Pion photo-production and weak production from a nucleon

Based on: arxiv:2502.12074

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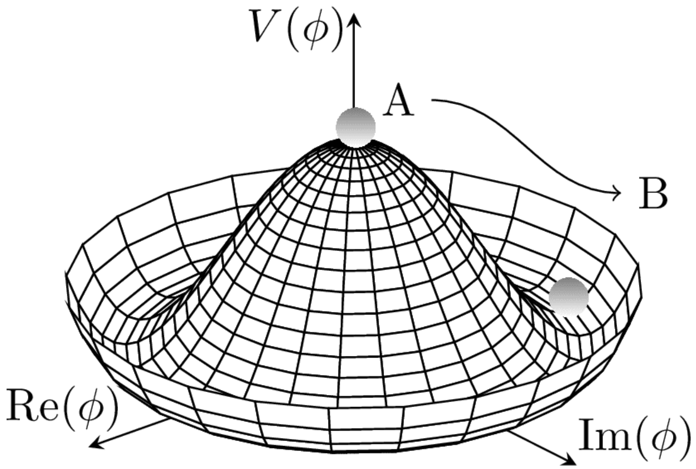
Background: testing QCD chiral dynamics

- Chiral symmetry breaking is an important feature in QCD

$$SU(3)_V \otimes SU(3)_A \xrightarrow{\textbf{SSB}} SU(3)_V$$



SSB

- Emergence of mesons

$$\pi^{\pm,0},\ K^{\pm},\ K^{0},\ \overline{K^{0}},\ \eta,\ \eta'\ldots$$

Lightest meson: $m_\pi \approx 140MeV \ll \Lambda_{QCD}$

Pion dynamics: Important ingredient to study the QCD Chiral symmtry SSB effects.

Background: testing QCD chiral dynamics

- Pion photoproduction is an applicable testground to explore the pion physics.
- Long history since 1970s, typical interplay between theory and experiment.

Experiment:

Virtual photon: By scattering $lN \to lN\pi$

Real photon: By scattering $\gamma N \to \pi N$

H.B. van den Brink et al., **PRL** , (1995)

D.Hornidge, et al.(A2,CB-TAPS),**PRL**, (2013)

? **Matching: Inconsistency still persist**

R. Lindgren, K. Chirapatimol, and L. C. Smith, **PoS CD12**, 073 (2013).

Theory:

Low energy theorem

Chiral perturbation theory…

First principle LQCD calculation Is necessary!

Background: necessary for neutrino experiment

- Future ν oscillation experiment also need high precision pion production data.
- DUNE designed uncertainty, 1) signal: 1%, 2) background: 5%

R. Acciarri *et al.* (DUNE), (2015)

$$\nu_\mu N \to \nu_\mu N\pi^0 \to \nu_\mu N e^+ e^-$$

$$\nu_\mu N \to \mu^- N\pi^0 \to \mu^- N e^+ e^-$$

$$\nu_\mu N \to \nu_\mu N\pi^\pm$$

Background	Normalization Uncertainty	Correlations
For $\nu_e/\bar{\nu}_e$ appearance:		
Beam ν_e	5%	Uncorrelated in ν_e and $\bar{\nu}_e$ samples
NC	5%	Correlated in ν_e and $\bar{\nu}_e$ samples
ν_μ CC	5%	Correlated to NC
ν_τ CC	20%	Correlated in ν_e and $\bar{\nu}_e$ samples
For $\nu_\mu/\bar{\nu}_\mu$ disappearance:		
NC	5%	Uncorrelated to $\nu_e/\bar{\nu}_e$ NC background
ν_τ	20%	Correlated to $\nu_e/\bar{\nu}_e$ ν_τ background

- **Reducing these background error need precise theoretical input!**

Theory framework

- The central object is to calculating the matrix element:

V.Bernard,N.Kaiser,T.S.H.Lee, andU.-G.Meißner, **Phys.Rept**. (1994)

$$\mathcal{J}_\mu^{em} = \langle N\pi | \underline{J_\mu^{em}(0)} | N\rangle, \quad \mathcal{J}_\mu^{W(Z),A} = \langle N\pi | \underline{J_\mu^{W(Z),A}(0)} | N\rangle$$

Electromagnetic current

Weak current

Extracting S-wave multipole amplitudes

$$L_{0+}, \quad E_{0+}$$

(reduce one D.O.F. due to ward identity)

$$L_{0+}^W, \quad H_{0+}, \quad M_{0+}$$

$$[\mathcal{J}_0^{em}]_{s',s} = \alpha_m \frac{|\vec{k}|}{k_0} \xi_{s'}^\dagger (\vec{\sigma}\cdot\hat{k}) \xi_s \, L_{0+}$$

$$[\vec{\mathcal{J}}^{em}]_{s',s} = \alpha_m \xi_{s'}^\dagger \left\{ L_{0+}\, \hat{k}(\vec{\sigma}\cdot\hat{k}) + E_{0+}\left[\vec{\sigma} - \hat{k}(\vec{\sigma}\cdot\hat{k})\right]\right\}\xi_s$$

$$[\mathcal{J}_0^{W,A}]_{s',s} = \alpha_m \xi_{s'}^\dagger \xi_s \left(L_{0+}^{(W)} + \frac{k_0}{m} H_{0+} \right)$$

$$[\vec{\mathcal{J}}^{W,A}]_{s',s} = \alpha_m \xi_{s'}^\dagger \left(\frac{\vec{k}}{m} H_{0+} - i(\vec{\sigma}\times\hat{k}) M_{0+} \right)\xi_s$$

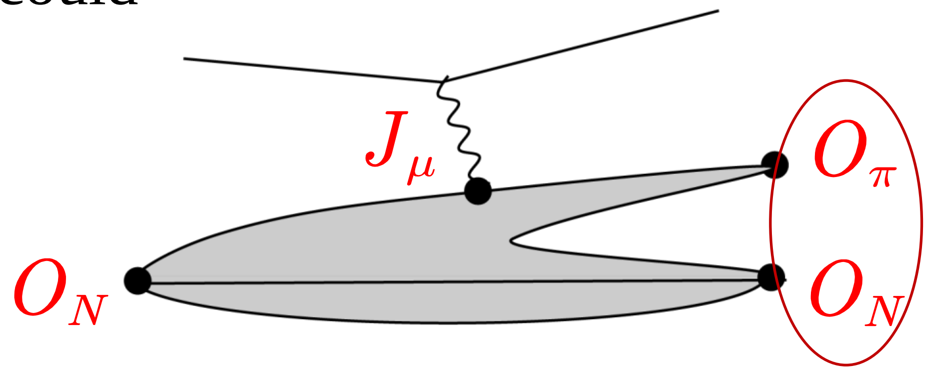
Theory framework

- To calculate these matrix element, one should calculate the relevant correlation function:

$$C_{N\pi JN}(t_f,t_J,t_i)=\left\langle O_{N\pi}^{I'',I_z''}(\vec{0},t_f)\tilde{J}_{\mu}^{I',I_z'}(\vec{k},t_J)\overline{O}_N^{I,I_z}(\vec{p},t_i)\right\rangle$$

- After spectra decomposition, the correlation function could be related to matrix element $\mathcal{J}_{s',s}$,

$$C_{N\pi JN}=\alpha_{N\pi JN}\sum_{s',s}\xi_{s'}\left[\mathcal{J}\right]_{s',s}\xi_s^{\dagger}$$



Multi-hadron $N\pi$ operator (demanding)

- $\alpha_{N\pi JN}$ contains the overlap amplitude of $O_{N\pi}$ and O_N, which could be extracted from two point functions.

$$C_{N\pi}(t_f,t_i)=\left\langle O_{N\pi}^{I,I_z}(\vec{0},t_f)\overline{O}_{N\pi}^{I,I_z}(\vec{0},t_i)\right\rangle,\ C_N(t_f,t_i)=\left\langle O_N^{I,I_z}(-\vec{p},t_f)\overline{O}_N^{I,I_z}(\vec{p},t_i)\right\rangle.$$

Theory framework

- To extract the multipole amplitudes, one should design projection operator for each. Take L_{0+} for example.

$$\frac{1}{N_R}\sum_{\hat{R}\in O_h}\frac{1}{2}Tr\left[\mathcal{P}_{L_{0+}}\cdot C^{0}_{N\pi VN}\right]\Big|_{\hat{R}\vec{k}} = \alpha_{N\pi JN}\alpha_m L_{0+}$$

Trace in 2×2 spin space

Momentum average in group O_h

Improve the signal !

$$\mathcal{P}_{L_{0+}} = \frac{k_0}{|\vec{k}|}\left(\hat{k}\cdot\vec{\sigma}\right)$$

- Similarly for other multipoles

$$\mathcal{P}_{L_{0+}} = \frac{k_0}{|\vec{k}|}\left(\hat{k}\cdot\vec{\sigma}\right), \quad \vec{\mathcal{P}}_{E_{0+}} = \frac{1}{2}\left(\vec{\sigma}-\left(\hat{k}\cdot\vec{\sigma}\right)\hat{k}\right),$$

$$\vec{\mathcal{P}}_{H_{0+}} = \frac{m}{|\vec{k}|}\hat{k}, \quad \mathcal{P}_{L^{(W)}_{0+},H_{0+}} = 1, \quad \vec{\mathcal{P}}_{M_{0+}} = \frac{i}{2}\left(\vec{\sigma}\times\hat{k}\right).$$

Control systematics: Finite volume effects

- Given finite volume state $|N\pi\rangle_V$, and infinite volume state $|N\pi\rangle_\infty$.

$$1=\sum_n |N\pi_n\rangle_V\langle N\pi_n|_V=\int dE\,\frac{dn}{dE}|N\pi_n\rangle_V\langle N\pi_n|_V=\int dE|N\pi_n\rangle_\infty\langle N\pi_n|_\infty$$

- The two states are related by the density of state $|N\pi\rangle_\infty=\left(\frac{dn}{dE}\right)^{1/2}|N\pi\rangle_V$
- From finite volume quantization condition

L. Lellouch and M. Lüscher, **Commun. Math. Phys**, (2001)

$$\frac{dn}{dE}=\frac{1}{\pi}\frac{d(\delta_1+\phi_{\mathbf{P}}^{\Gamma})}{dE}=\frac{E}{4\pi k^2}\left(k\frac{\partial\delta_1}{\partial k}+q\frac{\partial\phi_{\mathbf{P}}^{\Gamma}}{\partial q}\right)$$

Derivative of phase shift

Analytical known zeta function

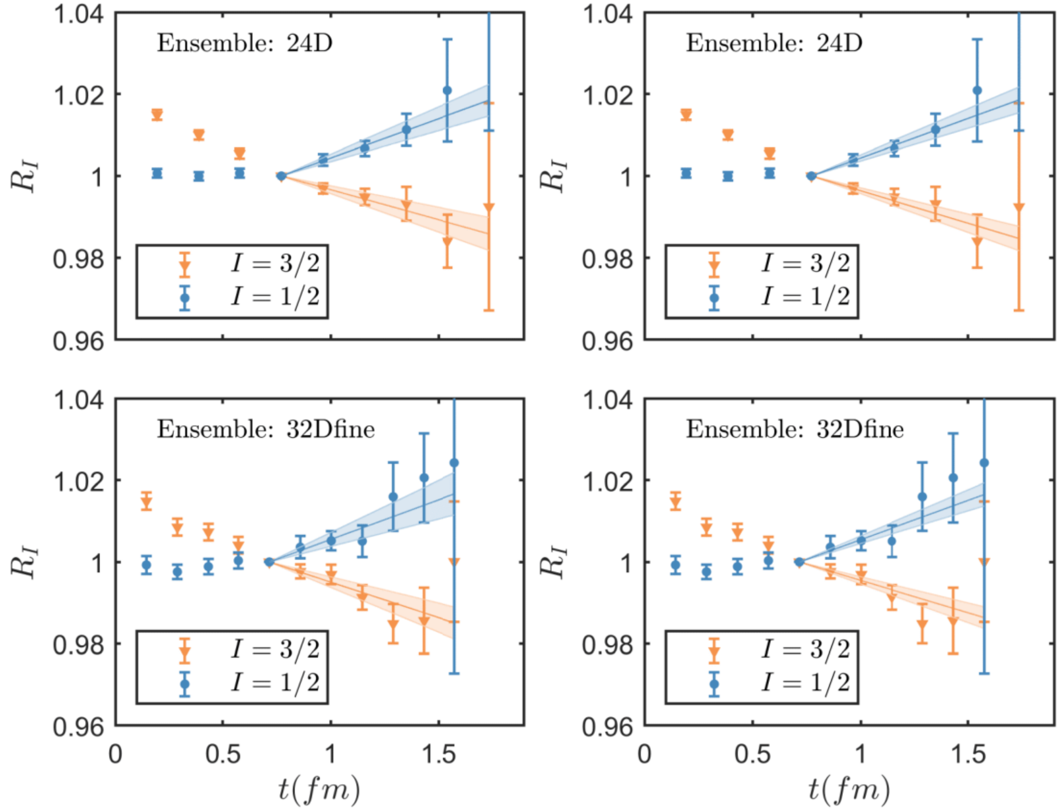
Need scattering length as input!

Control systematics: Finite volume effects

M. Lüscher, **Commun. Math. Phys.** (1986)

- Calculate the S-wave scattering length by Lüsher method, at $\vec{p}=0$.

$$R^I(t)=\frac{C^I_{N\pi}(t_f,t_i)}{C_N(t_f,t_i)C_\pi(t_f,t_i)}\approx A_I(1-\delta E_I\, t)\longrightarrow \delta E_I \xrightarrow{\text{Lüsher method}} a_0^I$$



	Ensemble	24D	32Dfine	Combined fit
$I=\frac{1}{2}$	χ^2/dof	0.26	0.47	0.34
	δE_I [MeV]	−3.78(79)	−3.9(1.2)	−3.80(66)
	$M_\pi a_0^I$	0.157(37)	0.157(56)	0.157(31)
$I=\frac{3}{2}$	χ^2/dof	0.26	0.51	0.38
	δE_I [MeV]	2.91(80)	3.40(90)	3.13(60)
	$M_\pi a_0^I$	−0.098(25)	−0.111(27)	−0.104(18)

Only one previous LQCD at physical point: $M_\pi a^{3/2}=-0.13(4)$

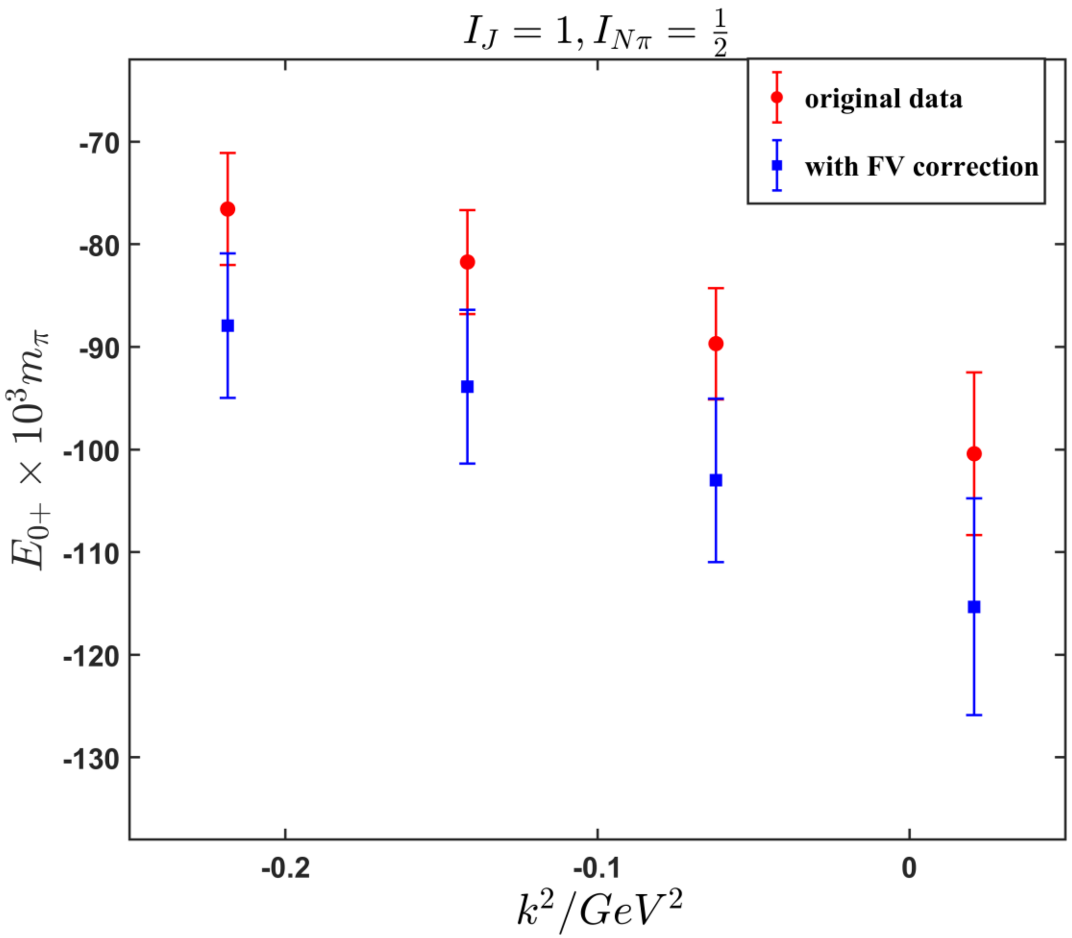
Data driven analysis: $M_\pi a^{3/2}=-0.087(2),\ M_\pi a^{1/2}=0.170(2)$

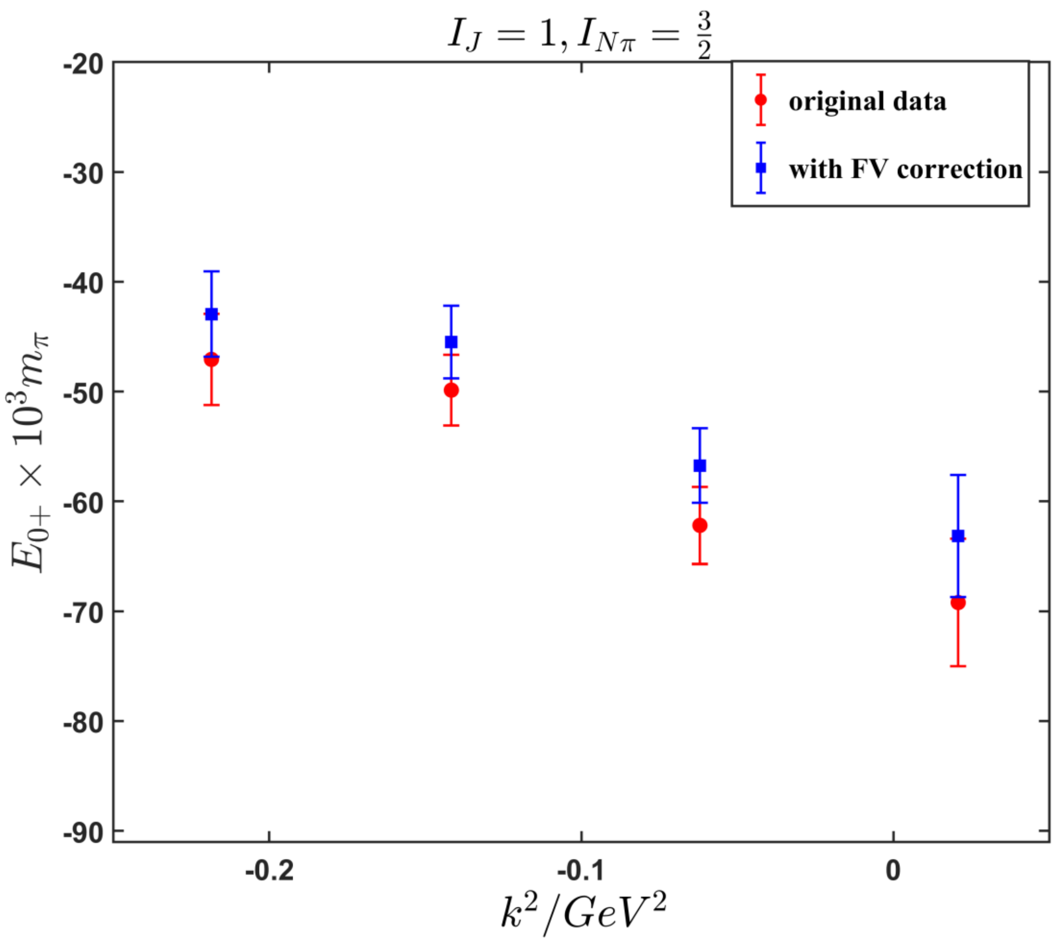
M. Hoferichter et al, **PLB** (2023)

C. Alexandrou et al, **PRD** (2024)

Control systematics: Finite volume effects

- The influence of finite volume correction is around 10%
- Shift the central value about $1 \sim 2\sigma$





Control systematics: Excited state's contamination

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- The operator $O_{N\pi}$ and O_N could couple to any states with same quantum number

$$O_{N(0)\pi(0)} \longrightarrow |N(p)\pi(-p)\rangle,\ p \in \left\{0,\ \frac{2\pi}{L},\ \sqrt{2}\frac{2\pi}{L},\ \sqrt{3}\frac{2\pi}{L}\right\}$$

$$O_{N(p)} \longrightarrow |N(p)\rangle,\ |N(p)\pi(0)\rangle,\ |N(0)\pi(p)\rangle$$

- Only contribution from ground state $|N(p)\rangle,\ |N(0)\pi(0)\rangle$ is needed.

- To remove the excited state contribution, we design optimized operator

$$\tilde{O}_{N(0)\pi(0)} = O_{N(0)\pi(0)} + c_2 O_{N(1)\pi(-1)} + c_3 O_{N(\sqrt{2})\pi(-\sqrt{2})} + c_4 O_{N(\sqrt{3})\pi(-\sqrt{3})}$$

$$\tilde{O}_{N(p)} = O_{N(p)} + d_2 O_{N(p)\pi(0)} + d_3 O_{N(0)\pi(p)}$$

- They're expected to have suppressed overlap with excited states.

Control systematics: Excited state's contamination

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- The coefficients $c_2,\ c_3,\ c_4$ and $d_2,\ d_3$ are solved by generalized eigenvalue problem (GEVP) method separately.

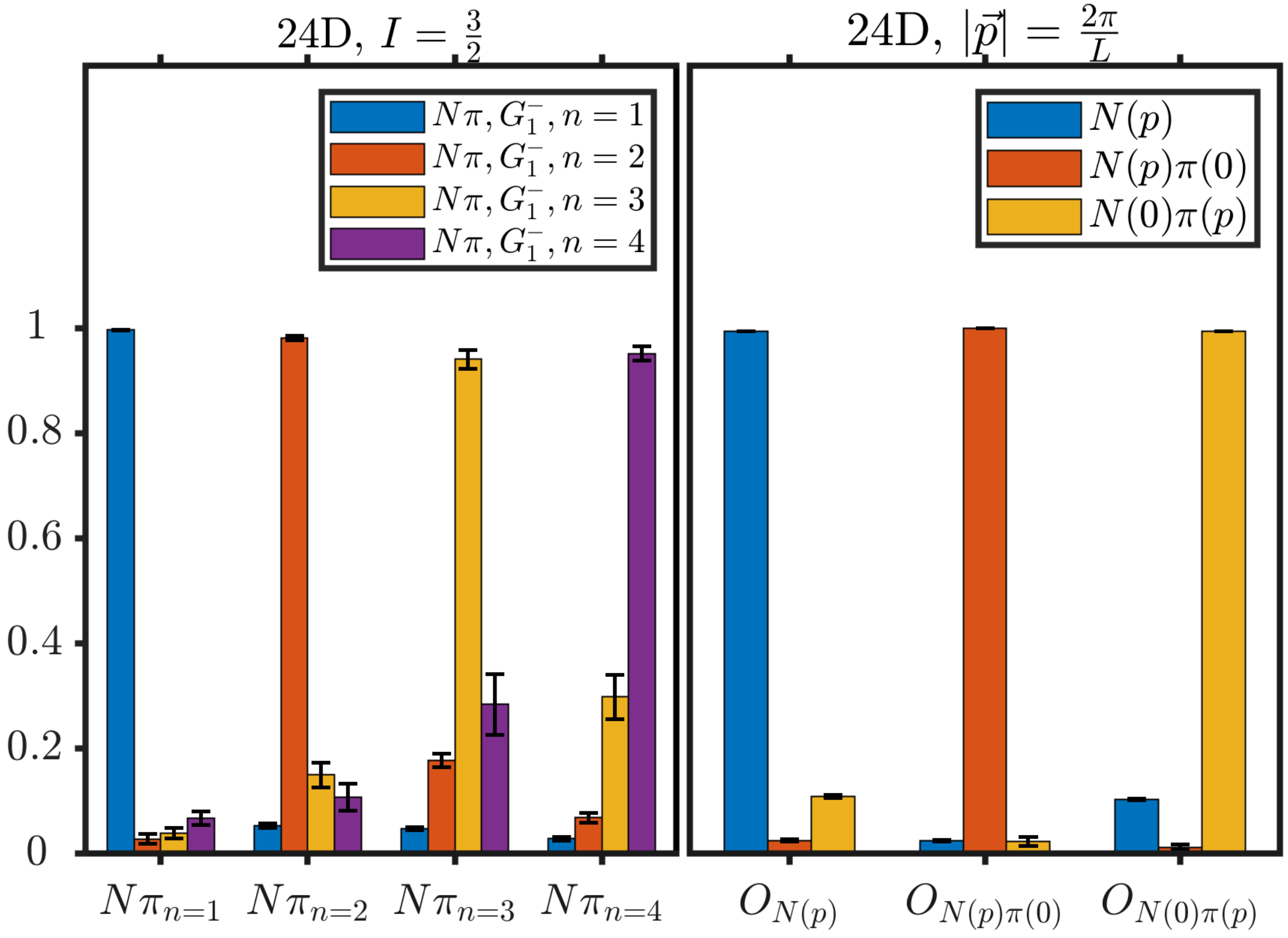
$$C_{ij}=\left\langle O_i(t)\,\overline{O}_j(0)\right\rangle,\ i,j\in\left\{N(p)\pi(-p)|\ p=0,\ \frac{2\pi}{L},\ \sqrt{2}\frac{2\pi}{L},\ \sqrt{3}\frac{2\pi}{L}\right\}\ \xrightarrow{\textbf{GEVP}}\ c_2,\ c_3,\ c_4$$

$$C_{ij}=\left\langle O_i(t)\,\overline{O}_j(0)\right\rangle,\ i,j\in\{N(p),\ N(p)\pi(0),\ N(0)\pi(p)\}\ \xrightarrow{\textbf{GEVP}}\ d_2,\ d_3$$

- The GEVP has non-negligible influence to both **scattering length** and **matrix element**.

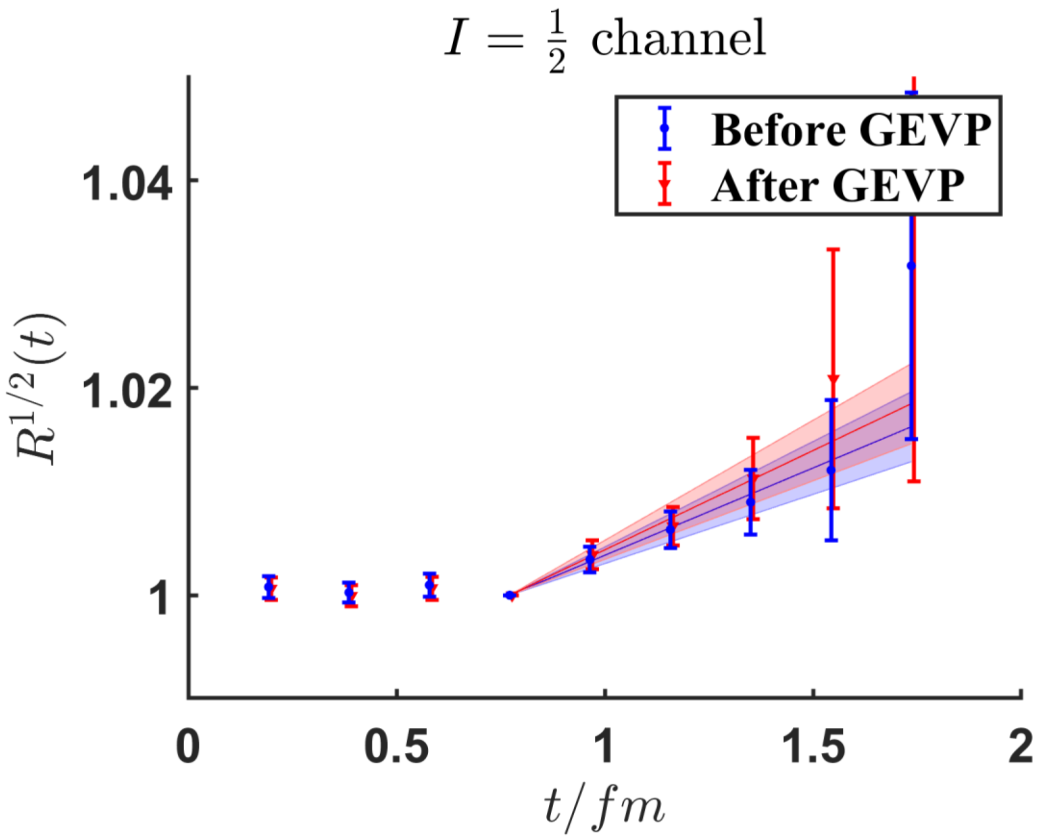
Control systematics: Excited state's contamination

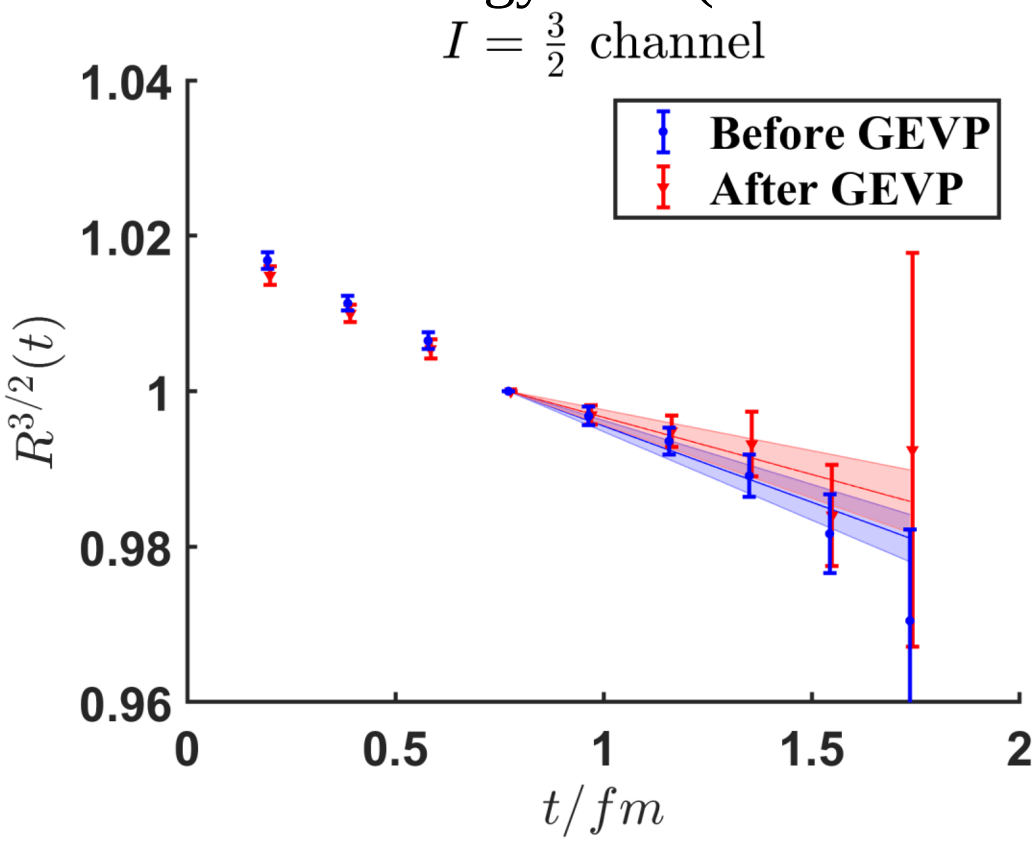
- The contribution of different states for the operators.



Control systematics: Excited state’s contamination

- The influence of GEVP to extracting of $N\pi$ S-wave energy shift (24D ensemble).





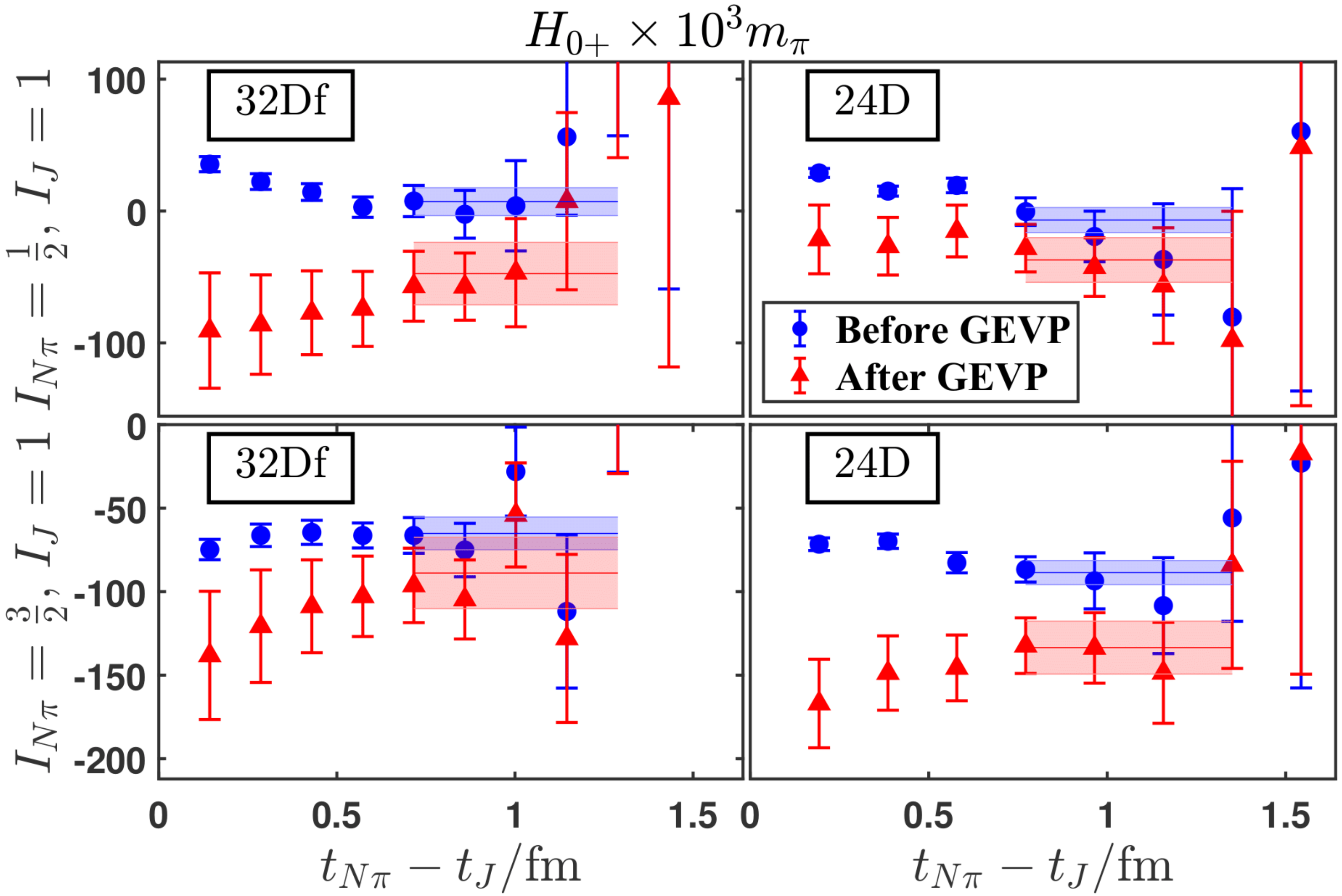
GEVP: $M_\pi a_0^{1/2} = 0.136(32) \rightarrow M_\pi a_0^{1/2} = 0.157(37)$ $\quad M_\pi a_0^{3/2} = 0.127(18) \rightarrow M_\pi a_0^{1/2} = 0.098(25)$

Data Driven: $M_\pi a^{1/2} = 0.170(2)$ $\quad M_\pi a^{3/2} = -0.087(2)$

Control systematics: Excited state's contamination

- Comparing the multipole extracted by optimize operator $\tilde{O}_{N(0)\pi(0)}$, and original operator $O_{N(0)\pi(0)}$.

- Shift the central value around $1\sim3\sigma$



Control systematics: Excited state's contamination

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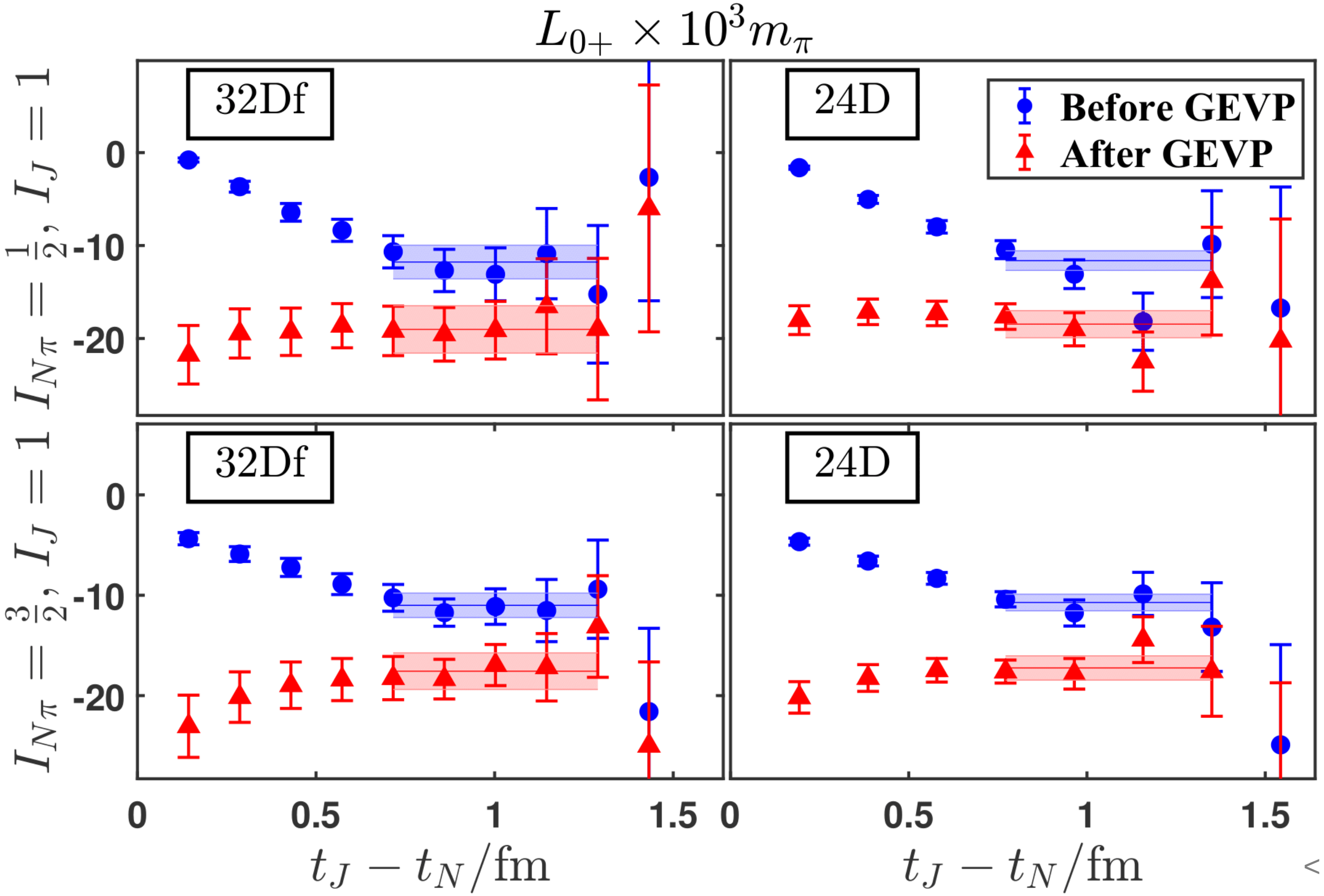
- Comparing the multipole extracted by optimize operator $\tilde{O}_{N(p)}$, and original operator $O_{N(p)}$.

- Shift the central value around $3 \sim 6\sigma$

- Better plateau behaviour



Numerical analysis: Data generating

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- We use two DWF gauge ensembles with physical pion mass[1].

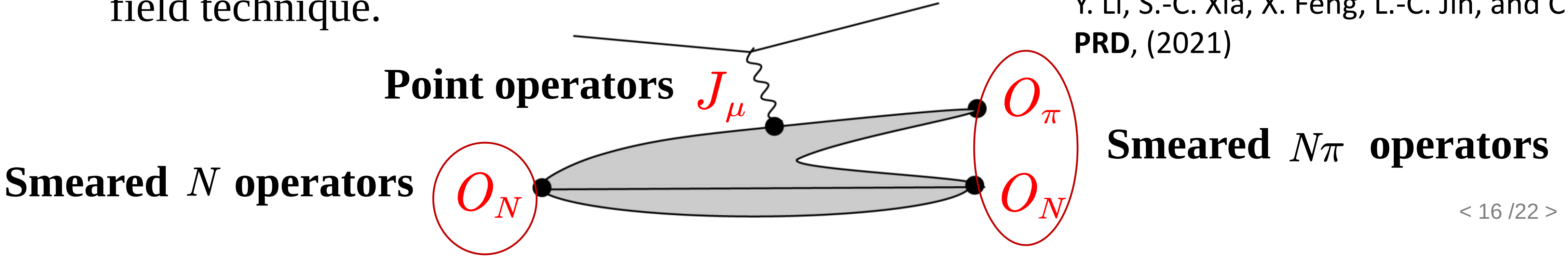
[1]T. Blum et al. (RBC, UKQCD), **PRD**, **93**, 074505

Ensemble	m_π [MeV]	L/a	T/a	a^{-1} [GeV]	N_{conf}
24D	142.6(3)	24	64	1.023(2)	207
32Dfine	143.6(9)	32	64	1.378(5)	69

- The two ensemble has the same physical size (L=4.6fm) and different lattice spacing, where we could check the lattice artifacts of the result.

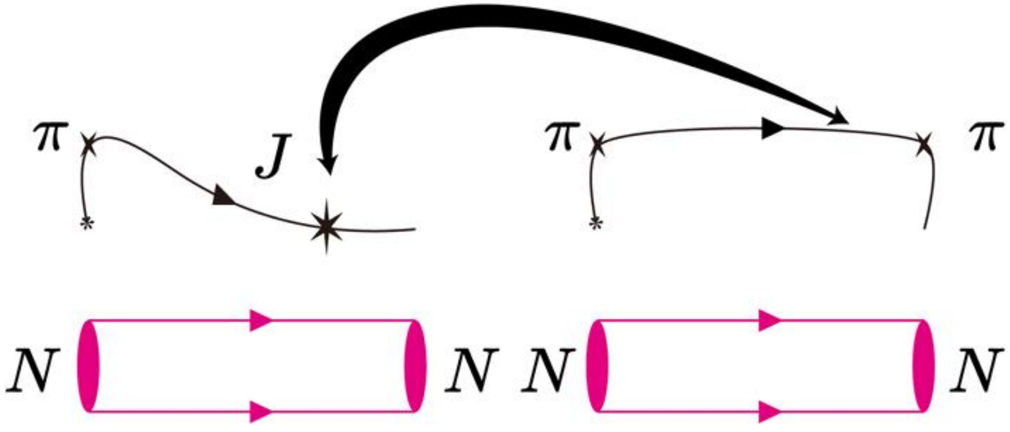
- The relevant nucleon four point functions are calculated using random sparse field technique.

Y. Li, S.-C. Xia, X. Feng, L.-C. Jin, and C. Liu, **PRD**, (2021)

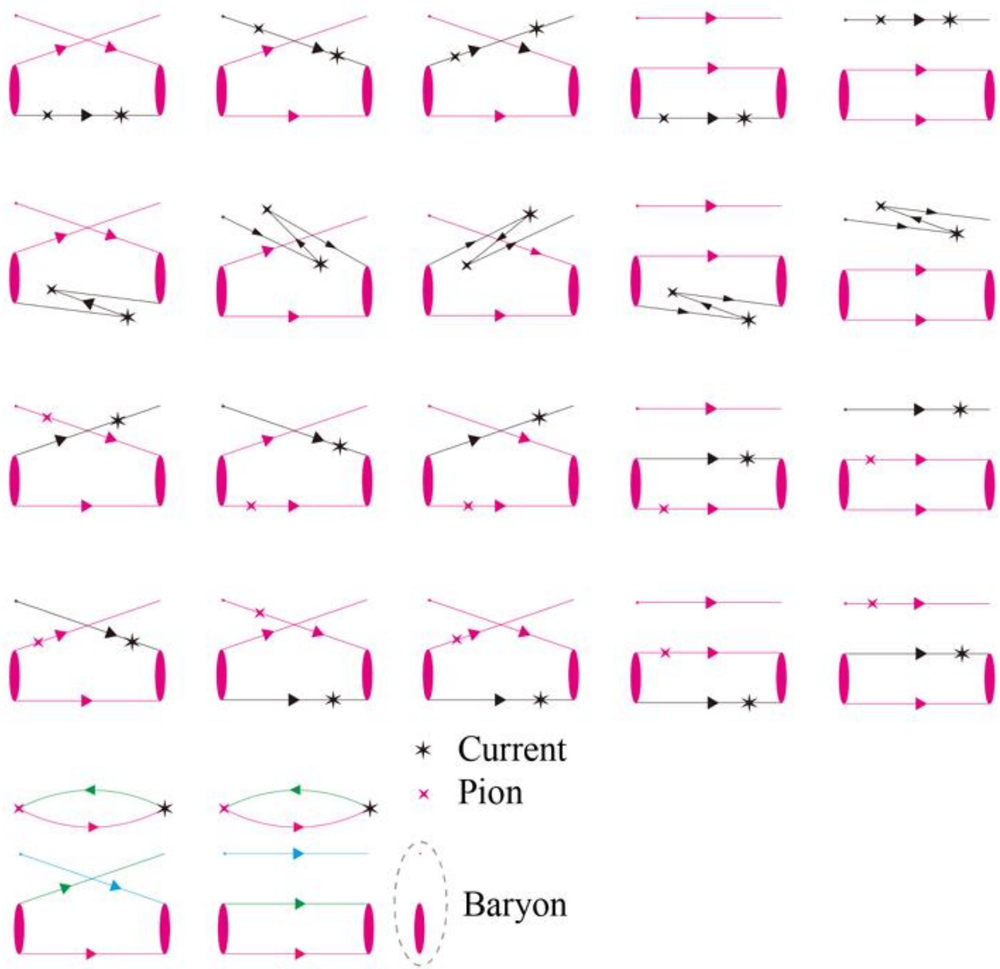


Numerical analysis: Data generating

- Wick contraction: 22 topologies are considered.



22 topologies digrams should be considered.

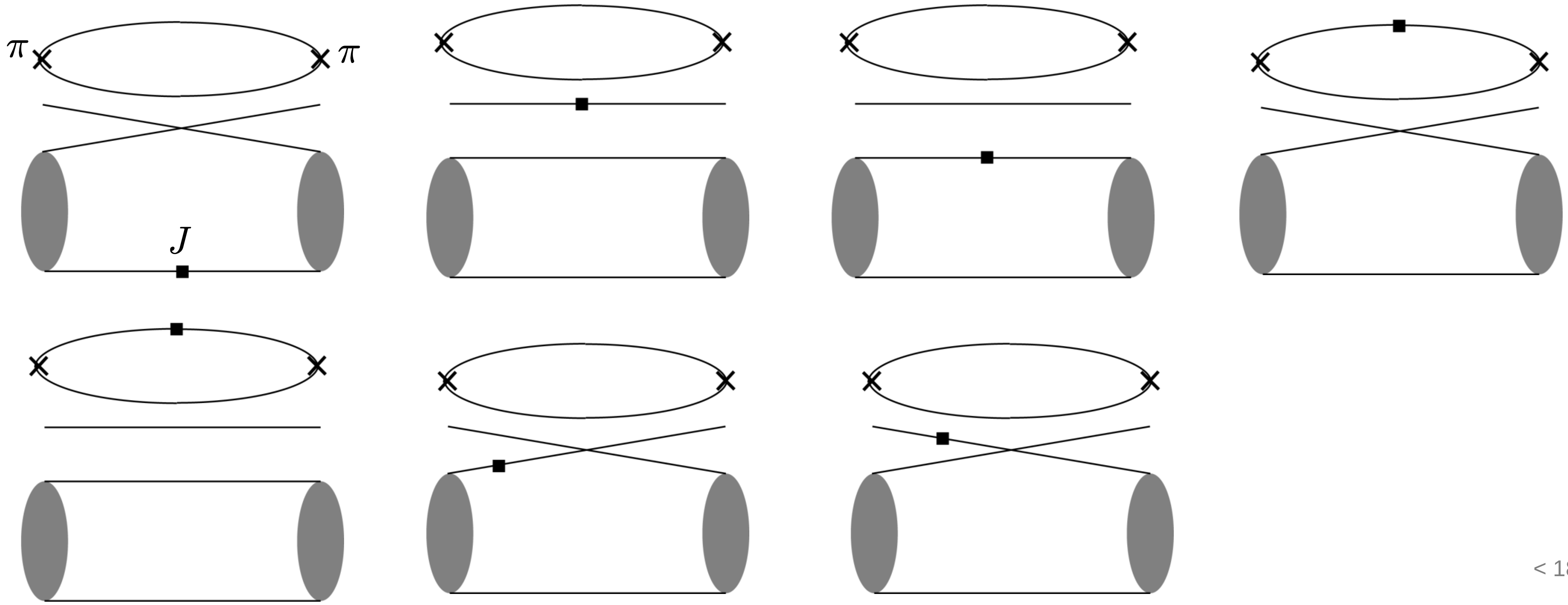


Figures by Yusheng Gao

Numerical analysis: Data generating

- Note that one five point correlation function is used in analysing excited state effects of $N(p)$, in which only 7 disconnected diagrams is calculated.

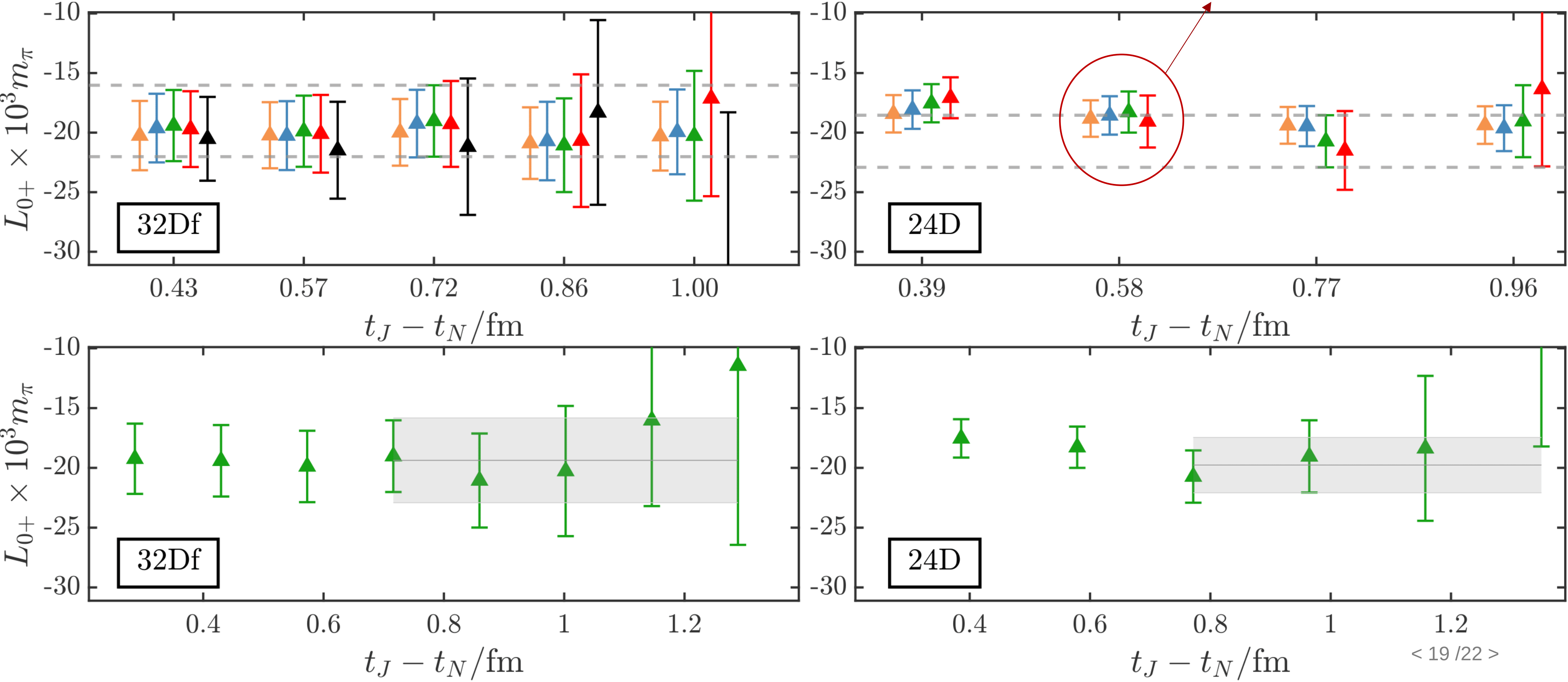
d_2 term: $\left\langle O_{N(0)\pi(0)} \;\; J_\mu \;\; \overline{O}_{N(p)\pi(0)} \right\rangle$ d_3 term: $\left\langle O_{N(0)\pi(0)} \;\; J_\mu \;\; \overline{O}_{N(0)\pi(p)} \right\rangle$



Numerical analysis: Result of multipoles

- We perform a plateau fit from $N\pi$ and N side separately.

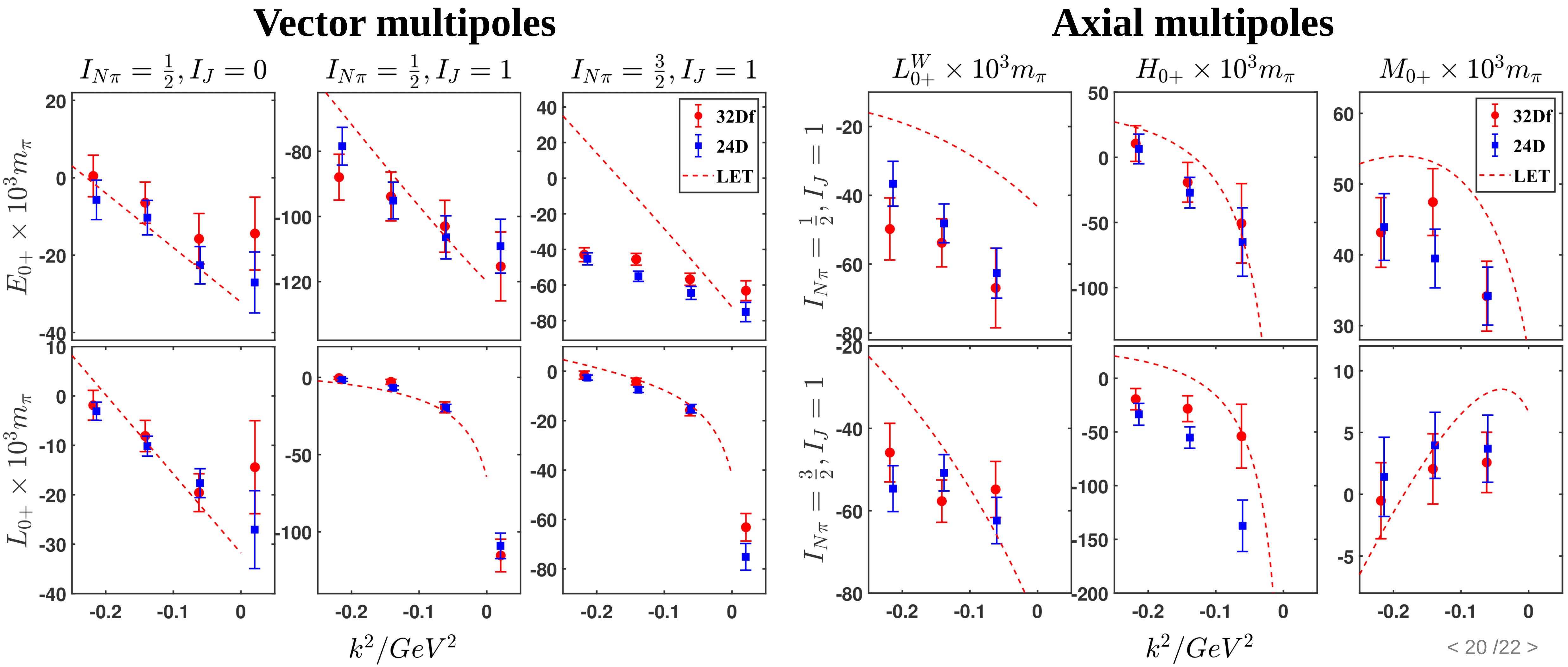
Different $t_{N\pi} - t_J$



Numerical analysis: Result of multipoles

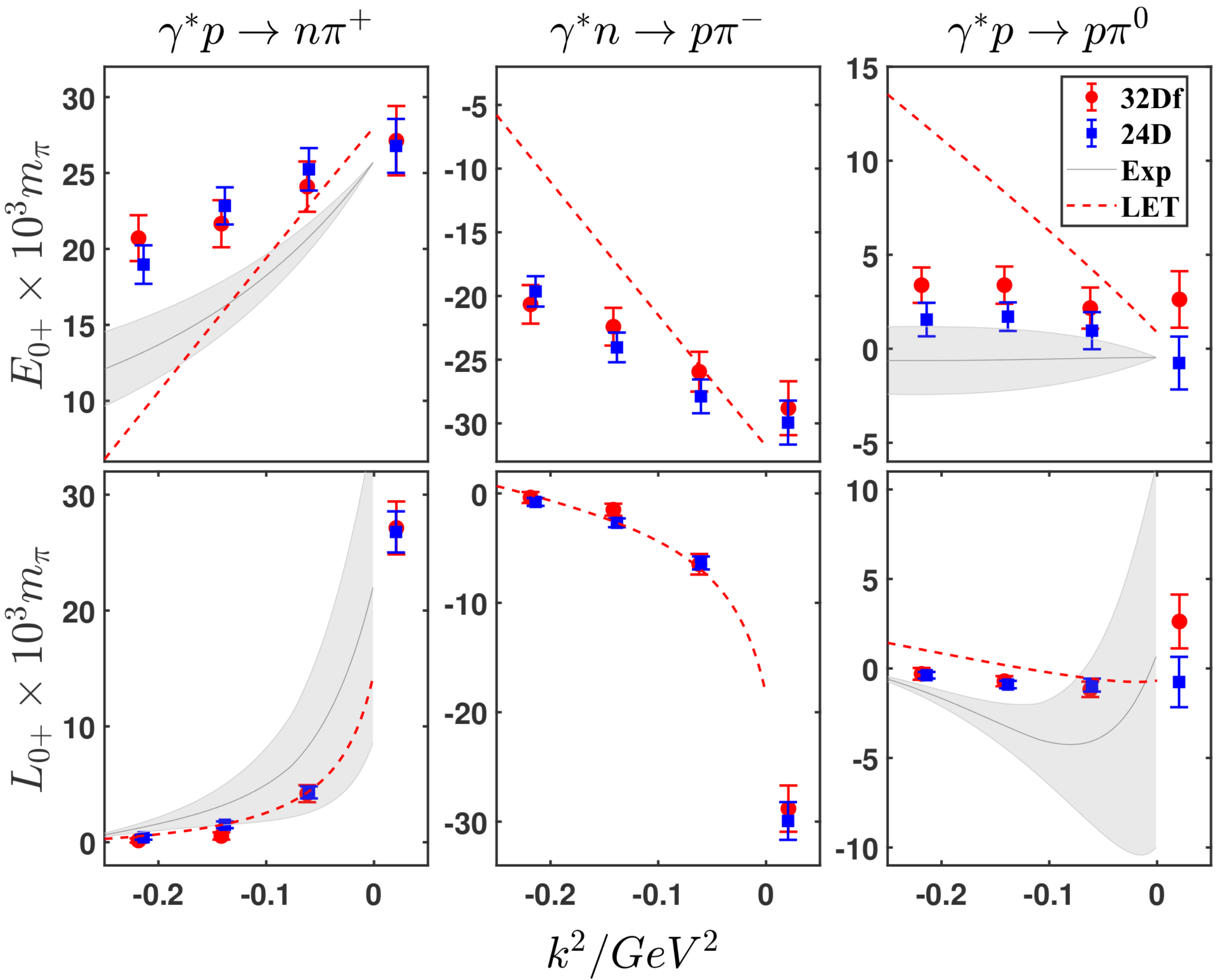
- The multipoles given in isospin basis.

V.Bernard,N.Kaiser,T.S.H.Lee, andU.-G.Meißner, **Phys.Rept**. (1994)



Numerical analysis: Result of multipoles

- The pion photoproduction multipoles in $\gamma^* p \to p\pi^0, \gamma^* p \to n\pi^+, \gamma^* n \to p\pi^-$ reactions.



- The LQCD data shows good agreement with Low energy theorem and experiment.

V.Bernard,N.Kaiser,T.S.H.Lee, andU.-G.Meißner, **Phys.Rept**. (1994)

M. Mai, J. Hergenrather, M. Döring, T. Mart, U. G. Meißner, D. Rönchen, and R. Workman, **EPJA**, (2023)

Possible future prospect

- In this work, we use two isospin symmetric ensembles with the same volume, which is impossible to explore the isospin breaking effects and volume dependence.

- Future work could be move forward to the DIS region of scattering:
  - $N\pi$ production with higher invariant mass and different partial waves.
  - Multi-pions production process, even for $N\pi\pi$, the finite volume quantization formulism is absent.
  - Resonance production process, most promising $\Delta(1232)$ resonance, with almost the total branch ratio decay to p-wave $N\pi$.

Thank you for listening!