

Gravitational Waves of GUT Phase Transition during Inflation

胡禧贺 huxihe23@mails.ucas.ac.cn 2501.01491

2025粒子物理标准模型及新物理精细计算研讨会 河北大学,2025年3月30日



School of Fundamental Physics and Mathematical Sciences







Quantum Fluctuations in Early Universe

國科大杭州髙等研究院 **Motivation—Cosmic Phase Transition** Hangzhou Institute for Advanced Study, UCAS

First Order Phase Transition (FOPT)





quantum tunneling with temperature

Sources I: GW sources during phase transition

Sources II: Scalar-induced GWs of PT

2402.04158, 2503.01962





國科大杭州髙等研究院 **Motivation—Cosmic Phase Transition** Hangzhou Institute for Advanced Study, UCAS Sources III: After PT, Topological Defects — monopole, cosmic string, domain wall **GW sources!** cosmic string usually from broken U(1)

domain wall usually from broken Z_n





 $t = 71t_i$

 $t = 31t_i$













1002.1555







Hangzhou Institute for Advanced Study, UCAS

Super-High Scale Phase Transition (SHSPT) $\Lambda_{PT} > 10^9 \text{ GeV}$

New Physics: GUT, Intermediate Symmetries of GUT, Seesaw Model, New Gauge Symmetries, Flavor Symmetries, Extra Dimensions, SUSY

Case II Case	e I
reheating	Phase Transition at a_{\star} f_{peak}
Inflation RI) MD ΛD
$f_{\rm peak} \propto T_{\rm Rh} e^{-N_{\star}} f_{\rm peak} q^{-N_{\star}}$	$\mathbf{x} T_{\star}$
SPT during RD: $\Lambda_{\rm PT} \sim T_{\star}$	$f_{\text{peak}} > 10^4 \text{ Hz}, \text{Unobservab}$
SPT during inflation: $\Lambda_{\text{PT}} \gtrsim \Lambda_{2}$	$\begin{cases} \bullet f_{\text{peak}} \text{ is redshifted by infla}\\ \bullet \text{ GWs have special feature} \end{cases}$
	H. P. An, et al., 2009.12381, 220
Possibly observable! Testing GUT in foreseeable future! XHH , Zhou 2501.02	



Production and Propagation of GWs Conformal FLRW Metric $ds^2 = a^2(\tau) \left| d\tau^2 - (\delta_i) \right|$ $\tilde{h}_{ii}''(\tau, \mathbf{k}) + 2\mathcal{H}\tilde{h}_{ii}'(\tau, \mathbf{k}) + k^2\tilde{h}_{ii}(\tau, \mathbf{k}) = 16\pi G_{\mathrm{N}}a^2(\tau)\tilde{\sigma}_{ii}(\tau, \mathbf{k})$ **E.O.M of CGW** $h^{\text{Inf}}(\tau, \mathbf{k}) =$ $\tilde{h}_{ij}(\tau, \mathbf{k}) = C_{ij,1} h_1(\tau, \mathbf{k}) + C_{ij,2} h_2(\tau, \mathbf{k}) \qquad h^{\text{RD}}(\tau, \mathbf{k}) =$

 $h^{\mathrm{MD}}(\tau, \mathbf{k})$

Energy density of SGWB $\frac{d\rho_{GW}}{d\log k} = \frac{1}{64\pi^3 G_N} \frac{k^3}{V}$ **Inside Horizon** $k^2 \gg \left| \frac{a''}{a} \right|, \quad \lambda \ll \frac{1}{\mathcal{H}}$ **Outside Horizon** $k^2 \ll \left| \frac{a''}{a} \right|, \quad \lambda \gg \frac{1}{\mathcal{H}}$

國科大杭州髙等研究院 Hangzhou Institute for Advanced Study, UCAS

$$_{ij} + h_{ij}(\tau, \mathbf{x})) \mathrm{d}x^i \mathrm{d}x^j$$

Traceless and transverse gauge

 \mathcal{H} is conformal Hubble factor

$$= \cos k\tau + k\tau \sin k\tau, \ \sin k\tau - k\tau \cos k\tau \qquad \text{Inflation, } \Lambda \text{D}: \mathscr{H} =$$

$$= \frac{\cos k\tau}{k\tau}, \ \frac{\sin k\tau}{k\tau} \qquad \text{RD}: \mathscr{H} = 1/\tau$$

$$= \frac{\cos k\tau + k\tau \sin k\tau}{(k\tau)^3}, \ \frac{\sin k\tau - k\tau \cos k\tau}{(k\tau)^3} \qquad \text{MD}: \mathscr{H} = 2/\tau$$

$$\stackrel{3}{\xrightarrow{-1}} \frac{1}{a^2(t)} \int_{T_\tau} \frac{d\tau}{T_\tau} \left| \tilde{h}'_{ij}(\tau, \mathbf{k}) \right|^2, \quad h^2 \Omega_{\text{GW}}(f) = \frac{h^2}{\rho_{\text{c}}} \frac{d\rho_{\text{GW}}}{d\log k} \Big|_{k=1}^{t=1}$$

$$\stackrel{5}{\xrightarrow{-1}} \frac{1}{a^2(t)} \int_{T_\tau} \frac{d\tau}{T_\tau} \left| \tilde{h}'_{ij}(\tau, \mathbf{k}) \right|^2, \quad h^2 \Omega_{\text{GW}}(f) = \frac{1}{\alpha} \frac{d\rho_{\text{GW}}}{d\log k} \Big|_{k=1}^{t=1}$$

$$\stackrel{5}{\xrightarrow{-1}} \frac{h^{\text{in}}_{ij}(\tau, \mathbf{k}) \simeq \tilde{h}^{\text{in}}_{ij} \frac{\sin(k\tau + \phi)}{a(\tau)}, \quad \frac{d\rho_{\text{GW}}}{d\log k} \propto \frac{1}{a^4(\tau)} \qquad \text{Wav}$$

$$\stackrel{6}{\xrightarrow{-1}} \frac{h^{\text{out}}_{ij}(\tau, \mathbf{k}) \simeq \tilde{h}_{ij}(\tau_0, \mathbf{k}) + \tilde{h}'_{ij}(\tau_0, \mathbf{k}) \int_{\tau_0}^{\tau} \frac{d\tau'}{a^2(\tau')}$$















The GW of all frequency is inside horizon evolution during RD



國科大杭州髙等研究院 Hangzhou Institute for Advanced Study, UCAS

RD
$$\Re$$
 (HZ) MD $\mathscr{H} \sim \begin{cases} 10^8 \sim 10^{-16} (HZ) & RD \\ 10^{-16} \sim 10^{-18} (HZ) & MD \\ 10^{-18} (HZ) & \Lambda D \end{cases}$

GW in this frequency range is inside horizon evolution during RD, MD and AD!

$$\tilde{h}_{ij}^{\text{Inf}}(\tau, \mathbf{k}) \Big|_{\text{Rh}} = \tilde{h}_{ij}^{\text{RD}}(\tau, \mathbf{k}) \Big|_{\text{Rh}}$$
$$\partial_t \tilde{h}_{ij}^{\text{Inf}}(\tau, \mathbf{k}) \Big|_{\text{Rh}} = \partial_t \tilde{h}_{ij}^{\text{RD}}(\tau, \mathbf{k}) \Big|_{\text{Rh}}$$







 h_0 dominates GW in UV and IR, h_1 dominates GW in utral UV



Hangzhou Institute for Advanced Study, UCAS

Conventions

$$\tilde{r}_{ij}(\mathbf{k}) \times \left[h_0(\tau, \mathbf{k}) + h_1(\tau, \mathbf{k})\right] \qquad y = \frac{k}{a_\star H_\star}, \ \epsilon = \frac{a_\star}{a_\mathrm{R}}$$

$$\frac{\mathrm{in}[k\tau - y\epsilon]}{y^3} \times \left\{ \cos[y(1-\epsilon)] - \frac{\sin[y(1-\epsilon)]}{y} \right\}$$

$$\left\{ \frac{1-\epsilon}{y^3} \cos[k\tau + y(1-2\epsilon)] - \left(\frac{1+y\epsilon}{y^4}\right) \sin[k\tau + y(1-\epsilon)] \right\}$$

$$v \approx 1, \ \mathrm{Far} = \frac{1}{y}$$

$$v \approx 1, \ \mathrm{Far} = \frac{1}{y}$$





Inflated GWs from short-time sources

Inflated Gravitational Waves

$$\frac{d\rho_{GW}}{d\log k} = \frac{d\rho_{GW}^{\text{flat}}}{d\log k} \times \frac{a_{Rh}^4}{a^4(t)} \times \mathcal{S}(t,k) \qquad \begin{array}{c} \text{relative}\\ \text{instant} \end{array}$$

$$\frac{d\rho_{GW}}{d\log k} \times \frac{d\rho_{GW}^{\text{flat}}}{a^4(t)} \times \mathcal{S}(t,k) \qquad \begin{array}{c} \text{flat}\\ \text{instant} \end{array}$$

$$\frac{d\rho_{GW}}{d\log k} \times \frac{d\rho_{GW}}{a^4(t)} \times \mathcal{S}(t,k) \qquad \begin{array}{c} \text{flat}\\ \text{instant} \end{array}$$

$$\frac{d\rho_{GW}}{d\log k} \times \frac{d\rho_{GW}}{a^4(t)} \times \mathcal{S}(t,k) \qquad \begin{array}{c} \text{flat}\\ \text{instant} \end{array}$$

$$\frac{d\rho_{GW}}{d\log k} \times \frac{d\rho_{GW}}{a^4(t)} \times \mathcal{S}(t,k) \qquad \begin{array}{c} \text{flat}\\ \text{instant} \end{array}$$

$$S(f) = \mathcal{S}(t_0, 2\pi a_0 f) = S_0(f) + S_1(f)$$
$$S_0(f) = \left\{ \frac{\cos[y(1-\epsilon)]}{2} - \frac{\sin[y(1-\epsilon)]}{2} \right\}^2$$

$$S_{0}(f) = \left\{ \frac{\cos[y(1-\epsilon)]}{y^{2}} - \frac{\sin[y(1-\epsilon)]}{y^{3}} \right\} \quad S(f) \text{ Oscillates!} \quad y = \frac{2\pi a_{0}f}{a_{\star}H_{\star}}, \ \epsilon = \frac{a_{\star}}{a_{Rh}}$$
$$S_{1}(f) = y\epsilon \times \left\{ \left[\frac{1}{y^{2}} + 2\epsilon - 1 \right] \frac{\sin[2y(1-\epsilon)]}{y^{4}} - \left[\frac{2-\epsilon}{y^{2}} + \epsilon \right] \frac{\cos[2y(1-\epsilon)]}{y^{3}} + \frac{\epsilon^{3}}{y} \left(\frac{1}{y^{2}} + 1 \right) \right\}$$

 S_0 dominates GW in IR ($y \ll 1$) and UV ($y \gg 1, y \in \ll 1$), S_1 dominates GW in FUV ($y \in \gg 1$)

tive via t sources

$$h_{ij}^{\text{flat}}(\tau, \mathbf{k}) = 16\pi G_{\text{N}} \tilde{\sigma}_{ij}(\mathbf{k}) \sin[k(\tau - \tau_{\star})]$$

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\mathrm{d}\log k} = \frac{2G_{\mathrm{N}}}{\pi} \frac{k^3}{V} \left| \tilde{\sigma}_{ij}(\mathbf{k}) \right|^2$$

GW in flat spacetime

uninflated SGWB

lation

$$h^2 \widetilde{\Omega}_{\text{GW}}(\tilde{f}) = \frac{h^2}{\rho_c} \frac{d\rho_{\text{GW}}^{\text{flat}}}{d\log k} \times \frac{a_{\text{Rh}}^4}{a_0^4} \quad \text{Waves!}$$

decreasing from in-horizon evolution











國科大杭州髙等研究院 Inflated GWs from GUT Phase Transition Hangzhou Institute for Advanced Study, UCAS

GUT phase transition and Monopoles

 σ — field of PT, ϕ — inflaton

cubic coupling potential

 $V_{\rm PT}(\phi, \sigma) = D(\phi^2 - \phi_0^2) \,\sigma^2 - \mu \,\sigma^3 + \frac{\lambda}{4} \sigma^4$ possible physical origins $SO(10): 45 \times 45 \times 45 = 1 + 45 + \dots$ $SU(5): 24 \times 24 \times 24 = 1 + 24 + \dots$

Coleman-Weinberg potential

$$V_{\rm PT}(\phi,\sigma) = D(\phi^2 - \phi_0^2)\,\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{4}\sigma^4\log\frac{\sigma^2}{\Lambda^2}$$

 $n_{\star} \simeq L_{\text{mono}}^{-3} \sim H_{\star}^{-3}$ monopoles density

monopoles problem

$$\frac{\Omega_{\text{mono}}}{10^{-6}} \sim \left(\frac{T_{\text{Rh}}}{10^{15} \text{ GeV}}\right)^4 e^{-3(N_{\star} - 15)} \lesssim 1$$

GW from GUT PT

Plasma is unnecessary during inflation, so we only considering bubble collisions during inflation, rather than sound waves or MHD turbulence.

bubble collision — envelope approximation astro-ph/9310044

The energy-momentum is mainly contained in envelope of bubbles rather than intersection $\frac{0.62\,\beta}{1.8 - 0.1v_{\rm w} + v_{\rm w}^2}$ duration of PT $f_{\star}^{\mathrm{peak}} =$ $\kappa(\alpha) = \frac{0.715\alpha + 0.181\sqrt{\alpha}}{1 + 0.715\alpha} n_{\beta}$ $\sqrt{2}$ $\Omega_{\text{GW}\star}^{\text{peak}} = \frac{0.11 v_{\text{W}}^3}{0.42 + v_{\text{W}}^2} \times \kappa^2 \left(\frac{\rho_{\text{PT}}}{\rho_{\text{tot}}}\right)^2 \div \left(\frac{\beta}{H_{\star}}\right)^2$ $v_w = 1, \alpha \to \infty, \kappa \to 1 \longrightarrow$ strong phase transition simulation 0806.1828 $\Omega_{\rm GW\star}(f_{\star}) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}^{\rm flat}}{d\log f_{\star}} = \Omega_{\rm GW\star}^{\rm peak} \frac{(a+b)(f_{\star}/f_{\star}^{\rm peak})^a}{a(f_{\star}/f_{\star}^{\rm peak})^{a+b} + b}$











Inflated GWs from GUT Phase Transition



 $h^2 \Omega_{\rm GW}$

Hangzhou Institute for Advanced Study, UCAS

 $f [H_{\mathbb{Z}}]$









Parameters of inflated GW from PT:

Inflated GWs from GUT PT: $T_{Rh} = 10^{15} \text{ GeV}$



國科大杭州髙等研究院 Hangzhou Institute for Advanced Study, UCAS

reheating temperature $T_{\rm Rh}$, e-folds number of sources N_{\star} , PT velocity β/H_{\star} , PT vacuum energy density $\rho_{\rm PT}/\rho_{\rm tot}$

 N_{\star} only affects frequency, $\rho_{\rm PT}/\rho_{\rm tot}$ only affects amplitude, and β/H_{\star} affects shape and amplitude.





Inflated GWs from PT of intermediate symmetries of GUT or other BSM below GUT scale



f [Hz]

國科大杭州髙等研究院 Hangzhou Institute for Advanced Study, UCAS



Inflated GWs from Phase Transition



 $T_{\rm Rh}$ and N_{\star} affect the peak frequency, β/H_{\star} and $\rho_{\rm PT}/\rho_{\rm tot}$ don't affect peak frequency. Considering in- and out- horizon, the peak frequency is not simply redshifted for the peak of uninflated GWs.

國科大杭州髙等研究院

Hangzhou Institute for Advanced Study, UCAS







Inflated GWs from Phase Transition











- After solving monopole problem, GWs from GUT phase transition during inflation, if it is first-order, can be redshifted and deformed to oscillate, and thus might be observable today and in foreseeable future.
- The general correlation between inflated GWs and uninflated GWs is established for short time sources.
- There are three region IR, UV and far UV for inflated GWs, where IR and UV usually aren't extremely depressed, and FUV is depressed by $a_{\star}^4/a_{\rm Rh}^4$.
- The inflated GWs from transitory phase transition have $f^{\text{peak}} \sim 60 \text{ MHz} \times \left(\frac{T_{\star}}{10^{15} \text{ GeV}}\right) \times e^{-N_{\star}},$



Results

$$h^2 \Omega_{\rm GW}^{\rm peak} \sim 6 \times 10^{-7} \times \left(\frac{H_{\star}}{\beta}\right)^5 \left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2,$$

where some parameters' regions can be tested today and in future as shown on the above.





Thanks!

huxihe23@mails.ucas.ac.cn

where and a state where the state of the sta





同

Hangzhou Institute for Advanced Study, UCAS

