第二届江门中微子暑期学校

Second JUNO Neutrino Summer School (JNSS 2025)

Theories of neutrino mass and mixing

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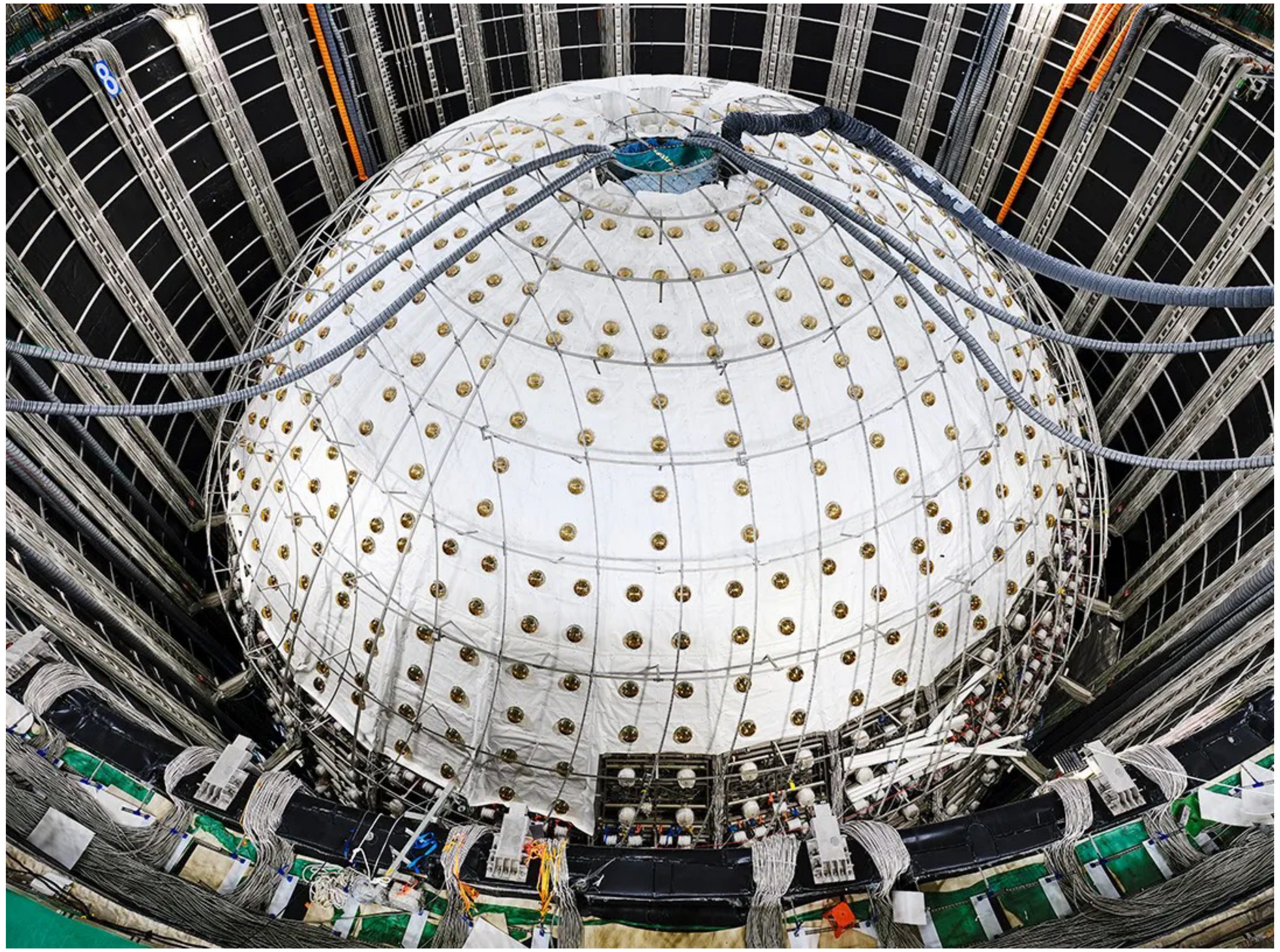


预备知识——自然单位制

约定

$$c = 3 \times 10^8 \mathrm{m/s} = 1$$

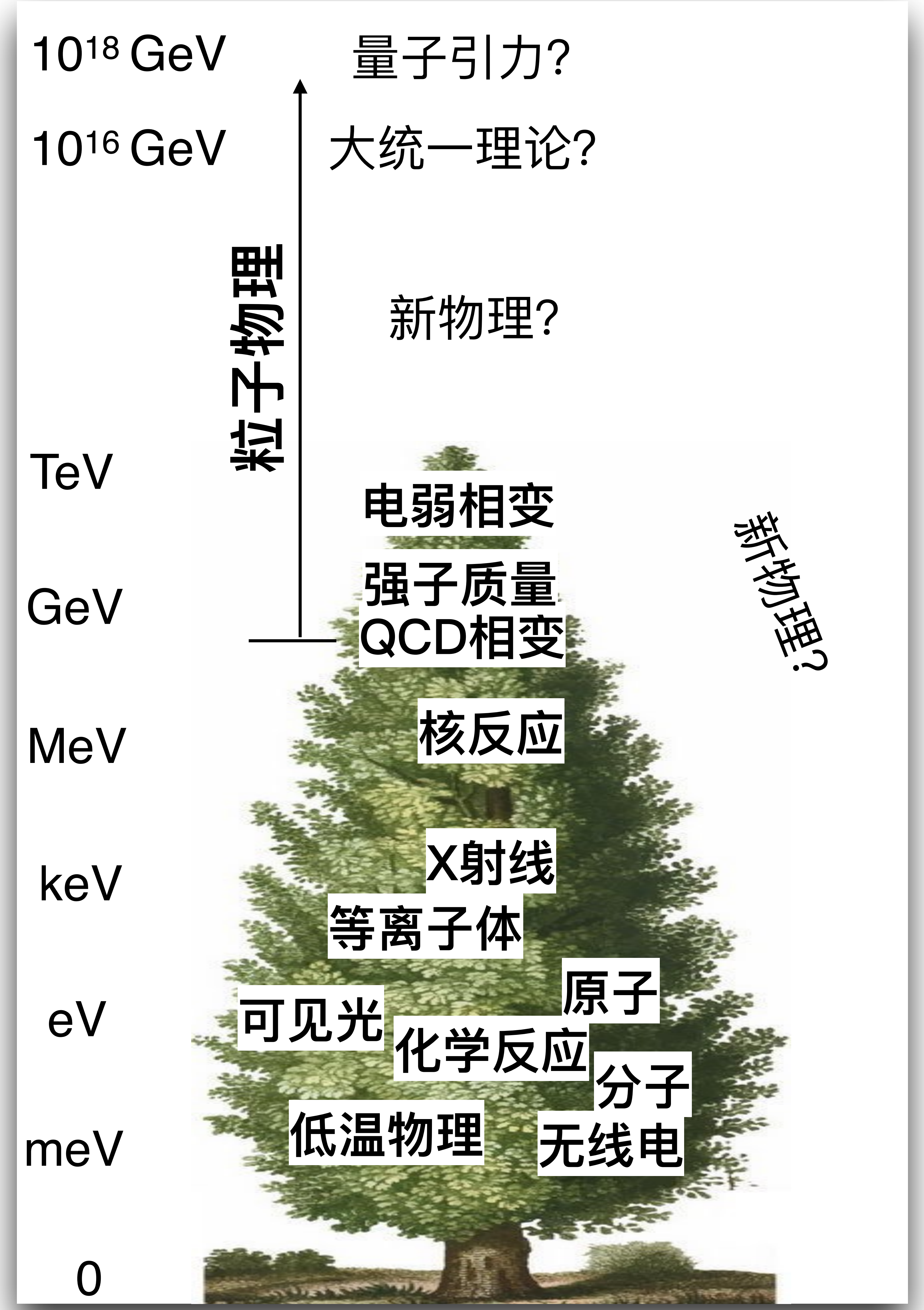
$$\hbar = 6.58 \times 10^{-15} \mathrm{eV \cdot s} = 1$$

$$k_{\mathrm{B}} = 8.617 \times 10^{-5} \mathrm{eV \cdot K^{-1}} = 1$$

所有物理量的单位都可以用电子伏特(eV)及其幂次来表示。

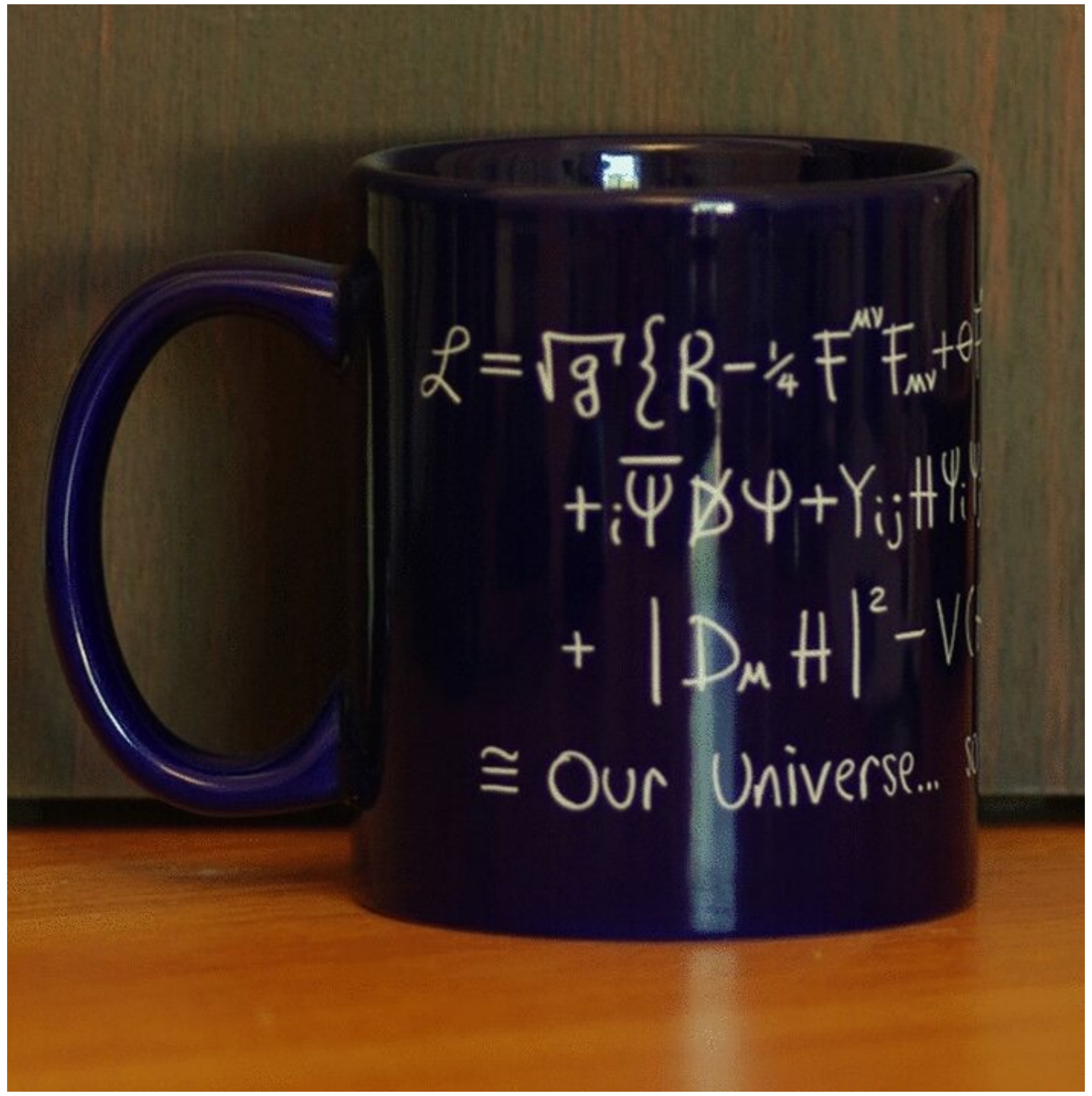
物理量	自然单位制	国际单位制
能量	eV	$1.602176634 \times 10^{-19}$ J
质量	eV	1.782662×10^{-36} kg
动量	eV	5.344286×10^{-28} kg·m/s
温度	eV	$1.160451812 \times 10^{4}$ K
时间	eV^{-1}	6.582119×10^{-16} s
长度	eV^{-1}	1.97327×10^{-7} m

能标



Neutrino in the Standard Model

The Standard Model (SM)



Credit: ICTP Souvenir and Gift Shop

- Symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$

  Strong Electroweak

- Gauge bosons g $W^{\pm}$ Z^0 γ

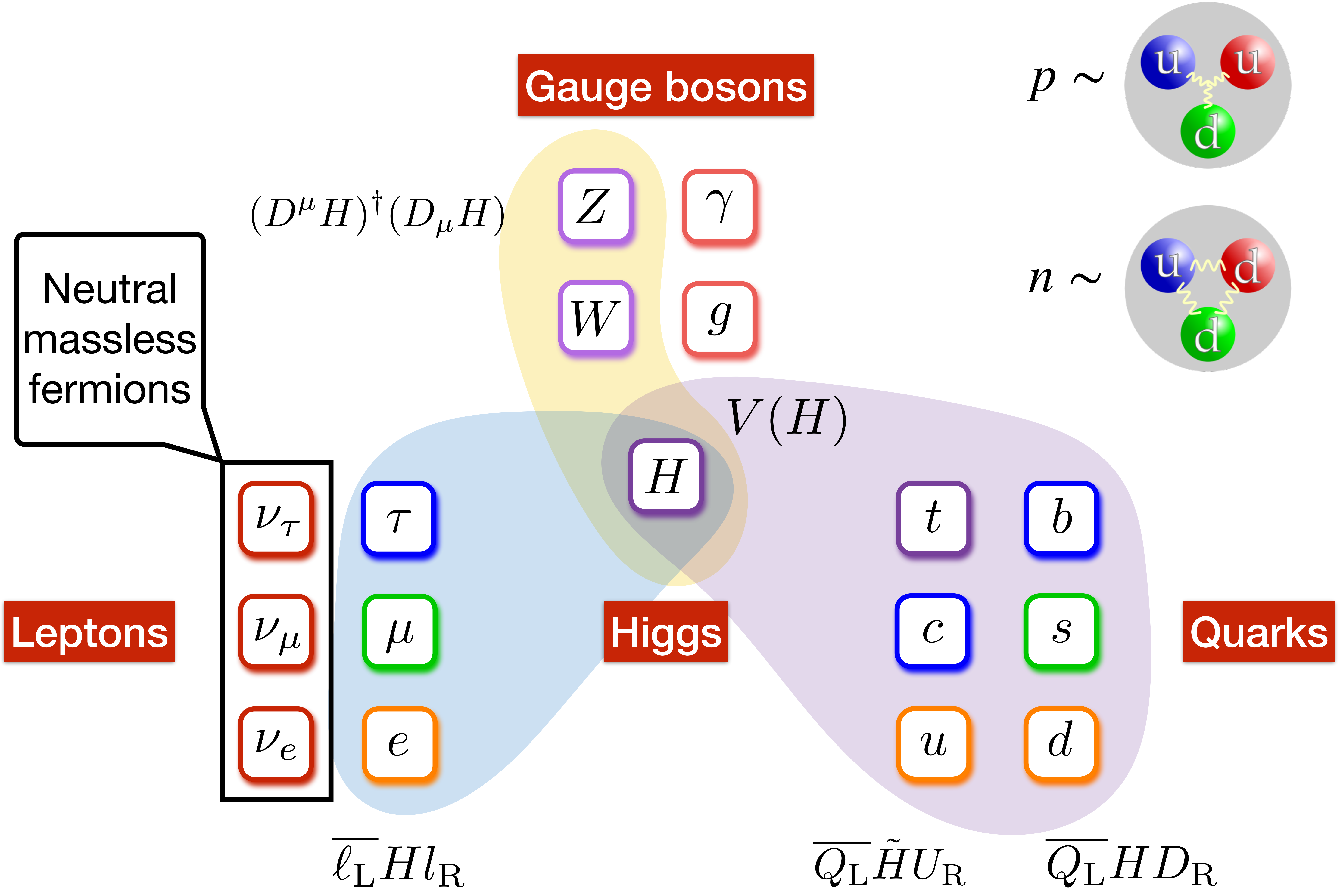
- The Higgs boson H

- Fermions / matter particles

3 copies

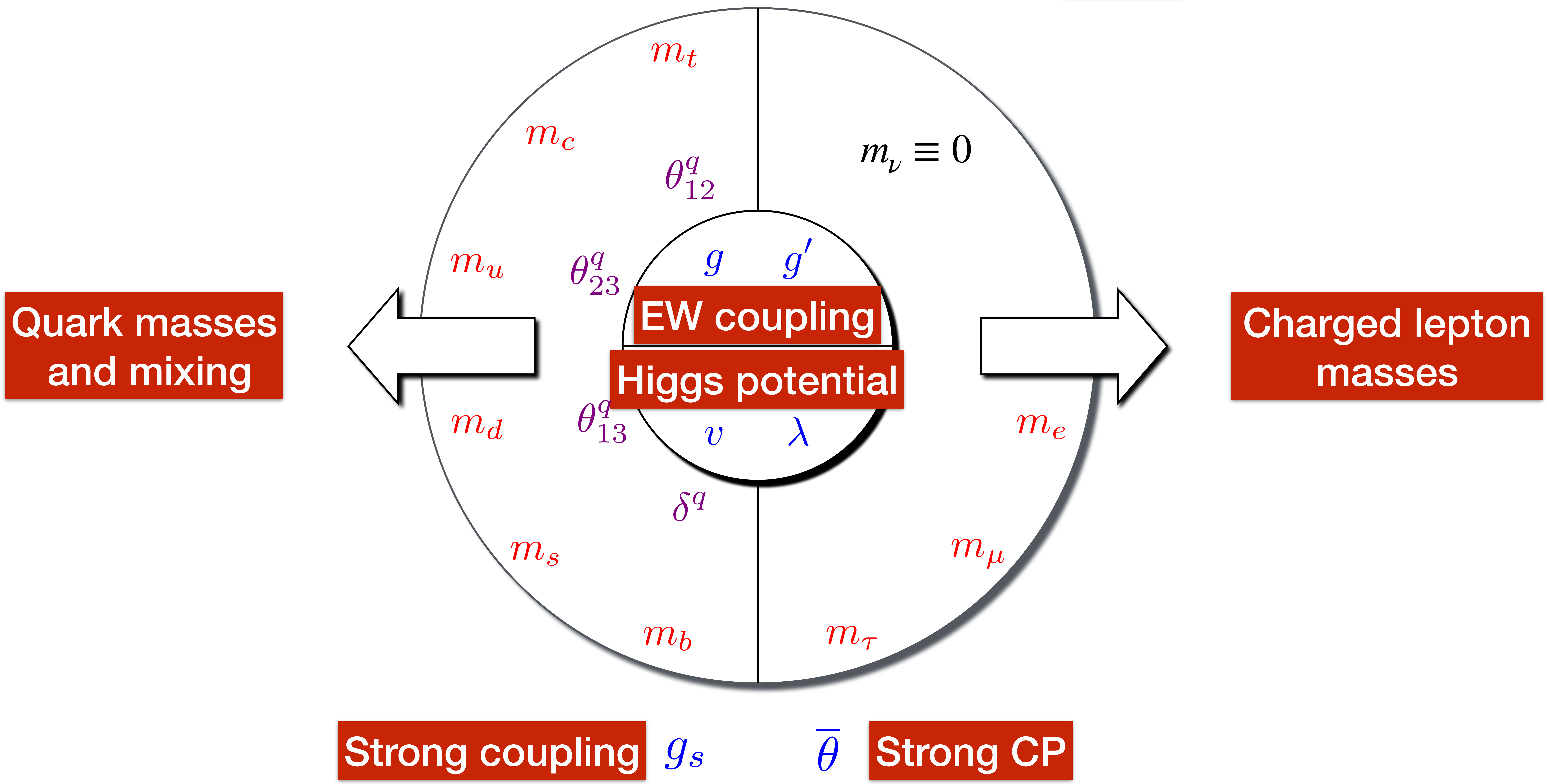
ν_τ	τ	t	b
ν_μ	μ	c	s
ν_e	e	u	d

The Standard Model (SM)



The Standard Model (SM)

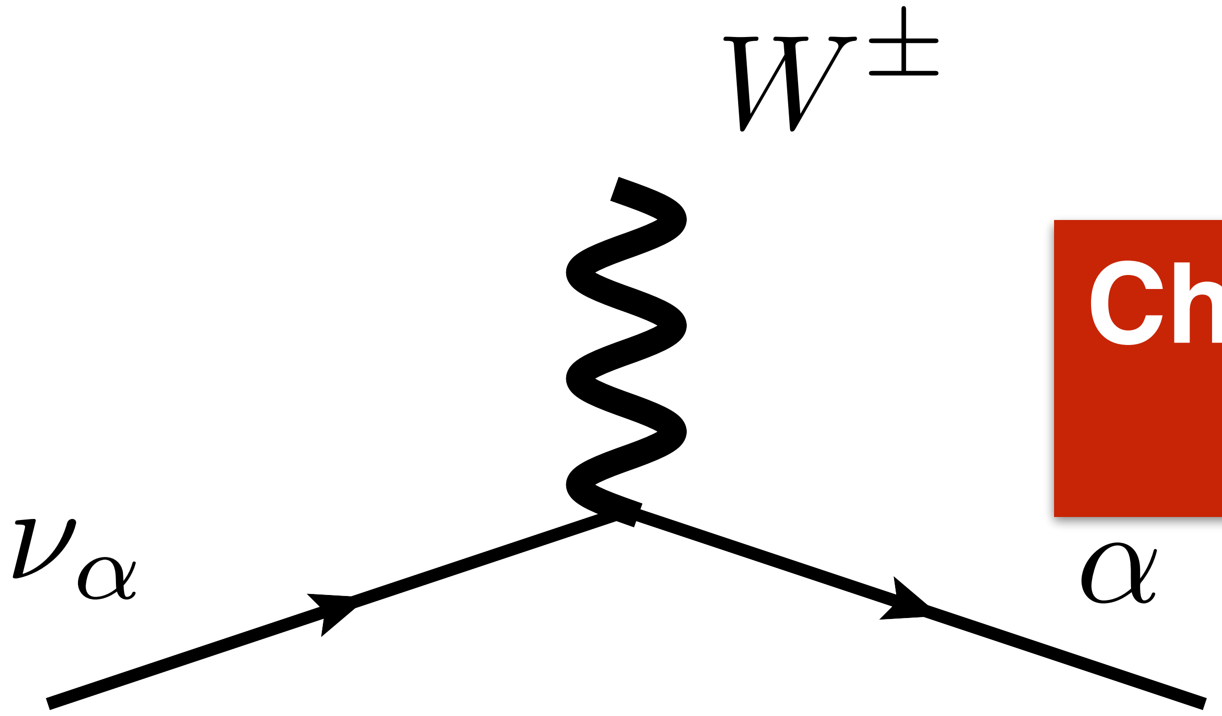
17 + 2 fundamental para.



Neutrinos in the SM

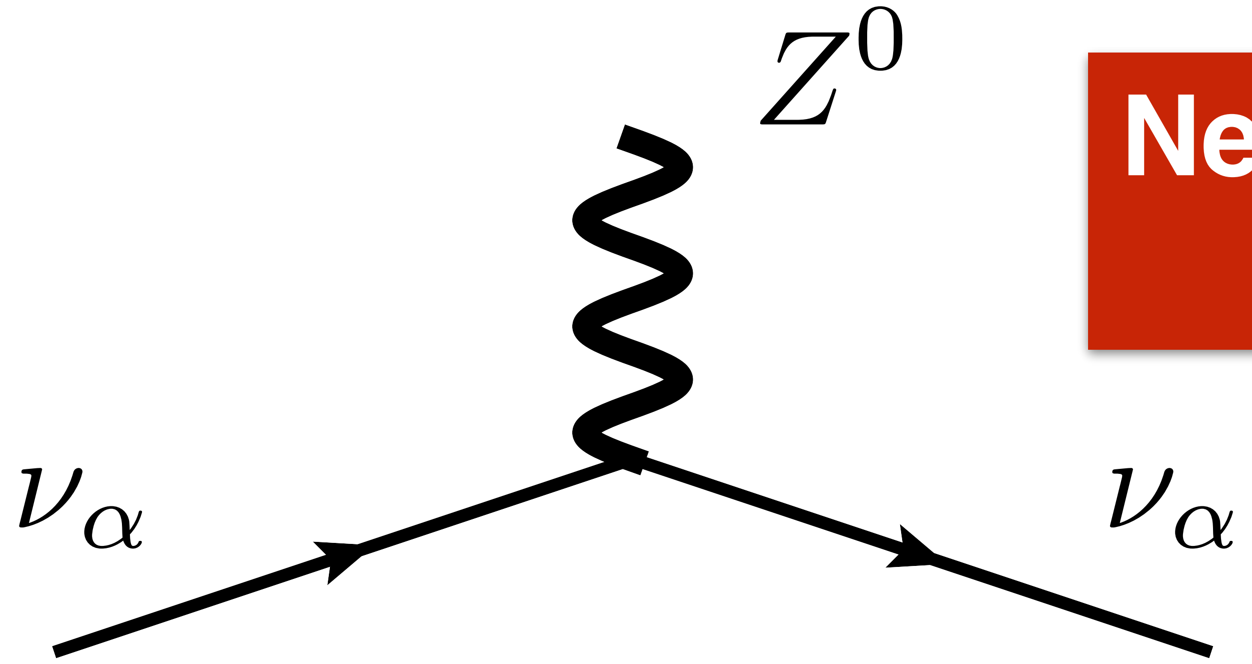
- Fermion, spin 1/2
- Massless
- No electric charge

Only weak interactions



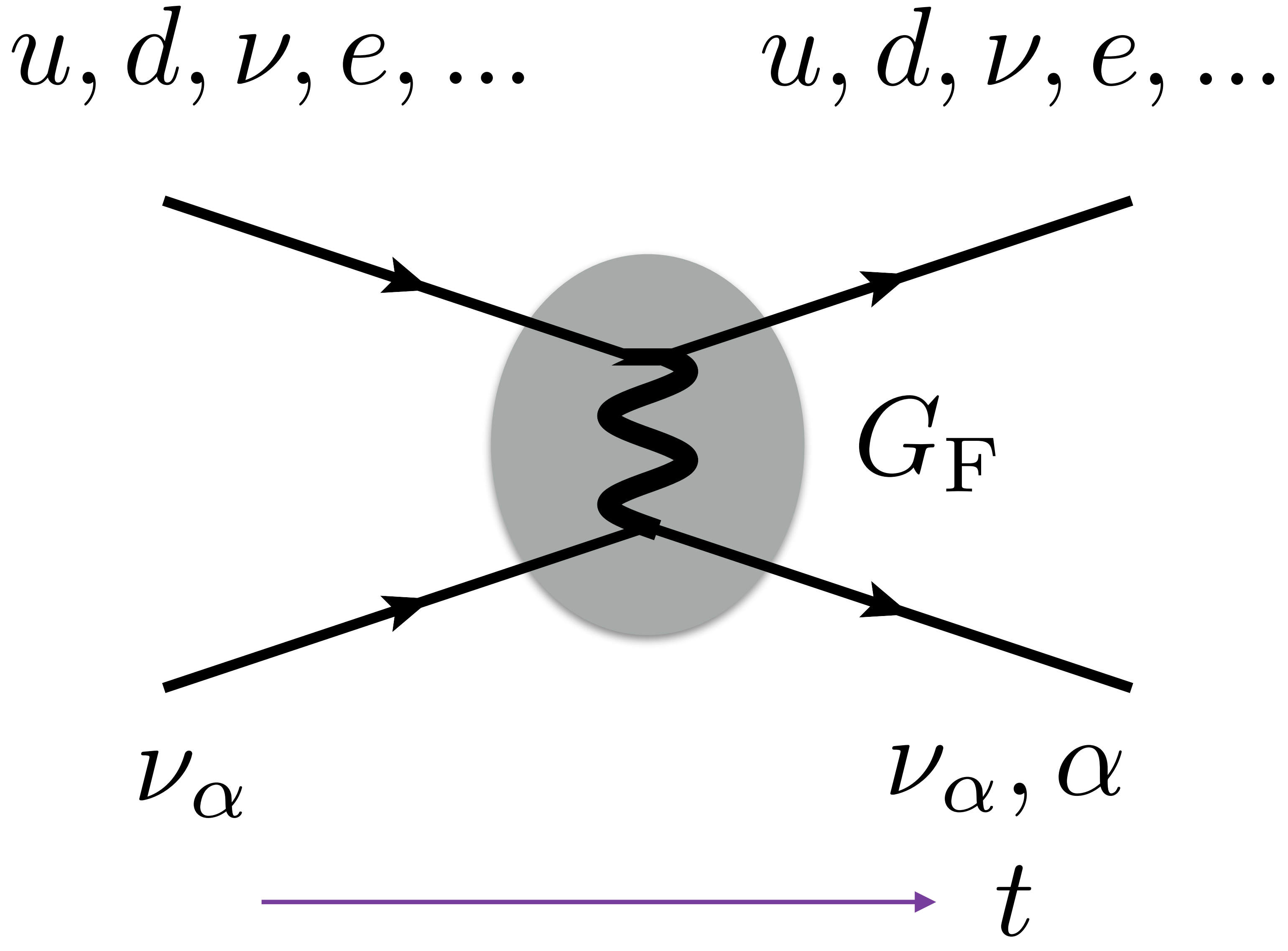
Charged current (CC)

$$\alpha = e, \mu, \tau$$



Neutral current (NC)

4-Fermi interaction



Lepton flavours and lepton number

	L_e	L_μ	L_τ	$L=L_e+L_\mu+L_\tau$
e^-, ν_e	1	0	0	1
$e^+, \overline{\nu}_e$	-1	0	0	-1
μ^-, ν_μ	0	1	0	1
$\mu^+, \overline{\nu}_\mu$	0	-1	0	-1
τ^-, ν_τ	0	0	1	1
$\tau^+, \overline{\nu}_\tau$	0	0	-1	-1

Flavour mixing ⇒ lepton flavour violation (LFV) $\nu_i = U^*_{ei}\nu_e + U^*_{\mu i}\nu_\mu + U^*_{\tau i}\nu_\tau$

- Typical LFV processes
  - Neutrino oscillation
  - $\mu^- \to e^- e^- e^+, \tau^- \to \mu^- \mu^- \mu^+, \mu^- e^- e^+$
  - $\mu^- \to e^- \gamma, \tau^- \to \mu^- \gamma$

Neutrino masses and mixing

Adding mass terms to neutrinos

- Lagrangian for a massive Dirac fermion

$$\mathscr{L}_\mathrm{D} = \overline{\psi}\, i\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi$$

4-component spinor

$$= \overline{\psi_L}\, i\gamma^\mu \partial_\mu \psi_L + \overline{\psi_R}\, i\gamma^\mu \partial_\mu \psi_R - m\overline{\psi_L}\psi_R - m\overline{\psi_R}\psi_L$$

$\psi = \psi_L + \psi_R$ Split into left-handed and right-handed fermions

$$\psi_L = P_L\psi = \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R = P_R\psi = \frac{1}{2}(1+\gamma_5)\psi$$

- Lagrangian for a massive Majorana fermion $\psi = \psi_L + \psi_L^c$ $\quad \psi_L^c = C\overline{\psi_L}^T, \quad C = i\gamma_0\gamma_2$

$$\mathscr{L}_\mathrm{M} = \frac{1}{2}\overline{\psi}\, i\gamma^\mu \partial_\mu \psi - \frac{1}{2} m\, \overline{\psi}\psi$$

$$= \overline{\psi_L}\, i\gamma^\mu \partial_\mu \psi_L - \frac{1}{2} m\, \overline{\psi_L}\psi_L^c - \frac{1}{2} m\, \overline{\psi_L^c}\psi_L$$

Adding mass terms to neutrinos

- Lepton mass terms in the 3-dim flavour space

$$\mathscr{L} \supset -\overline{(\tilde{l}_1, \tilde{l}_2, \tilde{l}_3)_L} \begin{pmatrix} m^l_{11} & m^l_{12} & m^l_{13} \\ m^l_{21} & m^l_{22} & m^l_{23} \\ m^l_{31} & m^l_{32} & m^l_{33} \end{pmatrix} \begin{pmatrix} \tilde{l}_1 \\ \tilde{l}_2 \\ \tilde{l}_3 \end{pmatrix}_R - \frac{1}{2}\overline{(\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3)_L} \begin{pmatrix} m^\nu_{11} & m^\nu_{12} & m^\nu_{13} \\ m^\nu_{12} & m^\nu_{22} & m^\nu_{23} \\ m^\nu_{13} & m^\nu_{23} & m^\nu_{33} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \\ \tilde{\nu}_3 \end{pmatrix}^c_L + \frac{g}{\sqrt{2}} \overline{(\tilde{l}_1, \tilde{l}_2, \tilde{l}_3)_L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \gamma^\mu \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \\ \tilde{\nu}_3 \end{pmatrix}_L W^-_\mu + \mathrm{h.c.}$$

Dirac mass matrix M_l | Majorana mass matrix M_ν | Charged-current interaction

Diagonalization

$$M_l = U_l \widehat{M}_l U_l'^\dagger \qquad \widehat{M}_l = \mathrm{diag}\{m_e, m_\mu, m_\tau\}$$

$$M_\nu = U_\nu \widehat{M}_\nu U_\nu^T \qquad \widehat{M}_\nu = \mathrm{diag}\{m_1, m_2, m_3\}$$

- Transform to mass basis $\begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_L = U_l^\dagger \begin{pmatrix} \tilde{l}_1 \\ \tilde{l}_2 \\ \tilde{l}_3 \end{pmatrix}_L \qquad \begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R = U_l'^\dagger \begin{pmatrix} \tilde{l}_1 \\ \tilde{l}_2 \\ \tilde{l}_3 \end{pmatrix}_R \qquad \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_R = U_\nu^\dagger \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \\ \tilde{\nu}_3 \end{pmatrix}_R$

$$\mathscr{L} \supset -\overline{(l_e, l_\mu, l_\tau)_L} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m^\tau \end{pmatrix} \begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R - \overline{(\nu_1, \nu_2, \nu_3)_L} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}^c_L + \frac{g}{\sqrt{2}} \overline{(l_e, l_\mu, l_\tau)_L} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu \quad + \mathrm{h.c.}$$

Mixing arises in CC interaction $U = U_l^\dagger U_\nu$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing

Flavour basis $-\overline{(l_e, l_\mu, l_\tau)_L}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m^\tau \end{pmatrix}\begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R -\frac{1}{2}\overline{(\nu_1, \nu_2, \nu_3)_L}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L^c + \frac{g}{\sqrt{2}}\overline{(l_e, l_\mu, l_\tau)_L}\gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \mathrm{h.c.}$

Mass basis $-\overline{(l_e, l_\mu, l_\tau)_L}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m^\tau \end{pmatrix}\begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R -\frac{1}{2}\overline{(\nu_e, \nu_\mu, \nu_\tau)_L}\begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L^c + \frac{g}{\sqrt{2}}\overline{(l_e, l_\mu, l_\tau)_L}\gamma^\mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L W_\mu^- + \mathrm{h.c.}$

In the neutrino sector, each flavour eigenstate is an hyperposition of neutrino mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Maki, Nakagawa, Sakata, 1962

Pontecorvo, 1957

and verse visa

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Parametrization of PMNS matrix

$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$ is a 3×3 unitary matrix, it satisfies $(U^\dagger U)_{ij} = \delta_{ij}$. Then it should have 9 free parameters.

But physically, there are only 6 physical: 3 mixing angles + 1 Dirac phase + 2 Majorana phases.

WHY? $\Rightarrow$ Parametrization

$c_{ij} = \cos\theta_{ij}, \; s_{ij} = \sin\theta_{ij}$

$$U = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Unphysical phases | **Atmospheric angle θ_{23}** | **Reactor angle θ_{13} & Dirac phase δ** | **Solar angle θ_{12}** | **Majorana phases ρ, σ**

Confirm why!

Unphysical for Dirac neutrinos

$$-\overline{(l_e, l_\mu, l_\tau)_L} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m^\tau \end{pmatrix} \begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R - \frac{1}{2}\overline{(\nu_1, \nu_2, \nu_3)_L} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L^c + \frac{g}{\sqrt{2}} \overline{(l_e, l_\mu, l_\tau)_L} \gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu \quad + \text{h.c.}$$

Parametrization of PMNS matrix

Generalize Euler rotations with complex phase

$$R_{12}(\theta)=\begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \to e^{i\varphi}$$

$$R_{23}(\sigma)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\sigma & s_\sigma \\ 0 & -s_\sigma & c_\sigma \end{pmatrix}$$

$$R_{31}(\tau)=\begin{pmatrix} c_\tau & 0 & s_\tau \\ 0 & 1 & 0 \\ -s_\tau & 0 & c_\tau \end{pmatrix}$$

[Fritzsch, Xing, hep-ph/9708366]

9 different parametrizations

Phases on both sides not included

$P1$: $V=R_{12}(\theta)R_{23}(\sigma,\varphi)R_{12}^{-1}(\theta')$

$$\begin{pmatrix} s_\theta s_{\theta'} c_\sigma + c_\theta c_{\theta'} e^{-i\varphi} & s_\theta c_{\theta'} c_\sigma - c_\theta s_{\theta'} e^{-i\varphi} & s_\theta s_\sigma \\ c_\theta s_{\theta'} c_\sigma - s_\theta c_{\theta'} e^{-i\varphi} & c_\theta c_{\theta'} c_\sigma + s_\theta s_{\theta'} e^{-i\varphi} & c_\theta s_\sigma \\ -s_{\theta'} s_\sigma & -c_{\theta'} s_\sigma & c_\sigma \end{pmatrix}$$

$P2$: $V=R_{23}(\sigma)R_{12}(\theta,\varphi)R_{23}^{-1}(\sigma')$

$$\begin{pmatrix} c_\theta & s_\theta c_{\sigma'} & -s_\theta s_{\sigma'} \\ -s_\theta c_\sigma & c_\theta c_\sigma c_{\sigma'} + s_\sigma s_{\sigma'} e^{-i\varphi} & -c_\theta c_\sigma s_{\sigma'} + s_\sigma c_{\sigma'} e^{-i\varphi} \\ s_\theta s_\sigma & -c_\theta s_\sigma c_{\sigma'} + c_\sigma s_{\sigma'} e^{-i\varphi} & c_\theta s_\sigma s_{\sigma'} + c_\sigma c_{\sigma'} e^{-i\varphi} \end{pmatrix}$$

$P3$: $V=R_{23}(\sigma)R_{31}(\tau,\varphi)R_{12}(\theta)$

$$\begin{pmatrix} c_\theta c_\tau & s_\theta c_\tau & s_\tau \\ -c_\theta s_\sigma s_\tau - s_\theta c_\sigma e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & s_\sigma c_\tau \\ -c_\theta c_\sigma s_\tau + s_\theta s_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} & c_\sigma c_\tau \end{pmatrix}$$

$P4$: $V=R_{12}(\theta)R_{31}(\tau,\varphi)R_{23}^{-1}(\sigma)$

$$\begin{pmatrix} c_\theta c_\tau & c_\theta s_\sigma s_\tau + s_\theta c_\sigma e^{-i\varphi} & c_\theta c_\sigma s_\tau - s_\theta s_\sigma e^{-i\varphi} \\ -s_\theta c_\tau & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} \\ -s_\tau & s_\sigma c_\tau & c_\sigma c_\tau \end{pmatrix}$$

$P5$: $V=R_{31}(\tau)R_{12}(\theta,\varphi)R_{31}^{-1}(\tau')$

$$\begin{pmatrix} c_\theta c_\tau c_{\tau'} + s_\tau s_{\tau'} e^{-i\varphi} & s_\theta c_\tau & -c_\theta c_\tau s_{\tau'} + s_\tau c_{\tau'} e^{-i\varphi} \\ -s_\theta c_{\tau'} & c_\theta & s_\theta s_{\tau'} \\ -c_\theta s_\tau c_{\tau'} + c_\tau s_{\tau'} e^{-i\varphi} & -s_\theta s_\tau & c_\theta s_\tau s_{\tau'} + c_\tau c_{\tau'} e^{-i\varphi} \end{pmatrix}$$

$P6$: $V=R_{12}(\theta)R_{23}(\sigma,\varphi)R_{31}(\tau)$

$$\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & s_\theta c_\sigma & s_\theta s_\sigma c_\tau + c_\theta s_\tau e^{-i\varphi} \\ -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\theta c_\sigma & c_\theta s_\sigma c_\tau - s_\theta s_\tau e^{-i\varphi} \\ -c_\sigma s_\tau & -s_\sigma & c_\sigma c_\tau \end{pmatrix}$$

P7: $V=R_{23}(\sigma)R_{12}(\theta,\varphi)R_{31}^{-1}(\tau)$

$$\begin{pmatrix} c_\theta c_\tau & s_\theta & -c_\theta s_\tau \\ -s_\theta c_\sigma c_\tau + s_\sigma s_\tau e^{-i\varphi} & c_\theta c_\sigma & s_\theta c_\sigma s_\tau + s_\sigma c_\tau e^{-i\varphi} \\ s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} & -c_\theta s_\sigma & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$$

$P8$: $V=R_{31}(\tau)R_{12}(\theta,\varphi)R_{23}(\sigma)$

$$\begin{pmatrix} c_\theta c_\tau & s_\theta c_\sigma c_\tau - s_\sigma s_\tau e^{-i\varphi} & s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} \\ -s_\theta & c_\theta c_\sigma & c_\theta s_\sigma \\ -c_\theta s_\tau & -s_\theta c_\sigma s_\tau - s_\sigma c_\tau e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$$

$P9$: $V=R_{31}(\tau)R_{23}(\sigma,\varphi)R_{12}^{-1}(\theta)$

$$\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\sigma s_\tau \\ s_\theta c_\sigma & c_\theta c_\sigma & s_\sigma \\ -s_\theta s_\sigma c_\tau - c_\theta s_\tau e^{-i\varphi} & -c_\theta s_\sigma c_\tau + s_\theta s_\tau e^{-i\varphi} & c_\sigma c_\tau \end{pmatrix}$$

PMNS mixing vs CKM mixing

Lepton masses and mixing

$$-\overline{(l_e, l_\mu, l_\tau)_L}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}\begin{pmatrix} l_e \\ l_\mu \\ l_\tau \end{pmatrix}_R -\frac{1}{2}\overline{(\nu_1, \nu_2, \nu_3)_L}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}^c_L + \frac{g}{\sqrt{2}}\overline{(l_e, l_\mu, l_\tau)_L}\gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu + \text{h.c.}$$

PMNS matrix

Quark masses and mixing

$$-\overline{(q_d, q_s, d_b)_L}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}\begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix}_R -\frac{1}{2}\overline{(q_u, q_c, q_t)_L}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}\begin{pmatrix} q_u \\ q_c \\ q_t \end{pmatrix}_R + \frac{g}{\sqrt{2}}\overline{(q_u, q_c, q_t)_L}\gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix}_L W^+_\mu + \text{h.c.}$$

Cabibbo–Kobayashi–Maskawa (CKM) matrix

$$V = \begin{pmatrix} e^{i\alpha_1^q} & 0 & 0 \\ 0 & e^{i\alpha_2^q} & 0 \\ 0 & 0 & e^{i\alpha_3^q} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^q & s_{23}^q \\ 0 & -s_{23}^q & c_{23}^q \end{pmatrix}\begin{pmatrix} c_{13}^q & 0 & s_{13}^q e^{-i\delta^q} \\ 0 & 1 & 0 \\ -s_{13}^q e^{i\delta^q} & 0 & c_{13}^q \end{pmatrix}\begin{pmatrix} c_{12}^q & s_{12}^q & \\ -s_{12}^q & c_{12}^q & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} e^{i\beta_1^q} & 0 & 0 \\ 0 & e^{i\beta_2^q} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Unphysical

Cabibbo anale: $\theta_C = \theta_{12}^q$

Unphysical

Wolfenstein Parametrization

PRL,1983

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda = \sin\theta_C \simeq 0.2$

$A, \rho, \eta \sim \mathcal{O}(1)$

Current data

NuFIT 6.0 (2024)

IC19 without SK atmospheric data		Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \to 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \to 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \to 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \to 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \to 8.91$
	$\delta_{\rm CP}/^\circ$	177^{+19}_{-20}	$96 \to 422$	285^{+25}_{-28}	$201 \to 348$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \to +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \to -2.438$

Current data

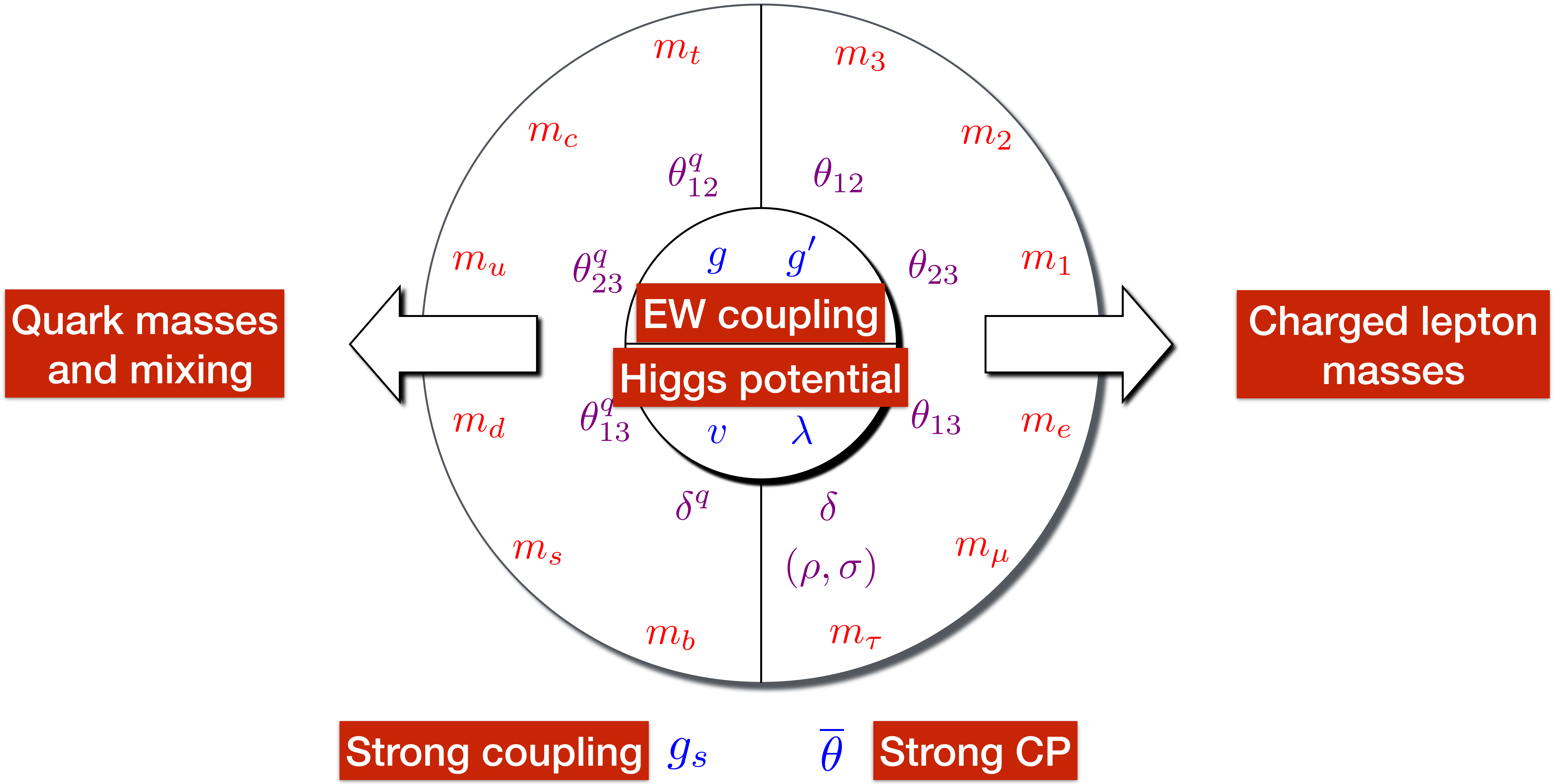
Do not be too confident!

NuFIT 5.0 (2020)

without SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
	$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
	$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{\rm CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

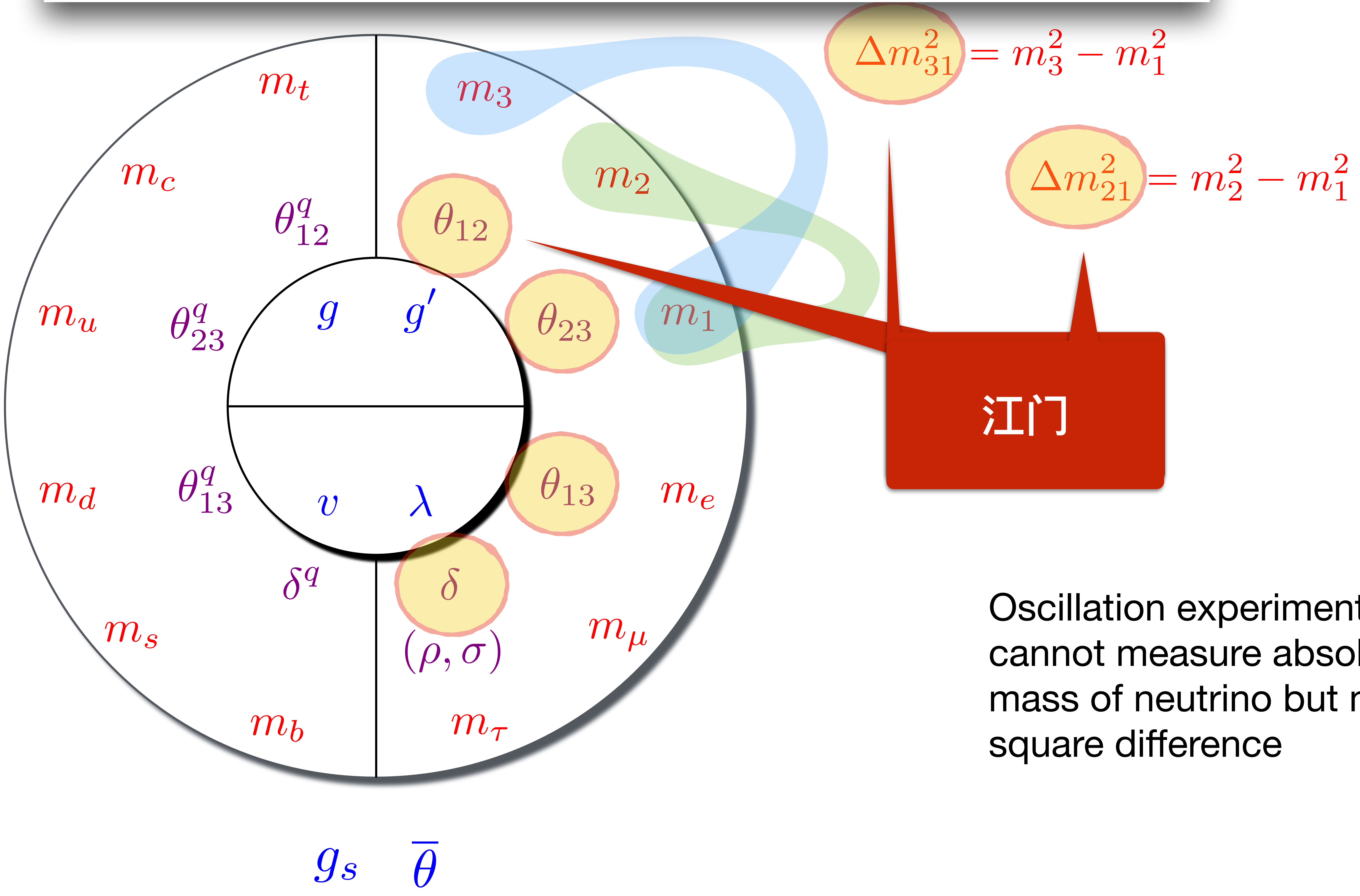
SM+massive neutrinos

24 (26) + 2 fundamental para.



中微子振荡实验能测量的基本参数

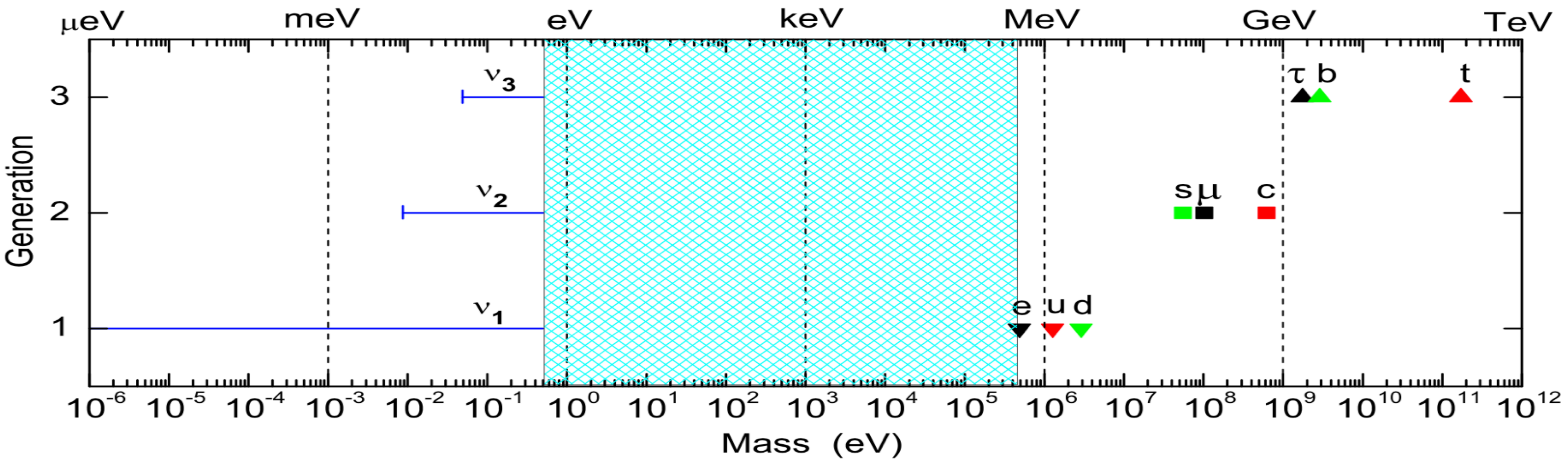
$$P_{\bar{e}\bar{e}} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21} - c_{12}^2 \sin^2 2\theta_{13} \sin^2 \Delta_{31} - s_{12}^2 \sin^2 2\theta_{13} \sin^2 \Delta_{32}$$



Oscillation experiments cannot measure absolute mass of neutrino but mass-square difference

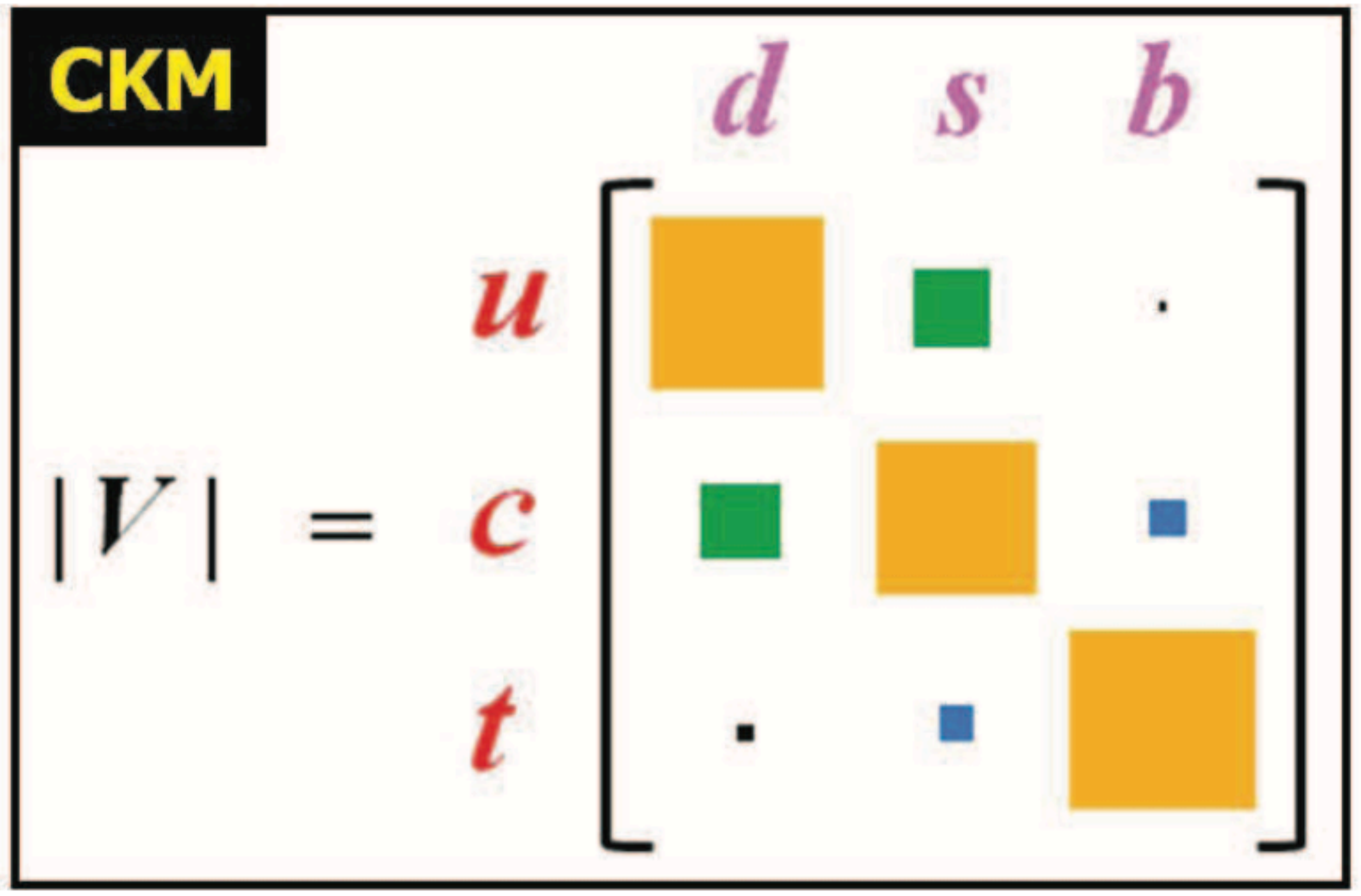
Fundamental questions in the neutrino sector

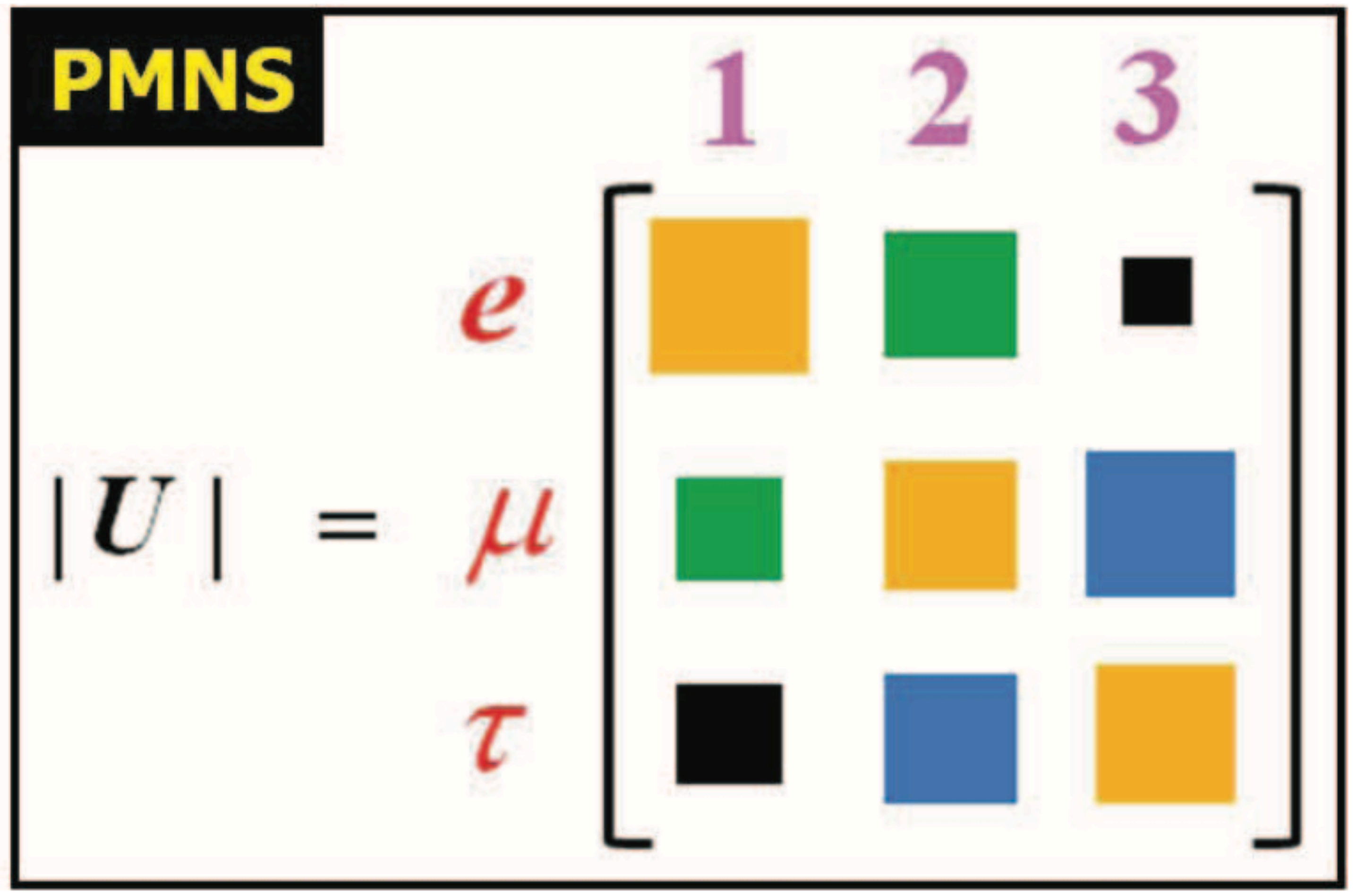
- Large gap between neutrino masses and other fermion masses



⇒ Origin of neutrino masses

- Mismatch between quark mixing and lepton mixing





⇒ Mechanism of flavour mixing

Origin of neutrino masses

The absence of neutrino masses in the SM

- Constructing operators in the SM $\quad S = \int \mathrm{d}^4x\, \mathscr{L}$

Mass dim [M] : $0 = (-4) + 4$

Renormalizable $[M] \leqslant 4$

$$\overline{\ell_L} i\gamma^\mu D_\mu \ell_L + \overline{l_R} i\gamma^\mu D_\mu l_R + D_\mu H^\dagger D^\mu H$$

$\overline{\ell_L} H l_R \Rightarrow$ Dirac mass for charged lepton

$$H^\dagger H, (H^\dagger H)^2$$

	$SU(3)_c$	$SU(2)_L$	Y	Mass dim
$\ell_L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L$	**1**	**2**	-1/2	3/2
l_R	**1**	**1**	-1	3/2
H	**1**	**2**	1/2	1
D_μ	**1**	**1**	0	1

- Higher-dimensional operators $\quad \mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\mathcal{L}_{\rm d=5}}{\Lambda} + \frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} + \cdots$

[M] = 5

$$\frac{\lambda}{\Lambda} \overline{\ell_L} \tilde{H} \tilde{H} \ell_L^c \xrightarrow{\langle H \rangle = v_{\rm EW}/\sqrt{2}} \frac{1}{2} m_\nu \overline{\nu_L} \nu_L^c, \quad m_\nu = \frac{\lambda}{\Lambda} v_{\rm EW}^2$$

L & B-L violating **Majorana mass**

[M] = 6 B-L conversing

[M] = 7 B-L violating, Majorana mass

Seesaw mechanism

- Seesaw mechanism (跷跷板机制)

$$-y_l \overline{\ell_L} H l_R \;-\; y_D \overline{\ell_L} H \nu_R \;-\; \frac{1}{2} m_N \overline{\nu_R^c} \nu_R$$

$$\langle H \rangle = v_{\rm EW}/\sqrt{2} \quad \Downarrow$$

$$-m_l \overline{l_L} l_R \;-\; m_D \overline{\nu_L} \nu_R \;-\; \frac{1}{2} m_N \overline{\nu_R^c} \nu_R \qquad m_l = y_l v_{\rm EW}/\sqrt{2}, \quad m_D = y_D v_{\rm EW}/\sqrt{2}$$

$$\frac{1}{2} \overline{(\nu_L, \nu_R^c)} \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

	$SU(3)_c$	$SU(2)_L$	Y	Mass dim
$\ell_L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L$	**1**	**2**	-1/2	3/2
l_R	**1**	**1**	-1	3/2
H	**1**	**2**	1/2	1
D_μ	**1**	**1**	0	1
$N\ (\nu_R)$	**1**	**1**	0	3/2

Mass eigenvalues: $m_{1,2} = \frac{1}{2}\left(m_N \mp \sqrt{m_N^2 + 4m_D^2}\right) \xrightarrow{m_D \ll m_N} m_1 \approx -\frac{m_D^2}{m_N}, \quad m_2 \approx m_N$

Taking m_ν ~ 0.1 eV and m_D ~ 10^2 GeV, we obtain m_N ~ 10^{14} GeV → canonical seesaw scale

Since ν_R is considered as much heavier than other particles, it is usually denoted as N

Seesaw with 3 flavours

$$\underbrace{\begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}}_{6\times 6} = \underbrace{\begin{pmatrix} U & R \\ S & V \end{pmatrix}}_{\mathcal{U}} \begin{pmatrix} \hat{M}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix} \begin{pmatrix} U & R \\ S & V \end{pmatrix}^T$$

$$0 = U\hat{M}_\nu U^T + R\hat{M}_N R^T$$

$$M_D = U\hat{M}_\nu S^T + R\hat{M}_N V^T \quad \approx R\hat{M}_N V^T$$

$$M_N = S\hat{M}_\nu S^T + V\hat{M}_N V^T \quad \approx V\hat{M}_N V^T$$

$$M_\nu \equiv U\hat{M}_\nu U^T = -R\hat{M}_N R^T = -R\hat{M}_N V^T (V\hat{M}_N V^T)^{-1} (R\hat{M}_N V^T)^T \approx -M_D M_N^{-1} M_D^T$$ ← seesaw formula in 3-flavour case

Casas-Ibarra parametrization
hep-ph/0103065

$$U\hat{M}_\nu U^T \approx -M_D (V^{-1})^T \hat{M}_N^{-1} V^{-1} M_D^T \qquad \hat{M}_\nu \approx -\underbrace{[U^{-1} M_D (V^{-1})^T]}_{\tilde{M}_D} \hat{M}_N^{-1} \underbrace{[V^{-1} M_D^T (U^{-1})^T]}_{\tilde{M}_D^T}$$

$$\Rightarrow \sqrt{\hat{M}_\nu} \cdot \sqrt{\hat{M}_\nu} \approx \tilde{M}_D \sqrt{\hat{M}_N^{-1}} \cdot \sqrt{\hat{M}_N^{-1}} \tilde{M}_D^T \qquad \Rightarrow \sqrt{\hat{M}_\nu}\mathcal{R} \approx \tilde{M}_D \sqrt{\hat{M}_N^{-1}}$$

$$\tilde{M}_D \approx \sqrt{\hat{M}_\nu}\mathcal{R}\sqrt{\hat{M}_N}$$

$\mathcal{R}$ is a complex orthogonal matrix, $\mathcal{R}^T\mathcal{R} = \mathcal{R}\mathcal{R}^T = 1$

In the flavour basis and heavy RHN mass basis $\quad M_D \approx U\sqrt{\hat{M}_\nu}\mathcal{R}\sqrt{\hat{M}_N} \quad \Rightarrow$ widely used in leptogenesis

Euler-like parametrization $\quad \mathcal{U} = (O_{56}O_{46}O_{36}O_{26}O_{16})(O_{45}O_{35}O_{25}O_{15})(O_{34}O_{24}O_{14})(O_{23}O_{13})O_{12}$

Xing, 0709.2220; 1110.0083

$$O_{12} = \begin{pmatrix} c_{12} & s_{12}e^{-i\delta_{12}} & 0 & \cdots \\ -s_{12}e^{i\delta_{12}} & c_{12} & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad O_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} & \cdots \\ 0 & 1 & 0 & \cdots \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Variation of seesaw models

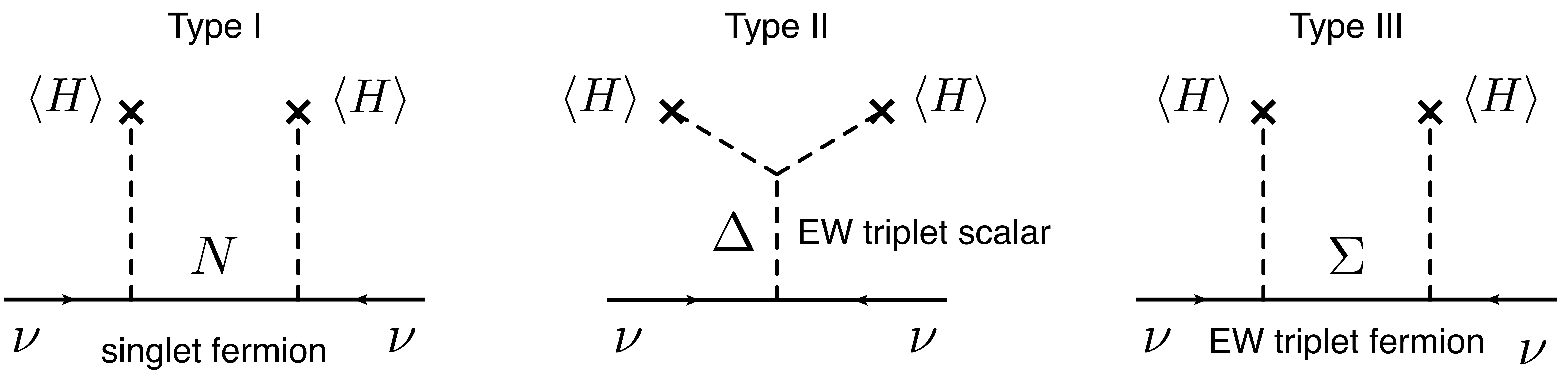
- Minimal seesaw SM + 2 N $m_{\rm lightest} = 0$

- Low-scale seesaw

  Taking m_ν ~ 0.1 eV and m_D ~ m_e, we obtain m_N ~ 10 TeV → TeV seesaw

  SM + N_R + N_L $\frac{1}{2}\overline{(\nu_L, N_R^c, N_L)}\begin{pmatrix} 0 & m_D & 0 \\ m_D & m_R & m'_D \\ 0 & m'_D & m_L \end{pmatrix}\begin{pmatrix} \nu_L^c \\ N_R \\ N_L^c \end{pmatrix}$ $M_\nu \approx (\frac{m_D}{m'_D})^2 m_L$ → Inverse seesaw

- Based on mediators



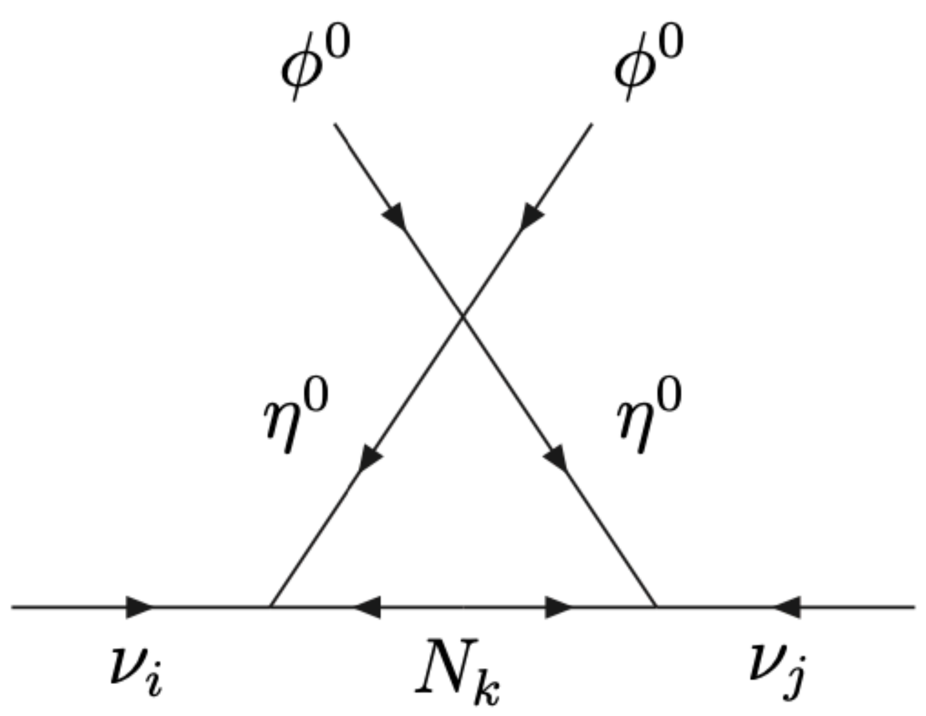
Other mechanisms

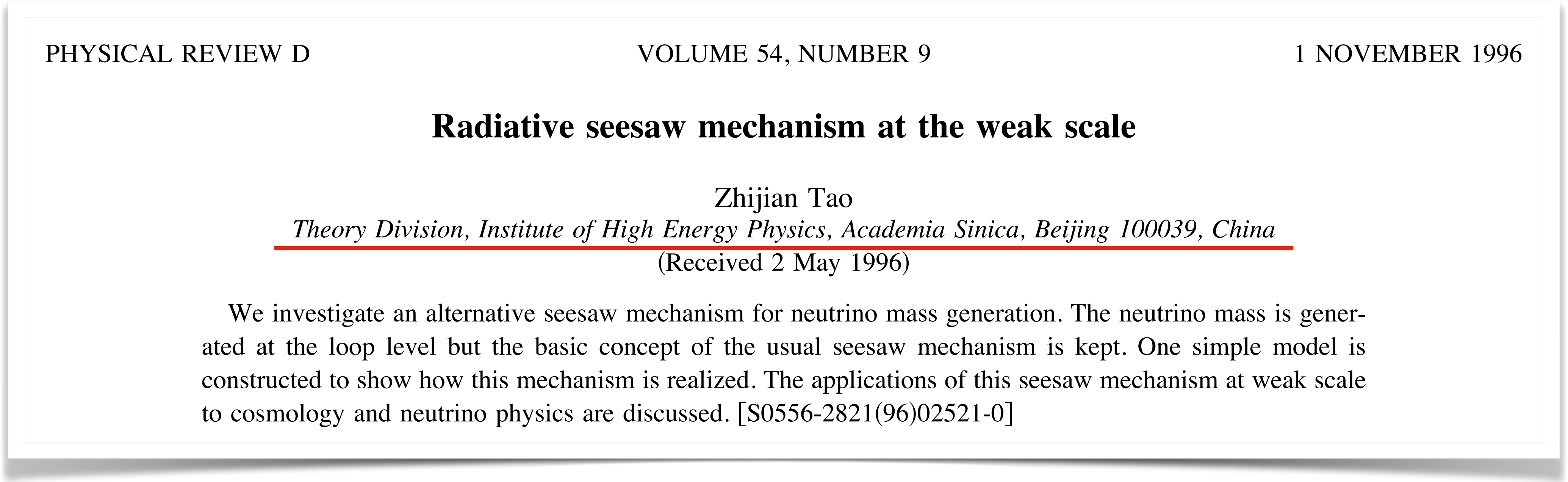
- Radiative models

SM + $N + \eta^0$ in Z_2

Scotogenic model

Tao(陶志坚), hep-ph/9603309



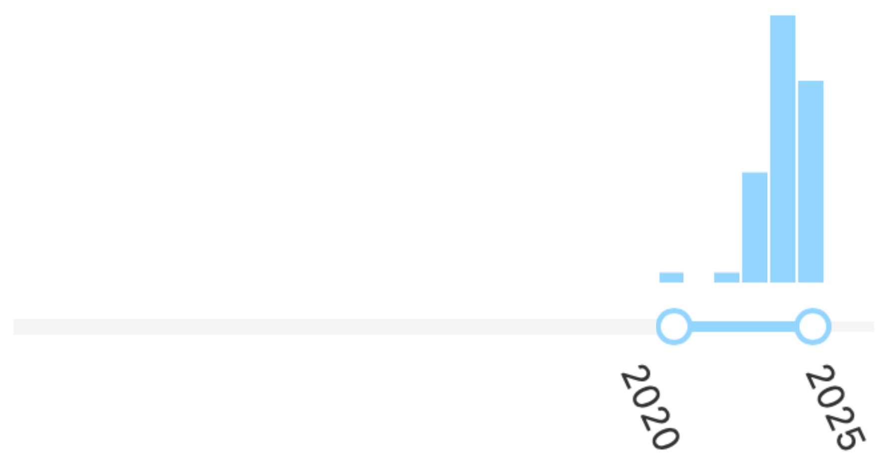

PHYSICAL REVIEW D VOLUME 54, NUMBER 9 1 NOVEMBER 1996

Radiative seesaw mechanism at the weak scale

Zhijian Tao
Theory Division, Institute of High Energy Physics, Academia Sinica, Beijing 100039, China
(Received 2 May 1996)

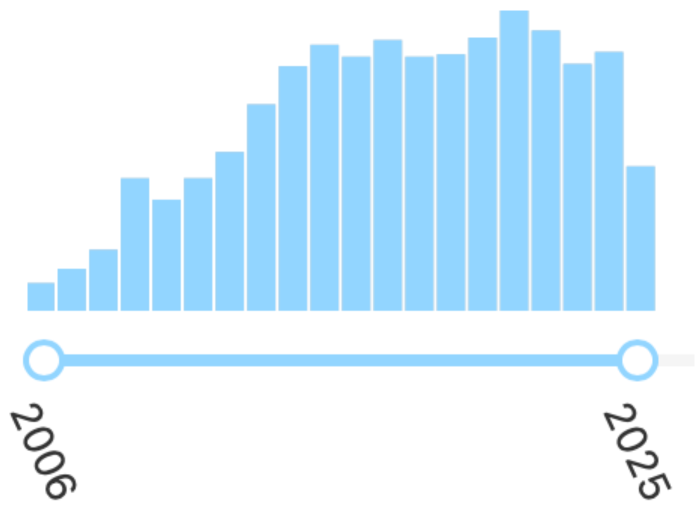
We investigate an alternative seesaw mechanism for neutrino mass generation. The neutrino mass is generated at the loop level but the basic concept of the usual seesaw mechanism is kept. One simple model is constructed to show how this mechanism is realized. The applications of this seesaw mechanism at weak scale to cosmology and neutrino physics are discussed. [S0556-2821(96)02521-0]

70+ citations



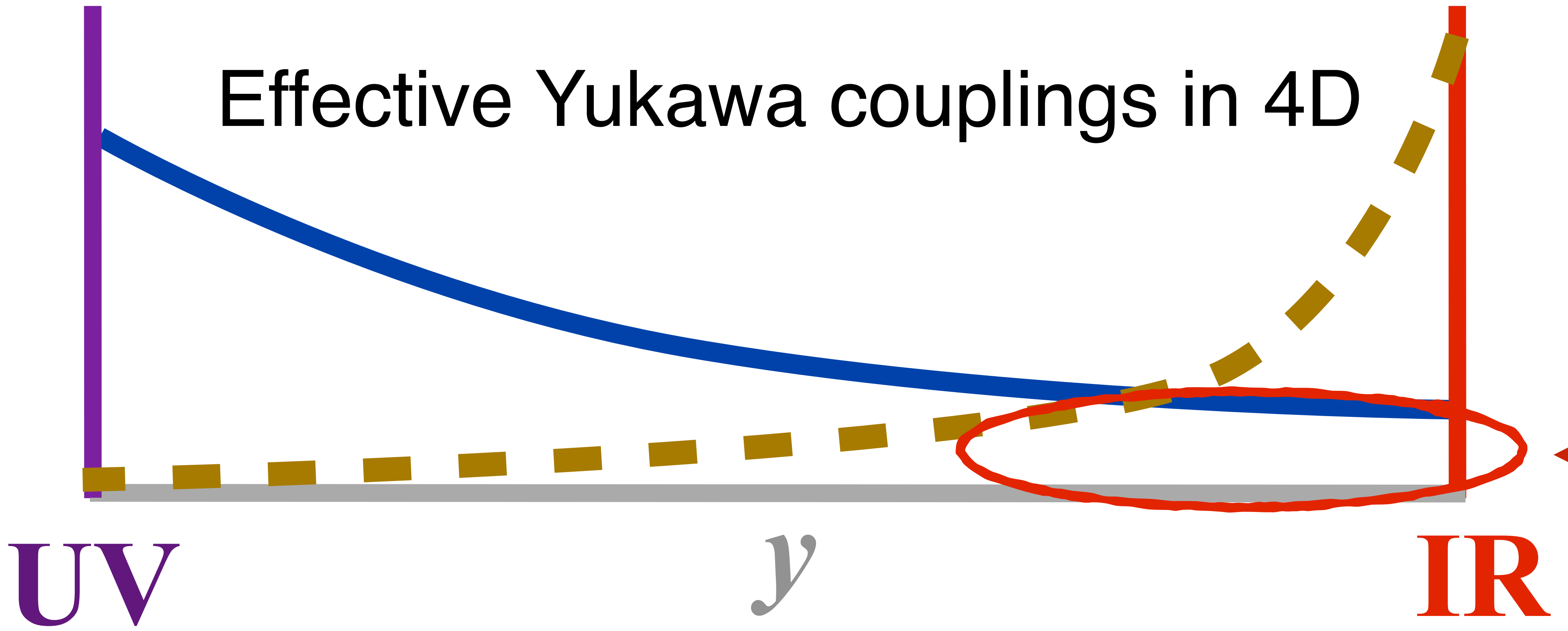
Ma, hep-ph/0601225

1600+ citations



- Models in extra dimension

Arkani-Hamed, Dimopoulos, Dvali, March-Russell, hep-ph/9811448



Dirac neutrinos

$\overline{\psi_{\rm L}} H \psi_{\rm R}$

Symmetries behind neutrino masses

- B-L is a good symmetry, but it is explicitly broken in seesaw models. One may guess seesaw might be an effective approach of a more fundamental theory.

$\Rightarrow$ The $U(1)_{B-L}$ model

Symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

Particles: SM particles + ν_R + Z_{BL} + ϕ

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B\text{-}L}$
$\ell_L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L$	**1**	**2**	-1/2	-1
ν_R	**1**	**1**	0	-1
l_R	**1**	**1**	-1	-1
$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L$	**3**	**2**	1/6	1/3
U_R	**3**	**1**	2/3	1/3
D_R	**3**	**1**	-1/3	1/3
H	**1**	**2**	1/2	0
ϕ	**1**	**1**	**1**	2

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- The $U(1)_{B-L}$ model is further embedded into left-right symmetric model (LRSM)

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Fields	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B\text{-}L}$
$\ell_L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L$	**1**	**2**	**1**	-1
$\ell_R = \begin{pmatrix} \nu \\ l \end{pmatrix}_R$	**1**	**1**	**2**	-1
$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L$	**3**	**2**	1	1/3
$Q_R = \begin{pmatrix} U \\ D \end{pmatrix}_R$	**3**	**1**	**2**	1/3
H	**1**	**2**	**2**	0
$\Delta_R\,(\Delta_L)$	**1**	**1 (3)**	**3 (1)**	2

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$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

- LRSM can be embedded into Pati-Salam model

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

Fields	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$
$F_L = (Q_L, \ell_L)$	**4**	**2**	**1**
$F_R = (Q_R, \ell_R)$	**4**	**1**	**2**
H	**1**	**2**	**2**
Φ	**15**	**2**	**2**
$\Delta_R\,(\Delta_L)$	**10**	**1 (3)**	**3 (1)**

Symmetries behind neutrino masses

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Symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

Particles: SM particles + ν_R + Z_{BL} + ϕ

- The $U(1)_{B-L}$ model is further embedded into left-right symmetric model (LRSM)

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

- LRSM can be embedded into Pati-Salam model

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

- They are all embedded into SO(10) GUT

Fields	$SO(10)$
Ψ_{16}	**16**
H_{10}	**10**
H_{126}	$\overline{\mathbf{126}}$
H_{120}	**120**

Neutrino masses in GUTs

- Grand unified theories (GUTs) are hypothetical theories trying to unify all fundamental forces in particle physics, including electromagnetic force, weak force and strong force.

- Typical examples are SU(5) and SO(10). Here we focus on SO(10) [Dutta, Mimura, Mohapatra, hep-ph/0406262]

- Field arrangements: fermion $\sim \mathbf{16}$, $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$ $\Rightarrow$ scalars $\sim \mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$

- Yukawa couplings $\mathscr{L}_Y = \Psi_{\mathbf{16}} \left[Y_{\mathbf{10}} H_{\mathbf{10}} + Y_{\overline{\mathbf{126}}} H_{\overline{\mathbf{126}}} + \mathrm{i} Y_{\mathbf{120}} H_{\mathbf{120}} \right] \Psi_{\mathbf{16}}$

  $(+ \Psi_{\mathbf{16}} \left[Y_{10} H^*_{\mathbf{10}} + \mathrm{i} Y^{ij}_{\mathbf{120}} H^*_{\mathbf{120}} \right] \Psi_{16})$ can be forbidden by the Peccei-Quinn symm

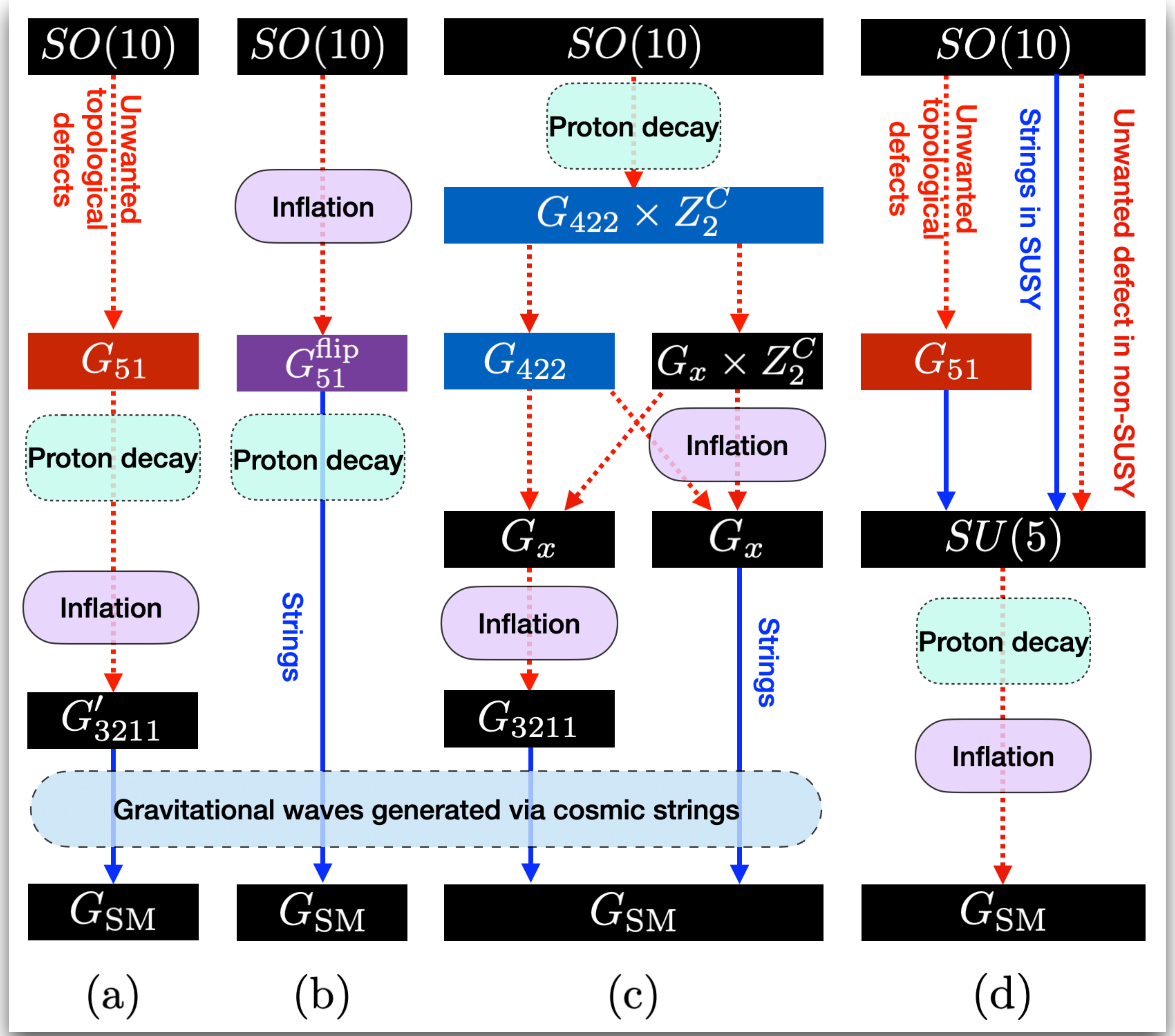
- Correlation between quark and lepton Yukawa couplings

$$M_u = Y_{10} v^u_{\mathbf{10}} + Y_{\overline{126}} v^u_{\overline{\mathbf{126}}} + \mathrm{i} Y_{120}(v^u_{\mathbf{120}} + v'^u_{\mathbf{120}}) \qquad M_D = Y_{10} v^u_{\mathbf{10}} - 3 Y_{\overline{126}} v^u_{\overline{\mathbf{126}}} + \mathrm{i} Y_{120}(v^u_{\mathbf{120}} - 3 v^{u'}_{\mathbf{120}})$$

$$M_d = Y_{10} v^d_{\mathbf{10}} + Y_{\overline{126}} v^d_{\overline{\mathbf{126}}} + \mathrm{i} Y_{120}(v^d_{\mathbf{120}} + v^{d'}_{\mathbf{120}}) \qquad M_N = Y_{\overline{126}} v_{BL}$$

$$M_l = Y_{10} v^d_{\mathbf{10}} - 3 Y_{\overline{126}} v^d_{\overline{\mathbf{126}}} + \mathrm{i} Y_{120}(v^d_{\mathbf{120}} - 3 v^{d'}_{\mathbf{120}}) \qquad M_\nu = - M_D M_N^{-1} M_D + Y_{\overline{126}} \Delta_L$$

GUT phenos



Unwanted topological defects: monopoles and domain walls

In any breaking chains, inflation has to been introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\rm flip} = SU(5)_{\rm flip} \times U(1)_{\rm flip}$$

$$Z_2^C: \quad \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

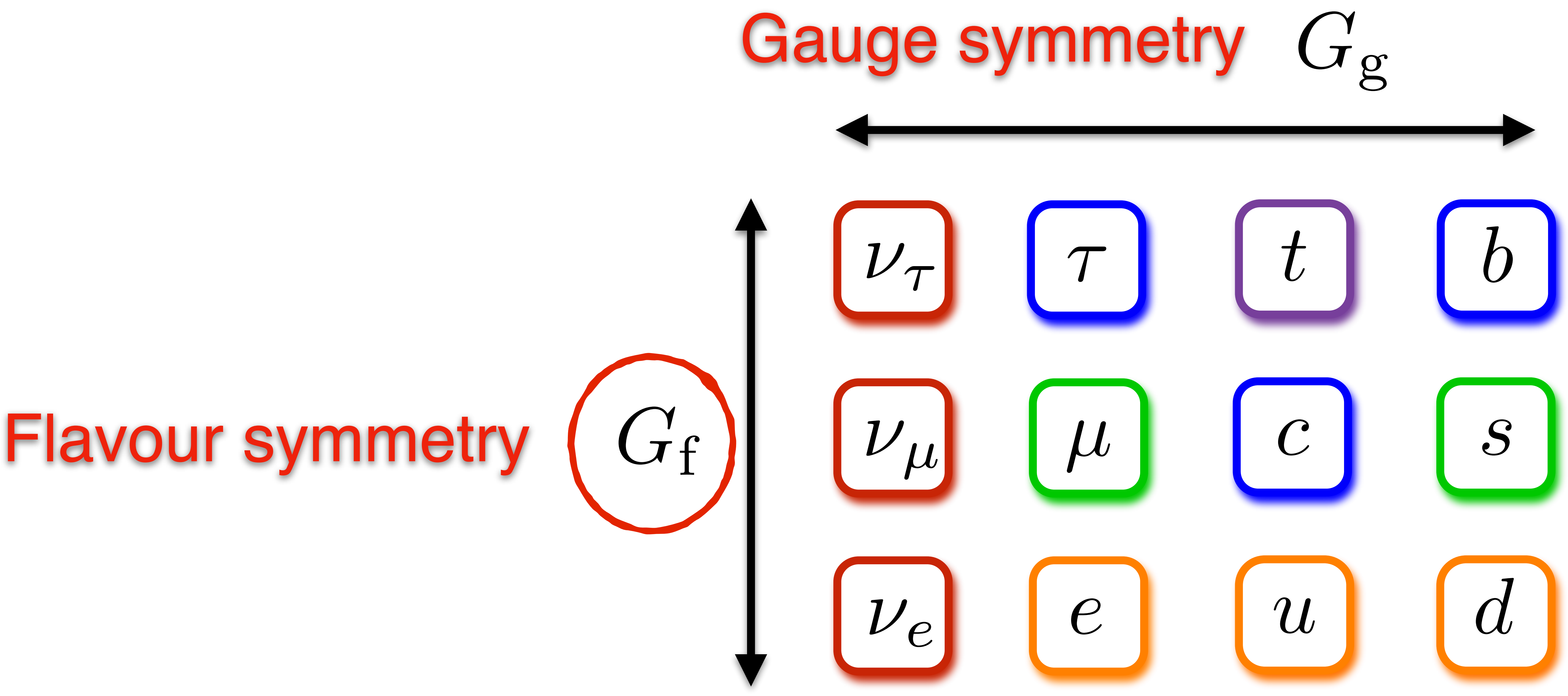
$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

King, Pascoli, Turner, YLZ, 2005.13549

Lepton flavour symmetries

Overview of flavour symmetries

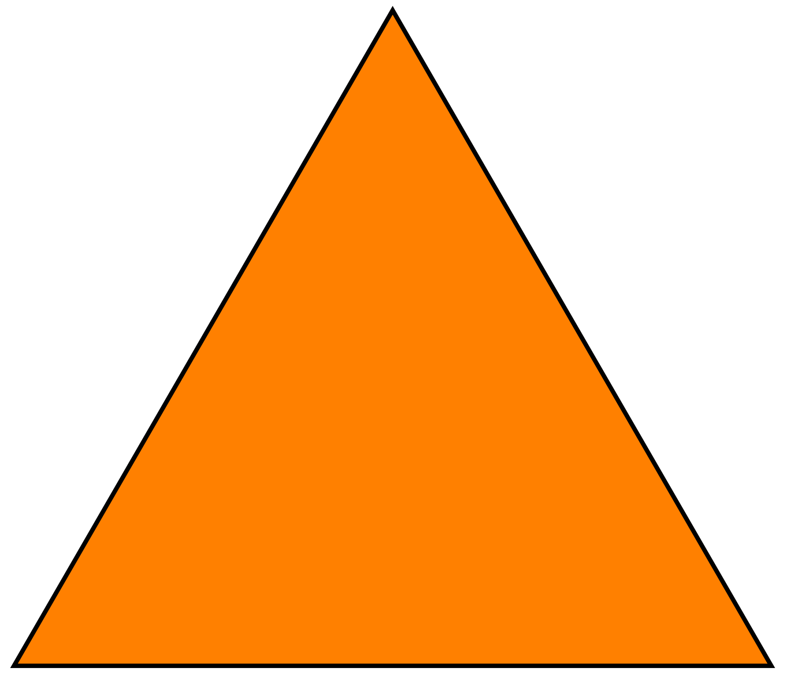


	Continuous	Discrete
Abelian	$U(1)$	Z_n
Non-Abelian	$SU(3), SO(3)\ldots$	$A_4, S_4, T', A_5, \Delta(48), \ldots$

Overview of flavour symmetries

$S_3 \simeq D_3$

Dihedral symmetry
with $n = 3$

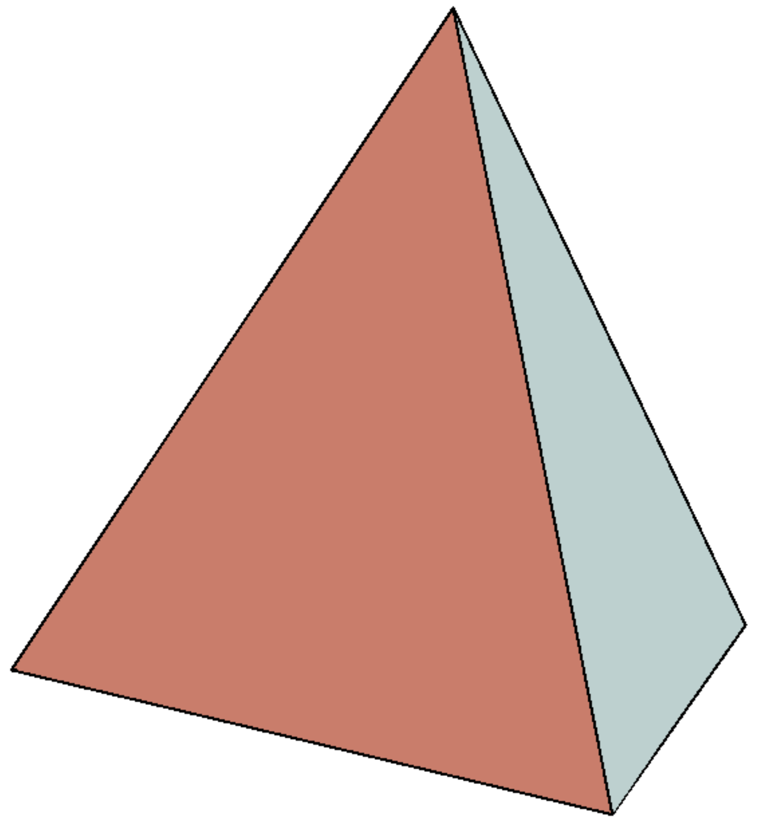


No triplet irrep

$A_4 \simeq T$

Tetrahedral symmetry

alternating subgroup of S_4

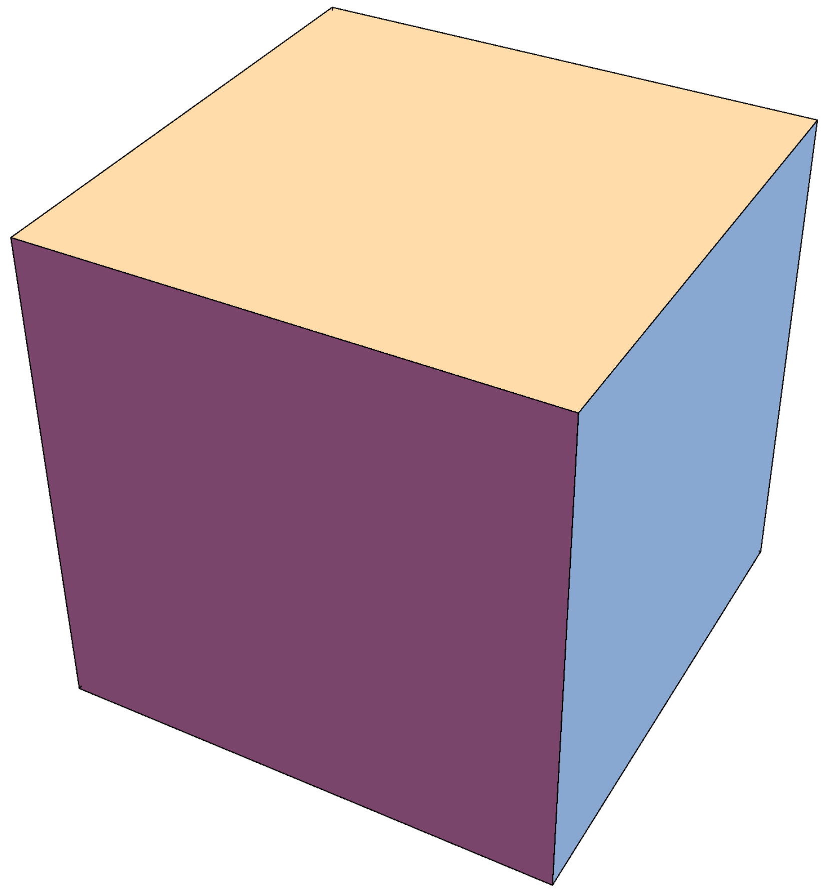


Tri-bimaximal mixing

$S_4 \simeq O$

Octahedral symmetry

symmetric group of 4 object permutation



Tri-bimaximal, Bimaximal,
Democratic mixing

Typical mixing patterns

- Tri-bimaximal (TBM) mixing

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott, 02

Xing, 02

$$3\sin^2\theta_{12} = 1 \qquad 2\sin^2\theta_{23} = 1 \qquad \sin^2\theta_{13} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ignore Majorana phases

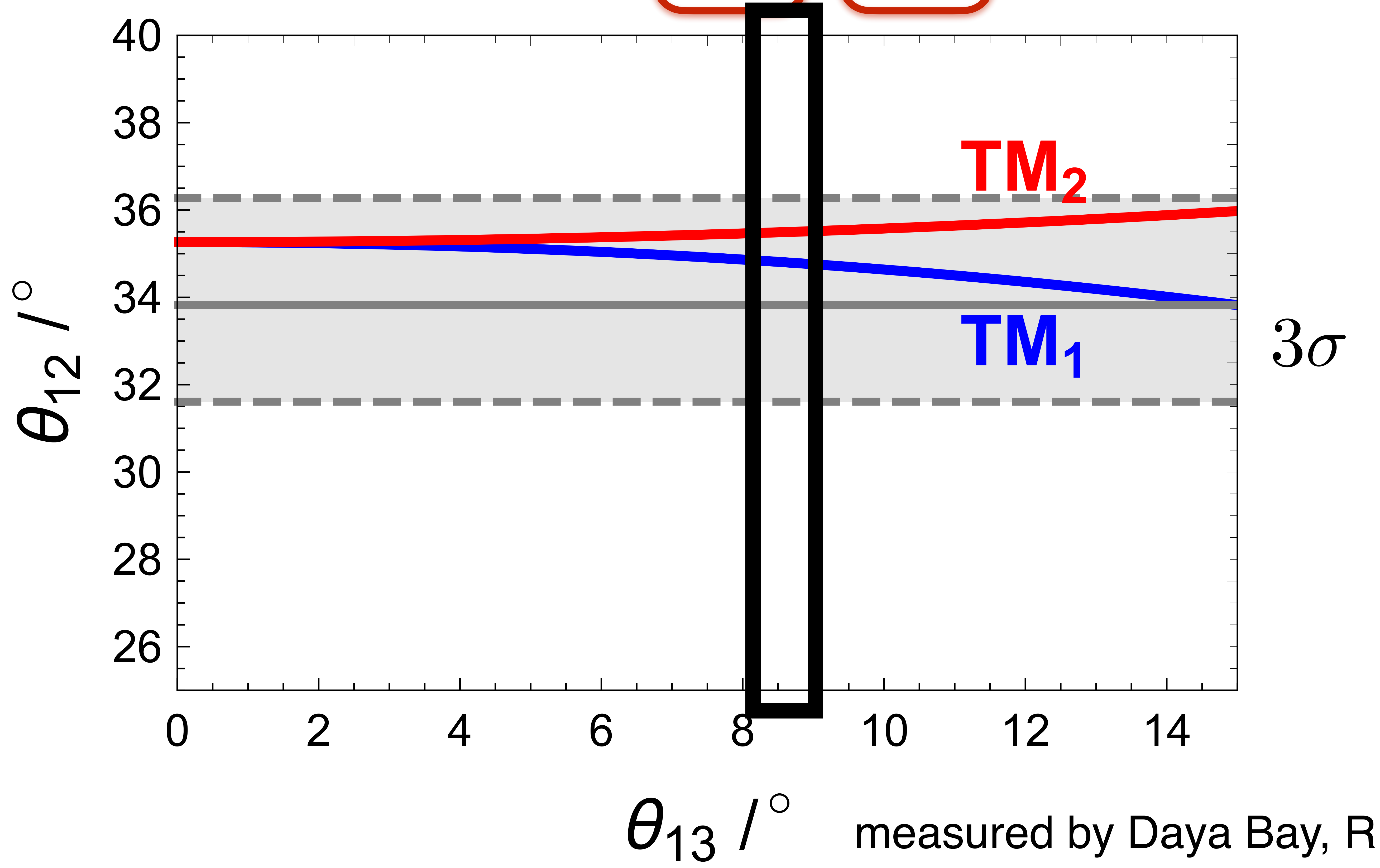
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{bmatrix},$$

From TBM to TM_i mixing

- Trimaximal mixing

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TM_1 / TM_2

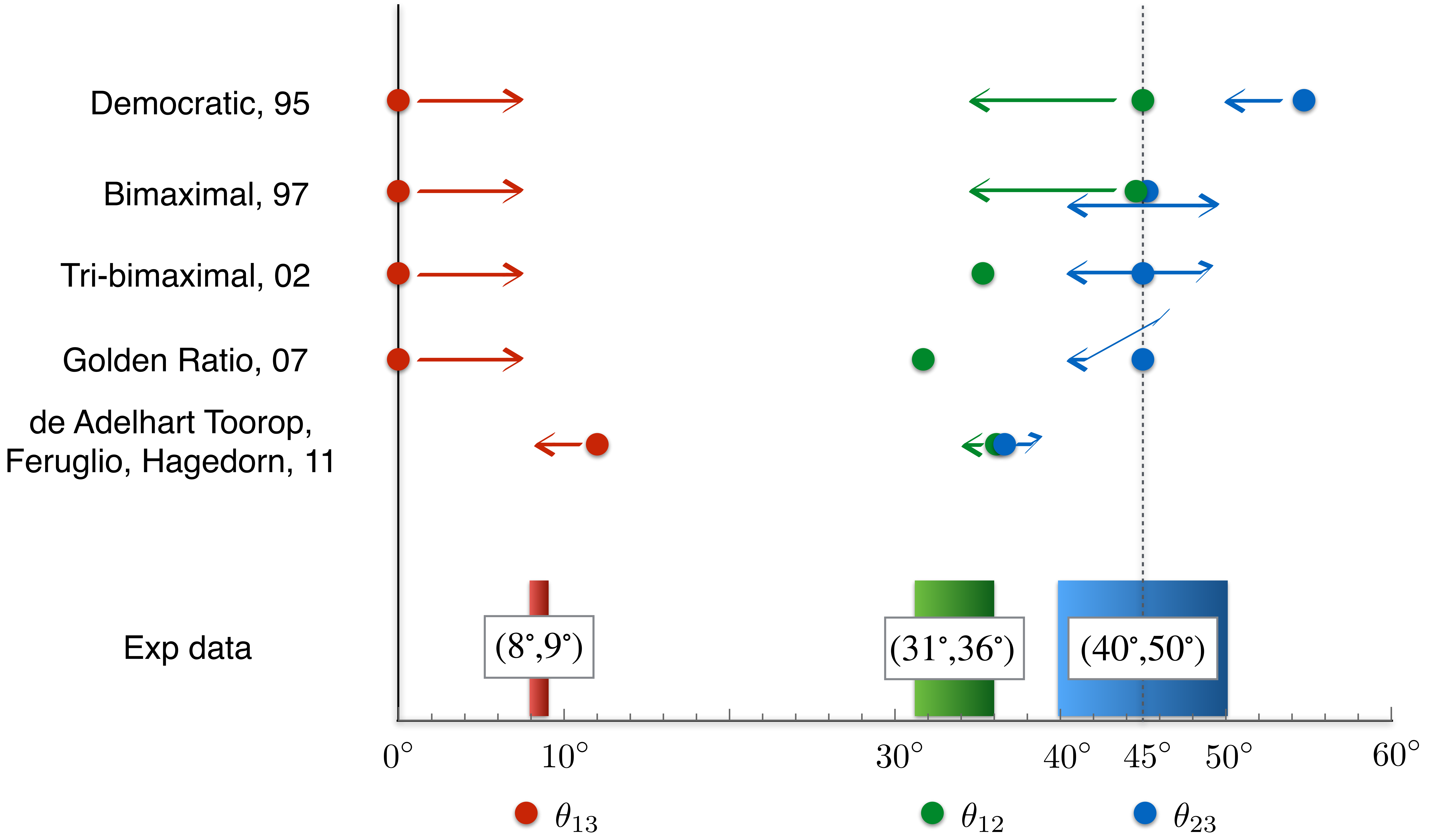


measured by Daya Bay, RENO,…

Bjorken, Harrison, Scott, 0511201; He, Zee, 0607163; Grimus, Lavoura, 0809.0226; 0810.4516

Xing, Zhou, 0607302; Lam, 0611017; Albright, Rodejohann, 0812.0436

Predictions of non-Abelian discrete symmetries

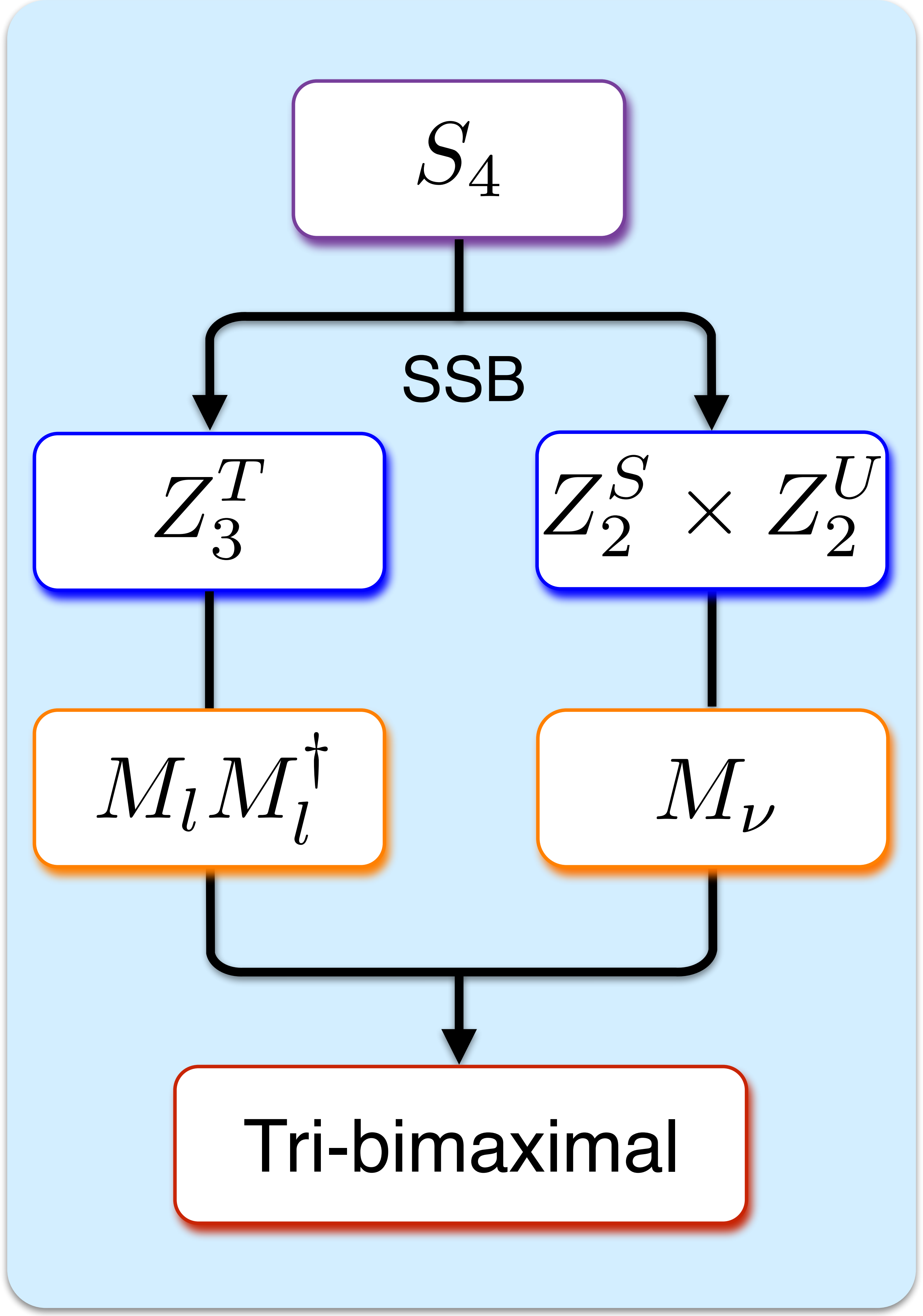


Residual symmetries vs flavour mixing

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Residual symm $\quad Z_3^T = \{\mathbf{1}, T, T^2\}$

$$T M_l M_l^\dagger T^\dagger = M_l M_l^\dagger$$



$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$Z_2^S = \{1, S\} \quad Z_2^U = \{1, U\}$

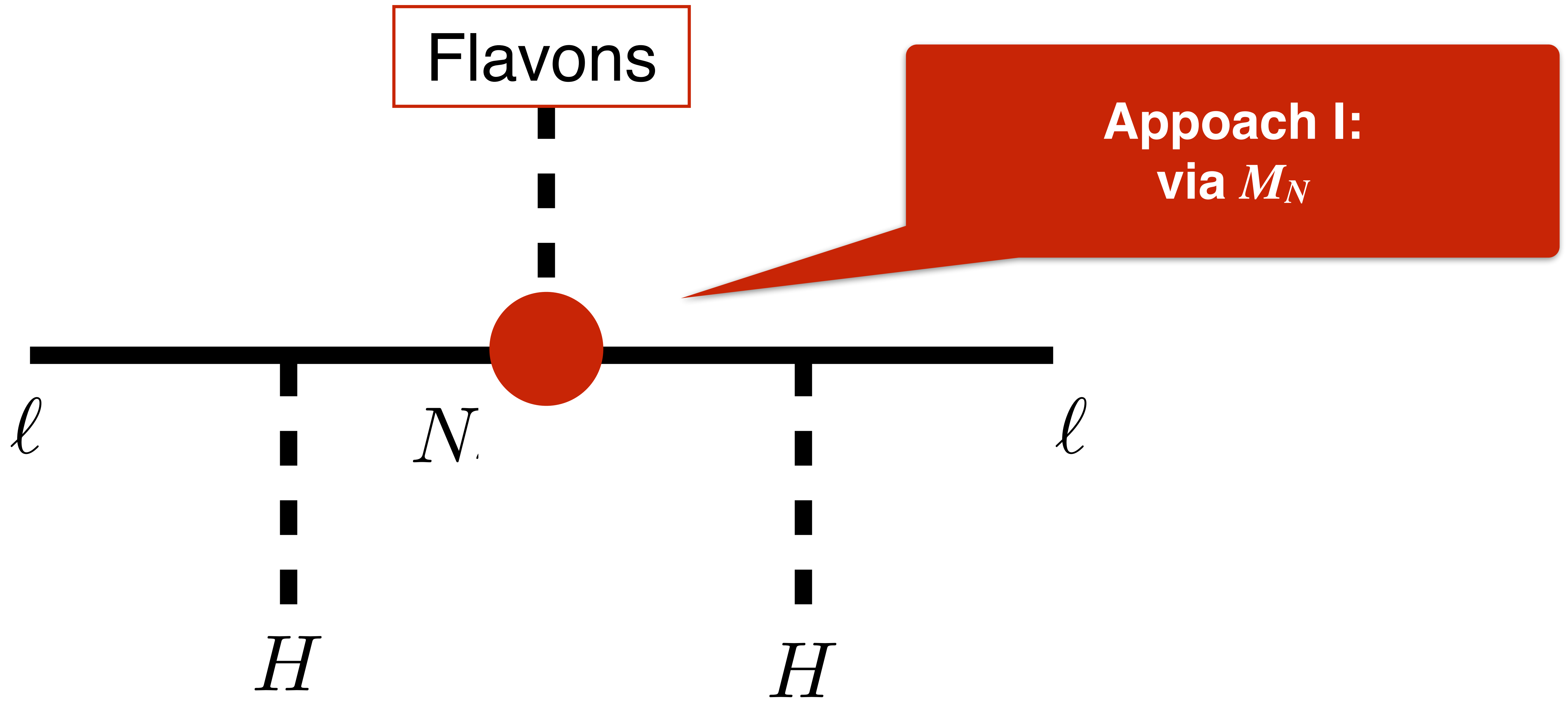
$Z_2^{SU} = \{\mathbf{1}, SU\}$

$$S M_\nu S^T = M_\nu$$
$$U M_\nu U^T = M_\nu$$

$$(Z_3^T, Z_2^{SU}) \to \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \qquad (Z_3^T, Z_2^S) \to \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad (Z_3^T, Z_2^U) \to \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

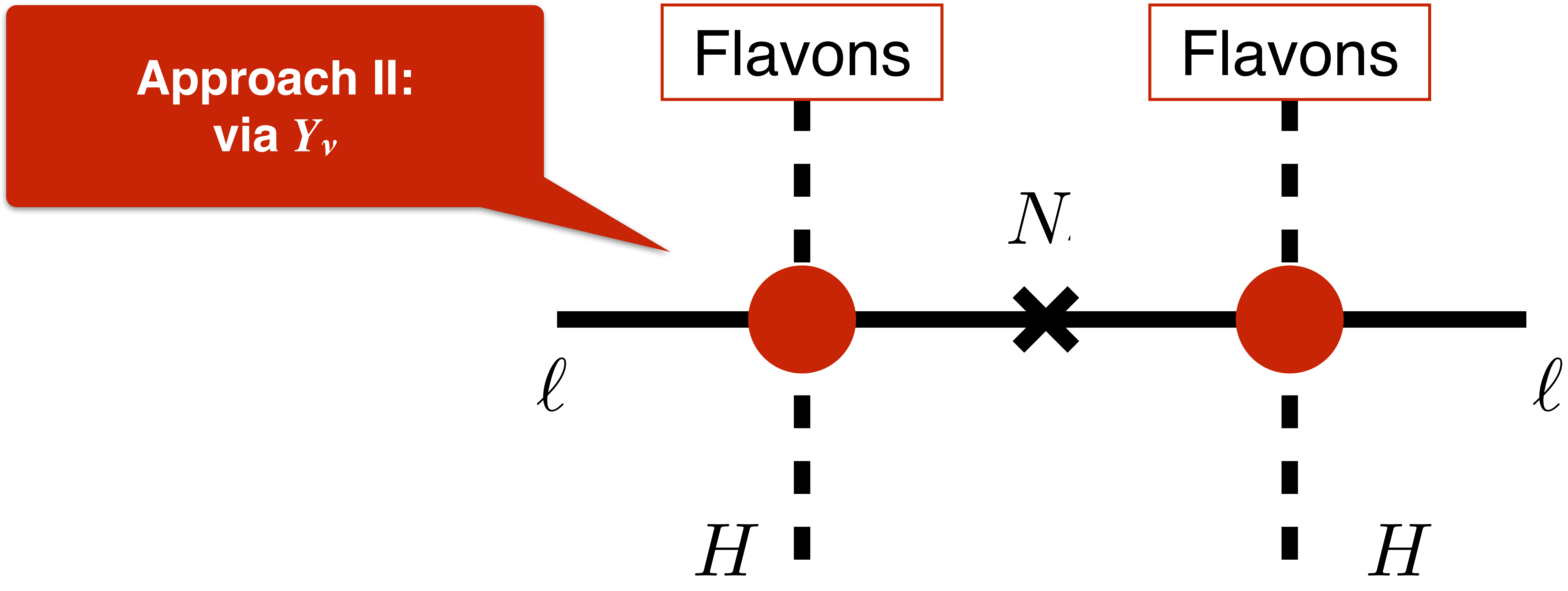
Model building

In the type-I seesaw framework



Review:

Alterelli, Feruglio, 1002.0211;
Ishimori, Kobayashi, et al, 1003.3552;
King, Luhn, 1301.1340;
King, Merle, et al, 1402.4271;
Xing, 1909.09610;
Feruglio, Romanino, 1912.06028,
Ding, King, 2311.09282;
Ding, Valle, 2402.16963



A toy model in A_4

- Field arrangement

Flavons $\quad \varphi = (\varphi_1, \varphi_2, \varphi_3)^T \sim \mathbf{3} \quad \chi = (\chi_1, \chi_2, \chi_3)^T \sim \mathbf{3}\,, \quad \eta \sim \mathbf{1}$

SM fields $\quad \ell_{\rm L} = (\ell_{e\rm L}, \ell_{\mu\rm L}, \ell_{\tau\rm L})^T \sim \mathbf{3}\,,\ e_{\rm R} \sim \mathbf{1}\,,\ \mu_{\rm R} \sim \mathbf{1}''\,,\ \tau_{\rm R} \sim \mathbf{1}'\,,\ H \sim \mathbf{1}$

$$N = (N_1, N_2, N_3)^T \sim \mathbf{3}$$

- Lagrangian terms

$$-\mathcal{L}_l = \frac{y_e}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}} e_R H + \frac{y_\mu}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}''} \mu_R H + \frac{y_\tau}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}'} \tau_R H + \text{h.c.} + \cdots \Rightarrow M_l$$

$$-\mathcal{L}_\nu = y_D(\overline{\ell_L}N)_{\mathbf{1}}\tilde{H} + \frac{y_1}{2}\left((\overline{N^c}N)_{\mathbf{3}_S}\chi\right)_{\mathbf{1}} + \frac{y_2}{2}(\overline{N^c}N)_{\mathbf{1}}\eta + \text{h.c.} + \cdots \Rightarrow M_\nu$$

- Vacuum alignment and achievement of flavour textures

$$\langle\varphi\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} v_\varphi \qquad \langle\chi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix} \frac{v_\chi}{\sqrt{3}} \qquad \langle\eta\rangle = v_\eta$$

$\Rightarrow$

$$M_l = \text{Diag}\{y_e, y_\mu, y_\tau\} \times y_\varphi \frac{v}{\sqrt{2}}$$

$$M_D = \mathbf{1} \times y_D \frac{v}{\sqrt{2}}$$

$$M_\nu = \begin{pmatrix} a+2b & -b & -b \\ -b & 2b & a-b \\ -b & a-b & 2b \end{pmatrix}$$

$$a = y_2 v_\eta\,, \quad b = \frac{1}{2\sqrt{3}} y_1 v_\chi$$

$\Rightarrow$ **TBM mixing**

Origin of discrete flavour symmetries

- SSB of a continues (gauge) symmetry

$SO(3) \to A_4, S_4, A_5$ Ovrut, 77; Etesi, 9706029; Berger, Grossman, 0910.4392; King, YLZ, 1809.10292

$SO(3)$	A_4	S_4	A_5
$\underline{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\underline{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$\underline{5}$	$\mathbf{1}' + \mathbf{1}'' + \mathbf{3}$	$\mathbf{2} + \mathbf{3}'$	$\mathbf{5}$
$\underline{7}$	$\mathbf{1} + \mathbf{3} + \mathbf{3}$	$\mathbf{1}' + \mathbf{3} + \mathbf{3}'$	$\mathbf{3}' + \mathbf{4}$
$\underline{9}$	$\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}$	$\mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3}'$	$\mathbf{4} + \mathbf{5}$
$\underline{11}$	$\mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3} + \mathbf{3}$	$\mathbf{2} + \mathbf{3} + \mathbf{3} + \mathbf{3}'$	$\mathbf{3} + \mathbf{3}' + \mathbf{5}$
$\underline{13}$	$\mathbf{1} + \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3} + \mathbf{3}$	$\mathbf{1} + \mathbf{1}' + \mathbf{2} + \mathbf{3} + \mathbf{3}' + \mathbf{3}'$	$\mathbf{1} + \mathbf{3} + \mathbf{4} + \mathbf{5}$

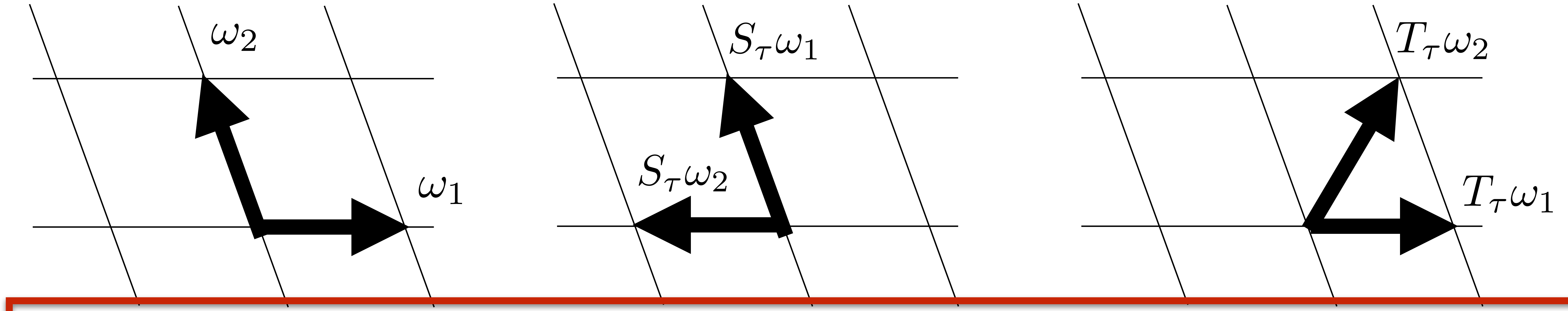
$SU(3) \to A_4$ e.g., Luhn, 1101.2417; Merle, Zwicky, 1110.4891

- Refection of discrete properties of spacetime with compact extra dimensions $\cdots$

Origin of discrete flavour symmetries

- Flavour symmetries from modular symmetries (模对称性) Ferrara, Lust, Theisen, 89

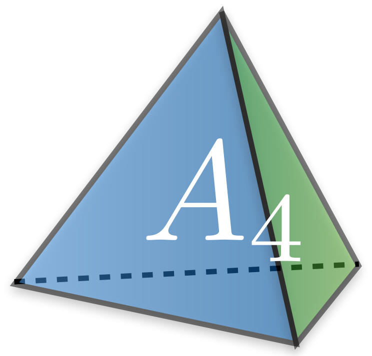
generated via two independent lattice vectors

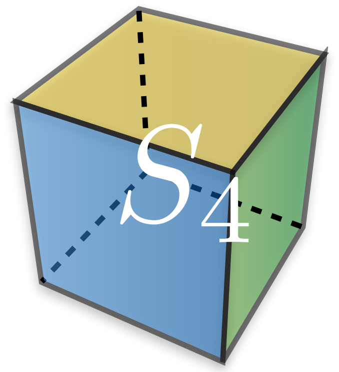


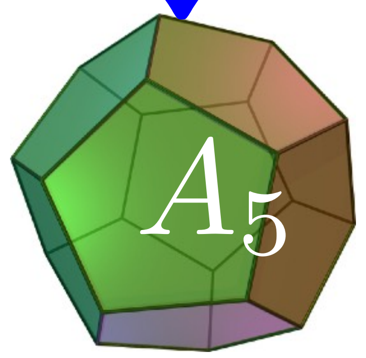
$\tau = \omega_2/\omega_1$ $\quad S_\tau : -1/\tau$ $\quad T_\tau : \tau \to \tau + 1$

- Finite modular symmetries (有限模群) $S_\tau^2 = (S_\tau T_\tau)^3 = \mathbf{1}$

$T_\tau^3 = 1$ $\quad T_\tau^4 = 1$ $\quad T_\tau^5 = 1$

$\Gamma_3 \simeq$ 

$\Gamma_4 \simeq$ 

$\Gamma_5 \simeq$ 

de Adelhart Toorop, Feruglio and Hagedorn, 1112.1340

Modular symmetries as direct origin of flavour mixing

- Transformation in flavour symmetry

$$\psi \to \rho_I(\gamma)\psi \qquad\qquad Y(\varphi_1, \varphi_2, ...) \to \rho_{I_Y}(\gamma) Y(\varphi_1, \varphi_2, ...)$$

- Transformation in modular symmetry

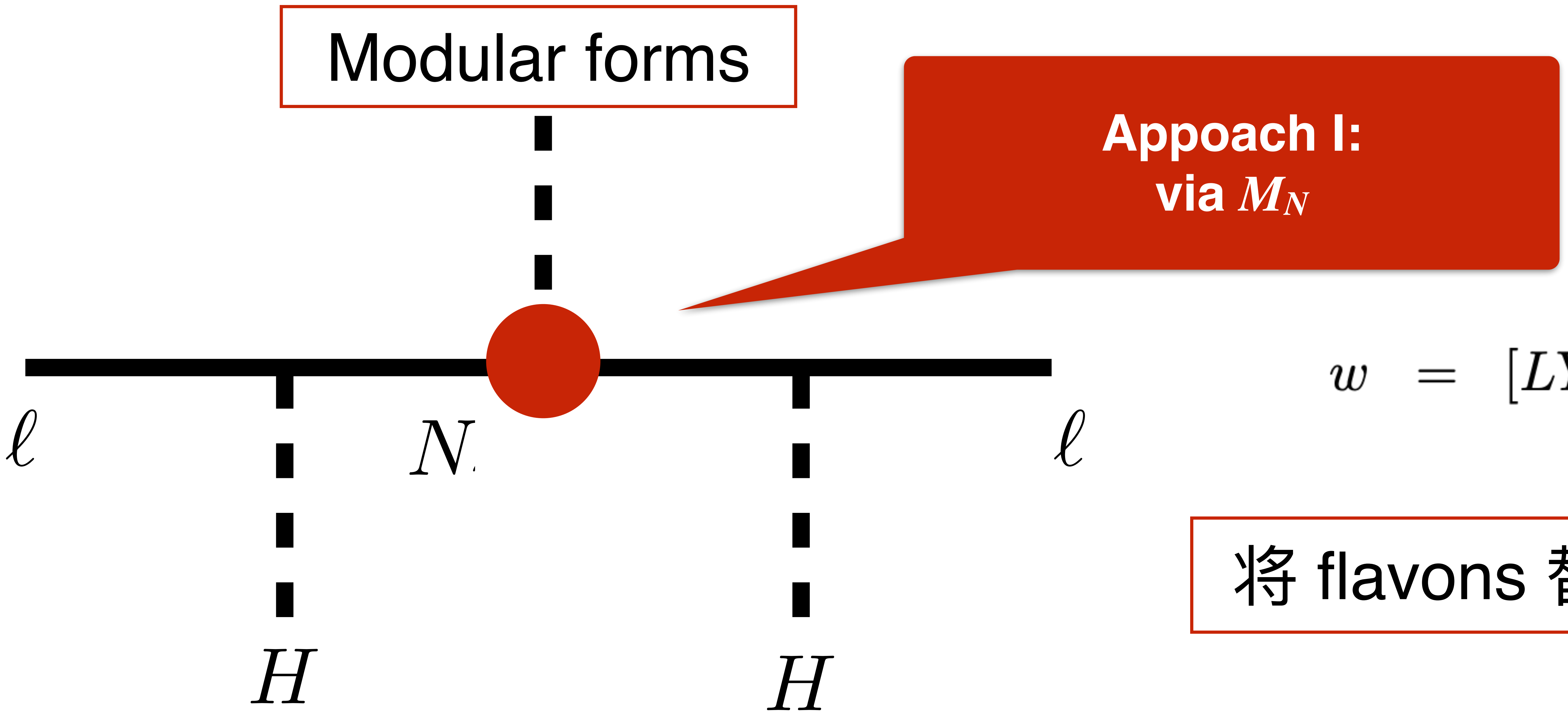
$$\gamma : \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$ in modular space τ with Im(τ) > 0

$$\psi \to (c\tau + d)^{2k} \rho_I(\gamma)\psi \qquad\qquad Y(\tau) \to (c\tau + d)^{2k_Y} \rho_{I_Y}(\gamma) Y(\tau)$$

- Once fermions have non-trivial modular weight (模权重), modular symmetry can act on the flavour space directly and be used to explain flavour mixing   Feruglio, 1706.08749

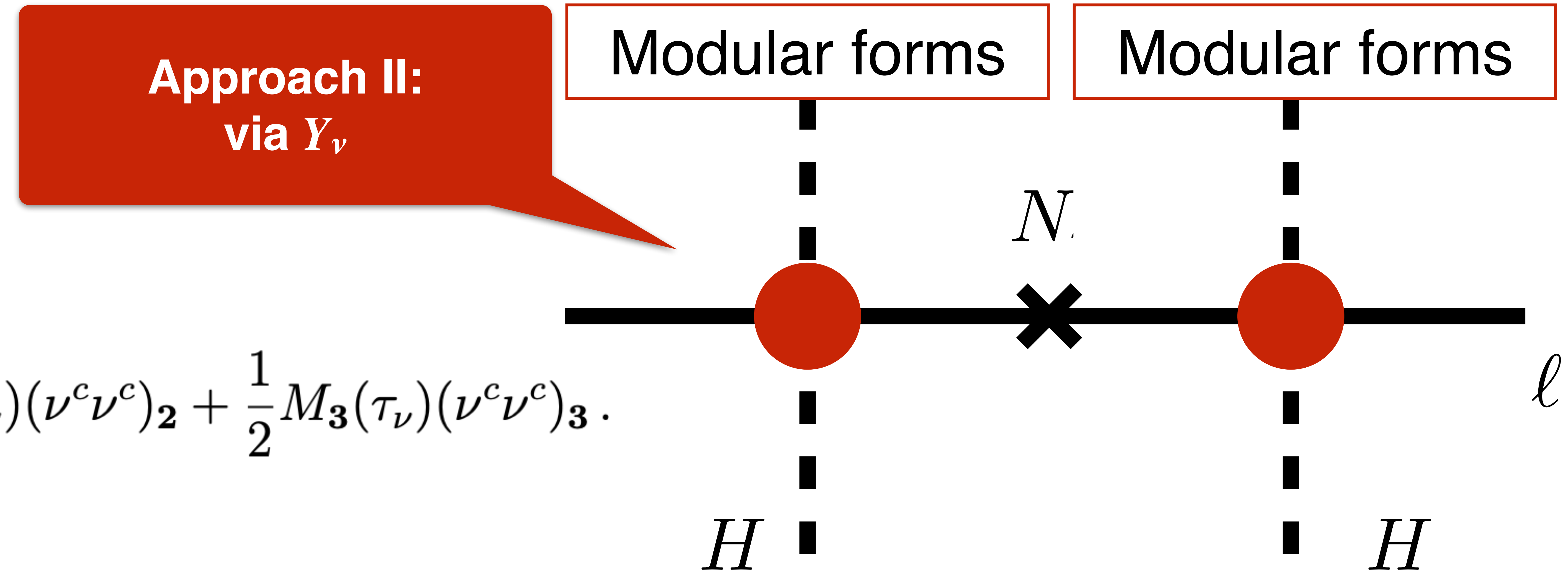
- From single modular symmetry to multiple modular symmetries

Flavour models in modular symmetries



$$w = [LY_e(\tau_l)e^c + LY_\mu(\tau_l)\mu^c + LY_\tau(\tau_l)\tau^c]\, H_d$$

将 flavons 替换为 Modular form （模形式）



$$\frac{y_\nu}{\Lambda} L\Phi\nu^c H_u + \frac{1}{2}M_{\mathbf{1}}(\tau_\nu)(\nu^c\nu^c)_{\mathbf{1}} + \frac{1}{2}M_{\mathbf{2}}(\tau_\nu)(\nu^c\nu^c)_{\mathbf{2}} + \frac{1}{2}M_{\mathbf{3}}(\tau_\nu)(\nu^c\nu^c)_{\mathbf{3}}\,.$$

Summary and conclusion

- **Neutrino in the Standard Model** massless
- **Neutrino masses and mixing** proved by neutrino oscillation experiments

  and consequent effective description and parametrization
- **Origin of neutrino masses**

  seesaw, variations, and other mechanisms, and symmetries behind
- **Lepton flavour symmetries** potential original of lepton flavour mixing

  mixing patterns, A_4, S_4, modular symmetries

Thanks for your interests