The three-body DD^*K system on the lattice

Qian Wang
South China Normal University

Frontiers in Nuclear Lattice EFT: From Ab Initio Nuclear Structure to Reactions
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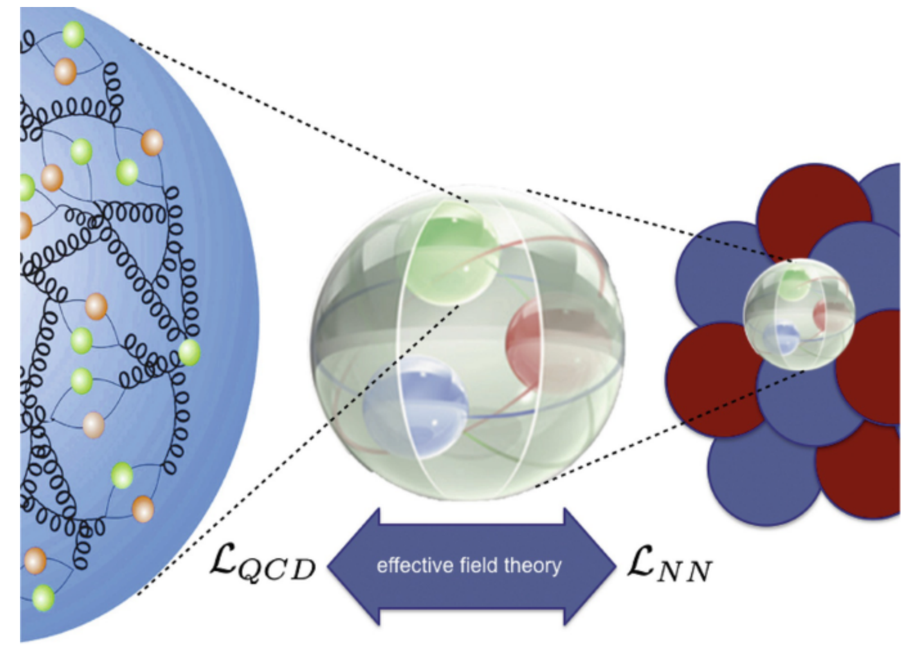
Z.Y. Zhang, X.Y. Hu, G. Zhao He, J. Kiu, J.A. Shi, B.N. Lu, QW, PRD111(2025)036002
W.J. Zhang, Z.Y. Zhang, J.F. Hu, B.N. Lu, J.Y. Pang, QW,…. in preparation

Introduction to LEFT

Lattice EFT=Chiral EFT+Lattice+Monte Carlo

- Hadrons are effective degrees of freedom
- EFT describes hadron interactions
- Lattice spacing is set around 1 fm



	LQCD	LEFT
Degrees of freedom	quarks & gluons	hadrons
lattice spacing	~0.1fm	~1fm
dispersion relation	relativistic	non-relativistic
continuum limit	✓	✗
model	Lagrangian	Schrödinger
solver	path integral	matrix compute

Dean Lee, Prog.Part.Nucl.Phys., 63(2009)117,

Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019).

Applications of LEFT

Applications in Nuclear Physics

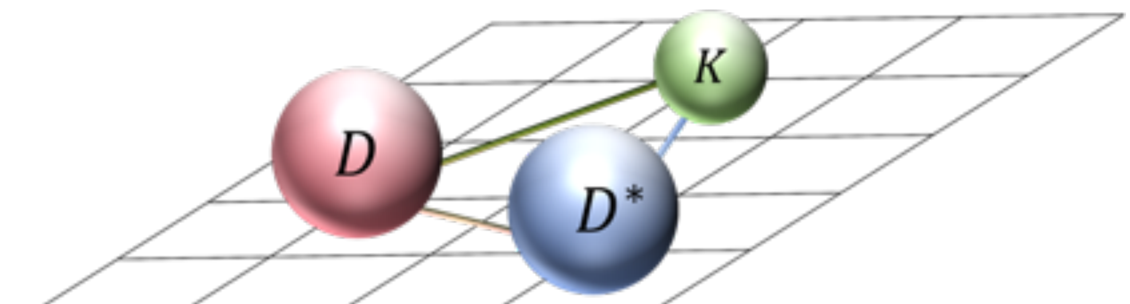
- Neutron-proton scattering — Phys.Rev.C, 98(2018)044002
- Nuclear binding — Phys.Rev.Lett., 117(2016)132501
- Alpha–alpha scattering — Phys.Rev.Lett., 111(2013)032502, Nature, 528(2015)111
- Nuclear thermodynamics — Phys.Rev.Lett., 125(2020)192502
- Properties of nuclei — Phys.Lett.B, 797(2019)134863
- Hoyle state — Phys.Rev.Lett., 109(2012)252501, Nat. Commun., 14(2023)2777

Applications of LEFT

Applications in Hadron Physics

TABLE XXXIX. Summary for heavy-flavor three-body states. Energies are in units of MeV.

Components	$I(J^P)$	Results (Method)	Decay modes
DNN	$\frac{1}{2}(0^-)$	BS $\sim 3500 - 15i$ (FCA, V) [836]	$\Lambda_c\pi^- p, \Lambda_c p$ [836]
$NDK, ND\bar{K}, ND\bar{D}$	$\frac{1}{2}(\frac{1}{2}^+)$	BS $\sim 3050, 3150, 4400$ (FCA) [837]	†
DD^*N	$\frac{1}{2}(\frac{1}{2}^+, \frac{3}{2}^+)$	BS $\sim 4773.2, 4790.7$ (GEM) [838]	$T_{cc}p, DDp + \pi(\gamma), \Xi_{cc} + \pi(\gamma)$, charmed baryon + charmed meson [838]
DD^*N	$\frac{3}{2}(-)$	difficult to form bound states (GEM) [838]	†
$DK\bar{K}$	$\frac{1}{2}(0^-)$	D-like state ~ 2845.5 (FCA) [821], D-like state ~ 2900 (QCDSR, χF) [839]	$\pi\pi D$ [821]
DKK	$\frac{1}{2}(0^-)$	no bound state (FCA) [821]	†
$\bar{D}\bar{K}\Sigma_c$	$1(\frac{1}{2}^+)$	BS ~ 4738.6 (GEM) [840]	$D\Xi', D_s\Sigma_c$ [840]
$D^{(*)}$ multi ρ	...	several $D_J^{(*)}$ states (FCA) [841, 842]	†
$\rho D\bar{D}$	$0(?), 1(?)$	BS $\sim 4241 - 10i$, $[4320 - 13i, 4256 - 14i]$ (FCA) [843]	†
DDK	$\frac{1}{2}(0^-)$	BS ~ 4162 (GEM) [273], 4140 (χF) [819], 4160 (FV) [820]	DD_s^*, D^*D_s [826]
$D\bar{D}K$	$\frac{1}{2}(0^-)$	BS ~ 4181.2 (GEM) [822], 4191 (FCA) [825]	$D_s\bar{D}^*, J/\psi K$ [822]
DD^*K	$\frac{1}{2}(1^-)$	BS ~ 4317.9 (BO) [823]	†
$D\bar{D}^*K$	$\frac{1}{2}(1^-)$	BS ~ 4294.1 (GEM) [822], 4317.9 (BO) [823], 4307 (FCA) [824]	$D_s^{(*)}\bar{D}^{(*)}, J/\psi K^*$ [823, 844]
$D^*D^*\bar{K}^*$	$\frac{1}{2}(0^-, 1^-, 2^-)$	BS $\sim$ $4845 - 40i$, $[4850 - 46i, 4754 - 50i]$, (FCA) [845] $[4840 - 43i, 4755 - 50i]$	$D^*D^*\bar{K}^*$, $D^*D^{(*)}\bar{K}^*$, $[D^*D^*\bar{K}^*, D^*D^{(*)}\bar{K}^*]$ [845]
$\bar{D}\bar{D}^*\Sigma_c$	$1(\frac{1}{2}^+, \frac{3}{2}^+)$	BS $\sim 6292.3, 6301.5$ (GEM) [829]	$J/\psi p\bar{D}^{(*)}, \bar{T}_{cc}\Lambda_c\pi$ [829]
$J/\psi K\bar{K}$	$0(1^-)$	$Y(4260) \sim 4150 - 45i$ (χF) [481]	†
DDD^*	$\frac{1}{2}(1^-)$	BS ~ 5742.2 (GEM) [833]	$DDD\pi(\gamma)$[833]
DD^*D^*	$\frac{1}{2}(0^-, 1^-, 2^-)$	several loosely bound states (GEM) [834]	charmed mesons + ... [834]
$D^*D^*D^*$	$\frac{1}{2}(0^-, 1^-, 2^-, 3^-)$	several loosely bound states (GEM)[834]	charmed mesons + ... [834]
	$\frac{1}{2}(0^-, 1^-, 2^-)$	BS $\sim 5790.9 - 49.8i, 5990.2, 5989.4$ (FCA) [835]	
$D^*D^*D^{(*)}$	$\frac{3}{2}(-)$	difficult to form bound states (GEM) [834]	†
$D^*D^*\bar{D}$	$\frac{1}{2}(2^-)$	BS ~ 5879 (F) [846]	†
$D^*D^*\bar{D}^*$	$\frac{1}{2}(3^-)$	BS ~ 6019 (F) [846]	†
$\Omega_{ccc}\Omega_{ccc}\Omega_{ccc}$	$?(\frac{3}{2}^+)$	no bound state (GEM) [847]	†
$\Xi_{cc}\Xi_{cc}\bar{K}$	$\frac{1}{2}(0^-)$	BS ~ 7641.8 (GEM) [848]	†



- Gaussian expansion method (GEM)
- QCD sum rule (QCDSR)
- Born-Oppenheimer approximation
- Fixed center approximation (FCA)
- Faddeev equation (F)

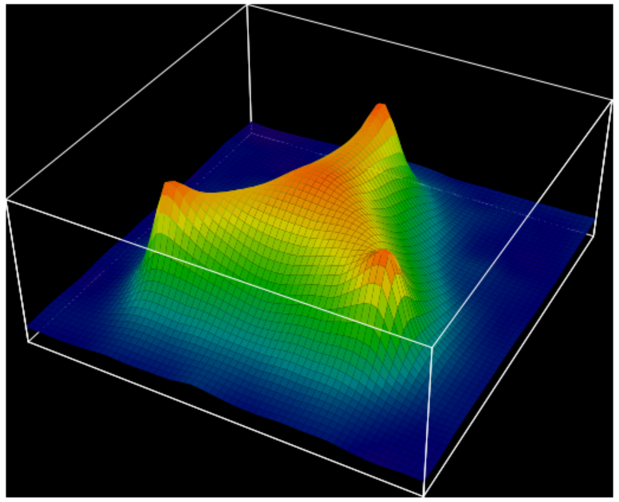
Without 3-body force!

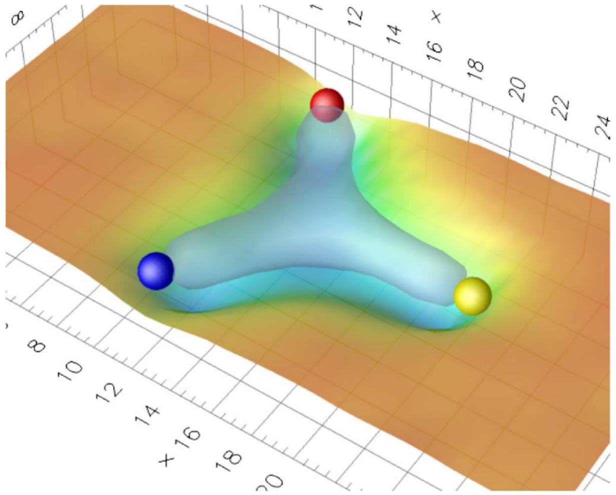
Liu et al., Phys.Rept.1108(2025)1

Advantages of LEFT

- Include three-body force directly

- Three-body force in baryon from LQCD





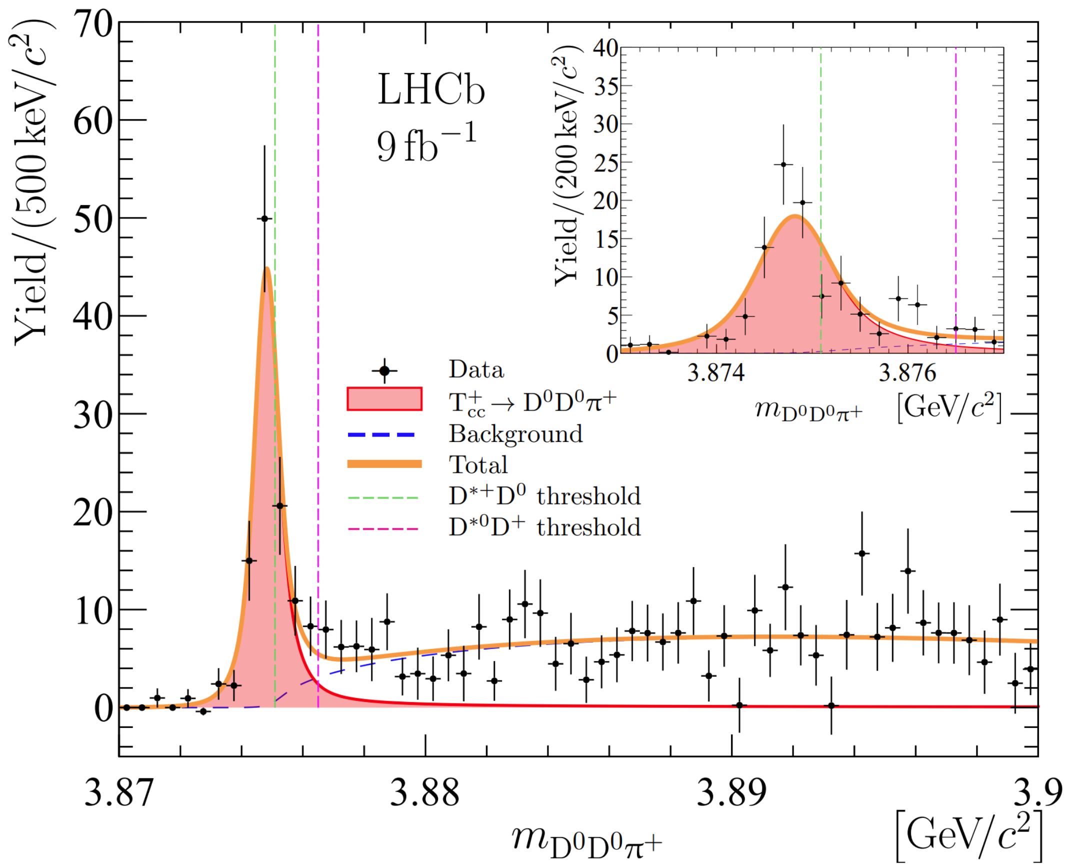
H. Ichie et al., Nucl. Phys.A 721(2003)C899-C902, F. Bissey et al., Phys.Rev.D 76(2007)114512

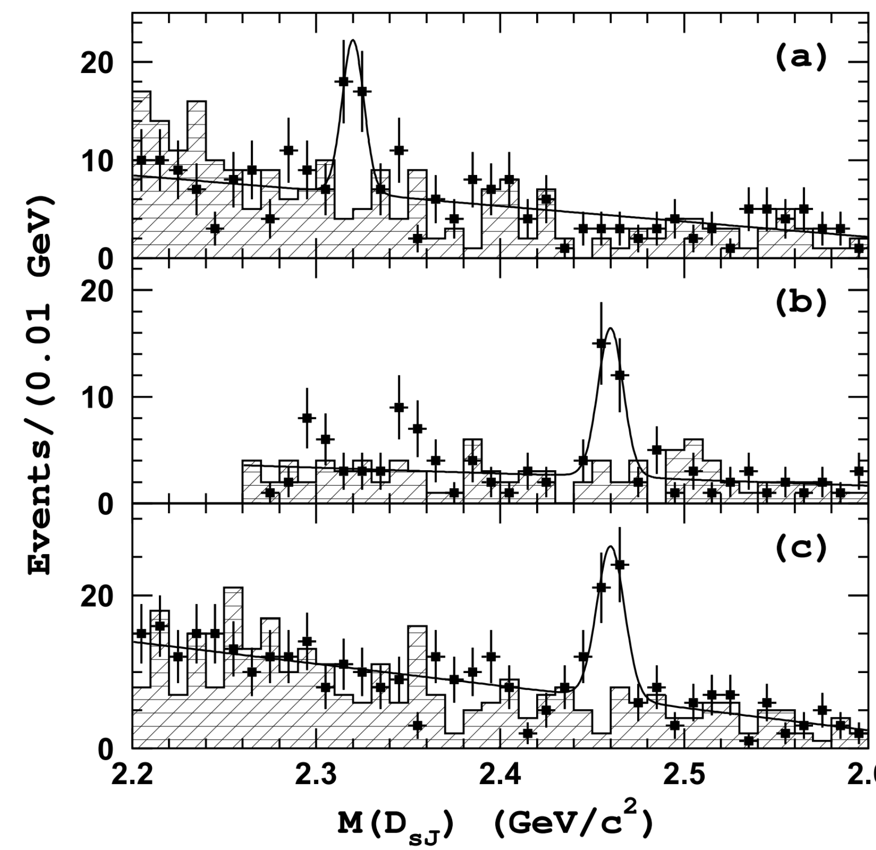
- Three-body force makes nucleus system ren. group invariant

Λ (MeV)	250	275	300	325	350	375	400	Exp.
c_E	5.170	2.763	1.538	0.890	0.561	0.412	0.380	
$E_{2NF}(^3H)$	-6.17(4)	-6.63(4)	-7.05(2)	-7.39(2)	-7.64(1)	-7.77(1)	-7.78(1)	-8.482
$E_{2NF+3NF}(^3H)$	-8.482	-8.482	-8.489	-8.485	-8.483	-8.483	-8.483	-8.482
$E_{2NF}(^4He)$	-30.6(7)	-30.3(6)	-30.7(4)	-30.0(4)	-29.8(4)	-29.4(4)	-29.2(4)	-28.34
$E_{2NF+3NF}(^4He)$	-29.8(7)	-29.5(6)	-29.9(4)	-29.2(4)	-29.0(4)	-28.6(4)	-28.4(4)	-28.34
$E_{2NF}(^{16}O)$	-144.0(21)	-135.1(14)	-136.3(11)	-139.1(9)	-140.6(8)	-141.7(8)	-141.8(9)	-127.6
$E_{2NF+3NF}(^{16}O)$	-135.8(20)	-124.8(14)	-124.5(11)	-126.3(9)	-127.3(8)	-128.1(8)	-128.1(8)	-127.6

The motivation of the DD^*K system

- The observation of $T_{cc}^+, D_{s0}^*(2317), D_{s1}(2460)$ in experiment





R. Aaij et al. (LHCb), Nature Phys. 18 (2022)751 P. Krokovny et al. (Belle), Phys. Rev. Lett. 91(2003) 262002

- Close to the DD^*, DK, D^*K thresholds
- Could be hadronic molecular candidates
- Be used to constraint the two-body force

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- The observation of $T_{cc}^+, D_{s0}^*(2317), D_{s1}(2460)$ in experiment

Meng et al., PRD104(2021)051502,

Agaev et al., NPB975(2022)115650,

Feijoo et al., PRD104(2021)114015,

Yan et al., PRD105(2022)014007,

Albaladejo et al., PLB829(2022)137052,

Du et al., PRD105(2022)014024,

Padmanath et al., PRL129(2022)032002,

Chen et al., PLB833(2022)137391,

Lyu et al., PRL131(2023)161901,……

Guo et al, EPJA40(2009)171,

Liu et al., PRD87(2013)014508,

Guo et al., PRD98(2018)014510,

Liu et al., PRD109(2024)5,

Kim et al., PTEP2024(2024)073D01,

Gil-Dominguez et al., PLB843(2023)137997,

Asokan et al., EPJC83(2023)850,

Fu et al., EPJA58(2022)70,

Kong et al., PRD104(2021)094012,……

R. Aaij et al. (LHCb), Nature Phys. 18 (2022)751   P. Krokovny et al. (Belle), Phys. Rev. Lett. 91(2003) 262002

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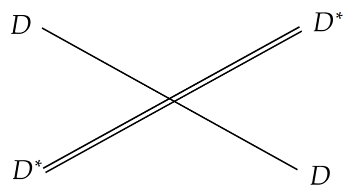
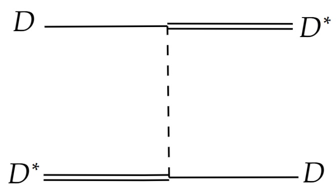
Two-body interactions

- The observation of $T_{cc}^{+}, D_{s0}^{*}(2317), D_{s1}(2460)$ in experiment

  - DD^* interaction: LO+OPE

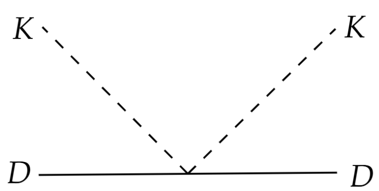
    Du et al, PRD105(2022)014024

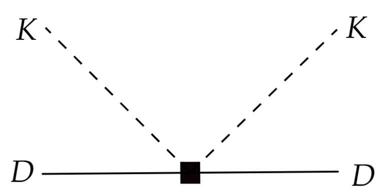




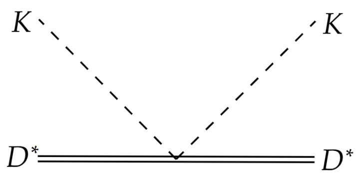
  - DK interaction: LO+NLO

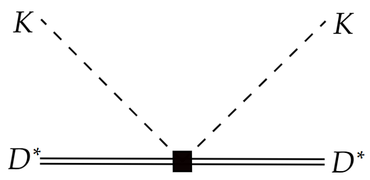
    Guo et al, EPJA40(2009)171





  - D^*K interaction: LO+NLO





  - Single particle regulator is used to obtain a better ren. Group invariant

    Lu et al, arXiv:2308.14559

Two-body interactions

- The observation of $T_{cc}^{+}, D_{s0}^{*}(2317), D_{s1}(2460)$ in experiment

- DD^{*} interaction: LO+OPE

Du et al, PRD105(2022)014024

$$V_{DD^*}^{\mathrm{Con}} = v_0 \epsilon \cdot \epsilon^* \qquad V_{DD^*}^{OPE}(\boldsymbol{q}) = -\frac{3g^2}{4f_\pi^2}\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{q}\boldsymbol{\epsilon}^*\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\mu^2}$$

- DK interaction: LO+NLO

Guo et al, EPJA40(2009)171

$$V_{\mathrm{LO}}^{DK}\left(p_i\right) = \frac{-1}{2f_\pi^2}\left(p_1\cdot p_2 + p_1'\cdot p_2' + p_1\cdot p_2' + p_2\cdot p_1'\right)$$

$$V_{\mathrm{NLO}}^{DK}\left(p_i\right) = -\frac{8M_K^2}{3f_\pi^2}h_1 + \frac{4}{f_\pi^2}h_3 p_2\cdot p_2' + h_5\left(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1'\right)\Bigg)$$

- $D^{*}K$ interaction: LO+NLO

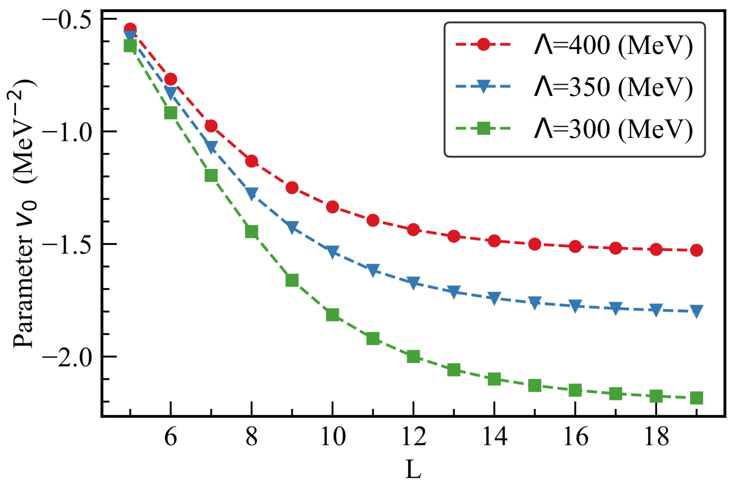
$$V_{\mathrm{NLO}}^{D^*K}\left(p_i\right) = \left(-\frac{8M_K^2}{3f_\pi^2}h_1^* + \frac{4}{f_\pi^2}\left(h_3' p_2\cdot p_2' + h_5^*\left(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1'\right)\right)\right)\boldsymbol{\epsilon}\cdot\epsilon^*$$

- Single particle regulator is used to obtain a better ren. Group invariant

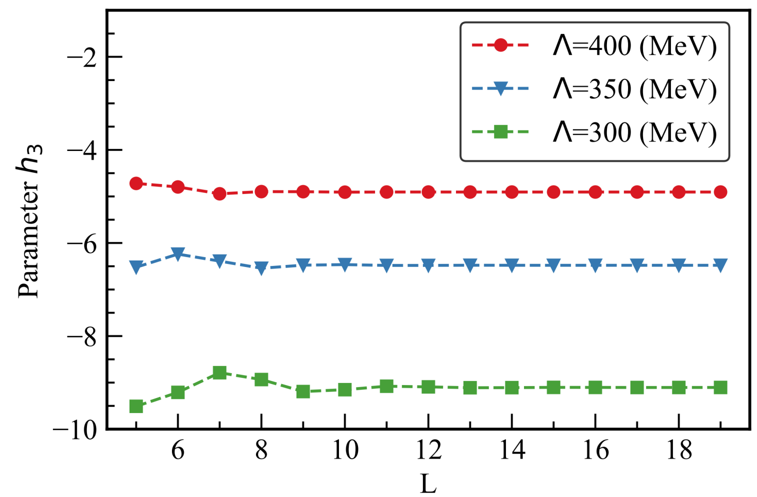
Lu et al, arXiv:2308.14559

The two-body parameters

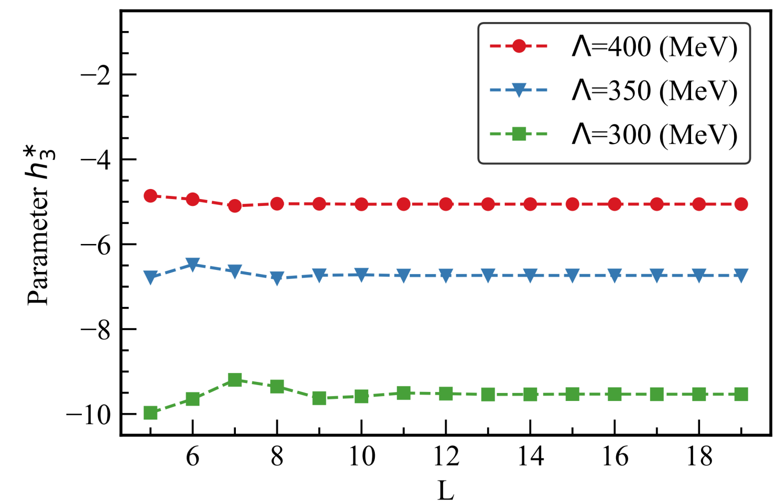
$T_{cc}^{+} \Rightarrow v_0$



$D_{s0}^{*}(2317) \Rightarrow h_3$



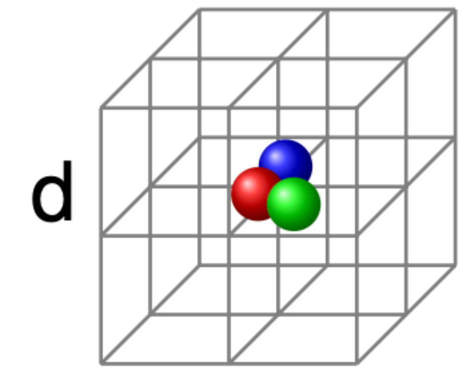
$D_{s1}(2460) \Rightarrow h_3^{*}$



- Cubic lattice $L^3 = 5^3 \cdots 19^3$
- Cutoff $\Lambda = 300,\ 350,\ 400$ MeV in the regulator
- Lattice spacing $a = 1/200$ MeV ~ 0.99 fm
- v_0 converges slow $\Leftarrow$ long-ranged force+shallow bound state
- $\Lambda = 400$ MeV converges quickly

Zhang et al., Phys.Rev.D111(2025)036002

The three-body interactions



- DD^*K Lag. $\mathscr{L} = c_3 \left\langle H\mathscr{D}_\mu H^\dagger H\mathscr{D}^\mu H^\dagger \right\rangle + c_3' \left\langle H\mathscr{A}_\mu H^\dagger H\mathscr{A}^\mu H^\dagger \right\rangle$

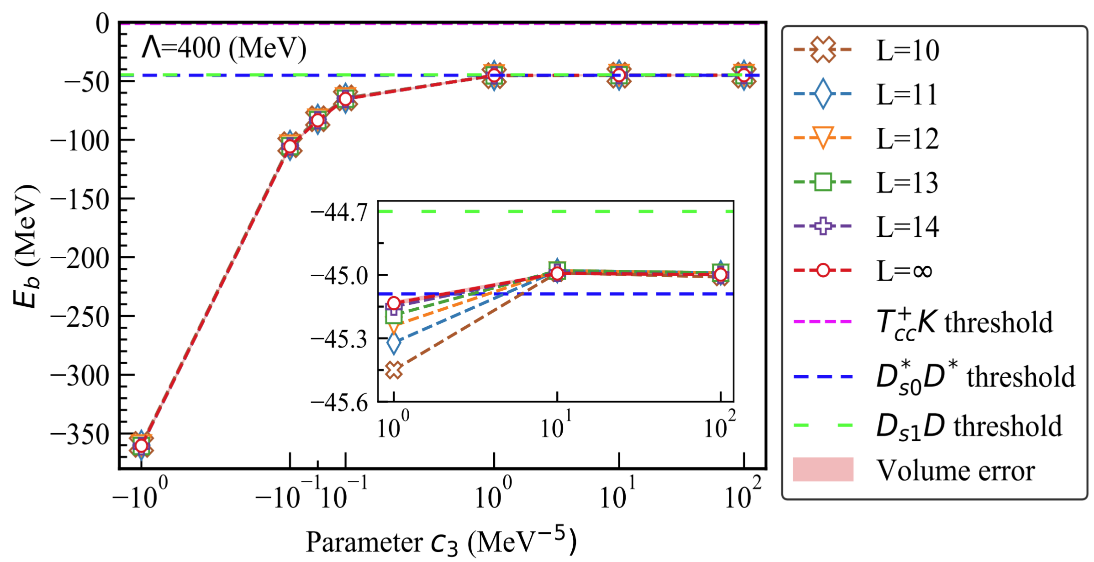
- DD^*K three-body force

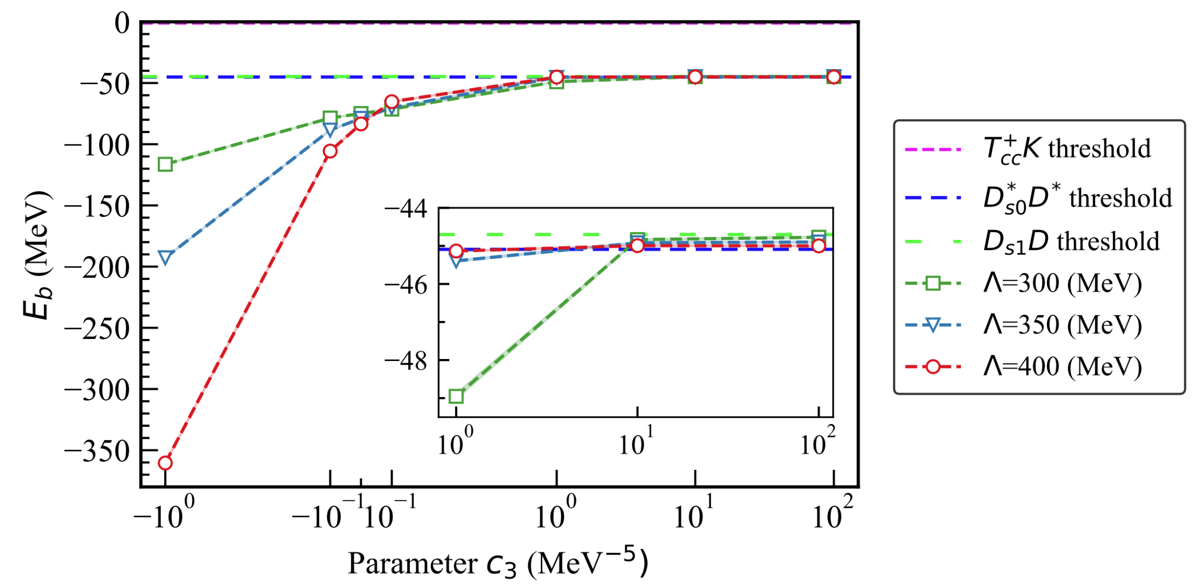
From Serdar's talk

$$V_{DD^*K}(p_i) = \frac{c_3}{4f_\pi^2}\left(p_1\cdot p_3 + p_1\cdot p_3' + p_2\cdot p_3 + p_2\cdot p_3' + p_1'\cdot p_3 + p_1'\cdot p_3' + p_2'\cdot p_3 + p_2'\cdot p_3'\right)\epsilon\cdot\epsilon^*$$

- DD^*K binding energy

Zhang et al., Phys.Rev.D111(2025)036002





- Extrapolate to infinite volume

Meng et al., PRD98(2018)014508

$$\frac{\Delta E}{E_T} = -(\kappa L)^{-3/2}\sum_{i=1}^{3} C_i \exp\left(-\mu_i \kappa L\right)$$

- Switch off three-body force, the result is consistent with that in

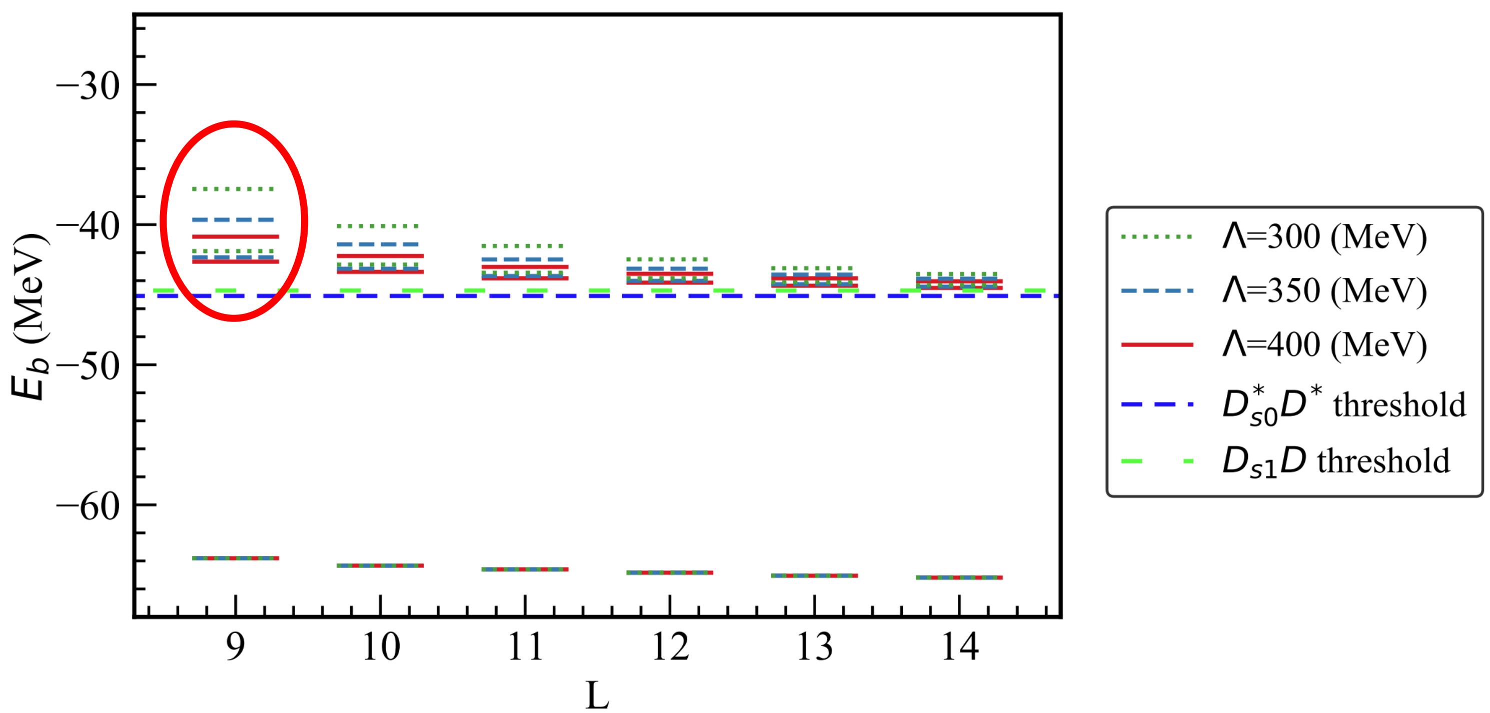
Ma et al., CPC43(2019)014102

The first excited state

- No experimental data ⇒ binding energy with $\Lambda = 400$ MeV as input
- The parameter c_3 at various cubic

Λ (MeV)	Parameter	L						State
		9	10	11	12	13	14	
400		0.100	0.100	0.100	0.100	0.100	0.100	Input
350	c_3 (MeV^{-5})	0.170	0.162	0.164	0.164	0.163	0.163	Fitted
300		0.328	0.305	0.281	0.278	0.281	0.280	Fitted

Zhang et al., Phys.Rev.D111(2025)036002



- $E^{\text{excited}}_{\Lambda_1} - E^{\text{excited}}_{\Lambda_2}$ decreases
- ρ-type and λ-type excitation
- The standard angular momentum and parity projection technique is used

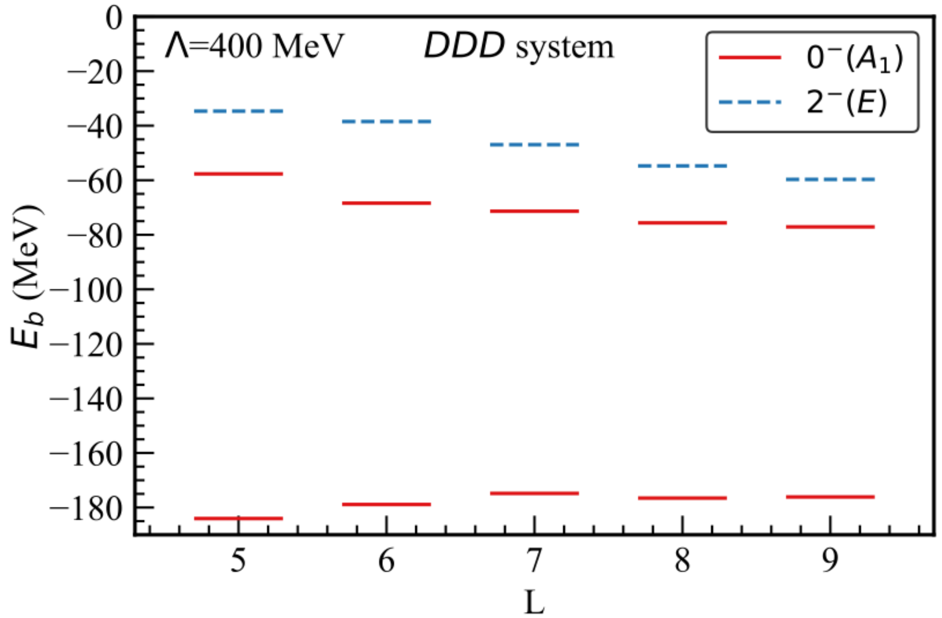
- $J^P = 1^-$ $\Leftarrow$

$$\left|\Psi_A\right\rangle = \frac{d_n}{24}\sum_{i=1}^{24}\chi_n\left(\Omega_i\right)R\left(\Omega_i\right)\left|\Psi_0\right\rangle$$

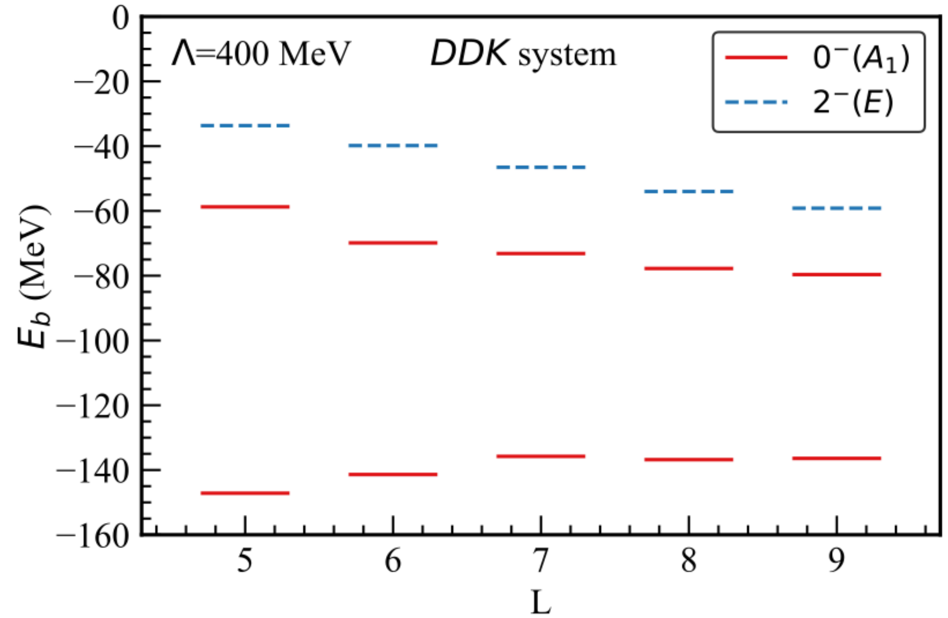
Lu et al., PRD90(2014)034507

The splitting of the first excited state

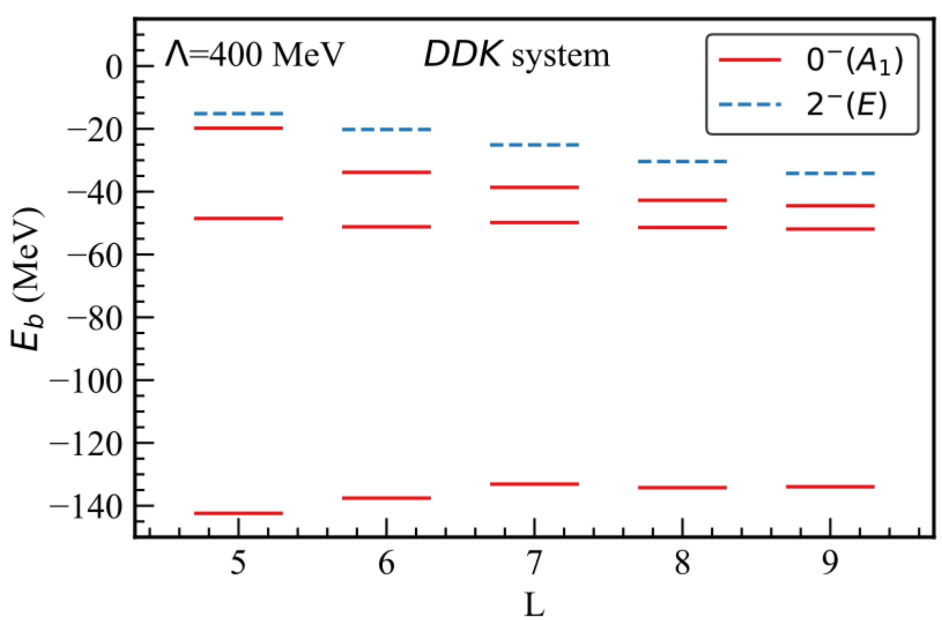
Zhang et al., Phys.Rev.D111(2025)036002



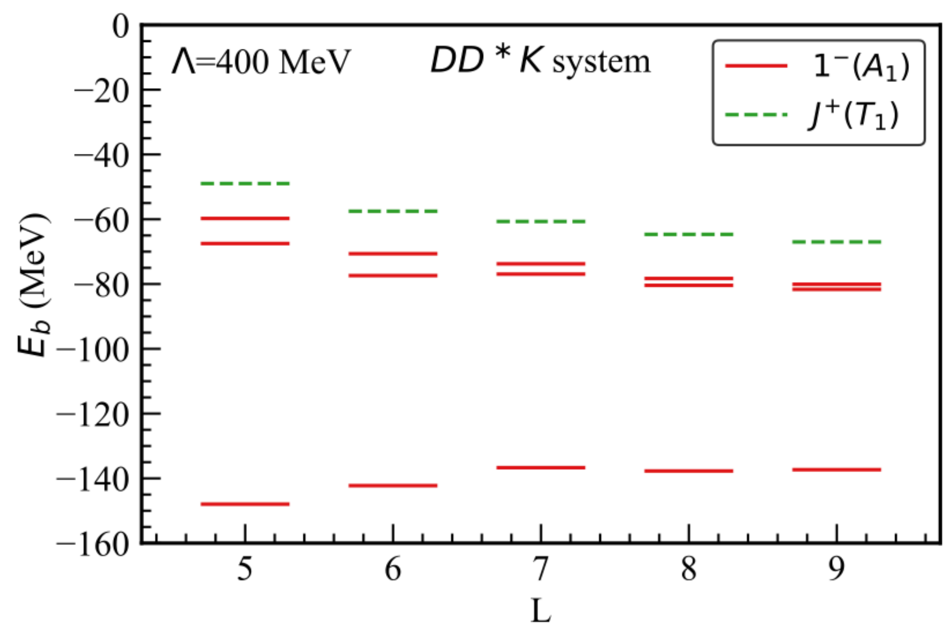
(a) The ground state and excited states of the DDD system with the two-body contact interaction strength $c_{DD} = -7$ MeV^{-2}.



(b) The ground state and the first excited states of the DDK system with the two-body contact interaction strengths $c_{DD} = -1$ MeV^{-2} and $c_{DK} = -10$ MeV^{-2}.



(c) The ground state and the first excited states of the DDK system with the two-body contact interaction strengths $c_{DD} = -5$ MeV^{-2} and $c_{DK} = -8$ MeV^{-2}.

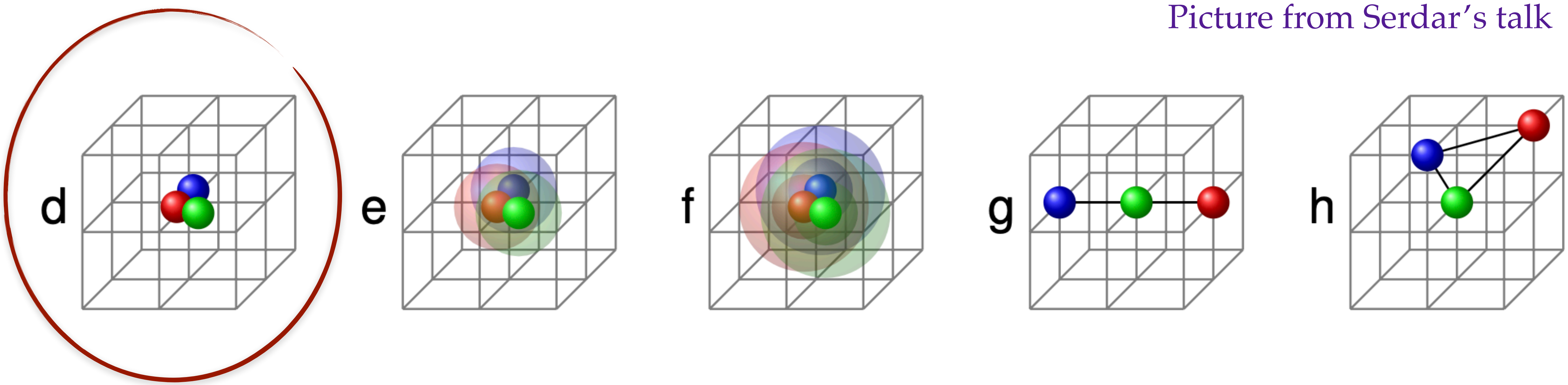


(d) The ground state and the first excited states of the DD^*K system with the two-body contact interaction strengths $c_{DD^*} = -1$ MeV^{-2}, $c_{DK} = c_{D^*K} = -10$ MeV^{-2}.

Summary and outlook

- The number of particles is conserved
- Find a $J^P = 1^- \; DD^*K$ bound state with binding energy in the $(-84, -44)$ MeV region
- We checked the ren. group invariance
- Two energy levels of excited state are ρ-type and λ-type
- Lattice EFT has more advantage for many hadrons system

Picture from Serdar's talk



A question from Akaki on FB23 conference

"why is the extrapolation formula for long-range interaction exponential form?"

ML in the FV extrapolation formula

- The FV extrapolation formula for short-range interaction is well established from Luescher formula
- How about the long-range interaction?
- Use symbolic regression approach

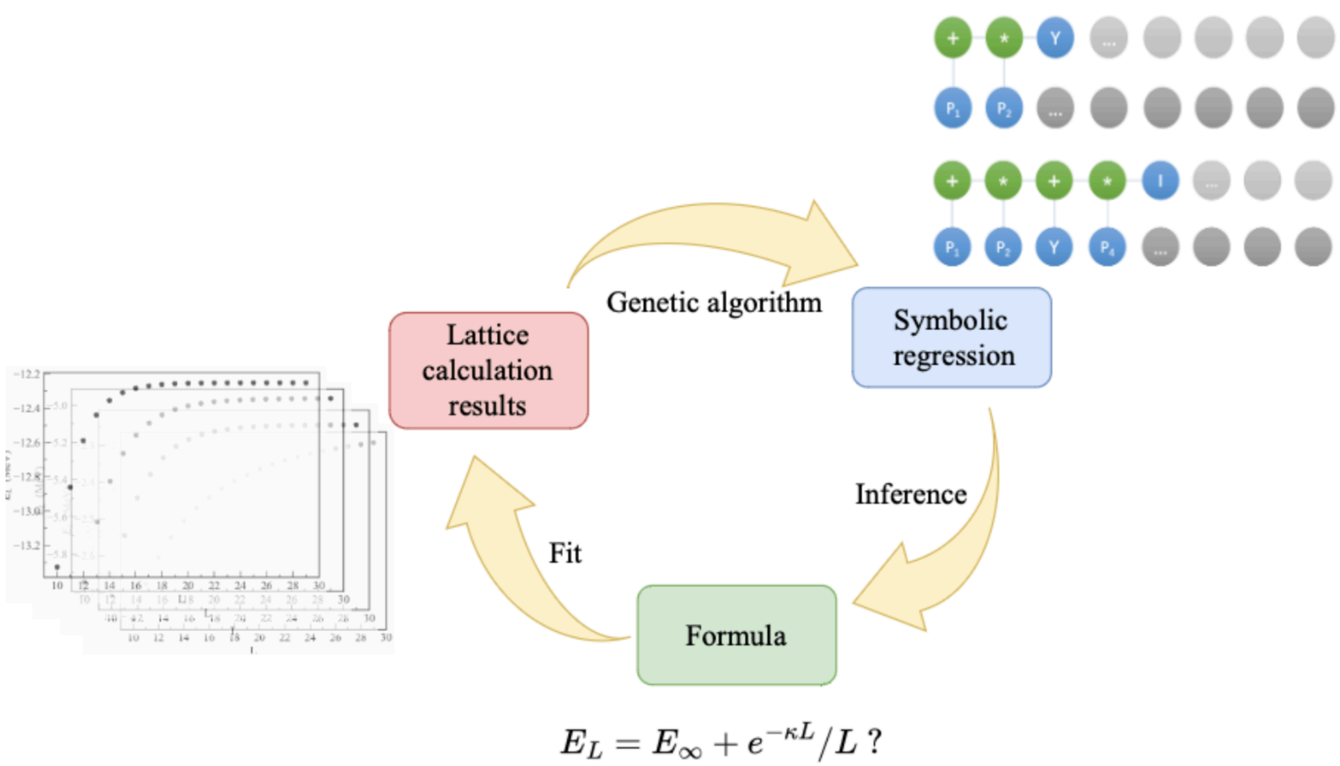


FIG. 1: Workflow of using SR, LEFT and formula.

Short-range interaction

Samples generation

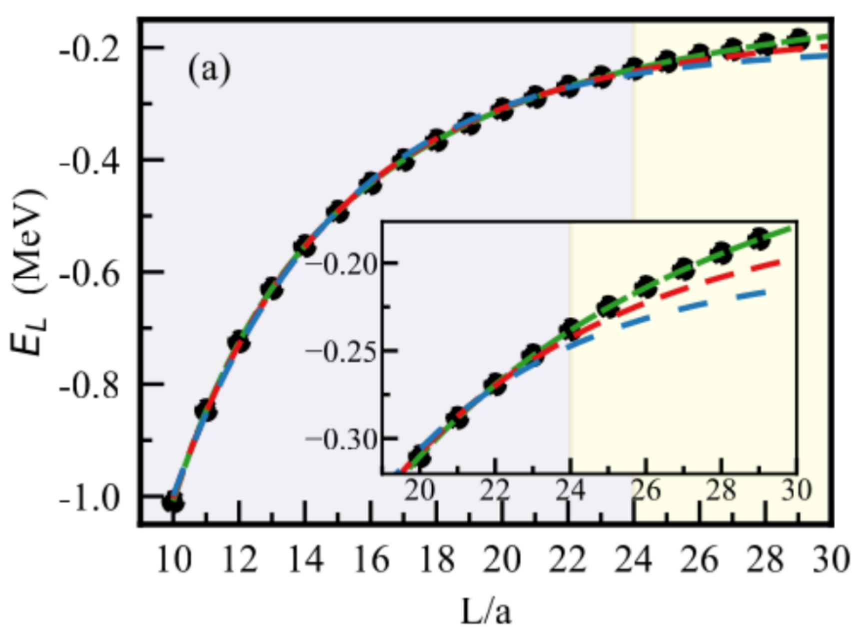
- The short-range potential (two identical particles with $m = 1969$ MeV)

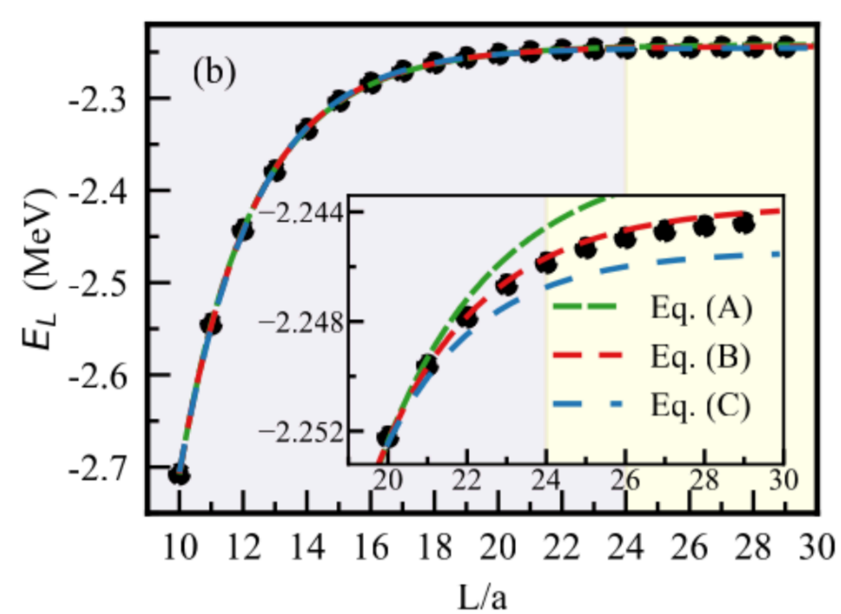
$$H = \sum_{i=1}^{2} \frac{\boldsymbol{p}_i^2}{2m_i} + f(\boldsymbol{p_1}, \boldsymbol{p_2})V(p)$$

$$f(\boldsymbol{p_1}, \boldsymbol{p_2}) = \prod_{i=1}^{2} g_\Lambda(\boldsymbol{p}_i) g_\Lambda(\boldsymbol{p}_i') \qquad V(\boldsymbol{r}) = -C_0 \delta^3(\boldsymbol{r})$$

Single-particle regulator — Direct contact potential

- Solve Schrödinger Equation $H_L\psi = E_L\psi$ in cubic box L^3





Short-range interaction

Symbolic regression

- The PySR model samples the space of analytic expression (the sign of operators, input variables and constants terms)
- The operators: addition, subtraction, multiplication, division, exponential, logarithm and square etc.
- We mutate over 50 iterations of 50 different population samples
- Two elements for the goodness of the output formula, i.e. loss and score

$$\text{Loss} = \sum_{i=1}^{N} (E_{\text{PySR}}(L_i) - E_L(L_i))^2/N \qquad \text{Score} = -\frac{\Delta \ln(\text{Loss})}{\Delta\ C}$$

C is the complexity, defined as the total numbers of operations, variables and constants in a formula, Δ the difference between the two iterations.

Short-range interaction

The results

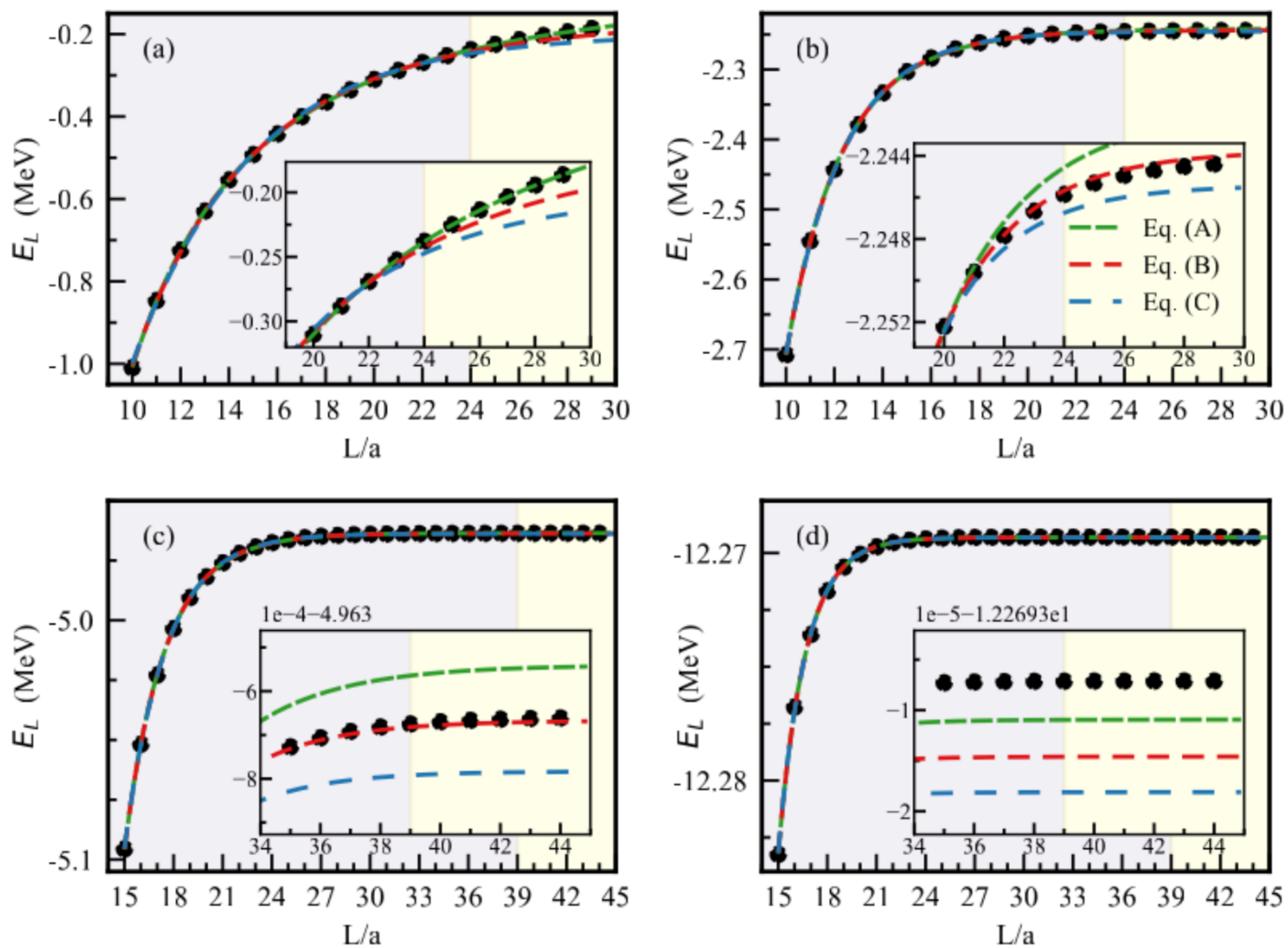
- Box size 10 ~ 30 fm

$$E_L = C_1 + C_2 e^{-C_3 L}/L^2 \quad \text{(A)}$$

$$E_L = C_1 + C_2 e^{-C_3 L}/L \quad \text{(B)}$$

$$E_L = C_1 + C_2 e^{-C_3 L} \quad \text{(C)}$$

- The value of C_3 is exactly the binding momentum



- Recover the formula of short-range interaction successfully

Long-range interaction

Samples generation

- The long-range potential (two identical particles with $m = 1969$ MeV)

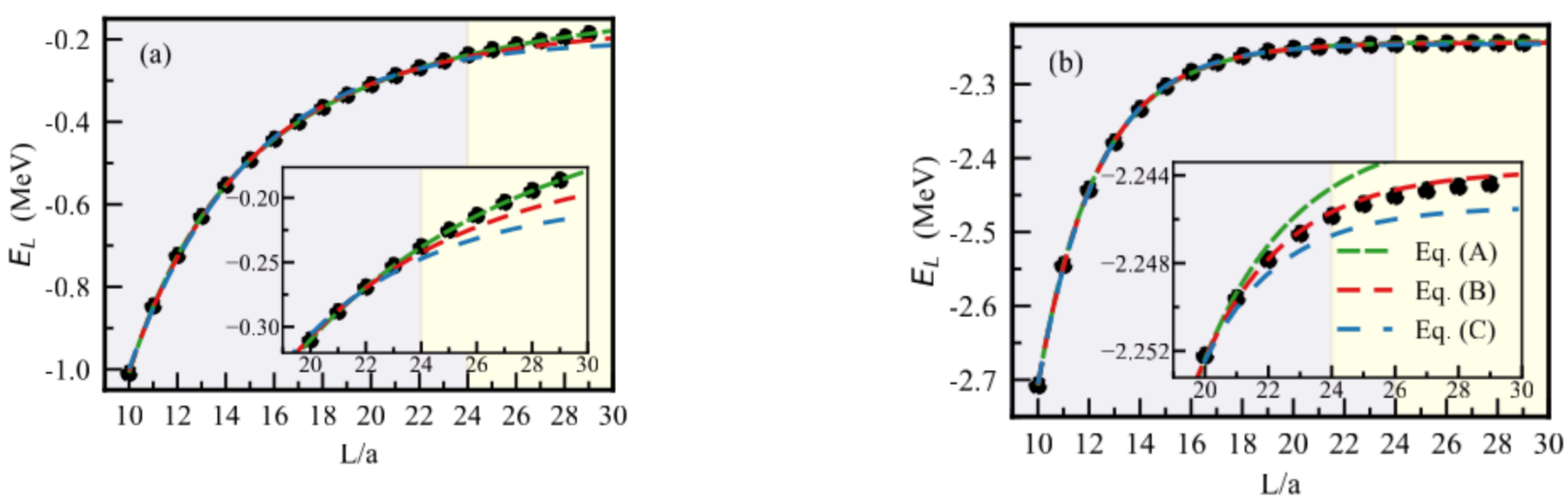
$$V(\boldsymbol{r}) = -C_{01}\delta^3(\boldsymbol{r}) - C_{02}\frac{e^{-\mu r}}{r} \qquad C_{01} = C_{02}$$

$$f(\boldsymbol{p_1},\boldsymbol{p_2}) = \prod_{i=1}^{2} g_\Lambda(\boldsymbol{p_i})g_\Lambda(\boldsymbol{p_i'}) \qquad \hat{f}(\boldsymbol{q}) = exp(-(\boldsymbol{q}^2+\mu^2)/\Lambda^2)$$

Single-particle regulator

Range parameter $\mu = 20$ MeV

- The parameter $C_{01},\ C_{02}$ is tuned to have the same binding energies



Long-range interaction

The results

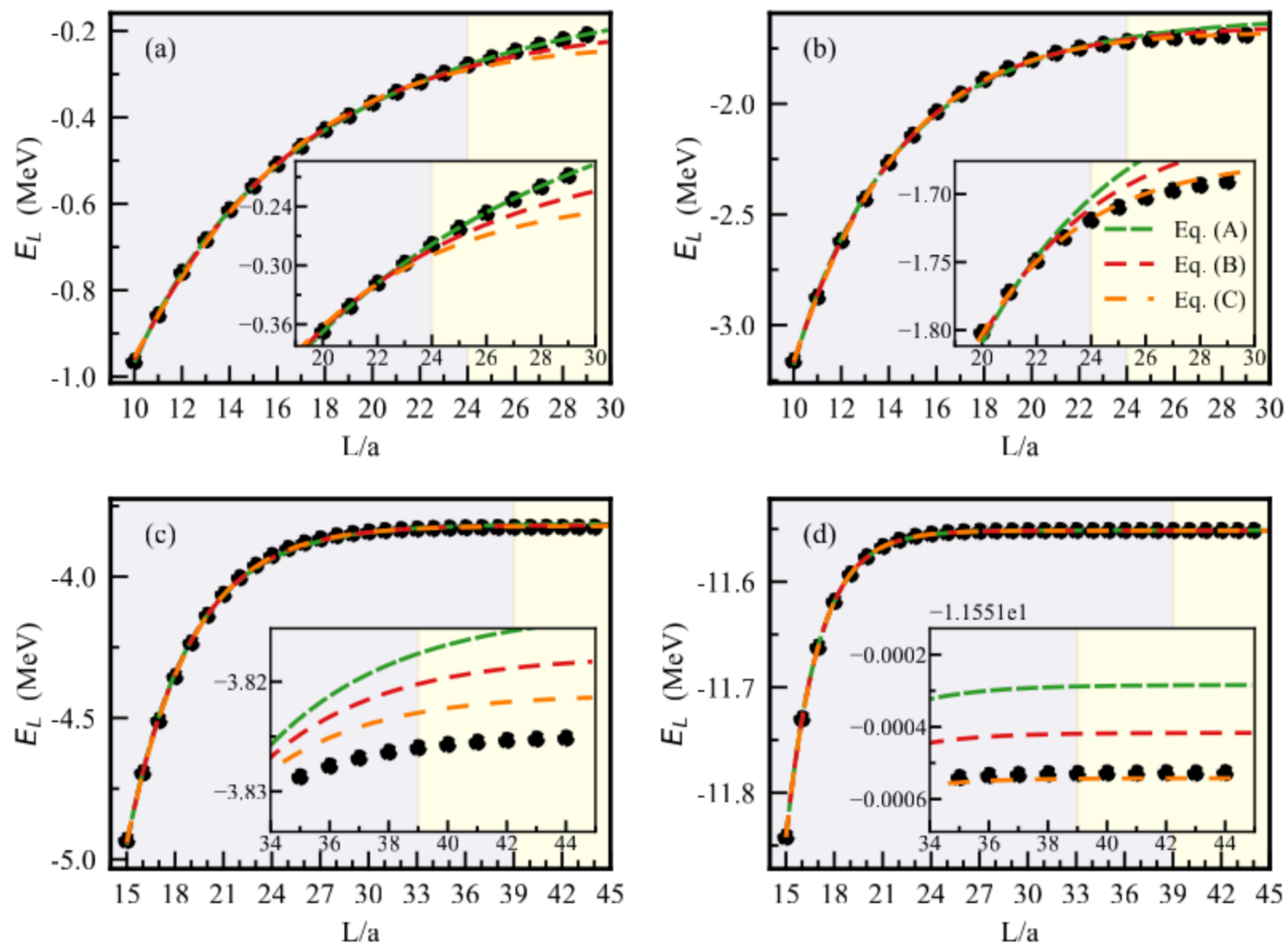
- Box size 10 ~ 30 fm

$E_L = C_1 + C_2 e^{-C_3 L}/L$ (A)

$E_L = C_1 + C_2 e^{-C_3 L}$ (B)

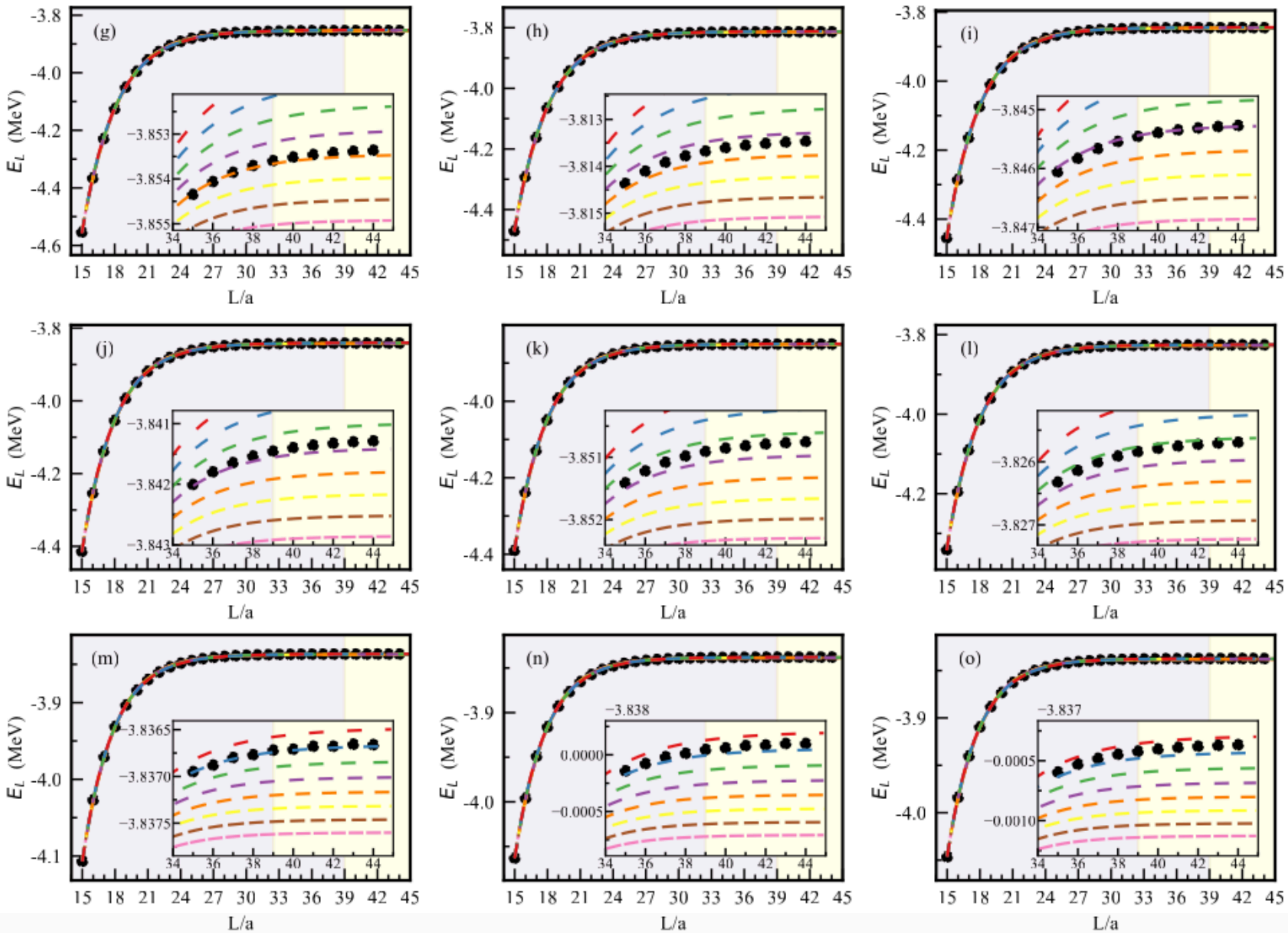
$E_L = C_1 + C_2 e^{-C_3 L} L$ (C)

- The power of L increases 2 for 10 fm range interaction



- The power depends on the range of the force $E_L = C_1 + C_2 e^{-C_3 L} L^n$

Long-range interaction

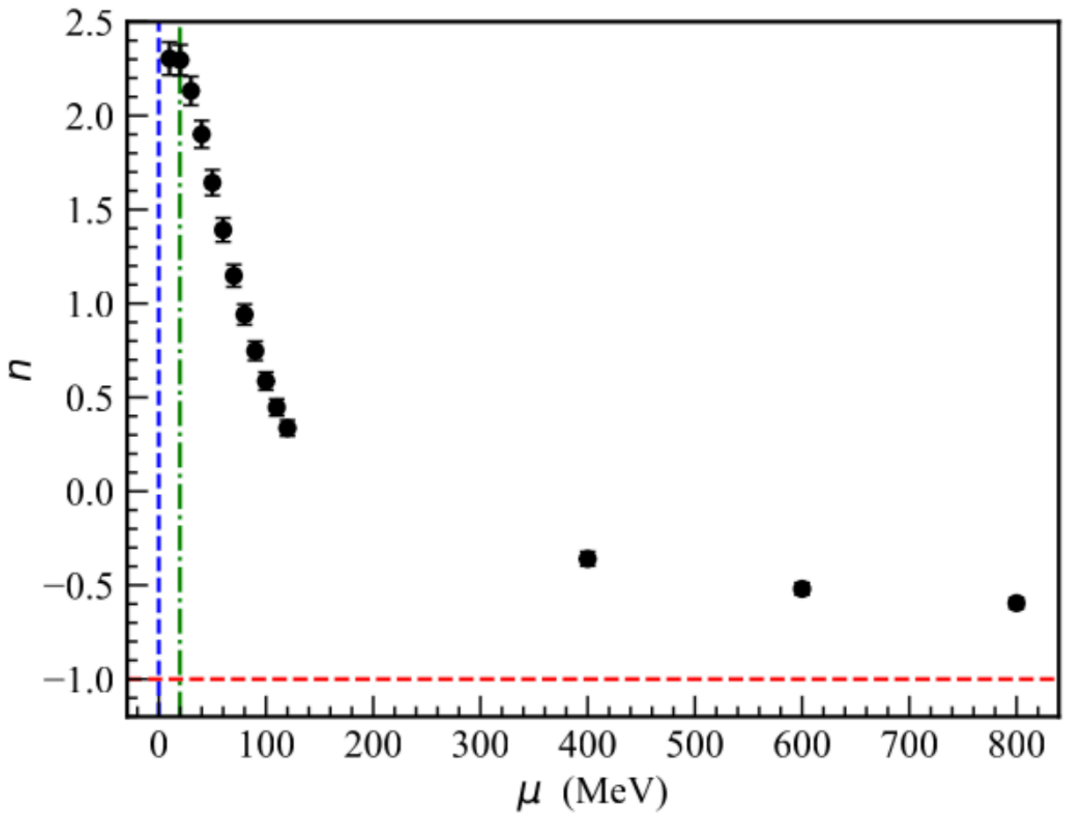


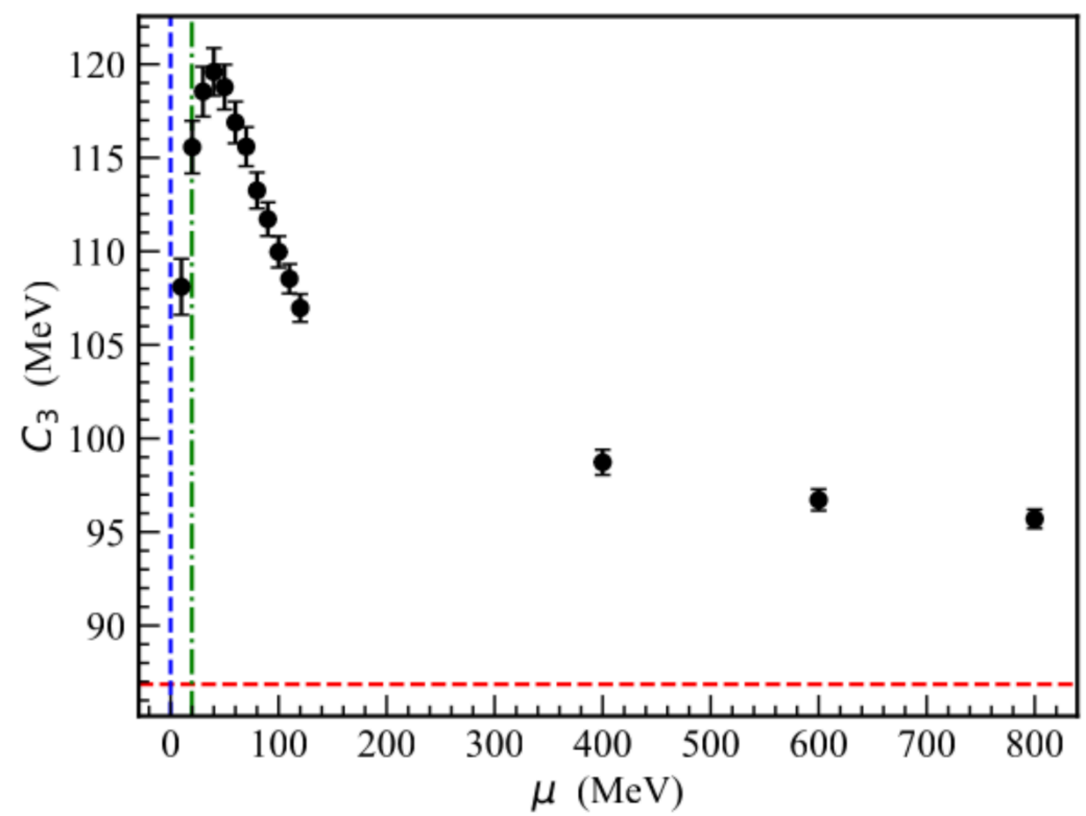
$\mu = 10,\ 20,\ldots,\ 120,\ 400,\ 600,\ 800\text{MeV} \qquad n = 5/2,\ 2,\ 3/2,\ 1,\ 1/2,\ 0,\ -1/2,\ -1$

Long-range interaction

The n and C_3 values

$\mu = 10, 20, \ldots, 120, 400, 600, 800\text{MeV}$





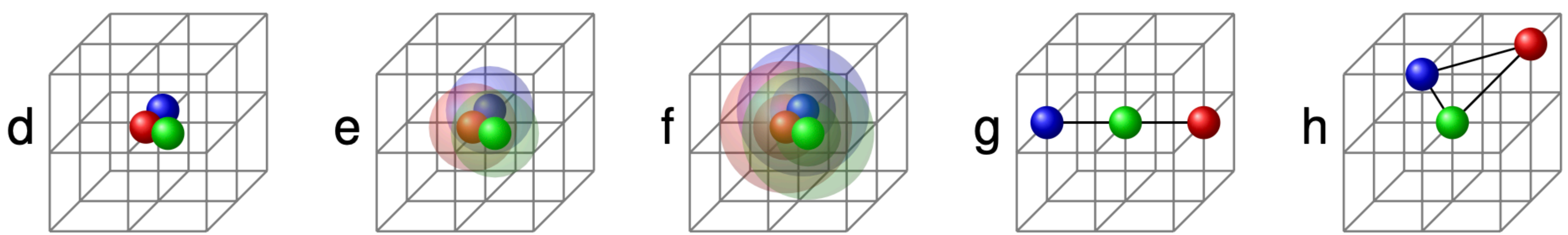
- $n = -1$
- $\mu = 0$
- $\hbar c / 10\ \text{fm} \sim 20\ \text{MeV}$

- κ_B
- $\mu = 0$
- $\hbar c / 10\ \text{fm} \sim 20\ \text{MeV}$

- The regressed formula recovers the short-range limit and indicates the long-range tendency

Summary and outlook

- SR unveils the power law of FV extrapolation formula for the two-body system with long-range interaction
- Our formula can recover the behaviors for infinity short- and long-range interactions
- How about the 3-body systems, especially for pion-exchanged 3-body force?



Thank you very much for your attention!