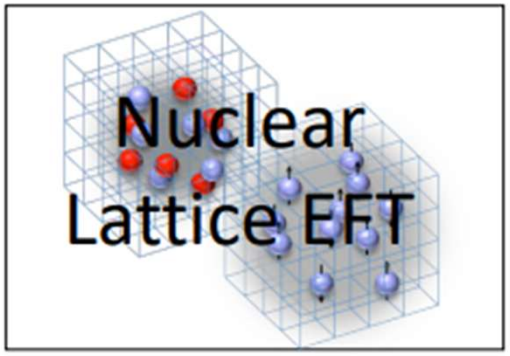


Carbon and Oxygen isotopes in Nuclear Lattice Effective Field Theory

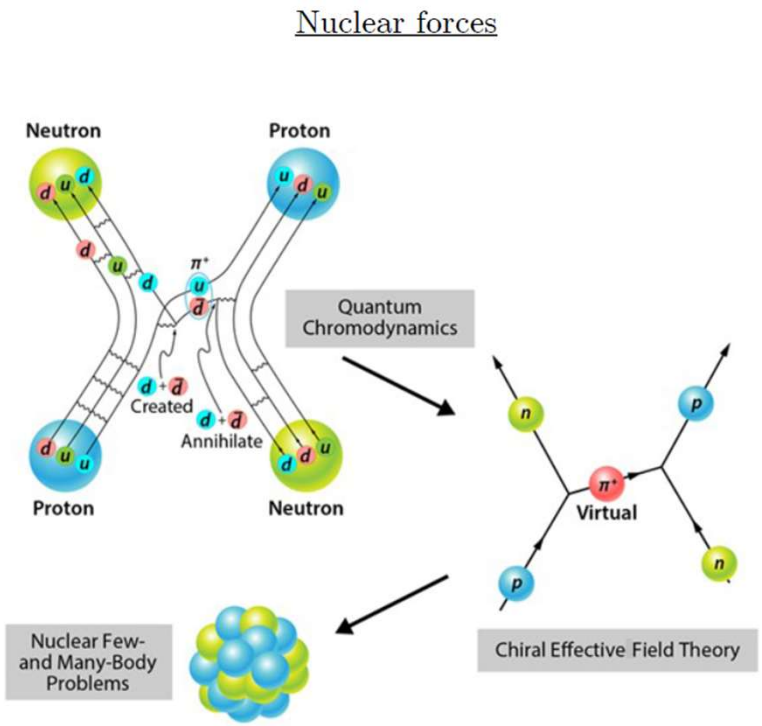
Young-Ho Song (IRIS, IBS)

NLEFT collaboration(Myungkuk Kim, Youngman Kim, Kihyeon Cho, Serdar Elhatisari, Dean Lee, Yuan-Zhuo Ma, Ulf-G. Meißner)

ibs 기초과학연구원 Institute for Basic Science

2025.03.01.-03., Frontiers in NLEFT , Beijing

Ab-initio method



Dream: Lattice QCD
➔ NN , NNN interaction
➔ Effective interactions/models (shell model, DFT, optical potential, collective excitation)

- *ab-initio* Nuclear Physics
  - (1) nucleon degrees of freedom
  - (2) nucleon-nucleon interaction

Goal: predict wide range of nuclear phenomena (without parameter fitting, model assumption) from nuclear interaction (for 2-body,3-body, many-body, based on QCD)

- Direct connection between

Nuclear Force ↔ Nuclear Phenomena

- Consistent approach :
- NN scattering, bound nuclei, reaction, nuclear matter

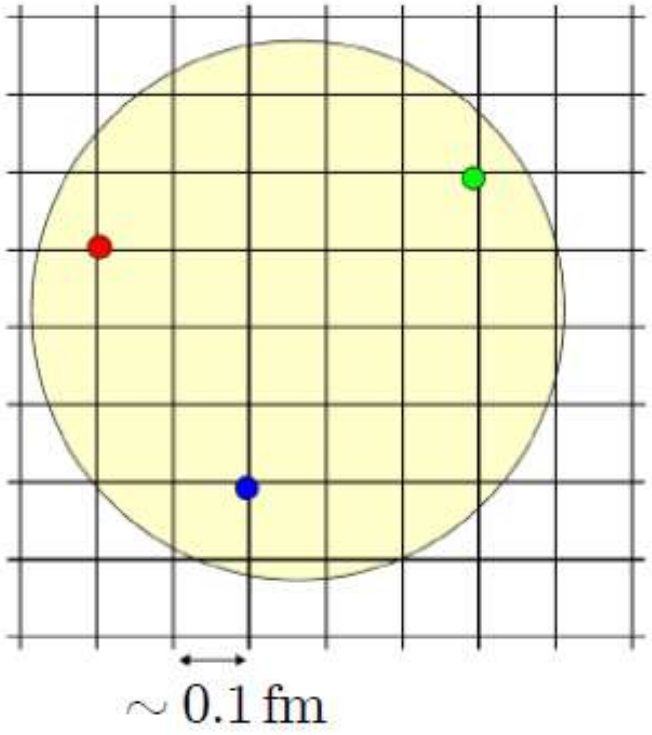
Ab initio Quantum many-body

- Challenge to ab initio quantum many-body problem.
- Requires:
- 1. Reliable theoretical tools
  - NLEFT : Auxiliary field Monte Carlo Method
  - → Sign problem !
  - Goal 1 : reduce the sign problem in NLEFT calculation
- 2. Nuclear interaction which explain simultaneously scattering, binding energies, charge radius of wide range of nuclei and nuclear matter, neutron matter
  - Goal 2: find out what properties of nuclear interactions are necessary.

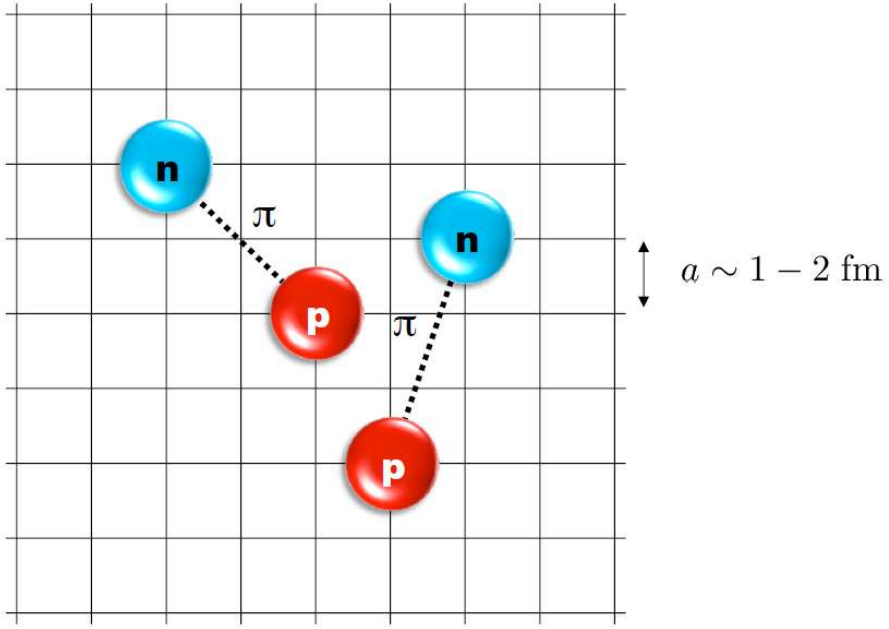
Nuclear Lattice Effective Field Theory

- One of ab initio method for many fermion system

Nucleon in LQCD



Nucleons as point particles on the lattice



Path integral

Correlator function for A Nucleons $\quad Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

Slater Determinants for A free Nucleons

Ground state energy by time derivative of the correlator

$$E(t) = -\frac{d}{dt} \ln Z_A(t)$$

At large time only ground states survive

$$E_A^0 = \lim_{t \to \infty} E_A(t)$$

$$|\Psi_A\rangle = \sum_n c_n |n\rangle_A,$$

$$Z_A(t) = \langle \Psi_A | e^{-tH} | \Psi_A \rangle = \sum_n c_n e^{-tE_n}$$

Transfer matrix

Transfer matrix operator formalism $\hat{M} =: \exp(-H_{\mathrm{LO}}\, a_t) :$

: O : represent Normal ordering.

$$Z(L_t) = \langle \psi_I \,|\hat{M}(L_t - 1)\, \hat{M}(L_t - 2)\, \ldots \hat{M}(1)\, \hat{M}(0) \,|\, \psi_I \rangle$$

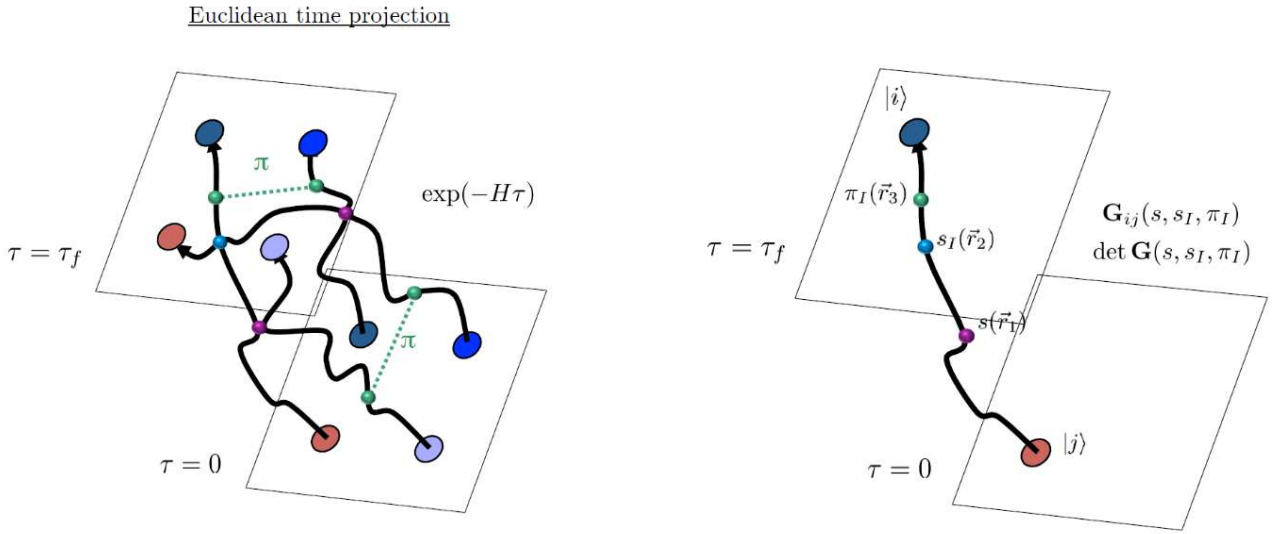
$$\lim_{L_t \to \infty} \frac{Z(L_t + 1)}{Z(L_t)} = e^{-E_0\, a_t}$$

$$\lim_{L_t \to \infty} \frac{\langle \psi_I |\, \hat{M}^{L_t/2}\, H_{\mathrm{LO}}\, \hat{M}^{L_t/2} \,|\psi_I\rangle}{\langle \psi_I |\, \hat{M}^{L_t} \,|\psi_I\rangle} = E_0$$

Auxiliary Field Monte Carlo

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right] \quad \times \quad (N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right] \quad \rangle \quad sN^\dagger N$$



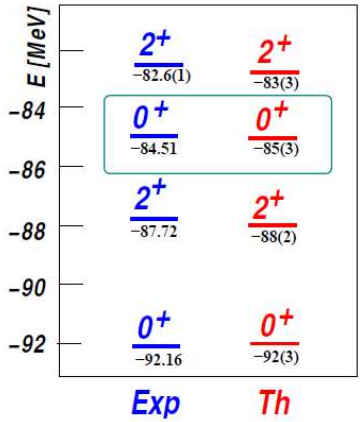
Interacting nucleons
→ Nucleons interacts with auxiliary fields
(no direct interaction between nucleons)

computing fermion Correlator amplitude
→ Integration over auxiliary fields
→ M.C. integral (sampling auxiliary field)

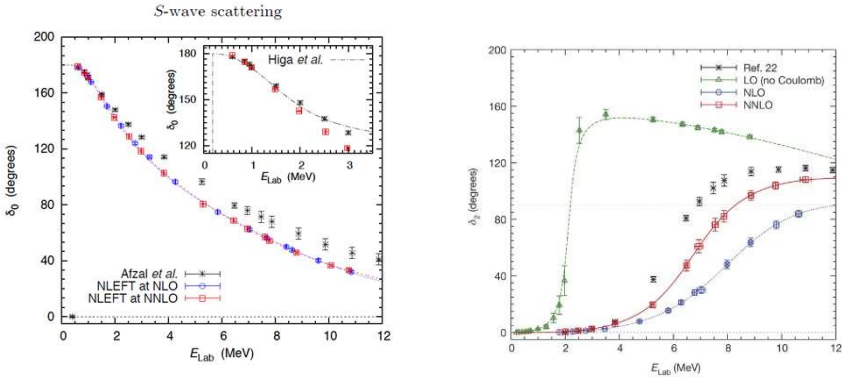
Applications of NLEFT

- Has been successfully applied to
  - Nuclear matter, Cold atom, dilute fermion system
  - Finite nuclei (A<=50)
  - First ab-initio calculation of Hoyle state
  - Cluster structure of ^{12}C and ^{16}O
  - NN scattering, N-D scattering
  - Alpha-alpha scattering
  - radiative capture, fusion
  - Etc.

The first ab-initio calculation of Hoyle state

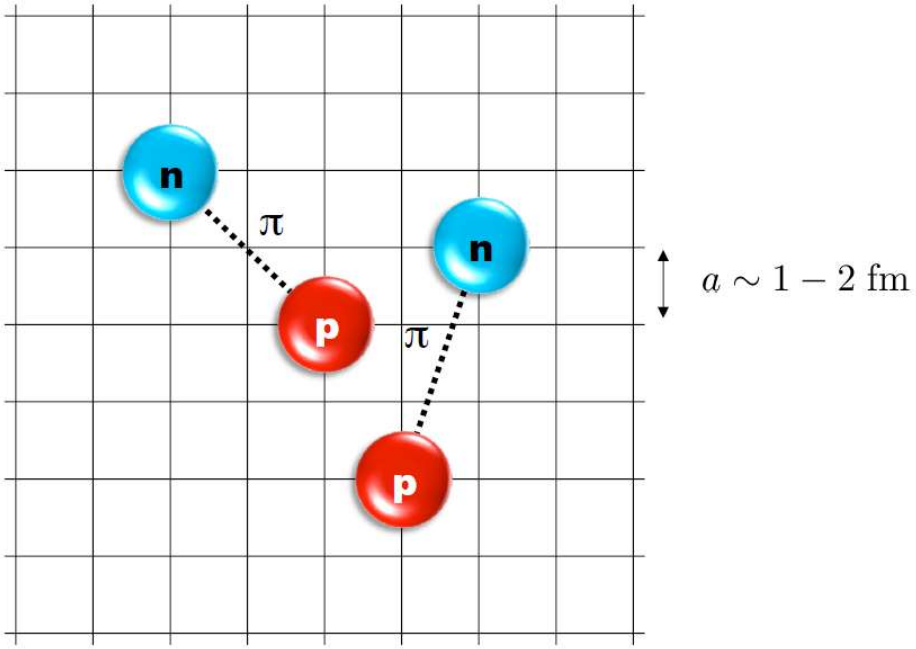


Epelbaum, Krebs, Lähde, Lee, Meißner: Phys. Rev. Lett. 109, 252501 (2012)



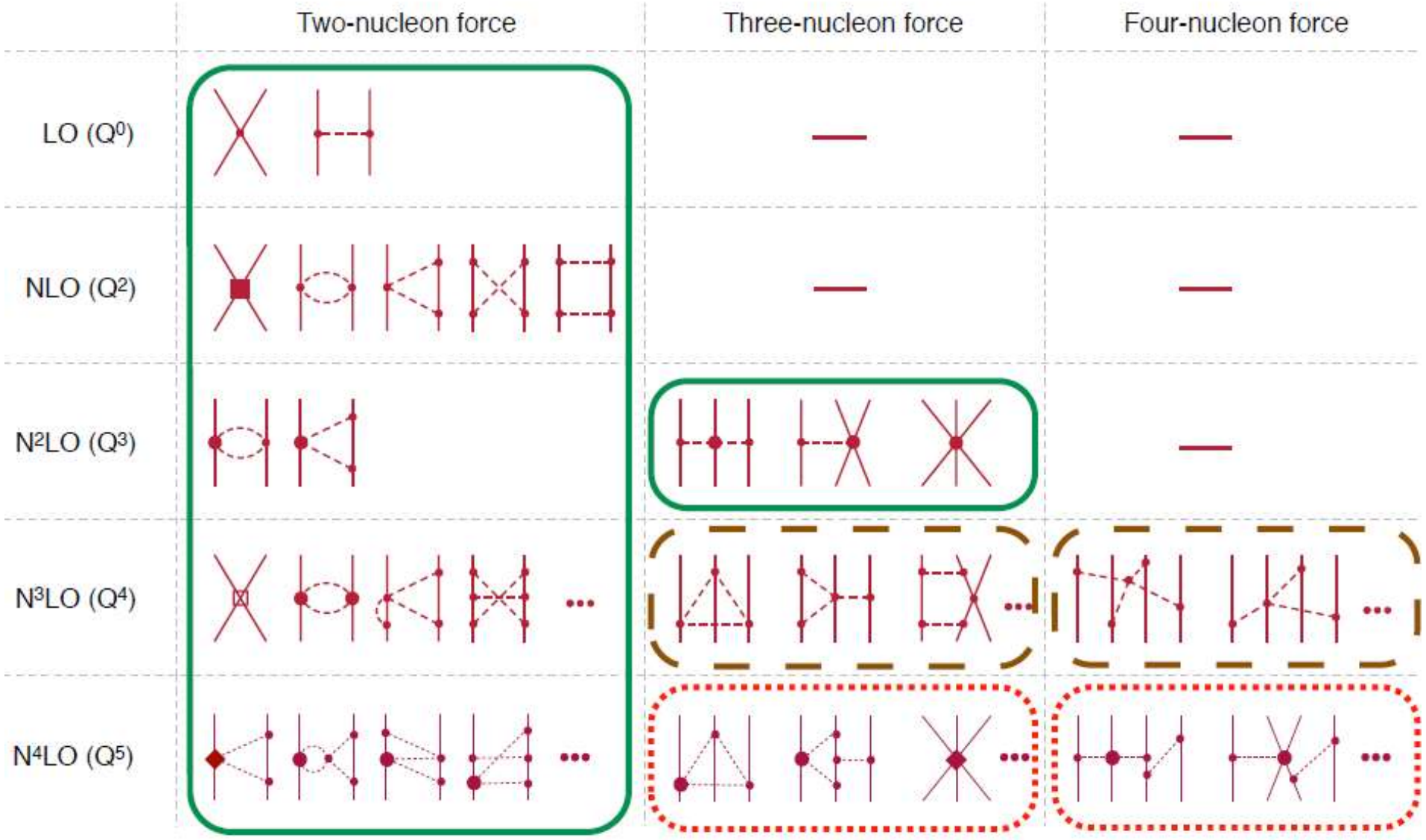
Ab initio alpha-alpha scattering
(Nature 528, 111-114(2015))

Lattice Hamiltonian



- We need to introduce a lattice scale in space and time:
- momentum space cutoff ~ 150 MeV → lattice spacing a= 1.316 fm
- Time cutoff ~ 1000 MeV
- We need to determine coefficients of interaction for the lattice size. (regularization scale.)
- Two-body interaction coefficients can be determined from phase shifts of np scattering.
- Three-body interaction can be fixed from binding energy of A>=3.

Chiral Effective Field Theory



Lattice chiral Hamiltonian at Leading order

- At Leading order, kinetic energy + contact interaction + one pion exchange

$$H = H_{\rm free} + V_{\rm 2N}^{\rm short} + V_{\rm 2N}^{\rm long}.$$

$$H_{\rm free} = \frac{49}{12m}\sum_{\mathbf{n}} a^\dagger(\mathbf{n})a(\mathbf{n}) - \frac{3}{4m}\sum_{\mathbf{n},i}\sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle_i} a^\dagger(\mathbf{n}')a(\mathbf{n}) + \frac{3}{40m}\sum_{\mathbf{n},i}\sum_{\langle\langle \mathbf{n}'\,\mathbf{n}\rangle\rangle_i} a^\dagger(\mathbf{n}')a(\mathbf{n}) - \frac{1}{180m}\sum_{\mathbf{n},i}\sum_{\langle\langle\langle \mathbf{n}'\,\mathbf{n}\rangle\rangle\rangle_i} a^\dagger(\mathbf{n}')a(\mathbf{n}).$$

- Kinetic

$$V_{\rm OPE} = -\frac{g_A^2}{8F_\pi^2}\sum_{\mathbf{n}',\mathbf{n},S',S,I} :\rho_{S',I}(\mathbf{n}')f_{S'S}(\mathbf{n}'-\mathbf{n})\rho_{S,I}(\mathbf{n}):,$$

- Long range OPE

$$V = \frac{C}{2}\int d^3\boldsymbol{r} :[\rho(\boldsymbol{r})]^2:,$$

$$V_{I^2} = \frac{C_{I^2}}{2}\sum_{I=1,2,3}\int d^3\boldsymbol{r} :[\rho_I(\boldsymbol{r})]^2:,$$

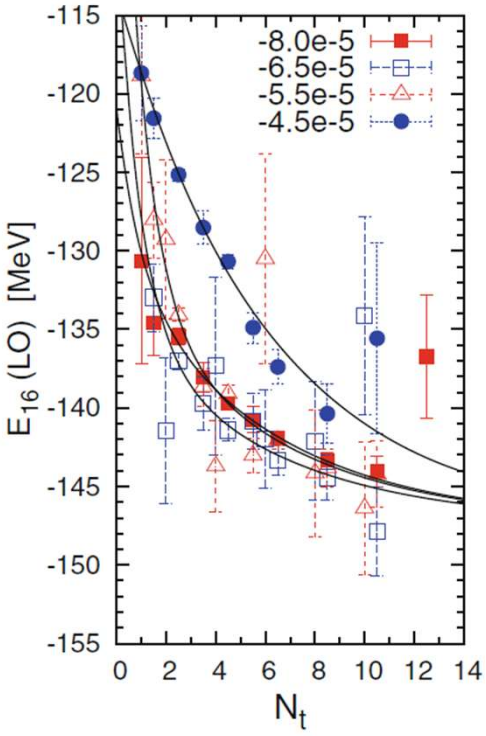
- Contact

Sign problem in NLEFT

- However, there is a difficulty in auxiliary MC calculation

$$Z(t) = \int \mathcal{D}s Z(s,t), \quad Z(s,t) = e^{-\frac{s^2}{2}} \det X(s,t),$$
$$= \int \mathcal{D}s e^{i\theta(s,t)} |Z(s,t)|$$

$$\langle O \rangle = \frac{\langle O e^{i\theta} \rangle_{pq}}{\langle e^{i\theta} \rangle_{pq}}, \quad \langle O \rangle_{pq} \equiv \frac{\int ds |\det X(s)| O}{\int ds |\det X(s)|}.$$



- We needs a large Euclidean time extrapolation
- If the denominator's sign oscillates rapidly
- → large uncertainty in the expectation value
- → sign problem
- SU(4) symmetric interaction in isospin symmetric system
- → No sign problem

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Essential elements for nuclear binding

Bing-Nan Lu [a], Ning Li [a], Serdar Elhatisari [b,c], Dean Lee [a,*], Evgeny Epelbaum [d], Ulf-G. Meißner [b,e,f]

$$H_{\mathrm{SU(4)}} = H_{\mathrm{free}} + \frac{1}{2!}C_2 \sum_{\boldsymbol{n}} \tilde{\rho}(\boldsymbol{n})^2 + \frac{1}{3!}C_3 \sum_{\boldsymbol{n}} \tilde{\rho}(\boldsymbol{n})^3,$$

$$\tilde{\rho}(\boldsymbol{n}) = \sum_i \tilde{a}_i^\dagger(\boldsymbol{n})\tilde{a}_i(\boldsymbol{n}) + s_L \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} \sum_i \tilde{a}_i^\dagger(\boldsymbol{n}')\tilde{a}_i(\boldsymbol{n}'),$$

$$\tilde{a}_i(\boldsymbol{n}) = a_i(\boldsymbol{n}) + s_{NL} \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} a_i(\boldsymbol{n}')$$

Minimal nuclear interaction

Which reproduce

(1) Light nuclei

(2) medium mass nuclei

(3) neutron matter

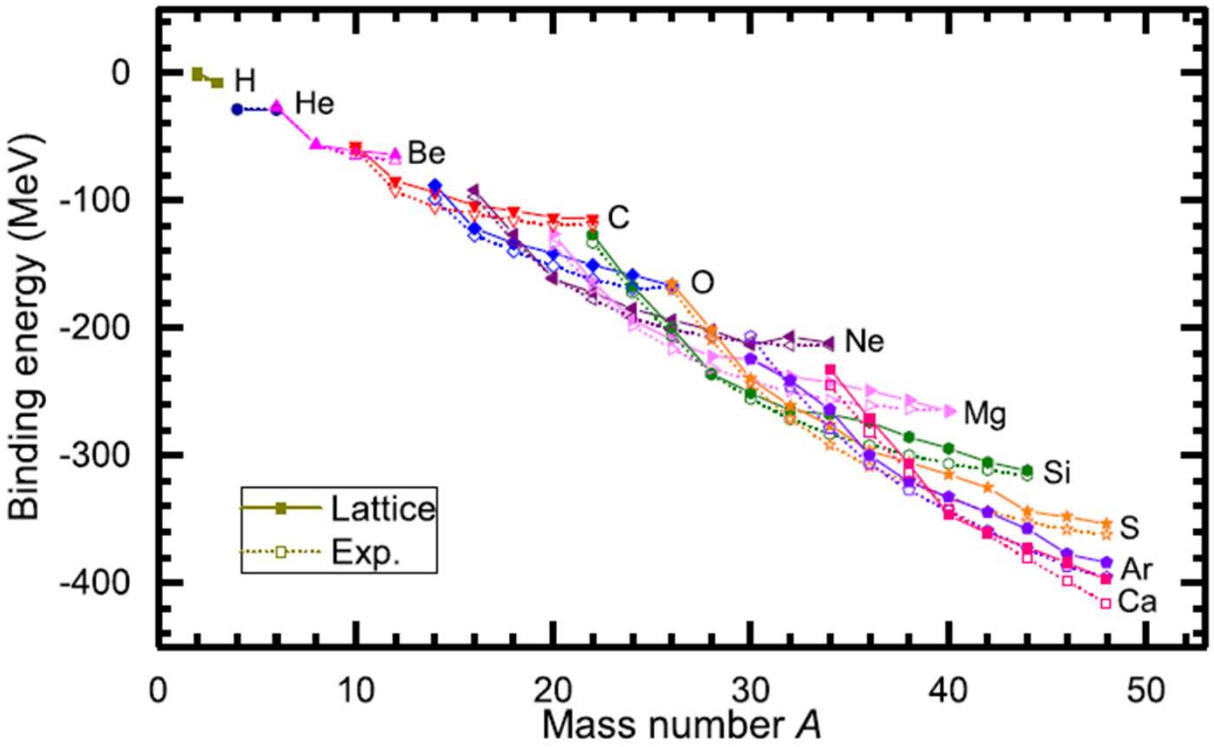
simultaneously up to few percent error in binding energy and charge radius

Only Four parameters

1. Strength of the two-nucleon S-wave interaction
2. Range of the two-nucleon S-wave interaction
3. Strength of three-nucleon contact interaction
4. Range of the local part of the two-nucleon interaction

Except for the Coulomb potential, the interaction is invariant under Wigner's SU(4) symmetry.

No sign problem



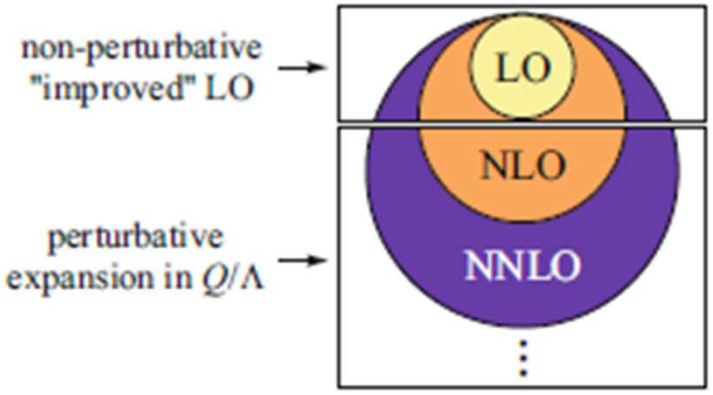
	B	Exp.	$R_{\rm ch}$	Exp.
^{3}H	8.48(2)(0)	8.48	1.90(1)(1)	1.76
^{3}He	7.75(2)(0)	7.72	1.99(1)(1)	1.97
^{4}He	28.89(1)(1)	28.3	1.72(1)(3)	1.68
^{16}O	121.9(1)(3)	127.6	2.74(1)(1)	2.70
^{20}Ne	161.6(1)(1)	160.6	2.95(1)(1)	3.01
^{24}Mg	193.5(02)(17)	198.3	3.13(1)(2)	3.06
^{28}Si	235.8(04)(17)	236.5	3.26(1)(1)	3.12
^{40}Ca	346.8(6)(5)	342.1	3.42(1)(3)	3.48

Can we improve the agreement by
Including higher order corrections?

Lattice chiral Hamiltonian (N3LO)

Full N3LO Hamiltonian includes

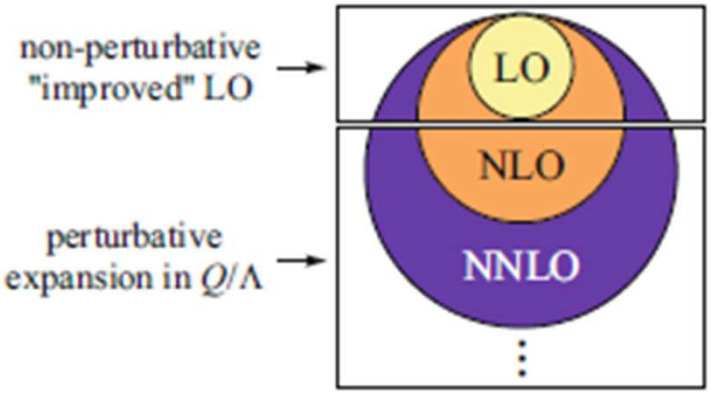
- Free Hamiltonian(Kinetic term)
- Short range (nonlocal smeared) contact interactions up to Q^4 order
- Isospin-breaking short range interactions
- One pion exchange potential
- Two pion exchange potential up to Q^4 order
- Coulomb interaction
- Long range isospin breaking interaction(isospin dependence in OPE)
- Galilean Invariance Restoration (GIR) term (because of non-local interaction.)
- Three nucleon interaction



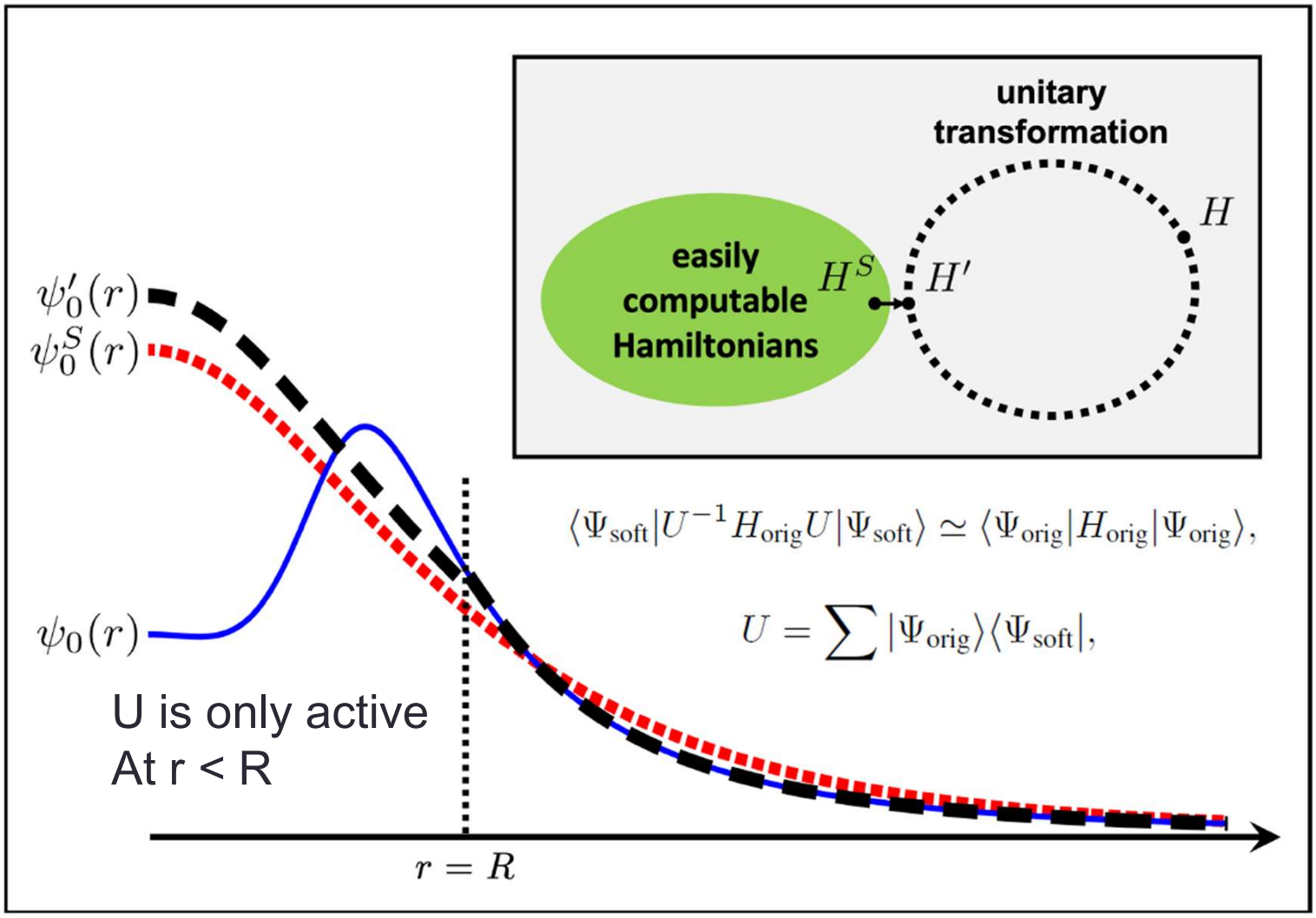
Difficulty with full chiral interaction

- Sign problem
  - NLEFT suffers sign problems at large Euclidean time limit
  - Large cancellation between positive and negative contributions→ large uncertainty.
  - SU(4) symmetric interaction does not have sign problem
  - One pion exchange and higher order chiral interaction, short range repulsion
  - Difficulty with Asymmetric nuclei
  - Needs a remedy to extend to neutron rich isotopes.

- A new approach to reduce the sign problem.
- → Wave function matching



Wave function matching

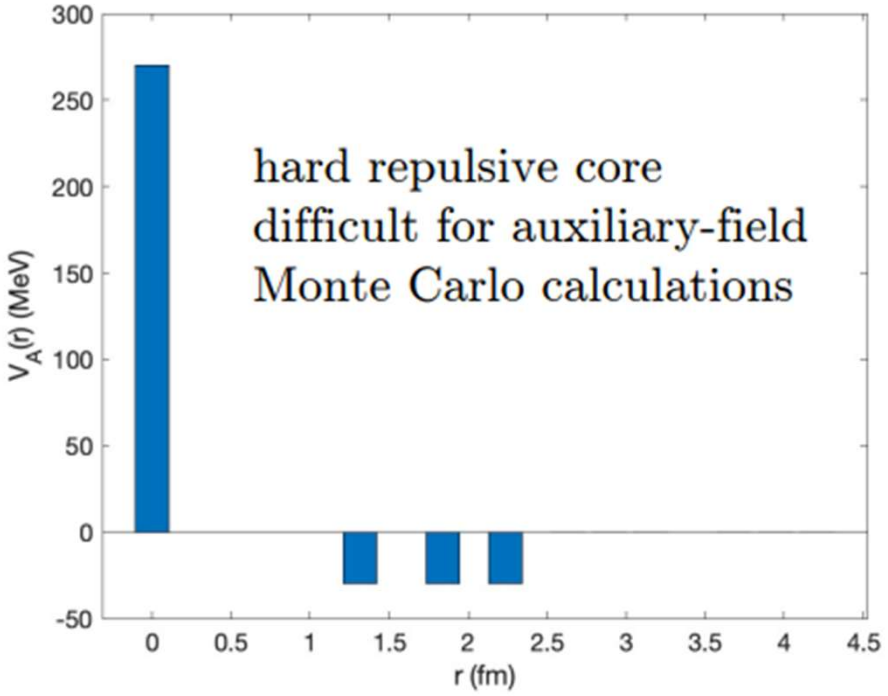


H and H' are fully equivalent to two-body observables

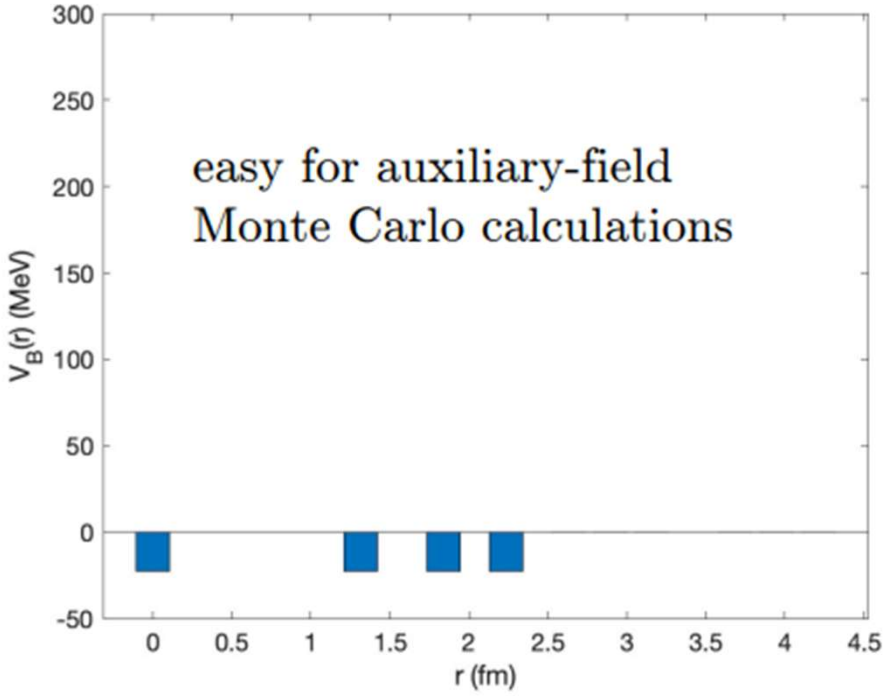
The goal is to make the perturbation expansion from "simple" wave function gives a good convergence

Wave Function Matching

$V_A(r)$



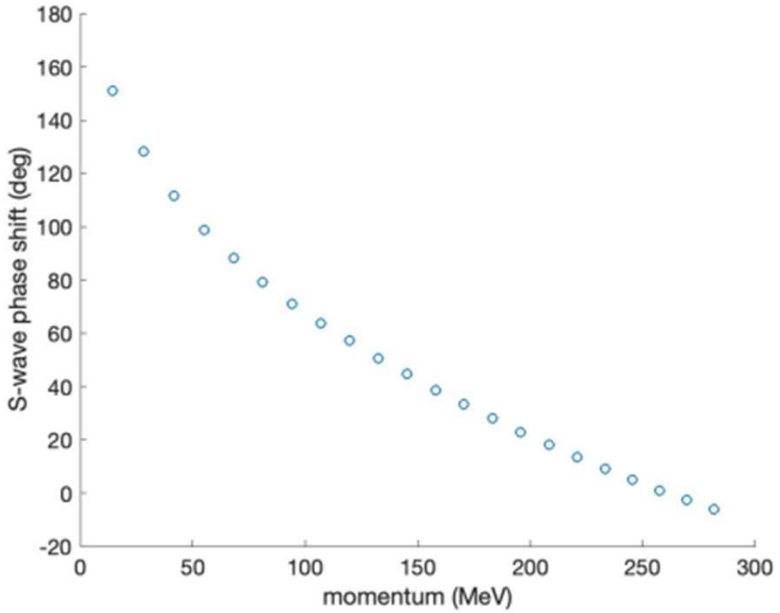
$V_B(r)$



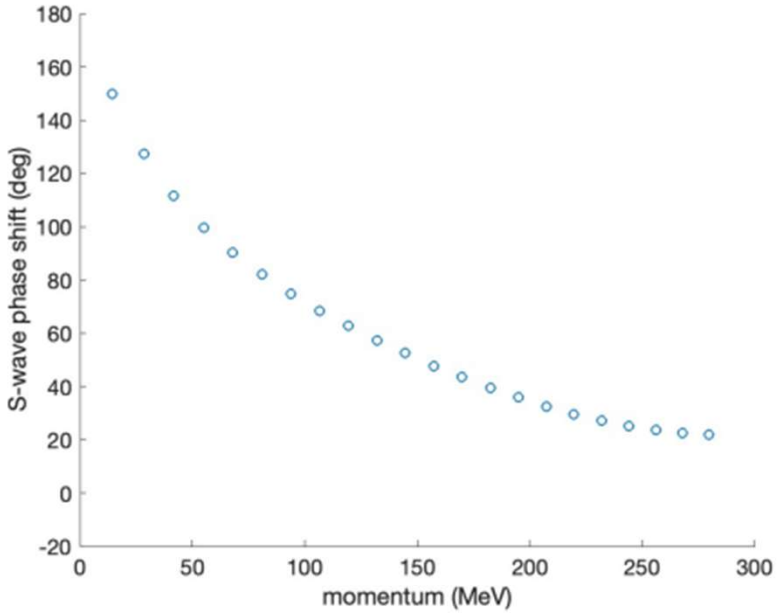
From Dean Lee

Wave Function Matching

$V_A(r)$



$V_B(r)$



Phase shifts from two interactions are different

From Dean Lee

Wave Function Matching

Introduce short range Unitary transformation

$$U : P\left|\psi_A^0\right\rangle / \| P\left|\psi_A^0\right\rangle \| \rightarrow P\left|\psi_B^0\right\rangle / \| P\left|\psi_B^0\right\rangle \|$$

$$U : H_A \rightarrow H'_A = U^\dagger H_A U$$

$$V'_A = H'_A - K = U^\dagger H_A U - K$$

Two potentials V_A and V'_A are unitarily equivalent.

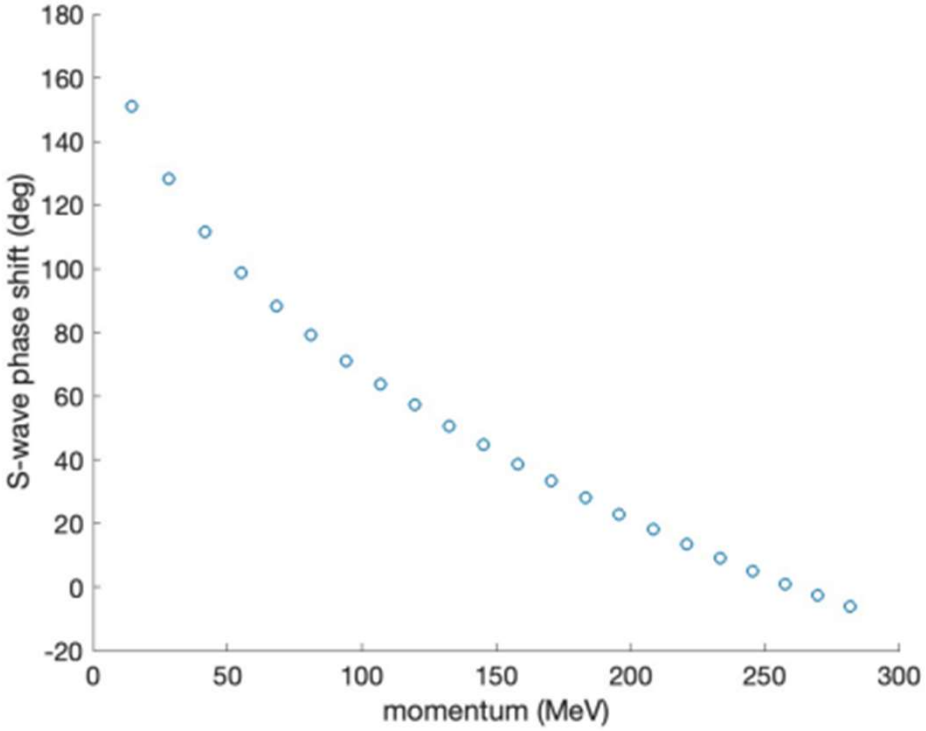
→ Same phase shifts.

→ H'_A has eigenvectors of H_B, but eigenvalues of H_A

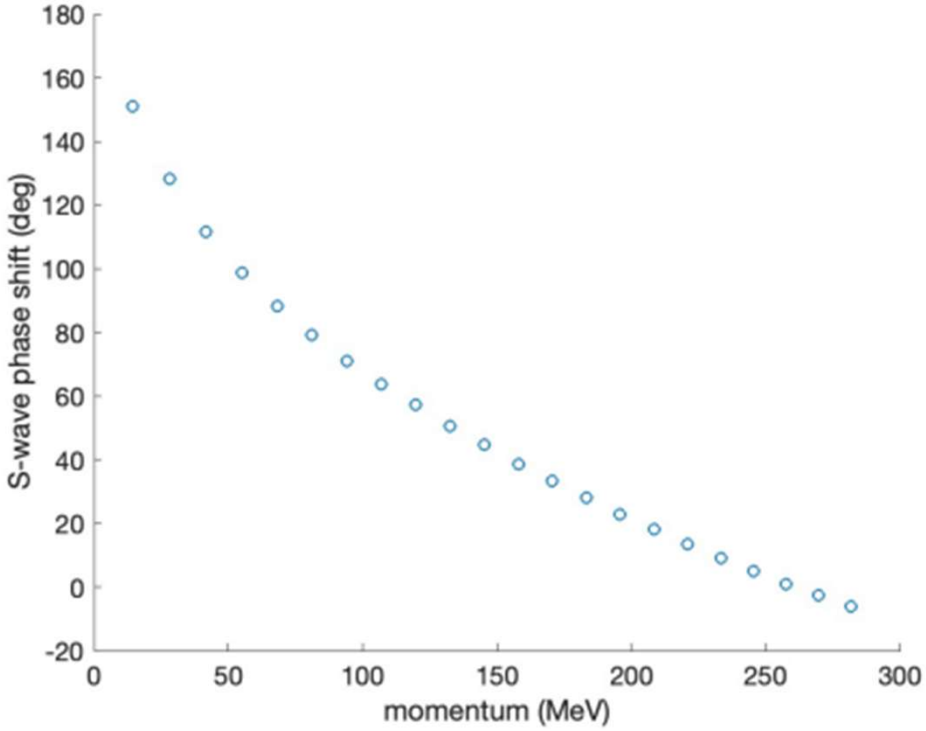
From Dean Lee

Wave Function Matching

$V_A(r)$



$V'_A(r, r')$



Two potentials V_A and V'_A are unitarily equivalent.

→ Same phase shifts.

→ H'_A has eigenvectors of H_B, but eigenvalues of H_A

From Dean Lee

Wave Function Matching

$R = 2.6$ fm

$E_{A,n} = E'_{A,n}$ (MeV)	$\langle\psi_{B,n}\|H_A\|\psi_{B,n}\rangle$ (MeV)	$\langle\psi_{B,n}\|H'_A\|\psi_{B,n}\rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

Energy from Perturbation theory with H'_A works better.
Difference H_A- H'_A can be treated as a correction.

From Dean Lee

Wave function matching Hamiltonian

- "Simple" Hamiltonian

$$H^S = K + \frac{c_{\mathrm{SU}(4)}}{2} \sum_{\vec{n}} : \left[\tilde{\rho}^{(1)}(\vec{n})\right]^2 : + \frac{c_I}{2} \sum_{I,\vec{n}} : \left[\tilde{\rho}_I^{(1)}(\vec{n})\right]^2 : + V_{\mathrm{OPE}}^{\Lambda_\pi},$$

$$\tilde{\rho}^{(d)}(\vec{n}) = \sum_{i,j=0,1} \tilde{a}_{i,j}^\dagger(\vec{n})\, \tilde{a}_{i,j}(\vec{n}) + s_{\mathrm{L}} \sum_{|\vec{n}-\vec{n}'|^2=1}^{d} \sum_{i,j=0,1} \tilde{a}_{i,j}^\dagger(\vec{n}')\, \tilde{a}_{i,j}(\vec{n}'),$$

$$\tilde{\rho}_I^{(d)}(\vec{n}) = \sum_{i,j,j'=0,1} \tilde{a}_{i,j}^\dagger(\vec{n})\, [\tau_I]_{j,j'}\, \tilde{a}_{i,j'}(\vec{n}) + s_{\mathrm{L}} \sum_{|\vec{n}-\vec{n}'|^2=1}^{d} \sum_{i,j,j'=0,1} \tilde{a}_{i,j}^\dagger(\vec{n}')\, [\tau_I]_{j,j'}\, \tilde{a}_{i,j'}(\vec{n}').$$

$$\tilde{a}_{i,j}(\vec{n}) = a_{i,j}(\vec{n}) + s_{\mathrm{NL}} \sum_{|\vec{n}'-\vec{n}|=1} a_{i,j}(\vec{n}').$$

(local,non-local smeared operators)

Wave function matching Hamiltonian

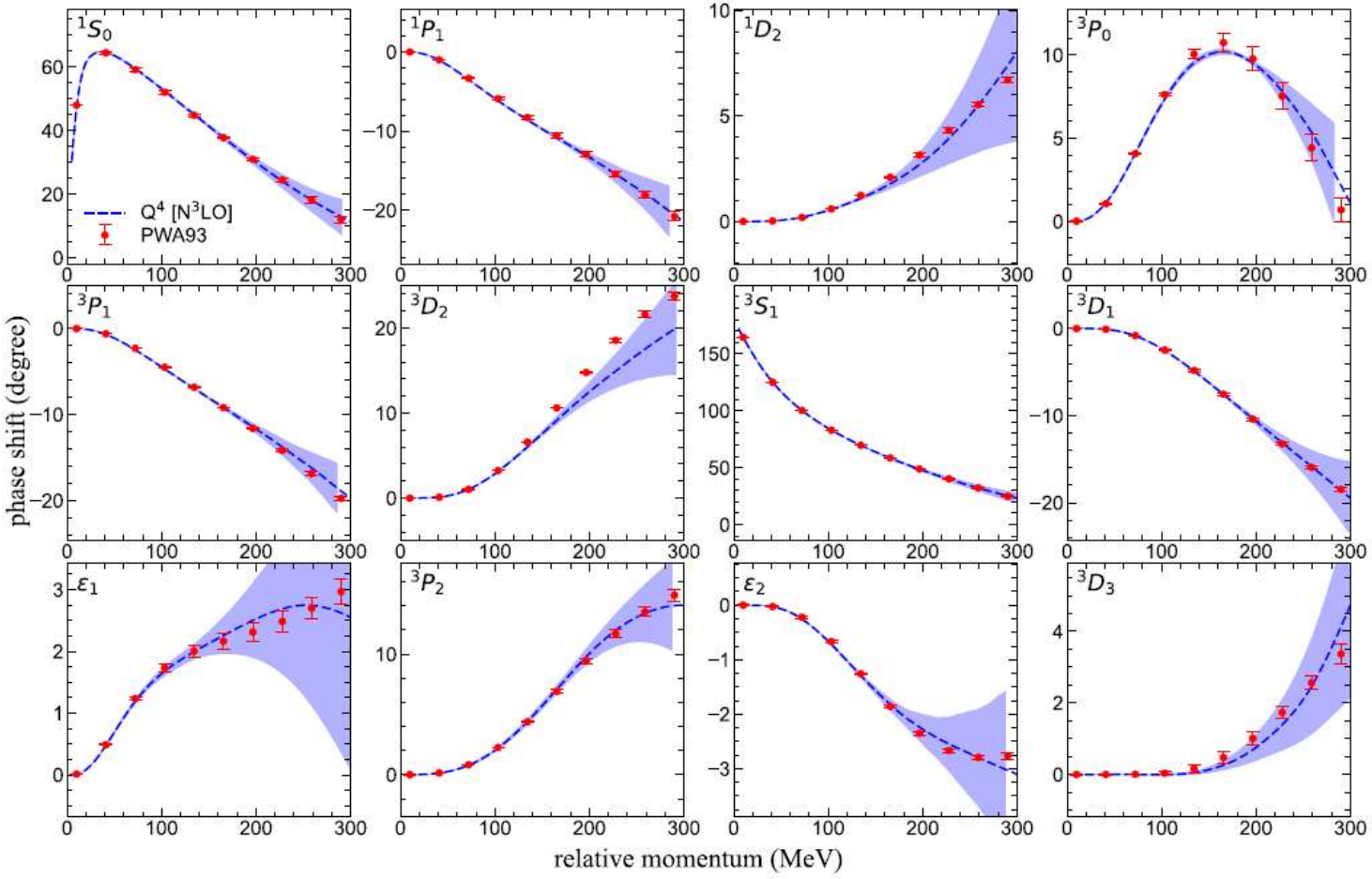
- N3LO Hamiltonian

$$H = K + V_{\mathrm{OPE}}^{\Lambda_\pi} + V_{\mathrm{C}_\pi}^{\Lambda_\pi} + V_{\mathrm{Coulomb}} + V_{\mathrm{3N}}^{\mathrm{Q}^3} + V_{\mathrm{2N}}^{\mathrm{Q}^4} + W_{\mathrm{2N}}^{\mathrm{Q}^4} + V_{\mathrm{2N,WFM}}^{\mathrm{Q}^4} + W_{\mathrm{2N,WFM}}^{\mathrm{Q}^4},$$

$$V_{\mathrm{3N}}^{\mathrm{Q}^3} = V_{c_E}^{(l)} + V_{c_E}^{(t)} + V_{c_E}^{(d)} + V_{c_D}^{(d)} + V_{\mathrm{3N}}^{(\mathrm{TPE})},$$

V_2N : short range NN interactions
W_2N: GIR restoration term for V_2N
V_2N,WFM : difference from H'-H
W_2N,WFM: GIR restoration correction to V_2N,WFM
V_3N : contains short range 3N interaction parameters(to be fitted)
and two pion exchange correction to 3N

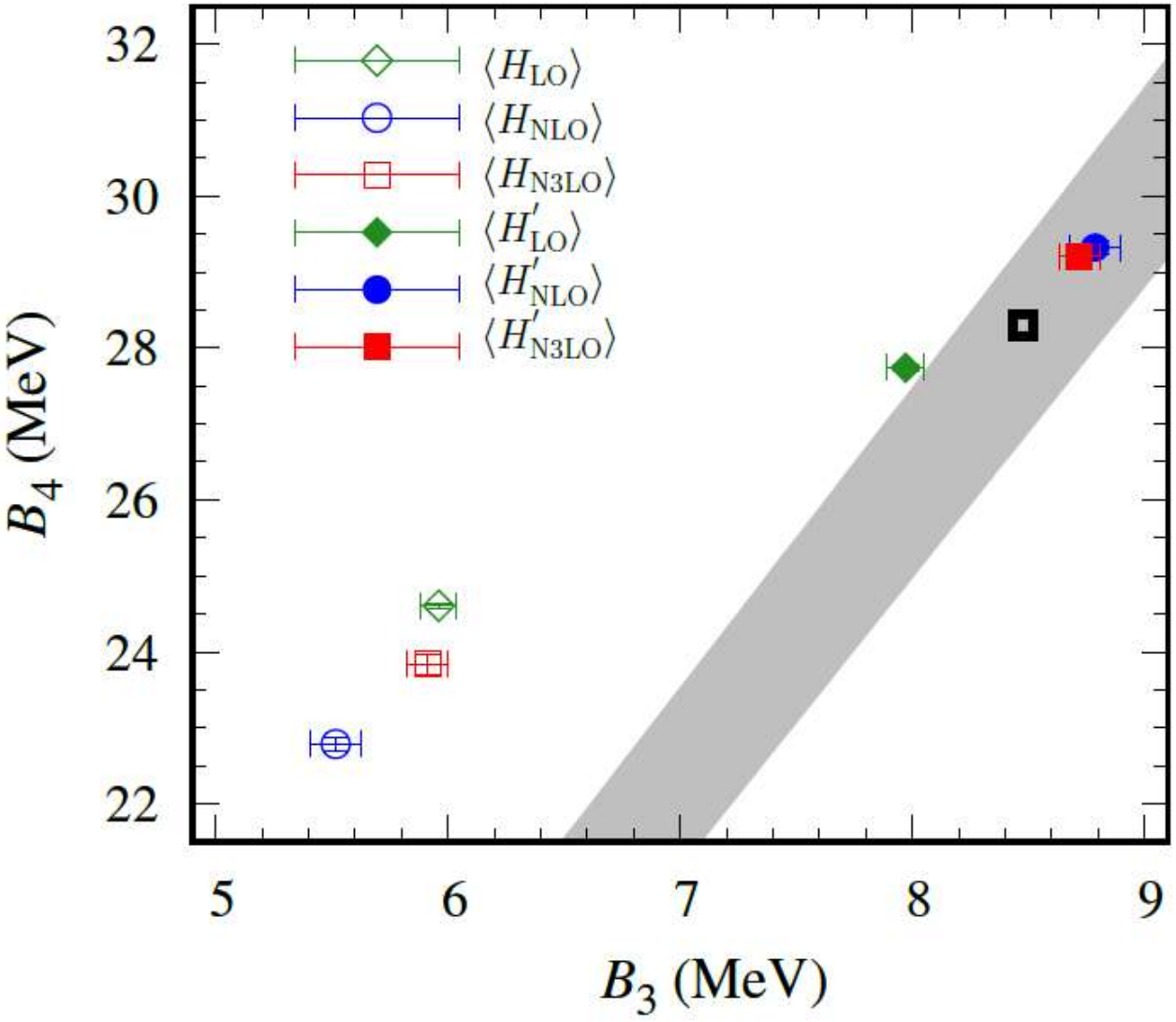
NN phase shifts from N3LO interaction



Original Chiral Hamiltonian, H, is fitted to phase shifts. (up to N3LO)

H' is computed : equivalent to original Hamiltonian(gives the same phase shifts).

Tjon line from WFM method



Realistic two-body interaction
→ Correlation between 3-body and 4-body binding energy
→ Tjon Line

WFM approach gives binding e nergy lies on the Tjon line.

→ Accurate reproduction of B. E. requires 3-body force.

Tale of two interactions

Nearly identical two interactions can give quite different phase:
sensitivity to range and locality of interaction

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{8}Be	−58.61(14)	−59.73(6)	−56.51(14)	−57.29(7)	−56.591
^{12}C	−88.2(3)	−95.0(5)	−84.0(3)	−89.9(5)	−92.162
^{16}O	−117.5(6)	−135.4(7)	−110.5(6)	−126.0(7)	−127.619
^{20}Ne	−148(1)	−178(1)	−137(1)	−164(1)	−160.645

$$\frac{E_{^{8}\mathrm{Be}}}{E_{^{4}\mathrm{He}}} = 1.997(6)$$

$$\frac{E_{^{12}\mathrm{C}}}{E_{^{4}\mathrm{He}}} = 3.00(1)$$

$$\frac{E_{^{16}\mathrm{O}}}{E_{^{4}\mathrm{He}}} = 4.00(2)$$

$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^{4}\mathrm{He}}} = 5.03(3)$$

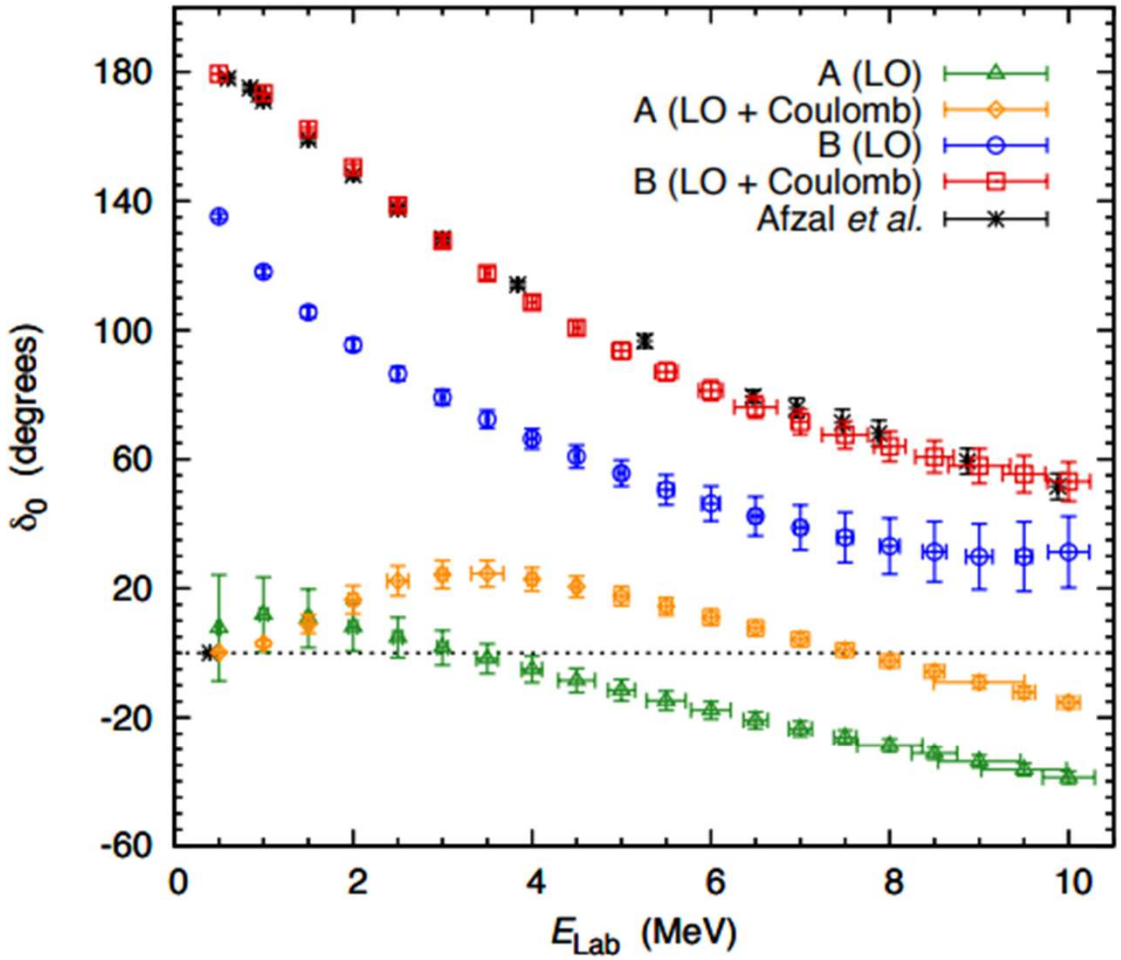
PRL 117, 132501 (2016)

From presentation file by Dean Lee

Tale of two interactions

Nearly ident
sensitivity

Nucleus
^{8}Be
^{12}C
^{16}O
^{20}Ne



ulomb)	Experiment
7)	−56.591
5)	−92.162
7)	−127.619
)	−160.645

$$\frac{E_{^{8}\mathrm{Be}}}{E_{^{4}\mathrm{He}}} = 1.9$$

$$\frac{E_{^{12}\mathrm{C}}}{E_{^{4}\mathrm{He}}} = 3.($$

$$\frac{E_{^{16}\mathrm{O}}}{E_{^{4}\mathrm{He}}} = 4.($$

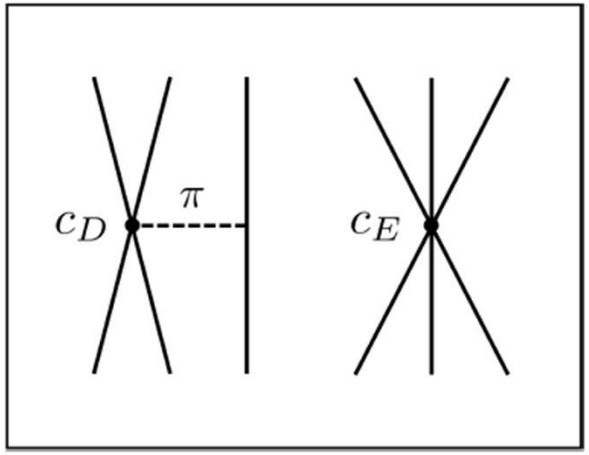
$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^{4}\mathrm{He}}} = 5.03(3)$$

Very different alpha-alpha scattering

PRL 117, 132501 (2016)

From presentation file by Dean Lee

3-body force



NNLO

$$V_{\rm 3N}^{\rm Q^3} = V_{c_E}^{(l)} + V_{c_E}^{(t)} + V_{c_E}^{(d)} + V_{c_D}^{(d)} + V_{\rm 3N}^{\rm (TPE)} ,$$

$$V_{c_E}^{(d)} = \frac{1}{6}\frac{c_E^{(d)}}{2f_\pi^4\Lambda_\chi} : \sum_{\vec{n}} \left[\rho^{(d)}(\vec{n})\right]^3 : ,$$

$$V_{c_D}^{(d)} = -\frac{c_D^{(d)}\, g_A}{4f_\pi^4\Lambda_\chi} \sum_{\vec{n},S,I} \sum_{\vec{n}',S'} : \rho_{S',I}^{(0)}(\vec{n}')\, f_{S',S}(\vec{n}'-\vec{n})\, \rho_{S,I}^{(d)}(\vec{n})\, \rho^{(d)}(\vec{n}) : ,$$

$$V_{c_E}^{(l)} = c_E^{(l)} \sum_{\vec{n},\vec{n}',\vec{n}''} \rho^{(0)}(\vec{n})\, \rho^{(0)}(\vec{n}')\, \rho^{(0)}(\vec{n}'')\, \delta_{|\vec{n}-\vec{n}'|,1}\, \delta_{|\vec{n}-\vec{n}''|,1}\, \delta_{|\vec{n}'-\vec{n}''|,2},$$

$$V_{c_E}^{(t)} = c_E^{(t)} \sum_{\vec{n},\vec{n}',\vec{n}''} \rho^{(0)}(\vec{n})\, \rho^{(0)}(\vec{n}')\, \rho^{(0)}(\vec{n}'')\, \delta_{|\vec{n}-\vec{n}'|,\sqrt{2}}\, \delta_{|\vec{n}-\vec{n}''|,\sqrt{2}}\, \delta_{|\vec{n}'-\vec{n}''|,\sqrt{2}}.$$

At N3LO:
TPE and
Adjustments to
cD and cE terms.

$$\rho^{(d)}(\vec{n}) = \sum_{i,j=0,1} a_{i,j}^\dagger(\vec{n})\, a_{i,j}(\vec{n}) + s_{\rm L} \sum_{|\vec{n}-\vec{n}'|^2=1}^{d} \sum_{i,j=0,1} a_{i,j}^\dagger(\vec{n}')\, a_{i,j}(\vec{n}').$$

$V_{c_E}^{(0)}$ and $V_{c_D}^{(0)}$ $V_{c_E}^{(l)}$ and $V_{c_E}^{(t)}$ $V_{c_E}^{(1)}$ and $V_{c_D}^{(1)}$ $V_{c_E}^{(2)}$ and $V_{c_D}^{(2)}$

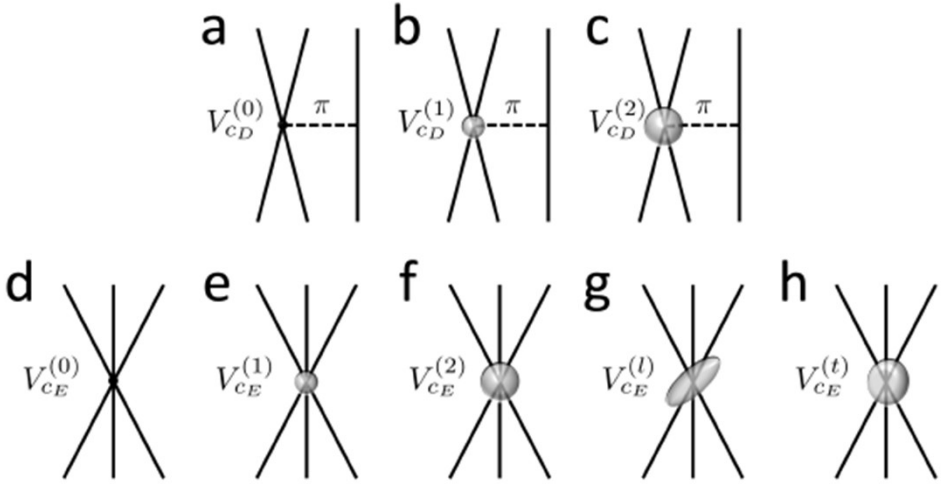
3-body force

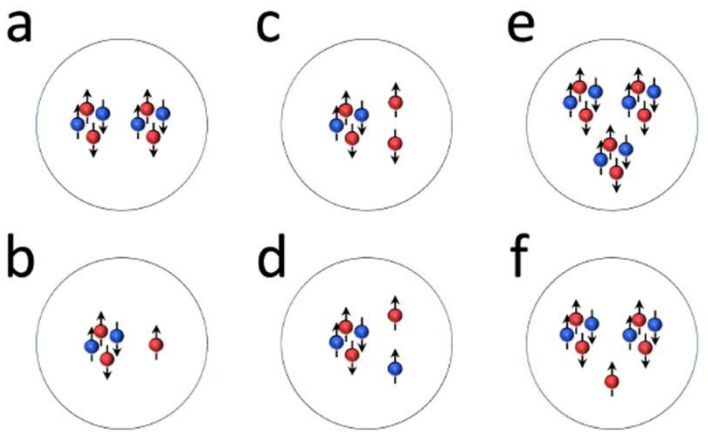
Tune 3-body interaction to minimize errors
in binding energy

Just one additional parameter,
RMSD for the E/A
drops from 1.2 MeV to 0.4 MeV

Energies
→ Significant sensitivity to
the locality of 3N interactions.

We interpret they are related with
effective interactions
between alphas and nucleons.

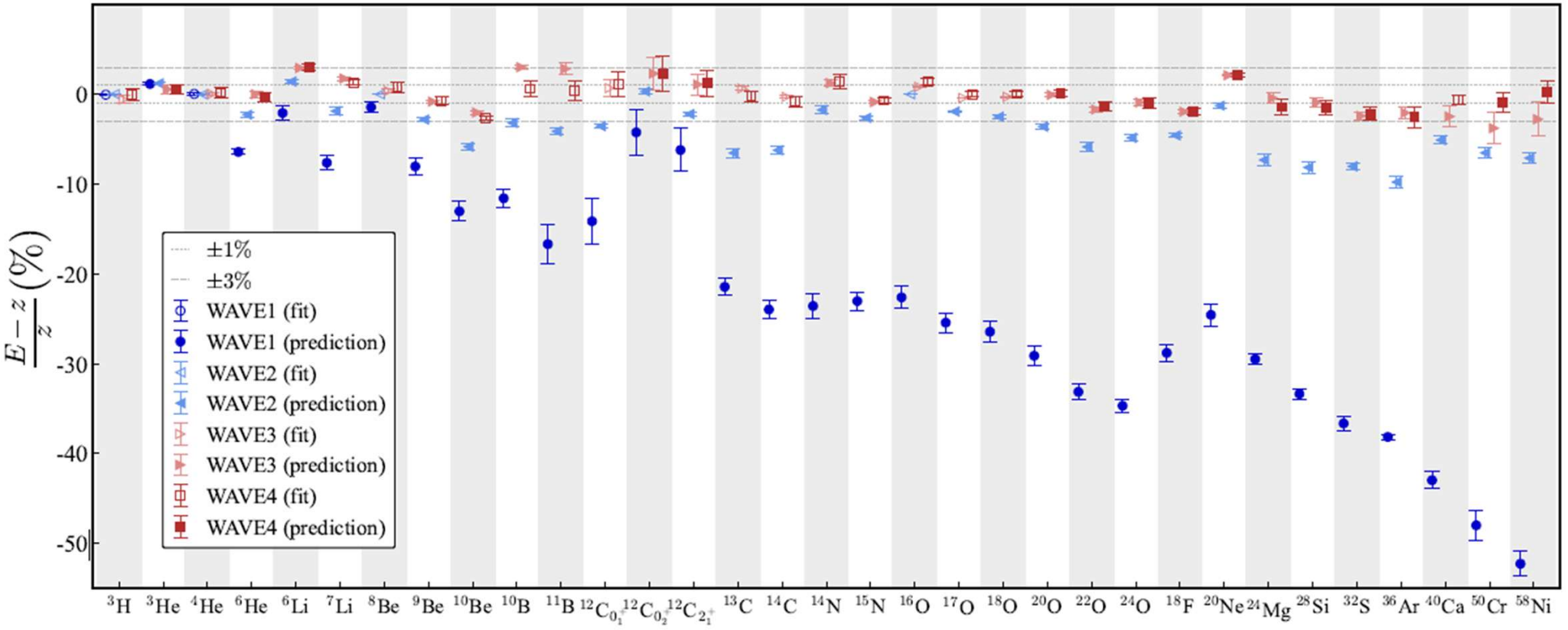




$E^6_{12,0^+_1} - 3E^2_4$, $E^8_{16} - 4E^2_4$, $E^2_6 - E^2_4$, $E^4_9 - E^4_8$, $E^6_{13} - E^6_{12}$, and $E^6_{14} - E^6_{12}$

Significantly high w/o additional 3-body terms
: alpha-alpha should be more attractive.

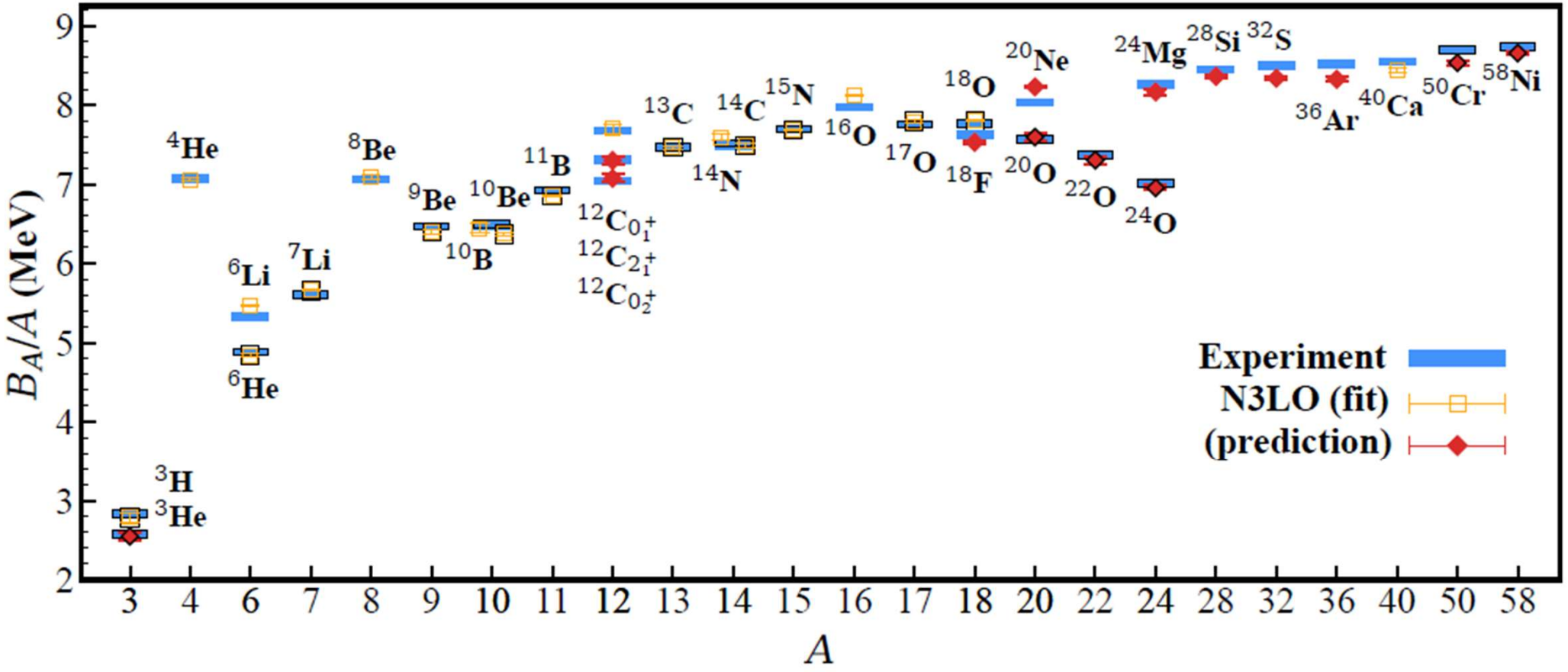
Determine 3-body force parameters



: $V_{cE}^{(0)}$ and $V_{cD}^{(0)}$ $V_{cE}^{(l)}$ and $V_{cE}^{(t)}$ $V_{cE}^{(1)}$ and $V_{cD}^{(1)}$ $V_{cE}^{(2)}$ and $V_{cD}^{(2)}$

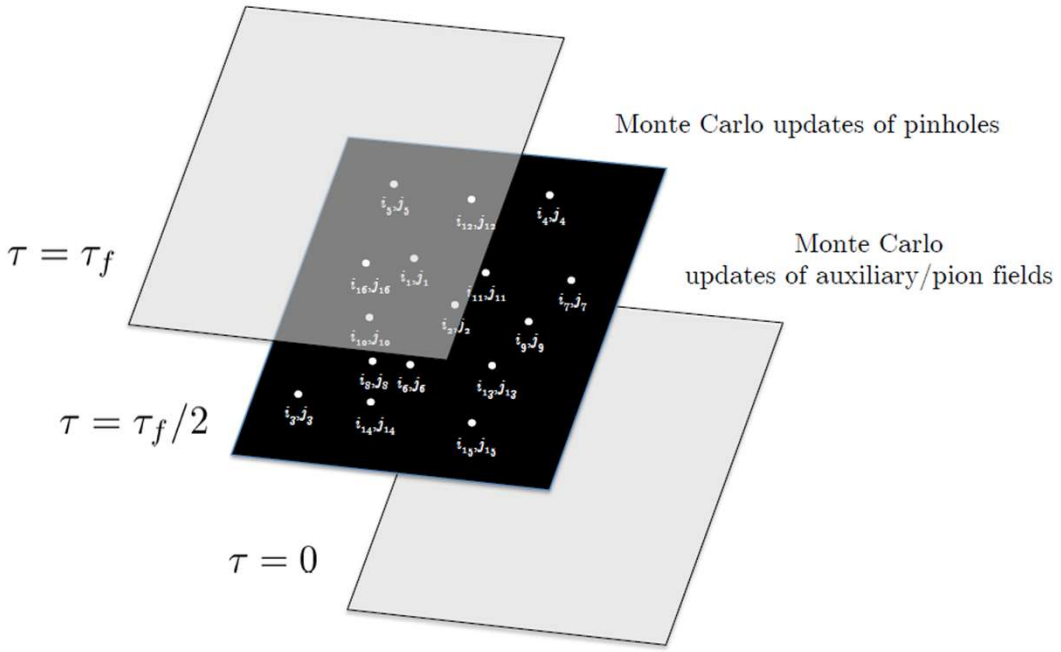
RMSD (MeV/nucleon): 1.2 → 0.3 →0.109 → 0.079

BE/A from WFM

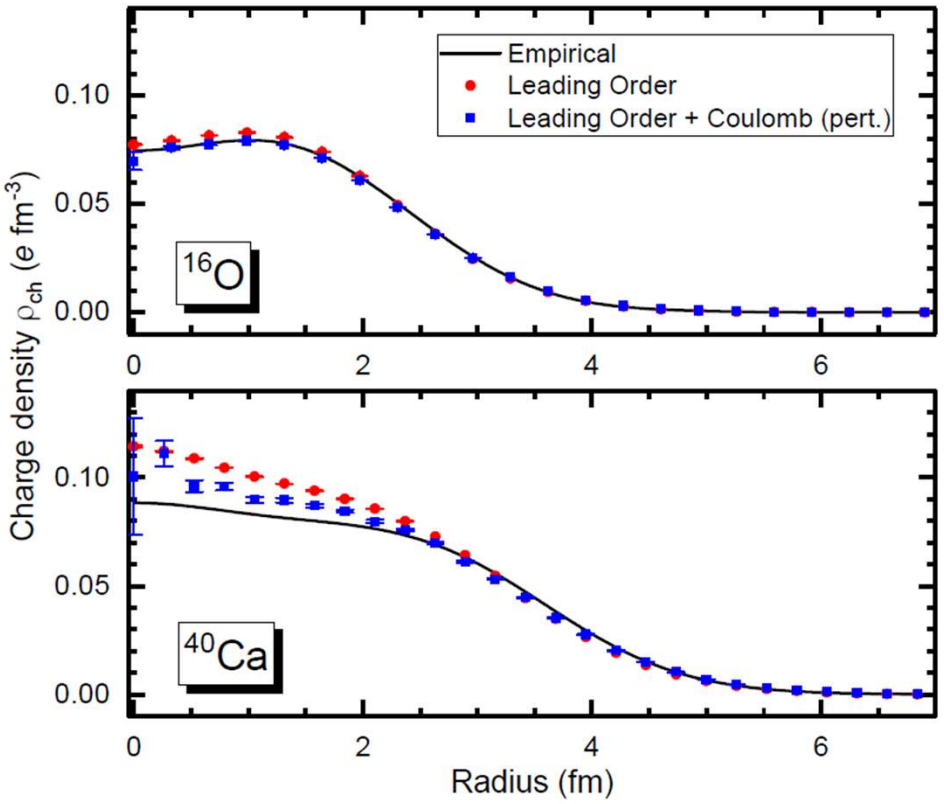


RMSD ~ 0.1 MeV per nucleon

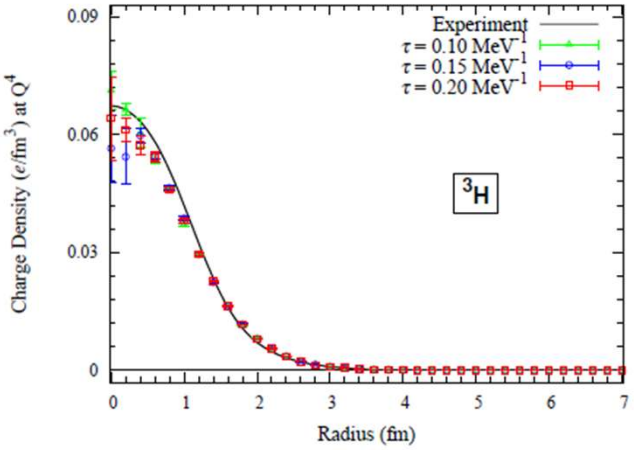
Pinhole Algorithm

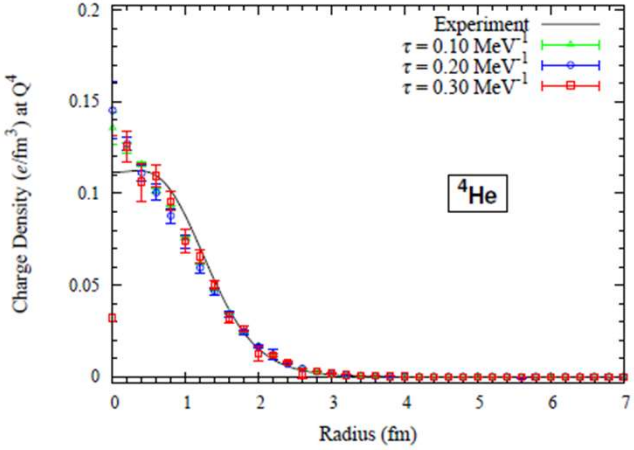


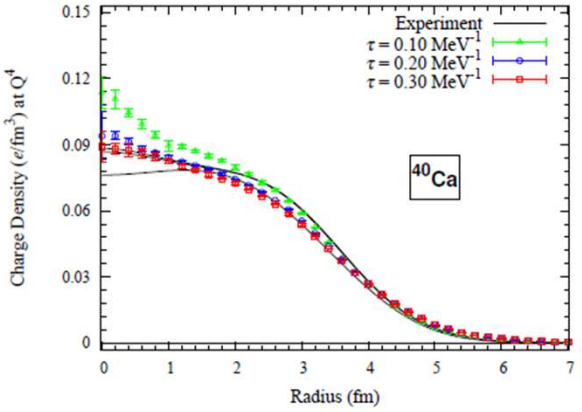
Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)

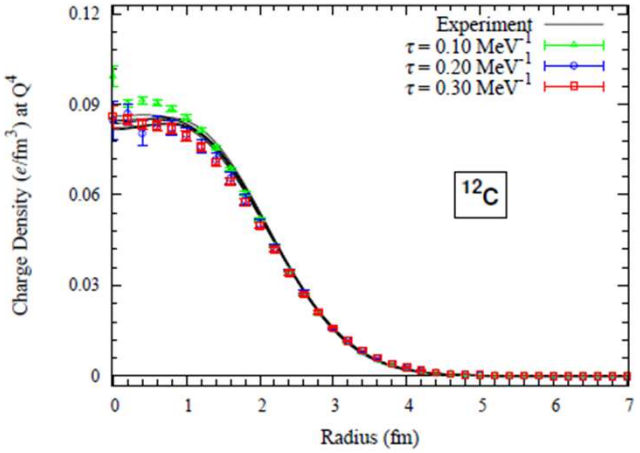


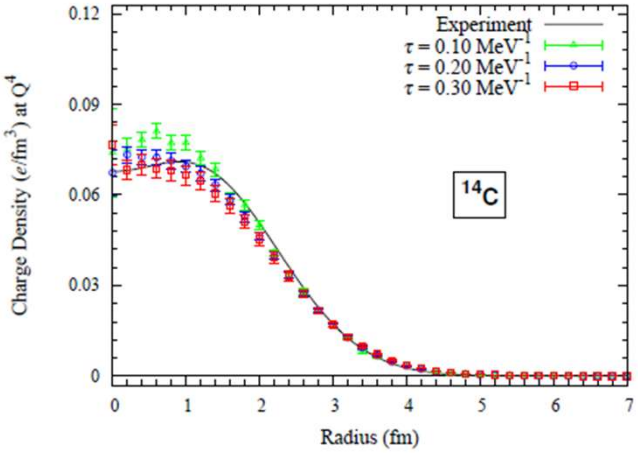
Charge density from WFM (Pinhole algorithm)



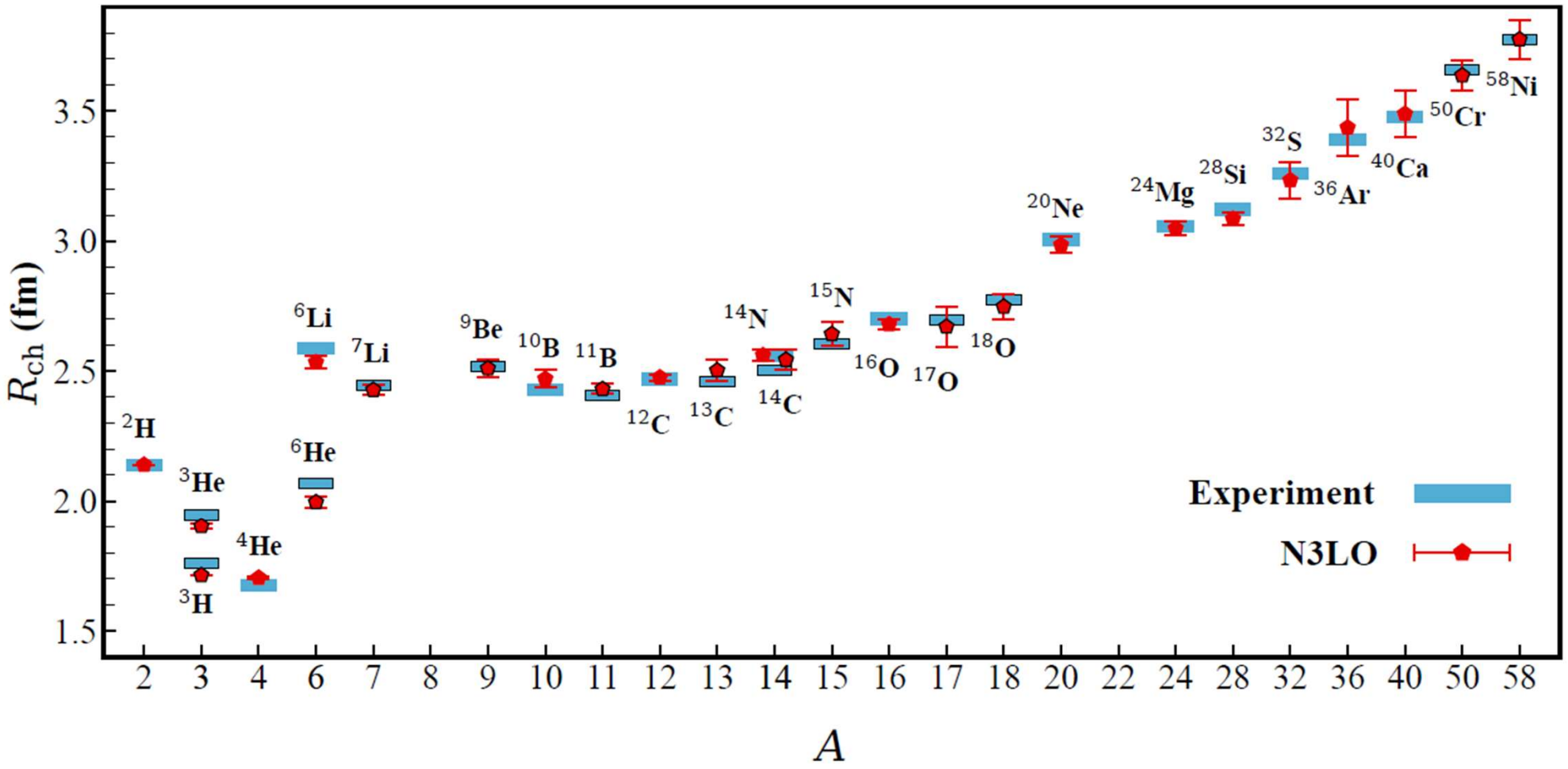






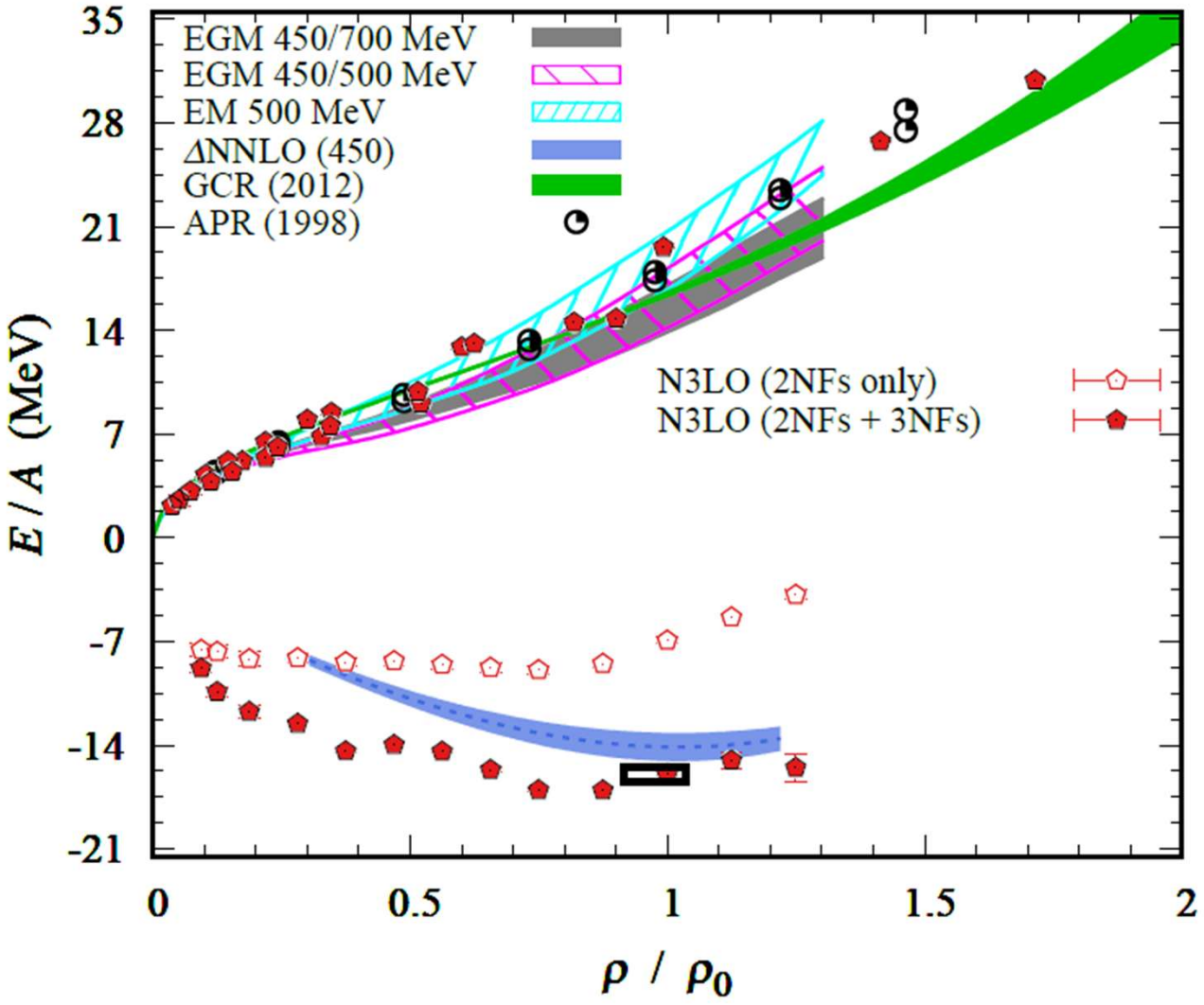


Charge Radius



No fitting! RMSD~ 0.03 fm

Nuclear/Neutron Matter

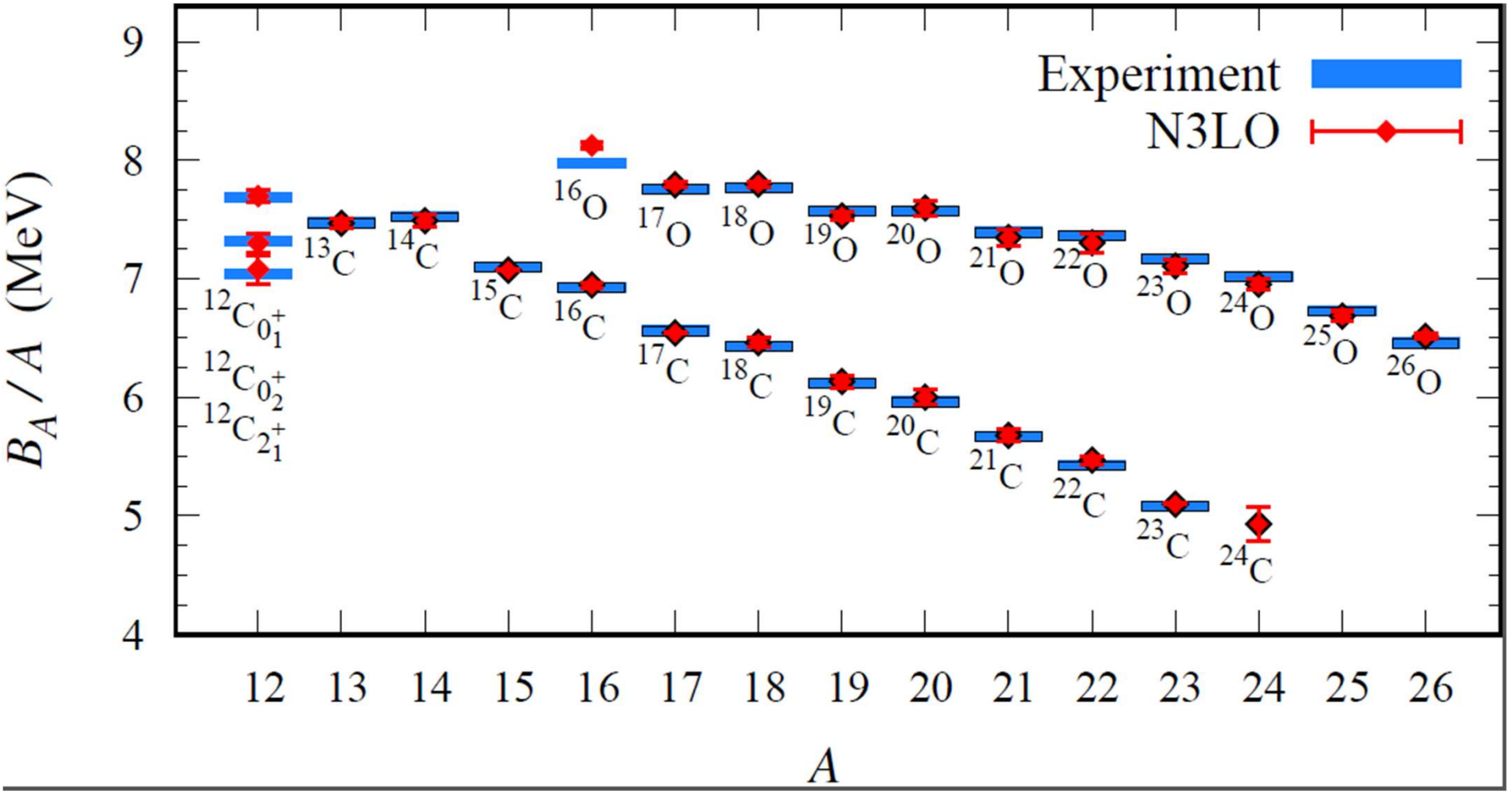


Neutron matter:
A=4~80
box size 6.6 ~ 13.2 fm.

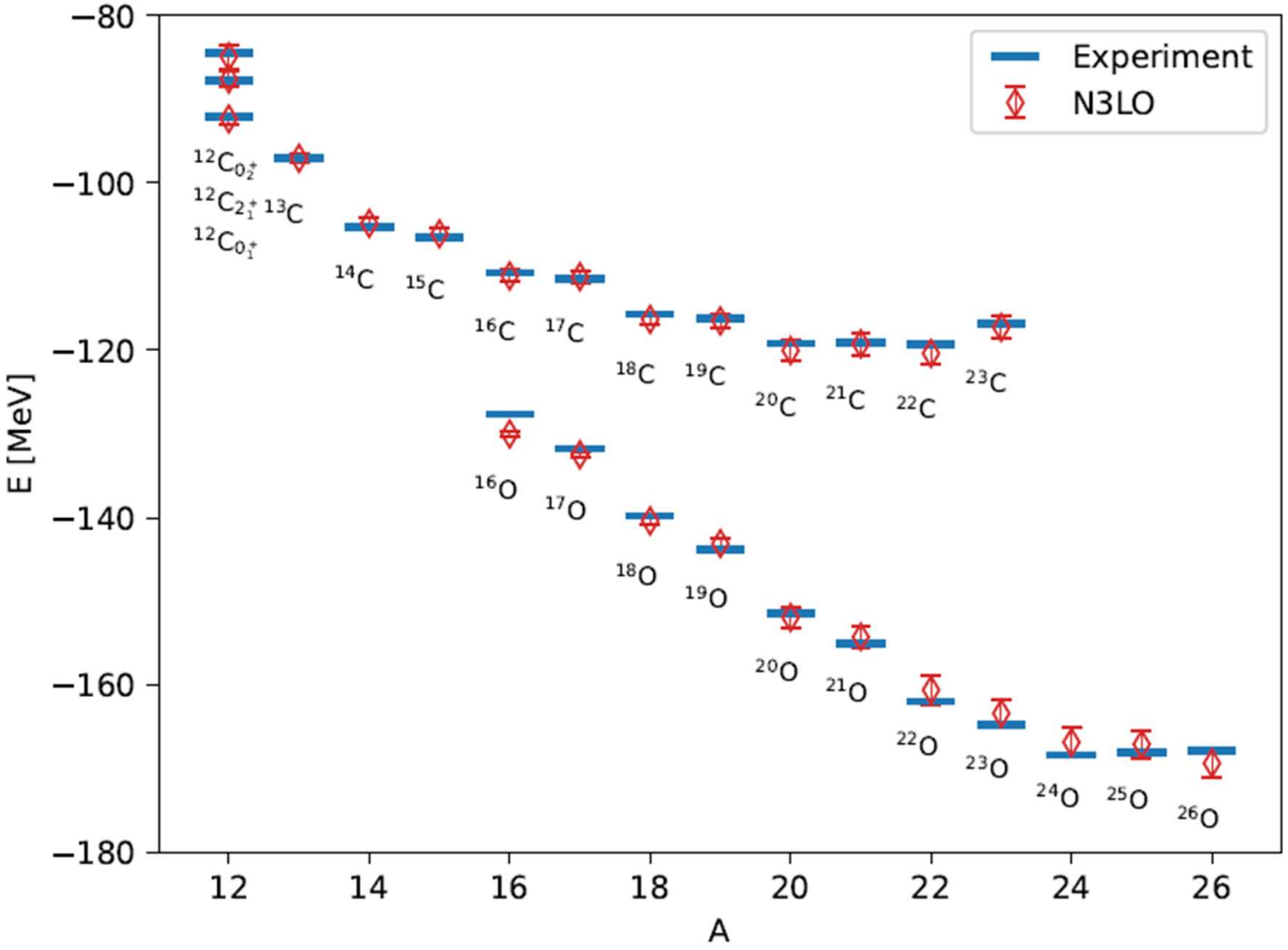
Nuclear matter:
A=4 ~ 160
Box size 7.92~9.24 fm.

- Uncertainties from finite system size correction

Carbon and Oxygen



Carbon and Oxygen



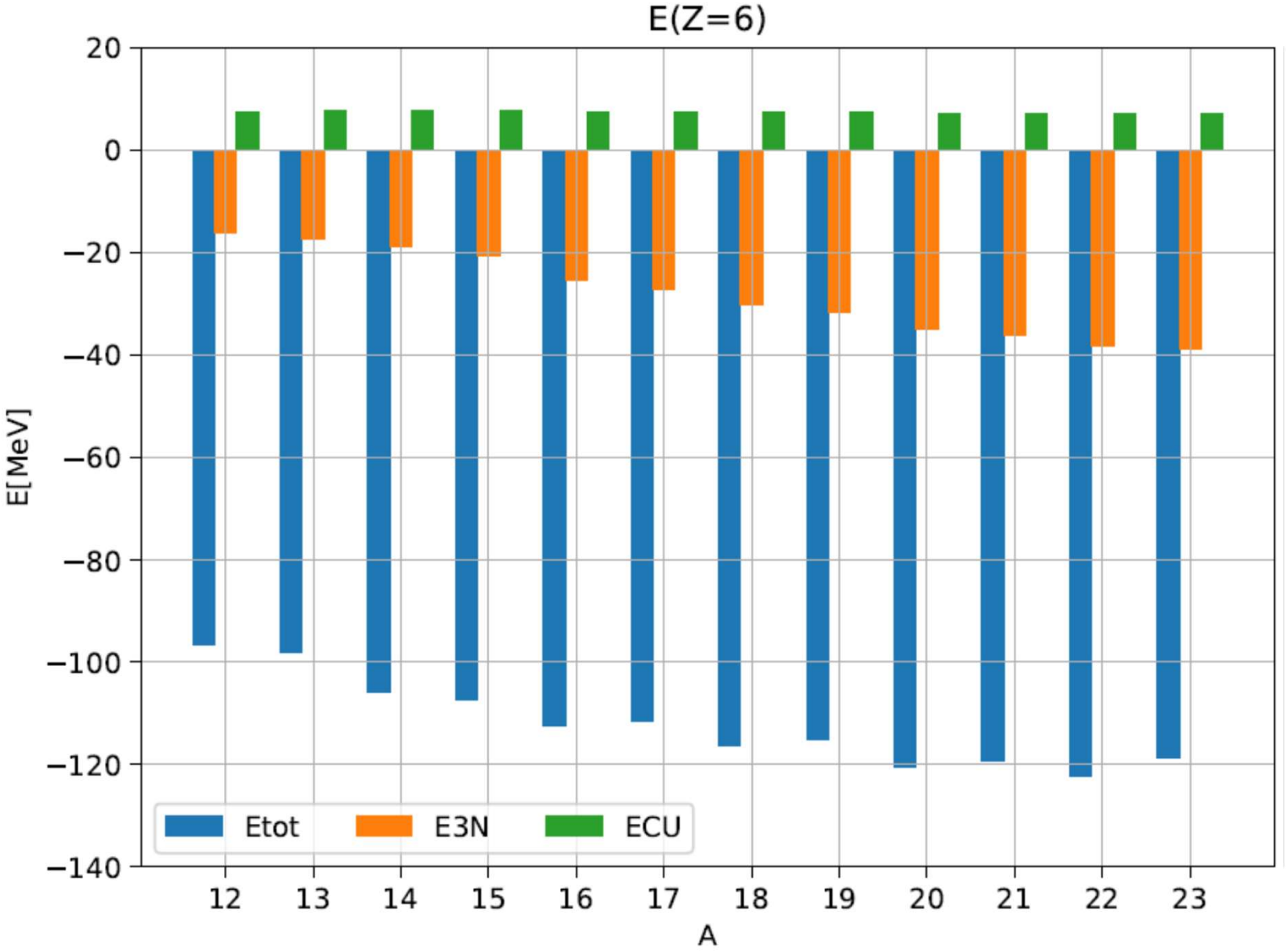
Finite Volume Effect

Lt=200	L=10			L=11			Exp	
	E	err	S2n	E	err	S2n	E	S2n
24O	-167.600	1.126		-171.880	0.501		-168.952	6.920
25O	-168.790	0.424		-169.670	0.551		-168.195	3.453
26O	-171.530	0.587	4.430	-172.380	0.459	0.500	-168.934	-0.018
27O	-169.960	0.450	1.170	-166.930	0.566	-2.740	-166.995	-1.200

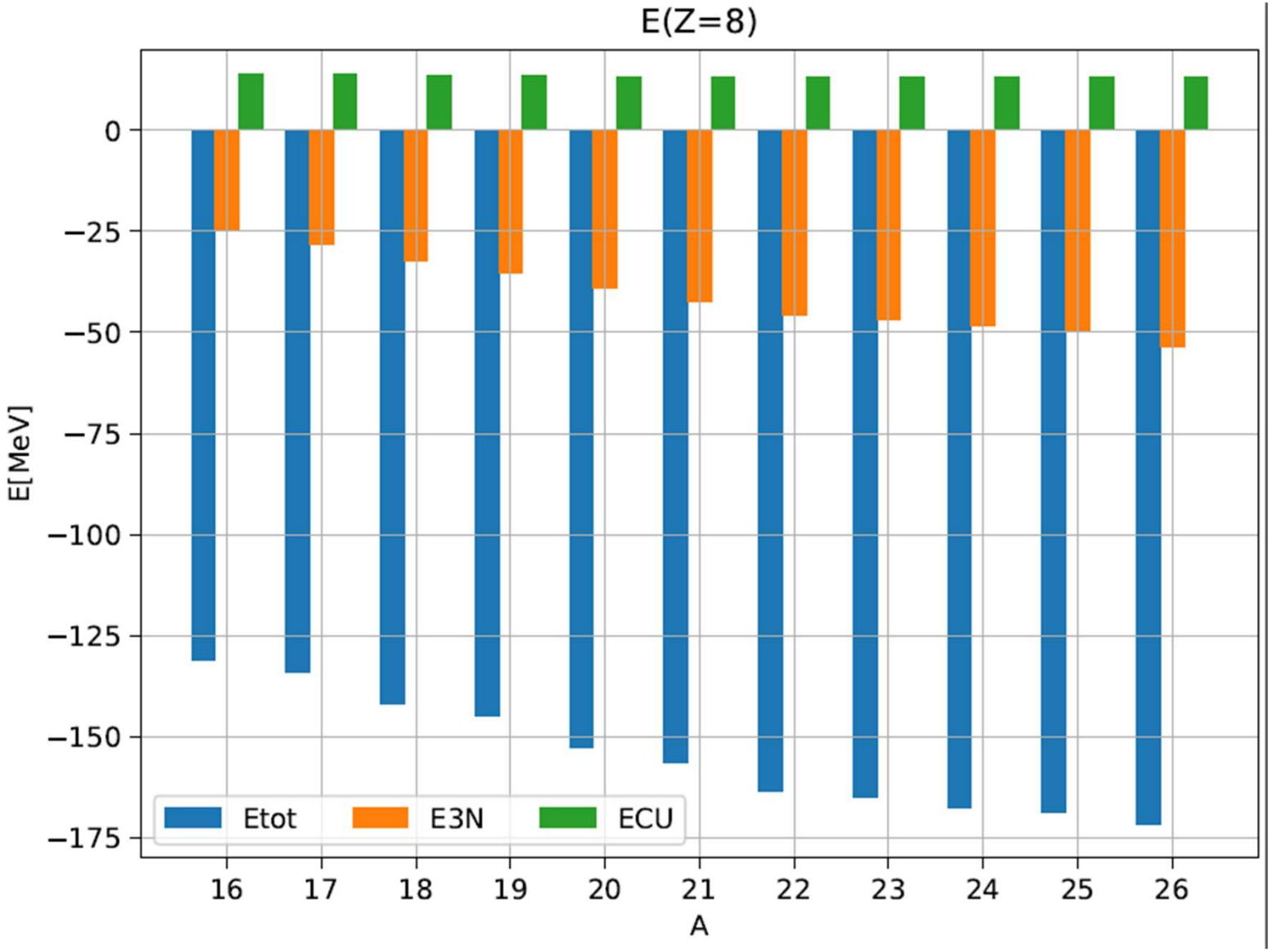
- Lt=200 result. (Not extrapolated results.)
- Systematic Error from finite volume effects need to be studied.
- L=11 improved Separation energy of 26O.

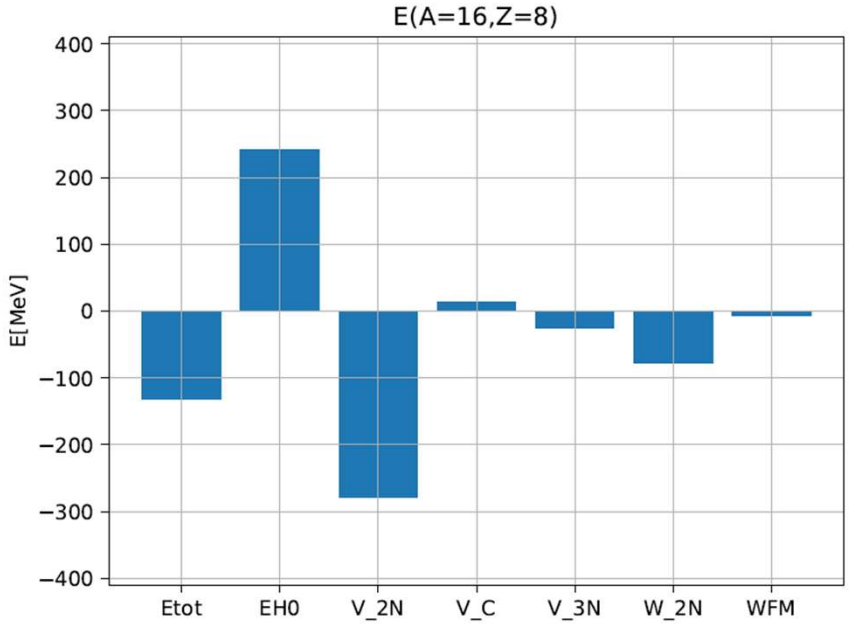
Preliminary

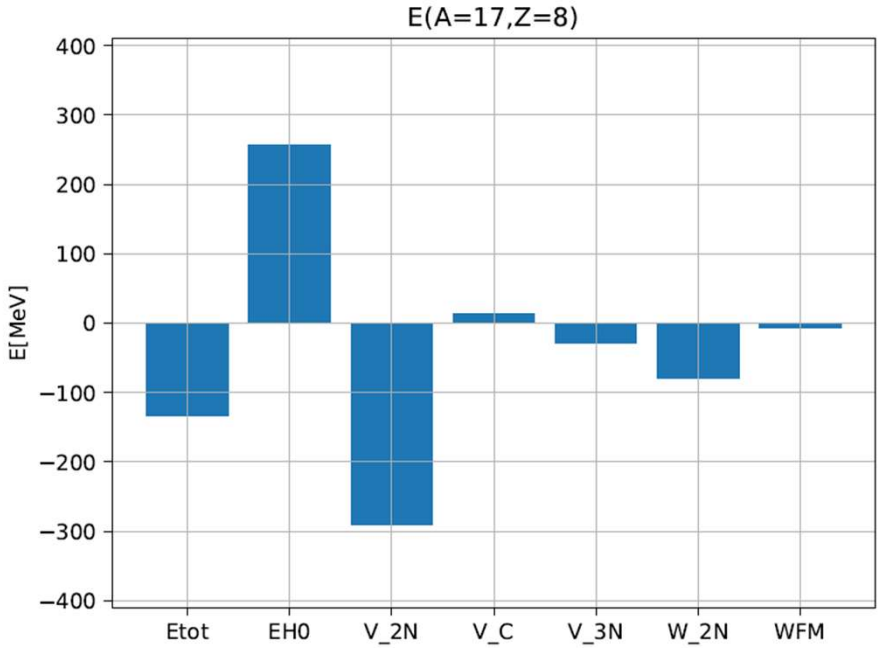
Carbon isotopes (Lt=200)

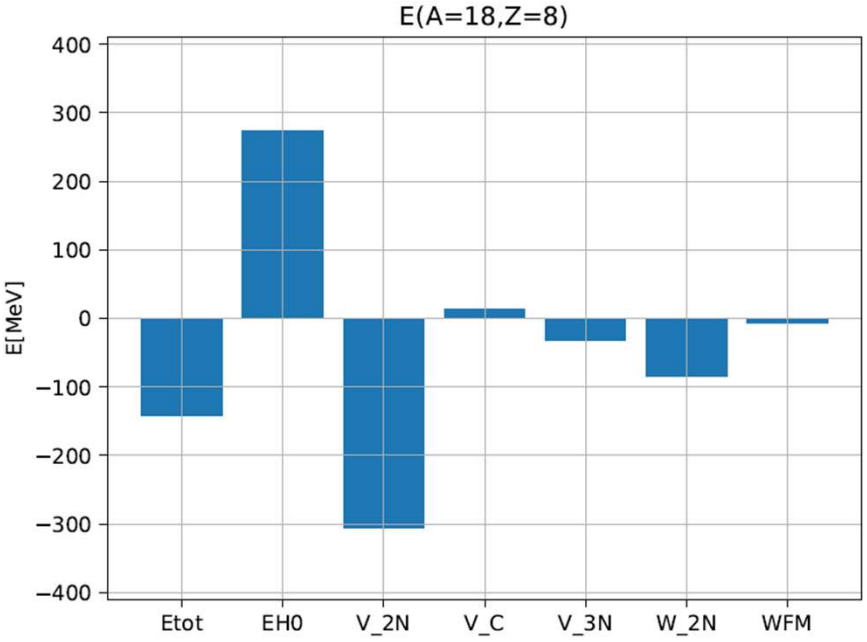


Oxygen isotopes (Lt=200)




E(A=16,Z=8)
E[MeV]
400
300
200
100
0
−100
−200
−300
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM


E(A=17,Z=8)
E[MeV]
400
300
200
100
0
−100
−200
−300
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM


E(A=18,Z=8)
E[MeV]
400
300
200
100
0
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−200
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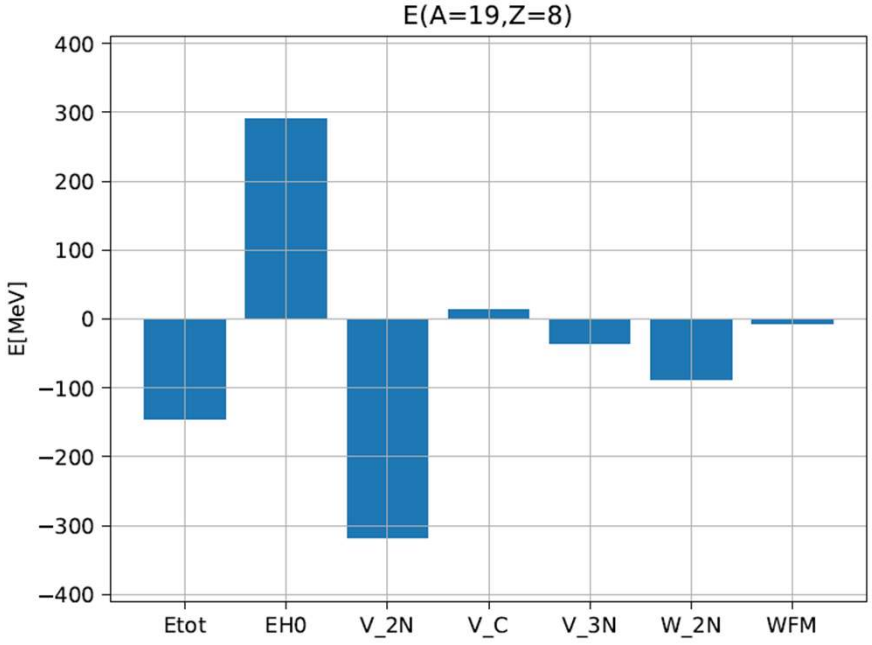
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM


E(A=19,Z=8)
E[MeV]
400
300
200
100
0
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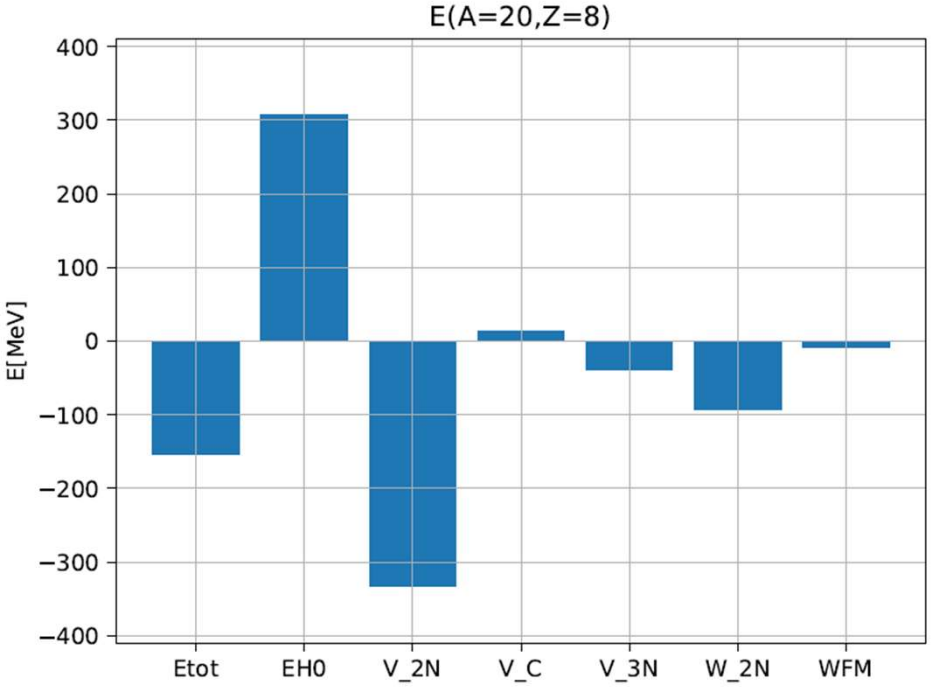
−200
−300
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM


E(A=20,Z=8)
E[MeV]
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Etot
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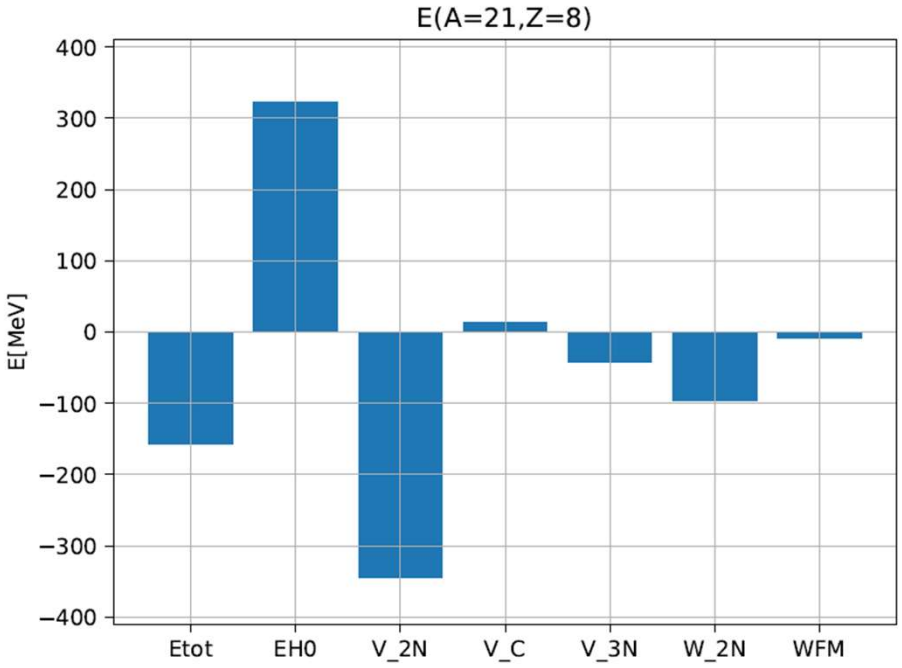
V_2N
V_C
V_3N
W_2N
WFM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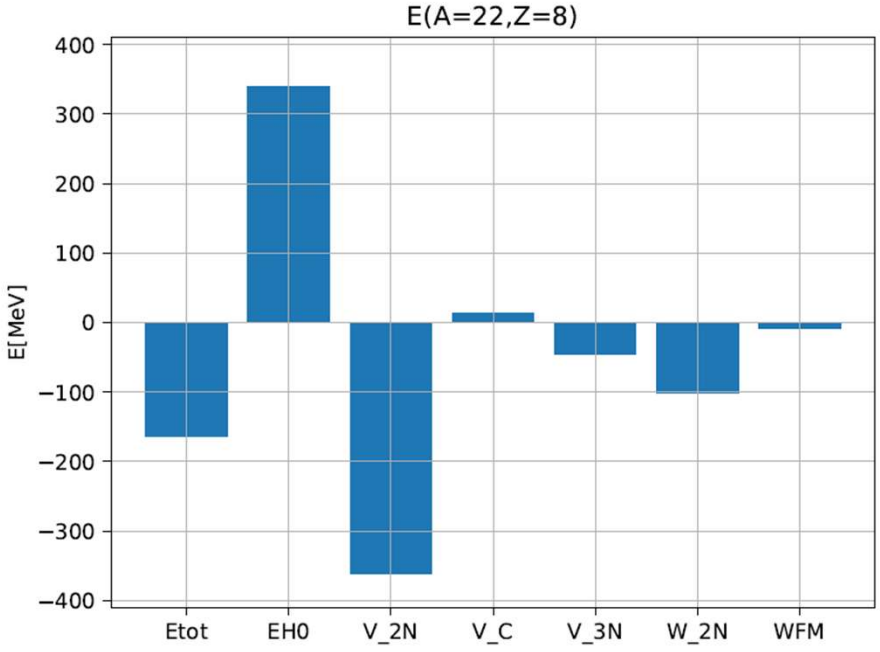
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM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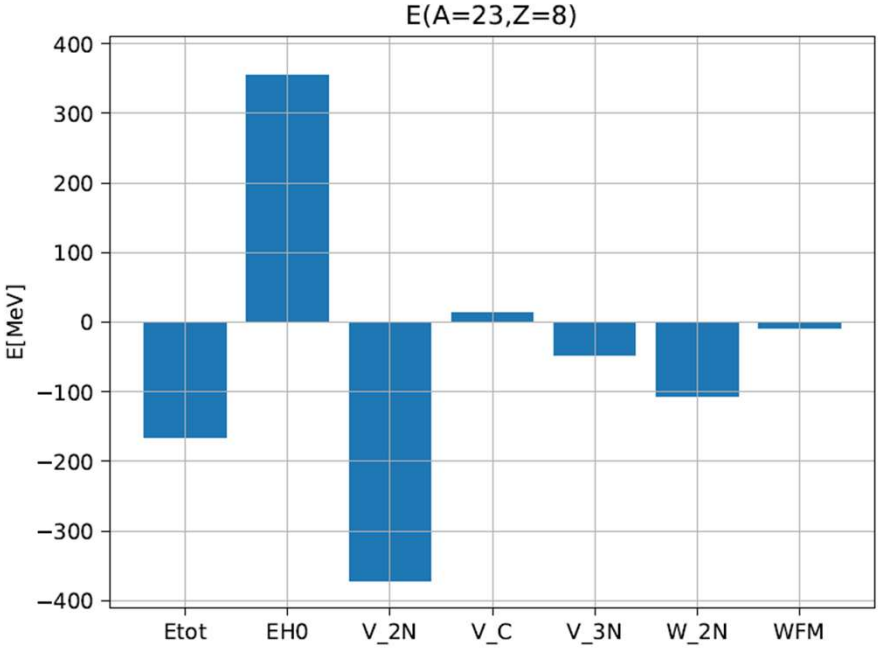
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM


E(A=23,Z=8)
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300
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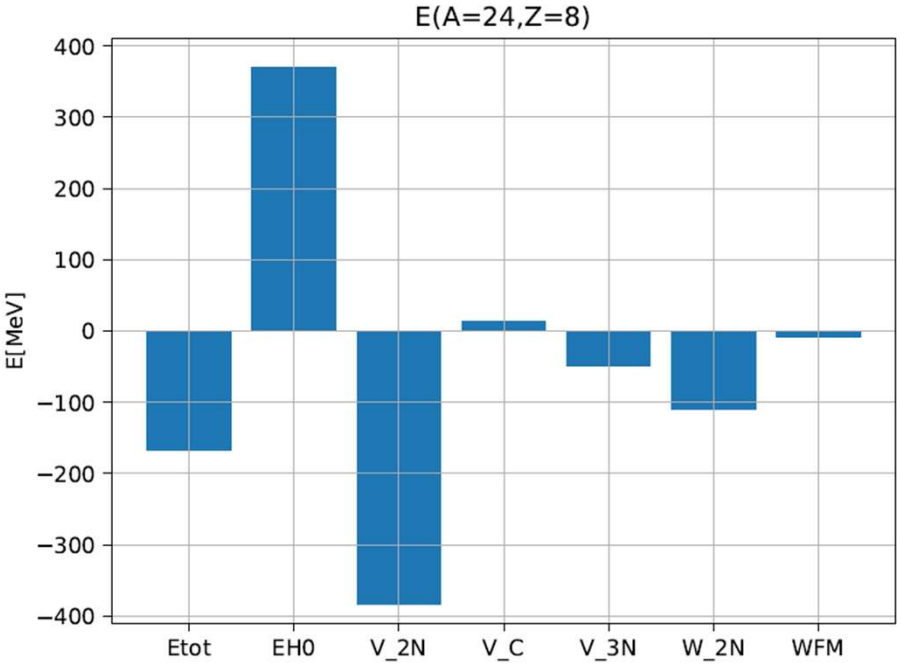
−200
−300
−400
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM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E[MeV]
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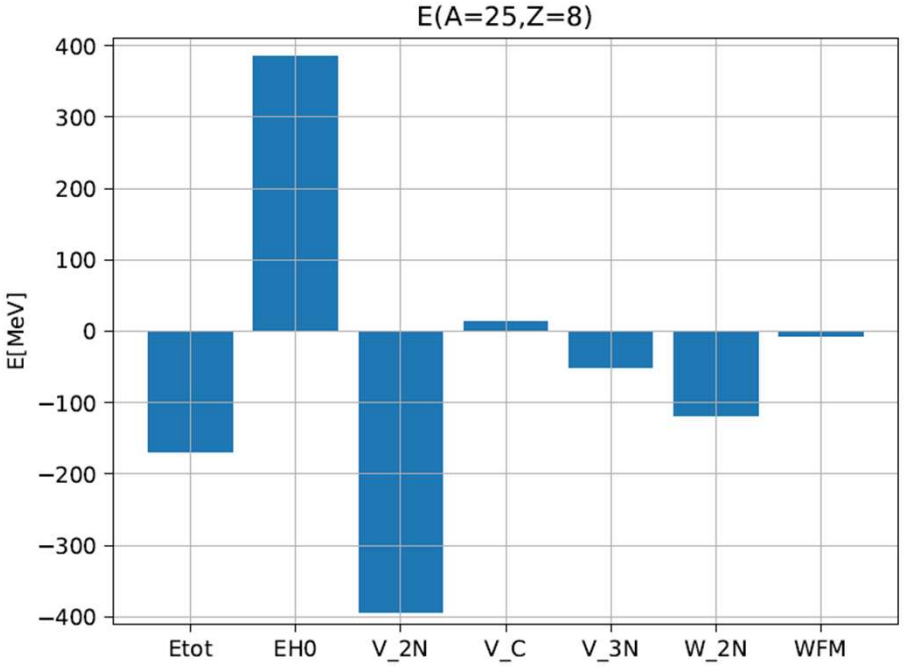
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM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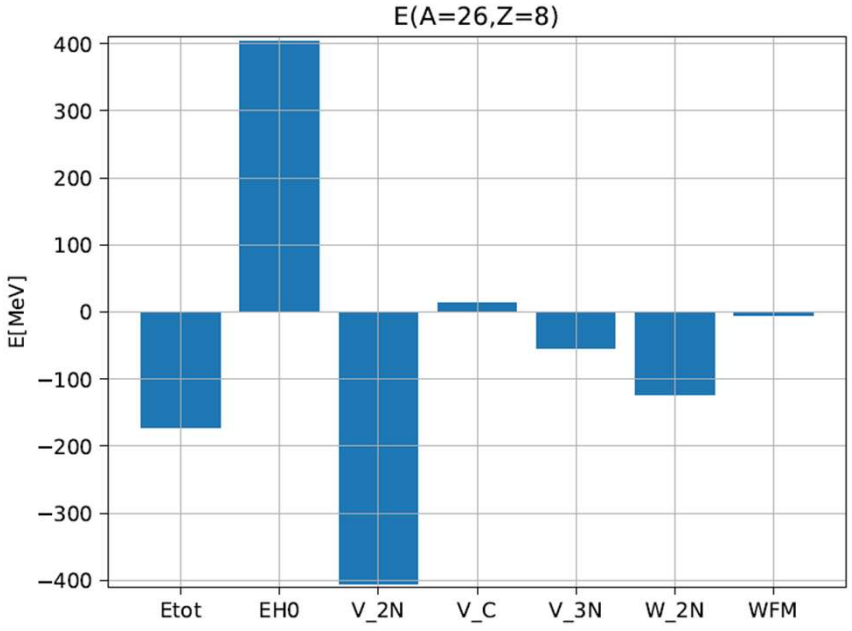
Etot
EH0
V_2N
V_C
V_3N
W_2N
WFM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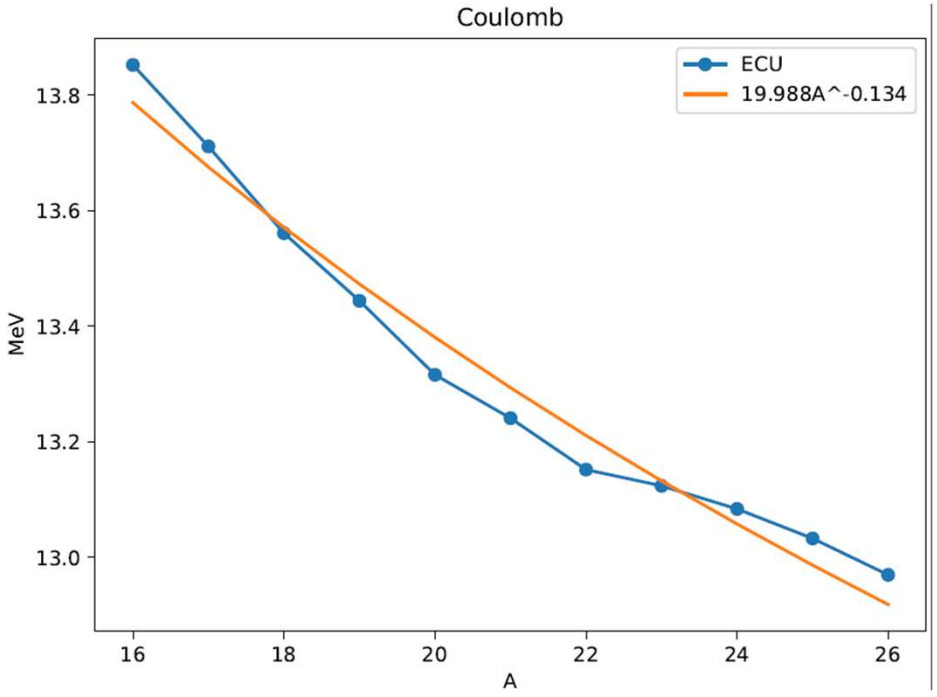
EH0
V_2N
V_C
V_3N
W_2N
WFM


Coulomb
ECU
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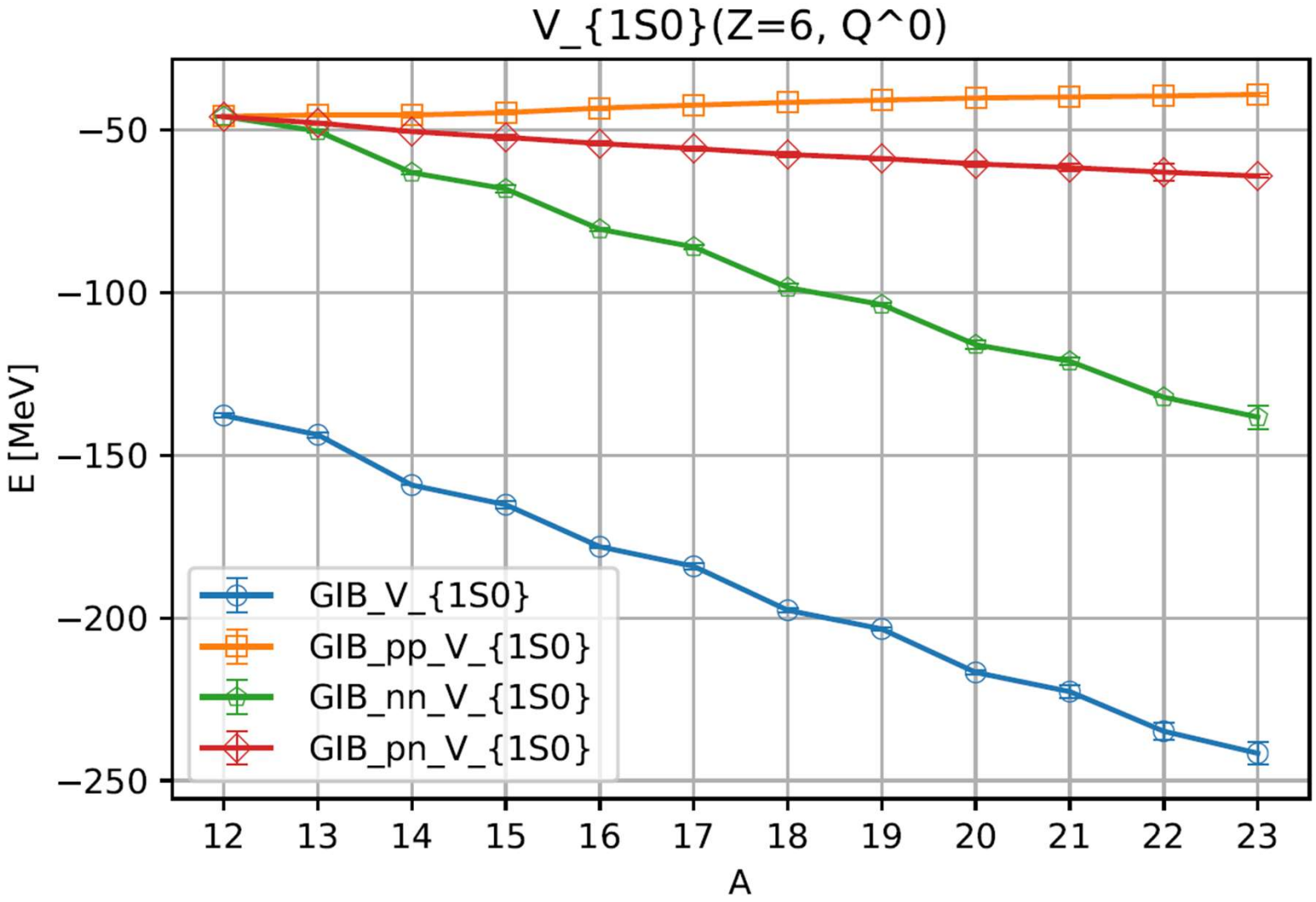
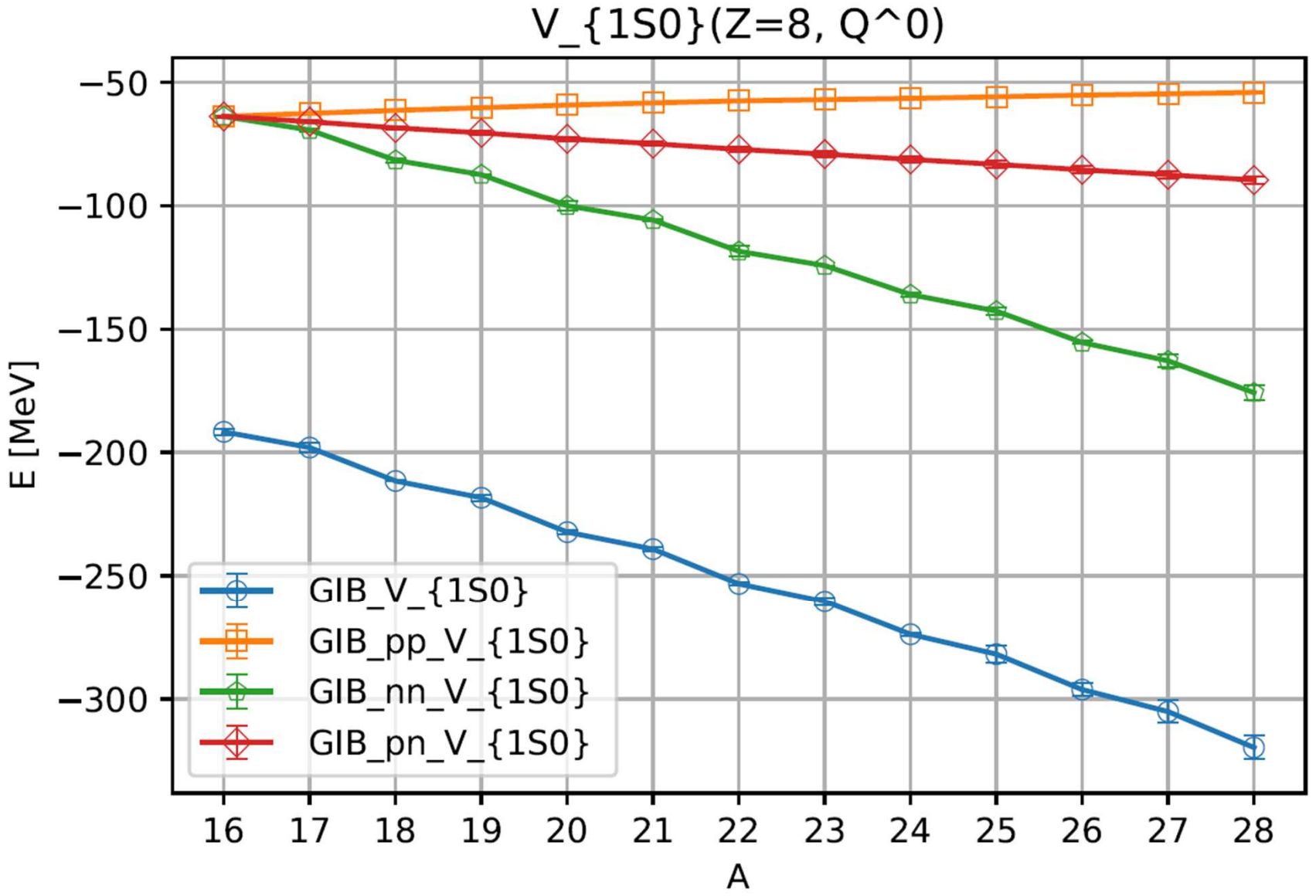
MeV
13.8
13.6
13.4
13.2
13.0
16
18
20
22
24
26
A

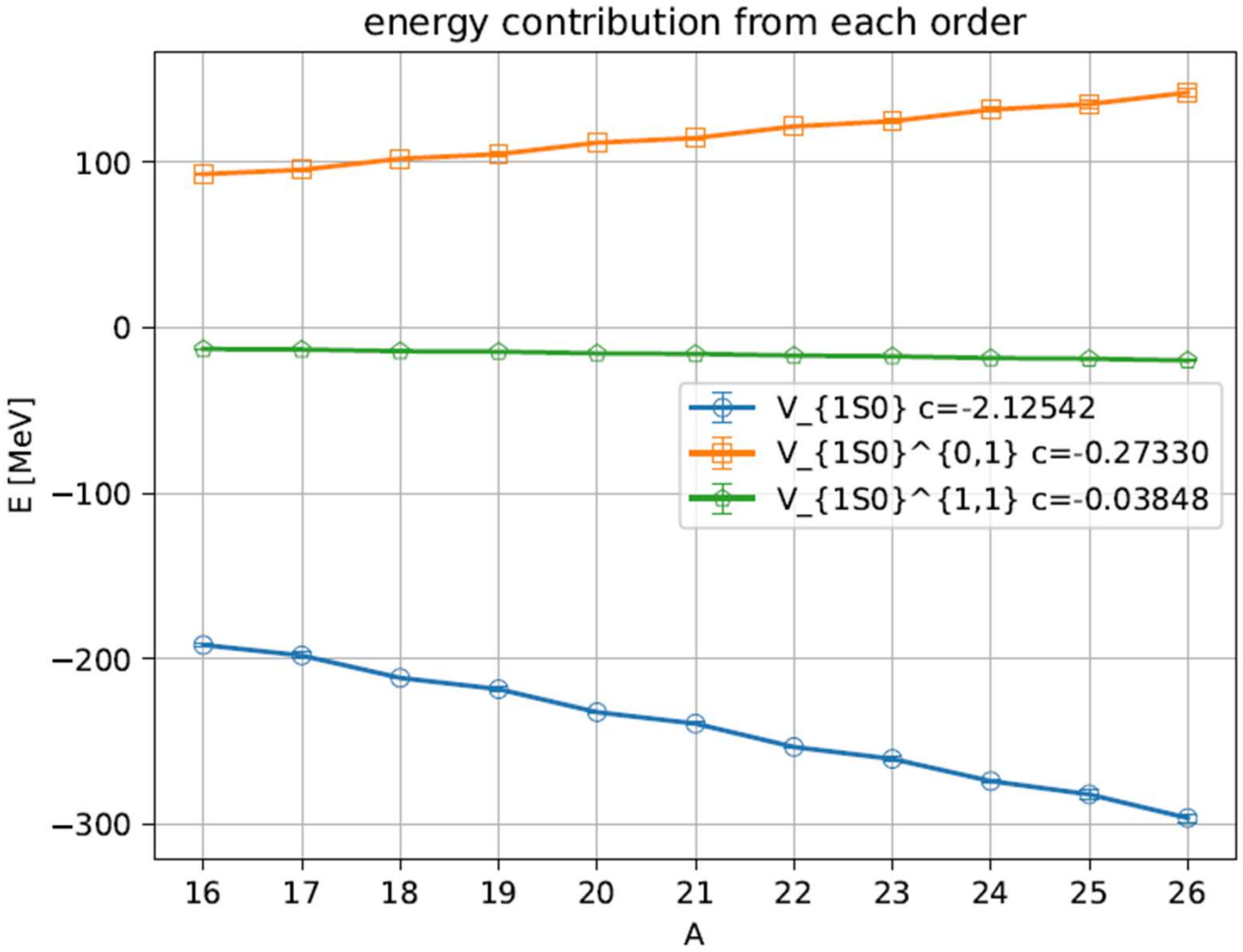
Partial waves contributions (Lt=200)



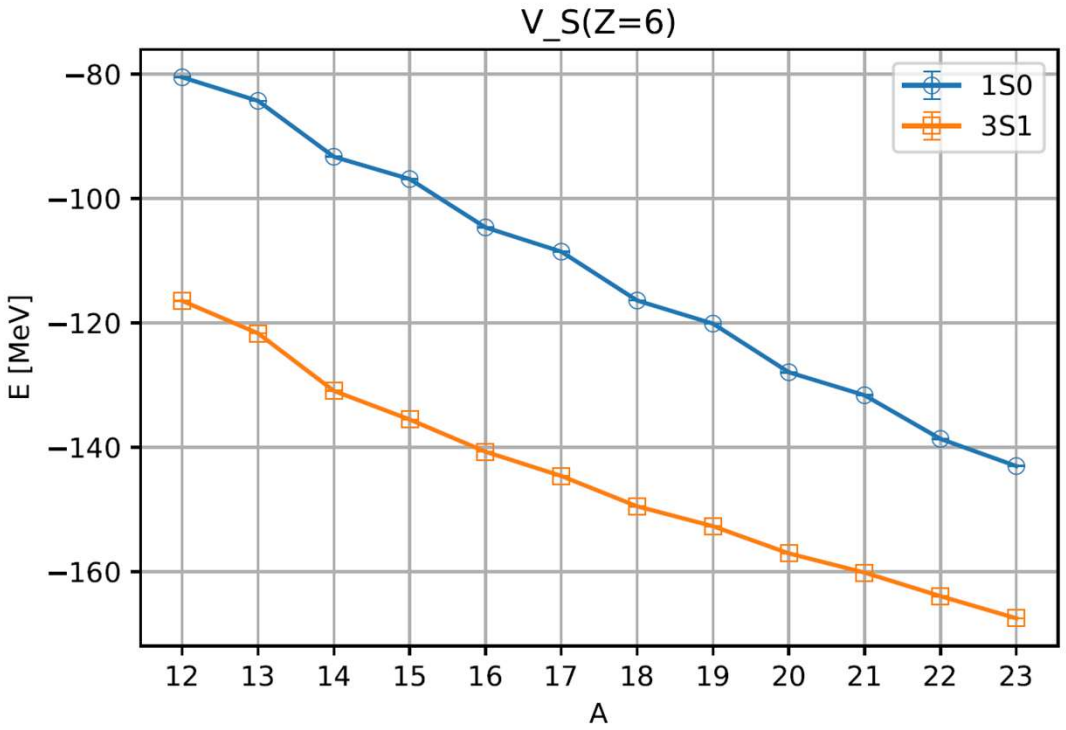
Partial waves contributions (Lt=200)

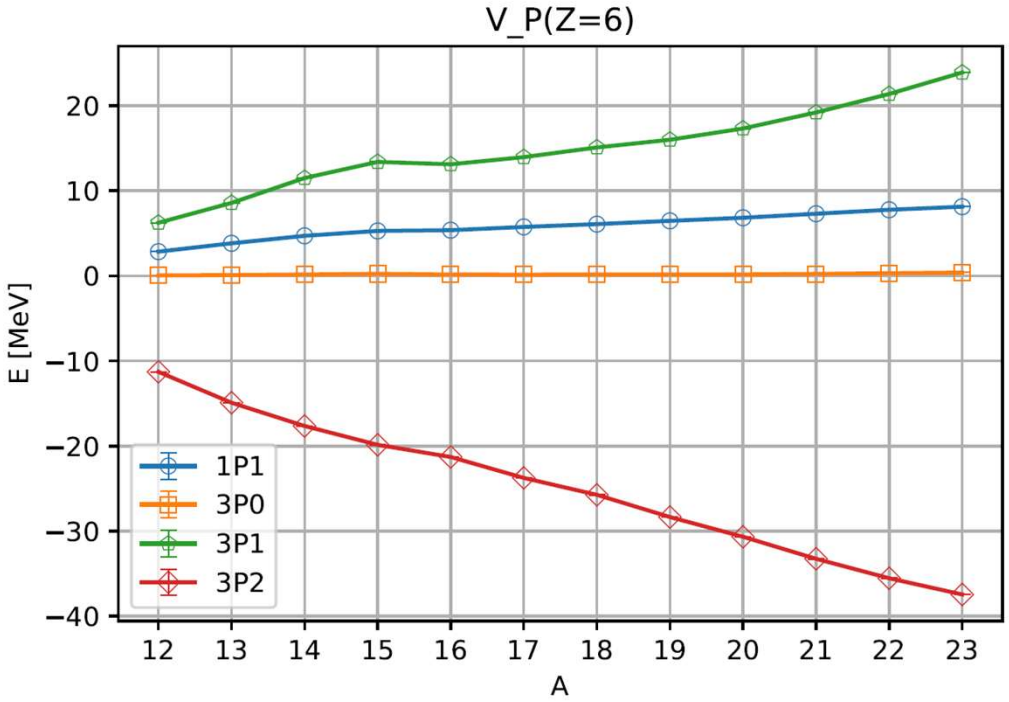


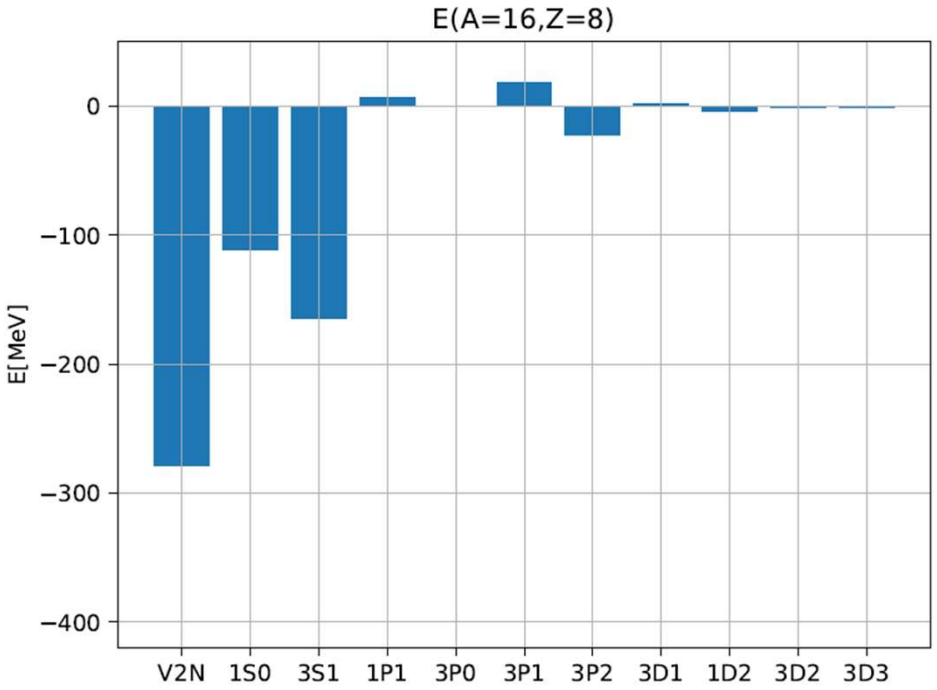
Partial waves contributions (Lt=200,Z=8)



Partial waves contributions (Lt=200)



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E[MeV]
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−100
−200
−300
−400
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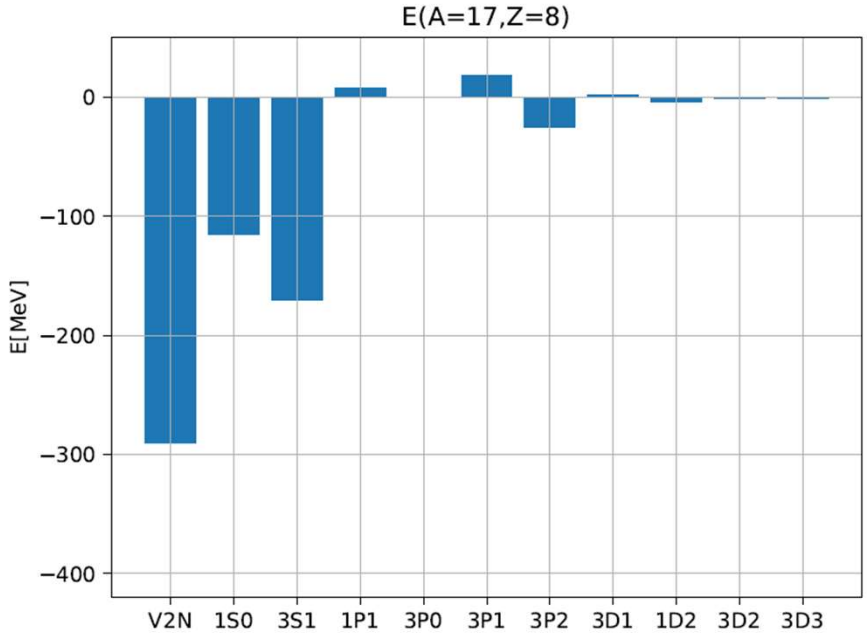
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=17,Z=8)
E[MeV]
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−100
−200
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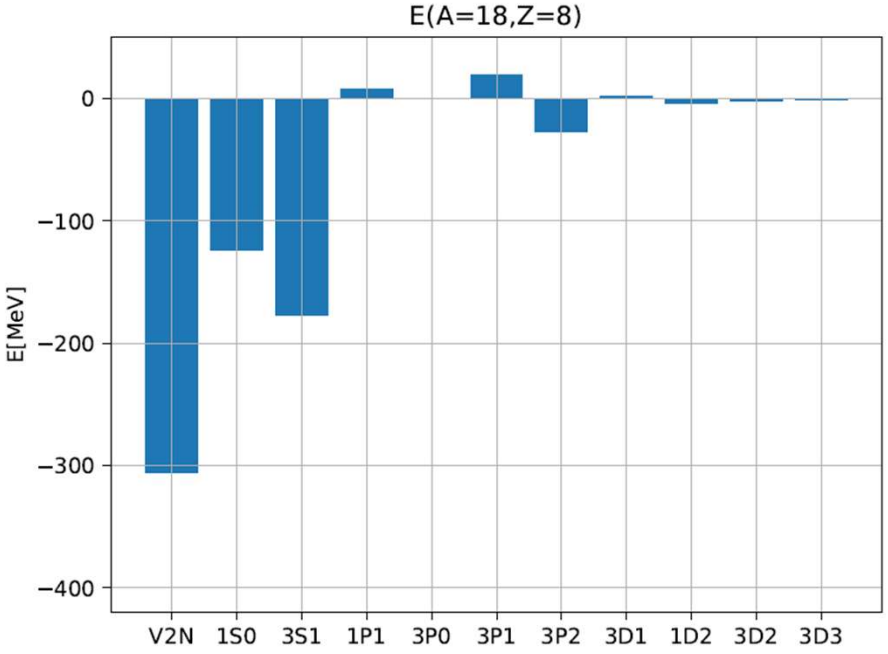
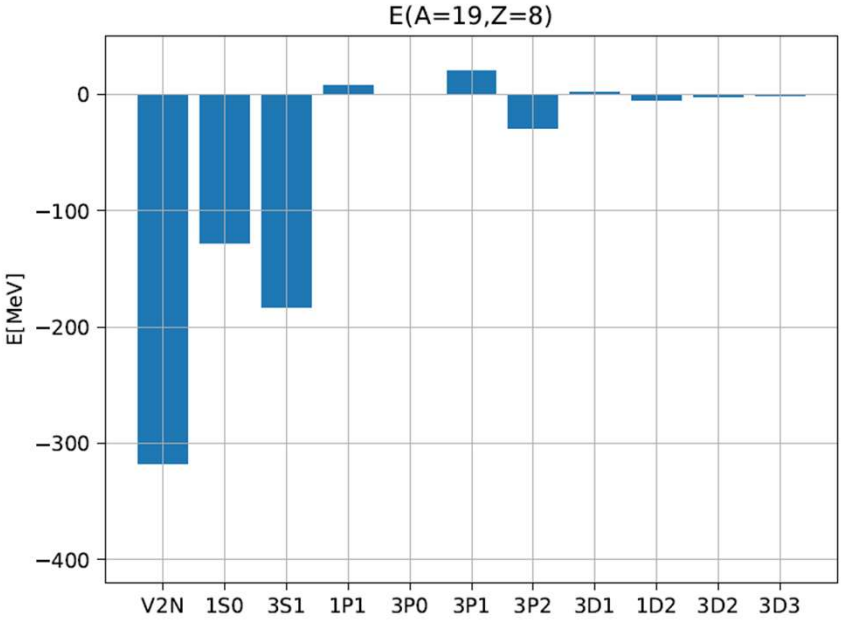
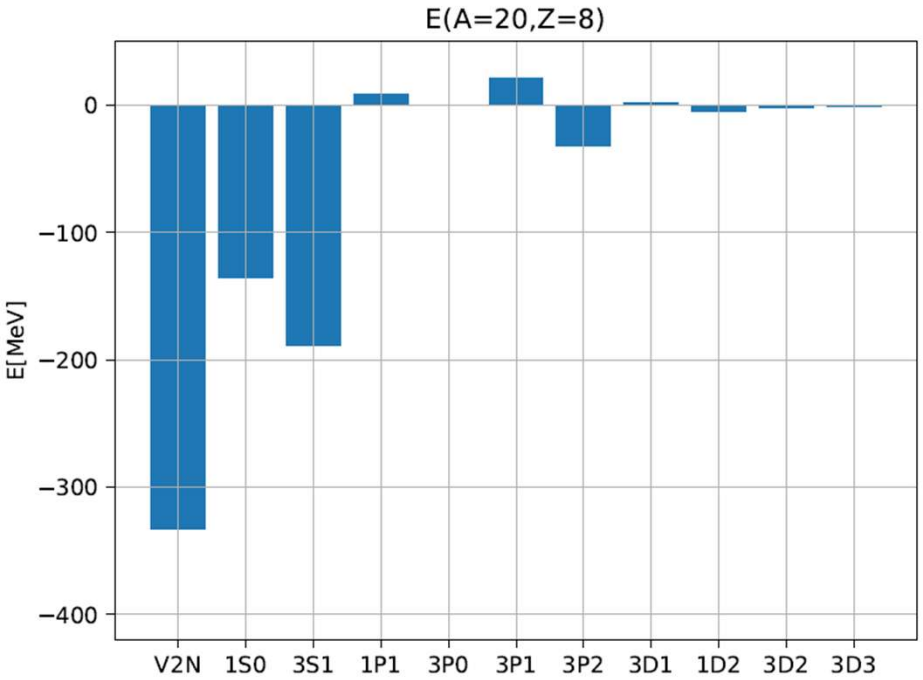
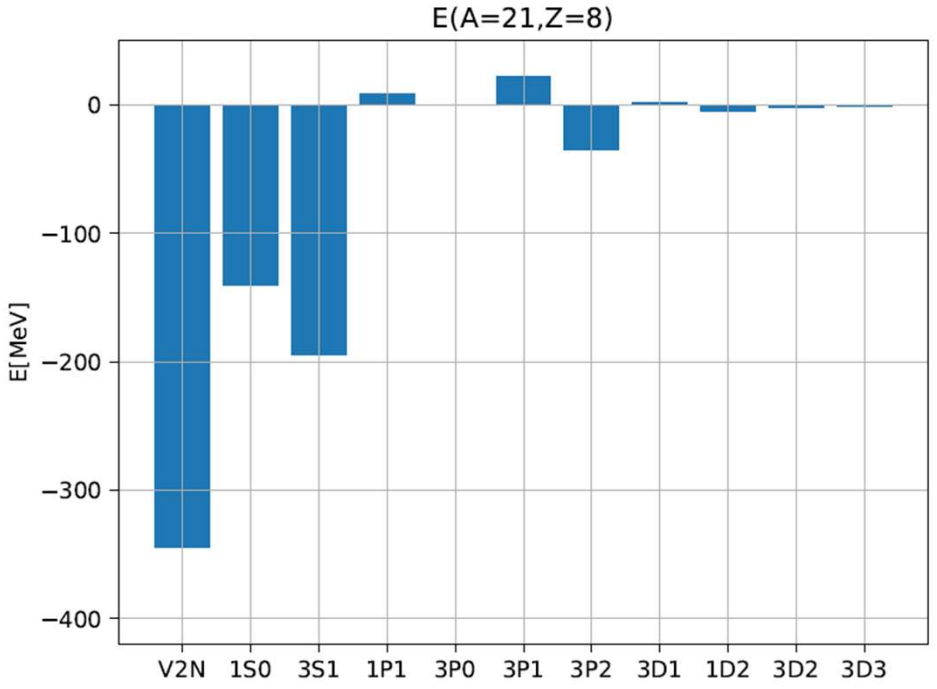
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=18,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=19,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=20,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=21,Z=8)
E[MeV]
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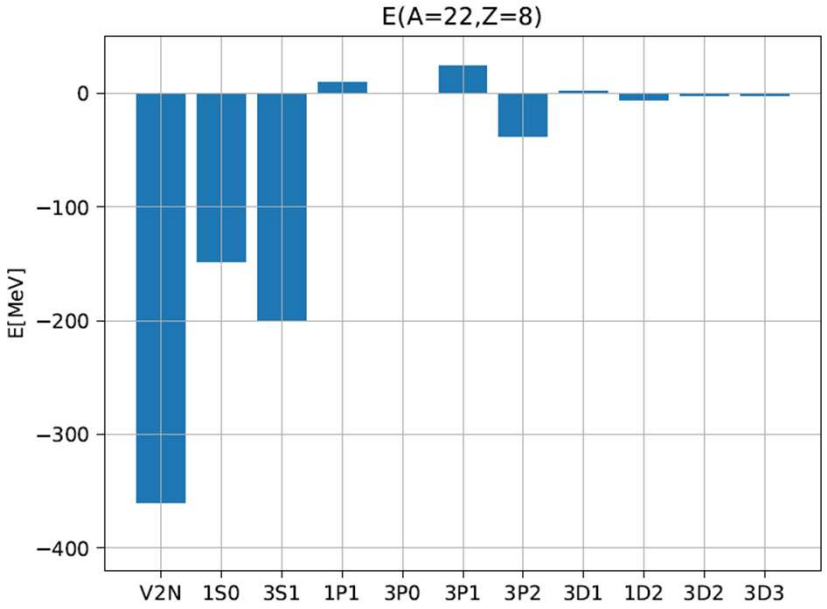
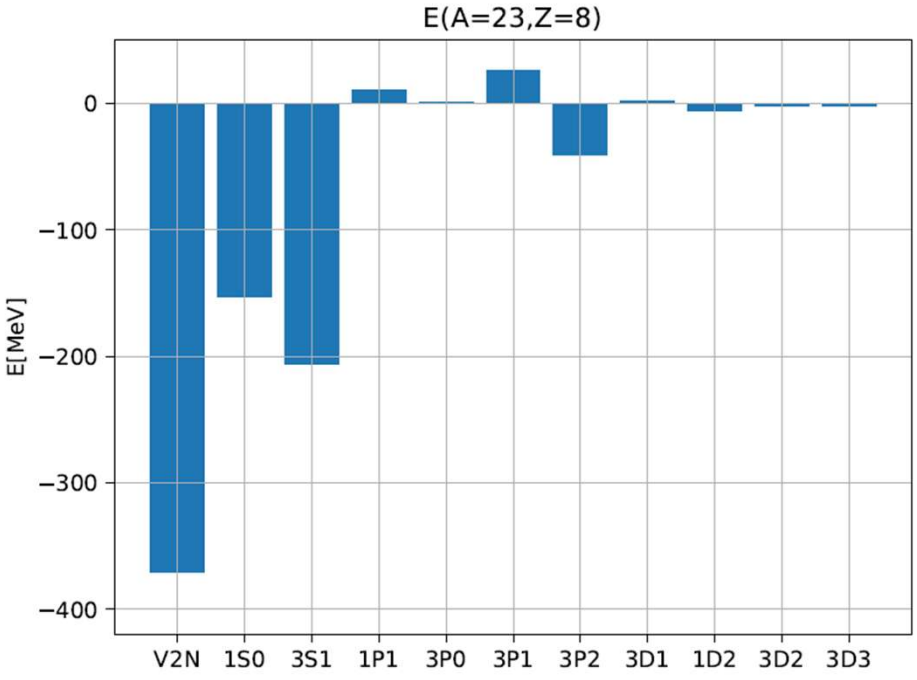
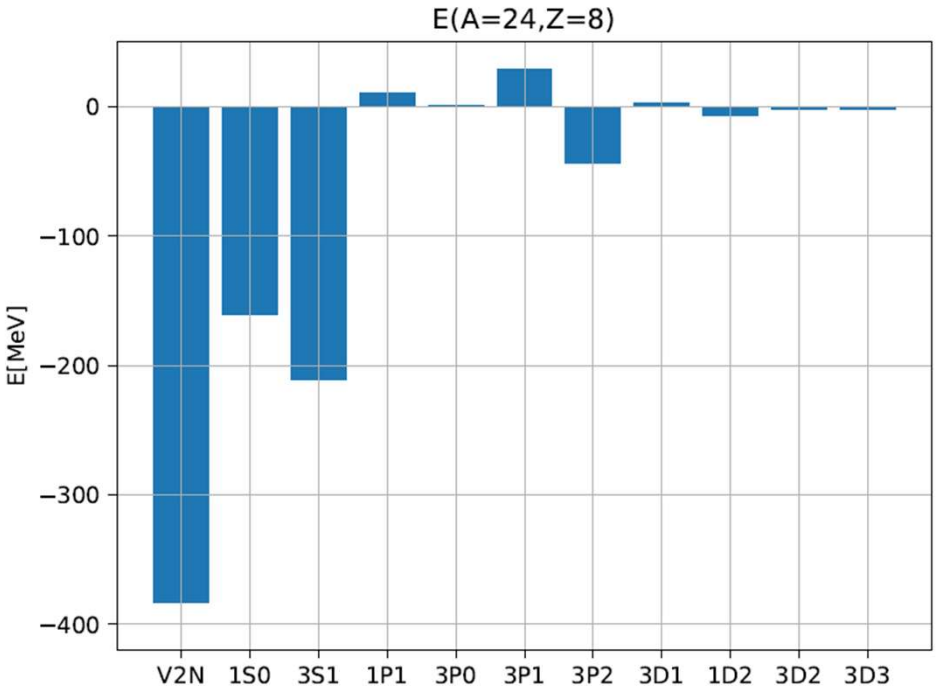
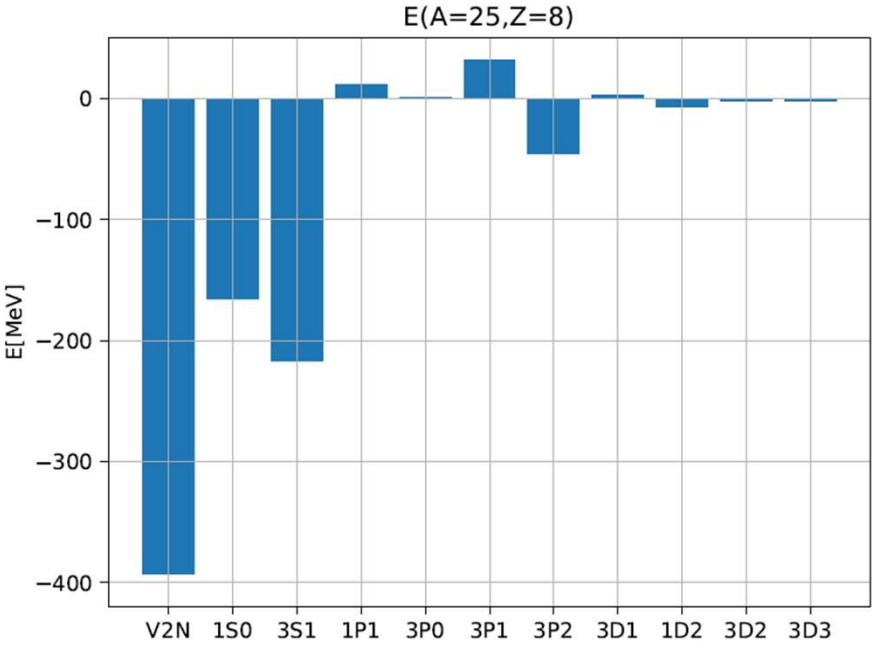
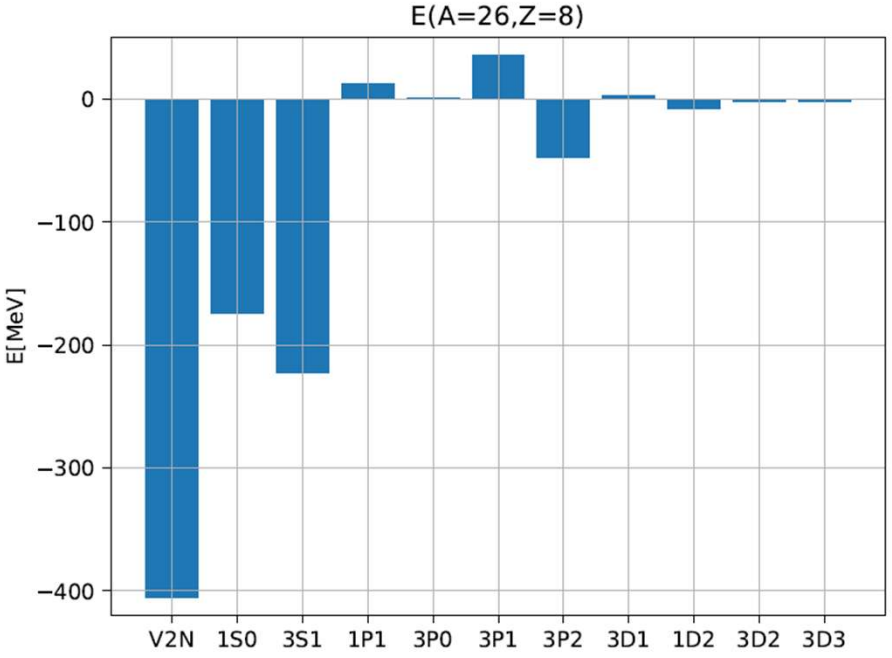
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=22,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=23,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=24,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=25,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3


E(A=26,Z=8)
E[MeV]
0
−100
−200
−300
−400
V2N
1S0
3S1
1P1
3P0
3P1
3P2
3D1
1D2
3D2
3D3

Measure of correlation among extra neutrons

$$f(T_z) = \langle \Psi_{(1/2,-1/2)} | A^{\dagger}_{(1,T_z)} A_{(1,T_z)} | \Psi_{(1/2,-1/2)} \rangle \, .$$

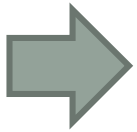
A(1,Tz) is a two nucleon annihilation operator
which gives isospin change by (1, Tz)

f(-1) : two proton operators (two-body interaction) expectation value
f(0) : proton-neutron two-body interaction expectation value
f(1) : two neutron operators (two-body interaction) expectation value

$$f(-1) = f_{3/2}$$

$$f(0) = \tfrac{2}{3} f_{3/2} + \tfrac{1}{3} f_{1/2}$$

$$f(1) = \tfrac{1}{3} f_{3/2} + \tfrac{2}{3} f_{1/2}$$



$$f(1) = 2f(0) - f(-1).$$

Measure of correlation among extra neutrons

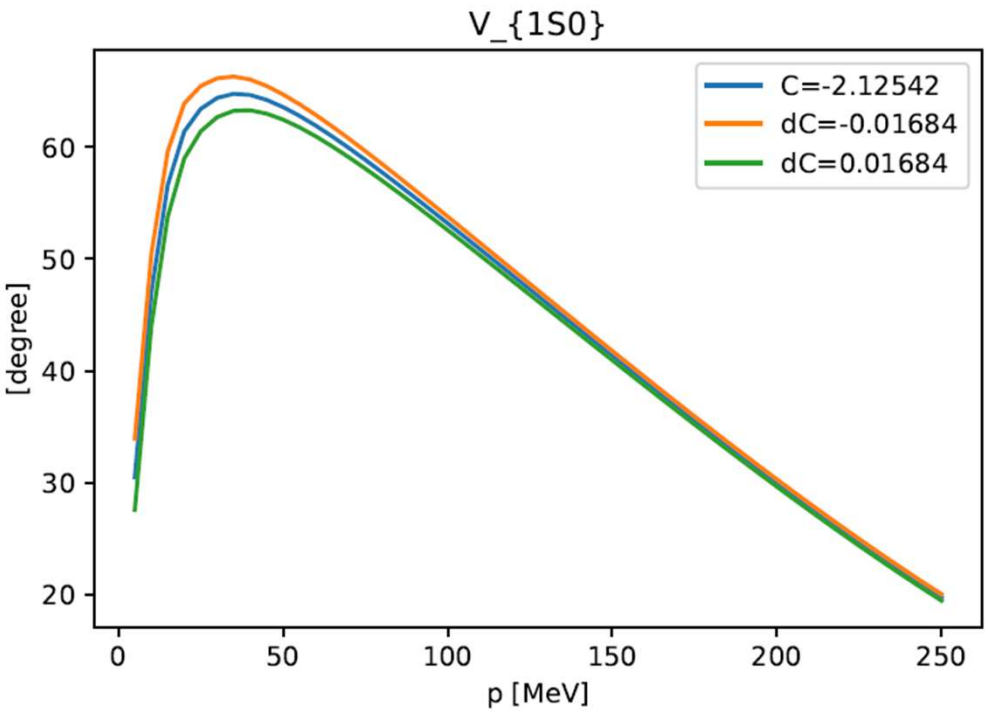
$$f(T_z) = \langle \Psi | A^{\dagger}_{(1,T_z)} A_{(1,T_z)} | \Psi \rangle$$

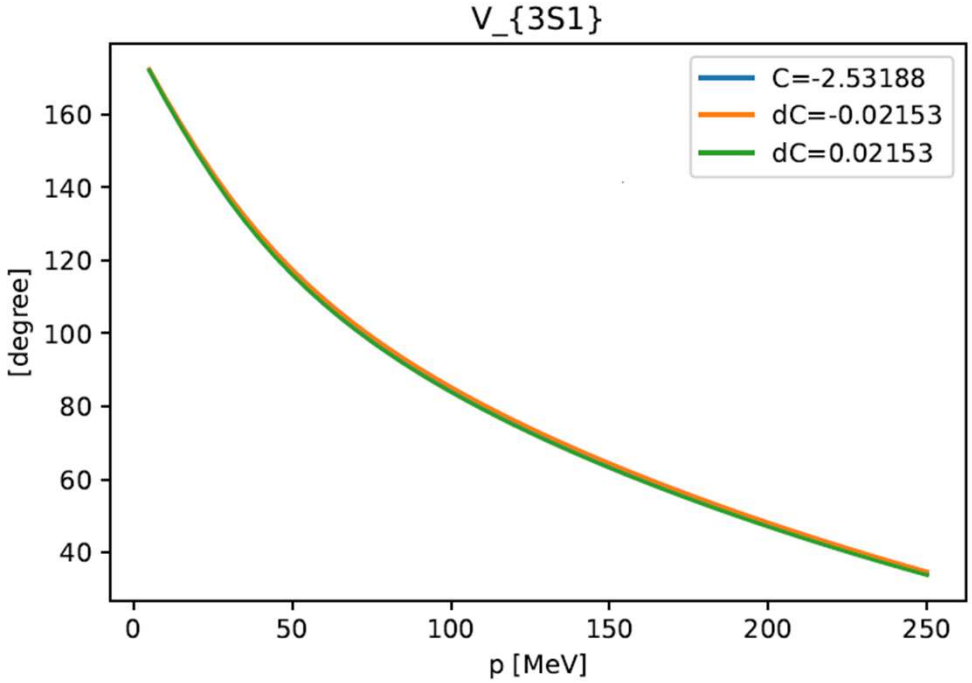
If each of extra neutrons are uncorrelated with each other
→ Additional correlations produced by extra neutrons will be additive.
(only correlation with core neutrons)

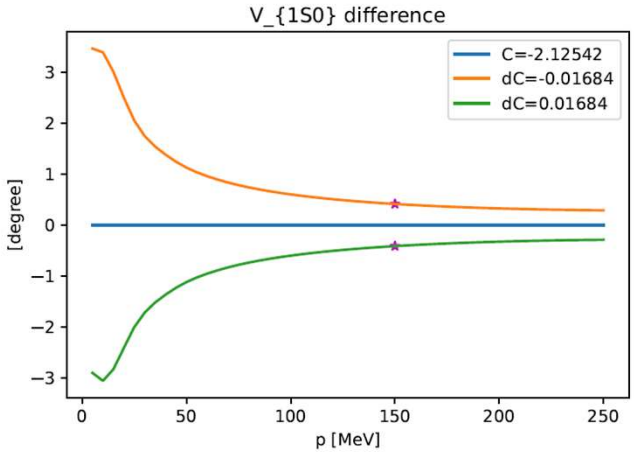
$$f(1) = 2f(0) - f(-1).$$

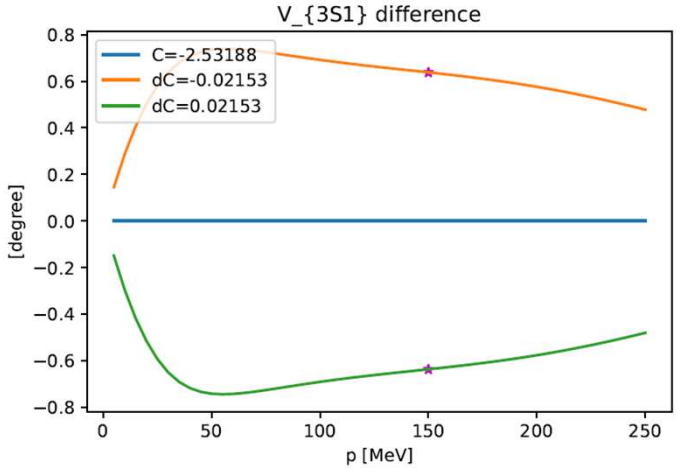
→ Difference between left and right is
a measure of correlation between additional neutrons.

Phase Shift Changes (1 percent)

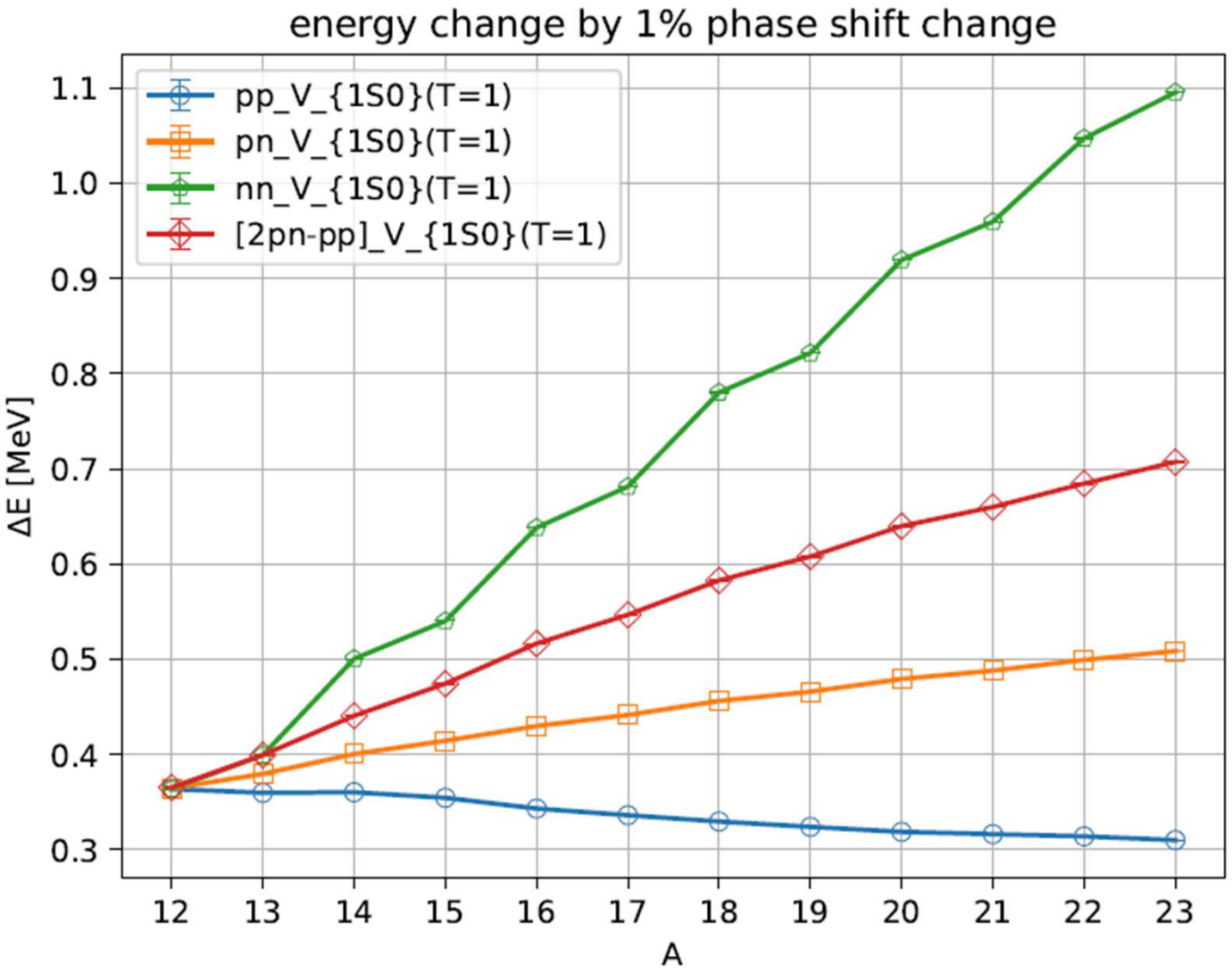








LEC(or phase shift fit) dependence (Lt=200)



Z= 6

- <pp1S0>

→ near constant

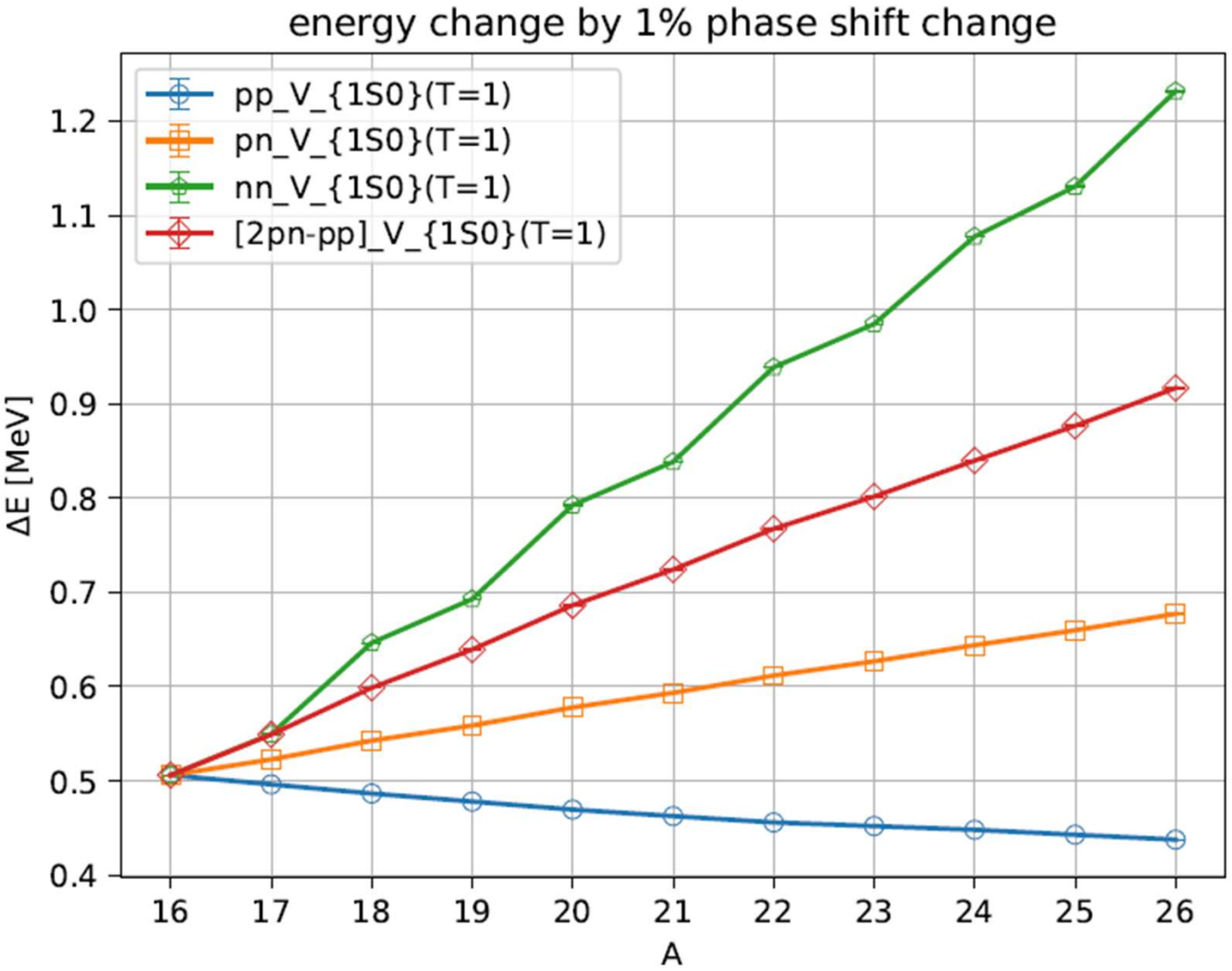
- <nn1S0>

→ strong correlation among paired neutrons

- Adding unpaired one neutron gives similar slope for <nn> and <2pn-pp>

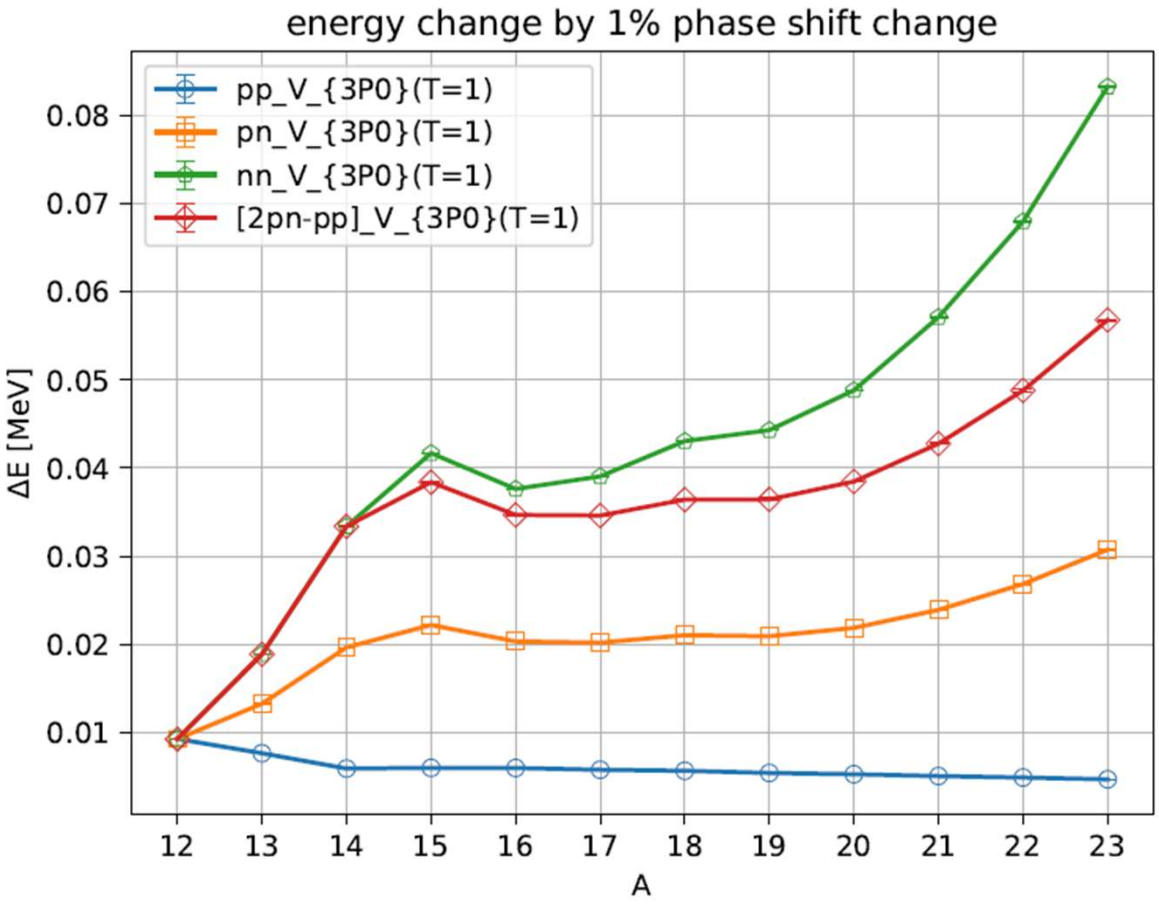
→ Unpaired neutron correlation with core nucleons.

LEC(or phase shift fit) dependence (Lt=200)



Z= 8

LEC(or phase shift fit) dependence (Lt=200)

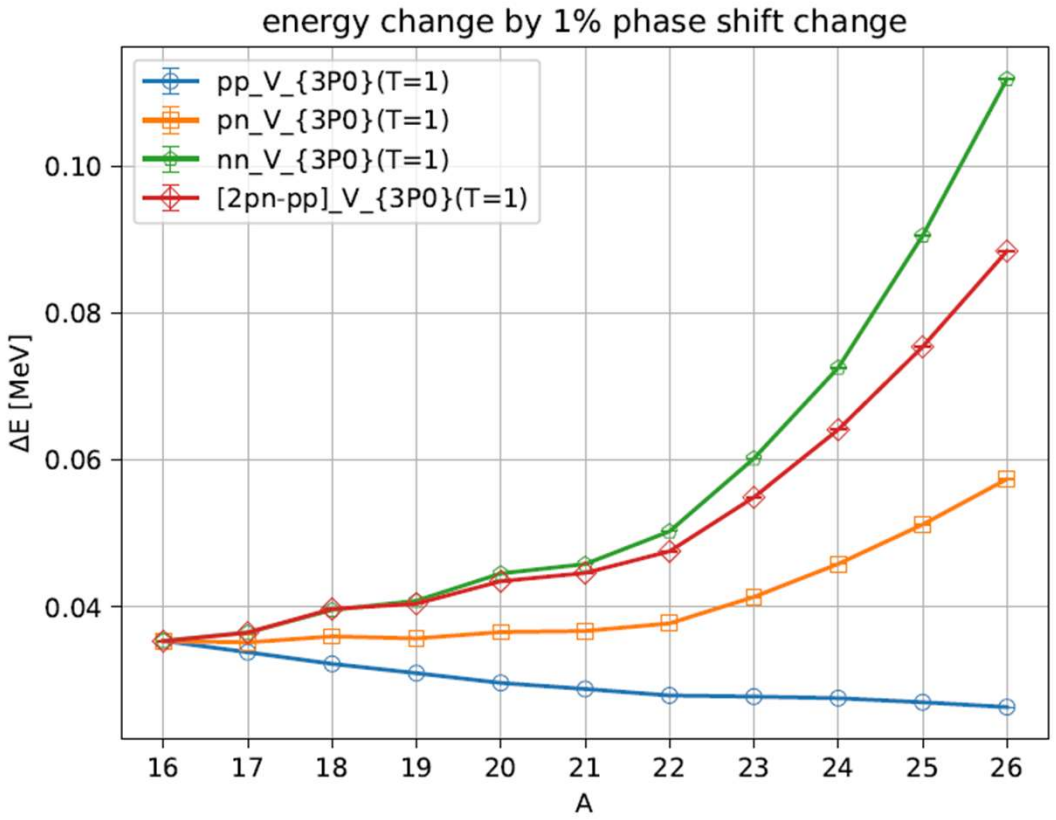


Z= 6

- <pp 3P0> decrease faster than <pp 1S0>

→ Correlation between alpha clusters may become weaker

LEC(or phase shift fit) dependence (Lt=200)



Z= 8

- Slope change in 22O

→ closure of 1d5/2 subshell

Summary

- Wave function matching method seems to be promising
- Carbon, Oxygen isotope up to dripline shows good agreement with experimental data
  - systematic error(finite volume effects) estimation may be necessary.

- Additional neutrons in 1S0 have strong pairing correlation
- Unpaired neutrons correlated primarily to the nuclear core.

two neutrons in 6He

How two neutrons are distributed in 6He?

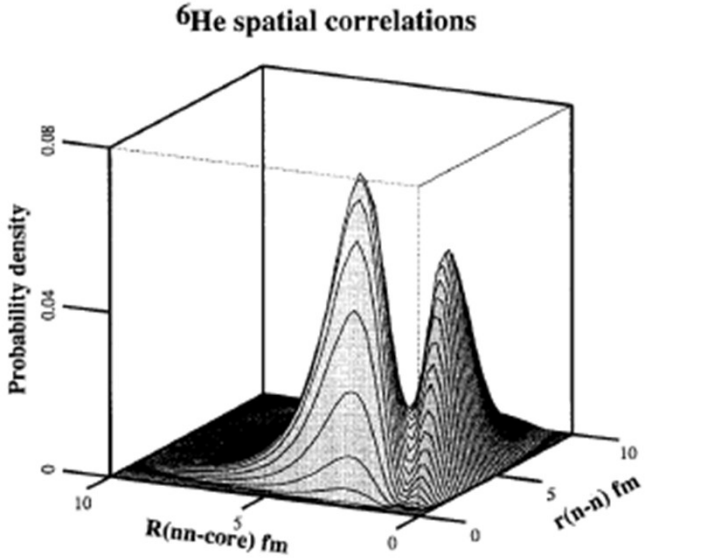


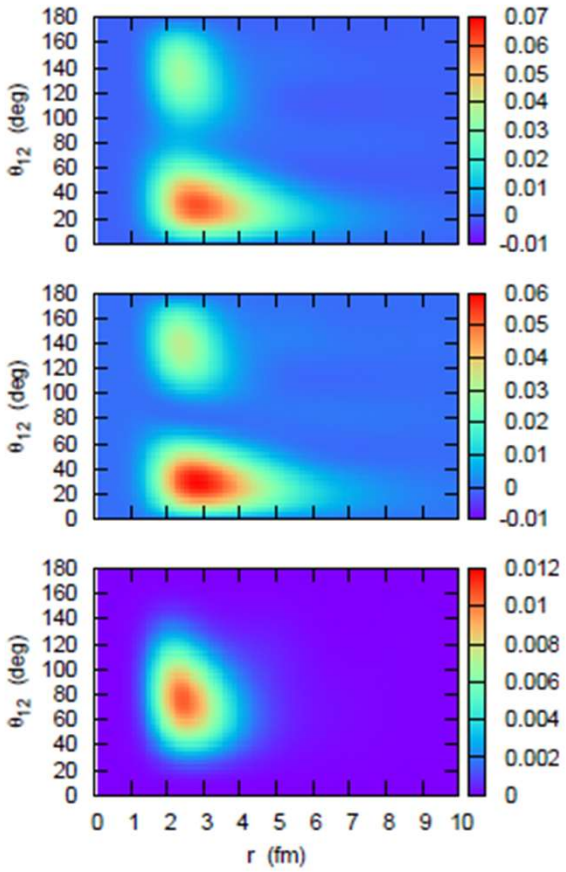
Fig. 4. Correlation density plot for the ground state of ^{6}He in the (nn) and (nn)α variables.

Zhukov et al. (1993)

3 body model calculations

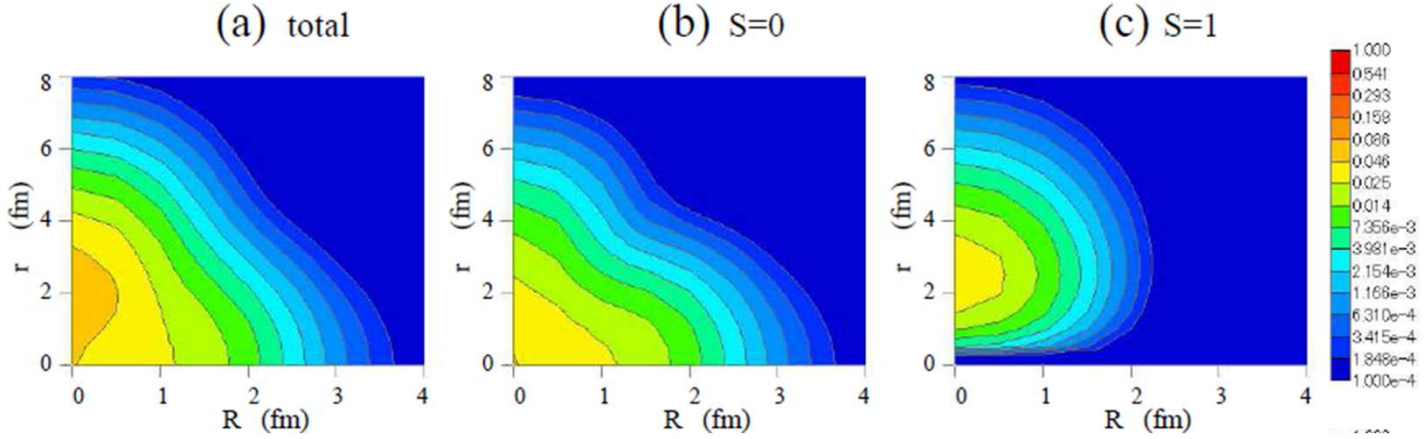
	^{6}He	
J	0^+	2^+
E (MeV)	−0.70	0.88
E^{expt} (MeV)	−0.98	0.83
B_Λ (MeV)		
B_Λ^{expt} (MeV)		
$\overline{r}_{core\text{-}n}$ (fm)	4.55	
$\overline{r}_{core\text{-}2n}$ (fm)	3.79	
$\overline{r}_{n\text{-}n}$ (fm)	4.68	

Hiyama et al.(1996)



Hagino, Sagawa (2005)

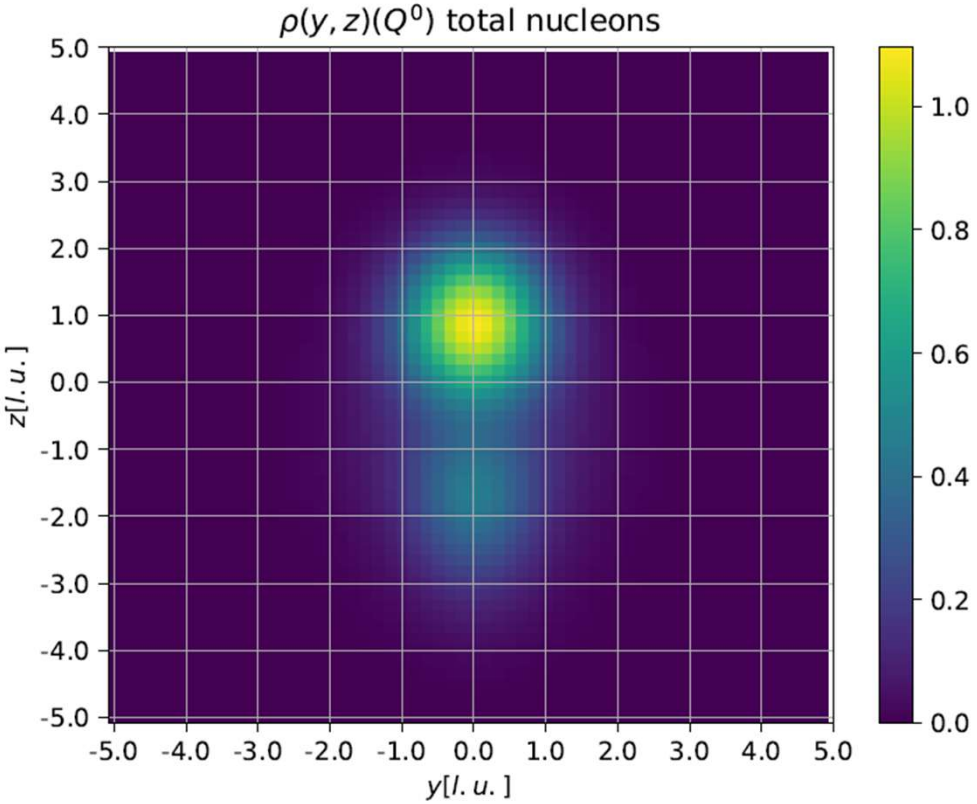
Total
S=0
S=1

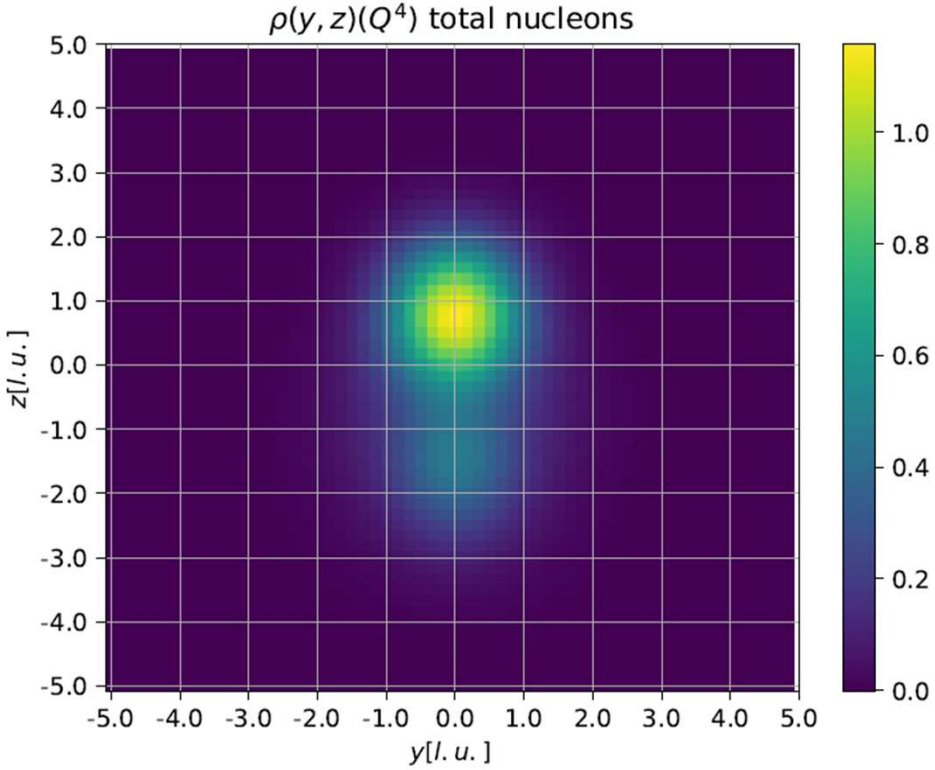


AMD 6-body calculation
Kanada-Enyo et al. (2011)

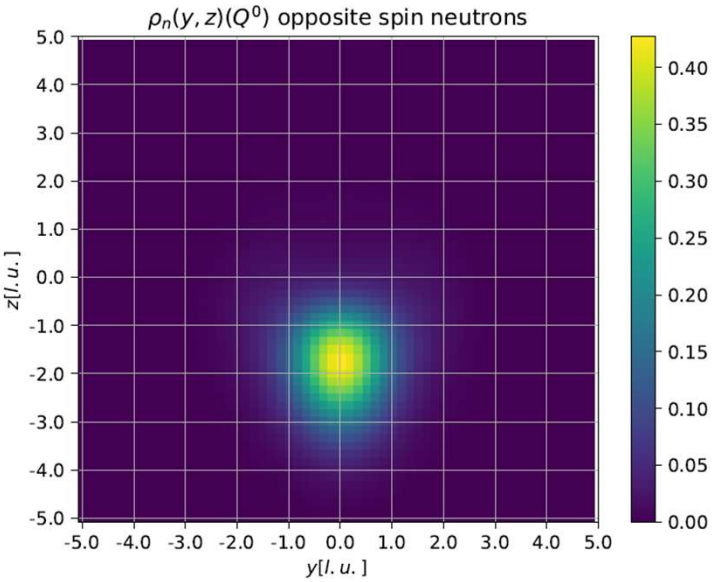
total two-neutron density

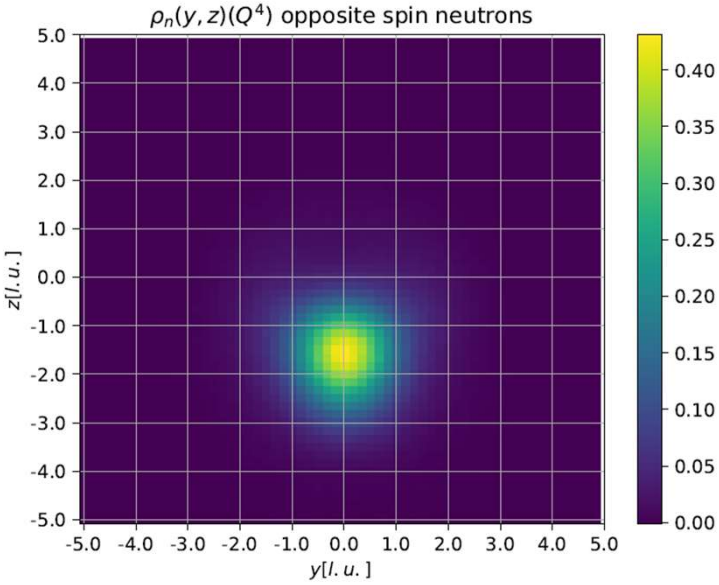
6He pinhole calculation(WFM)

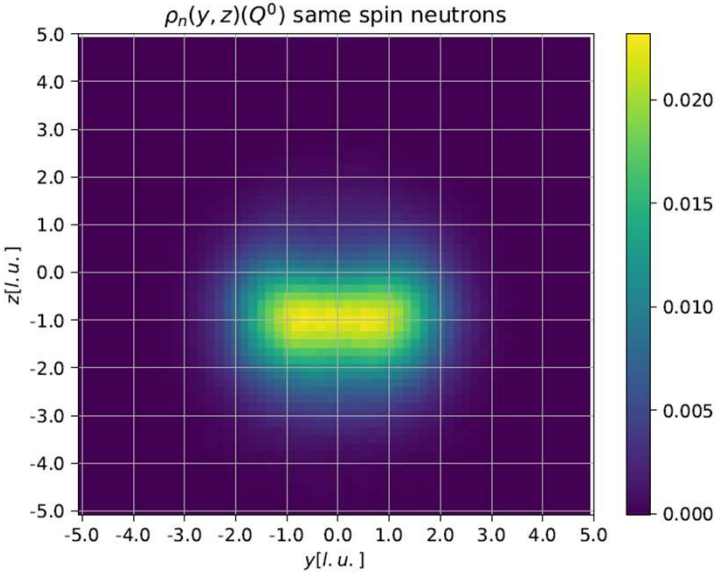


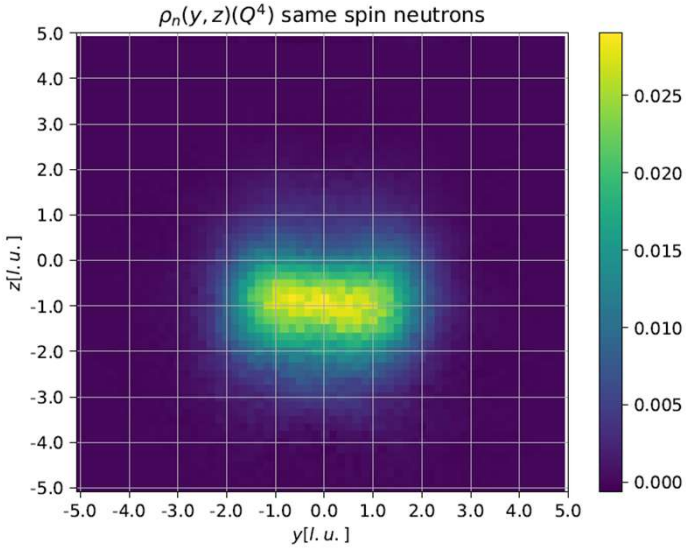


6He pinhole calculation(WFM)

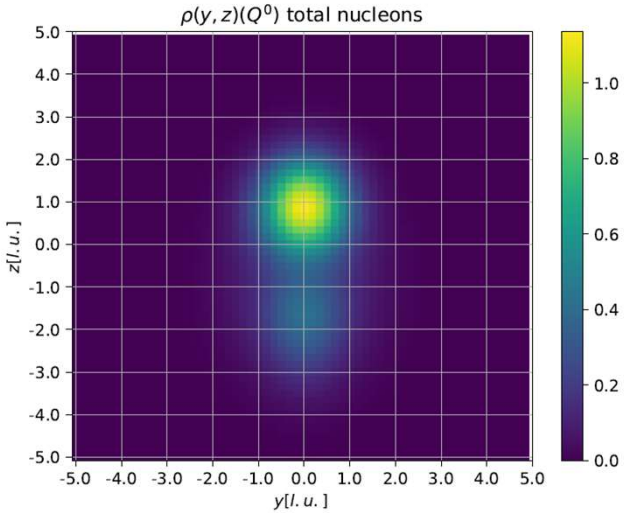


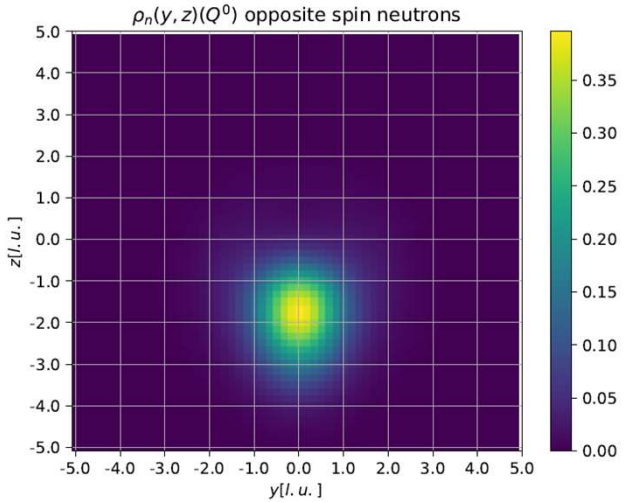


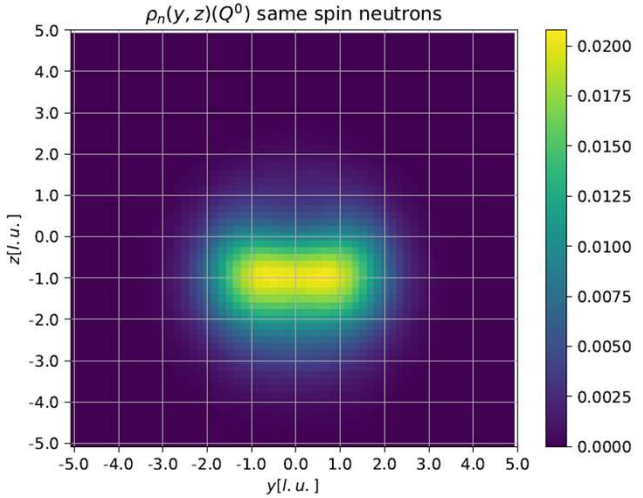


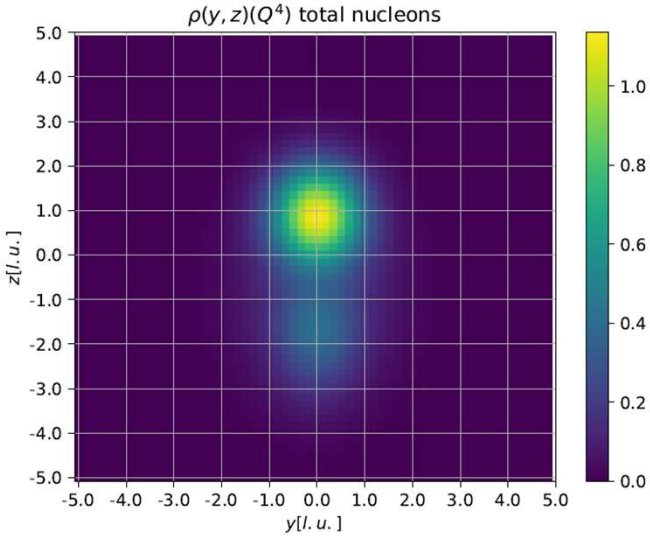


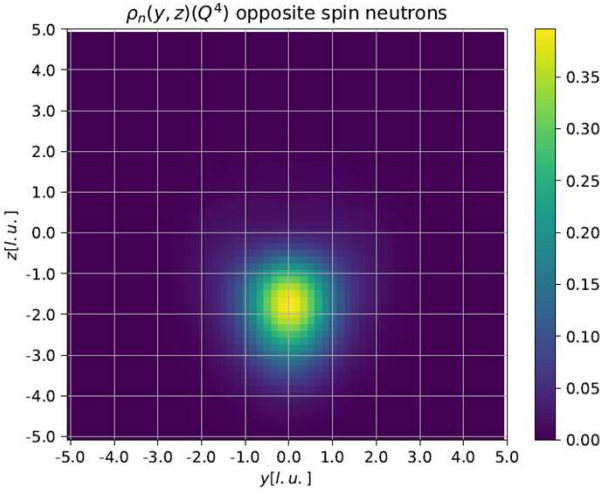
6He pinhole calculation(SU4 “essential”)

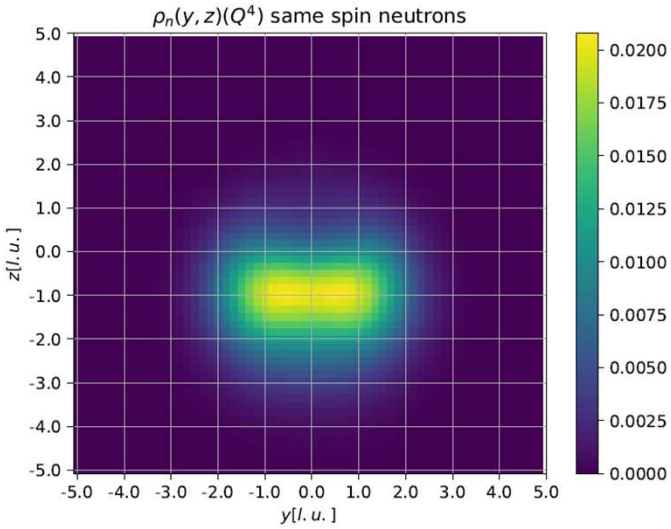




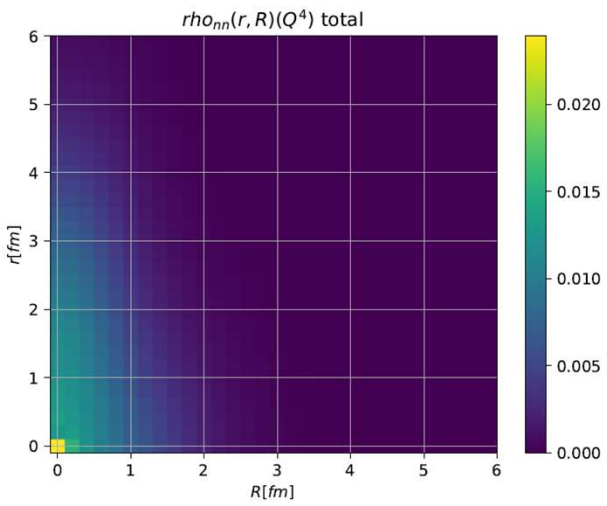


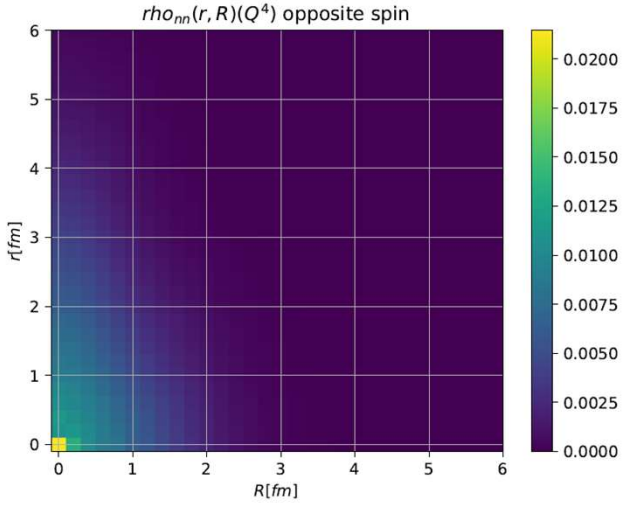


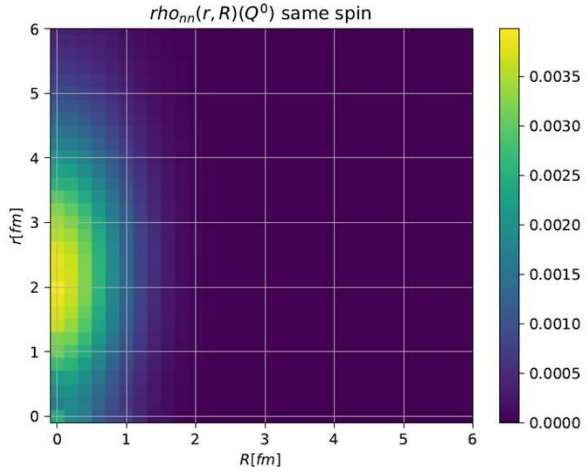


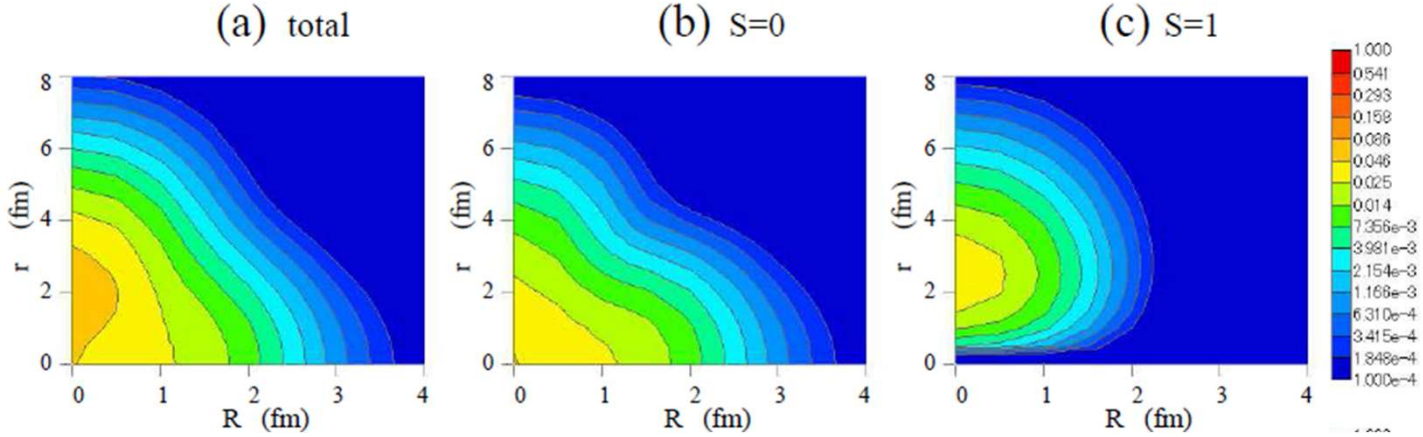


6He pinhole calculation(SU4 “essential”)

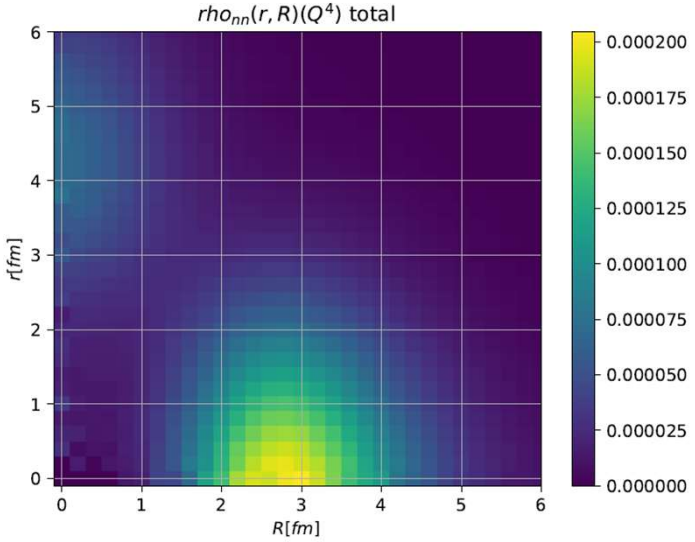


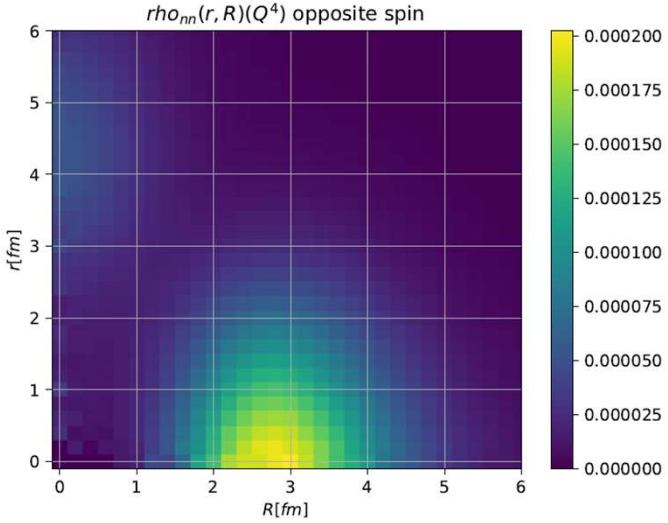


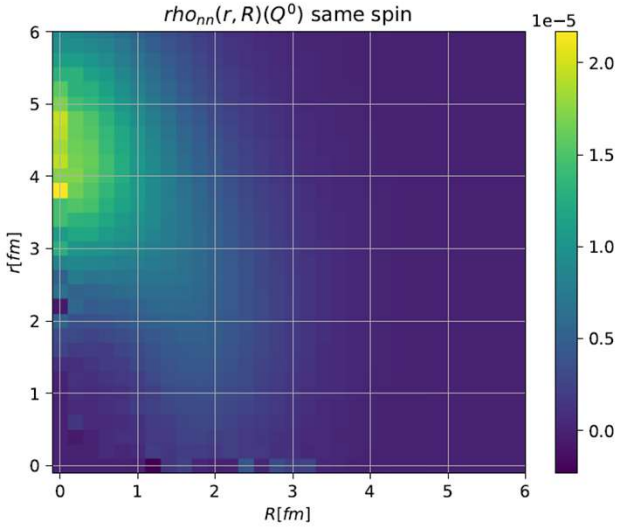




6He pinhole calculation(SU4 “essential”)







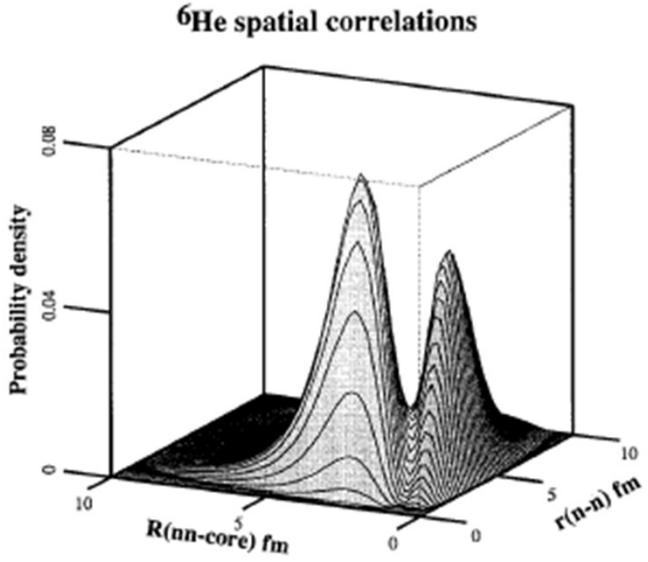


Fig. 4. Correlation density plot for the ground state of ^{6}He in the (nn) and (nn)α variables.

Charge radii

2	He	3	1.9661	0.0030
		4	**1.6755**	**0.0028**
		6	2.0660	0.0111
		8	1.9239	0.0306
6	C	12	2.4702	0.0022
		13	2.4614	0.0034
		14	2.5025	0.0087
8	O	16	2.6991	0.0052
		17	2.6932	0.0075
		18	2.7726	0.0056
10	Ne	**20**	**3.0055**	**0.0021**
		21	2.9695	0.0033
		22	2.9525	0.0040

Alpha clusters + neutrons

Unfortunately,
NURION machine is
too slow to use
full WFM interaction
For pinhole calculations
Of O, Ne isotopes.

Charge radii: SU(4) interaction

	LO	LO+Coul	Essential	Exp.
4He	1.7168(5)	1.7239(5)	1.72(1)(3)	1.6755(28)
6He	2.0667(9)	2.0731(10)		2.0660(111)
12C	2.7987(12)	2.8379(13)		2.4702(22)
13C	2.7517(12)	2.7868(13)		2.4614(34)
14C	2.7043(11)	2.7359(11)		2.5025(87)
16O	2.7054(9)	2.7481(9)	2.74(1)(1)	2.6991(52)
17O				2.6932(75)
18O	2.7807(12)	2.8247(13)		2.7726(56)
20Ne	3.0674(12)	3.1250(14)	2.95(1)(1)	3.0055(21)
21Ne	3.0742(15)	3.1308(17)		2.9695(33)
22Ne	3.0876(17)	3.1431(19)		2.9525(40)

- "Essential" binding Interaction(2N+3N)
- Lt=300 results

→ SU(4) is
Not accurate enough.
May need WFM.

→ Euclidean time extrapolation?