Frontiers in Nuclear Lattice EFT: From Ab Initio Nuclear Structure to Reactions

An accurate relativistic chiral nuclear force

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Cooperator: Dr. Qingyu Zhai, Dr. Yang Xiao, Prof. Lisheng Geng

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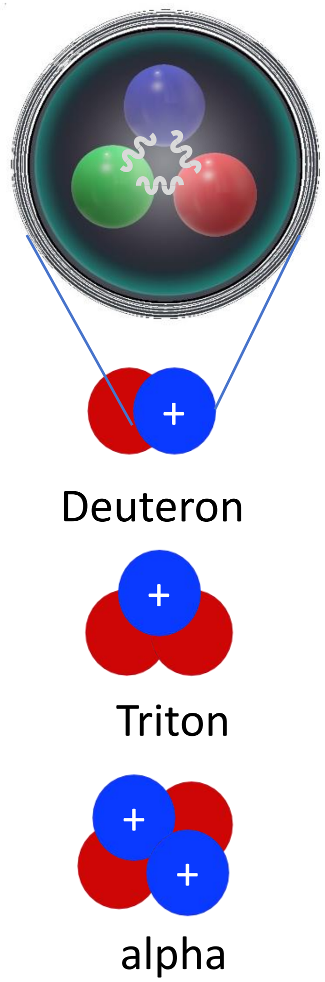
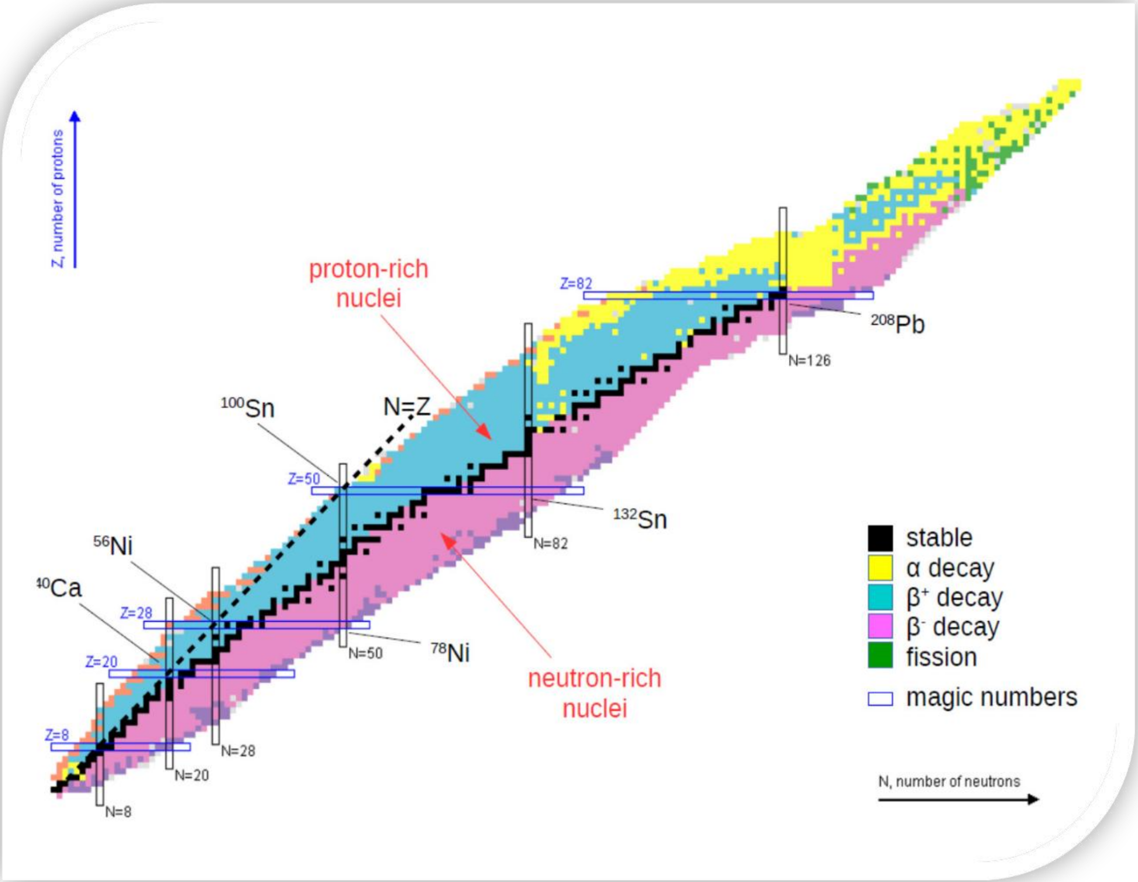
- A brief introduction of nuclear force
- Relativistic chiral nuclear force: framework and status
- Recent development
  - $\mathrm{N^3LO}$ two-body forces
  - Relativistic 3-body formulism
- summary and prospect

☐ The key for nuclear physics

- Nuclear matter, structure, reaction, astrophysics…





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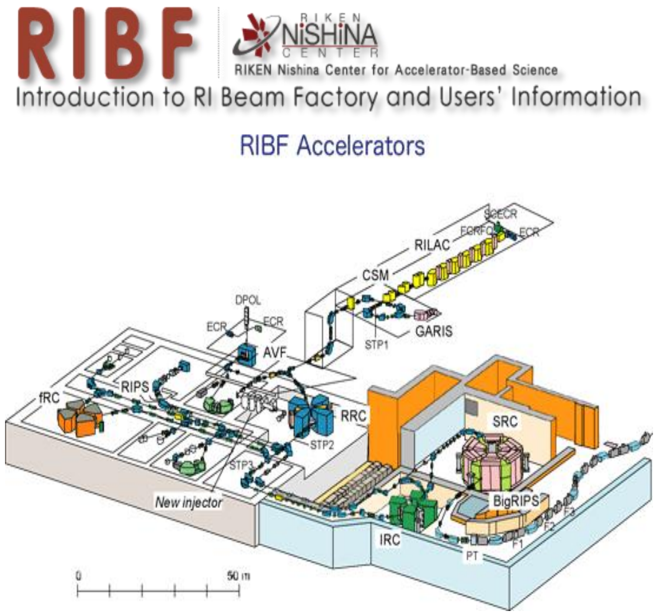
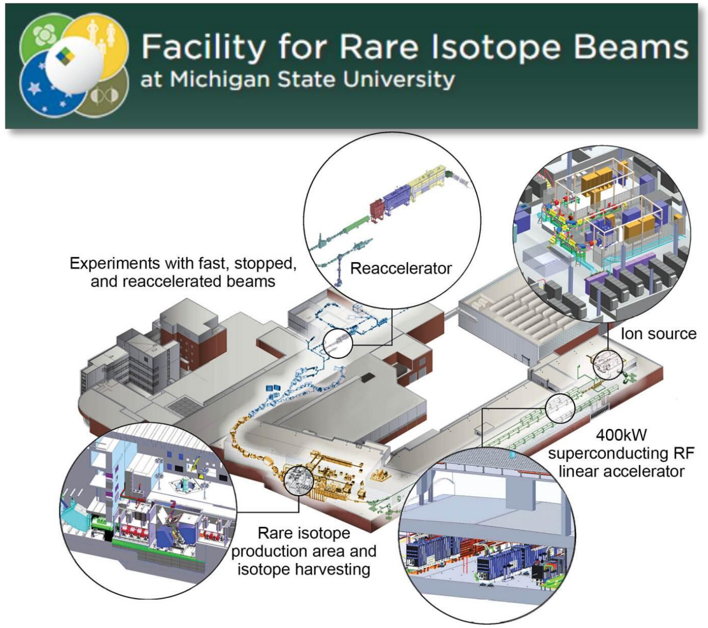
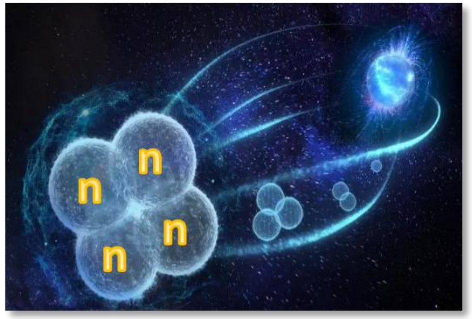
The major concern of various facilities



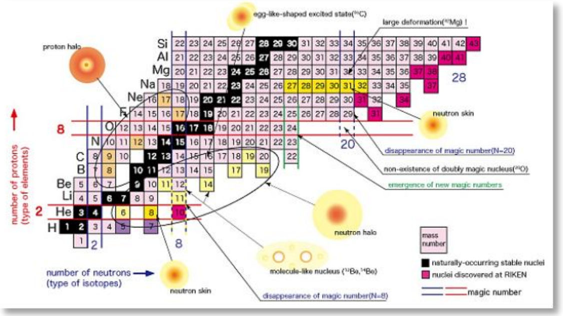


A new era of discovery----Exotic nuclei

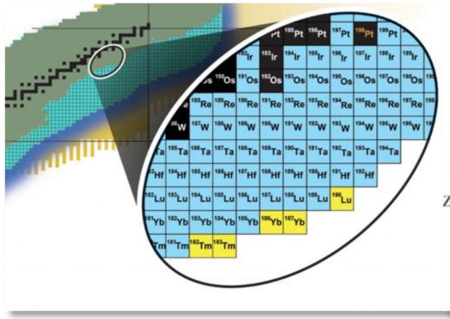
four-neutron system



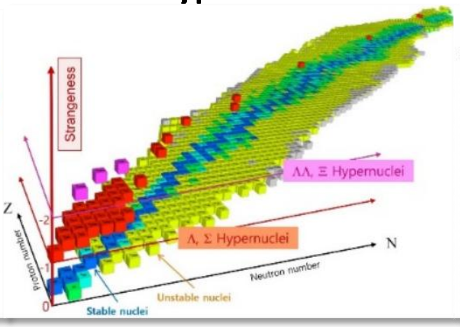
halo nucleus



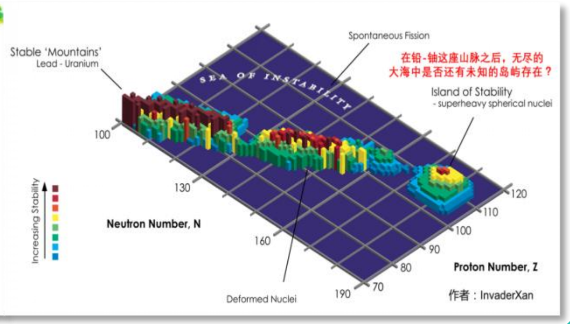
nuclei near drip line



Hyper-nuclei



Super heavy elements



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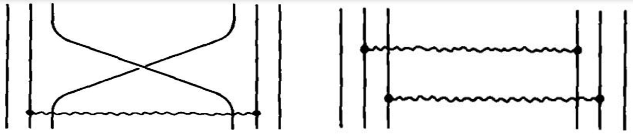
One of the most difficult problem

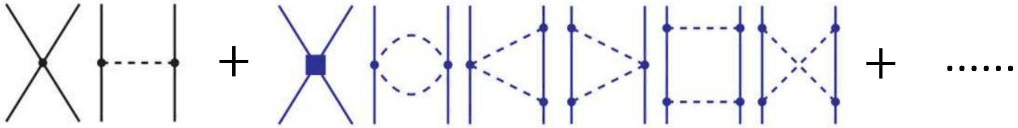
- 1935 Yukawa, Meson theory
- 1950-1960' One pion exchange, One boson exchange
- 1980' Quark model



- 1990' Weinberg, **ChEFT**

S. Weinberg, PLB1990

Modern NF

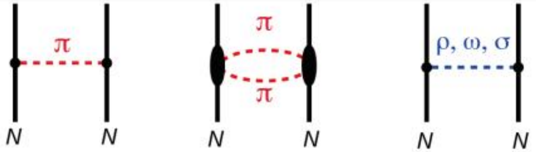


- 1994 **High precision pheno. Models**: **AV18/Reid93 (operator parameterization)**

V. Stoks, PRC1994 R. Wiringa,PRC1994

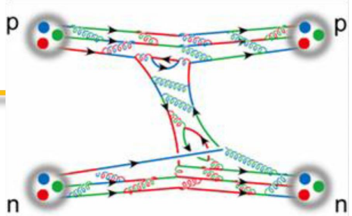
$$V_{NN} = V_c(r)\hat{\mathbf{1}} + V_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{LS}(r)\,\boldsymbol{L}\cdot\boldsymbol{S} + V_T(r)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q} + \cdots\cdots$$

- 2001 **High precision pheno. Models** : **CD-Bonn (Meson Exchange)**



R. Machleidt, PRC2001

- 2006 **Lattice QCD with full dynamics**



S.R.Beane PRL2006

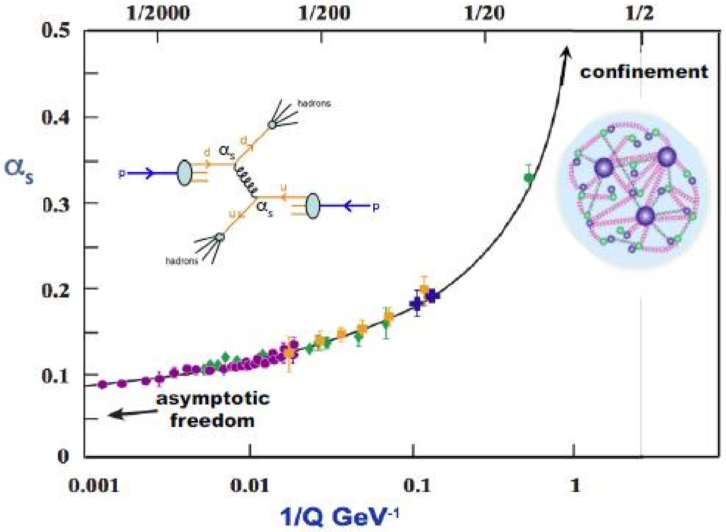
Hans Bethe **Yukawa**

Nobel Prize 1967/1949

❑ QCD: Asymptotic freedom and confinement

➢ In higher energy regions
- "free" quarks
- perturbative QCD

➢ In lower energy regions
- **non-perturbative nature**
- **hadrons** as degrees of freedom for interactions



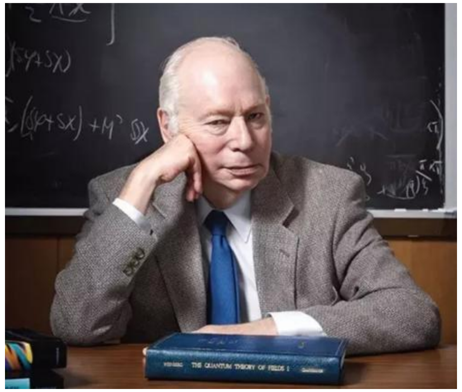
❑ Chiral effective field theory

One has to write **the most general Lagrangian** consistent with **the assumed symmetry principles**, particularly the **(broken) chiral symmetry of QCD**.

➢ Chiral symmetry and its spontaneously breaking——Goldstone bosons

➢ EFT——Interactions in low-energy regime do not depend on the details of those in high-energy regimes

closely related to QCD	Degree of freedom	Expansion parameters
$\mathcal{L}_{QCD} \to$ $\mathcal{L}_{\chi EFT} = \sum c_i O_i$	Quarks & Gluons → **Hadrons**	$Q/\Lambda \ll 1$ • **Hard scale Λ** • **Soft scale Q**



Steven Weinberg

Nobel Prize 1979

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❑ Perturbative treatment of low-energy strong interaction

- In powers of external momenta and light quark masses——**power counting rule**

$$p_i \to \alpha\, p_i \qquad m_q \to \alpha^2\, m_q$$

$$\mathcal{A}(\alpha p_i, \alpha^2 m_q) = \alpha^D \mathcal{A}(p_i,\, m_q)$$

$$\boxed{D = 4L + \sum_n n\, V_n - 2N_M - N_B}$$

- D: chiral order of certain diagram
- L: number of loops
- N_M: number of meson propagators
- N_B: number of baryon propagators
- V_n: number of n-th order vertices

❑ Advantages

- Closer connected to QCD
- Systematically improvement
- Uncertainty estimation
- Self-consistent 3-body force

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Advantages

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For baryon systems —— power counting breaking

- Non-vanishing baryon mass at chiral limit

Heavy baryon (HB)

- Heavy static nucleon
- Non-relativistic $p^\mu = m_B v^\mu + k^\mu$

Jenkins and Manohar, **PLB** 255 (1991) 558.

Infrared regularization (IR)

- Covariance
- Analytical problem

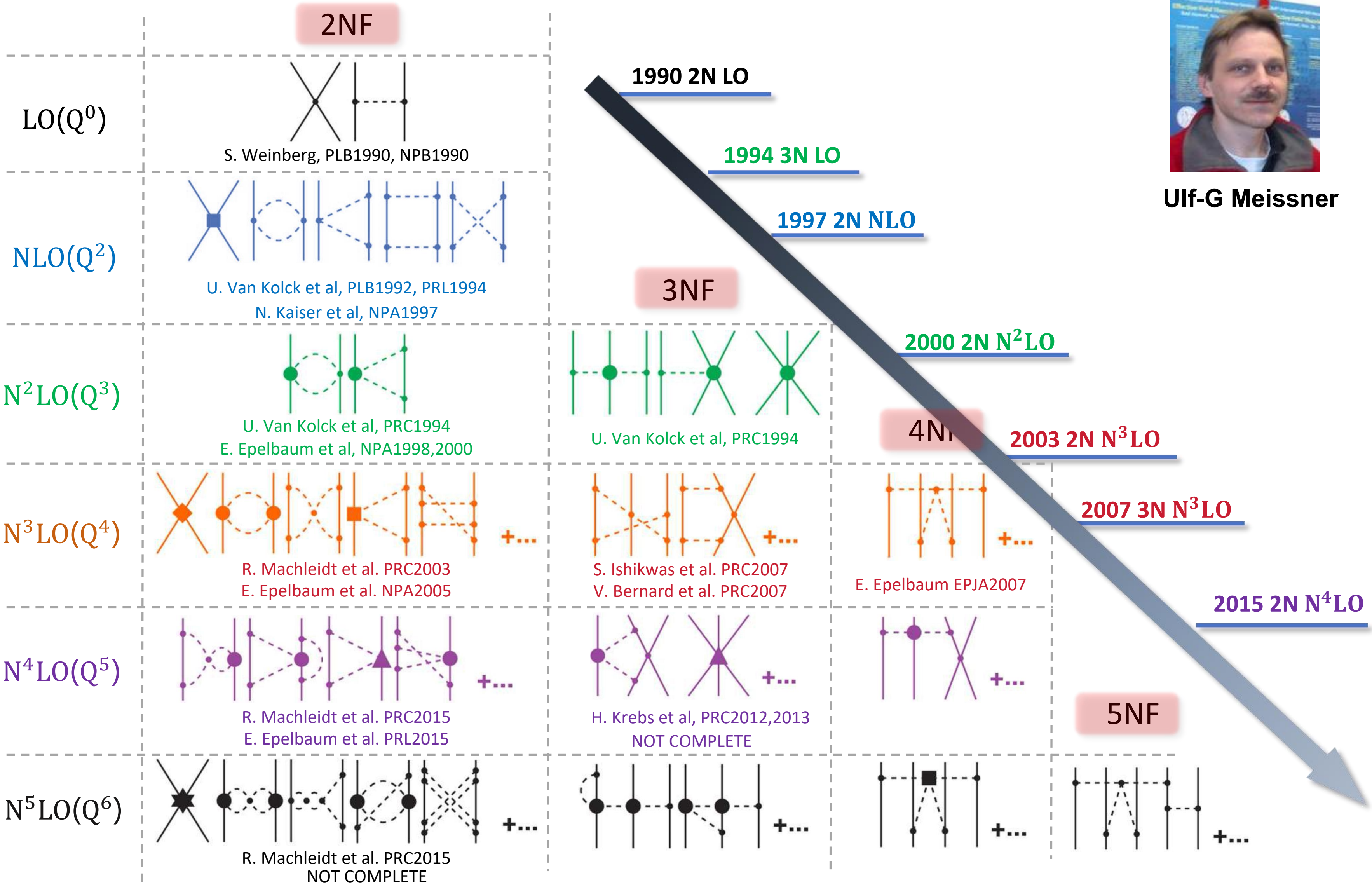
Becher and Leutwyler, EPJC 9 (1999) 643

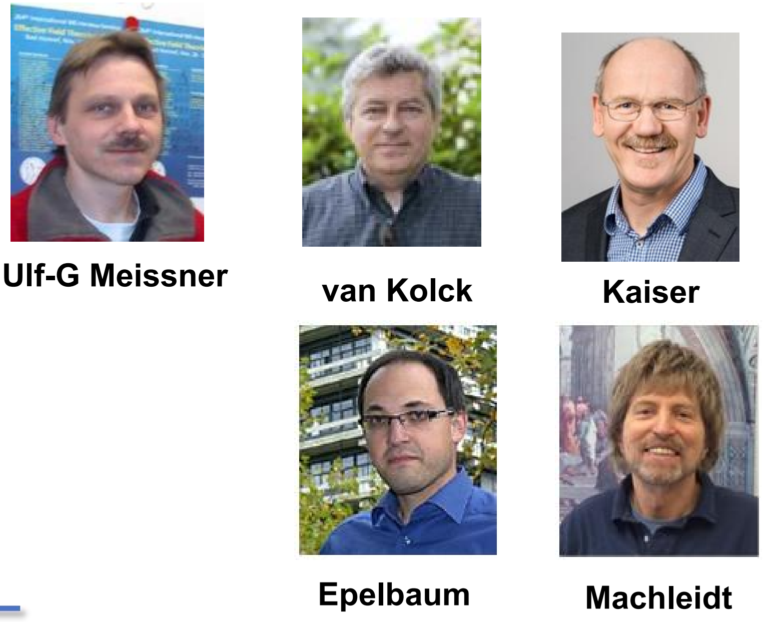
Extended-on-mass-shell (EOMS)

- Covariance
- Faster convergence

Gegelia, Fuchs et al. in **PRD60**(1999), **PRD68** (2003)

Non-relativistic chiral nuclear force





Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

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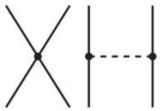
Non-relativistic chiral nuclear force

2NF

LO(Q^0)



S. Weinberg, PLB1990, NPB1990

1990 2N LO

1994 3N LO

TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [38] below 290 MeV by various *np* potentials.

Bin (MeV)	# of data	N^{3}LO[a]	NNLO[b]	NLO[b]	AV18[c]
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

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0–290	2057	1.50	35.4	80.1	1.38

D.R. Entem, R. Machleidt et al. PRC2003

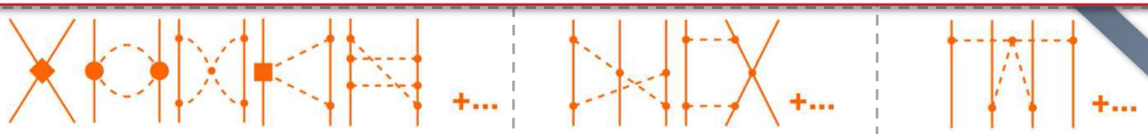
n Kolck

Kaiser

elbaum

Machleidt

N^{3}LO(Q^4)



R. Machleidt et al. PRC2003
E. Epelbaum et al. NPA2005

S. Ishikwas et al. PRC2007
V. Bernard et al. PRC2007

E. Epelbaum EPJA2007

2007 3N N^{3}LO

2015 2N N^{4}LO

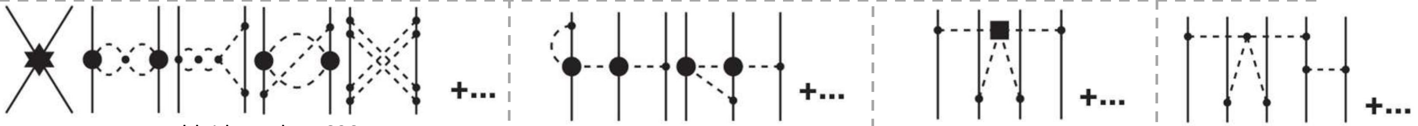
N^{3}LO: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

R. Machleidt et al. PRC2015
E. Epelbaum et al. PRL2015

H. Krebs et al, PRC2012,2013
NOT COMPLETE

5NF

N^{5}LO(Q^6)



R. Machleidt et al. PRC2015
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Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

Non-relativistic chiral nuclear force

2NF

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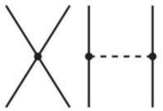

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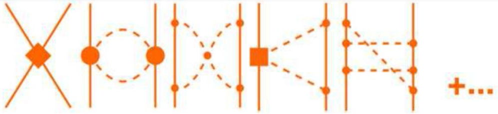
D.R. Entem, R. Machleidt et al. PRC2003

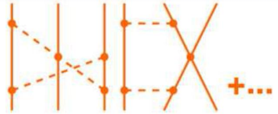
elbaum

Machleidt

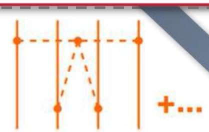
$\mathrm{N^3LO}(Q^4)$



R. Machleidt et al. PRC2003
E. Epelbaum et al. NPA2005



S. Ishikwas et al. PRC2007
V. Bernard et al. PRC2007



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$\mathrm{N^3LO}$: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

R. Machleidt et al. PRC2015

H. Krebs et al. PRC2012,2013

5NF

Convenience for ab-initio calculation

- **Local/non-local/semi-local**
- **Momentum/Coordinate**
- **Optimization**
- **Softening(SRG)**

R. Machleidt et al. PRC2015

NOT COMPLETE

Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

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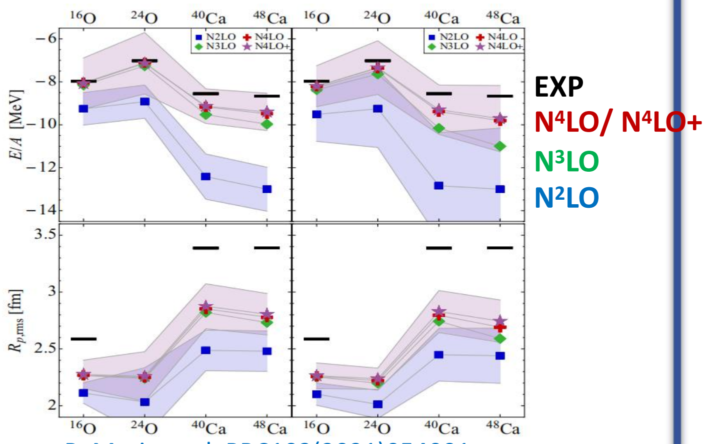
Huge success, but still less satisfying

Slow convergence

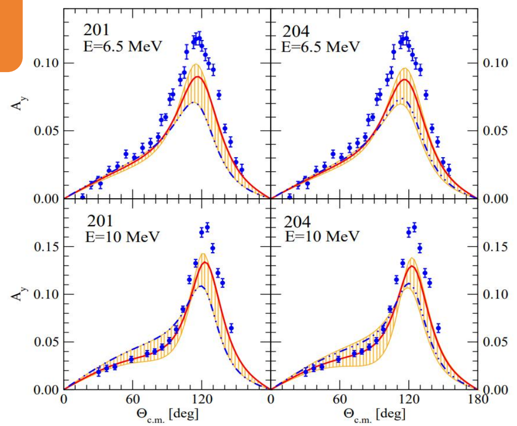
At least N⁴LO: To achieve desired accuracy



P. Maris et al. PRC103(2021)054001

Ay puzzle in n-d scattering

Consistent N³LO:
2N+3N
does not solve the puzzle

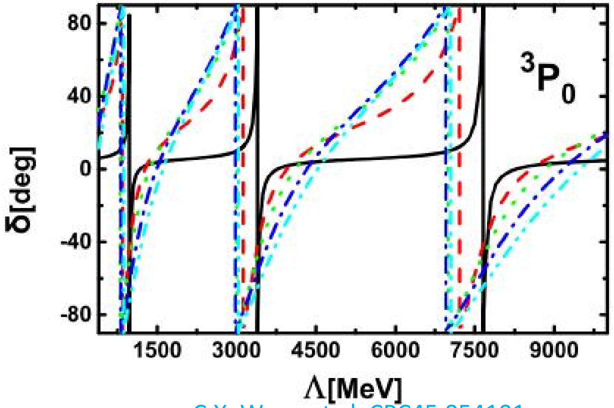


Red: N3LO chiral NN potential
Blue: N3LO chiral NN + full 3N potential

L. Girlanda et al. PRC2019 J. Golak et al. EPJA50,177

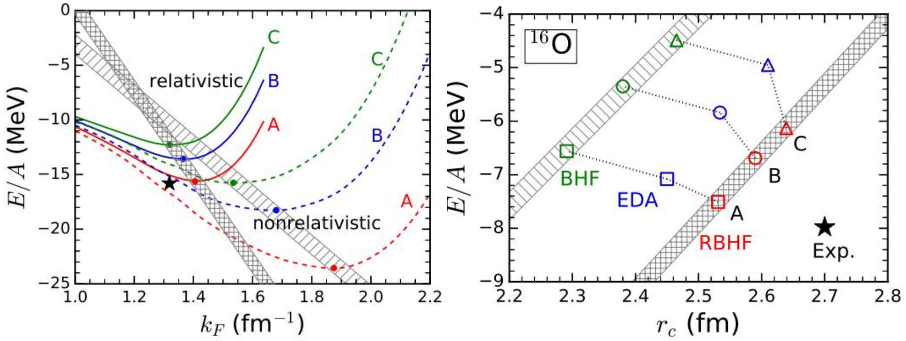
Renormalization group invariance

LS equation is NOT renormalizable when the potentials are truncated up to certain order



C.X. Wang et al. CPC45,054101

Advantages of relativistic ab-initio calculations

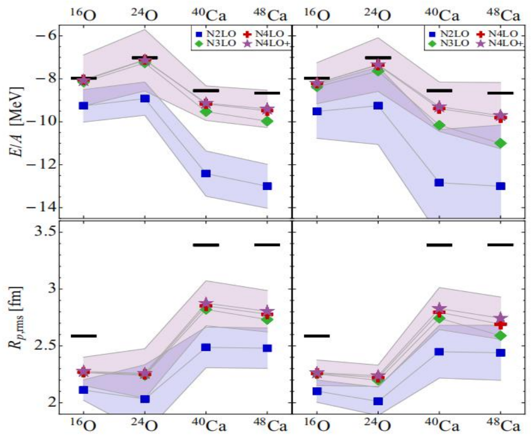


Equation of state of symmetric nuclear matter   Energy per nucleon

□ Huge success, but still less satisfying

Slow convergence

At least N⁴LO:
To achieve desired accuracy



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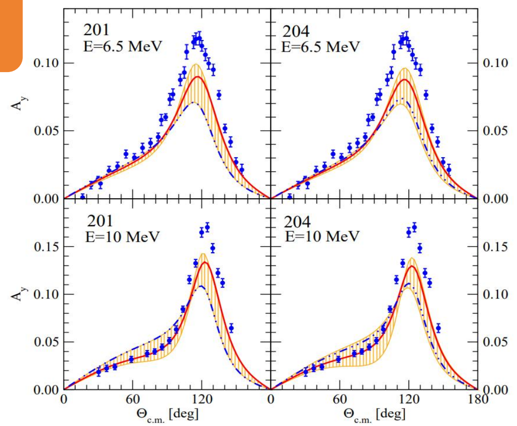

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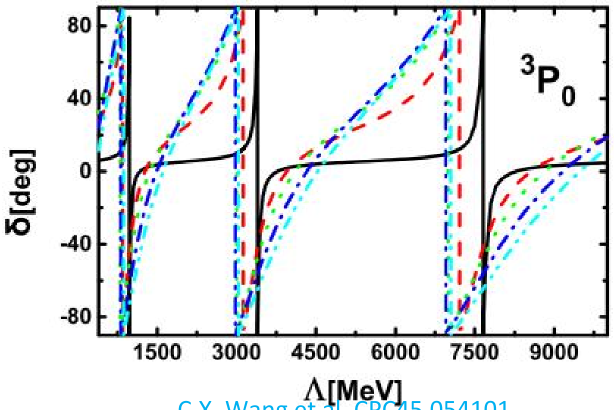

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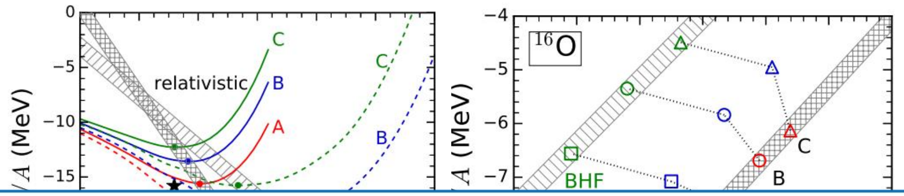

But now, the only available input for relativistic ab-initio calculations —— Bonn potential

Call for accurate relativistic nuclear force!

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- ❑ A brief introduction of nuclear force
- ❑ **Relativistic chiral force: framework and status**
- ❑ Recent development
  - ➢ $\mathrm{N^3LO}$ two-body force
  - ➢ Relativistic 3-body formulism
- ❑ summary and prospect

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Relativistic framework

Dynamically and kinematically

- ✓ Lorentz invariance
- ✓ Faster convergence
- ✓ Improved renormalization group invariance
- ✓ Available for relativistic ab-initio calculations

Chiral effective lagrangian: Dirac Spinor/Clifford algebra

$$u(p,s) = N_p \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m} \end{pmatrix} \chi_s$$

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{O}$	0	1	0	0	0	−	0	1
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+

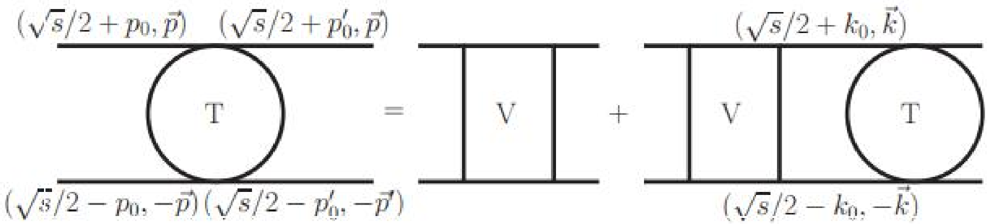
Covariant scheme to restore the broken power counting

Extended-on-mass-shell(EOMS) scheme

$$C_i = C_i^b + C_i^d R + C_i^{\mathrm{PCB}}$$

C_i: LEC C_i^b: the bare value C_i^d: for UV divergence C_i^{PCB}: for PCB terms

Non-perturbative treatment: relativistic scattering equation (Blankenbecler-Sugar equation)



$$\mathcal{T}(p',p|W) = \mathcal{A}(p',p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p',k|W) G(k|W) \mathcal{T}(k,p|W),$$

$$\mathcal{T} = \mathcal{V} + \mathcal{V} g \mathcal{T},$$
$$\mathcal{V} = \mathcal{A} + \mathcal{A}(G-g)\mathcal{V}$$

$$g = \frac{\pi i \delta(k^0) \Lambda_+^1(\boldsymbol{k}) \Lambda_+^2(-\boldsymbol{k})}{2E_k(E_k^2 - s/4 - i\epsilon)}$$

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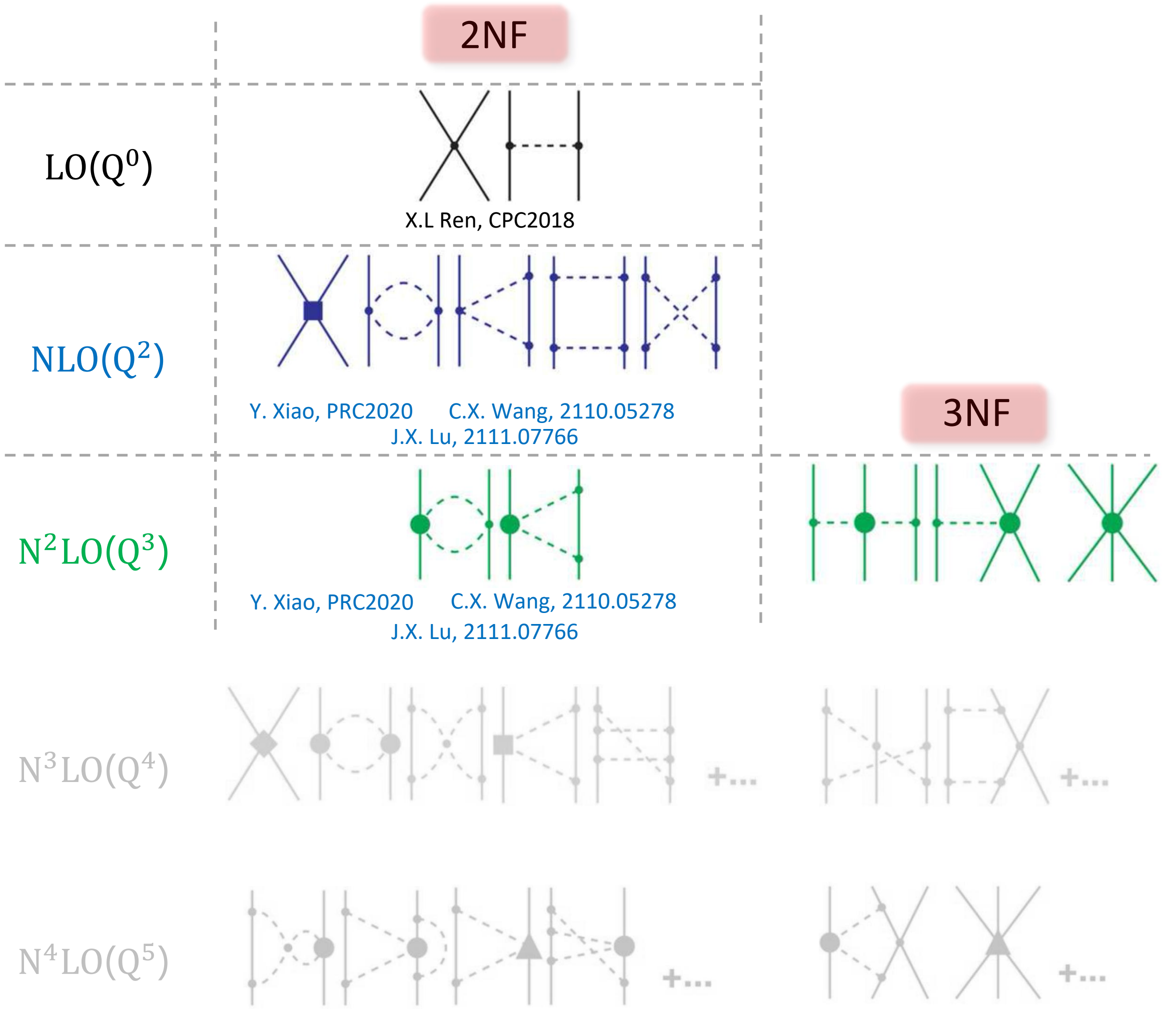
Relativistic chiral nuclear force



Relativistic Nuclear force

2018: ***Leading order*** *relativistic chiral nucleon-nucleon interaction*
X.L Ren et al CPC42,014103

2019: *Covariant chiral nucleon-nucleon contact* ***Lagrangian*** *up to order* $\mathcal{O}(q^4)$
Y. Xiao et al Phys.Rev.C 99,024004

2020: ***Two-pion exchange*** *contributions to the nucleon-nucleon interaction in covariant baryon ChPT*
Y. Xiao et al Phys.Rev.C 102,054001

2021: ***Non-perturbative two-pion exchange*** *contributions to the nucleon-nucleon interaction in covariant baryon ChPT*
C.X. Wang et al Phys.rev.C 105,014003

An accurate *relativistic chiral nucleon-nucleon interaction up to NNLO*
J.X Lu et al Phys.rev.lett. 128,142002

2023: ***Saturation of nuclear matter*** *in the relativistic Brueckner-Hatree-Fock approach with* ***a leading order covariant chiral nuclear force***
W.J. Zou et al. Phys.Lett.B 854 (2024) 138732

2024: ***Antinucleon-nucleon interactions*** *in covariant chiral effective field theory*
Y. Xiao et al. arXiv: 2406.01292

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◻ NNLO 2N np scattering

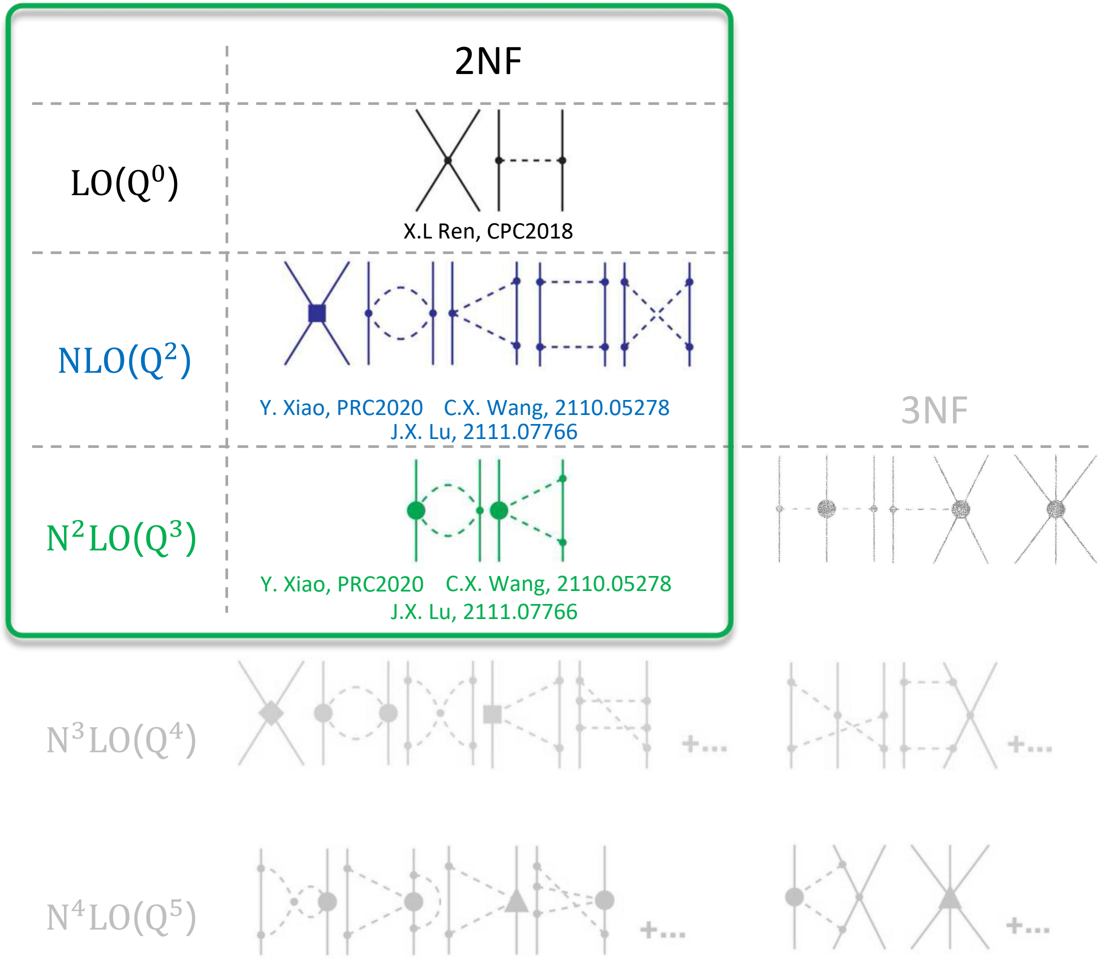
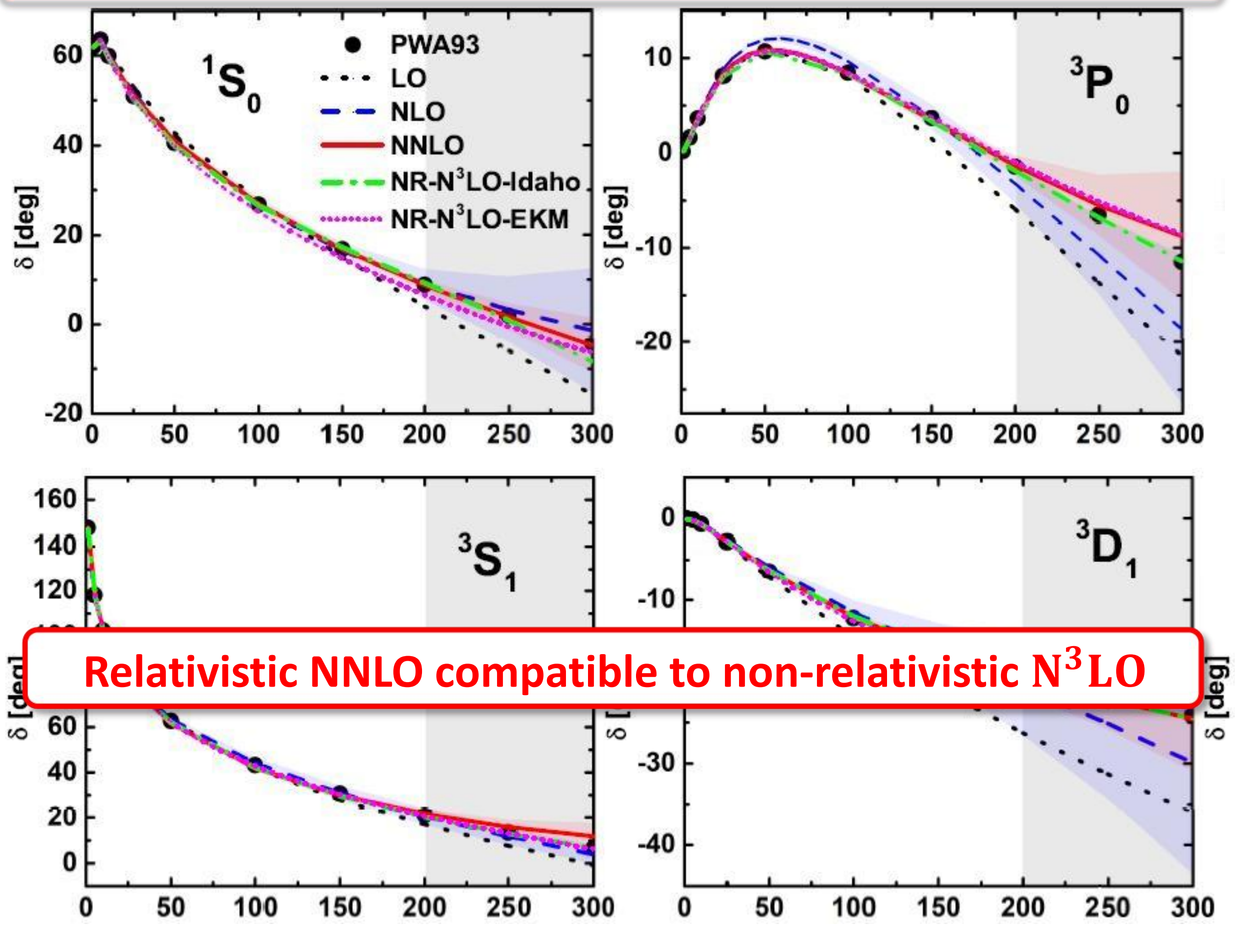


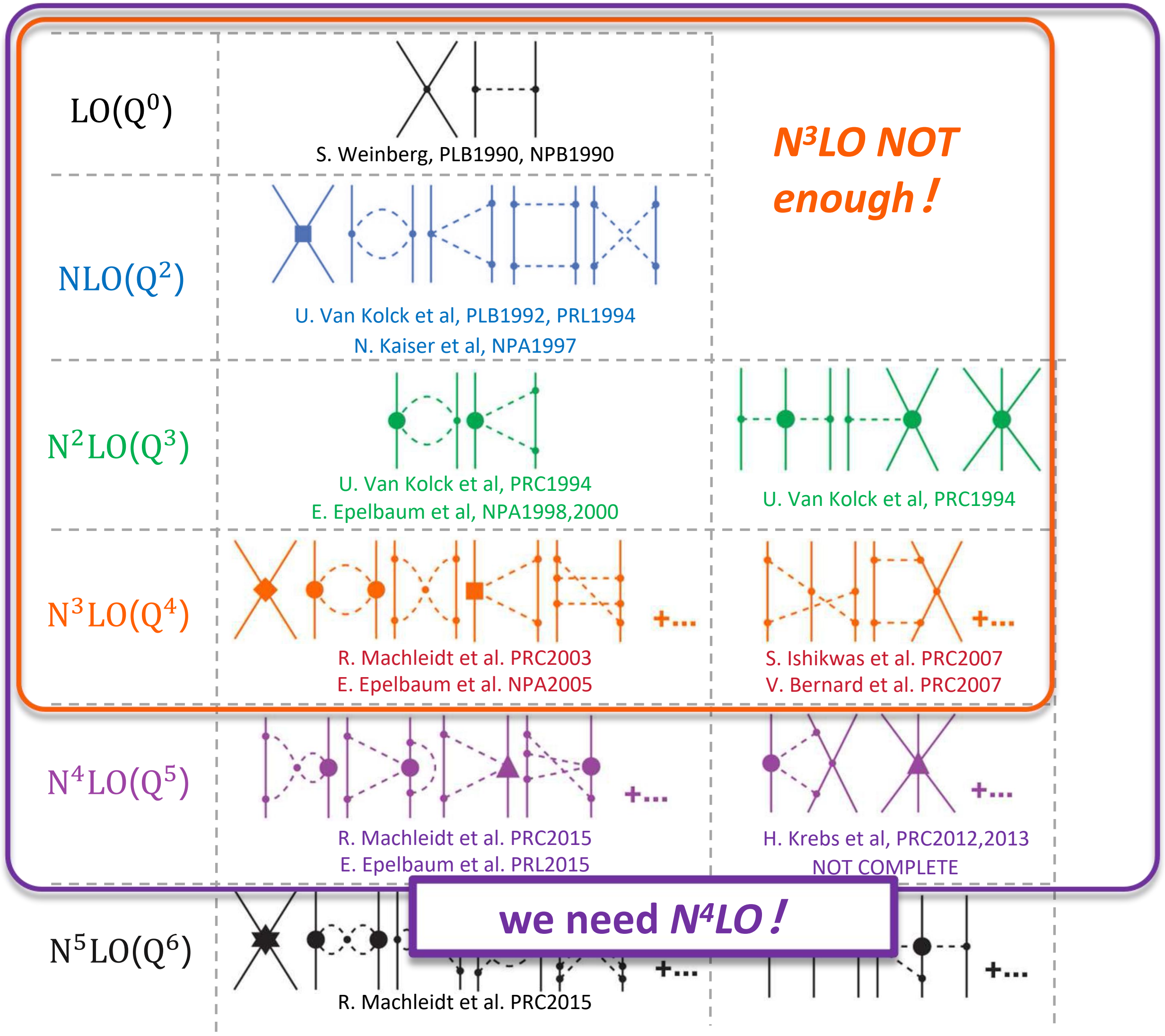
TABLE III. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\mathrm{PWA93}})^2$ of different chiral forces for partial waves up to $J \leq 2$.

	Total	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
NLO	17.02	1.02	7.04	0.46	0.33	1.80	1.69	0.15	2.18	1.35	0.95	0.01	0.04
NNLO	16.61	0.18	0.30	1.07	1.55	3.36	0.26	0.03	0.01	9.56	0.01	0.27	0.01
NR-N^{3}LO-Idaho	8.84	1.53	0.30	2.41	0.04	2.33	1.00	0.02	0.57	0.42	0.17	0.03	0.02
NR-N^{3}LO-EKM	16.08	13.45	0.29	0.34	0.06	0.01	0.13	0.01	0.02	0.43	0.12	1.22	0.00

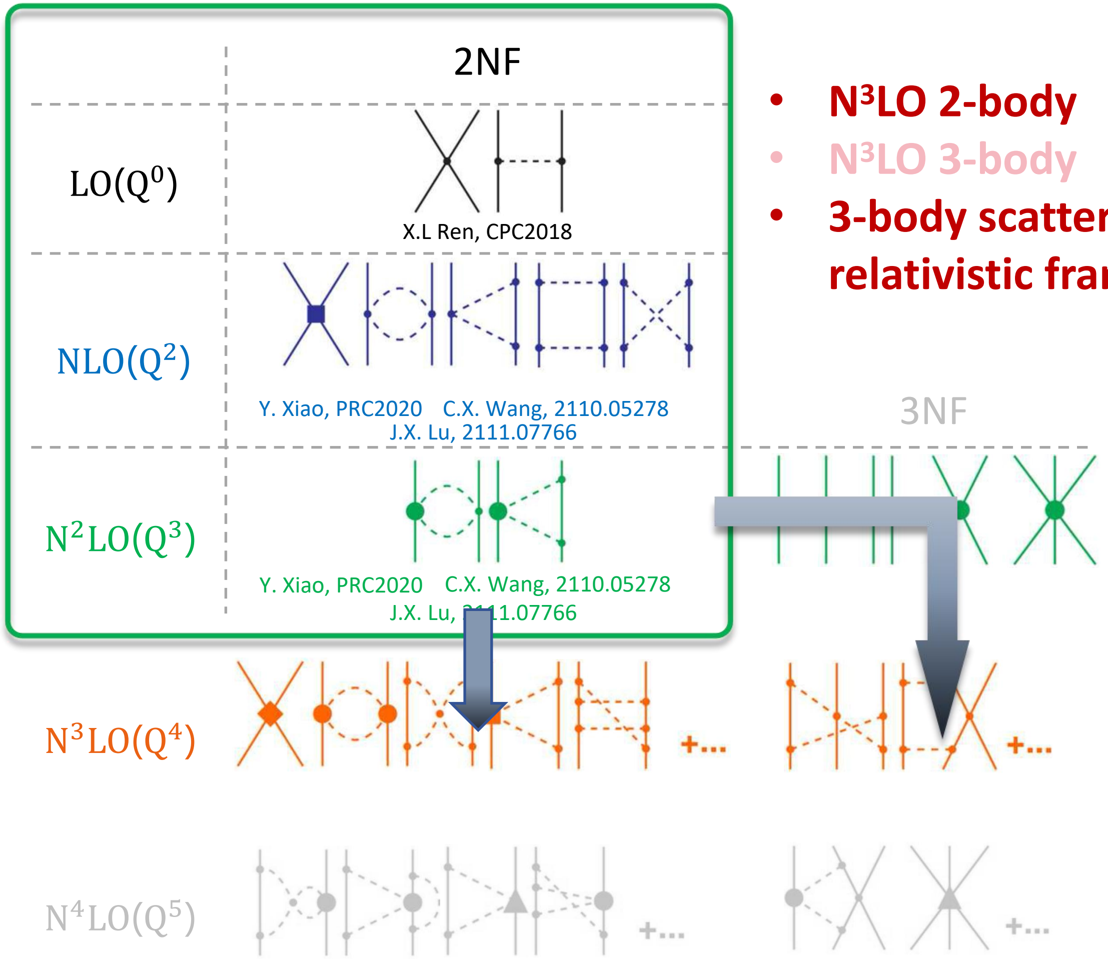


Relativistic NNLO compatible to non-relativistic N^{3}LO

☐NR-ChNF: N³LO not enough!



☐R-ChNF: go to N³LO



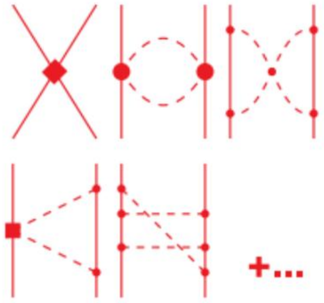
- **N³LO 2-body**
- **N³LO 3-body**
- **3-body scattering in relativistic frame**

R-ChNF-N³LO ongoing !

- A brief introduction of nuclear force
- Relativistic chiral force
- **Recent development**
  - **$\mathrm{N}^3\mathrm{LO}$ two-body force**
  - Relativistic 3-body formulism
- summary and prospect

Relativistic N^3LO 2N force

N^3LO
$(Q/\Lambda_\chi)^4$



- ✓ **Contact term** **(23 terms)**
- ✓ **One Pion** **Renormalization**
- ✓ **Two Pion** **One + Two loop**
- ✓ **Three Pion** **negligible**

$$V_{1\pi} = V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \cdots$$

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$$V_{3\pi} = V_{3\pi}^{(4)} + \cdots,$$

Isospin breaking effect

➢ Contact terms (23 terms)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar\psi\gamma^{\mu}\psi)(\bar\psi\gamma_{\mu}\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}\psi)\partial^2\partial_{\alpha}\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^{\mu}\psi)(\bar\psi\gamma_5\gamma_{\mu}\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^{\beta}\partial_{\nu}\left(\bar\psi\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar\psi\psi)\partial^4(\bar\psi\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar\psi\gamma^{\mu}\psi)\partial^4(\bar\psi\gamma_{\mu}\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^{\mu}\psi)\partial^4(\bar\psi\gamma_5\gamma_{\mu}\psi)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}\psi)\partial^4(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_{\alpha}\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^{\mu}\psi)\partial^2\left(\bar\psi\gamma_{\mu}\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^{\alpha}\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^{\mu}\psi)\partial^2\left(\bar\psi\gamma_5\gamma_{\mu}\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_{\alpha}\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\psi)\partial^2(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_{\nu}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

Y. Xiao et al Phys.Rev.C 99,024004

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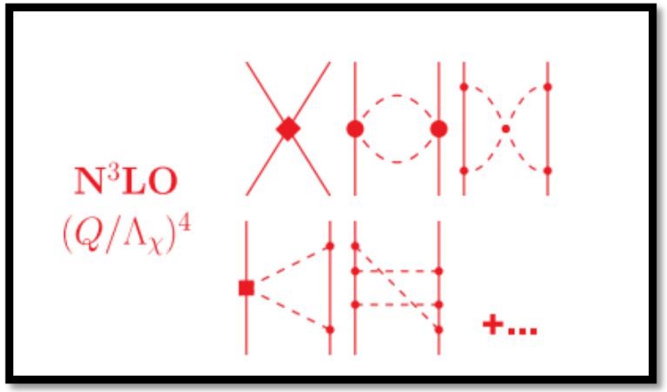

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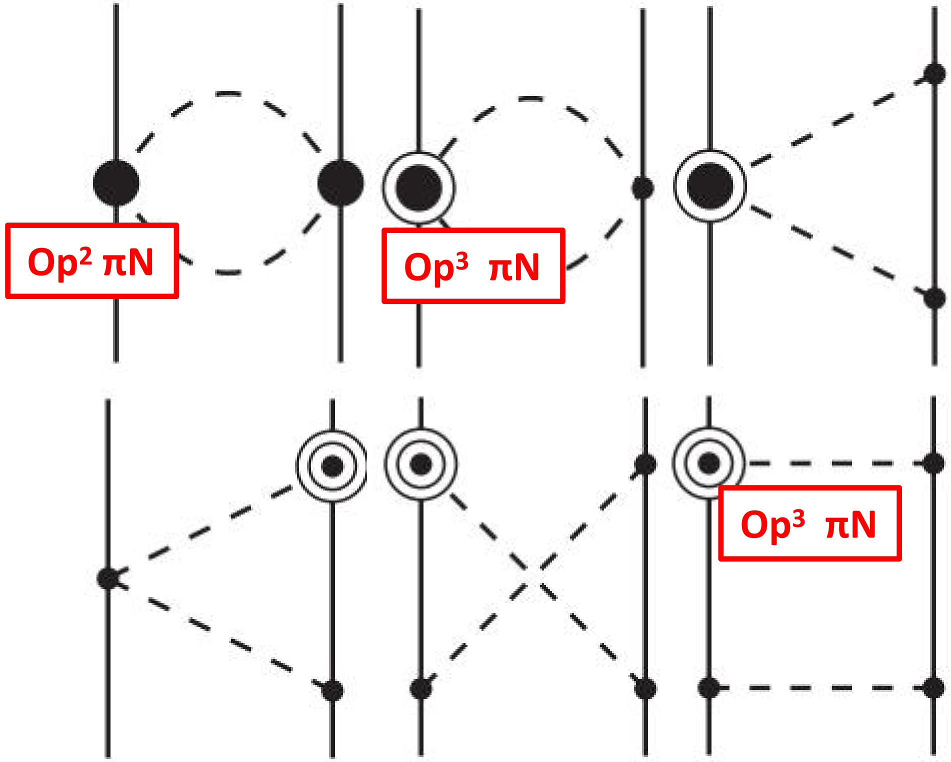
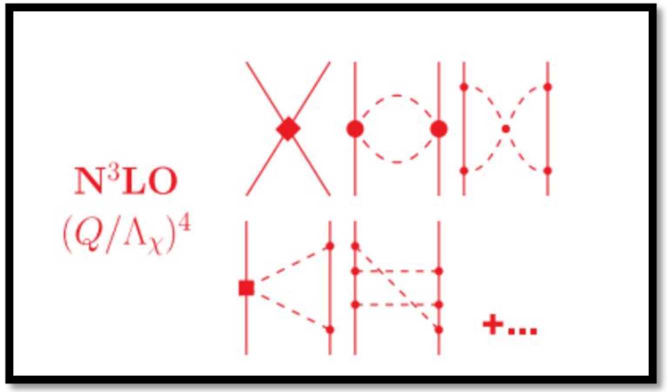
Isospin breaking effect

- TPE (One-loop part)



Relativistic N^3LO 2N force



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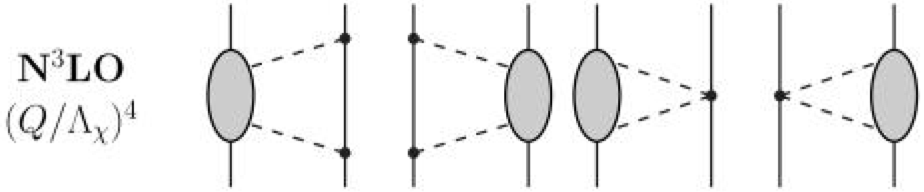
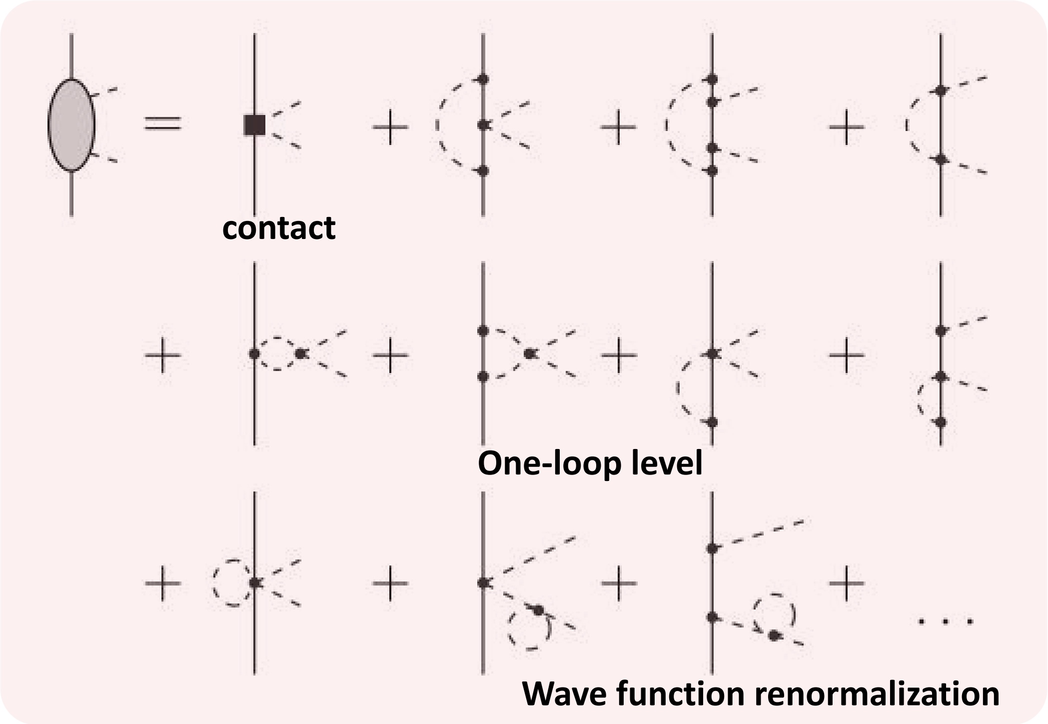
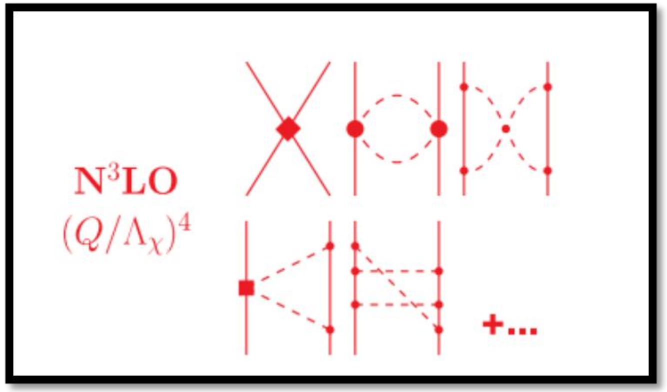
Isospin breaking effect

- TPE (two-loop part)







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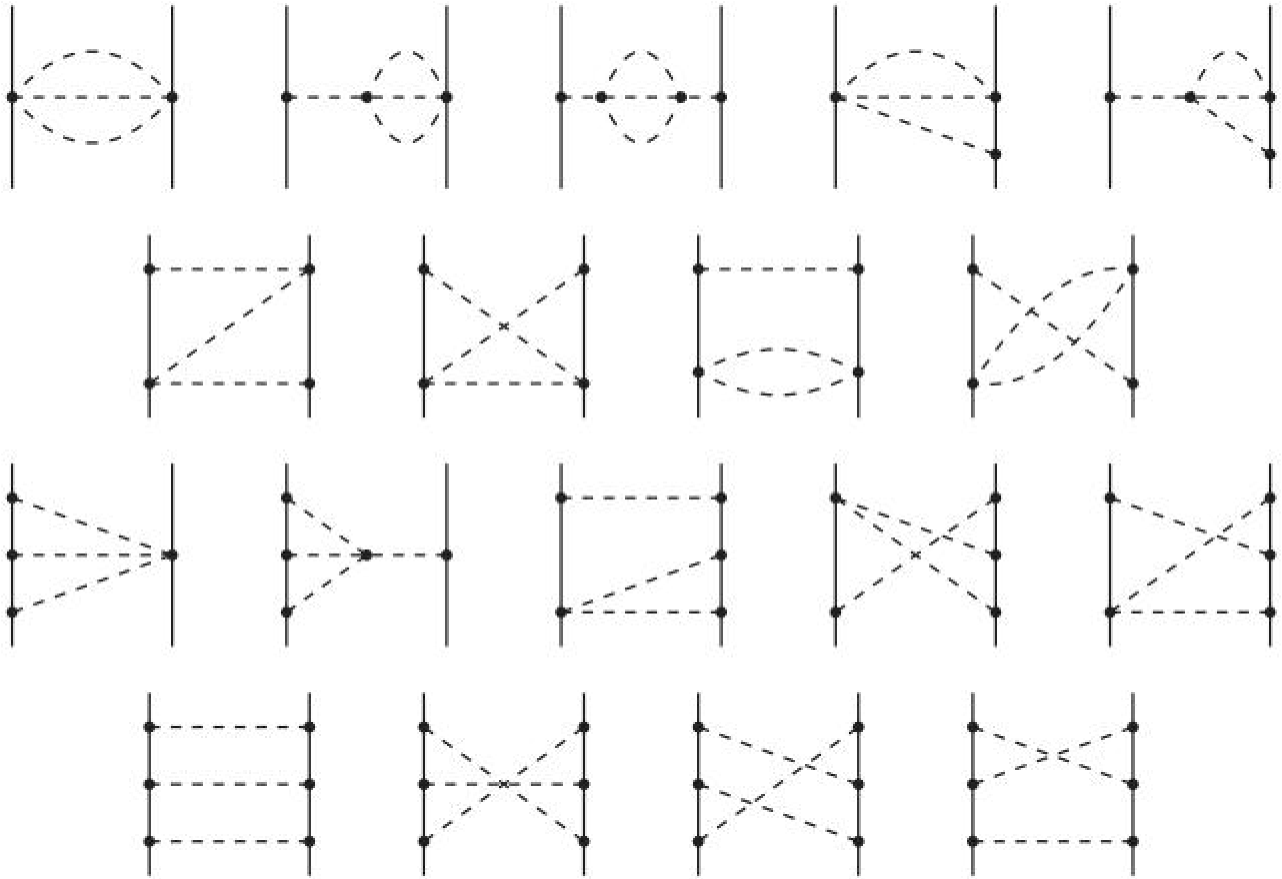
Isospin breaking effect

- Three Pion Exchange



R. Machleidt, D.R. Entem / Physics Reports 503 (2011) 1–75

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Two-loop —— most challenging part in N^{3}LO 2N force

Dimensional regularization

- **Convenient / straightforward**(FeynCalc, FeynArts, PackageX etc.)
- **Full amplitudes**—non-analytical parts and polynomial parts
- The loop momentum—pion momentum are integrated up to infinity
- Extremely complicated for two-loop, especially in NF involving massive fermions

So far, the two-loop contributions to NF with the DR method remain untouched

Spectral representation

- **Analytical structure & dispersion relation**—— Mandelstam representation
- Mostly **only** non-analytical parts due to subtraction
- Convenient to introduce cutoffs to the loop momentum
- From the experience of NR, it should be applicable for two-loops in the relativistic case as well.

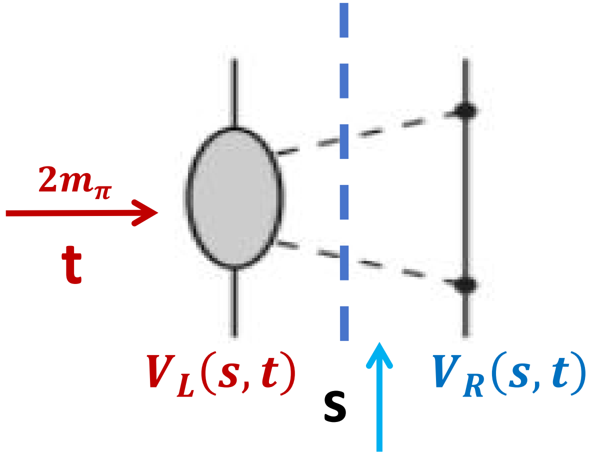
It has become the common choice in the newly developed higher-order NR nuclear forces

- **Semi-local N^{4}LO NR-NF** Reinert et al. *Eur.Phys.J.A* 54, 86 (2018)
- **Local position-space N^{3}LO NR-NF** Saha et al. *Phys.Rev.C* 107, 034002 (2023)

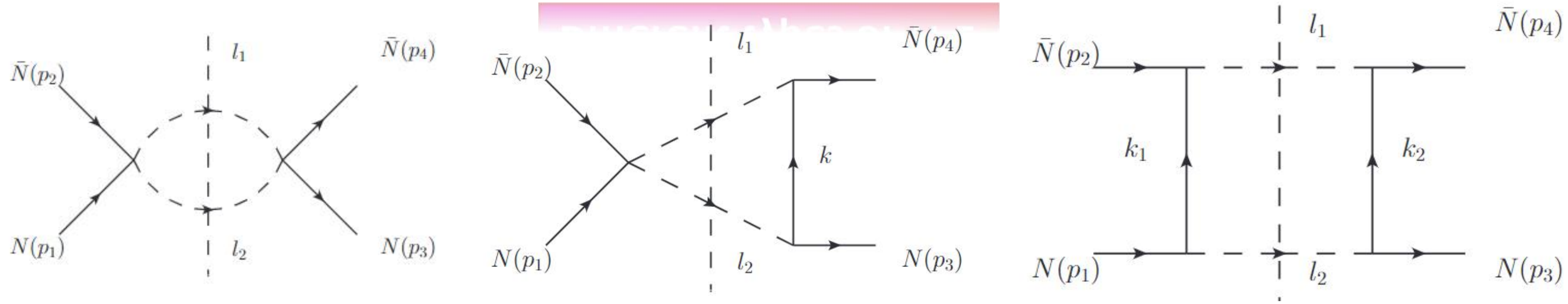
The spectral representation for loop diagrams

- **t-channel discontinuities in the multi-pion exchange diagrams—the Cutkosky rule**

  Equivalent to the s channel cut in $\mathbf{N\bar{N} \to \pi\pi \to N\bar{N}}$



Different types of TPE



$$\mathrm{Disc}[\mathcal{A}(s,t)] = i\int d^4l \frac{V_L(s,t)V_R(s,t)}{(2\pi)^4}(-2\pi i)^2\delta(l^2-m_\pi^2)\delta((l-q)^2-m_\pi^2)$$

$V_L(s,t)$, $V_R(s,t)$ are **πN amplitudes** at $\mathcal{O}(p^3)$ or $\mathcal{O}(p^1)$

- **insert into the dispersion relations**

$$\mathrm{Re}[\mathcal{A}(s,t)] = \frac{1}{2\pi i}\int d\mu^2 \frac{\boxed{\mathrm{Disc}[\mathcal{A}(s,\mu^2)]}}{\mu^2-t}$$

spectral function

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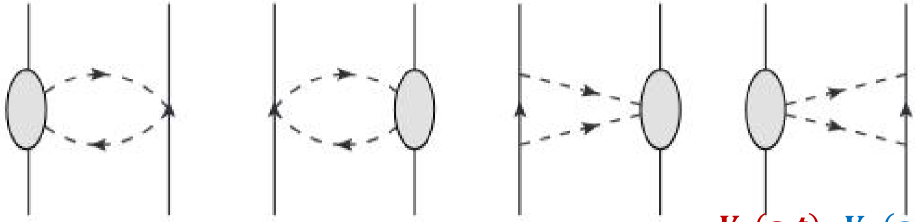
☐ Non-relativistic cases

N. Kaiser, Phys.Rev.C64,057001(2001)



- non-relativistic **πN amplitudes** $\left(\pi^a(q) + N(p) \rightarrow \pi^b(q') + N(p')\right)$: concise in the heavy static limit

$$T_{\pi N}^{ba} = \delta^{ba}\left[g^+(\omega,t) + i\vec{\sigma}\cdot(\vec{q}\,'\times\vec{q})\,h^+(\omega,t)\right] + i\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot(\vec{q}\,'\times\vec{q})\,h^-(\omega,t)\right]$$

$g^{\pm}(\omega,t)$ isoscalar/isovector non-spin-flip amplitude $h^{\pm}(\omega,t)$ isoscalar/isovector spin-flip amplitude **ω** the pion cms energy

e.g. $$g^+(\omega,t)_{\rm loop} = \frac{g_A^2}{32\pi F_\pi^4}\left\{-\frac{4\omega^2}{g_A^2}\sqrt{M_\pi^2-\omega^2} + (M_\pi^2-2t)\left[M_\pi + \frac{2M_\pi^2-t}{2\sqrt{-t}}\arctan\frac{\sqrt{-t}}{2M_\pi}\right] + \frac{4g_A^2}{3\omega^2}(2\omega^2+t-2M_\pi^2)\left[(M_\pi^2-\omega^2)^{3/2}-M_\pi^3\right]\right\}$$

Bernard et al. Nucl.Phys.A615,483(1997)

- **two-loop NN** amplitude (central part V_c as an example):

$$V_c(q) = -\frac{2q^6}{\pi}\int_{2m_\pi}^{\infty} d\mu \frac{\mathrm{Im}\,V_c(i\mu)}{\mu^5(\mu^2+q^2)} \qquad \mathrm{Im}\,V_c(i\mu) \propto \mathrm{Re}\; g^+(\omega,t)$$

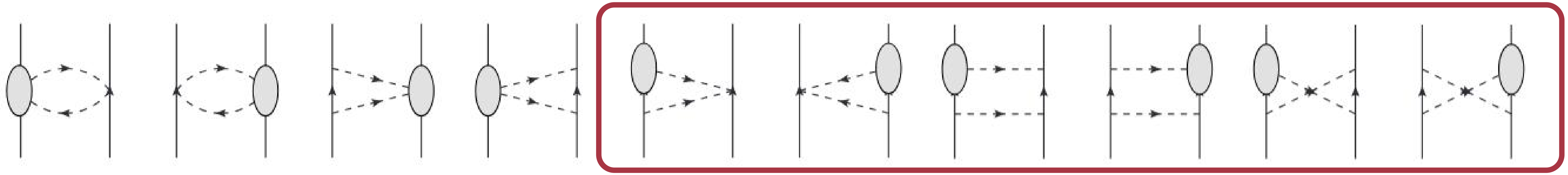
- **Take $\omega \rightarrow 0$ and $t = \mu^2$**

$$\mathrm{Im}\,V_C(i\mu) = \frac{3g_A^4(\mu^2-2m_\pi^2)}{\pi\mu(4f_\pi)^6}\boxed{\left\{(m_\pi^2-2\mu^2)\left[2m_\pi + \frac{2m_\pi^2-\mu^2}{2\mu}\ln\frac{\mu+2m_\pi}{\mu-2m_\pi}\right] + 4g_A^2 m_\pi(2m_\pi^2-\mu^2)\right\}}$$

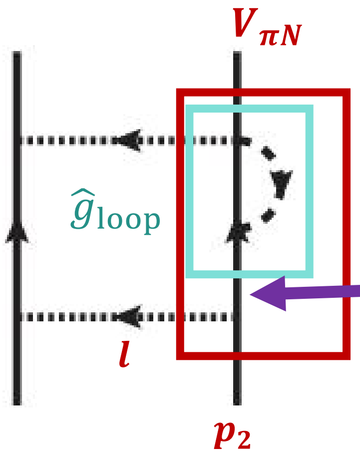
- similarly $V_{S,T}$ and $W_{C,S,T}$

relativistic framework

- more complicated



- **Totally 10 categories, 88 feynman diagrams**
- relativistic **πN amplitudes still contain nucleon propagators**



relativistic πN amplitude

$$V_{\pi\mathrm{N}} = \hat{g}_{\text{loop}} \frac{1}{(p_2 - l)^2 - m_N^2}$$

πN amplitude at heavy static limit

$$p^\mu = m\, v^\mu + k^\mu \qquad v^2 = 1$$

$$(p_2 - l)^2 - m_N^2 \sim 2\, m_N [v \cdot (p_2 - l) - m_N] \sim \omega$$

- Contain **pole-type terms, thus cannot be simply treated like a vertex**
- **Double spectral function is necessary for box-like diagrams**

$$T_G(s,t) = \frac{1}{\pi}\int_{(m_1+m_2)^2}^{\infty} ds' \frac{\Delta_s T_G(s',t)}{s'-s} = \frac{1}{\pi^2}\iint ds'\, dt' \frac{\rho_{st}(s',t')}{(s'-s)(t'-t)}$$

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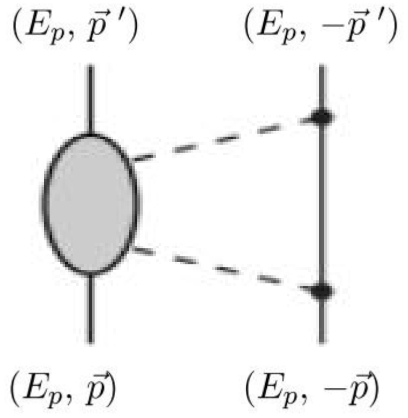
Preliminary results for two-loop diagrams



In general, the **relativistic amplitudes** can be expressed as

$$\mathcal{A} = \text{Loop integral} \times \text{Bilinear}$$

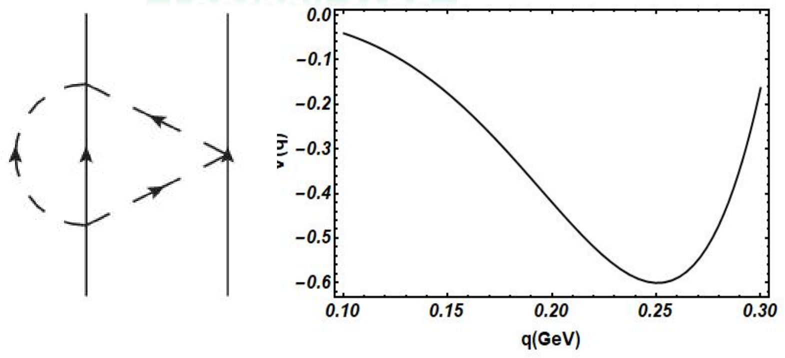
- Bilinear

$$\bar{u}_3 u_1 \cdot \bar{u}_4 u_2 \qquad \bar{u}_3 \not{p}_i u_1 \cdot \bar{u}_4 u_2 \qquad \bar{u}_3 \gamma^\mu u_1 \cdot \bar{u}_4 \gamma_\mu u_2$$

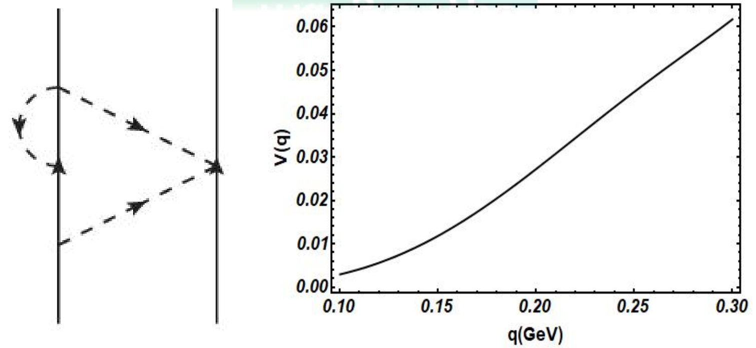
- Loop integral

$$\mathrm{Re}[\mathcal{A}(q)] = \frac{1}{2\pi} \int d\mu^2 \frac{\mathrm{Disc}[\mathcal{A}(\mu^2)]}{\mu^2 + q^2}$$

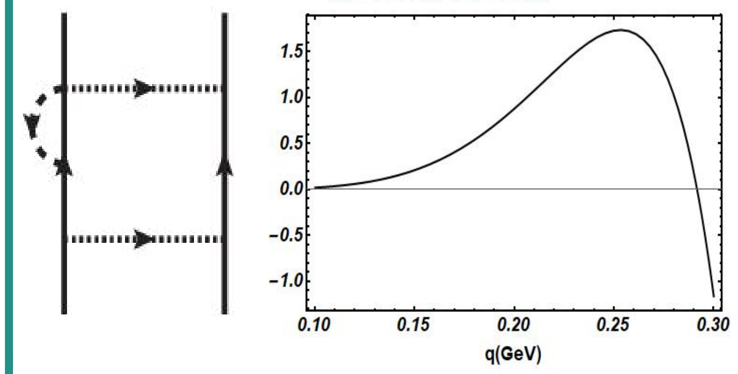
Football-type



Triangle-type



Box-type



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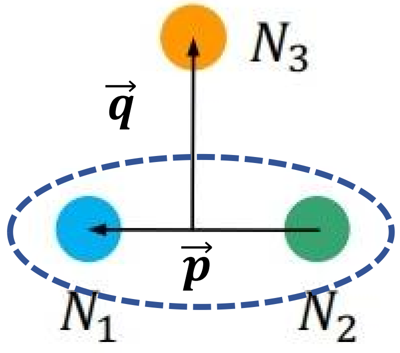
- A brief introduction of nuclear force
- Relativistic chiral force
- **Recent development**
  - $\mathrm{N^3LO}$ two-body force
  - **Relativistic 3-body formulism**
- summary and prospect

The standard scheme for rigorously solving the three-body problem——Faddeev equation

- **Empirically, needed in reproducing triton binding energies & nuclear matter saturation in the NR studies.**
- **Theoretically**, self-consistency from power counting, **naturally emerges when integrating out high-momentum modes**



$$t_i = v_i + v_i G_0 t_i$$

$$\langle \vec{p}_i \vec{q}_i | t_i(E) | \vec{p}_i{}' \vec{q}_i{}' \rangle = \delta(\vec{q}_i - \vec{q}_i{}') \langle \vec{p}_i | t_i(E - q_i^2/2\tilde{\mu}_i) | \vec{p}_i{}' \rangle$$

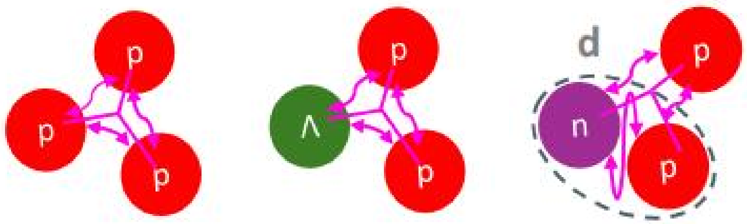
$$\begin{pmatrix} T^{(i)} \\ T^{(j)} \\ T^{(k)} \end{pmatrix} = \begin{pmatrix} t_i \\ t_j \\ t_k \end{pmatrix} + \begin{pmatrix} 0 & t_i & t_i \\ t_j & 0 & t_j \\ t_k & t_k & 0 \end{pmatrix} G_0 \begin{pmatrix} T^{(i)} \\ T^{(j)} \\ T^{(k)} \end{pmatrix}$$

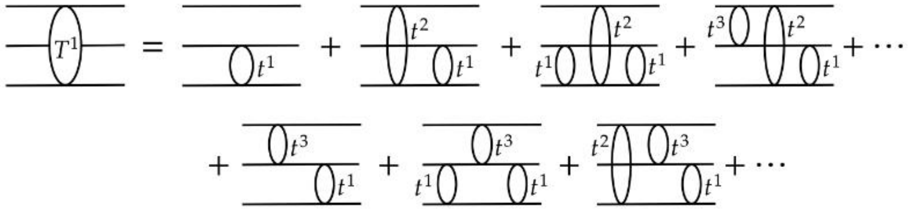
L.D. Faddeev
23 Mar. 1934 – 26 Feb. 2017

Faddeev L D. *Journal of Experimental and Theoretical Physics*, 1961, 12(5): 1014-1019

Ideal relativistic 3-body formulism

2B+3B forces + **(relativistic) 3-body scattering equation**





no complete relativistic 2N+3N forces

no relativistic Faddeev equation

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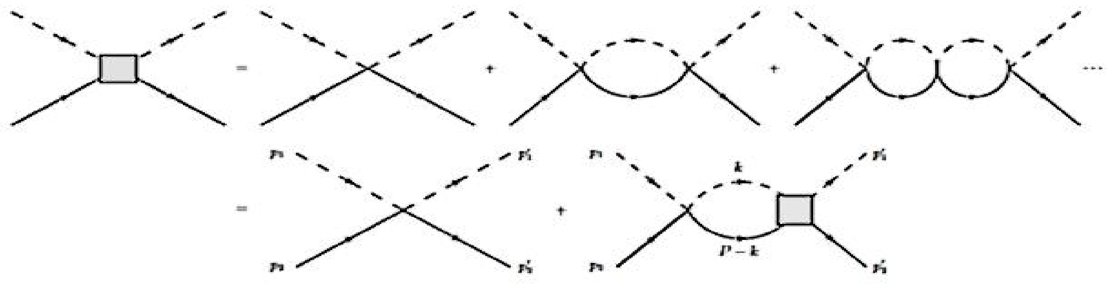
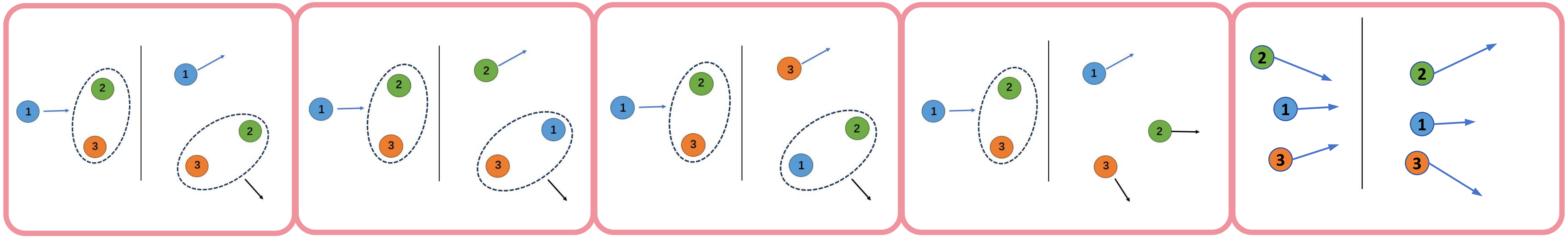
2-body rescattering process——**Bethe-Salpeter equation (BSE)**



$$T = \mathcal{V} + \mathcal{V}\mathfrak{g}T$$

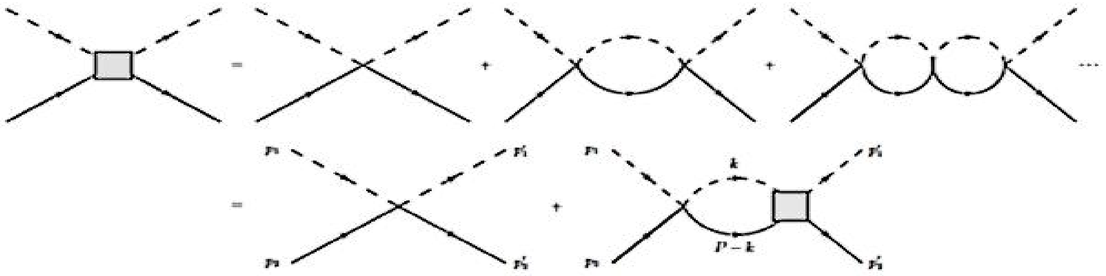
- $\mathcal{V}$: all irreducible connected Feynman diagrams
- $\mathfrak{g}$: relativistic propagators of nucleons

3-body rescattering process——**Faddeev equation**



elastic scattering $2 \to 2$	rearrangement type **I** $2 \to 2$	rearrangement type **II** $2 \to 2$	breakup process $2 \to 3$	true 3-body elastic scattering $3 \to 3$

2-body rescattering process——Bethe-Salpeter equation (BSE)

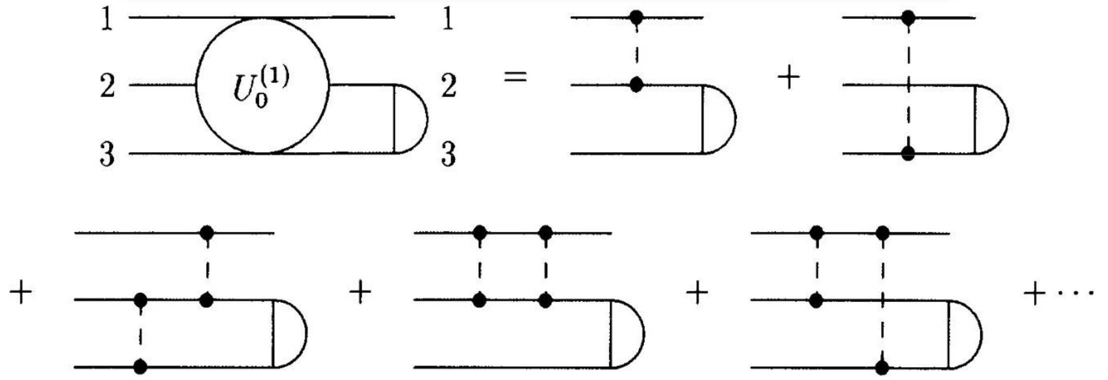


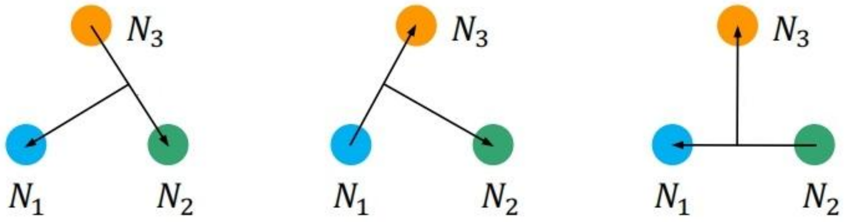
$$T = \mathcal{V} + \mathcal{V}\mathrm{g}T$$

- $\mathcal{V}$: all irreducible connected Feynman diagrams
- g: relativistic propagators of nucleons

3-body rescattering process——Faddeev equation

$N + d \to N + n + p$ breakup process





$$U = Pv + PtG_0U$$

- P: **permutation operator**

$$P = P_{12}P_{23} + P_{13}P_{23}$$

$$\langle k_1(k_2k_3)|P_{12}P_{23}|k_1(k_2k_3)\rangle = \langle k_1(k_2k_3)|k_2(k_3k_1)\rangle$$

- v: all irreducible connected Feynman diagrams (**2N+3N**)
- G_0 : **3-nucleon propagator**

Relativistic 3-body formulism

□ Faddeev equation: a tough task

$$
\langle pq\alpha|T|\phi\rangle = \langle pq\alpha|tP|\phi\rangle + \sum_{\alpha'}\sum_{\alpha''}\int_0^{\infty} dq'\, q'^2 \int_{-1}^{+1} dx\, \frac{\tilde{t}_{\bar{\alpha}\bar{\alpha}'}(p,\pi_1,E-(3/4m)q^2)}{\pi_1^{l'}} \times G_{\alpha'\alpha''}(qq'x)\, \frac{1}{E+i\epsilon-q^2/m-q'^2/m-qq'x/m}\, \frac{\langle \pi_2 q'\alpha''|T|\phi\rangle}{\pi_2^{l''}}
$$

Walter Gloeckle et al. *Phys. Rept.* 274 (1996) 107-285

special and complex interpolation due to the permutation operator

logarithmic singularities due to the 3-nucleon propagator

□ Wave-packet continuum discretization (WPCD)

$$
\langle A|B\rangle \cong \sum_{n=1} \int_{p_{n-1}}^{p_n} p^2 dp\, \langle A|p\rangle \langle p|B\rangle
$$

$$
\cong \sum_{n=1} \frac{1}{p_n - p_{n-1}} \left[\int_{p_{n-1}}^{p_n} p dp\, \langle A|p\rangle\right]\left[\int_{p_{n-1}}^{p_n} p dp\, \langle p|B\rangle\right]
$$

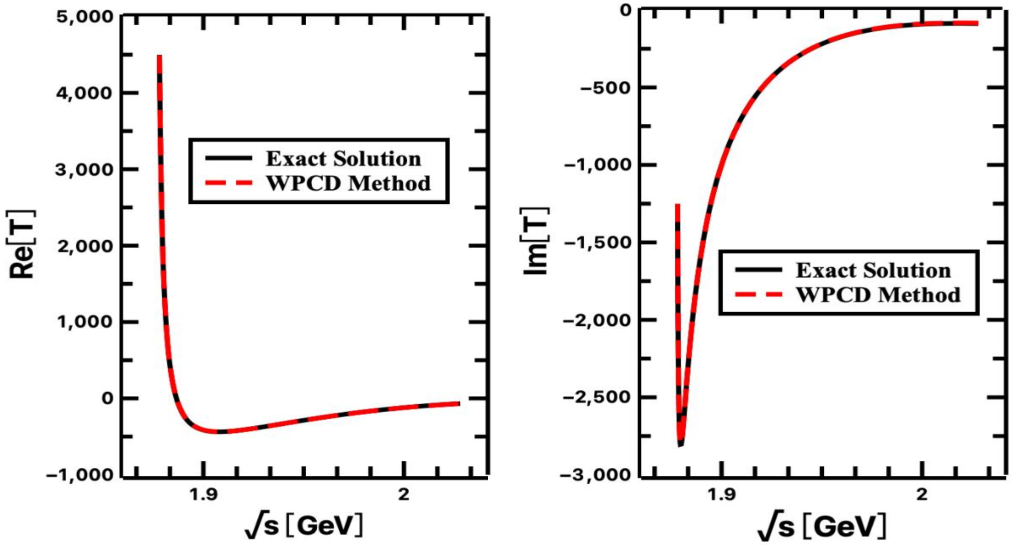
$$
= \sum_{n=1} \langle A|x[n]\rangle \langle x[n]|B\rangle
$$

Wave-packet basis $|x_i\rangle = \dfrac{1}{\sqrt{\Delta p_i}} \displaystyle\int_{p_{i-1}}^{p_i} p dp |p\rangle$

O.A. Rubtsova et al. 1501.02531

Sean B S Miller et al. 2002 *J. Phys. G: Nucl. Part. Phys.* 49 024001

When applied to the **two-body problem**, it almost **coincides with the exact solutions.**



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□permutation matrix in relativistic framework——the most challenging part

- **non-relativisitc situation** momentum part

3-body scattering basis $|pq\gamma\rangle = |\boldsymbol{pq}(\boldsymbol{ls})\boldsymbol{j}(\boldsymbol{\lambda s_1})\boldsymbol{I}(\boldsymbol{jI})\boldsymbol{JMtt_1T\tau_T}\rangle$

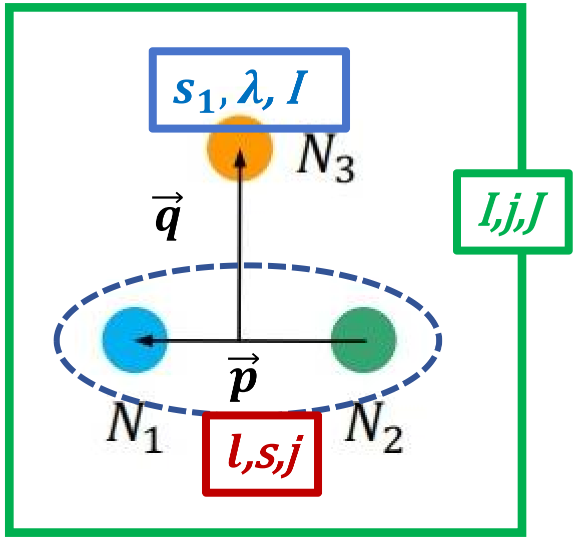
angular momentum & isospin part

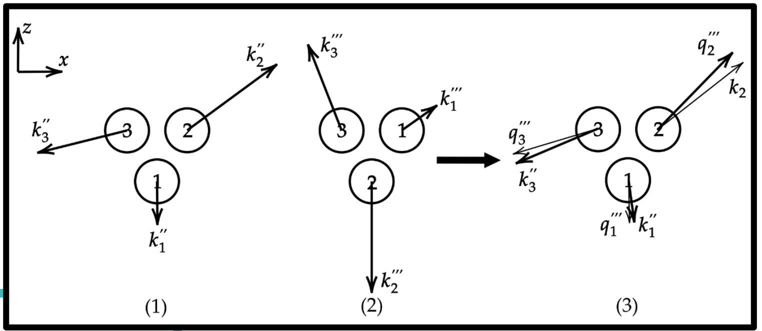
$$\langle pq(l\lambda)LM_L|P_{12}P_{23}|p'q'(l'\lambda')L'M_L\rangle$$
$$= \int d\vec{p}_1 d\vec{q}_1 d\vec{p}_1{}' d\vec{q}_1{}' \langle pq(l\lambda)LM_L|\vec{p}_1\vec{q}_1\rangle \langle \vec{p}_1\vec{q}_1|P_{12}P_{23}|\vec{p}_1{}'\vec{q}_1{}'\rangle \langle \vec{p}_1{}'\vec{q}_1{}'|p'q'(l'\lambda')L'M_L{}'\rangle$$

- **Relativisitc situation**
  - **Dirac spinor / Clifford algebra**
  - **Galilean transformation → Lorentz boost**
  - Helicity basis

$$|qJM, pjm, \lambda_1(\lambda_2\lambda_3)\rangle = \eta_J\eta_j \int \mathrm{d}S \int_0^\pi \mathrm{d}\tilde{\theta} \sin\tilde{\theta}\, \mathcal{D}^{(J)*}_{M,m-\lambda_1}(S) d^j_{m,\lambda_2-\lambda_3}(\tilde{\theta})$$
$$\times R_S \left\{|(q,\pi,\pi),\lambda_1\rangle \otimes Z_q R_{0,\tilde{\theta},0}|(p,0,0),\lambda_2\lambda_3\rangle\right\}$$

$$\langle p_m q_i \gamma'|P|p_n q_j \gamma\rangle$$
$$= N(m,i,n,j)\sum[m',\lambda_1',\lambda_2',\lambda_3',m,\lambda_1,\lambda_2,\lambda_3] \int_{p_{m-1}}^{p_m} p\mathrm{d}p \int_{p_{n-1}}^{p_n} p'\mathrm{d}p' \int_{q_{i-1}}^{q_i} q\mathrm{d}q \int_{q_{j-1}}^{q_j} q'\mathrm{d}q' \int_0^\pi \mathrm{d}\chi \sin\chi$$
$$\sqrt{\frac{pp'qq'}{E_pE_{p'}E_qE_{q'}}} C^{hh'}_{\gamma\gamma'} \frac{\sqrt{E^2+p_0'^2-q^2}}{p_0'^2 E} \frac{\sqrt{E^2+p_0^2-q'^2}}{p_0^2 E} \Big[\mathcal{D}^{(J)*}_{m'-\lambda_1',m-\lambda_2}(\pi,\chi,\pi) d^{j'}_{m',\lambda_2'-\lambda_3'}\left(\tilde{\theta}_0'^{(1)}\right) d^j_{m,\lambda_3-\lambda_1}\left(\tilde{\theta}_0^{(1)}\right)\mathcal{U}_{123}+$$
$$\mathcal{D}^{(J)*}_{m'-\lambda_1',m-\lambda_3}(0,\chi,0) d^{j'}_{m',\lambda_2'-\lambda_3'}\left(\tilde{\theta}_0'^{(2)}\right) d^j_{m,\lambda_1-\lambda_2}\left(\tilde{\theta}_0^{(2)}\right)\mathcal{U}_{132}\Big]\delta(p-p_0)\delta(p'-p_0')$$





dimensions $(N_{\rm in} \times N_{\rm in} \times N_{cc})^2$

- $N_{\rm in}$: number of interpolation for p,q
- N_{cc}: number of coupled channels
- **sparse matrix**

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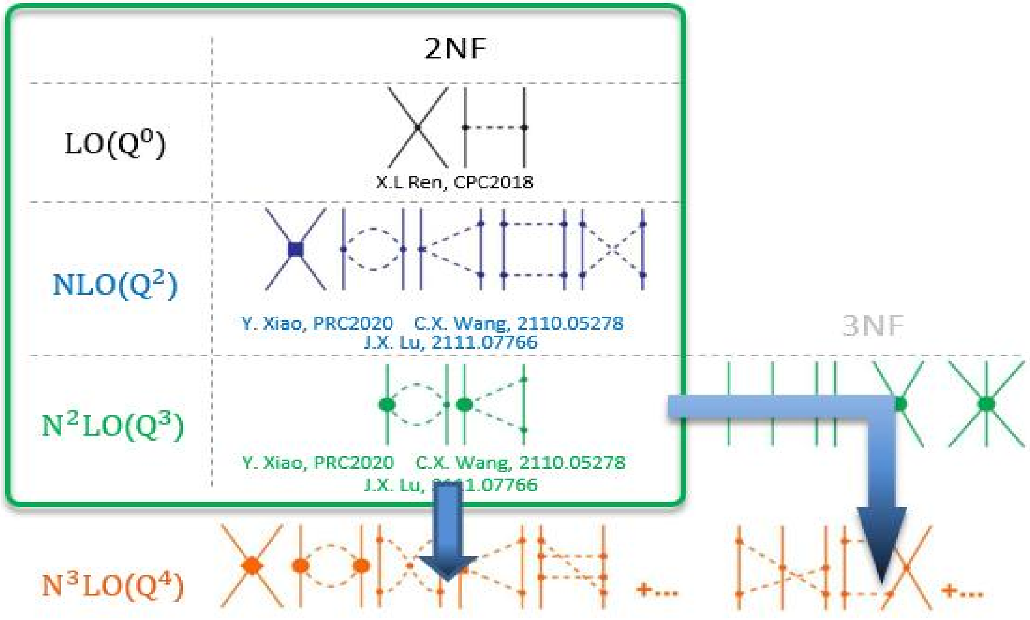
- the current relativistic chiral nuclear force
  - We maintain the relativity **both dynamically and kinematically**
    - **covariant lagrangian**
    - **covariant power counting**
    - **relativistic scattering equation**
  - For 2-body scattering, it is compatible with **non-relativistic N^3LO results** for lower energies.
    - **Lorentz invariance**
    - **Fewer parameters**
    - **Lower chiral order**

- ***R-ChNF-N³LO ongoing !***



Thanks for your attention

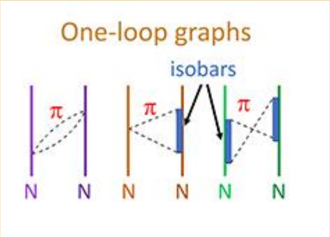
PHYSICAL REVIEW C 50th Anniversary **Milestone papers(41)**

“remain central to developments in the field of nuclear physics”

PHYSICAL REVIEW C 50 YEARS

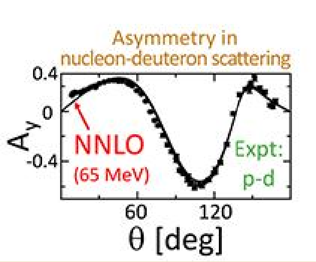
Chiral EFT

Chiral EFT sparked a renaissance in low-energy nuclear theory over the past quarter of a century



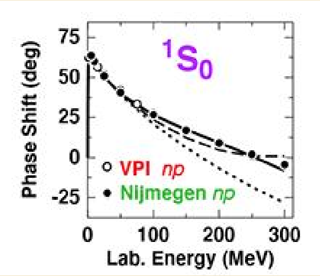
C.Ordóñez, L.Ray, and U.van Kolck

Phys. Rev. C 53, 2086 (1996)



E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, Ulf-G. Meißner, and H. Witała

Phys. Rev. C 66, 064001 (2002)

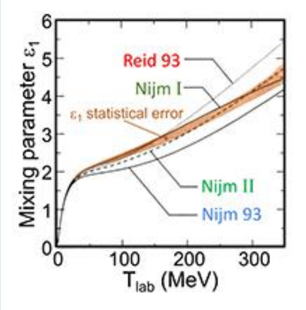


D. R. Entem and R. Machleidt

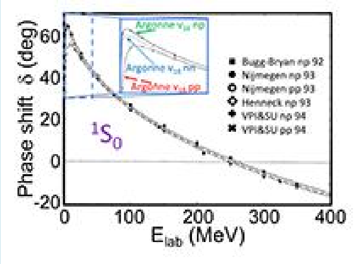
Phys. Rev. C 68, 041001 (2003)

Phenomenology

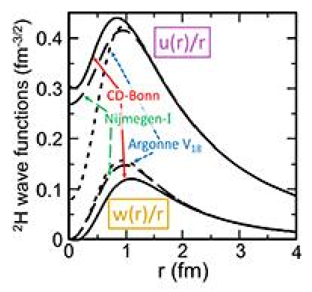
the basis of numerous significant few-body calculations



V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart
Phys. Rev. C 49, 2950 (1994)

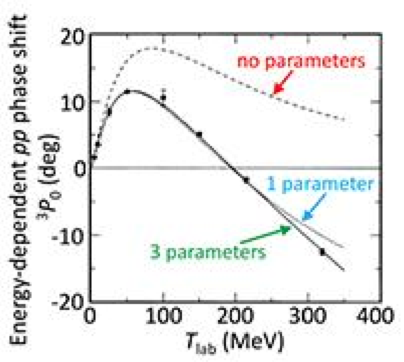


R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla
Phys. Rev. C 51, 38 (1995)



R. Machleidt
Phys. Rev. C 63, 024001 (2001)

Partial wave analysis



V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart

Phys. Rev. C 48, 792 (1993)

相对论手征核力