Chiral Interactions with Gradient Flow Regulator

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Frontiers in Nuclear Lattice EFT: From Ab Initio Nuclear Structure to Reactions
Beihang University, Beijing, China
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ERATO
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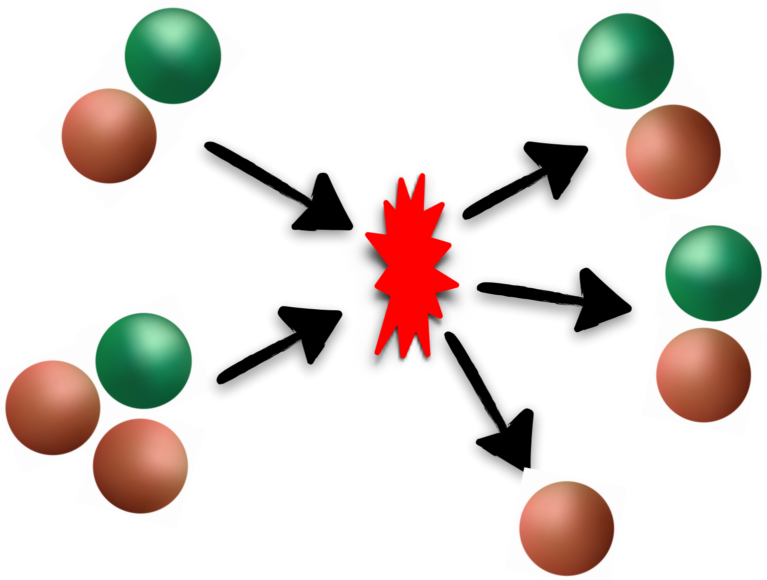
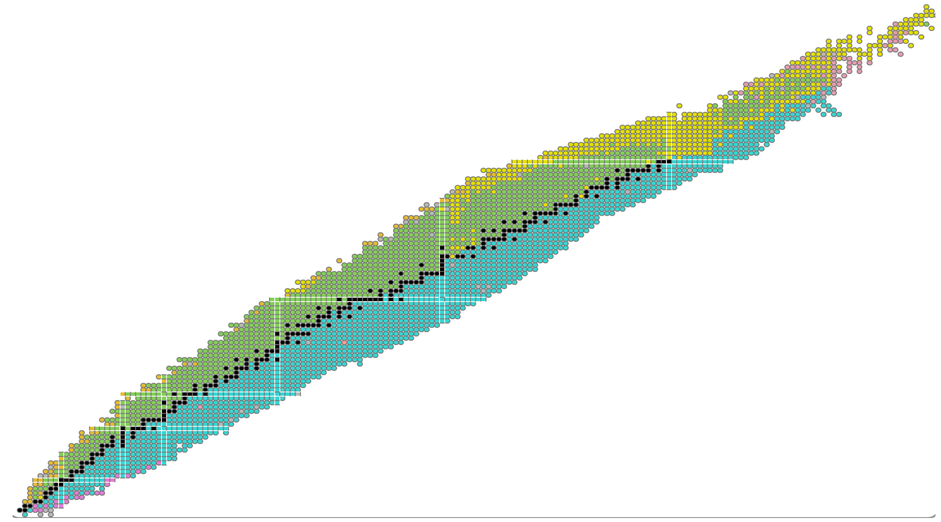
In collaboration with Evgeny Epelbaum

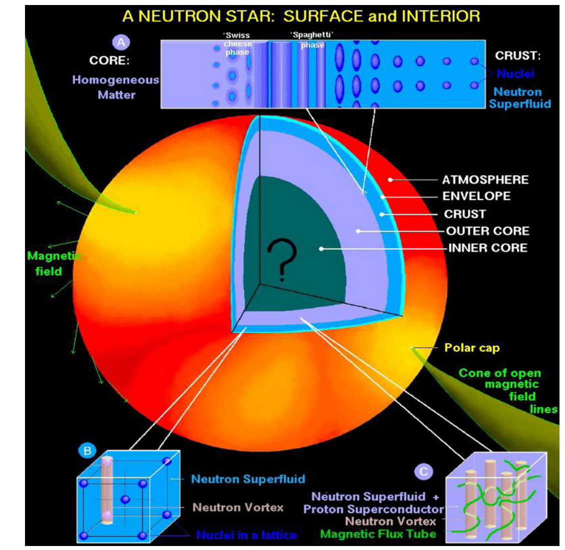
Outline

- Nuclear forces up to N^3LO
- Most general form of 3NF
- Method for derivation of nuclear forces in chiral EFT
- Status report on construction of 3NF





Livechart, IAEA: https://www-nds.iaea.org

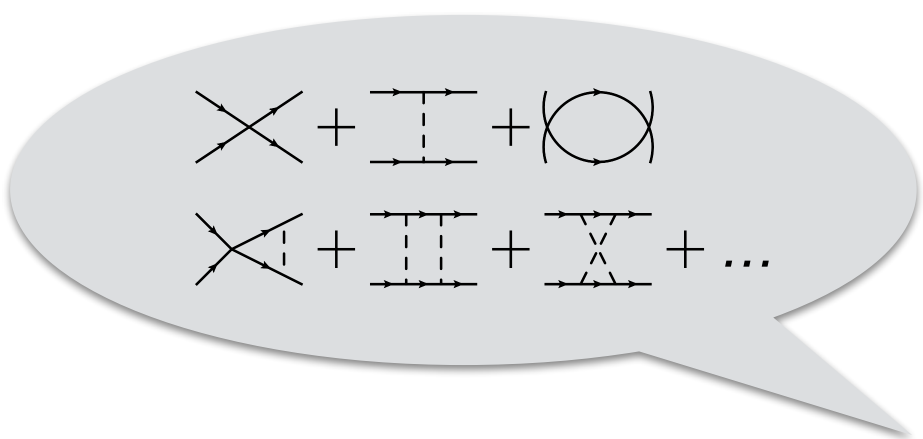


Lattimer: NAR54 (2010) 101

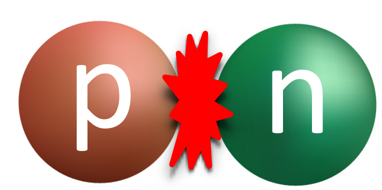
QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\dots}_{\textit{derived within ChPT}}\right]|\Psi\rangle=E|\Psi\rangle$$

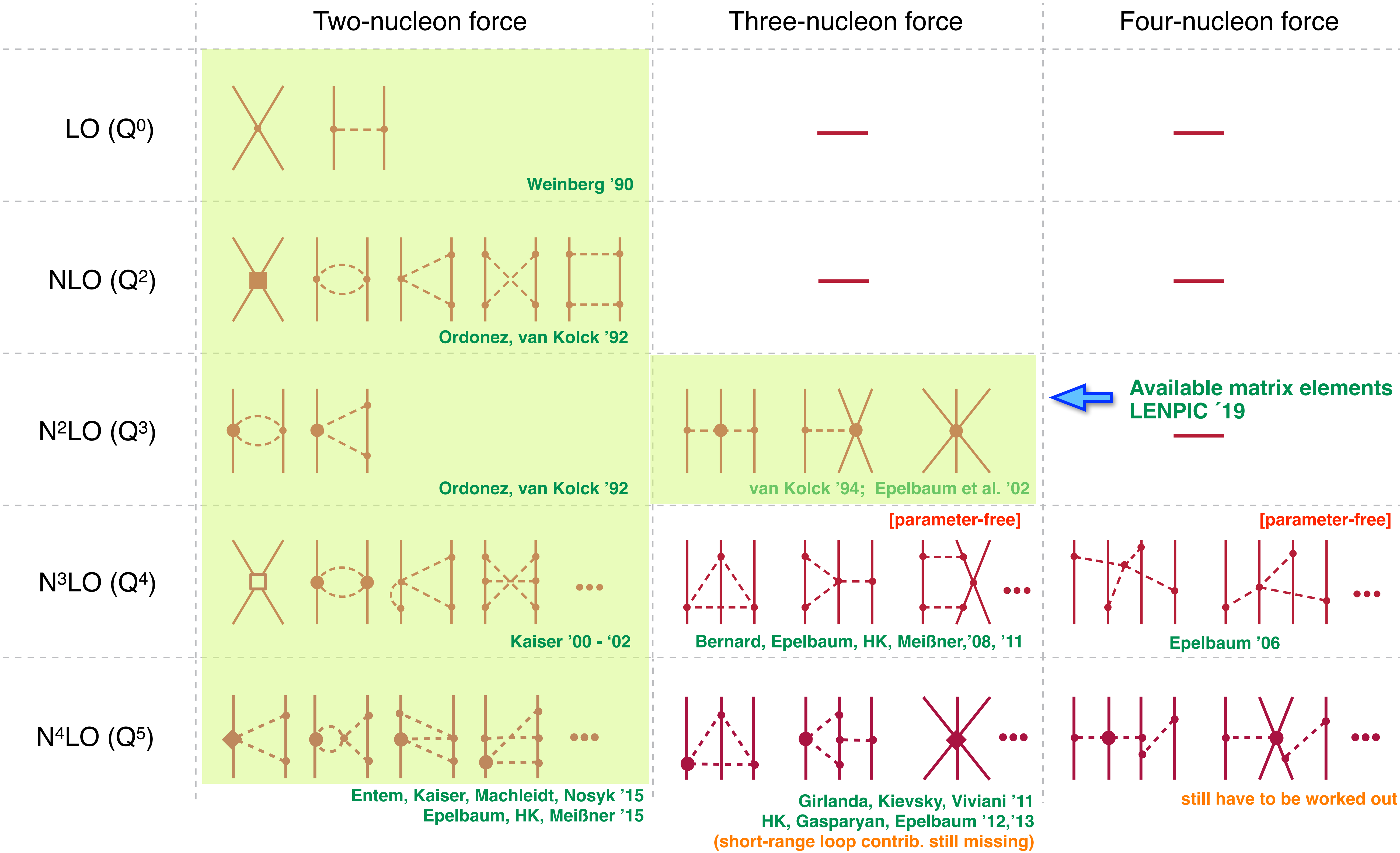
Weinberg '91



Chiral EFT is a systematic tool for derivation of nuclear forces below pion-production threshold



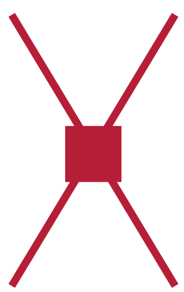
Chiral Expansion of the Nuclear Forces



Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)

NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)

N^2LO [Q^3]: no new terms

N^3LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)

N^4LO [Q^5]: + 5 IB operators

N^4LO^+ [Q^6]: + 4 operators (F-waves)

# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

Summary on NN

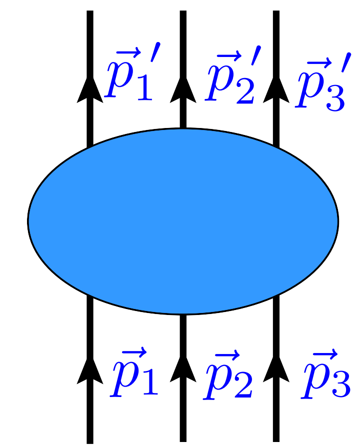
- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data
  χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

Three-Nucleon Forces

Most general Spin-Isospin-Momentum Structure

Most general structure of a local 3NF

Epelbaum, Gasparyan, HK, PRC87 (2013) 054007; Schat, Phillips, PRC88 (2013) 034002,
Epelbaum, Gasparyan, HK, Schat, EPJA51 (2015) 3, 36



$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i \qquad \vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

Complete set of local independent operators

$$\mathcal{G}_1 = 1$$
$$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$$
$$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$$
$$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$$
$$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \, \vec{q}_3 \cdot \vec{\sigma}_1$$
$$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_2$$
$$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_2$$
$$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{q}_3 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_2$$
$$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{q}_3 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_2$$
$$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{q}_2 \cdot \vec{\sigma}_1 \, \vec{q}_2 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{q}_3 \cdot \vec{\sigma}_2 \, \vec{q}_3 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_3$$
$$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$$
$$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_3 \cdot \vec{q}_1 \, \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$
$$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$$

Building blocks:

$$\vec{\sigma}_1, \vec{\sigma}_2, \vec{\sigma}_3, \boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \boldsymbol{\tau}_3, \vec{q}_1, \vec{q}_3$$

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance
- Rotation invariance in 3-dim

80 operators generated by 20 operators

$$V_{3N} = \sum_{i=1}^{20} \mathcal{G}_i F_i(q_1, q_2, q_3) + 5 \text{ perm}\,.$$

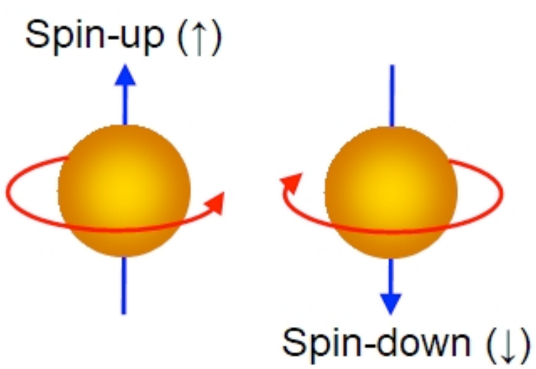
Most general structure of 3NF

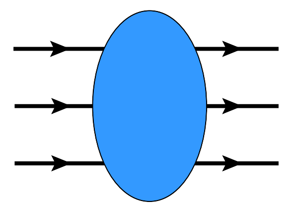
Possible spin-momentum structures in 3NF

Topolnicki, Golak, Skibinski, Witala, PRC96 (2017) 014611
Topolnicki, EPJA53 (2017) 9, 181

5 isospin structures:

$$1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)$$





2^3 incoming $\times\, 2^3$ outgoings
spin configurations

$2^3 \times 2^3 \times 5 = 320$
spin-isospin-momentum
structures

Assume that $\vec{q}_1$ & $\vec{q}_3$ are linear independent → $\vec{q}_1, \vec{q}_3, \vec{q}_1 \times \vec{q}_3$ is a basis of $\mathbb{R}^3$

$$\vec{k}_j = \vec{q}_1 \frac{\vec{k}_j \cdot \vec{q}_1\, q_3^2 - \vec{k}_j \cdot \vec{q}_3\, \vec{q}_1 \cdot \vec{q}_3}{(\vec{q}_1 \times \vec{q}_3)^2} + \vec{q}_2 \frac{\vec{k}_j \cdot \vec{q}_3\, q_1^2 - \vec{k}_j \cdot \vec{q}_1\, \vec{q}_1 \cdot \vec{q}_3}{(\vec{q}_1 \times \vec{q}_3)^2} + \vec{q}_1 \times \vec{q}_3 \frac{\vec{k}_j \cdot (\vec{q}_1 \times \vec{q}_3)}{(\vec{q}_1 \times \vec{q}_3)^2}$$

3NF can be described by local spin-isospin-momenum operators
multiplied with non-local scalar functions

$$V_{3\mathrm{N}} = \sum_{i=1}^{68} \mathcal{G}_i F_i(\vec{q}_1, \vec{q}_3, \vec{k}_1, \vec{k}_2, \vec{k}_3) + 5\,\mathrm{perm}\,.$$

By permutation of 68 local operators
one can generate all 320 operators

Most general structure of 3NF

Antisymmetrization reduces the number of structures

Epelbaum, HK, forthcoming

$$\mathcal{P}_1 = 1$$

$$\mathcal{P}_2 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\mathcal{P}_3 = \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{\sigma}_2 \cdot \vec{\sigma}_3 + \vec{q}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\mathcal{P}_4 = i\,(\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_1 \, \vec{\sigma}_2 \cdot \vec{\sigma}_3 + i\,(\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\mathcal{P}_5 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \vec{\sigma}_1 \cdot \vec{q}_1$$

$$\mathcal{P}_6 = i\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \vec{\sigma}_1 \cdot (\vec{q}_1 \times \vec{q}_3)$$

$$\mathcal{P}_7 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \vec{\sigma}_1 \cdot \vec{q}_3$$

$$\mathcal{P}_8 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \left(\vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_2 - \frac{1}{3} q_1^2 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right)$$

$$\mathcal{P}_9 = i\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \left(\frac{1}{2} \vec{q}_1 \cdot \vec{\sigma}_1 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_2 + \frac{1}{2} \vec{q}_1 \cdot \vec{\sigma}_2 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_1 \right)$$

$$\mathcal{P}_{10} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \left(\frac{1}{2} \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_2 + \frac{1}{2} \vec{q}_1 \cdot \vec{\sigma}_2 \, \vec{q}_3 \cdot \vec{\sigma}_1 - \frac{1}{3} \vec{q}_1 \cdot \vec{q}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right)$$

$$\mathcal{P}_{11} = i\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \left(\frac{1}{2} \vec{q}_3 \cdot \vec{\sigma}_1 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_2 + \frac{1}{2} \vec{q}_3 \cdot \vec{\sigma}_2 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_1 \right)$$

$$\mathcal{P}_{12} = \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_3 \left(\vec{q}_3 \cdot \vec{\sigma}_1 \, \vec{q}_3 \cdot \vec{\sigma}_2 - \frac{1}{3} q_3^2 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right)$$

$$\mathcal{P}_{13} = \frac{1}{3} \Big(\vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_2 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_3 + \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_3 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_2 + \vec{q}_1 \cdot \vec{\sigma}_2 \, \vec{q}_1 \cdot \vec{\sigma}_3 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_1$$
$$- \frac{1}{5} q_1^2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_3 + \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_2 + \vec{\sigma}_2 \cdot \vec{\sigma}_3 \, (\vec{q}_1 \times \vec{q}_3) \cdot \vec{\sigma}_1 \right] \Big)$$

$$\mathcal{P}_{14} = \frac{1}{3} \Big(\vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_2 \, \vec{q}_3 \cdot \vec{\sigma}_3 + \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{q}_1 \cdot \vec{\sigma}_3 \, \vec{q}_3 \cdot \vec{\sigma}_2 + \vec{q}_1 \cdot \vec{\sigma}_2 \, \vec{q}_1 \cdot \vec{\sigma}_3 \, \vec{q}_3 \cdot \vec{\sigma}_1$$
$$+ \frac{1}{5} q_1^2 \left[\vec{q}_3 \cdot \vec{\sigma}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{q}_3 \cdot \vec{\sigma}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_3 + \vec{q}_3 \cdot \vec{\sigma}_1 \vec{\sigma}_2 \cdot \vec{\sigma}_3 \right] - \frac{2}{5} \vec{q}_1 \cdot \vec{q}_3 \left[\vec{q}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{q}_1 \cdot \vec{\sigma}_2 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 + \vec{q}_1 \cdot \vec{\sigma}_1 \, \vec{\sigma}_2 \cdot \vec{\sigma}_3 \right]$$

$$V_{3\mathrm{N}} = \sum_{i=1}^{14} \mathcal{P}_i F_i(\vec{q}_1, \vec{q}_3, \vec{k}_1, \vec{k}_2, \vec{k}_3) + 5\,\mathrm{perm}\,.$$

- Number of spin-isospin-momentum structures in moving 3NF and NN is comparable: 14 vs 13
- Structure functions in 3NF are reacher due to five momenta compared to three in moving NN force
- Minimal basis is inconvenient for practical calculations due to kinematic singularities (instable)

Path-Integral Framework
for Derivation of Nuclear Forces

HK, Epelbaum, PRC110 (2024) 4, 044003

Illustration fo Yukawa Model

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x \left(\mathscr{L} + \eta^\dagger(x) N(x) + N^\dagger(x) \eta(x) \right) \right)$$

Yukawa toy-model:

$$\mathscr{L} = N^\dagger \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{g}{2F} \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right) N + \frac{1}{2} \left(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - M^2 \boldsymbol{\pi}^2 \right)$$

- Perform a Gaussian path-integral over the pion fields

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN] \exp\left(i S_N + i \int d^4x \left(\eta^\dagger(x) N(x) + N^\dagger(x) \eta(x) \right) \right)$$

$$S_N = \int d^4x \, N^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N(x) - V_{NN}$$

← Non-instant one-pion-exchange interaction

$$V_{NN} = - \frac{g^2}{8F^2} \int d^4x \, d^4y \, \vec{\nabla}_x \cdot \left[N^\dagger(x) \vec{\sigma} \boldsymbol{\tau} \right] N(x) \, \Delta_F(x-y) \, \vec{\nabla}_y \cdot \left[N^\dagger(y) \vec{\sigma} \boldsymbol{\tau} \right] N(y)$$

with non-instant pion propagator: $\Delta_F(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-i q \cdot x}}{q^2 - M^2 + i\epsilon}$

Instant Interactions from Path-Integral

To transform V_{NN} into an instant form we rewrite a pion propagator

$$\frac{1}{q_0^2-\omega_q^2} = -\frac{1}{\omega_q^2} + \frac{1}{q_0^2-\omega_q^2} + \frac{1}{\omega_q^2} = -\frac{1}{\omega_q^2} + q_0^2\frac{1}{\omega_q^2}\frac{1}{q_0^2-\omega_q^2}, \quad \omega_q = \sqrt{\vec{q}^2+M^2}$$

In coordinate space this corresponds to $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$ with

$$\Delta_S(x) = -\int\frac{d^4q}{(2\pi)^4}\frac{e^{-i\,q\cdot x}}{\omega_q^2} = -\,\delta(x_0)\int\frac{d^3q}{(2\pi)^3}\frac{e^{i\,\vec{q}\cdot\vec{x}}}{\omega_q^2}, \quad \Delta_{FS}(x) = \int\frac{d^4q}{(2\pi)^4}\frac{e^{-i\,q\cdot x}}{\omega_q^2(q_0^2-\omega_q^2)}$$

- The decomposition $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$ can be generalized

$G(x) = \int\frac{d^4q}{(2\pi)^4}e^{-i\,q\cdot x}\tilde{G}(q_0^2,q^2)$ and $\tilde{G}(q_0^2,q^2)$ is differentiable at $q_0=0$

Defining $G_S(x) = \int\frac{d^4q}{(2\pi)^4}e^{-i\,q\cdot x}\tilde{G}(0,q^2)$ and $G_{FS}(x) = \int\frac{d^4q}{(2\pi)^4}e^{-i\,q\cdot x}\frac{\tilde{G}(q_0^2,q^2)-\tilde{G}(0,q^2)}{q_0^2}$

$$\rightarrow \quad G(x) = G_S(x) - \frac{\partial^2}{\partial x_0^2}G_{FS}(x)$$

Instant Interactions from Path-Integral

Perform an instant decomposition of the pion propagator $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$

$$V_{NN} = -\frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\boldsymbol{\tau}\right]N(x)\,\Delta_F(x-y)\,\vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\boldsymbol{\tau}\right]N(y)$$

$$\rightarrow \quad V_{NN} = V_{OPE} + V_{FS}$$

$$V_{OPE} = -\frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\boldsymbol{\tau}\right]N(x)\,\Delta_S(x-y)\,\vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\boldsymbol{\tau}\right]N(y)$$ is instant

$$V_{FS} = \frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\boldsymbol{\tau}\right]N(x)\,\frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x-y)\,\vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\boldsymbol{\tau}\right]N(y)$$ is non-instant

V_{FS} is time-derivative dependent and thus can be eliminated by a non-polynomial field redefinition

$$N(x) \to N'(x) = N(x) + i\frac{g^2}{8F^2}\int d^4y\,[\vec{\sigma}\boldsymbol{\tau}N(x)]\cdot[\,\vec{\nabla}_x\frac{\partial}{\partial x_0}\Delta_{FS}(x-y)]\,\vec{\nabla}_y \cdot [N^\dagger(y)\vec{\sigma}\boldsymbol{\tau}N(y)]$$

$$N^\dagger(x) \to N'^\dagger(x) = N^\dagger(x) - i\frac{g^2}{8F^2}\int d^4y \vec{\nabla}_y \cdot [N^\dagger(y)\vec{\sigma}\boldsymbol{\tau}N(y)][\,\vec{\nabla}_y\frac{\partial}{\partial y_0}\Delta_{FS}(y-x)]\cdot[N^\dagger(x)\vec{\sigma}\boldsymbol{\tau}]$$

Instant Interactions from Path-Integral

Non-local field transformations remove time-derivative dependent two-nucleon interactions but generate time-derivative dependent three-nucleon interactions.

These contributions can be eliminated by similar field transformations

$$Z[\eta^\dagger,\eta] = \int [DN'^\dagger][DN'] \det\left(\frac{\delta(N'^\dagger,N')}{\delta(N^\dagger,N)}\right)\exp\left(i\,S_{N(N'^\dagger,N')} + i\int d^4x\left(\eta^\dagger(x)N(N'^\dagger,N')(x) + N(N'^\dagger,N')^\dagger(x)\eta(x)\right)\right)$$

$$\simeq \int [DN'^\dagger][DN'] \det\left(\frac{\delta(N'^\dagger,N')}{\delta(N^\dagger,N)}\right)\exp\left(i\,S_{N(N'^\dagger,N')} + i\int d^4x\left(\eta^\dagger(x)N'(x) + N'^\dagger(x)\eta(x)\right)\right)$$

Equivalence theorem: nucleon pole-structure is unaffected by the field-transf.

$$S_{N(N'^\dagger,N')} = \int d^4x\, N'^\dagger(x)\left(i\,\frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m}\right)N'(x) \;-\; V_{OPE} + \mathcal{O}(g^4)$$

$$V_{OPE} = -\frac{g^2}{8F^2}\int d^4x\, d^4y\, \vec{\nabla}_x \cdot \left[N'^\dagger(x)\vec{\sigma}\boldsymbol{\tau}\right]N'(x)\,\Delta_S(x-y)\,\vec{\nabla}_y \cdot \left[N'^\dagger(y)\vec{\sigma}\boldsymbol{\tau}\right]N'(y)$$

Instant one-pion-exchange interaction

One-Loop Corrections to Interaction

One loop corrections to NN & NNN interaction come from functional determinant

$$\det\left(\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right)=\exp\left(\mathrm{Tr}\log\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right)$$

Due to non-local structure of field transformations $\det\left(\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right)\neq 1$

$$S_{N(N'^{\dagger},N')}=\int d^4x\, N'^{\dagger}(x)\left(i\frac{\partial}{\partial x_0}+\frac{\vec{\nabla}^2}{2m}+\frac{3g^2M^3}{32\pi F^2}\right)N'(x)\;-\;V_{OPE}+\mathcal{O}(g^4)$$

Nucleon mass-shift **Gasser, Zepeda, NPB 174 (1980) 445**
is reproduced from functional determinant

Note: The Z-factor of the nucleon is equal to one. This is due to the replacement

$\eta^{\dagger}N+N^{\dagger}\eta\rightarrow\eta^{\dagger}N'+N'^{\dagger}\eta$ in the generating functional $Z[\eta^{\dagger},\eta]$

The original Z-factor of the nucleon is reproduced if we remove this replacement

$$Z=1-\frac{9M^2g^2}{2F^2}\left(\bar{\lambda}+\frac{1}{16\pi^2}\left(\log\frac{M}{\mu}+\frac{1}{3}-\frac{\pi}{2}\frac{M}{\mu})\right)\right)$$

Path-integral Approach

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x \left(\mathscr{L}_\pi + \mathscr{L}_{\pi N} + \mathscr{L}_{NN} + \mathscr{L}_{NNN} + \eta^\dagger(x) N(x) + N^\dagger(x) \eta(x) \right) \right)$$

- Integrate over pion fields via loop-expansion of the action

  → expansion of the action around the classical pion solution

- Perform instant decomposition of the remaining interactions between nucleons

- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction

- Calculate functional determinant to get one-loop corrections to few-nucleon forces

Checks in dimensional regularization

Unitary transformation (Okubo) & path-integral approaches lead to the same chiral EFT nuclear forces up to N^4LO

Symmetry Preserving Regulator

HK, Epelbaum, PRC 110 (2024) 4, 044004

Gradient-Flow Equation (GFE)

Balitsky, Yung, PL168B (1986) 113; Irwin, Manton, PLB 385 (1996) 187

Yang-Mills gradient flow in QCD: Lüscher, JHEP 04 (2013) 123

$\partial_\tau B_\mu = D_\nu G_{\nu\mu}$ with $B_\mu|_{\tau=0} = A_\mu$ & $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$

B_μ is a regularized gluon field

- Apply this idea to ChPT: HK, Epelbaum, PRC 110 (2024) 4, 044004

(Proposed in various talks by D. Kaplan for nuclear forces)

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \text{ with } w = \sqrt{W} \text{ and } \mathrm{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}(1-\alpha\boldsymbol{\phi}^2) - \frac{\boldsymbol{\phi}^2}{2}\left[1 + \left(\frac{1}{4} - 2\alpha\right)\boldsymbol{\phi}^2\right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty}\frac{1}{F^n}\phi_b^{(n)}$$

In the absence of external sources we have

$$\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\phi_b^{(1)}(x,\tau) = 0, \quad \phi_b^{(1)}(x,0) = \pi_b(x)$$

$$\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\phi_b^{(3)}(x,\tau) = (1-2\alpha)\partial_\mu\boldsymbol{\phi}^{(1)}\cdot\partial_\mu\boldsymbol{\phi}^{(1)}\phi_b^{(1)} - 4\alpha\partial_\mu\boldsymbol{\phi}^{(1)}\cdot\boldsymbol{\phi}^{(1)}\partial_\mu\phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha)\boldsymbol{\phi}^{(1)}\cdot\boldsymbol{\phi}^{(1)}\phi_b^{(1)}, \quad \phi_b^{(3)}(x,0) = 0$$

Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q,\tau) = \int d^4x\, e^{iq\cdot x}\phi_b^{(n)}(x,\tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2+M^2)}\tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}(2\pi)^4\delta(q-q_1-q_2-q_3)\int_0^\tau ds\, e^{-(\tau-s)(q^2+M^2)}e^{-s\sum_{j=1}^3(q_j^2+M^2)} \times\left[4\alpha\, q_1\cdot q_3 - (1-2\alpha)q_1\cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\pi}(q_1)\cdot\tilde{\pi}(q_2)\tilde{\pi}_b(q_3)$$

Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Properties under Chiral Transformation

Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathscr{L}_{\pi N}^{(1)} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D_w^0 + g\, w \cdot S \Big) N$$

Chiral transformation: by induction, one can show

$$U \to R U L^\dagger \longrightarrow W \to R W L^\dagger$$

- Regularized pion fields transform under τ - independent transformations

- Nucleon fields transform in τ - dependent way

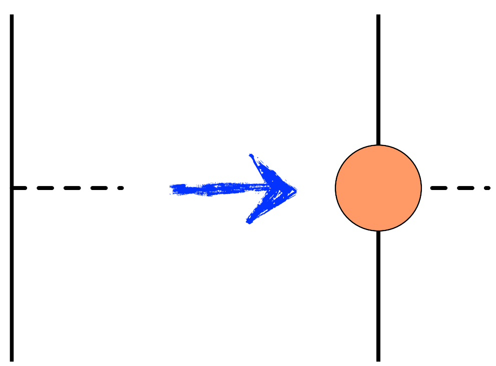
$$N \to K N, \quad K = \sqrt{L U^\dagger R^\dagger} R \sqrt{U} \longrightarrow N \to K_\tau N, \quad K_\tau = \sqrt{L W^\dagger R^\dagger} R \sqrt{W}$$

Regularization for Nuclear Forces

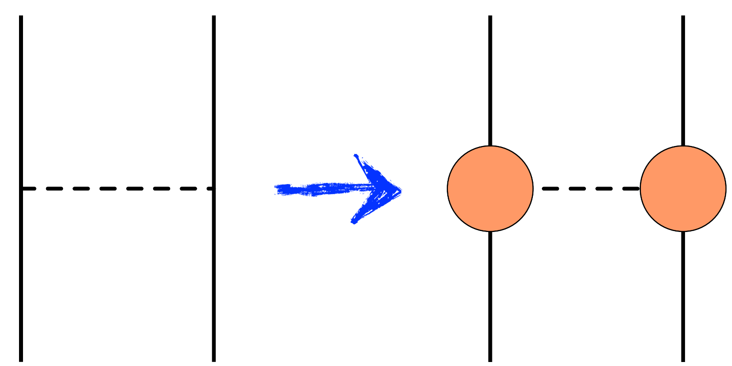
To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathscr{L}_\pi^{(2)}$ & $\mathscr{L}_\pi^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathscr{L}_{\pi N}^{(1)} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D_w^0 + g\, w \cdot S \Big) N$$



$\sim e^{-\tau(q^2+M^2)}$

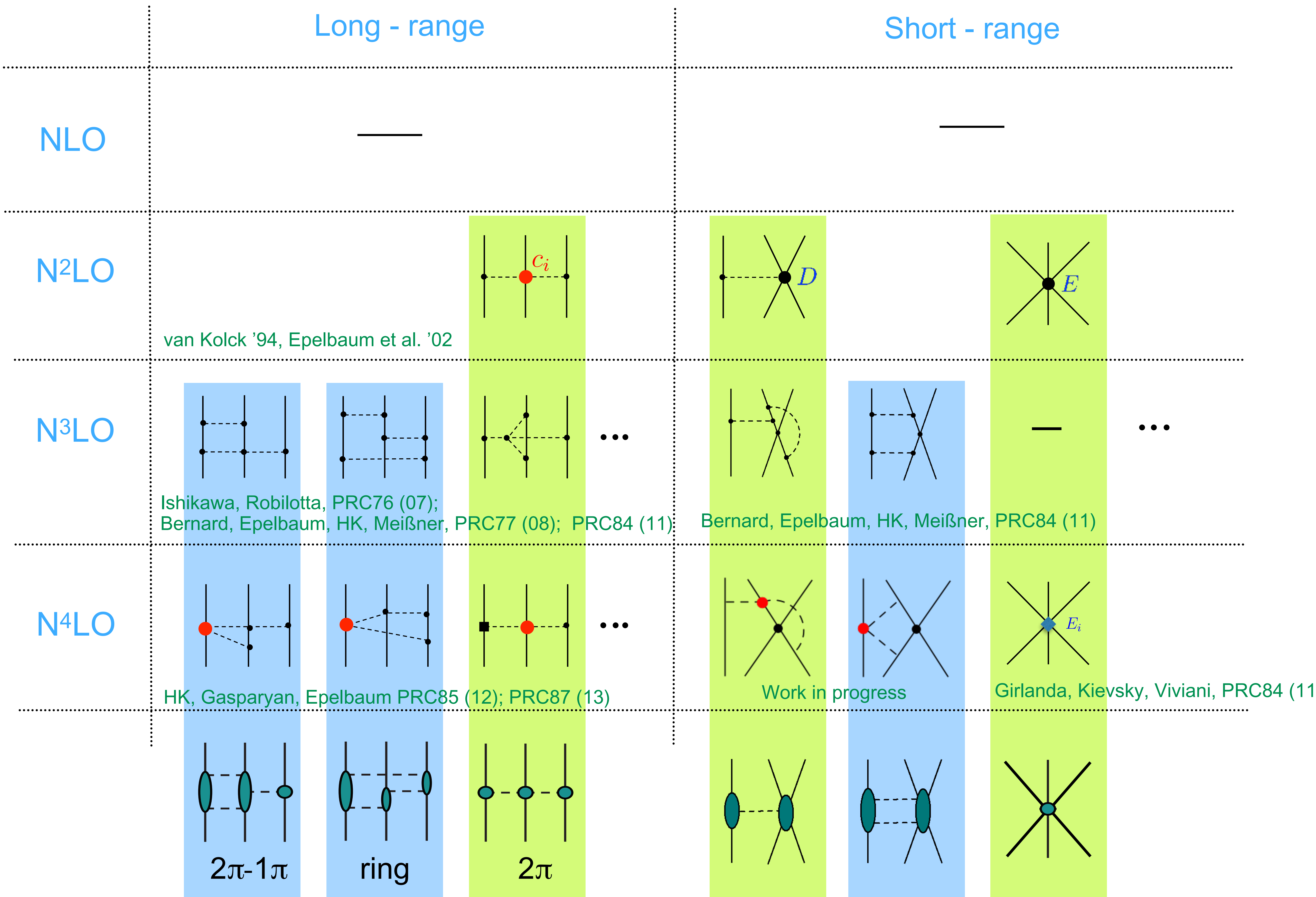


$\sim e^{-2\tau(q^2+M^2)} \dfrac{1}{q^2+M^2}$

For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

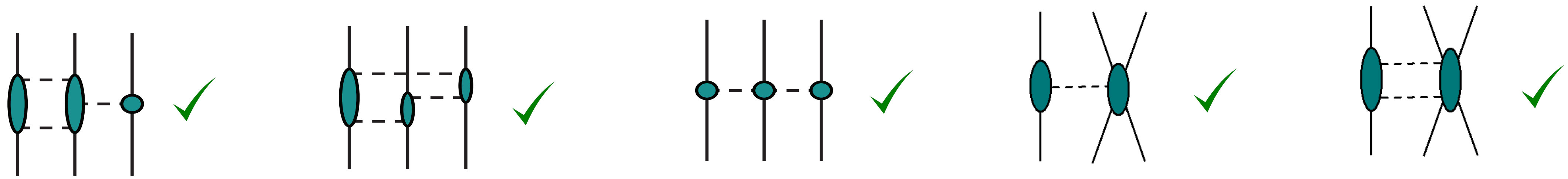
Status Report on 3NF

3NF up to N^4LO



Status Report on 3N at N^{3}LO

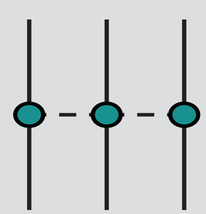
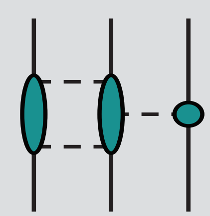
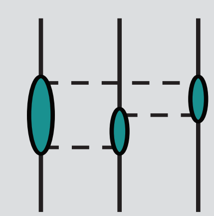
- We calculated all long- and short-range contributions to 3NF & 4NF at N^{3}LO



3NF's are given in terms of integrals over Schwinger parameters

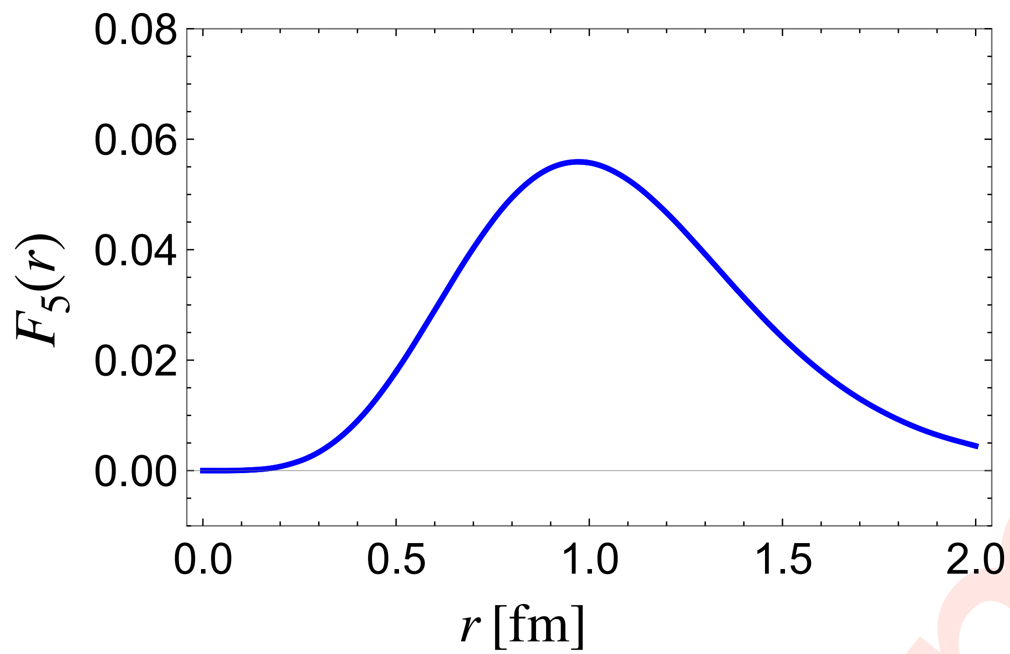
$$V_{3N}^{2\pi-1\pi} = \tau_1\cdot\tau_2\times\tau_3 \vec{q}_1\cdot\vec{\sigma}_1\times\vec{\sigma}_2 \vec{q}_3\cdot\vec{\sigma}_3 \frac{e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{q_3^2+M_\pi^2}\left(-\frac{g_A^4}{F_\pi^6}\frac{q_1}{2048\pi}\int_0^\infty d\lambda\, \mathrm{erfi}\left(\frac{q_1\lambda}{2\Lambda\sqrt{2+\lambda}}\right)\frac{\exp\left(-\frac{q_1^2+4M_\pi^2}{4\Lambda^2}(2+\lambda)\right)}{2+\lambda}+\ldots\right)+\ldots$$

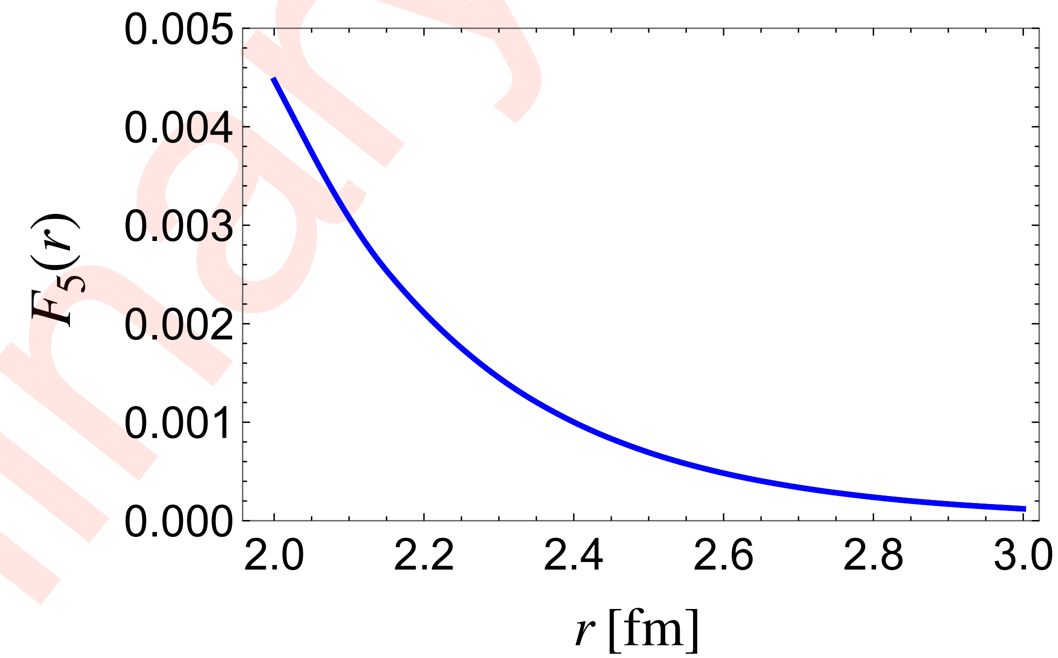
Dimension of integrals over Schwinger parameters depends on topology

Space			
Momentum	2	1	3
Coordinate	4	1	0

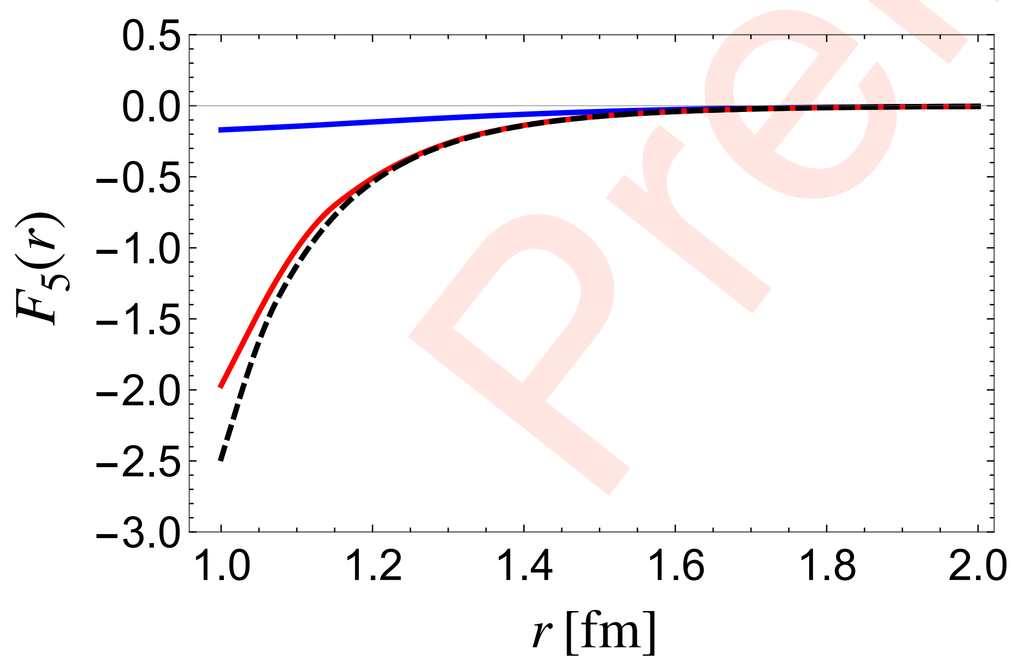
Selected Profile Functions

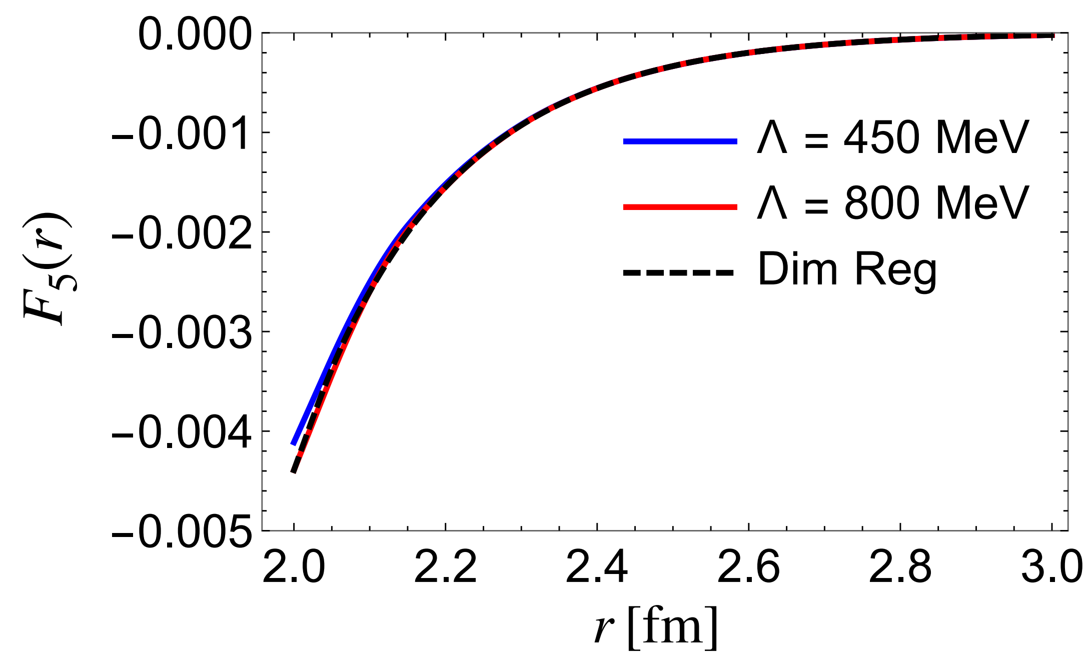
$V_{3N}^{\text{ring}} = F_1(r_{12}, r_{23}, r_{13}) + \ldots + \tau_2 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 F_5(r_{12}, r_{23}, r_{13}) + \ldots$ $F_5(r) = F_5(r, r, r)$ [MeV]





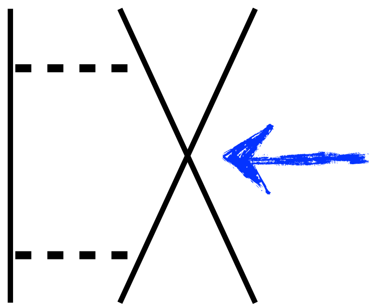
At $\Lambda \to \infty$ regularized 3NF reproduce dim. reg. results from **Bernard et al. PRC77 (08)**





Short Range 3NF at N³LO

Complication in calculation of short-range 3NF due to non-local regulator of LO NN



Non-local regulator of short-range NN at LO introduces additional momentum in loop functions

Structure functions of short-range 3NF can become complex

Time-reversal transformation (T): $\vec{\sigma}_j \to -\vec{\sigma}_j,\ \tau_j^y \to -\tau_j^y,\ \vec{q}_j \to \vec{q}_j,\ \vec{k}_j \to -\vec{k}_j$

Hermitian conjugation (h.c.): $\vec{\sigma}_j \to \vec{\sigma}_j,\ \boldsymbol{\tau}_j \to \boldsymbol{\tau}_j,\ \vec{q}_j \to -\vec{q}_j,\ \vec{k}_j \to \vec{k}_j$

$$\exp\Big(-\frac{(2(\vec{k}_2-\vec{k}_3)+\vec{q}_2)^2}{8\Lambda^2}\Big)+\exp\Big(-\frac{(2(\vec{k}_2-\vec{k}_3)-\vec{q}_2)^2}{8\Lambda^2}\Big)$$ Invariant under T and h.c.

$$i\left[\exp\Big(-\frac{(2(\vec{k}_2-\vec{k}_3)+\vec{q}_2)^2}{8\Lambda^2}\Big)-\exp\Big(-\frac{(2(\vec{k}_2-\vec{k}_3)-\vec{q}_2)^2}{8\Lambda^2}\Big)\right]$$ Invariant under T and h.c.

- Combination of these functions are allowed to appear in structure functions

→ Structure functions might be complex: not related to unitarity cut (phase)

Short Range 3NF at N^3LO

Complex structure functions of short-range part of 3NF require complex PWD

Solution 1: Is there a nucleon-field transformation which would make 3NF's real?

Idea: Constrain field transformations needed to make interactions instant

Every ϵ_{ijk} in field transformations should be accompanied with an „i“

→ Indeed, we achieved with these transformations an instant 3NF and get real structure functions for short-range 3NF

Solution 2: Change the regulator of short-range NN interaction at LO to local one

→ Short-range 3NF's at N^3LO becomes local and automatically real

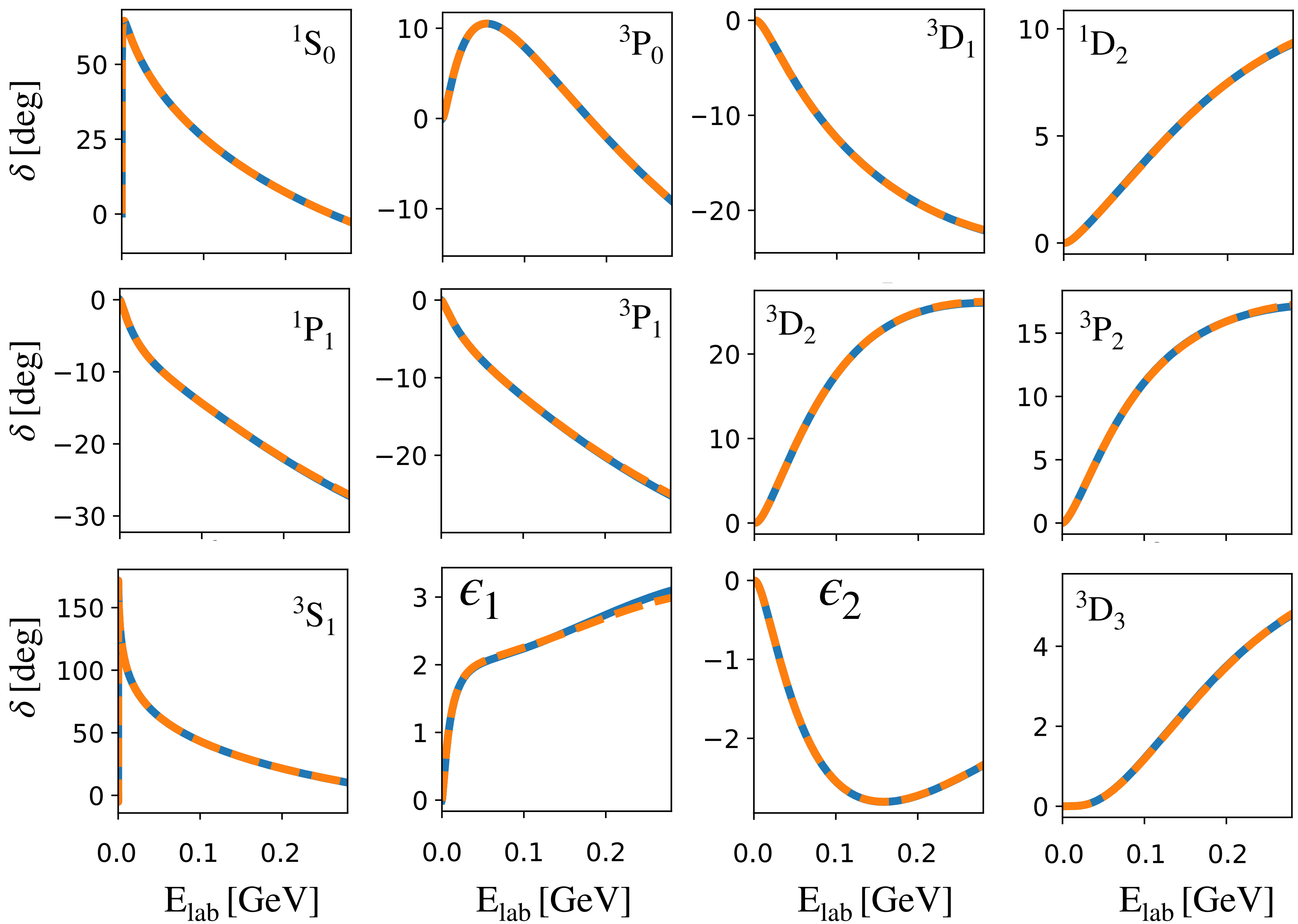
→ Expressions for local short-range 3NF's at N^3LO are simpler

→ PWD of local 3NF's is less expensive

But: we need to generate a new NN force

NN phase shifts and mixing angles

Heihoff et al. : forthcoming



$\Lambda = 450\,\mathrm{MeV}$

- Quality of nuclear force does not change when we change the regulator of the LO short-range NN interaction
- Local regularization of the LO short-range NN leads to simpler 3NF at N³LO

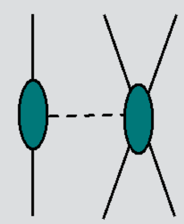
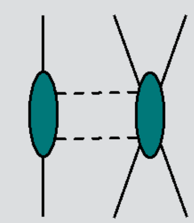
Local short-range regulator at LO: $\exp(-q^2/\Lambda^2)$, $\chi^2 = 1.0069$

Non-local short-range regulator at LO: $\exp\big(-(p'^2+p^2)/\Lambda^2\big), \chi^2 = 1.0062$

Short Range 3NF at N^3LO

We followed both paths and provide two versions of 3NF

Version 1: Non-local short-range 3NF which can be used with SMS potential

Space		
Momentum	1	1

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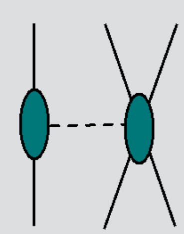
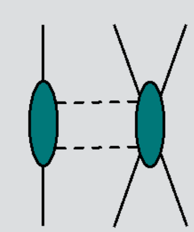
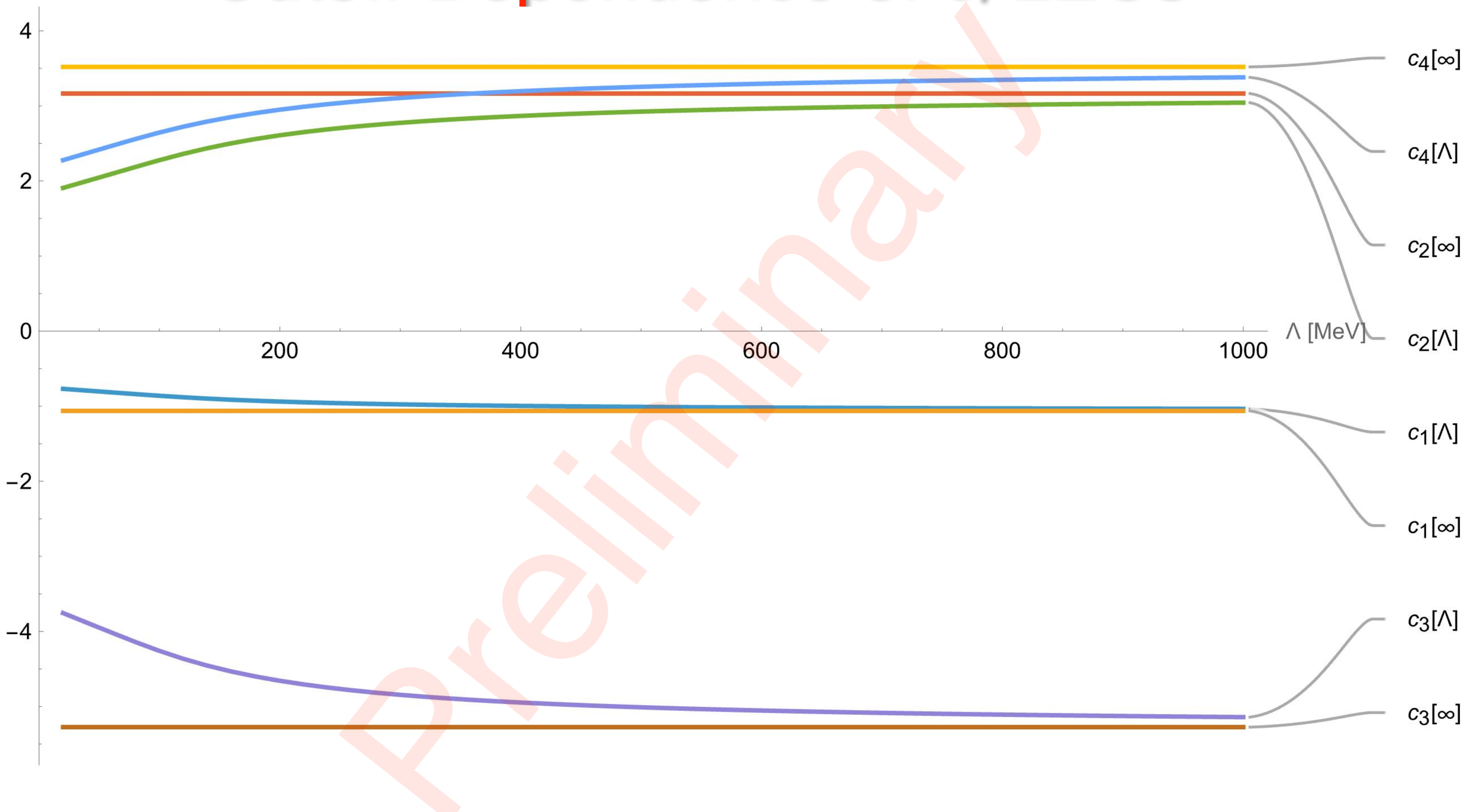
Version 2: Local short-range 3NF to be used with the new NN potential

Space		
Momentum	1	1
Coordinate	0	0

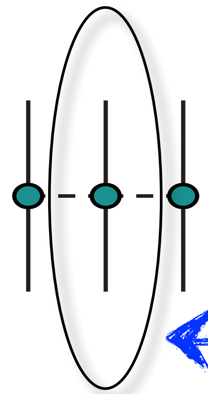
0.4 MB

Cutoff Dependence of c_i LECs



- Saturation towards dim-reg results ($\Lambda \to \infty$) is fast
- For $\Lambda \sim 500\,\mathrm{MeV}$ the absolute value of c_i is smaller compared to c_i in dim-reg.

Pion-Nucleon Scattering up to Q^3

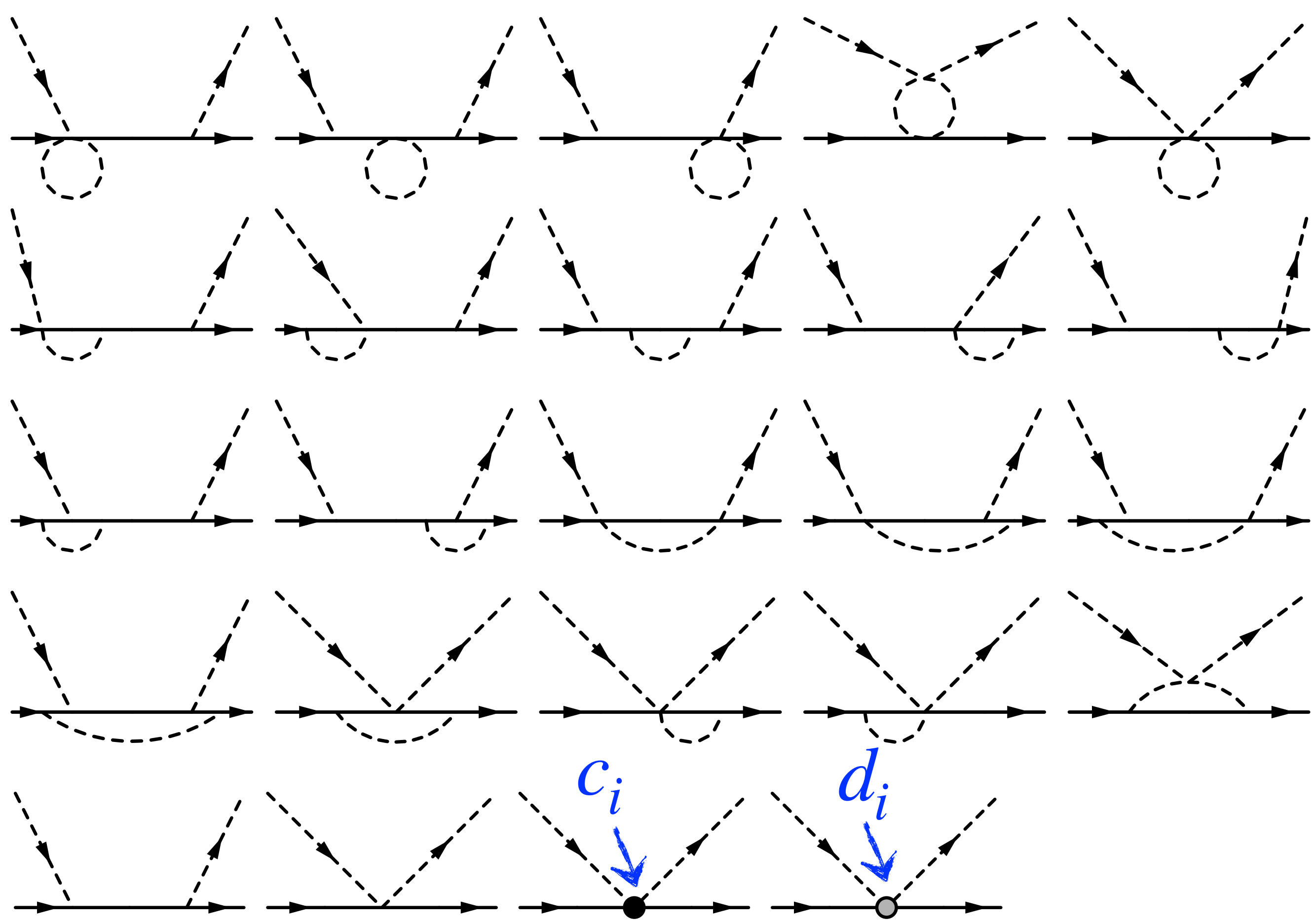


- TPE topology includes pion-nucleon amplitude as a subprocess

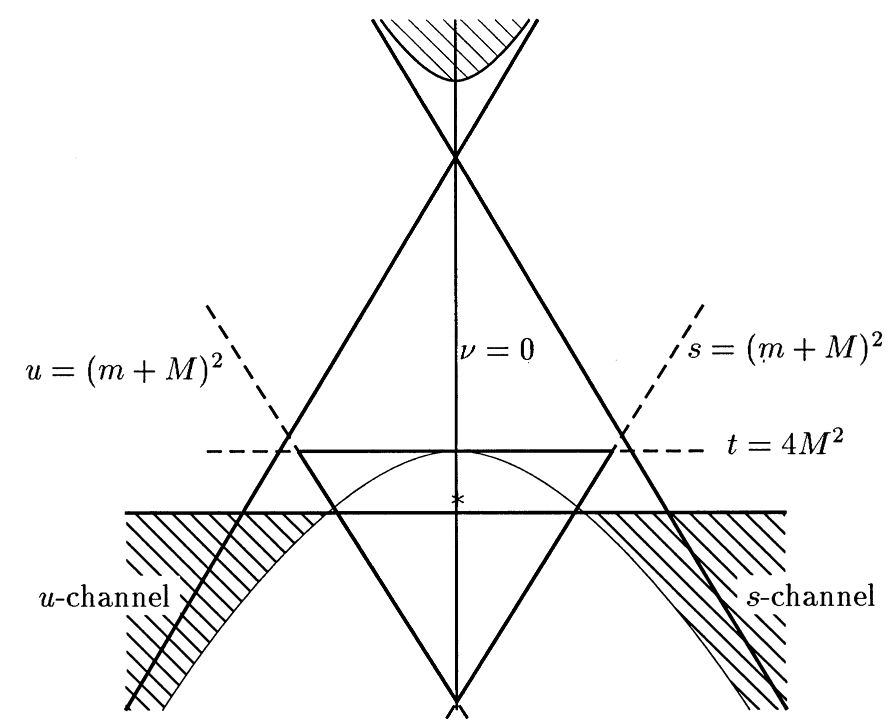
Pion-nucleon amplitude with gradient-flow regulator depends on c_i's

Calculation of pion-nucleon scattering with gradient-flow regulator required

→ Patrick Walkowiak's master thesis



Fit LECs to pion-nucleon sub-threshold coefficients which are determined from Roy-Steiner equation



Summary

- General 3NF via 14 spin-isospin-momentum operators
- Calculation of gradient-flow regularized 3NF at N^3LO is finished
  - Two versions for short-range 3NF at N^3LO
    - With non-local regulator in LO NN (SMS potential)
    - With local regulator in LO NN (new NN required)

Outlook

- Partial wave decomposition (PWD): K. Hebeler, A. Nogga & K. Topolnicki

  PWD is computationally more expensive, due to higher dimension of integrals over Schwinger parameters

Why a new Framework?

Difficulties in formulation of regularized chiral EFT

- Regularization should preserve chiral and gauge symmetries
- Regularization should not affect long-range pion physics

Pion-propagator in Euclidean space: $q^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2} = \frac{1}{q^2 + M_\pi^2} - \frac{1}{\Lambda^2} + \frac{q^2 + M_\pi^2}{2\Lambda^4} + \ldots$$

all $1/\Lambda$-corrections are short-range interactions

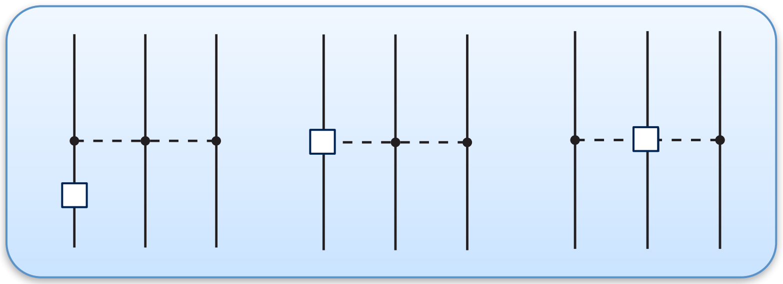
q_0 - dependence in exponential requires second and higher order time-derivatives in pion field in the chiral Lagrangian

→ Canonical quantization of the regularized theory becomes difficult (Ostrogradski - approach, Constrains, …)

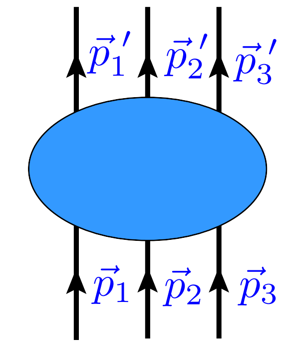
Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, Front. in Phys. 8 (2020) 98



← 1/m - corrections to TPE 3NF $\sim g_A^2$



$$V^{g_A^2}_{2\pi,1/m} = i\,\frac{g_A^2}{32 m F_\pi^4}\,\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\,\tau_1\cdot(\tau_2\times\tau_3)\left(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2\right)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i \qquad \vec{k}_i = \frac{1}{2}\left(\vec{p}_i{}' + \vec{p}_i\right)$$

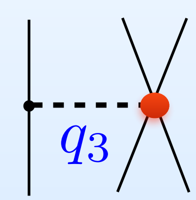
Naive local cut-off regularization of the current and potential

$$V^{g_A^2,\Lambda}_{2\pi,1/m} = V^{g_A^2}_{2\pi,1/m}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V^{Q^0,\Lambda}_{1\pi} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\,\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

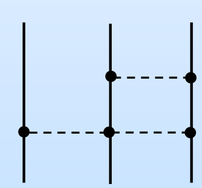
First iteration with OPE NN potential

$$V^{g_A^2,\Lambda}_{2\pi,1/m}\frac{1}{E-H_0+i\epsilon}V^{Q^0,\Lambda}_{1\pi} + V^{Q^0,\Lambda}_{1\pi}\frac{1}{E-H_0+i\epsilon}V^{g_A^2,\Lambda}_{2\pi,1/m} = \Lambda\,\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}\left(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3\right)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2} + \ldots$$

No such D-like term in chiral Lagrangian →



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi}$ =  + … is finite

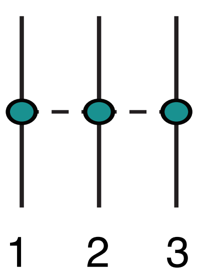
Subtraction Scheme

Choice of the short-range scheme

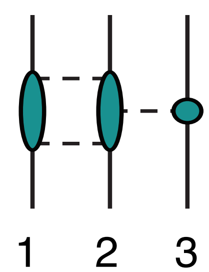
- NN case: local part of NN force vanishes if distance between nucleons vanishes

  → leads to natural size of LECs

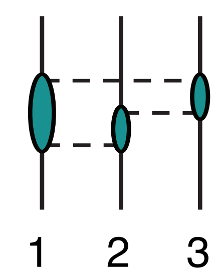
- 3N case: vanishing of the local part of 3NF is topology dependent



$= 0$ if $r_{12} = 0$ or $r_{23} = 0$

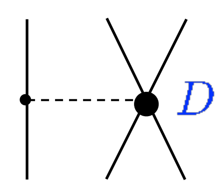


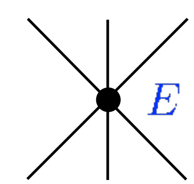
$= 0$ if $r_{12} = 0$



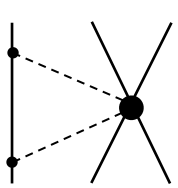
$= 0$ if $r_{12} = r_{23} = 0$

Can be achieved by adjustment of D- and E-like terms:





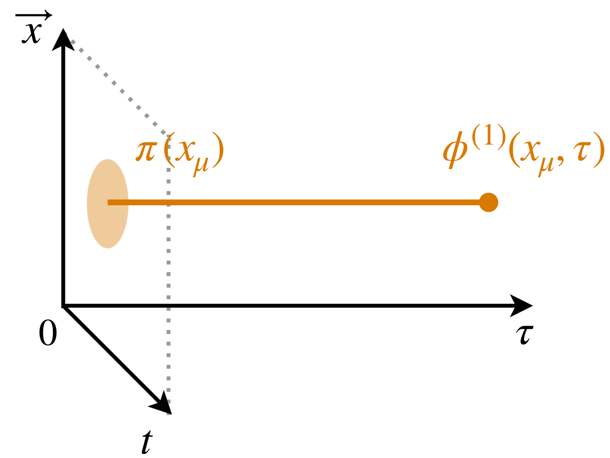
Vanishing of 3NF for any $r_{ij} = 0$ would require inclusion of two-pion-contact terms



Appear first at N^{5}LO and are expected to be small

Iterative solution in Coordinate Space

$\phi(x_\mu, \tau) \quad =$



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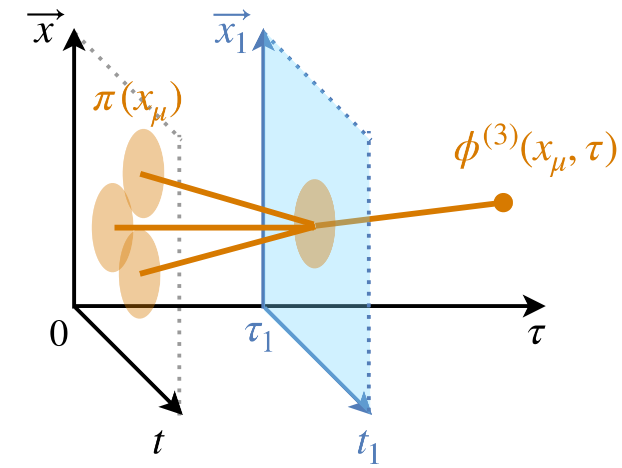

$+ \quad \ldots$

[integrated over $\vec{x}_1$, t_1, τ_1]

Light-shaded area visualizes smearing in Euclidean space of size $\sim \sqrt{2\tau}$

Solid line stands for Green-function:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right] G(x - y, \tau - s) = \delta(x - y)\delta(\tau - s)$$

$$G(x, \tau) = \theta(\tau) \int \frac{d^4 q}{(2\pi)^4} e^{-\tau(q^2 + M^2)} e^{-i\, q \cdot x}$$

$$\phi^{(1)}_b(x, \tau) = \int d^4 y \, G(x - y, \tau) \pi_b(y)$$

$$\phi^{(3)}_b(x, \tau) = \int_0^\tau ds \int d^4 y \, G(x - y, \tau - s) \big[(1 - 2\alpha) \partial_\mu \phi^{(1)}(y, s) \cdot \partial_\mu \phi^{(1)}(y, s) \phi^{(1)}_b(y, s)$$
$$- 4\alpha \, \partial_\mu \phi^{(1)}(y, s) \cdot \phi^{(1)}(y, s) \partial_\mu \phi^{(1)}_b(y, s) + \frac{M^2}{2} \phi^{(1)}(y, s) \cdot \phi^{(1)}(y, s) \phi^{(1)}_b(y, s)\big]$$