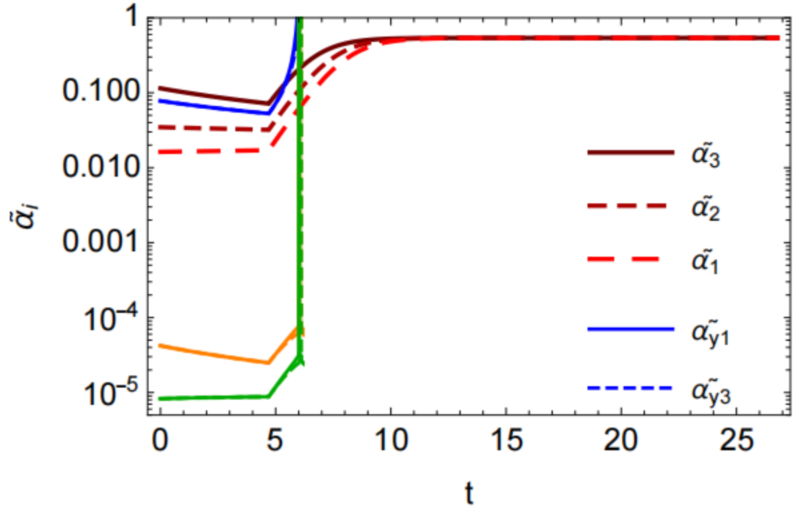
Asymptotic unification in 5-dimension spacetime of gauge couplings: these gauge couplings gradually approach the same value in the deep UV limit.

Asymptotic freedom in 5-dimension spacetime of Yukawa couplings: these Yukawa couplings gradually approach zero in the deep UV limit.

5D $SU(5)$



5D $SO(10)$ with only H_{10}



1. Yukawa couplings flow to Landau pole
2. Cannot explain fermion masses

Not realistic!

Cacciapaglia, et al, 2012.14732, 2210.03596

Whether there exist a realistic 5D $SO(10)$ GUT with more than one Higgs field?

Framework

Breaking chain: 5D $SO(10) \xrightarrow[\text{BC}]{M_{\text{KK}}}$ 4D Pati-Salam$(SU(4)_c \times SU(2)_L \times SU(2)_R) \xrightarrow[\mathbf{16}]{M_{\text{PS}}}$ SM

Energy scale	Symmetry	Fermion	Higgs
$\mu > M_{\text{KK}}$	$SO(10)$	$\Psi_{\mathbf{16}} \sim \mathbf{16}$ $\Psi_{\overline{\mathbf{16}}} \sim \overline{\mathbf{16}}$ $\nu_{\text{S}} \sim \mathbf{1}$	$H_{\mathbf{10}} \sim \mathbf{10}$, complex $H_{\mathbf{120}} \sim \mathbf{120}$, real $H_{\mathbf{16}} \sim \mathbf{16}$ $H_{\mathbf{45}} \sim \mathbf{45}$, real
$M_{\text{KK}} < \mu < M_{\text{PS}}$	G_{422}	$\psi_L \sim (4,2,1)$ $\psi_R \sim (4,1,2)$ $\nu_{\text{S}} \sim (1,1,1)$	$h_1, h_1' \sim (1,2,2), h_{15} \sim (15,2,2)$ $h_{3L} \sim (1,3,1), h_{3R} \sim (1,1,3)$ $h_{15}' \sim (15,1,1), h_{\bar{4}} \sim (\bar{4},1,2)$
$M_{\text{SM}} \ll \mu < M_{\text{KK}}$	G_{SM}	q_L, d_R, u_R $l_L, \nu_R, e_R, \nu_{\text{S}}$	h_{SM}

Table 1. Gauge symmetries and particle contents remnant of the model at different energy scales.

Economical choice to achieve fermion masses

Zero mode — No zero mode

$$\Psi_{\mathbf{16}} = \psi_L + \Psi_R^c$$
$$\Psi_{\overline{\mathbf{16}}} = \Psi_L^c + \psi_R$$

$$\psi_L = (q_L, l_L) \sim (4,2,1)\,,\ \Psi_R^c = (Q_R^c, L_R^c) \sim (\bar{4},1,2)$$
$$\Psi_L^c = (Q_L^c, L_L^c) \sim (\bar{4},2,1)\,,\ \psi_R = (q_R, l_R) \sim (4,1,2)$$

Boundary conditions (BC) for fields of zero modes in UV and IR branes:

left-handed fermions, scalars $\Rightarrow$ $(+,+)$

righg-handed fermions $\Rightarrow$ $(-,-)$

Framework

Yukawa coupling terms in 5D $SO(10)$:

$$-\mathcal{L}_Y = y_{\mathbf{10}}\overline{\Psi_{\mathbf{16}}}H_{\mathbf{10}}\Psi_{\overline{\mathbf{16}}} + y_{\mathbf{120}}\overline{\Psi_{\mathbf{16}}}H_{\mathbf{120}}\Psi_{\overline{\mathbf{16}}} + y_{\mathbf{16}}\overline{\nu_{\mathrm{S}}}H_{\mathbf{16}}\Psi_{\overline{\mathbf{16}}} + \frac{L}{2}\mu_{\mathrm{M}}\,\overline{\nu_{\mathrm{S}}}\nu_{\mathrm{S}}^c\,\delta(y-\pi R) + \mathrm{h.c.}$$

Yukawa coupling terms in 4D PS:

$$-\mathcal{L}_y \supset y_1\overline{\psi_L}h_1\psi_R + \overline{\psi_L}(y_1' h_1' + y_{15}h_{15})\psi_R + y_4\overline{\nu_{\mathrm{S}}}h_{\bar{4}}\psi_R + \frac{1}{2}\mu_{\mathrm{M}}\overline{\nu_{\mathrm{S}}}\nu_{\mathrm{S}}^c + \mathrm{h.c.}$$

Dirac mass matrices of fermions: $c = v/v_{\mathrm{EW}}$

$$y_t = \sqrt{2}\,y_{\mathbf{10}}c_{\mathbf{10}}^u + \sqrt{2}\,y_{\mathbf{120}}(c_{\mathbf{120}}^{d'} + \frac{1}{\sqrt{3}}c_{\mathbf{120}}^d)\,,$$

$$y_b = \sqrt{2}\,y_{\mathbf{10}}c_{\mathbf{10}}^d + \sqrt{2}\,y_{\mathbf{120}}(c_{\mathbf{120}}^{d'} + \frac{1}{\sqrt{3}}c_{\mathbf{120}}^d)\,,$$

$$y_\tau = \sqrt{2}\,y_{\mathbf{10}}c_{\mathbf{10}}^d + \sqrt{2}\,y_{\mathbf{120}}(c_{\mathbf{120}}^{d'} - \sqrt{3}c_{\mathbf{120}}^d)\,,$$

$$y_\nu = \sqrt{2}\,y_{\mathbf{10}}c_{\mathbf{10}}^u + \sqrt{2}\,y_{\mathbf{120}}(c_{\mathbf{120}}^{d'} - \sqrt{3}c_{\mathbf{120}}^d)\,.$$

$SO(10)$

Yukawa matching relation from $SO(10)$ to PS:

$$y_{\mathbf{10}} = \frac{1}{\sqrt{2}}y_1,\ y_{\mathbf{16}} = y_4\,,$$

$$y_{\mathbf{120}} = \frac{1}{\sqrt{2}}y_1' = \frac{1}{2\sqrt{2}}y_{15}$$

→

$$y_t = y_1c_{\mathbf{10}}^u + y_1'c_{\mathbf{120}}^{d'} + \frac{1}{2\sqrt{3}}y_{15}c_{\mathbf{120}}^d\,,$$

$$y_b = y_1c_{\mathbf{10}}^d + y_1'c_{\mathbf{120}}^{d'} + \frac{1}{2\sqrt{3}}y_{15}c_{\mathbf{120}}^d\,,$$

$$y_\tau = y_1c_{\mathbf{10}}^d + y_1'c_{\mathbf{120}}^{d'} - \frac{\sqrt{3}}{2}y_{15}c_{\mathbf{120}}^d\,,$$

$$y_\nu = y_1c_{\mathbf{10}}^u + y_1'c_{\mathbf{120}}^{d'} - \frac{\sqrt{3}}{2}y_{15}c_{\mathbf{120}}^d\,.$$

PS

Inverse seesaw:

$$\begin{pmatrix} 0 & m_{\mathrm{D}} & 0 \\ m_{\mathrm{D}} & 0 & m_{\mathrm{S}} \\ 0 & m_{\mathrm{S}} & \mu_{\mathrm{M}} \end{pmatrix} \longrightarrow m_\nu = \mu_{\mathrm{M}}\frac{m_{\mathrm{D}}^2}{m_{\mathrm{S}}^2},\ m_{\mathrm{S}} = y_{16}M_{\mathrm{PS}}$$

Gauge running and asymptotic unification

RG running for gauge couplings at different energy scales:

$$2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t}=b_i^{\mathrm{SM}}\alpha_i^2 \longrightarrow 2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t}=b_i^{\mathrm{PS}}\alpha_i^2 \longrightarrow 2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t}=b_i^{\mathrm{PS}}\alpha_i^2+(S(t)-1)b_{\mathbf{10}}\alpha_i$$

$$M_Z \longrightarrow M_{\mathrm{PS}} \longrightarrow M_{\mathrm{KK}} \longrightarrow$$

Express KK states contribution in a continuous approximation:

$$S(t)=\begin{cases}1 & \text{for } \mu<M_{\mathrm{KK}}\,,\\ \mu/M_{\mathrm{KK}}=M_Z e^t/M_{\mathrm{KK}} & \text{for } \mu>M_{\mathrm{KK}}\,.\end{cases}$$

Define effective 't Hooft coupling with respect to KK excitations:

$$\tilde{\alpha}_i(t)=\alpha_i(t)S(t)$$

$$2\pi\frac{\mathrm{d}\alpha_i}{\mathrm{d}t}=b_i^{\mathrm{PS}}\alpha_i^2+(S(t)-1)b_{\mathbf{10}}\alpha_i$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_i}{\mathrm{d}t}=2\pi\tilde{\alpha}_i+b_{\mathbf{10}}\tilde{\alpha}_i^2$$

$$\tilde{\alpha}_i=\frac{2\pi}{e^{-t+c_i}-b_{\mathbf{10}}}$$

$$\tilde{\alpha}_4,\tilde{\alpha}_{2L},\tilde{\alpha}_{2R}\xrightarrow{\mathrm{UV}}\tilde{\alpha}_{\mathbf{10}}^{\mathrm{UV}}=-\frac{2\pi}{b_{\mathbf{10}}}$$

$b_{\mathbf{10}}<0$ is crucial for gauge couplings existing asymptotically safe fixed point

Gauge running and asymptotic unification

β-coefficient of $SO(10)$ gauge group above the compactification scale M_{KK}:

$$b_{\mathbf{10}} = -(\tfrac{11}{3} - \tfrac{1}{6})C_2(SO(10)) + \tfrac{4}{3}\sum_F T(F_i) + \tfrac{1}{6}\sum_S T(S_i)$$

$H_{\mathbf{10}} \sim \mathbf{10}$, complex
$H_{\mathbf{120}} \sim \mathbf{120}$, real
$H_{\mathbf{16}} \sim \mathbf{16}$
$H_{\mathbf{45}} \sim \mathbf{45}$, real

$\longrightarrow b_{\mathbf{10}} = -5 \longrightarrow \tilde{\alpha}_4, \tilde{\alpha}_{2L}, \tilde{\alpha}_{2R} \xrightarrow{\text{UV}} \tilde{\alpha}_{\mathbf{10}}^{\text{UV}} = \dfrac{2\pi}{5}$

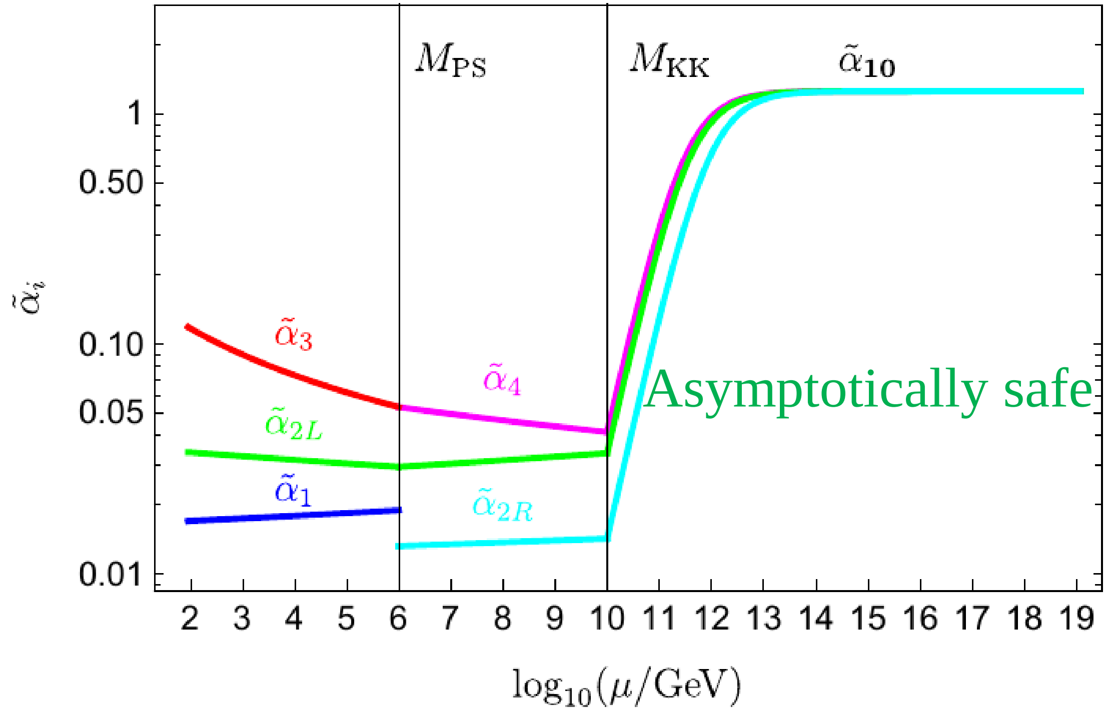


Figure 1. Running of the gauge couplings with $M_{\text{PS}} = 10^6$ GeV and $M_{\text{KK}} = 10^{10}$ GeV.

5D effective gauge coupling:

$$\left.\frac{\Omega(d)}{(2\pi)^d} 4\pi\tilde{\alpha}\right|_{d=5} = \frac{8\pi^2}{3(2\pi)^5} 4\pi\tilde{\alpha} = \frac{2}{15\pi} \sim 0.04 \ll 1$$

Well under perturbative control

RGEs of Yukawa couplings

Running of Yukawa couplings:

Only consider Yukawa couplings of the third generation fermions for simplicity

$$\mu: \quad M_Z \longrightarrow M_{\rm PS} \longrightarrow M_{\rm KK} \longrightarrow \text{UV}$$

$$\text{Yukawas}: \quad y_t, y_b, y_\tau \quad y_1, y_1', y_{15} \quad y_1, y_1', y_{15} \quad \boldsymbol{y_{10}}, \boldsymbol{y_{120}}$$

$$2\pi\frac{d\alpha_t}{dt} = \left[\frac{9}{2}\alpha_t + \frac{3}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{17}{20}\alpha_1 - 8\alpha_3\right]\alpha_t$$

$$2\pi\frac{d\alpha_b}{dt} = \left[\frac{3}{2}\alpha_t + \frac{9}{2}\alpha_b + \alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{1}{4}\alpha_1 - 8\alpha_3\right]\alpha_b$$

$$2\pi\frac{d\alpha_\tau}{dt} = \left[3\alpha_t + 3\alpha_b + \frac{5}{2}\alpha_\tau - \frac{9}{4}\alpha_{2L} - \frac{9}{4}\alpha_1\right]\alpha_\tau$$

$M_Z < \mu < M_{\rm PS}$, one-loop Yukawa RGE in SM

$$2\pi\frac{d\alpha_{y1}}{dt} = \left[10\alpha_{y1} + 4\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1}$$

$$2\pi\frac{d\alpha_{y1'}}{dt} = \left[2\alpha_{y1} + 12\alpha_{y1'} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y1'}$$

$$2\pi\frac{d\alpha_{y15}}{dt} = \left[9\alpha_{y15} + 2\alpha_{y1} - \frac{45}{4}\alpha_4 - \frac{9}{4}(\alpha_{2L} + \alpha_{2R})\right]\alpha_{y15}$$

$M_{\rm PS} < \mu < M_{\rm KK}$, one-loop Yukawa RGE in PS

$$-\mathcal{L}_Y = \boldsymbol{y_{10}}\overline{\boldsymbol{\Psi_{16}}}\boldsymbol{H_{10}}\boldsymbol{\Psi_{\overline{16}}} + \boldsymbol{y_{120}}\overline{\boldsymbol{\Psi_{16}}}\boldsymbol{H_{120}}\boldsymbol{\Psi_{\overline{16}}} + \boldsymbol{y_{16}}\overline{\nu_{\rm S}}\boldsymbol{H_{16}}\boldsymbol{\Psi_{\overline{16}}} + \text{h.c.}$$

$$y_6(\overline{\psi_L}H_6\Psi_L^c + \overline{\Psi_R^c}H_6\psi_R) + y_6'(\overline{\psi_L}H_{6L}\Psi_L^c + \overline{\Psi_R^c}H_{6R}\psi_R) + y_{10}(\overline{\psi_L}H_{10}\Psi_L^c + \overline{\Psi_R^c}H_{\overline{10}}\psi_R) + \text{h.c.}$$

+

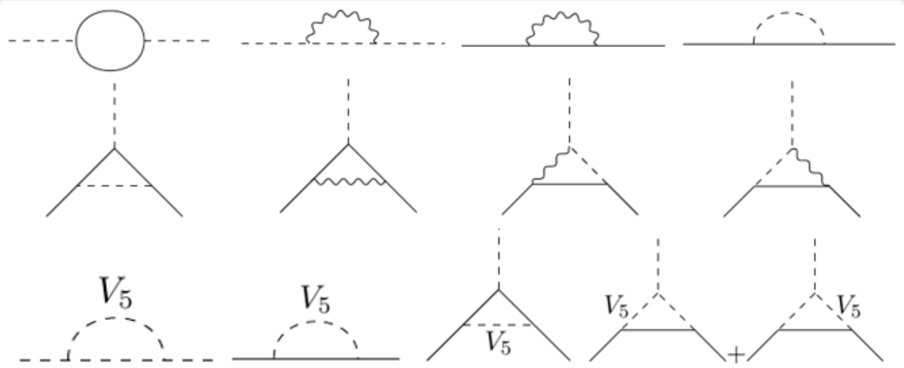
$$y_1\overline{\psi_L}h_1\psi_R + \overline{\psi_L}(y_1'h_1' + y_{15}h_{15})\psi_R + y_4\overline{\nu_{\rm S}}h_{\bar{4}}\psi_R + \text{h.c.}$$

$$2\pi\frac{d\alpha_{yr}}{dt} = 2\pi\frac{d\alpha_{yr}}{dt}\Big|_{\rm PS} + (S(t) - 1)\,2\pi\frac{d\alpha_{yr}}{dt}\Big|_{\rm KK}$$

$M_{\rm KK} < \mu$, $SO(10)$

$SO(10)$ Yukawa couplings ⟹ PS decomposition

RGEs of Yukawa couplings



$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t} = \left[2\pi + 42\tilde{\alpha}_{y\mathbf{10}} + 24\tilde{\alpha}_{y\mathbf{120}} - \frac{171}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{10}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t} = \left[2\pi + 10\tilde{\alpha}_{y\mathbf{10}} + 136\tilde{\alpha}_{y\mathbf{120}} - \frac{219}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{120}}$$

One-loop Yukawa RGE in 5D $SO(10)$

$$2\pi\frac{\mathrm{d}\alpha_{y1}}{\mathrm{d}t}\Big|_{\mathrm{KK}} = \Big[18\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 4\alpha_{y1'} - \frac{5}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{81}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\Big]\alpha_{y1}\,,$$

$$2\pi\frac{\mathrm{d}\alpha_{y1'}}{\mathrm{d}t}\Big|_{\mathrm{KK}} = \Big[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 20\alpha_{y1'} + \frac{15}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\Big]\alpha_{y1'}\,,$$

$$2\pi\frac{\mathrm{d}\alpha_{y15}}{\mathrm{d}t}\Big|_{\mathrm{KK}} = \Big[2\alpha_{y1} + \frac{3}{2}\alpha_{y6} + 11\alpha_{y15} + \frac{3}{2}\alpha_{y10} + \frac{9}{4}\alpha_{y6'} - \frac{129}{8}\alpha_4 - \frac{45}{8}(\alpha_{2L} + \alpha_{2R})\Big]\alpha_{y15}\,.$$

PS decomposition

Explicit unification: all Yukawa couplings have already been fully unified into their $SO(10)$ values at KK scale

$$\begin{aligned} \frac{1}{2}\alpha_{y1},\ \frac{1}{4}\alpha_{y6} &= \alpha_{y\mathbf{10}}\,, \\ \frac{1}{2}\alpha_{y1'},\ \frac{1}{8}\alpha_{y15},\ \frac{1}{8}\alpha_{y10},\ \frac{1}{16}\alpha_{y6'} &= \alpha_{y\mathbf{120}}\,, \end{aligned} \quad \text{at } \mu = M_{\mathrm{KK}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t} = \left[2\pi + 42\tilde{\alpha}_{y\mathbf{10}} + 24\tilde{\alpha}_{y\mathbf{120}} - \frac{81}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{10}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t} = \left[2\pi + 10\tilde{\alpha}_{y\mathbf{10}} + 136\tilde{\alpha}_{y\mathbf{120}} - \frac{129}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{120}}$$

RGEs of Yukawa couplings

One-loop Yukawa RGE in 5D $SO(10)$:

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t}=\left[2\pi+42\tilde{\alpha}_{y\mathbf{10}}+24\tilde{\alpha}_{y\mathbf{120}}-\frac{171}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{10}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t}=\left[2\pi+10\tilde{\alpha}_{y\mathbf{10}}+136\tilde{\alpha}_{y\mathbf{120}}-\frac{219}{8}\tilde{\alpha}_{\mathbf{10}}\right]\tilde{\alpha}_{y\mathbf{120}}$$

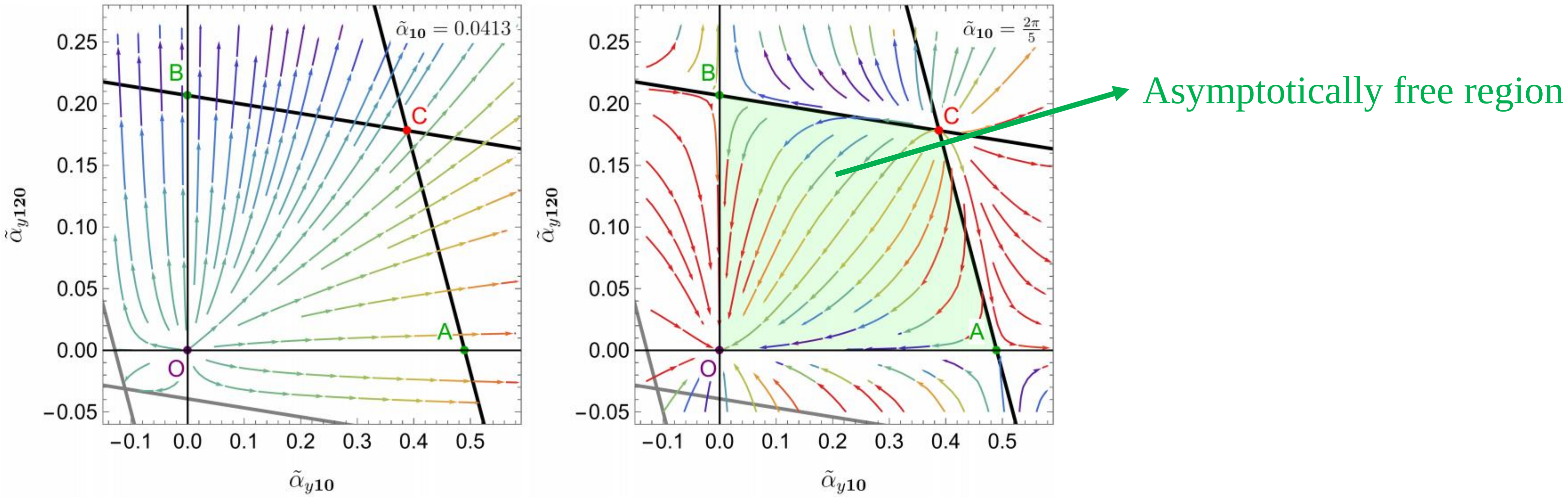


Figure 2. Stream plot of Yukawa couplings $\tilde{\alpha}_{y\mathbf{10}}, \tilde{\alpha}_{y\mathbf{120}}$ in 5D $SO(10)$ GUT.

Scan Yukawa couplings

Free parameters: $\{y_{\mathbf{10}}(M_{\rm KK}), y_{\mathbf{120}}(M_{\rm KK})\}$

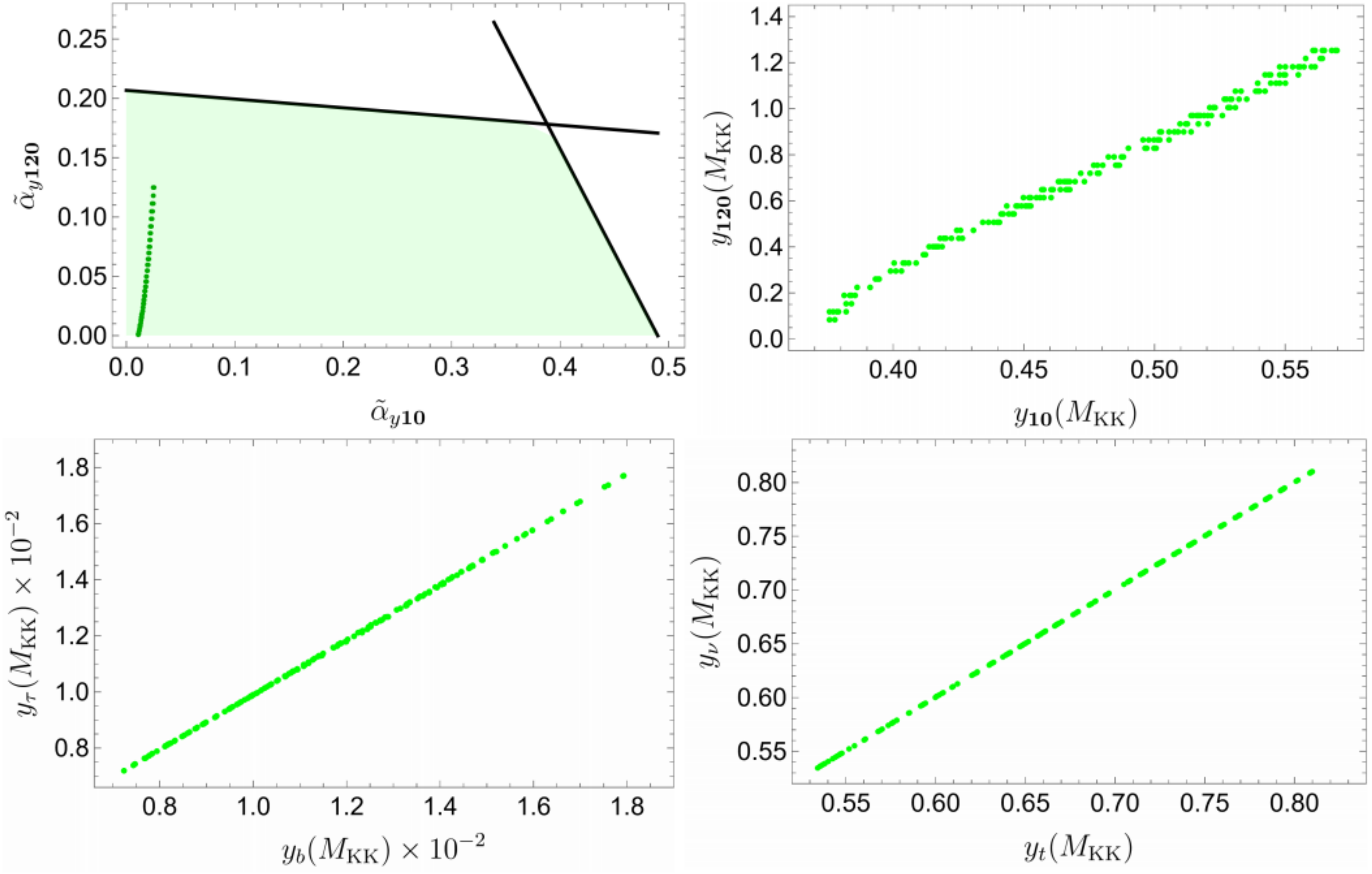
evolve couplings from $M_{\rm KK}$ to $M_{\rm PS}$ through one-loop Yukawa RGE in $SO(10)$

VEV constraint: $(c^u_{\mathbf{10}})^2 + (c^d_{\mathbf{10}})^2 + 2(c^d_{\mathbf{120}})^2 + 2(c^{d'}_{\mathbf{120}})^2 = 1$

Initial values for one-loop Yukawa RGE in PS to evolve couplings from the EW scale to $M_{\rm PS}$:

$$\{y_t, y_b, y_\tau\} = \{0.97, 0.016, 0.010\} \text{ at } \mu = M_Z$$

G.Y. Huang, S. Zhou, 2009.04851



$y_{\mathbf{10}}(M_{\rm KK}) \in (0.3757, 0.5699)$
$y_{\mathbf{120}}(M_{\rm KK}) \in (0.0822, 1.2519)$
$y_b(M_{\rm KK}) \in (0.0072, 0.0179)$
$y_\tau(M_{\rm KK}) \in (0.0072, 0.0177)$
$y_t(M_{\rm KK}) \in (0.5344, 0.8101)$
$y_\nu(M_{\rm KK}) \in (0.5344, 0.8098)$

Benchmark point

$$\mu > M_{\text{KK}} \quad 2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{10}}}{\mathrm{d}t} = \left[2\pi + 42\tilde{\alpha}_{y\mathbf{10}} + 24\tilde{\alpha}_{y\mathbf{120}} - \frac{81}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{10}}$$

$$2\pi\frac{\mathrm{d}\tilde{\alpha}_{y\mathbf{120}}}{\mathrm{d}t} = \left[2\pi + 10\tilde{\alpha}_{y\mathbf{10}} + 136\tilde{\alpha}_{y\mathbf{120}} - \frac{129}{8}\tilde{\alpha}_4 - \frac{45}{8}(\tilde{\alpha}_{2L} + \tilde{\alpha}_{2R})\right]\tilde{\alpha}_{y\mathbf{120}}$$

Negative gauge contribution ultimately surpassing the positive Yukawa contribution

Inputs	$y_{\mathbf{10}}(M_{\text{KK}})$ 0.376	$y_{\mathbf{120}}(M_{\text{KK}})$ 0.082		
	M_{PS} 10^6 GeV	M_{KK} 10^{10} GeV	μ_{M} 10 eV	$y_{\mathbf{16}}(M_{\text{KK}})$ 10^{-3}
Outputs	$c^u_{\mathbf{10}}$ 0.999	$c^d_{\mathbf{10}}$ 0.007	$c^d_{\mathbf{120}}$ 0.0002	$c^{d'}_{\mathbf{120}}$ 0.0317
	$y_1(M_{\text{PS}})$ 0.677	$y_1'(M_{\text{PS}})$ 0.174	$y_{15}(M_{\text{PS}})$ 0.338	m_ν 0.09 eV
	$y_b(M_{\text{KK}})$ 0.0075	$y_\tau(M_{\text{KK}})$ 0.0074	$y_t(M_{\text{KK}})$ 0.534	$y_\nu(M_{\text{KK}})$ 0.534

Table 2. Inputs and predictions of VEVs, Yukawa couplings, charged fermion masses and neutrino masses of one point.

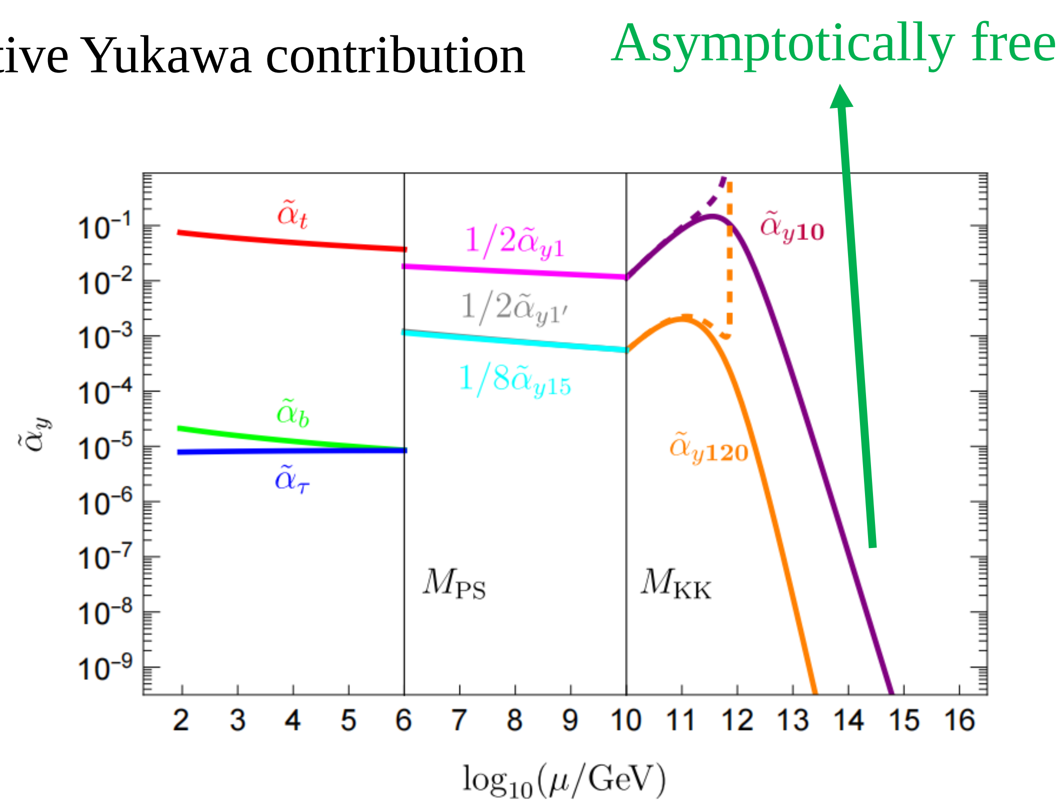


Figure 4. Running of the Yukawa couplings for the benchmark point with $M_{\text{PS}} = 10^6$ GeV and $M_{\text{KK}} = 10^{10}$ GeV. Solid line (with H_{45}), dashed line (without H_{45})

Conclusion

1. 5D $SO(10)$ GUT with PS as an intermediate scale can realize asymptotic unification of gauge couplings, which means these couplings gradually approach the same value in the deep UV limit.
2. Asymptotic freedom of the 't Hooft couplings of the Yukawa couplings can be realized.
3. 5D $SO(10)$ GUT with PS as an intermediate scale can recover experimental data on the masses of quarks and leptons.
4. We have first calculated one-loop Yukawa RGEs in $SO(10)$ group and its decomposition to PS group.
5. As the energy scale runs toward the UV limit, the ‘t Hooft couplings of all gauge couplings approach the UV fixed point $-2\pi/b_{10}$ regardless of their initial values. A negative b_{10} is crucial for realizing asymptotic unification of gauge couplings, which limits the redundancy of Higgs content in $SO(10)$ GUTs.
6. Separate left-handed and right-handed chiral fields into $\Psi_{\mathbf{16}}$ and $\Psi_{\overline{\mathbf{16}}}$, proton decay is naturally forbidden.

Thanks！