

**UNIVERSITY
OF WARWICK**



Accelerator and Atmospheric Neutrinos Part 1: The Interesting Part

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University of Warwick

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2025 August 22, Hangzhou



國科大杭州高等研究院

Hangzhou Institute for Advanced Study, UCAS

Outline

1. Neutrino Oscillations
 - a. Visualisation
2. Accelerator Neutrinos
 - a. Techniques
 - b. Key results
 - c. Future vision
3. Atmospheric Neutrinos
 - a. MSW matter effect
 - b. Flux
 - c. Key results

There are **Quizzes** and **Homework!**

Great lectures on neutrinos if you want to dive into the subject

❑ Steve Boyd, University of Warwick, *Neutrino Physics Lectures*, [Warwick Week](#), 2025

Great text on nuclear, particle, and neutrino physics

1. Mohapatra, R. N., Pal, P. B. (2004). *Massive Neutrinos in Physics and Astrophysics*. India: World Scientific.
2. Giunti, C., Kim, C. W. (2007). *Fundamentals of Neutrino Physics and Astrophysics*. United Kingdom: OUP Oxford.
3. Povh, B., Rith, K., Scholz, C., Zetsche, F., Rodejohann, W. (2015). *Particles and Nuclei: An Introduction to the Physical Concepts*. Germany: Springer Berlin Heidelberg. **7th Ed.**
4. Donnelly, T. W., Formaggio, J. A., Holstein, B. R., Milner, R. G., Surrow, B. (2017). *Foundations of Nuclear and Particle Physics*. United Kingdom: Cambridge University Press.
5. Zuber, K. (2020). *Neutrino Physics*. United Kingdom: CRC Press. **3rd Ed.**
6. Rubbia, A. (2022). *Phenomenology of Particle Physics*. United Kingdom: Cambridge University Press.

Part 1: Neutrino Oscillations

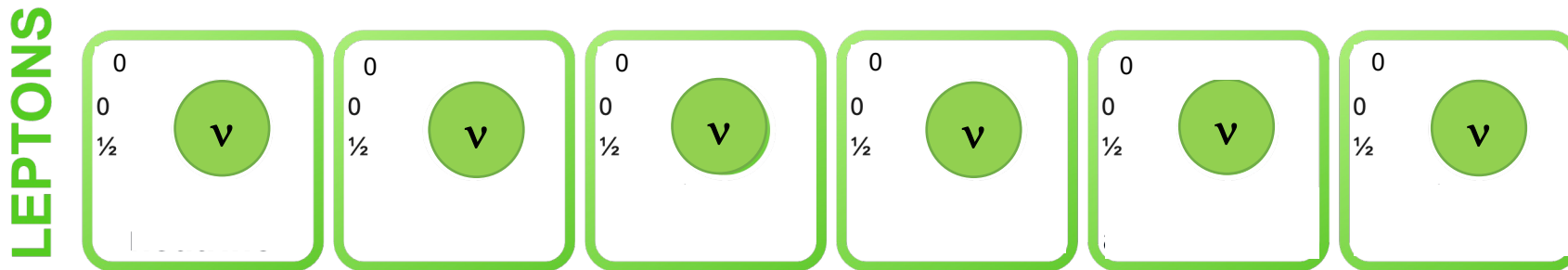
Fundamental matter in our current world view: Standard Model

	I	II	III	I	II	III
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
QUARKS	u up	c charm	t top	$\bar{u}$ antiup	$\bar{c}$ anticharm	$\bar{t}$ antitop
	$\approx 4.7 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ d down	$\approx 96 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ s strange	$\approx 4.18 \text{ GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ b bottom	$\approx 4.7 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{d}$ antidown	$\approx 96 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{s}$ antistrange	$\approx 4.18 \text{ GeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{b}$ antibottom
	$\approx 0.511 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ e electron	$\approx 105.66 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ μ muon	$\approx 1.7768 \text{ GeV}/c^2$ -1 $\frac{1}{2}$ τ tau	$\approx 0.511 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ e^+ positron	$\approx 105.66 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ $\bar{\mu}$ antimuon	$\approx 1.7768 \text{ GeV}/c^2$ 1 $\frac{1}{2}$ $\bar{\tau}$ antitau
LEPTONS	0 0 $\frac{1}{2}$ ν	0 0 $\frac{1}{2}$ ν	0 0 $\frac{1}{2}$ ν	0 0 $\frac{1}{2}$ ν	0 0 $\frac{1}{2}$ ν	0 0 $\frac{1}{2}$ ν

3 generations?!
Symmetric antimatter?

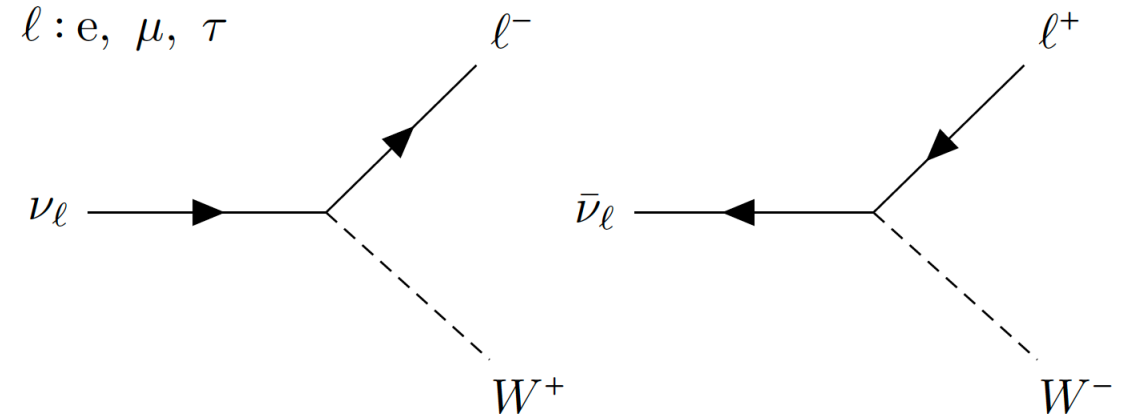
Neutrinos and antineutrinos in Standard Model

1. Charge neutral
 2. Massless
- ❖ How can we tell all 6 of them apart?



Neutrinos and antineutrinos in Standard Model

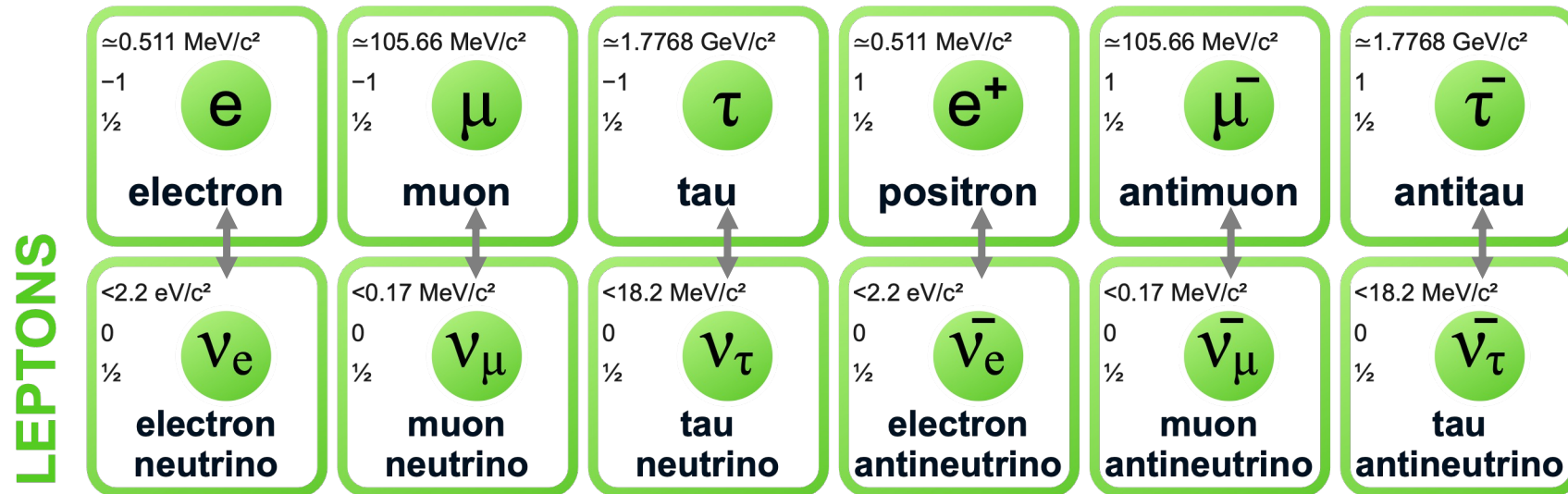
1. Charge neutral
2. Massless
- ❖ How can we tell all 6 of them apart?
3. Flavour, defined by charged-current interactions



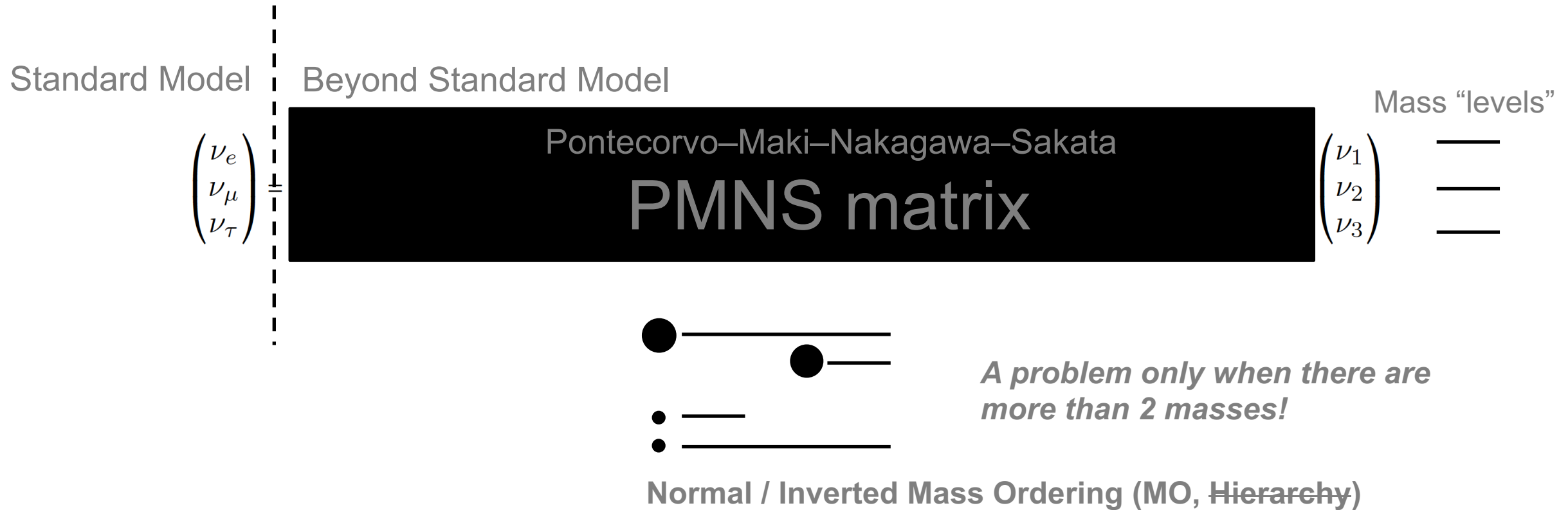
LEPTONS	$\approx 0.511 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ electron	$\approx 105.66 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ muon	$\approx 1.7768 \text{ GeV}/c^2$ -1 $\frac{1}{2}$ tau	$\approx 0.511 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ positron	$\approx 105.66 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ antimuon	$\approx 1.7768 \text{ GeV}/c^2$ 1 $\frac{1}{2}$ antitau
	0 0 $\frac{1}{2}$ electron neutrino	0 0 $\frac{1}{2}$ muon neutrino	0 0 $\frac{1}{2}$ tau neutrino	0 0 $\frac{1}{2}$ electron antineutrino	0 0 $\frac{1}{2}$ muon antineutrino	0 0 $\frac{1}{2}$ tau antineutrino

Neutrinos and antineutrinos in Nature

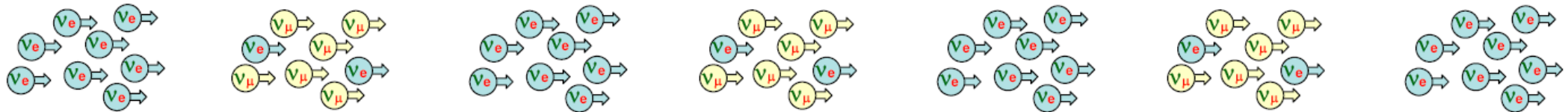
1. Charge neutral
2. ~~Massless~~ turns out to be tiny but not zero!
 - ❖ How can we tell all 6 of them apart? *One more way (too many) to tell the neutrinos apart!*
3. Flavour, defined by charged-current interactions



Neutrino Mass and Mixing



Mass gap (Δm^2) **and** mixing lead to neutrino flavour oscillations



[Mark Thomson, Particle Physics lecture notes](#)

Analogy: spin rotation in a magnetic field

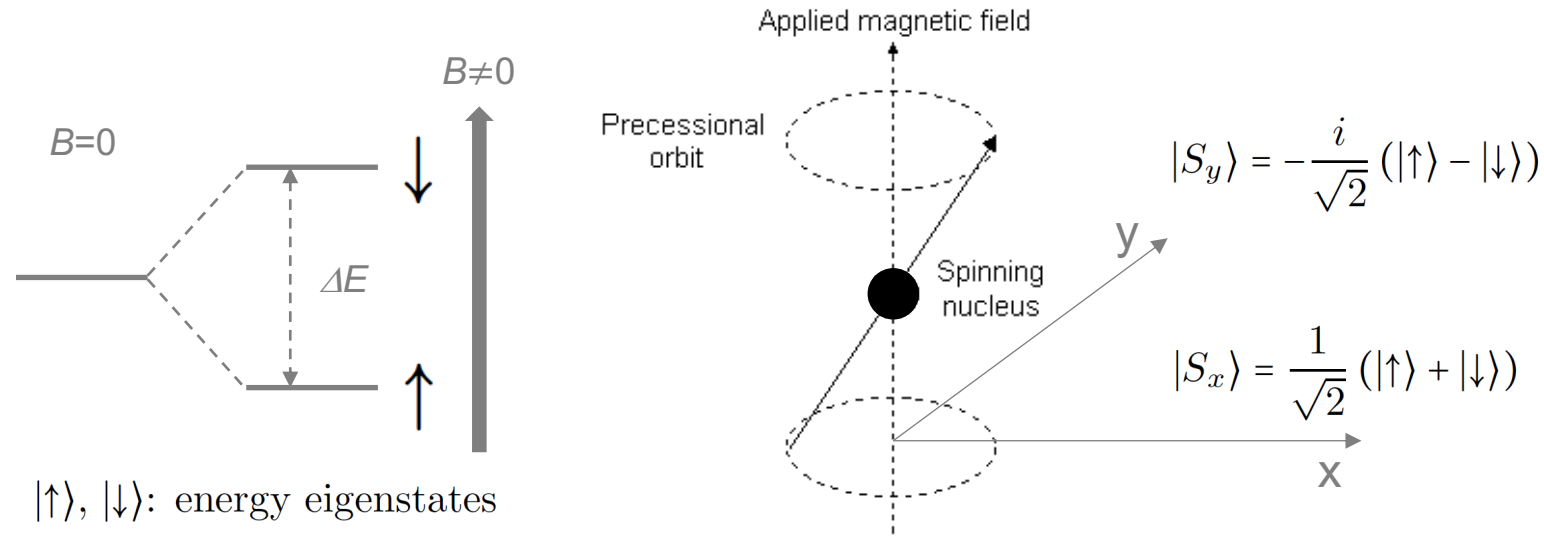


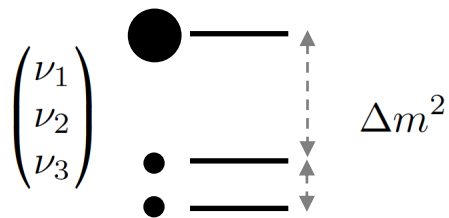
Image source: <https://teaching.shu.ac.uk/hwb/chemistry/tutorials/molspec/nmr1.htm>

Energy gap

$$\begin{pmatrix} |\uparrow\rangle_t \\ |\downarrow\rangle_t \end{pmatrix} = \begin{pmatrix} e^{i\frac{\Delta E}{2}t} & 0 \\ 0 & e^{-i\frac{\Delta E}{2}t} \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_0 \\ |\downarrow\rangle_0 \end{pmatrix}$$

Mixing

$$\begin{pmatrix} |S_x\rangle \\ |S_y\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$$



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \text{PMNS} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Homework: Show that

$$|S_x\rangle_t = \cos \frac{\Delta E}{2} t |S_x\rangle_0 - \sin \frac{\Delta E}{2} t |S_y\rangle_0$$

Oscillation probability

$$P_{S_x \rightarrow S_x}(t) = \cos^2 \frac{\Delta E}{2} t$$

$$P_{S_x \rightarrow S_y}(t) = \sin^2 \frac{\Delta E}{2} t$$

Survival

$$P_{\nu_\mu \rightarrow \nu_\mu} \left(\frac{L}{E} \right)$$

Appearance

$$P_{\nu_\mu \rightarrow \nu_e} \left(\frac{L}{E} \right)$$

Oscillation Phase: $[E^0]$

☐ Δm^2 : $[E^2]$

☐ L : propagation distance $[E^{-1}]$

☐ E : energy $[E]$

Phase $\sim \Delta m^2 L / E$

The phase that is responsible for oscillation is often written as (with c and $\hbar$ restored)

$$\frac{\Delta_{jk}(mc^2)^2 L}{4\hbar c E} = \frac{\text{GeV fm}}{4\hbar c} \times \frac{\Delta_{jk} m^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E} \approx 1.27 \times \frac{\Delta_{jk} m^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E}$$

Wiki: https://en.wikipedia.org/wiki/Neutrino_oscillation

PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta_{CP}} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{13} \neq 0 \rightarrow \delta_{CP}$ can be observed

Open questions

☐ What are the neutrino masses?

❖ Mass gaps (Δm_{21}^2 , $|\Delta m_{32}^2|$) and ordering ($\text{sgn } \Delta m_{32}^2$)?

☐ What are the mixing parameters?

❖ Mixing angles (θ_{12} , θ_{23} , θ_{13}) and CP-phase (δ_{CP})?

🙅 Not accessible through oscillation measurements:

☐ Absolute mass

☐ Dirac/Majorana nature

θ_{12} : mixing between ν_1 and ν_2

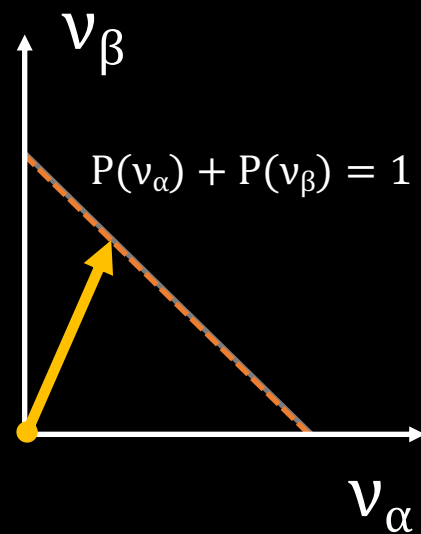
θ_{23} : mixing between ν_μ and ν_τ

θ_{13} : if 0, effective 2 flavour mixing

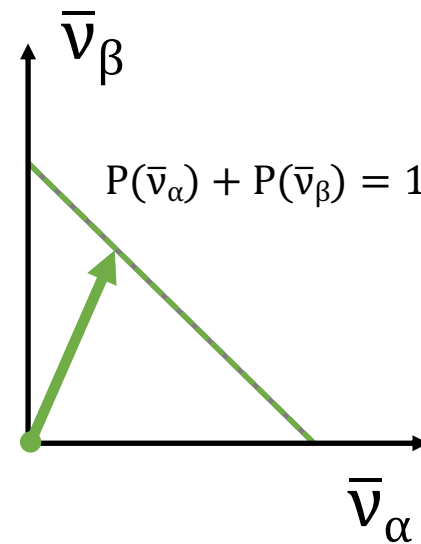
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu_3 \end{pmatrix}$$

$$\theta_{13} = 0$$

2-flavor oscillation



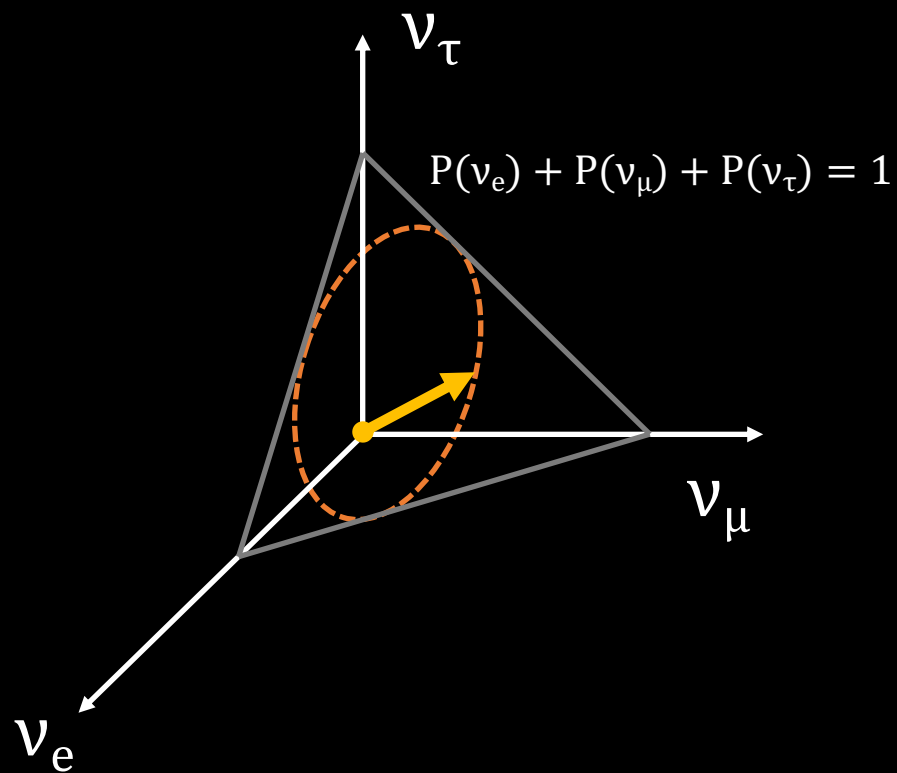
Antineutrinos



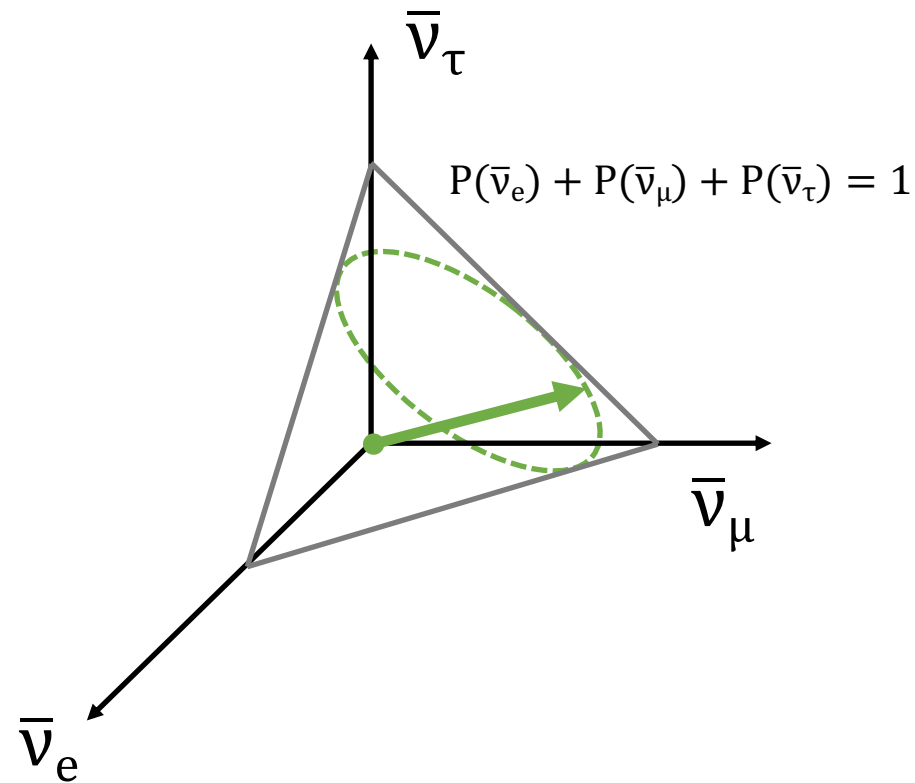
Oscillation as a function of *time*
line-in-**line** → same trivia

$$\theta_{13} \neq 0$$

3-flavor oscillation



Antineutrinos



Oscillation as a function of *time*
line-in-plane $\rightarrow$ CP-violation possible

PMNS

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Trimaximal mixing
— maximally CP-violating

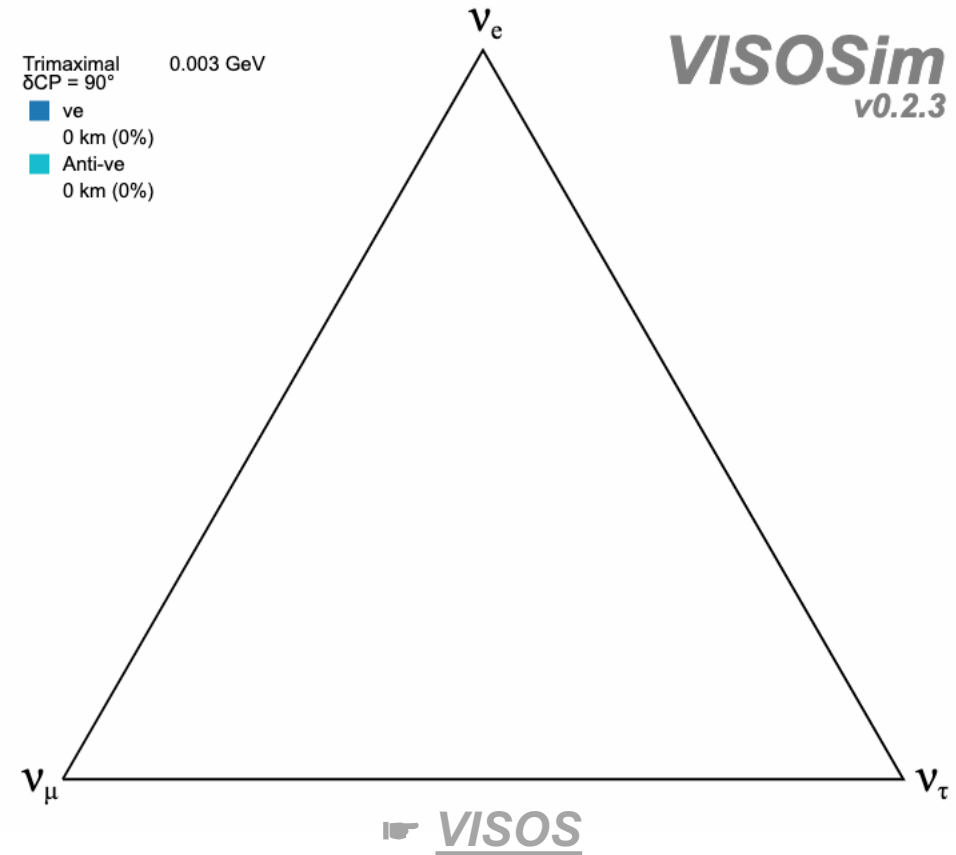
$$(|U_{i\alpha}|^2) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

PMNS

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Trimaximal mixing
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$$(|U_{i\alpha}|^2) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

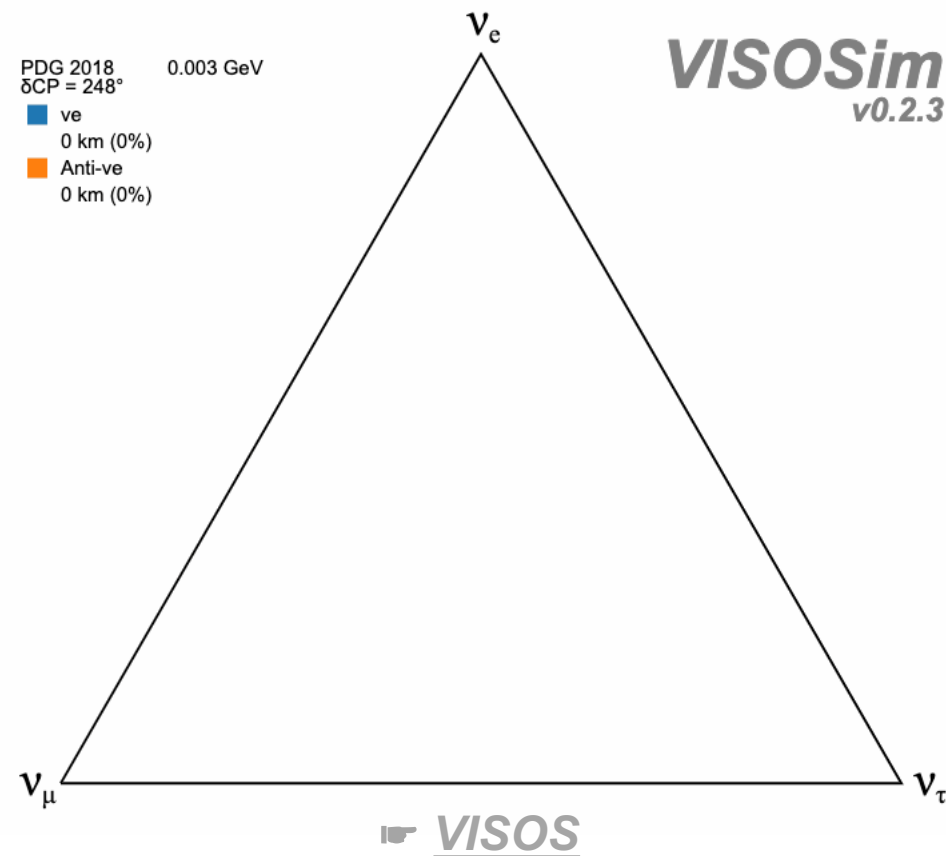


Beautiful but *not* how Nature works ͇_(ツ)_͇

How Nature might work:

$$\begin{bmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{bmatrix}$$
$$= \begin{bmatrix} 0.801 \dots 0.845 & 0.513 \dots 0.579 & 0.143 \dots 0.156 \\ 0.233 \dots 0.507 & 0.461 \dots 0.694 & 0.631 \dots 0.778 \\ 0.261 \dots 0.526 & 0.471 \dots 0.701 & 0.611 \dots 0.761 \end{bmatrix}$$

Quiz: Which experiment has a 50 km baseline observing few-MeV neutrinos?

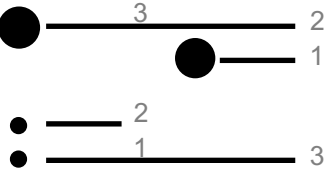


How Nature might work:

$$\begin{bmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{bmatrix}$$
$$= \begin{bmatrix} 0.801 \dots 0.845 & 0.513 \dots 0.579 & 0.143 \dots 0.156 \\ 0.233 \dots 0.507 & 0.461 \dots 0.694 & 0.631 \dots 0.778 \\ 0.261 \dots 0.526 & 0.471 \dots 0.701 & 0.611 \dots 0.761 \end{bmatrix}$$

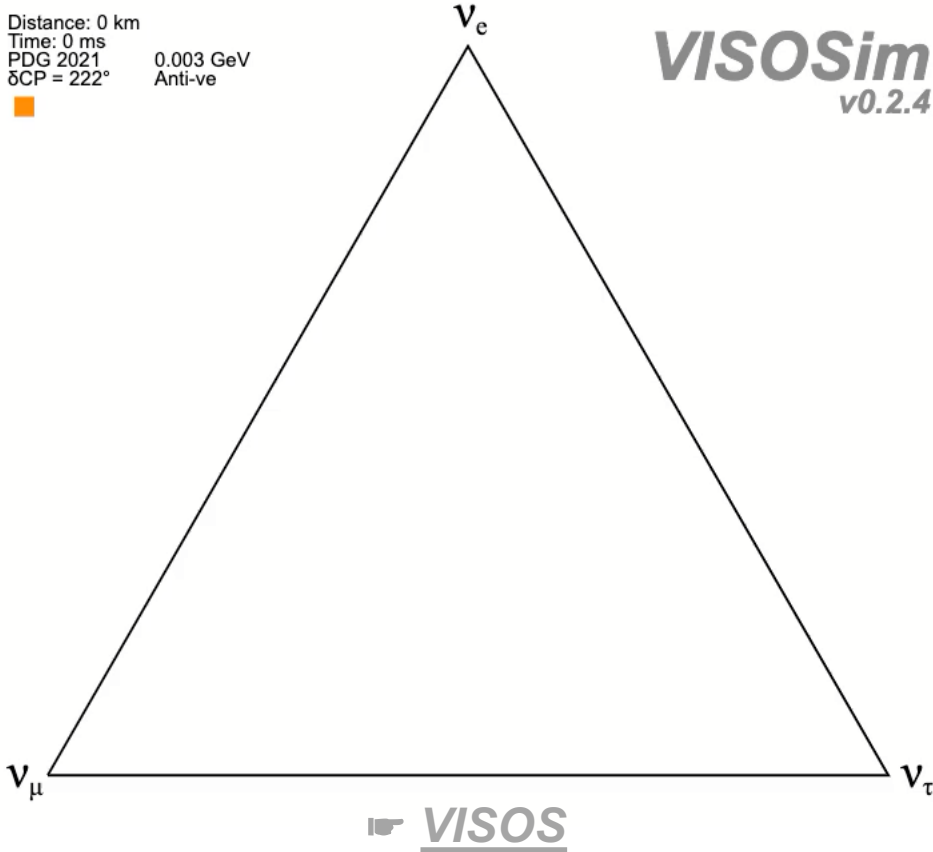
Quiz: Which experiment has a 50 km baseline observing few-MeV neutrinos?

Quiz: Is the oscillation driven by Δm^2_{21} fast or slow? Estimate $|\Delta m^2_{32}|/\Delta m^2_{21}$.



Normal / Inverted MO

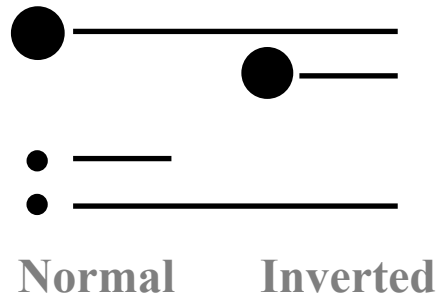
Homework: Estimate Δm^2_{21} and compare your result with PDG.



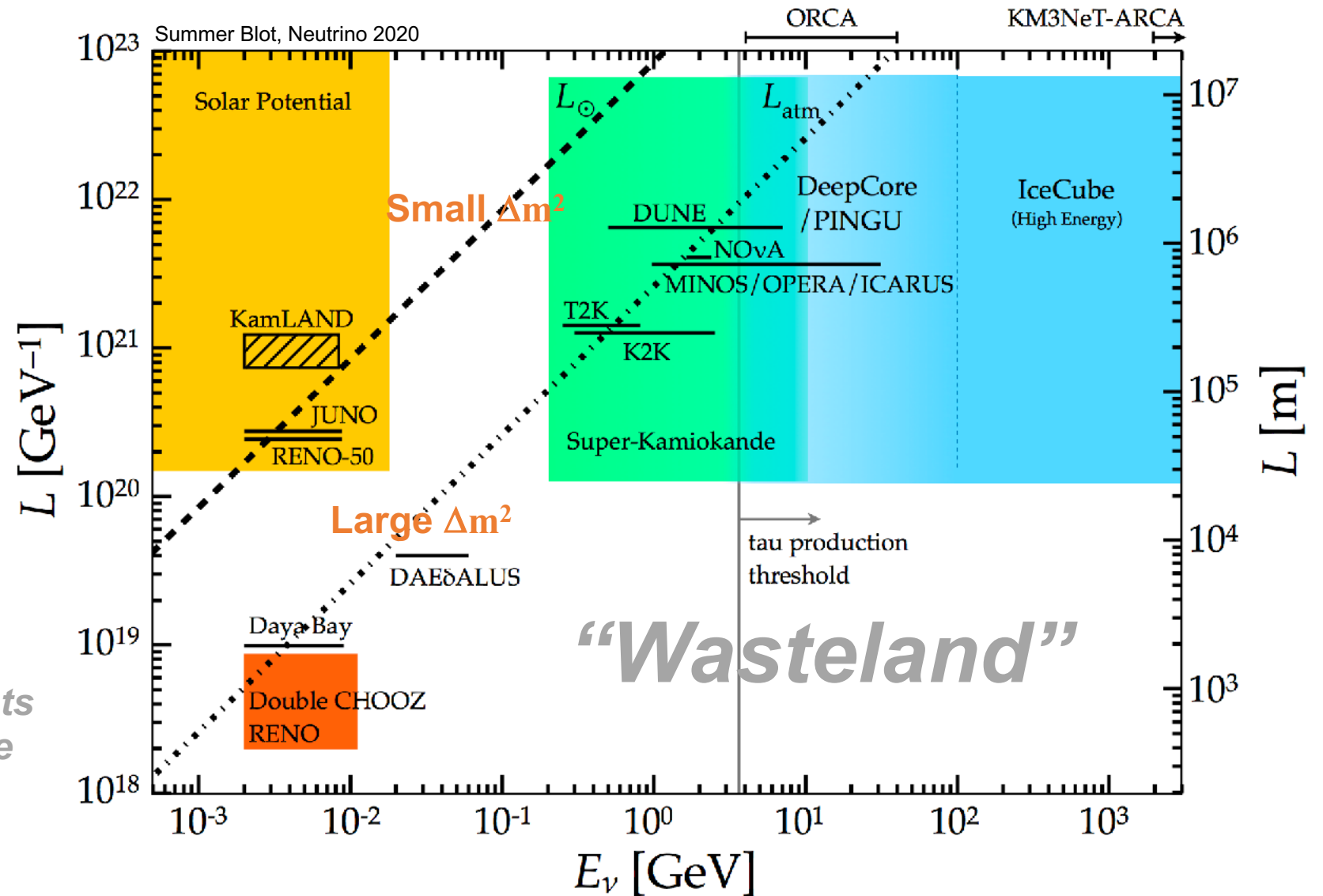
...will come back to IceCube later

Oscillation Phase $\sim \Delta m^2 L/E$

Only two Δm^2



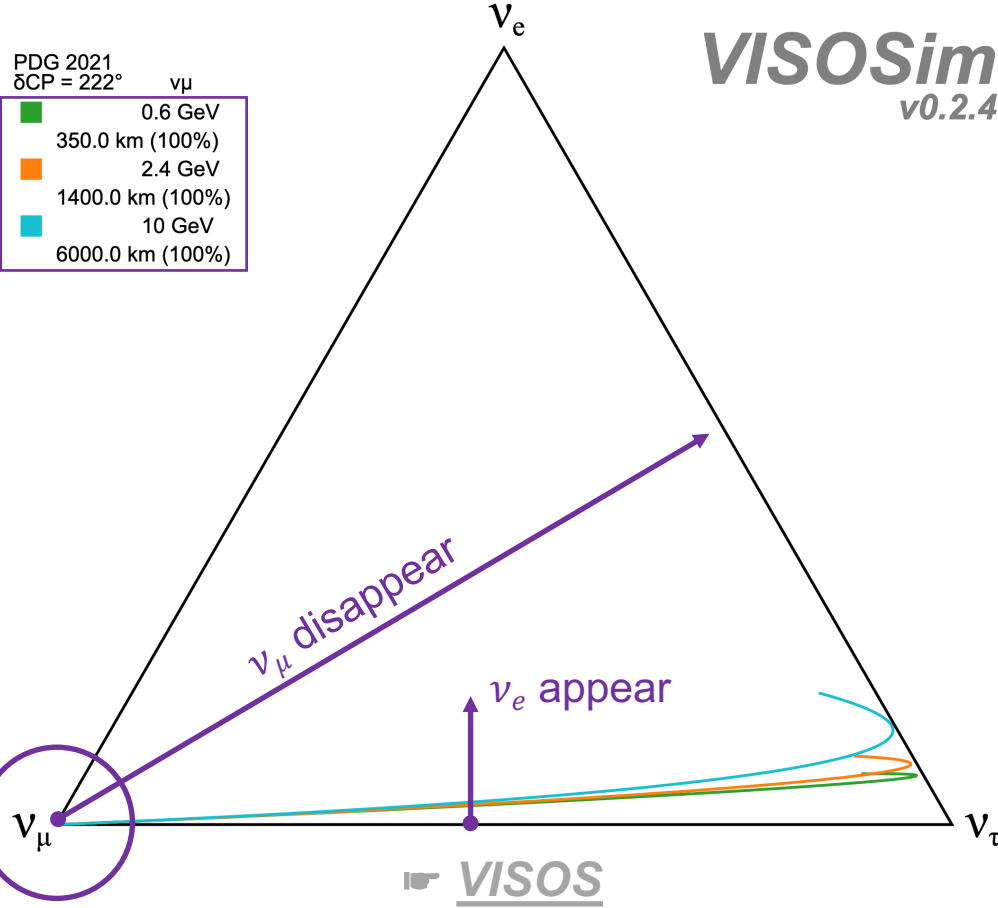
Quiz: Name a few experiments in the “wasteland”? What are they for?



Oscillation experiments

Future Oscillation Experiment	E_ν/GeV @Flux Peak	Detector Technology	Target Nuclei
T2K/HK	0.6	Water Che'	H ₂ O
NOvA	2	Liquid Scintillator	CH
DUNE	2.4	LAr TPC	Ar
IceCube	3-10 (NMO sensitive region)	Cherenkov in ice	H ₂ O
SK/KM3NeT		Water Che'	H ₂ O

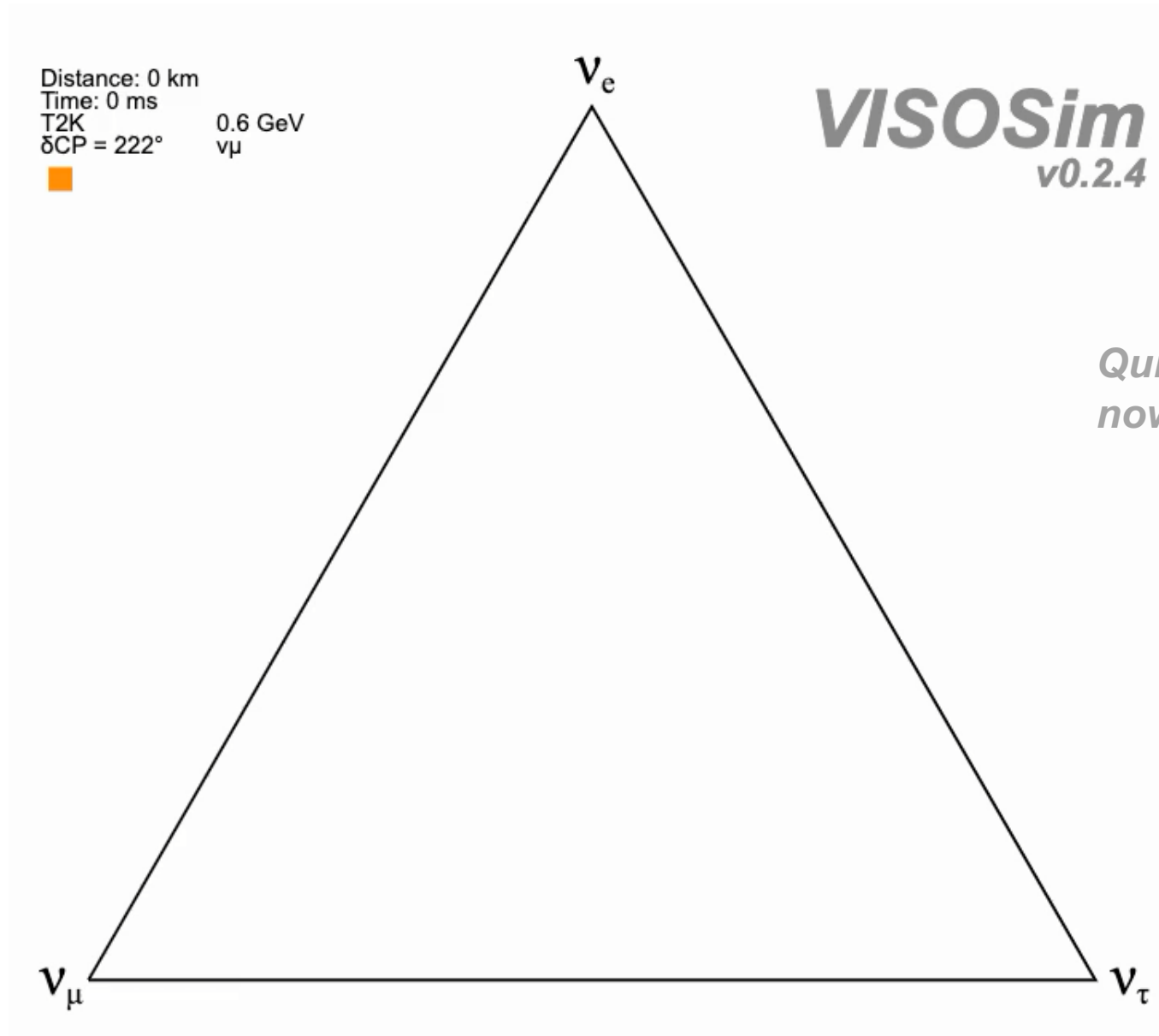
- Flux: ν_μ and $\bar{\nu}_\mu$
- Oscillation
 - ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)
 - ν_e and $\bar{\nu}_e$ appearance



Signal = (Flux · Oscillation probability · Cross section) ⊕ Detector effects
... to discuss tomorrow

Quiz: Is it driven by Δm_{21}^2 or Δm_{32}^2 ?

Oscillation experiments



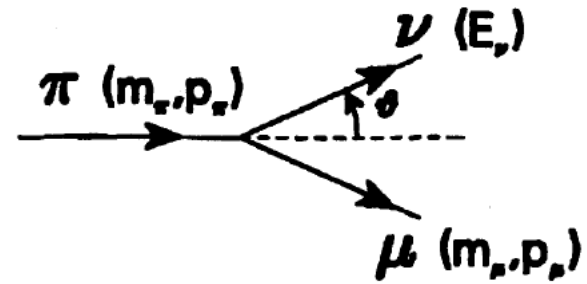
Quiz: What is $|\Delta m_{32}^2|/\Delta m_{21}^2$ now? Is it reasonable?

Part 2: Accelerator Neutrinos

Accelerator Neutrinos

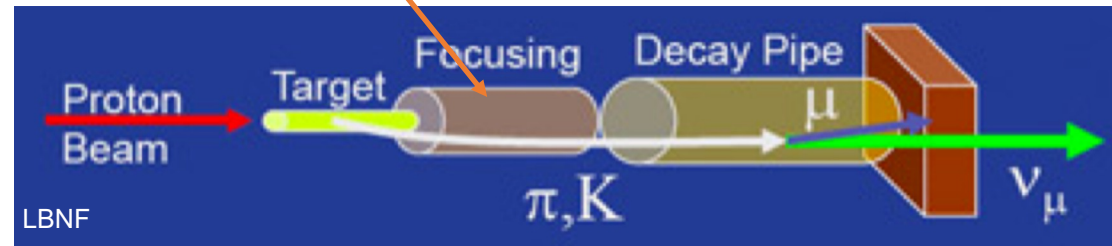
How to obtain a controlled sample of neutrinos?

Let's start from π decays



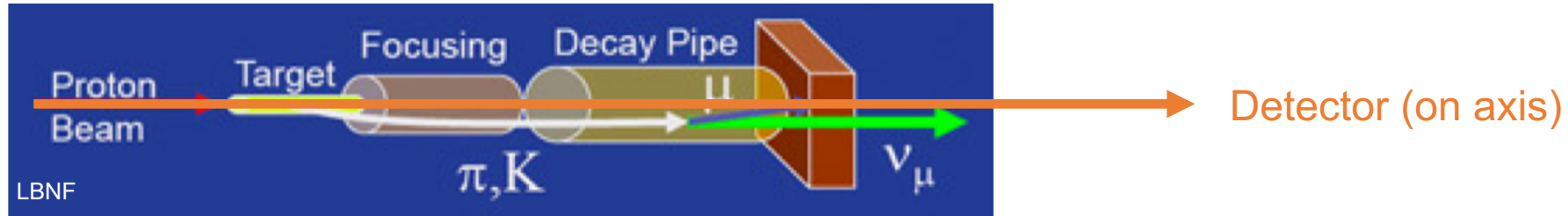
Accelerator Neutrinos

Magnetic “Horn”



Mostly ν_μ from π , few ν_e from K
Proton energy flow and horn $\rightarrow$ Directional
Horn charge selection on $\pi \rightarrow$ High purity ν or $\bar{\nu}$ beams

Accelerator Neutrinos



Mostly ν_μ from π , few ν_e from K

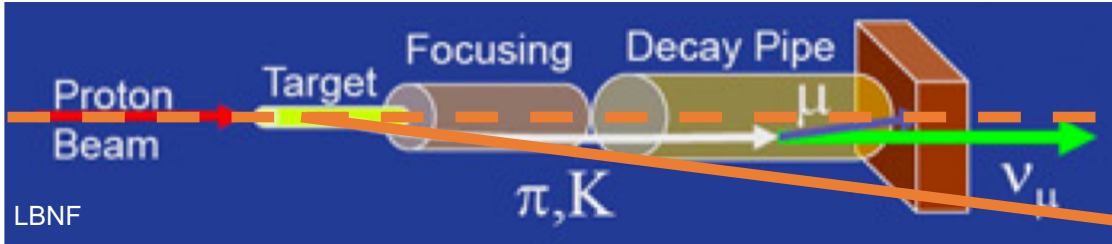
Proton energy flow and horn $\rightarrow$ Directional

Horn charge selection on $\pi \rightarrow$ High purity ν or $\bar{\nu}$ beams

Accelerator Neutrinos

Off-axis (OA) technique → Narrow-band beams

[D. Beavis, et al., P889: long baseline neutrino oscillation experiment at the AGS, Report No. BNL-52459, April, 1995](#)



Proton beam axis

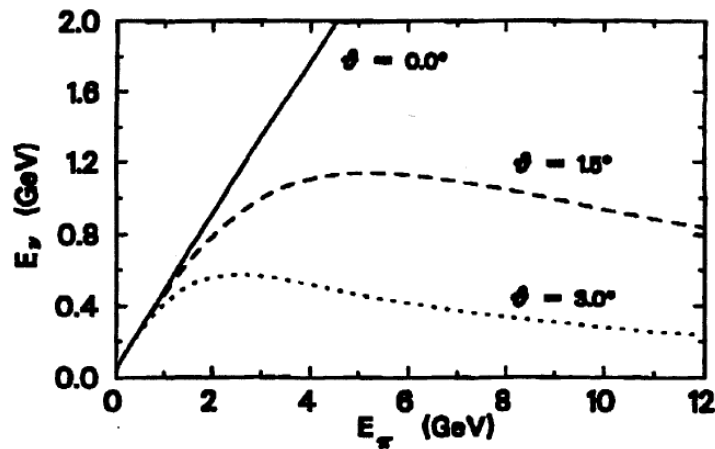
Off-axis angle

Detector (off axis)

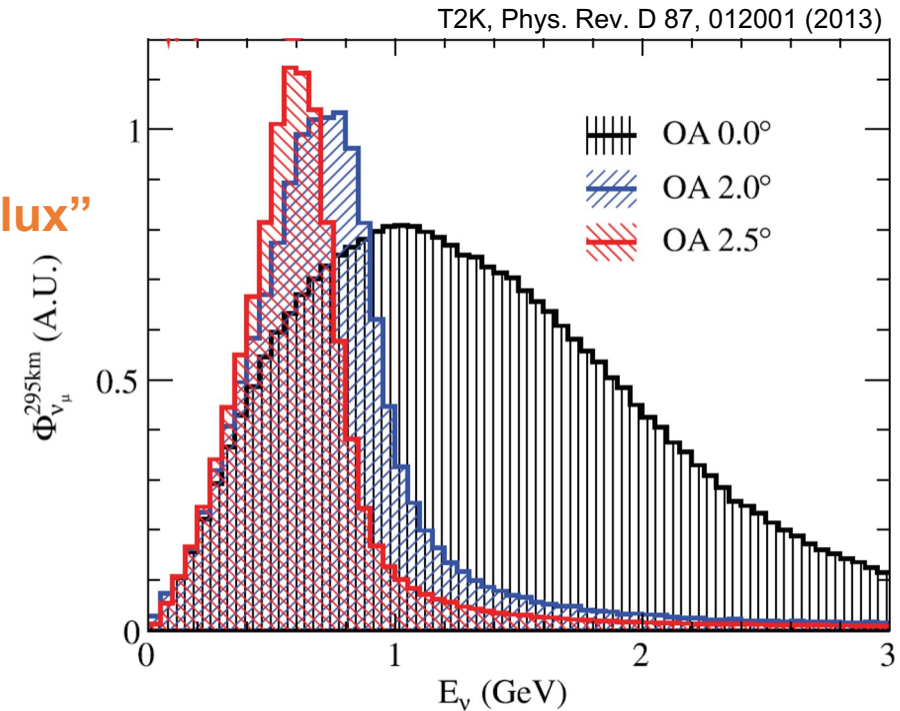
Mostly ν_μ from π , few ν_e from K
Proton energy flow and horn → Directional
Horn charge selection on π → High purity ν or $\bar{\nu}$ beams

From energy, momentum conservation

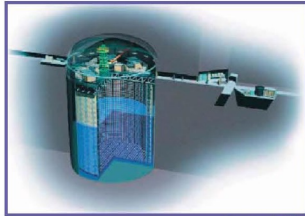
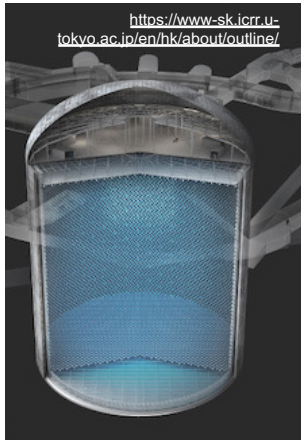
$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos \theta)}$$



“Flux”

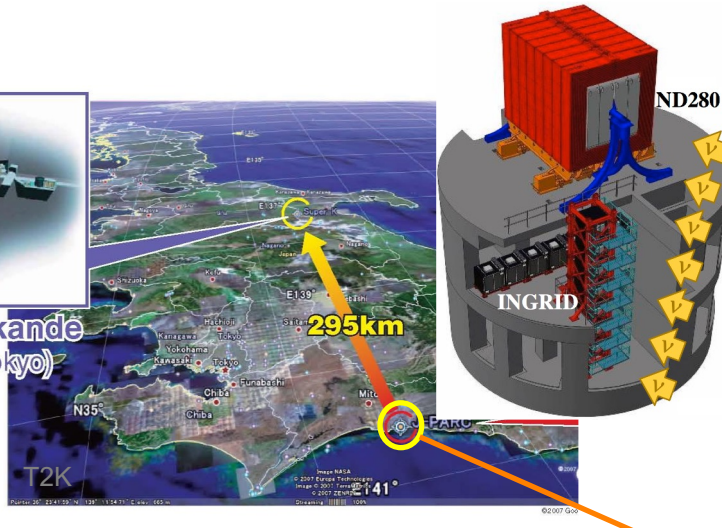


Accelerator Neutrino Experiments



Super-Kamiokande
(ICRR, Univ. Tokyo)

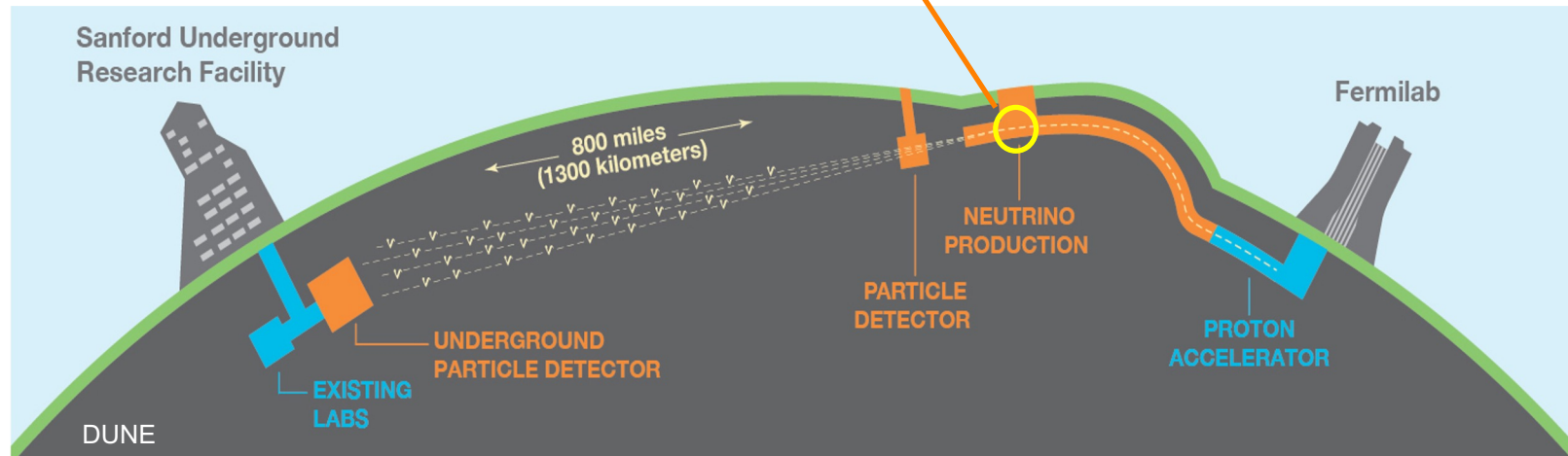
T2K / Hyper-K



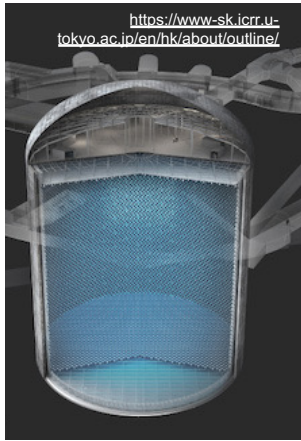
NOvA

ν beams via hadron decay

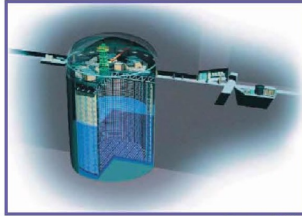
DUNE



Accelerator Neutrino Experiments

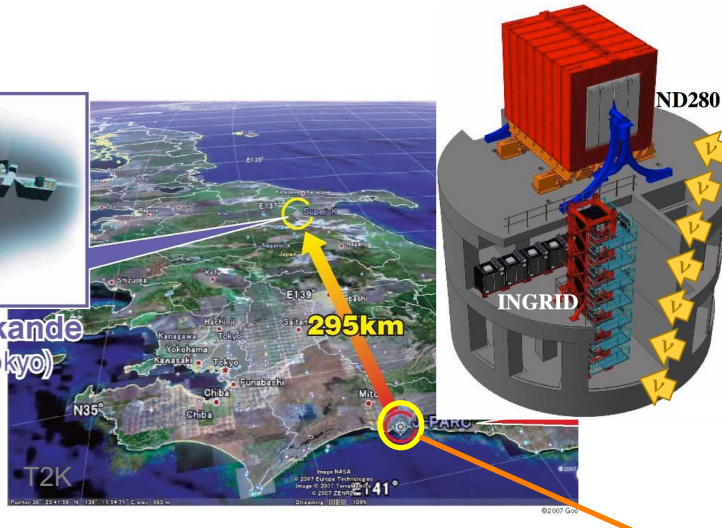


<https://www-sk.icrr.u-tokyo.ac.jp/en/hk/about/outline/>



Super-Kamiokande
(ICRR, Univ. Tokyo)

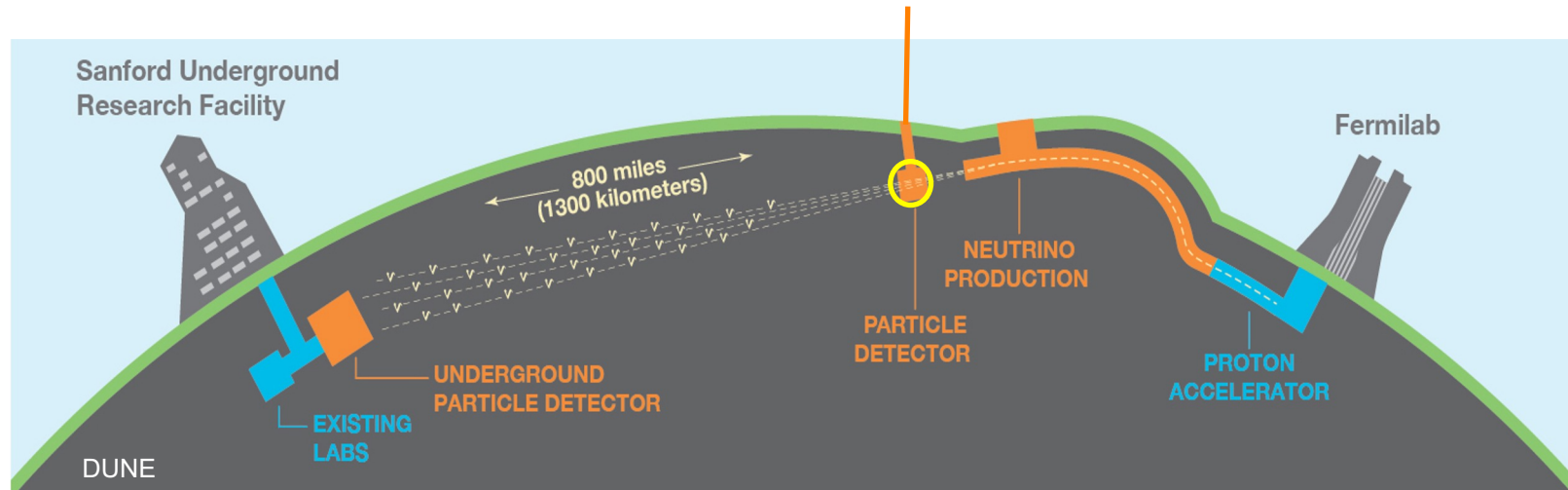
T2K / Hyper-K



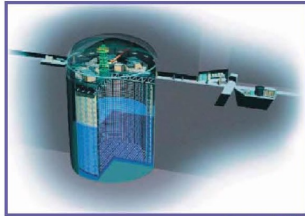
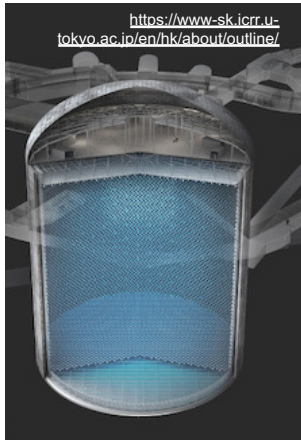
NOvA

Near Detectors (NDs) to constrain ν flux *and* cross section *together* (no oscillation)

DUNE

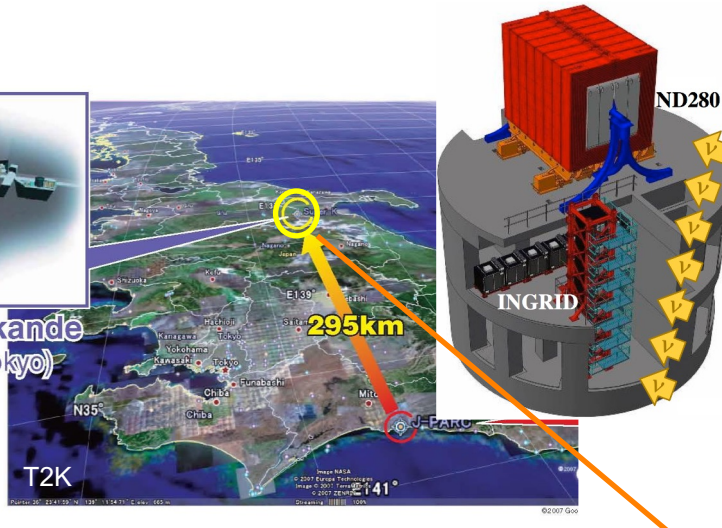


Accelerator Neutrino Experiments



Super-Kamiokande
(ICRR, Univ. Tokyo)

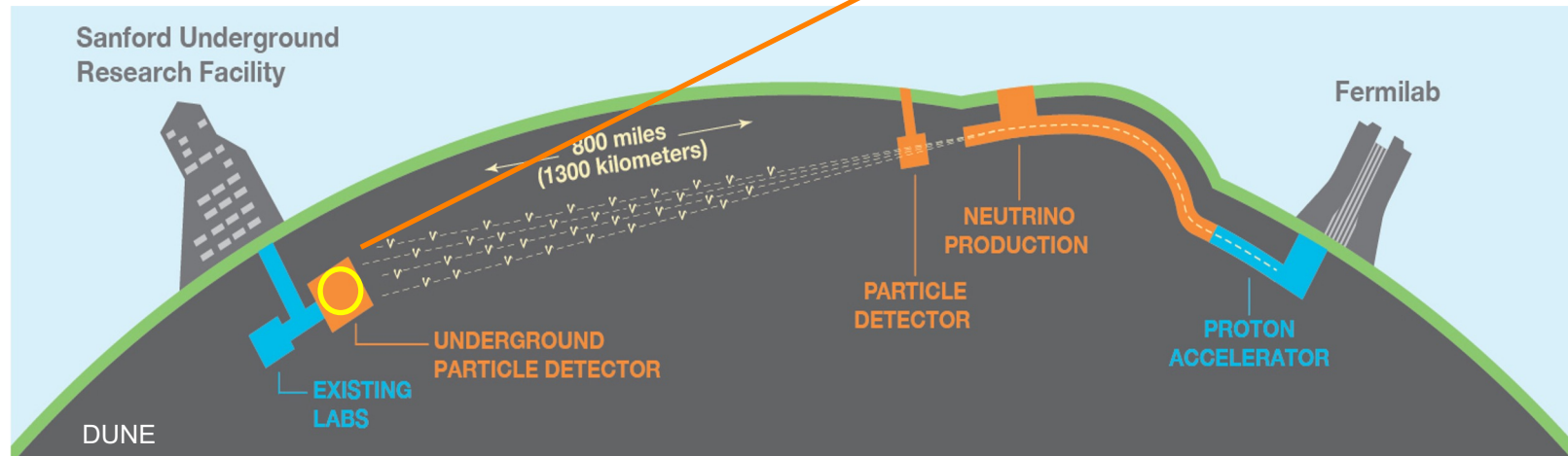
T2K / Hyper-K



NOvA

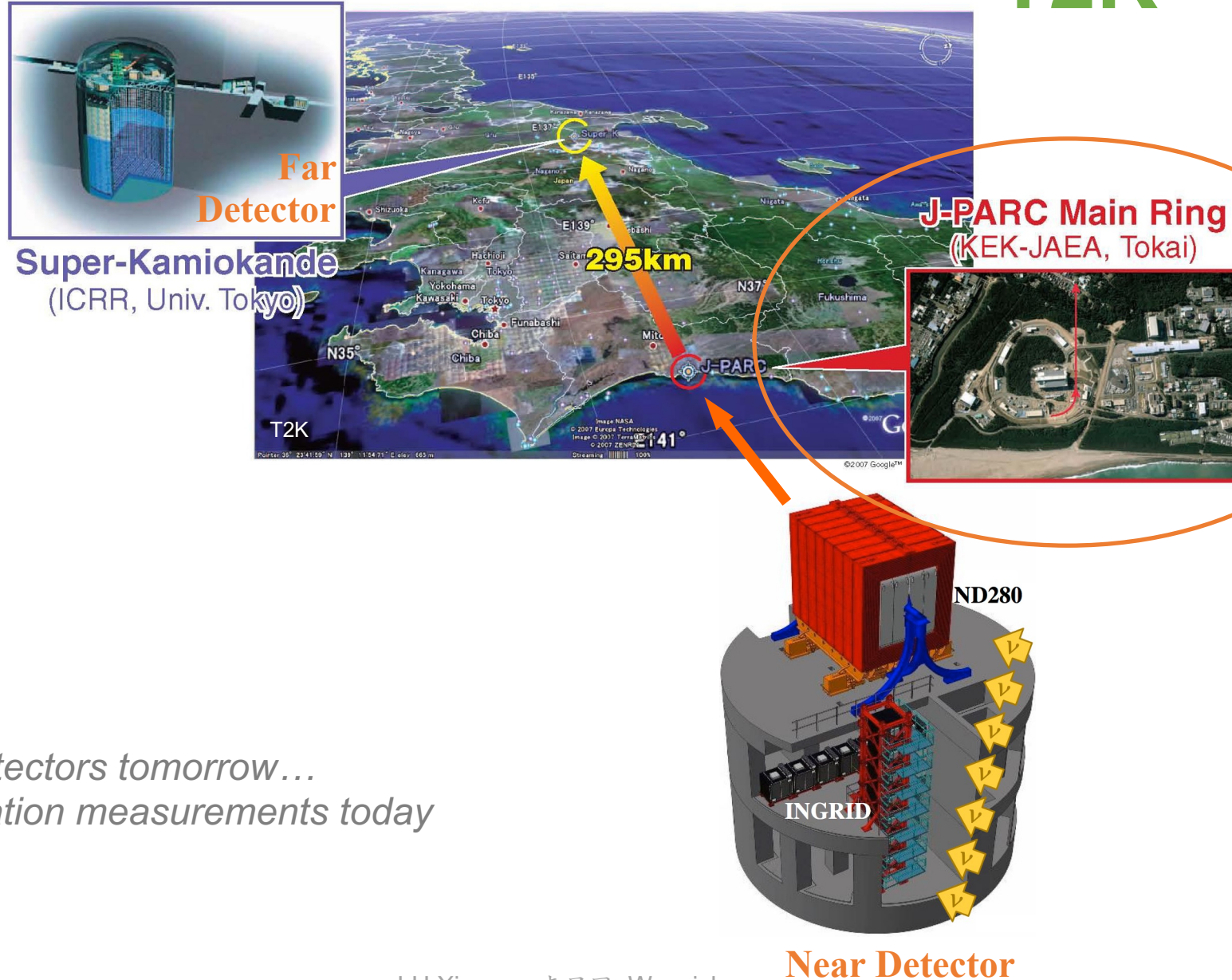
Far Detectors to measure ν oscillation

DUNE



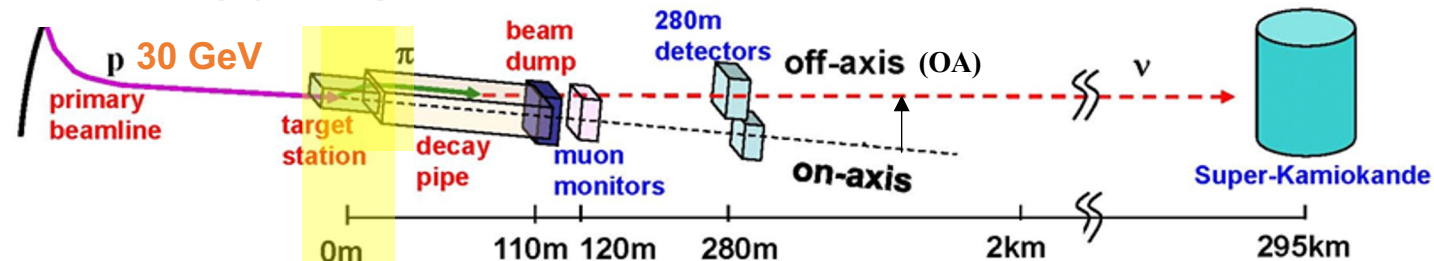
Accelerator Neutrino Experiments

T2K



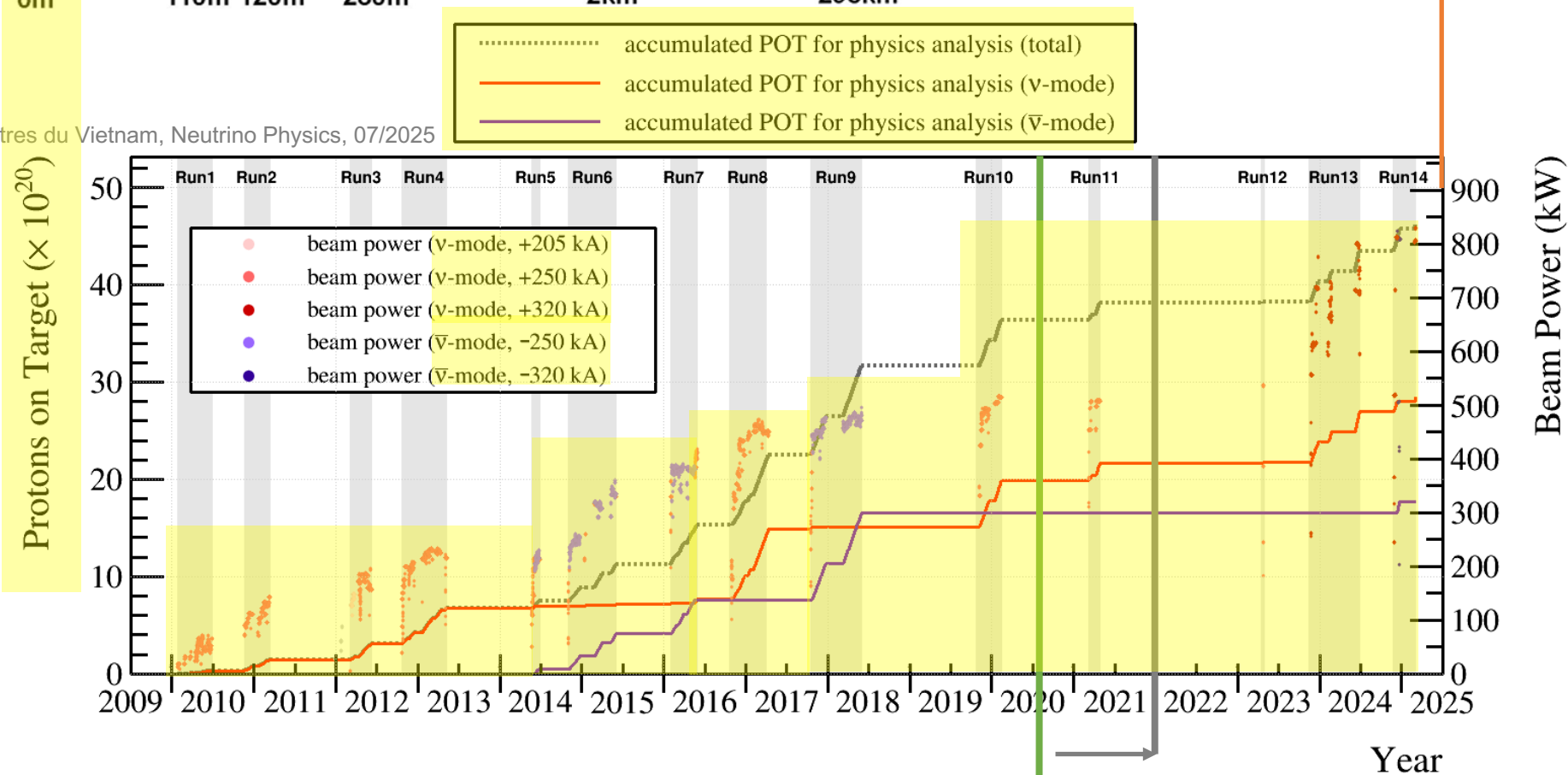
*Will check the detectors tomorrow...
... flux and oscillation measurements today*

T2K Beams



1.3 MW power near the beginning of the Hyper-Kamiokande data taking [HK, 2505.15019]

T2K, 21st Rencontres du Vietnam, Neutrino Physics, 07/2025



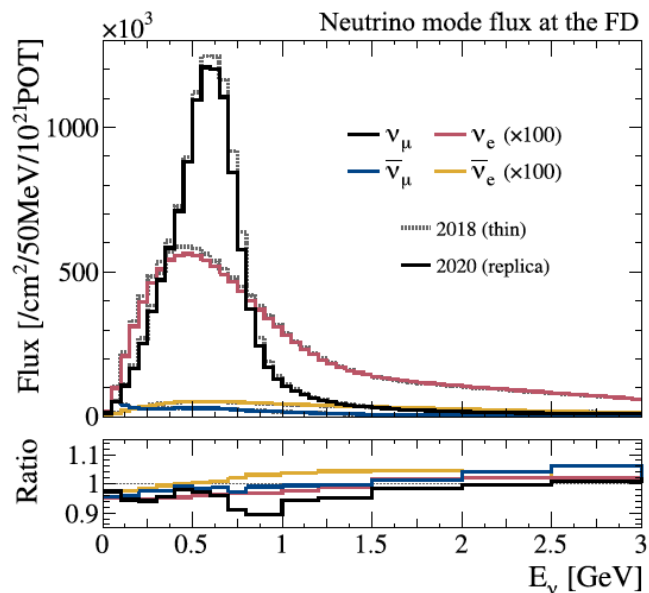
T2K, Eur.Phys.J.C 83, 9, 782 (2023)

Latest analysis (07/25)

T2K Beams

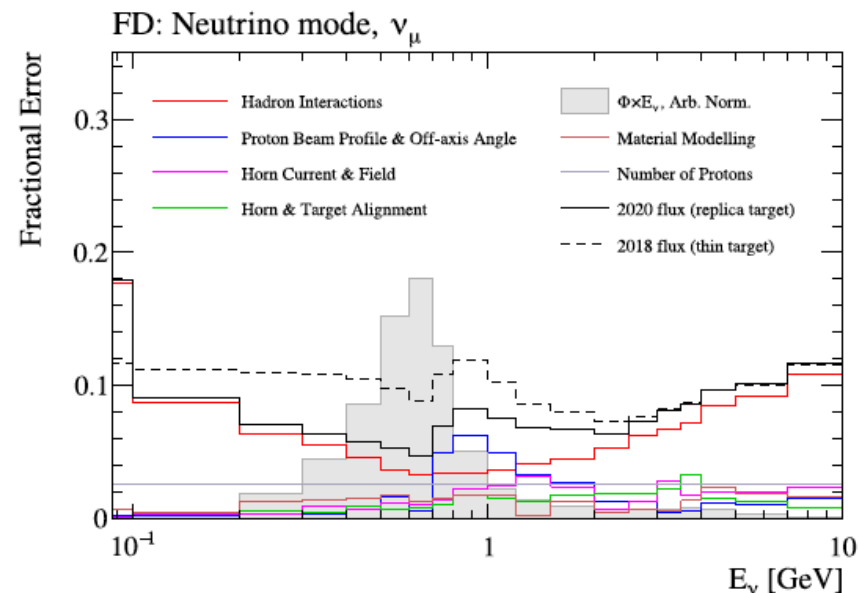
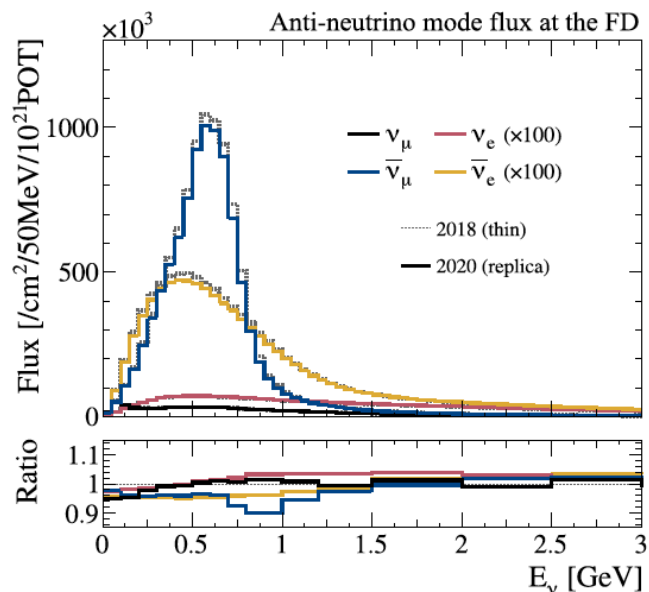
T2K, Eur.Phys.J.C 83, 9, 782 (2023)

For ν oscillations

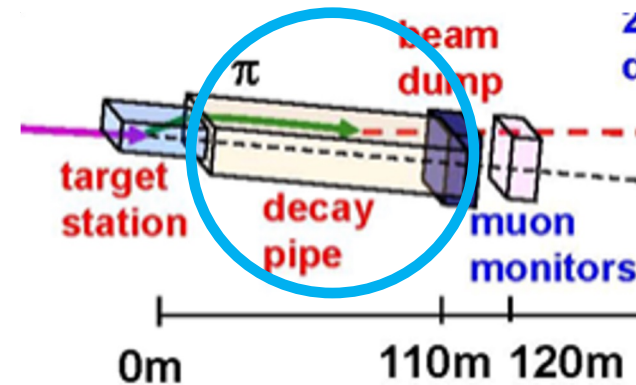


Bkg from 3 other types
... more on this later

For $\bar{\nu}$ oscillations



Flux model error 5%+
Extremely hard to get it further down



No control over neutrino parents (π , K) after focusing.
Everything relies on the horn.

... more on this later

Accelerator Neutrino Experiments

T2K

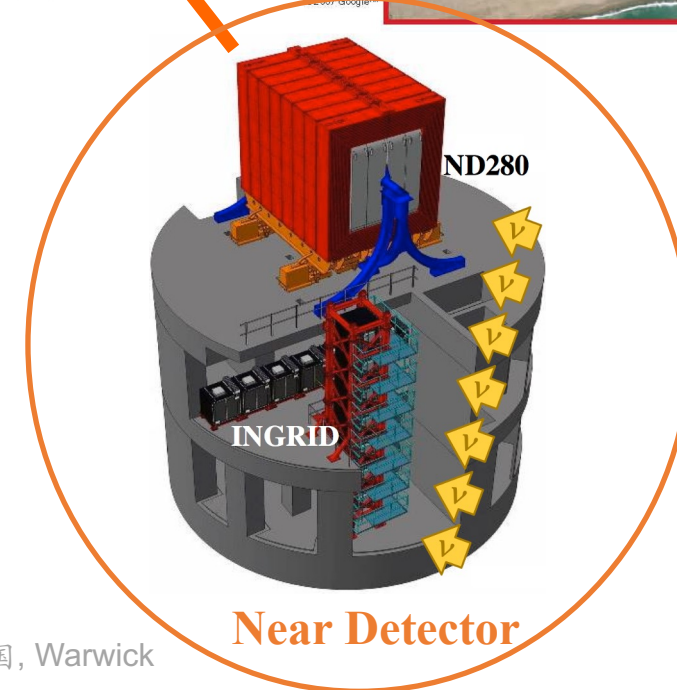


Far
Detector

Super-Kamiokande
(ICRR, Univ. Tokyo)



J-PARC Main Ring
(KEK-JAEA, Tokai)

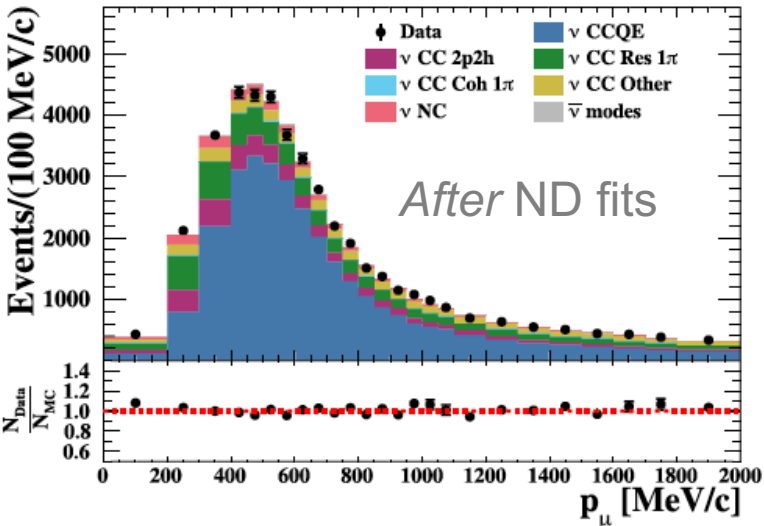
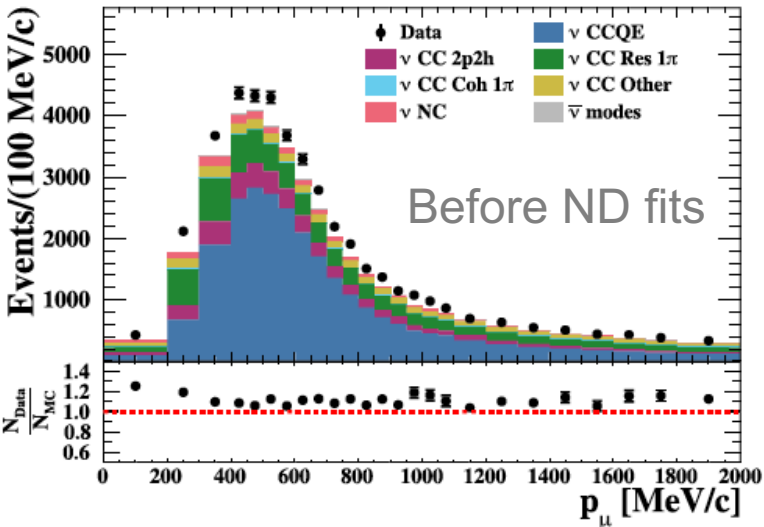


Near Detector

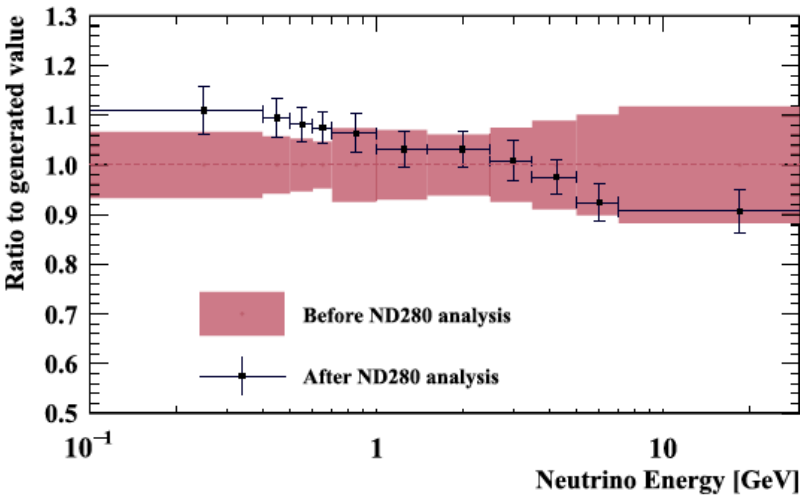
T2K Near Detector (ND) Fits

T2K, Eur.Phys.J.C 83, 9, 782 (2023)

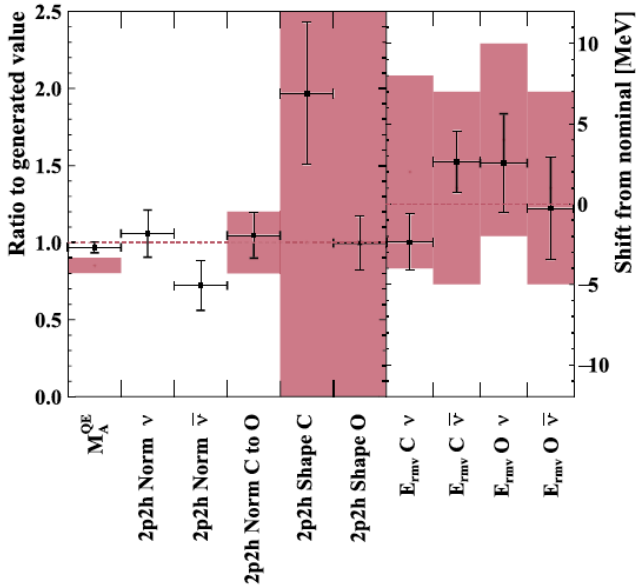
Signal = (Flux · Oscillation probability · Cross section) ⊕ Detector effects



(a) FGD1 ν -mode ν_μ CC0 π



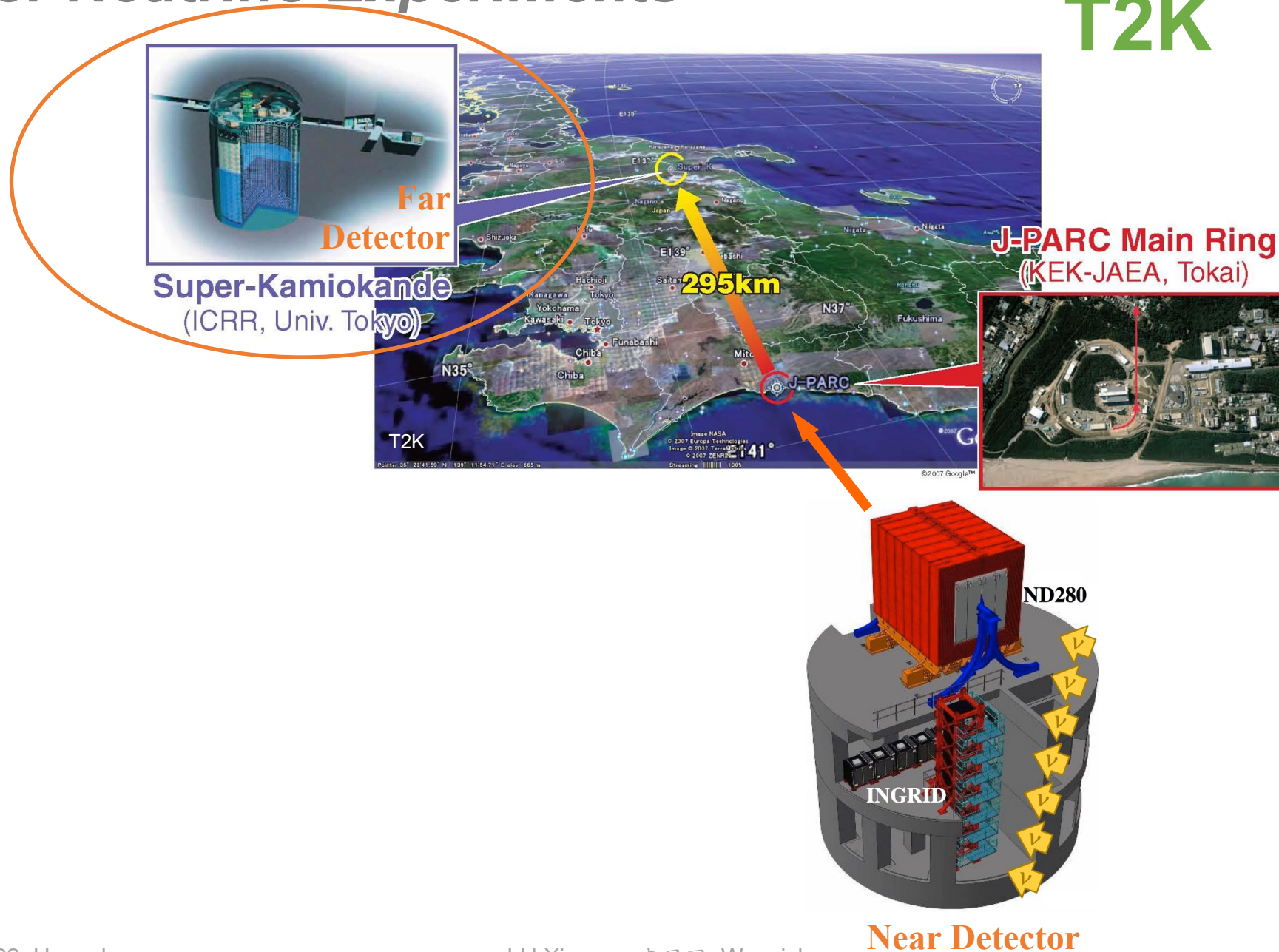
Flux prediction (mean and spread) improved after ND fits



Cross section models *now have constraints or are better constrained* after ND fits

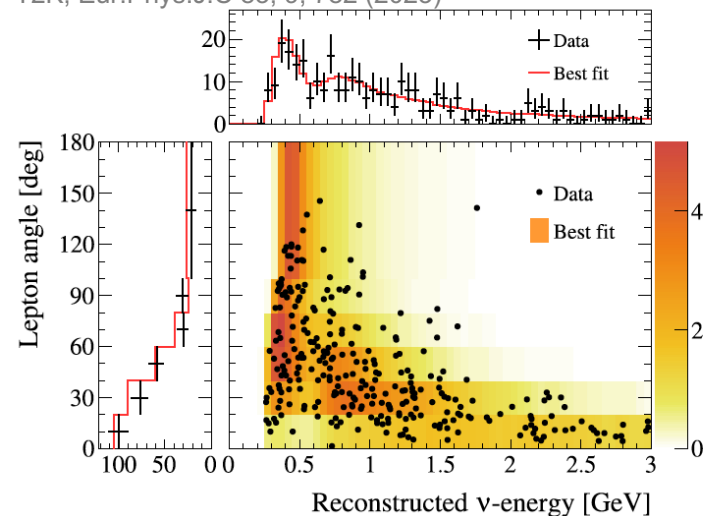
Accelerator Neutrino Experiments

T2K

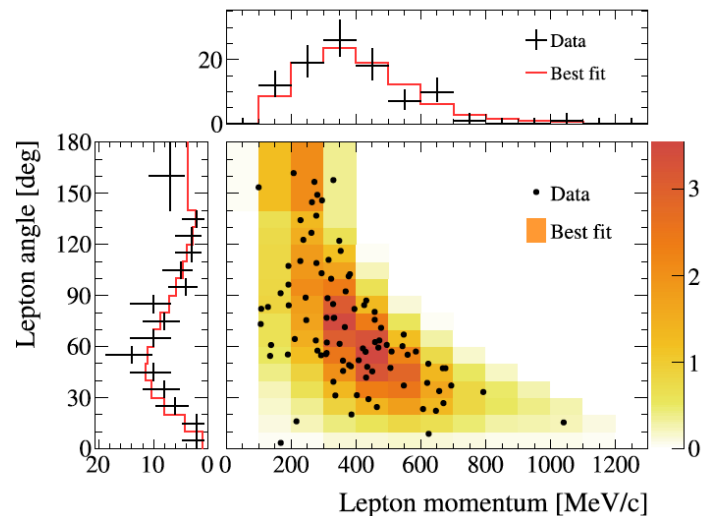


T2K Far Detector Events

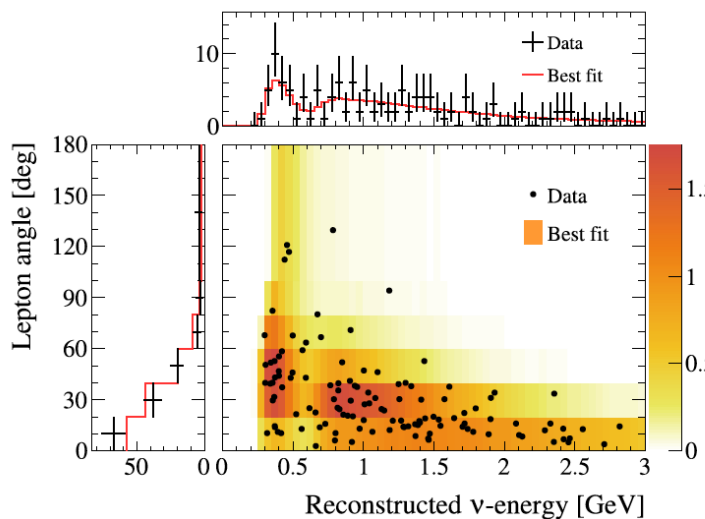
T2K, Eur.Phys.J.C 83, 9, 782 (2023)



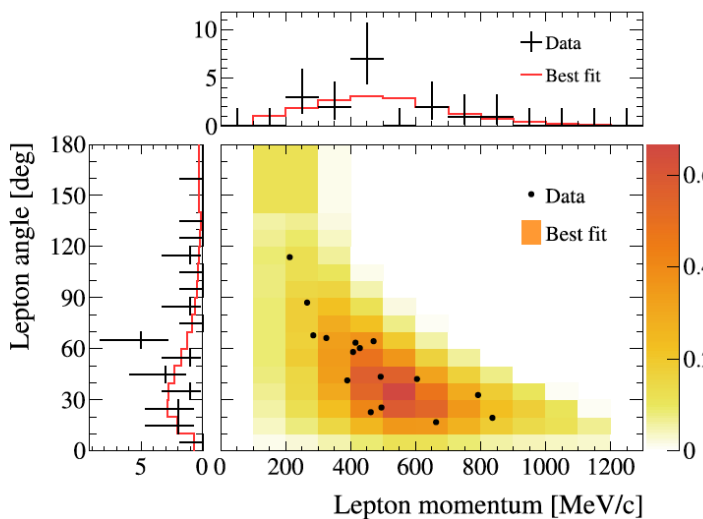
(a) ν -mode $1R\mu$



(b) ν -mode $1Re$



(d) $\bar{\nu}$ -mode $1R\mu$



(e) $\bar{\nu}$ -mode $1Re$

□ Flux: ν_μ and $\bar{\nu}_\mu$

□ Oscillation

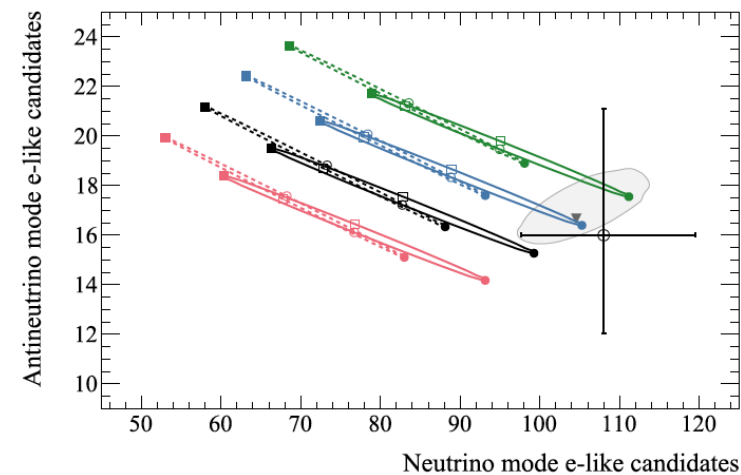
❖ ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)

❖ ν_e and $\bar{\nu}_e$ appearance, then CP violation

68% syst err. at best-fit
 ▼ Best-fit
 Data (68% stat err.)

$\sin^2\theta_{23} = 0.45, 0.50, 0.55, 0.60$
 $\Delta m_{32}^2 = 2.49 \times 10^{-3} \text{ eV}^2 \text{ (NO)}$
 $\Delta m_{31}^2 = -2.46 \times 10^{-3} \text{ eV}^2 \text{ (IO)}$

○ $\delta_{CP} = \pi$
 ■ $\delta_{CP} = +\pi/2$
 □ $\delta_{CP} = 0$
 ● $\delta_{CP} = -\pi/2$



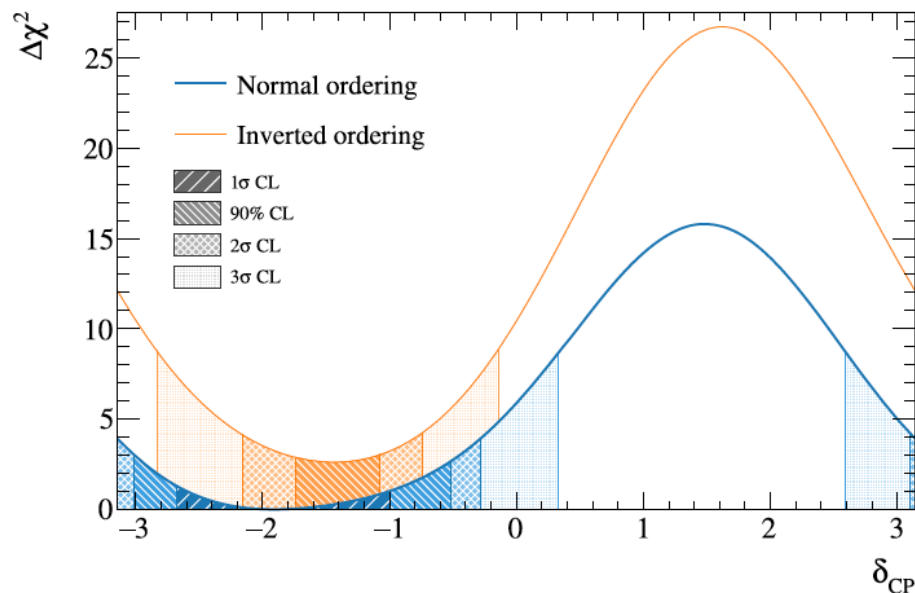
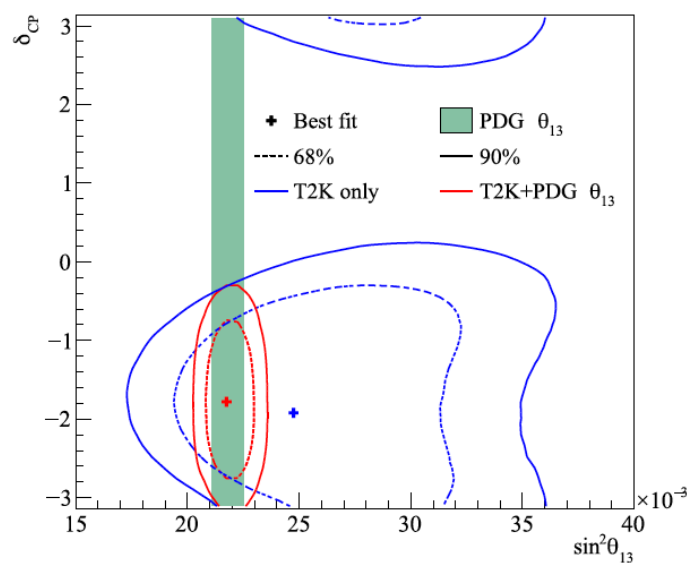
$\sim$ Normal ordering, $\delta_{CP} = -\frac{\pi}{2}$

Lucky fluctuations?

... next slide

T2K Oscillation Parameter Results

T2K, Eur.Phys.J.C 83, 9, 782 (2023)



θ_{13} relies on ν_e appearance, statistically very limited

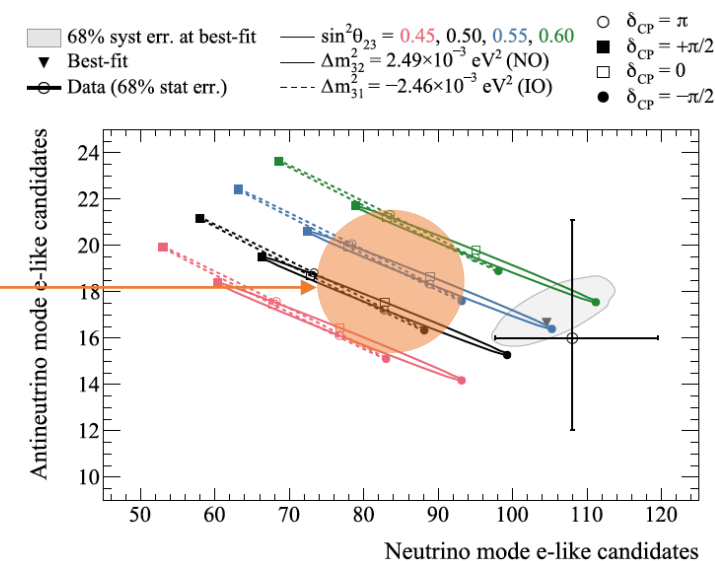
➤ Need reactor constraints

Similar δ_{CP} intervals for both MO

➤ What happens if Nature gives us different event rates here

➤ Need external constraints on MO

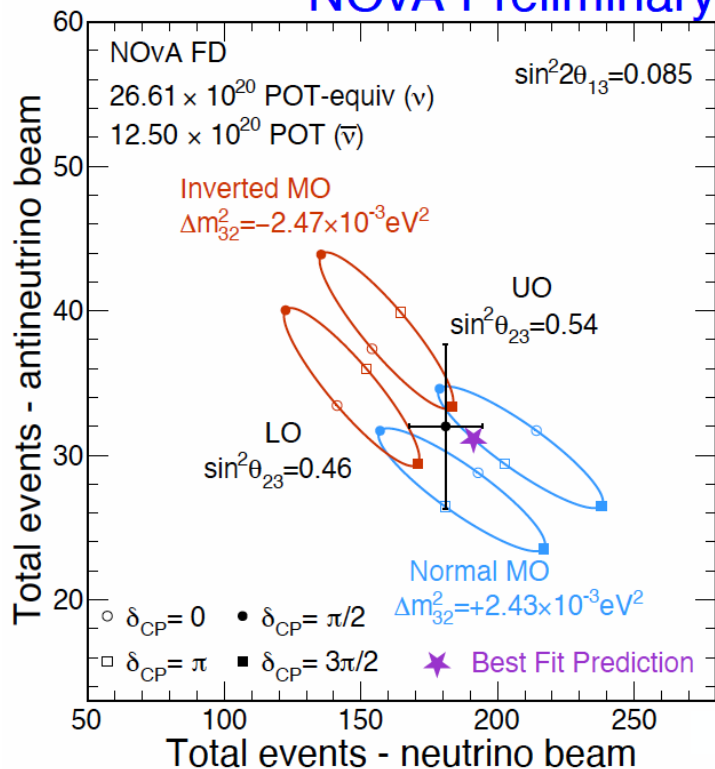
... will come to this later



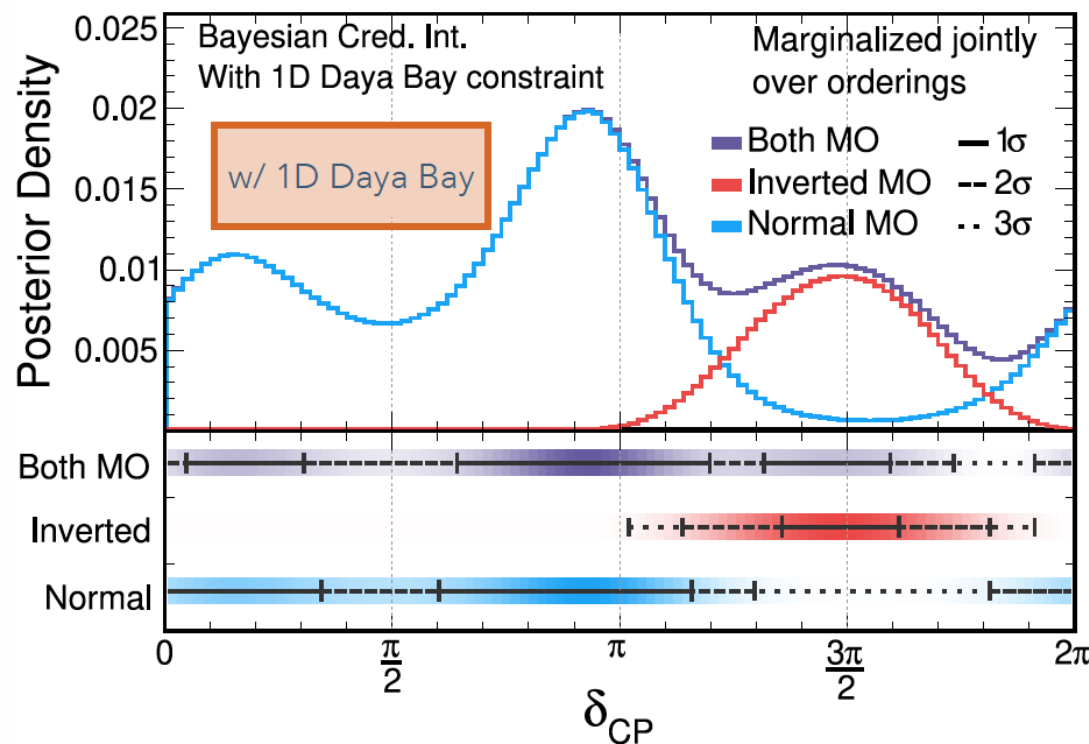
One slide about NOvA

NOvA, 21st Rencontres du Vietnam, Neutrino Physics, 07/2025

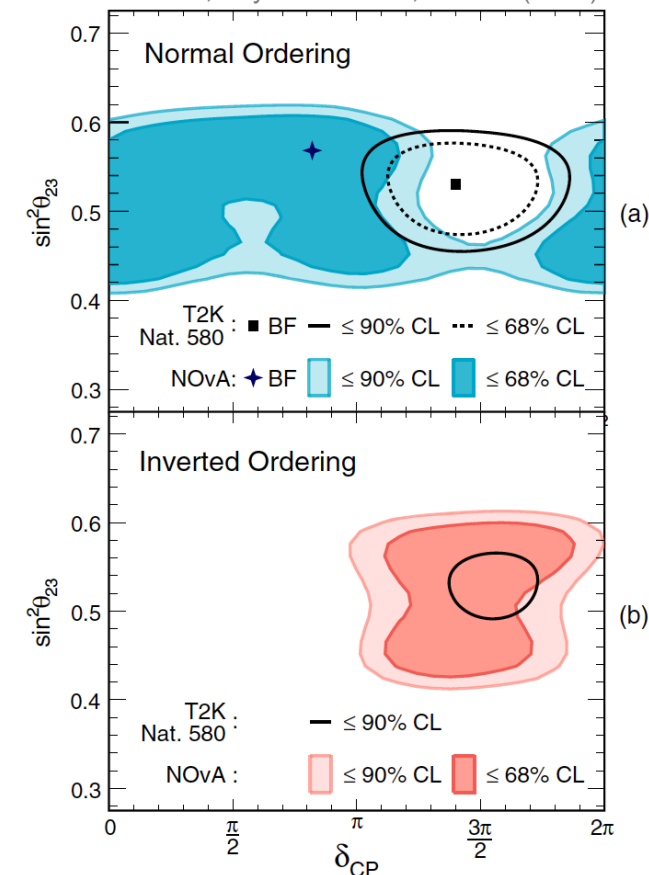
NOvA Preliminary



NOvA Preliminary

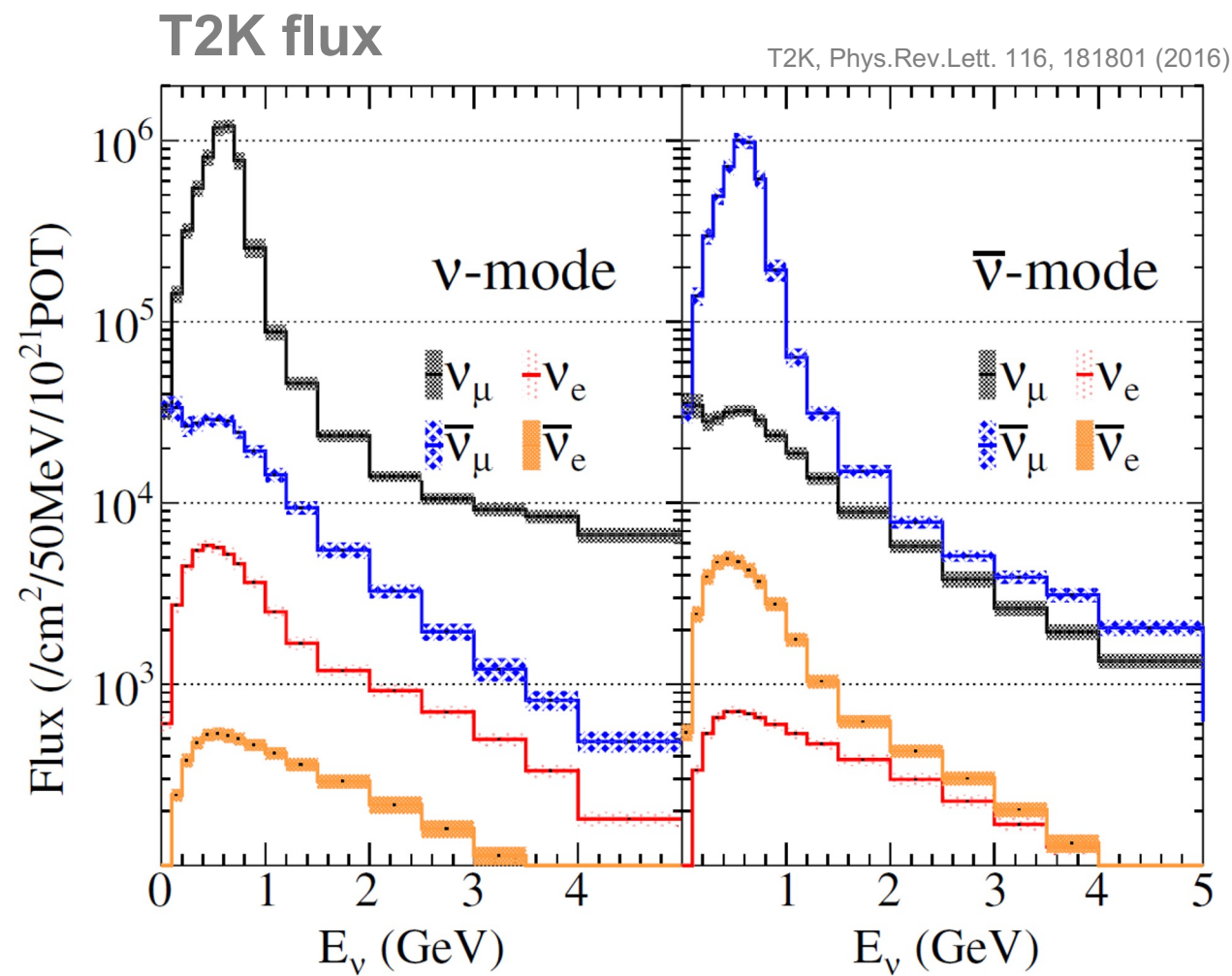


NOvA, Phys.Rev.D 106, 032004 (2022)



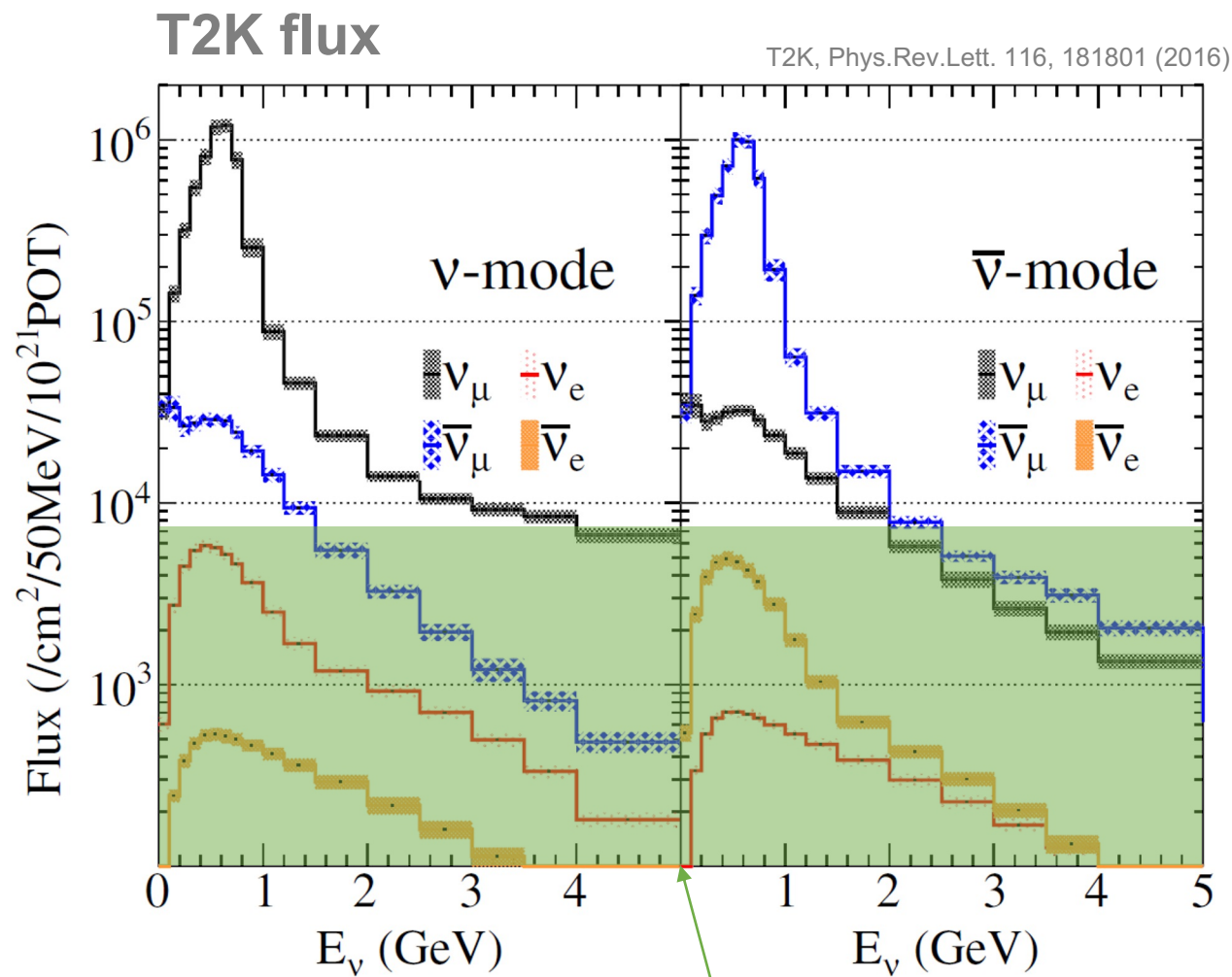
- Not knowing NMO makes it very difficult to determine δ_{CP}
- Already difficult even if NMO known

Coming back to the flux background...



□ Maximise ν_μ and $\bar{\nu}_\mu$ flux

Coming back to the flux background...



- ☐ Maximise ν_μ and $\bar{\nu}_\mu$ flux
- ☒ Minimise ν_e and $\bar{\nu}_e$ to observe appearance
- ☒ ν_e and $\bar{\nu}_e$ flux are background

Call for a GeV ν_e and $\bar{\nu}_e$ Machine

ν_e ($\bar{\nu}_e$) cross sections: major δ_{CP} systematics

- ❑ Very few ν_e scattering data
- ❑ $\delta_{CP} \sim \nu_e$ appearance $\sim$ no ν_e in beams
 - ❖ No *in situ* ν_e measurements
- ❑ ν_μ for ν_e via lepton universality, but higher precision needed for δ_{CP}

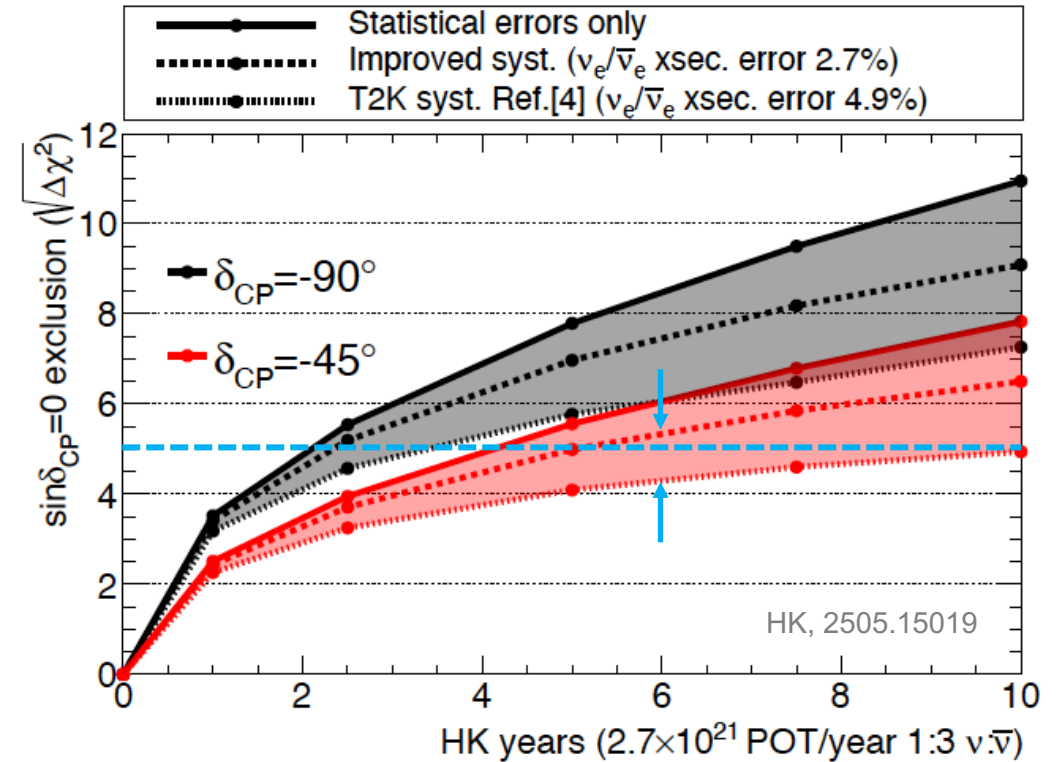
Hyper-K example

Improving error of $\nu_e/\bar{\nu}_e$ xsec ratio 4.9% $\rightarrow$ 2.7%

- ❑ Improve δ_{CP} sensitivity by $\sim 1 \sigma$ for 6 year
- ❑ **Significantly shorten running time to reach 5σ**

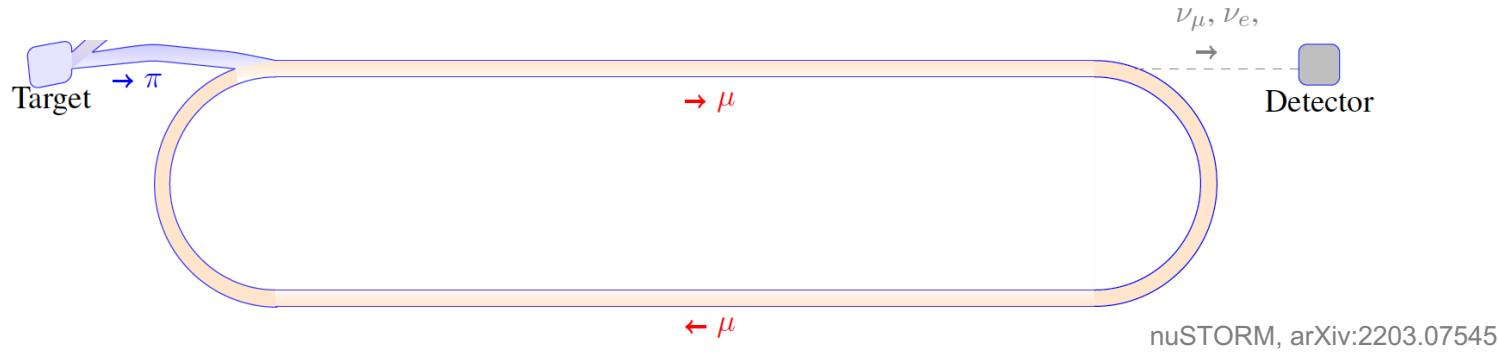
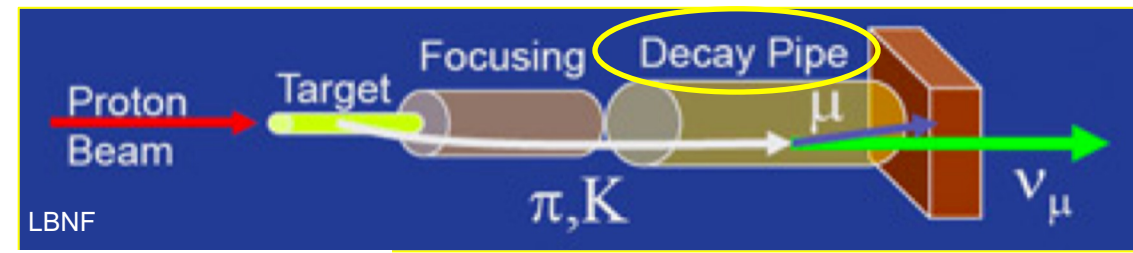
✓ Wishlist

1. ν_e and $\bar{\nu}_e$ beams with the relevant energy for appearance
2. Well-understood fluxes
3. High statistics
4. Low ν_μ background

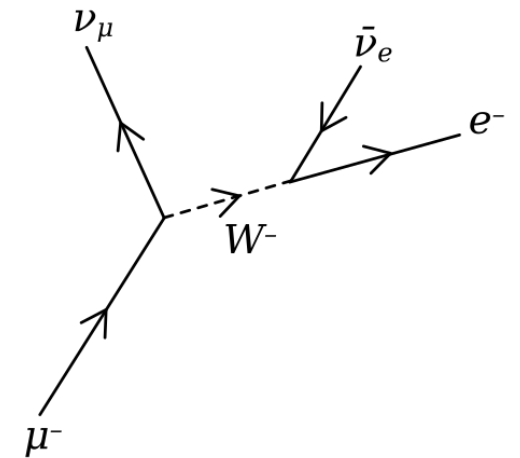


ν from *STORed* Muons (*nuSTORM*)

Coming back to the beam control...



- ❑ π captured by storage ring $\rightarrow \mu \rightarrow \nu$, instead of decay pipe $\rightarrow \nu$
- ❑ $\bar{\nu}_\mu + \nu_e$ and $\nu_\mu + \bar{\nu}_e$ fluxes from $\mu^\pm$ decays
 - ❖ **Storage ring: tunable fluxes**
 - ❖ **μ decay: perfect understanding of flux shape and normalisation**



Part 3: Atmospheric Neutrinos

More on Neutrino Mass: MSW Matter Effect

Before getting into atmospheric ν ...

All neutrinos experience the same NC matter potential in medium.

Additional CC potential for ν_e (“+”) and $\bar{\nu}_e$ (“-”)

$$V_{CC} = \pm\sqrt{2}G_F n_e$$

Electron density n_e

Fermi constant $\sqrt{2}G_F \simeq 7.63 \times 10^{-14} \frac{\text{eV cm}^3}{N_A}$

$$A_{CC} \equiv 2EV_{CC}$$

Homework: Write down the Feynman diagrams of ν_e and $\bar{\nu}_e$ CC interactions with electrons.

Extremely relativistic kinematics

$$E \simeq p + \frac{m^2}{2p}, \quad p \simeq E$$

As usual, $\Delta m^2 \equiv m_2^2 - m_1^2$

1. No mixing: $\nu_1 = \nu_e, \nu_2 = \nu_\mu$

ν_e Hamiltonian in matter

$$H = E + V_{CC} \equiv E + \frac{m_M^2}{2E}$$

Effective mass $m_M^2 = m^2 + A_{CC}$

(NC induces the same A_{NC} for all neutrino flavours)

Effective mass gap $\Delta m_M^2 = \Delta m^2 - A_{CC}$

- When the matter potential fills up the mass gap

$$A_{CC} = \Delta m^2$$

$$E [\text{GeV}] n_e \left[\frac{N_A}{\text{cm}^3} \right] = \frac{\Delta m^2 [10^{-5} \text{eV}^2]}{2 \times 7.63 \times 10^{-14} \left[\frac{\text{eV cm}^3}{N_A} \right]}$$

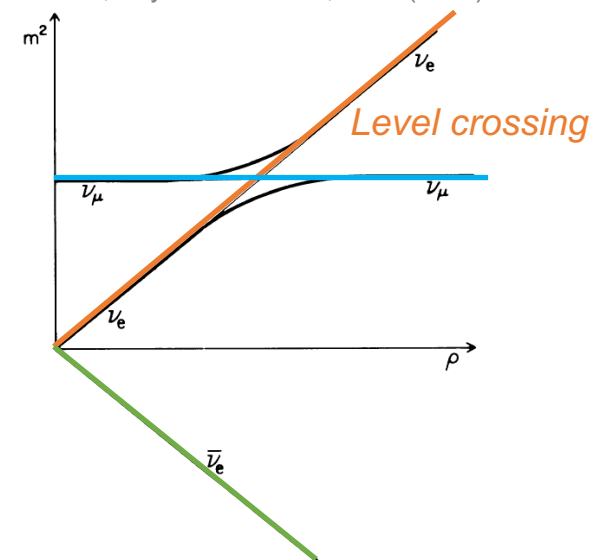
$$En_e \simeq \frac{\Delta m^2}{15}, \quad E \text{ in GeV}, n_e \text{ in } \frac{N_A}{\text{cm}^3}, \Delta m^2 \text{ in } 10^{-5} \text{eV}^2$$

$$\text{Water } n_e = \frac{5}{9} \frac{N_A}{\text{cm}^3}$$

$$\text{For } \Delta m^2 = 10^{-5} \text{eV}^2, E = 0.12 \text{ GeV}$$

- For $A_{CC} > \Delta m^2, \Delta m_M^2 < 0$
- For $A_{CC} \gg \Delta m^2, \Delta m_M^2 \simeq -A_{CC}$

Bethe, Phys.Rev.Lett. 56, 1305 (1986)



More on Neutrino Mass: MSW Matter Effect

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$$E n_e \simeq \frac{\Delta m^2}{15}, \quad E \text{ in GeV}, n_e \text{ in } \frac{N_A}{\text{cm}^3}, \Delta m^2 \text{ in } 10^{-5} \text{eV}^2$$

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- For $A_{CC} > \Delta m^2$, $\Delta m_M^2 < 0$
- For $A_{CC} \gg \Delta m^2$, $\Delta m_M^2 \simeq -A_{CC}$

2. Mixing: Vacuum Hamiltonian in $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ basis

$$H \supset \frac{\Delta m^2}{4E} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Full Hamiltonian in $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ basis

$$H' \supset \frac{\Delta m^2}{4E} U \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U^\dagger + \frac{A_{CC}}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\equiv \frac{\Delta m_M^2}{4E} U_M \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U_M^\dagger,$$

with effective mass gap Δm_M^2 and mixing

$$U_M \equiv \begin{pmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{pmatrix}$$

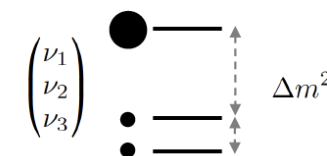
- When $A_{CC} \gg \Delta m^2$, U becomes irrelevant,

$$\frac{A_{CC}}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\Delta m_M^2}{4E} U_M \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U_M^\dagger$$

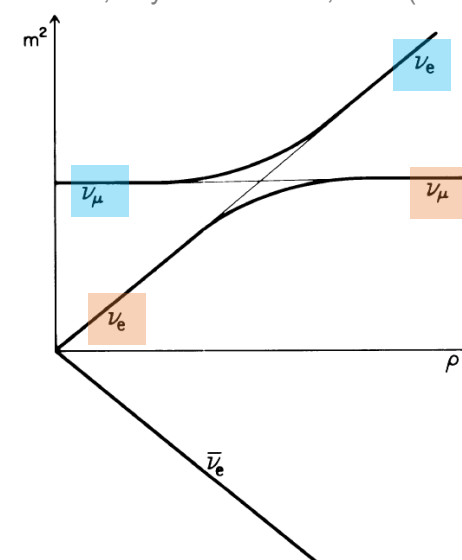
- $\Delta m_M^2 = -A_{CC}$, $\theta_M = 0$, falling back to the previous no mixing case, or
- $\Delta m_M^2 = A_{CC}$, $\theta_M = \pm \frac{\pi}{2}$, naming $\nu_1 = \nu_\mu$, $\nu_2 = \nu_e$.

Quiz: Is there oscillation when the mixing angle is $\pi/2$?

$$\begin{pmatrix} |\uparrow \rangle_t \\ |\downarrow \rangle_t \end{pmatrix} = \begin{pmatrix} e^{i \frac{\Delta E}{2} t} & 0 \\ 0 & e^{-i \frac{\Delta E}{2} t} \end{pmatrix} \begin{pmatrix} |\uparrow \rangle_0 \\ |\downarrow \rangle_0 \end{pmatrix}$$



Bethe, Phys.Rev.Lett. 56, 1305 (1986)



More on Neutrino Mass: MSW Matter Effect

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$$E \simeq p + \frac{m^2}{2p}, \quad p \simeq E$$

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$$A_{CC} = \Delta m^2$$

$$E [\text{GeV}] n_e \left[\frac{N_A}{\text{cm}^3} \right] = \frac{\Delta m^2 [10^{-5} \text{eV}^2]}{2 \times 7.63 \times 10^{-14} \left[\frac{\text{eV cm}^3}{N_A} \right]}$$

$$E n_e \simeq \frac{\Delta m^2}{15}, E \text{ in GeV}, n_e \text{ in } \frac{N_A}{\text{cm}^3}, \Delta m^2 \text{ in } 10^{-5} \text{eV}^2$$

$$\text{Water } n_e = \frac{5}{9} \frac{N_A}{\text{cm}^3}$$

$$\text{For } \Delta m^2 = 10^{-5} \text{eV}^2, E = 0.12 \text{ GeV}$$

- For $A_{CC} > \Delta m^2, \Delta m_M^2 < 0$
- For $A_{CC} \gg \Delta m^2, \Delta m_M^2 \simeq -A_{CC}$

2. Mixing: Vacuum Hamiltonian in $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ basis

$$H \supset \frac{\Delta m^2}{4E} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Full Hamiltonian in $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ basis

$$\begin{aligned} H' &\supset \frac{\Delta m^2}{4E} U \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U^\dagger + \frac{A_{CC}}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &\equiv \frac{\Delta m_M^2}{4E} U_M \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U_M^\dagger, \end{aligned}$$

with effective mass gap Δm_M^2 and mixing

$$U_M \equiv \begin{pmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{pmatrix}$$

- When $A_{CC} \gg \Delta m^2, U$ becomes irrelevant,

$$\frac{A_{CC}}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\Delta m_M^2}{4E} U_M \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U_M^\dagger$$

- $\Delta m_M^2 = -A_{CC}, \theta_M = 0$, falling back to the previous no mixing case, or
- $\Delta m_M^2 = A_{CC}, \theta_M = \pm \frac{\pi}{2}$, naming $\nu_1 = \nu_\mu, \nu_2 = \nu_e$.

- In general, with

$$U \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

The effective mass gap and mixing angle are

$$\begin{aligned} \Delta m_M^2 &= \sqrt{(\Delta m^2 \cos 2\theta - A_{CC})^2 + (\Delta m^2 \sin 2\theta)^2} \\ \tan 2\theta_M &= \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}} \quad \text{e.g. Giunti \& Kim (2007)} \end{aligned}$$

Mikheyev-Smirnov-Wolfenstein effect

Resonance at $A_{CC}^R = \Delta m^2 \cos 2\theta$

$$\theta_M^R = \frac{\pi}{4}, \text{ regardless of } \theta$$

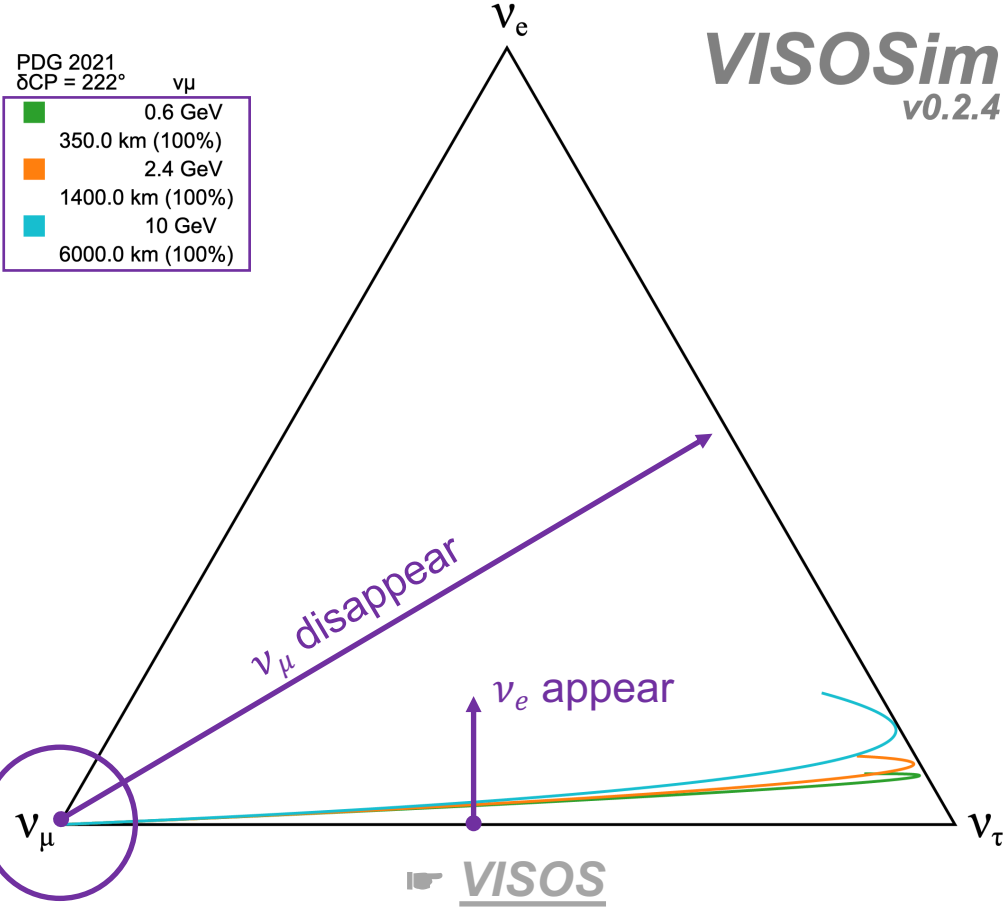
... will come back to this later

Homework: Recover the previous no-mixing and small gap cases with the general effective mass gap and mixing angle.

Oscillation experiments

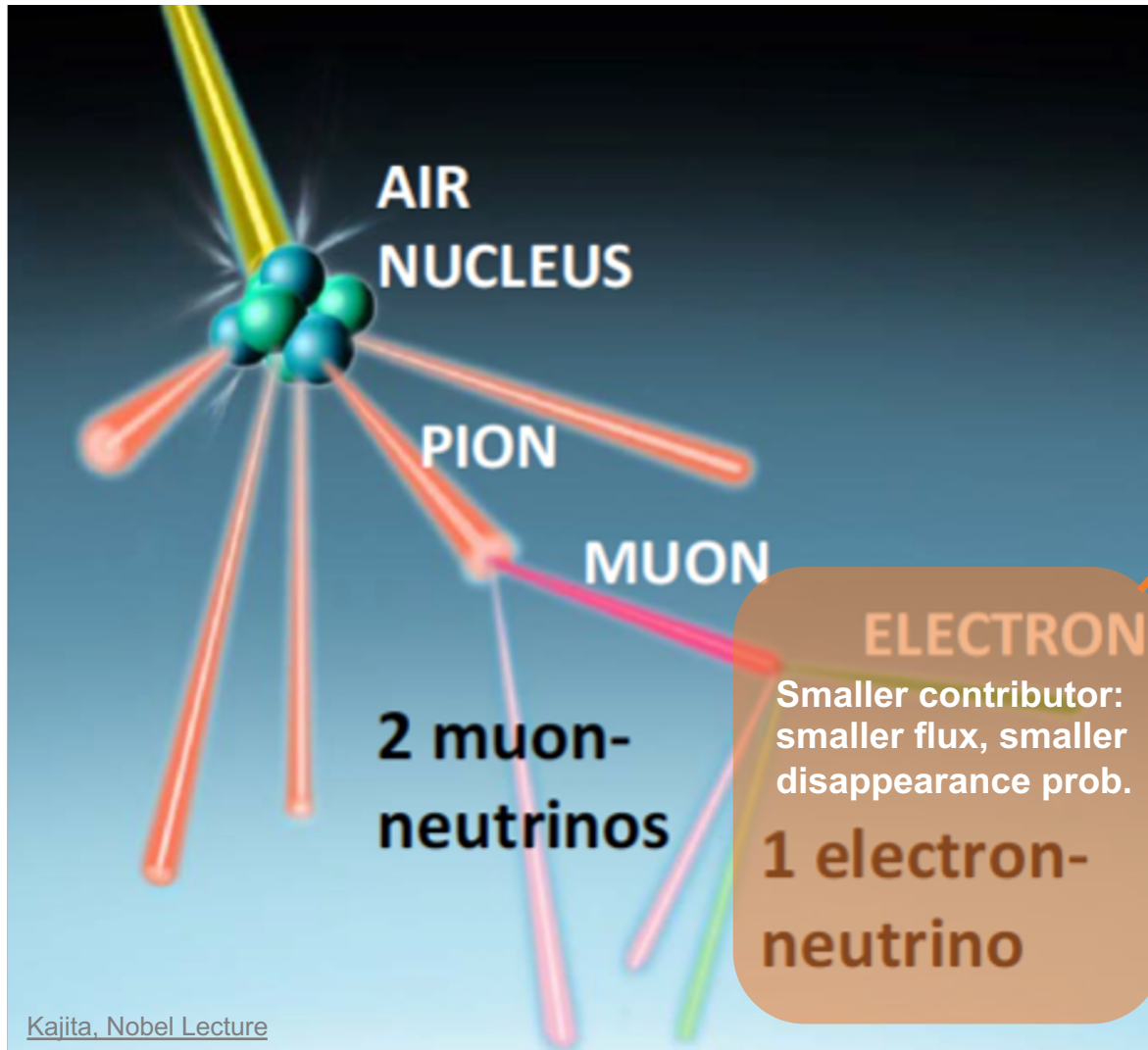
Future Oscillation Experiment	E_ν/GeV @Flux Peak	Detector Technology	Target Nuclei
T2K/HK	0.6	Water Che'	H ₂ O
NOvA	2	Liquid Scintillator	CH
DUNE	2.4	LAr TPC	Ar
IceCube	3-10 (NMO sensitive region)	Cherenkov in ice	H ₂ O
SK/KM3NeT		Water Che'	H ₂ O

- Flux: ν_μ and $\bar{\nu}_\mu$
- Oscillation
 - ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)
 - ν_e and $\bar{\nu}_e$ appearance

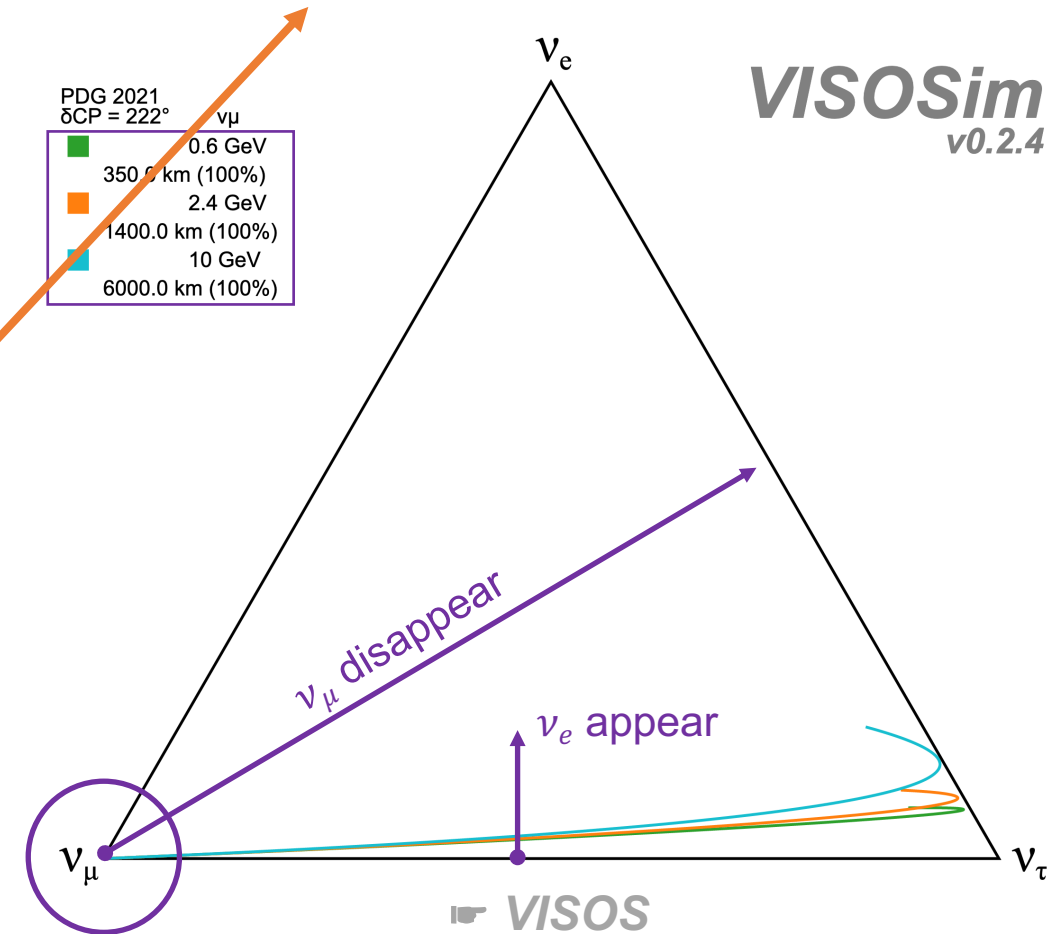


Signal = (Flux · Oscillation probability · Cross section) ⊕ Detector effects
... to discuss tomorrow

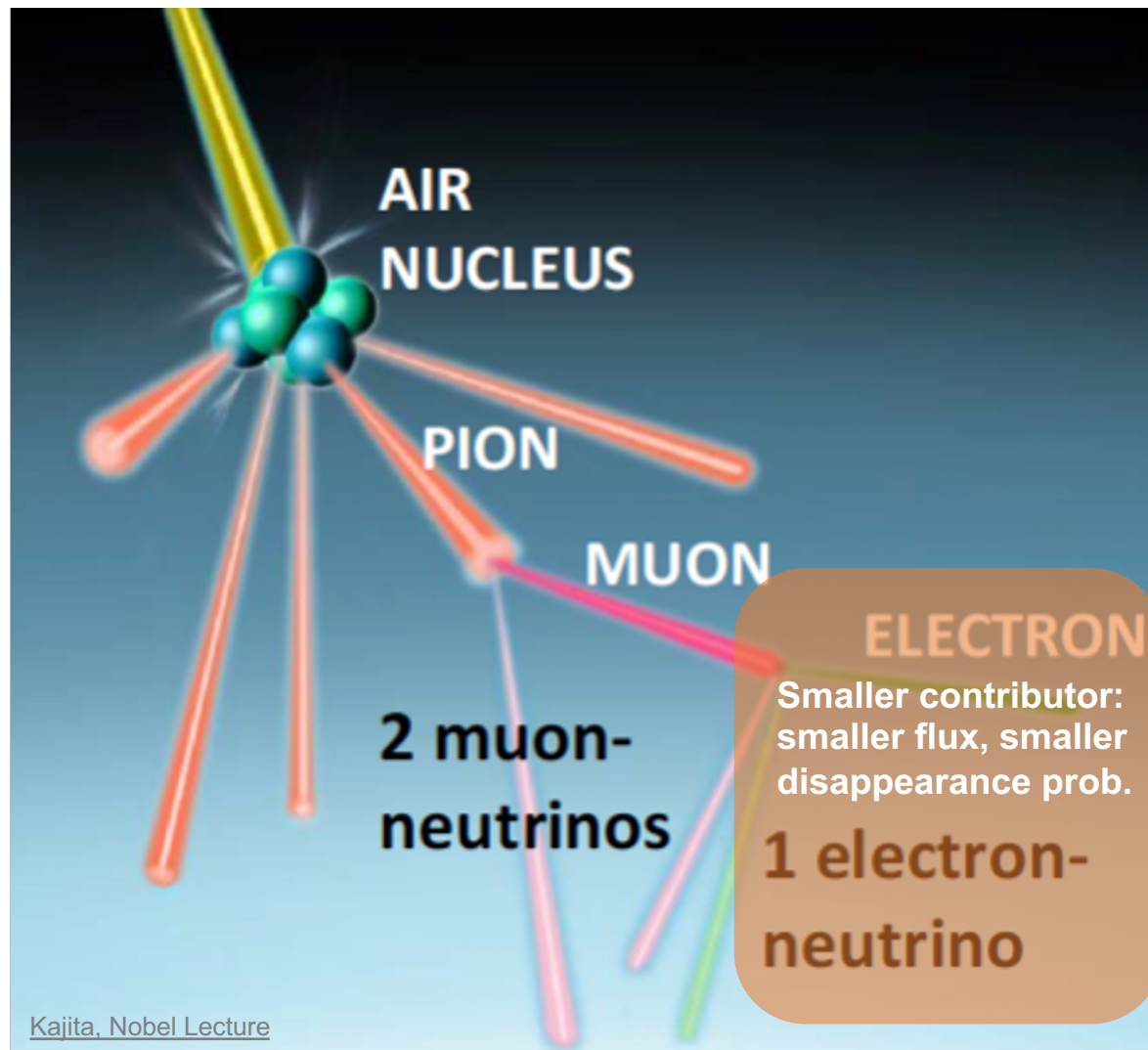
Atmospheric Neutrinos



- Flux: ν_μ and $\bar{\nu}_\mu$
- Oscillation
 - ❖ ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)
 - ❖ ν_e and $\bar{\nu}_e$ appearance



Atmospheric Neutrinos



□ Flux: ν_μ and $\bar{\nu}_\mu$

□ Oscillation

❖ ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)

❖ ν_e and $\bar{\nu}_e$ appearance

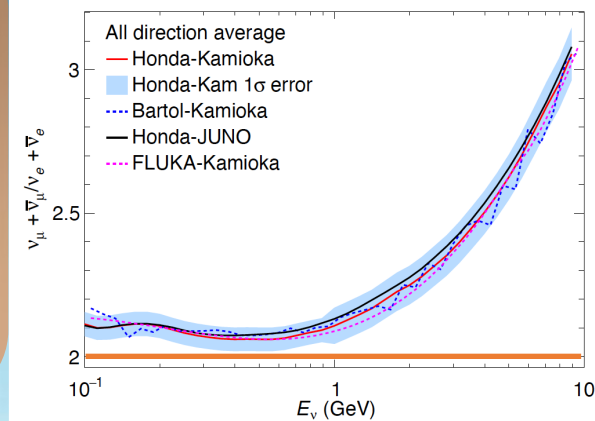
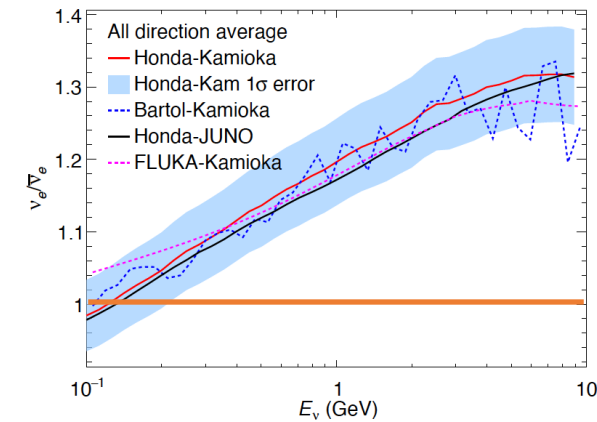
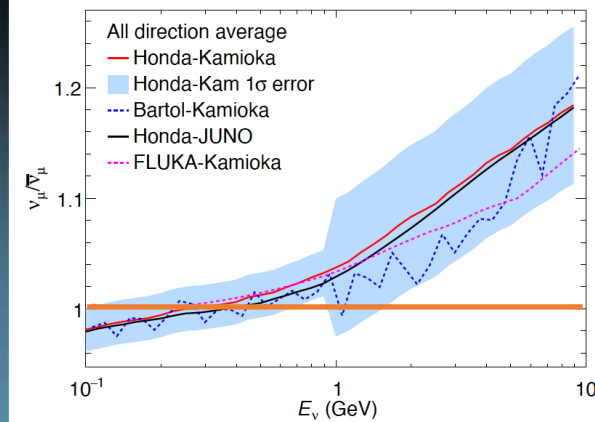


Figure courtesy: Z.-O. He, Warwick

Naïve expectation

$$\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} = 2, \quad \frac{\nu}{\bar{\nu}} = 1$$

Quiz: What causes
 $\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} = 2$ *to fail?*

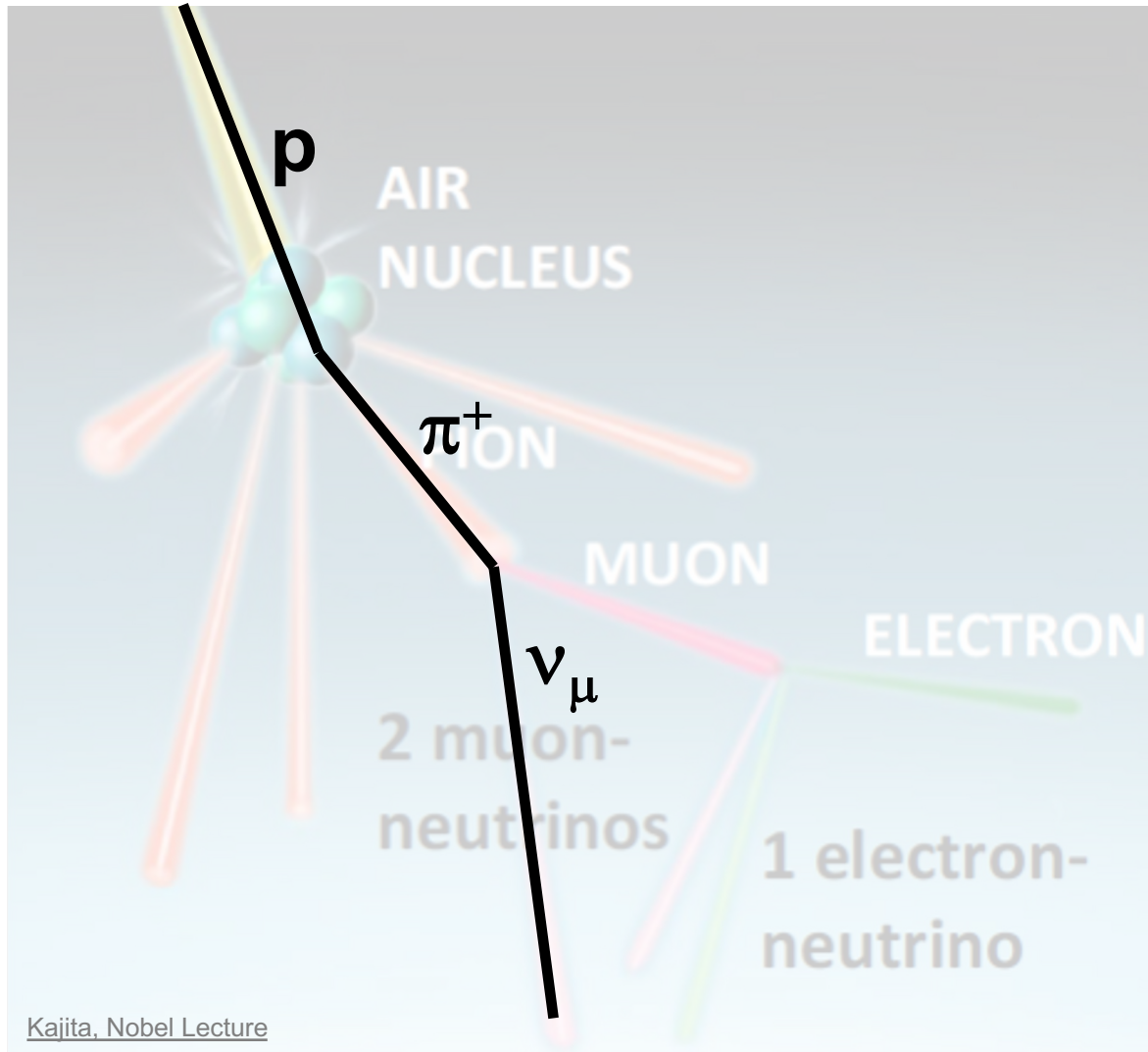
Atmospheric Neutrinos

Fully uncontrolled →

☐ Flux: ν_μ and $\bar{\nu}_\mu$

☐ Oscillation

- ❖ ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)
- ❖ ν_e and $\bar{\nu}_e$ appearance

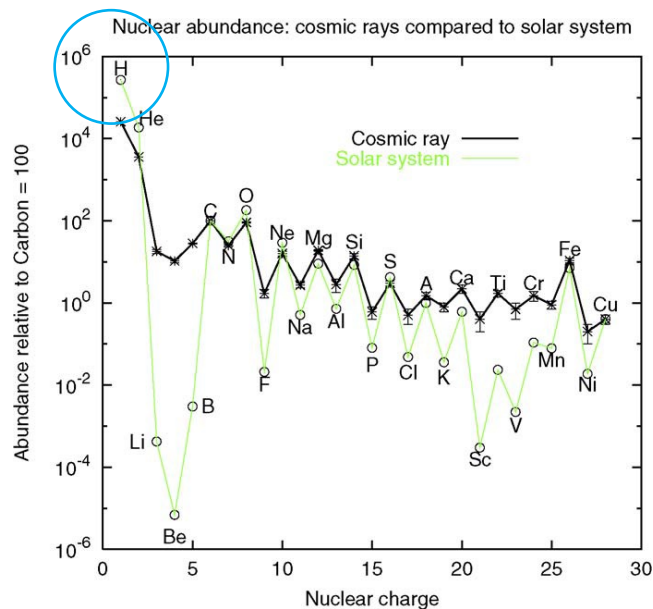
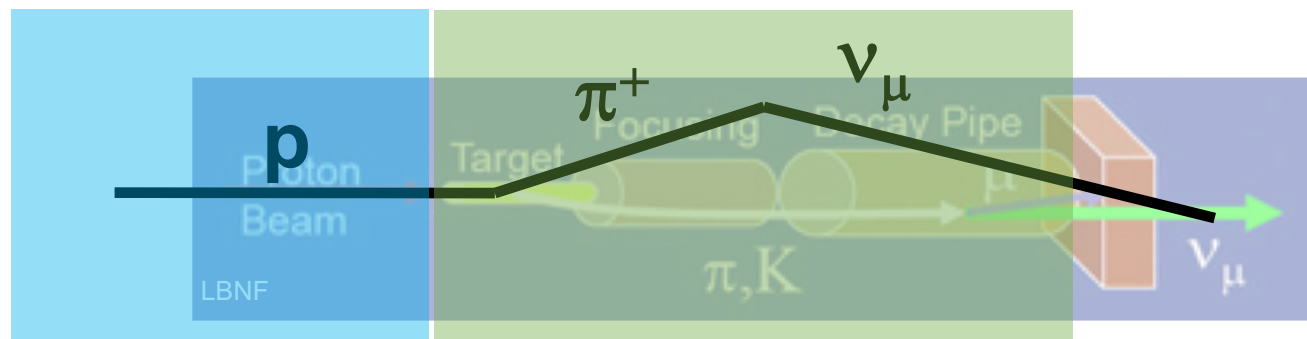


Atmospheric Neutrinos

Fully uncontrolled →

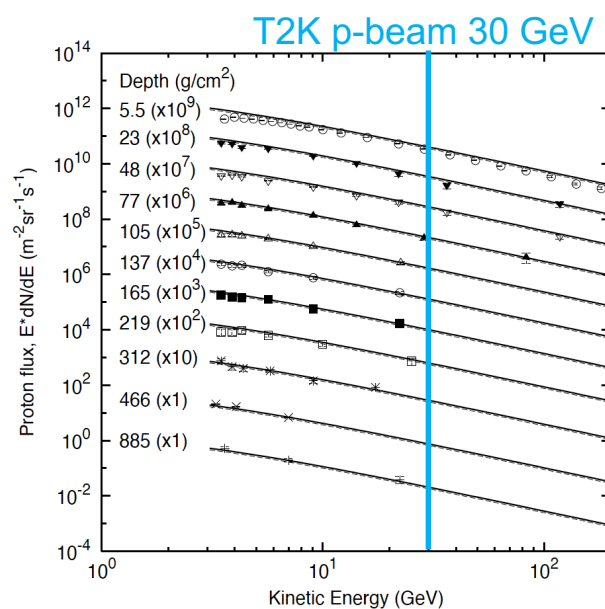
- Flux: ν_μ and $\bar{\nu}_\mu$
- Oscillation

- ❖ ν_μ and $\bar{\nu}_\mu$ disappearance (most oscillated to ν_τ and $\bar{\nu}_\tau$)
- ❖ ν_e and $\bar{\nu}_e$ appearance



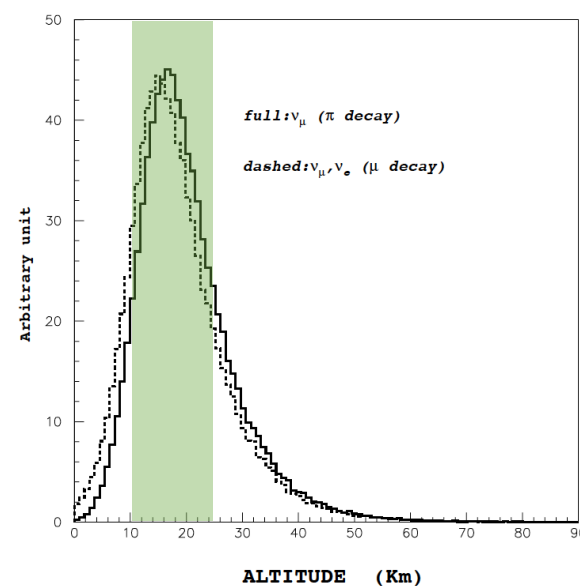
Gaisser and Stanev, Nucl.Phys.A 777, 98 (2006)

□ Cosmic rays: p + others



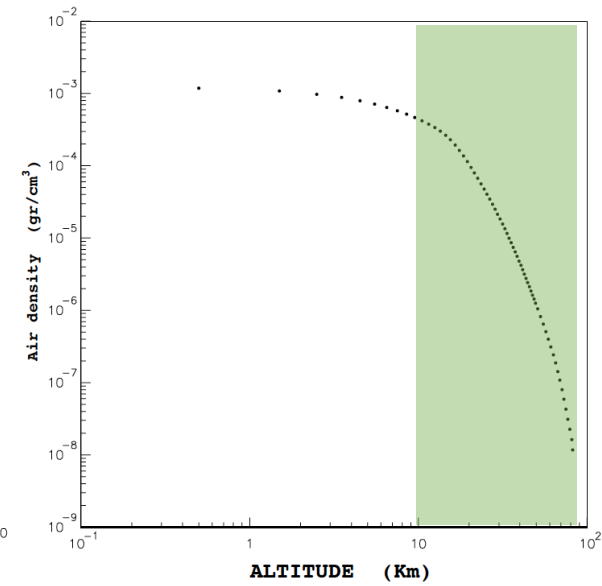
Gaisser, astro-ph/0501195

□ Not monoenergetic



Favier et al., Phys.Rev.D 68, 093006 (2003)

□ inhomogeneous target, non-localised production



Atmospheric neutrino problem

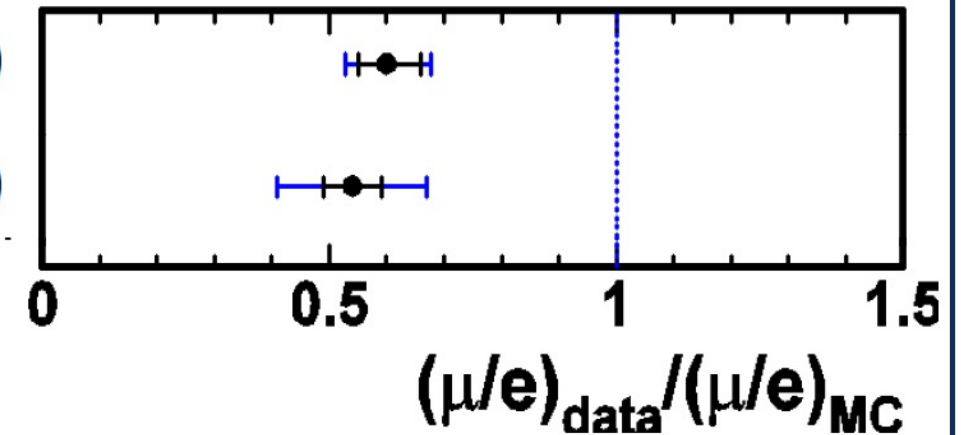
- ✓ Proton decay experiments in the 1980's observed many atmospheric neutrino events.
- ✓ Because atmospheric neutrinos were the most serious background to the proton decay searches, it was necessary to understand atmospheric neutrino interactions.
- ✓ During these studies, a significant deficit of the muon-neutrinos events was observed.



Kamiokande (1988, 92, 94)

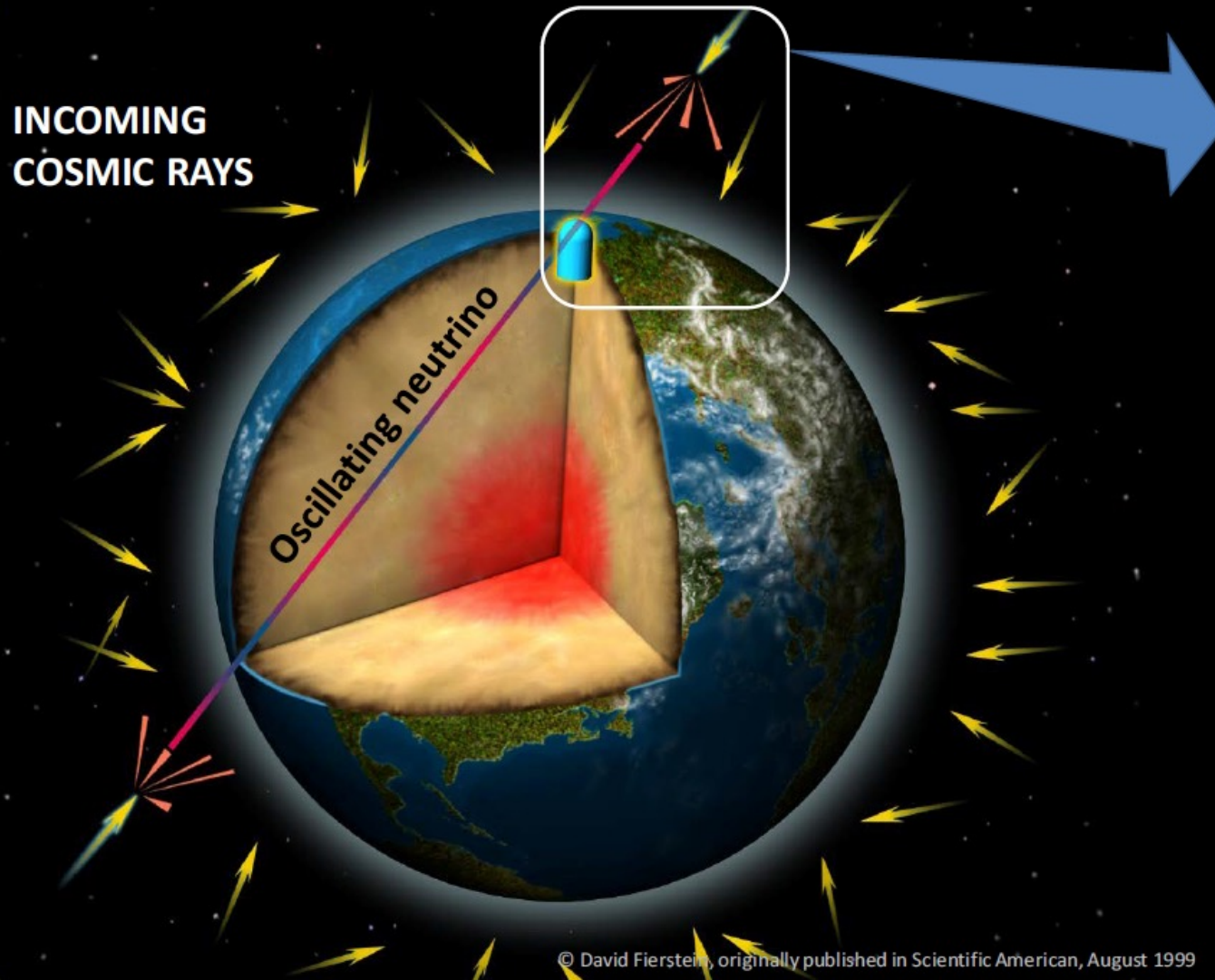


IMB (1991, 92)

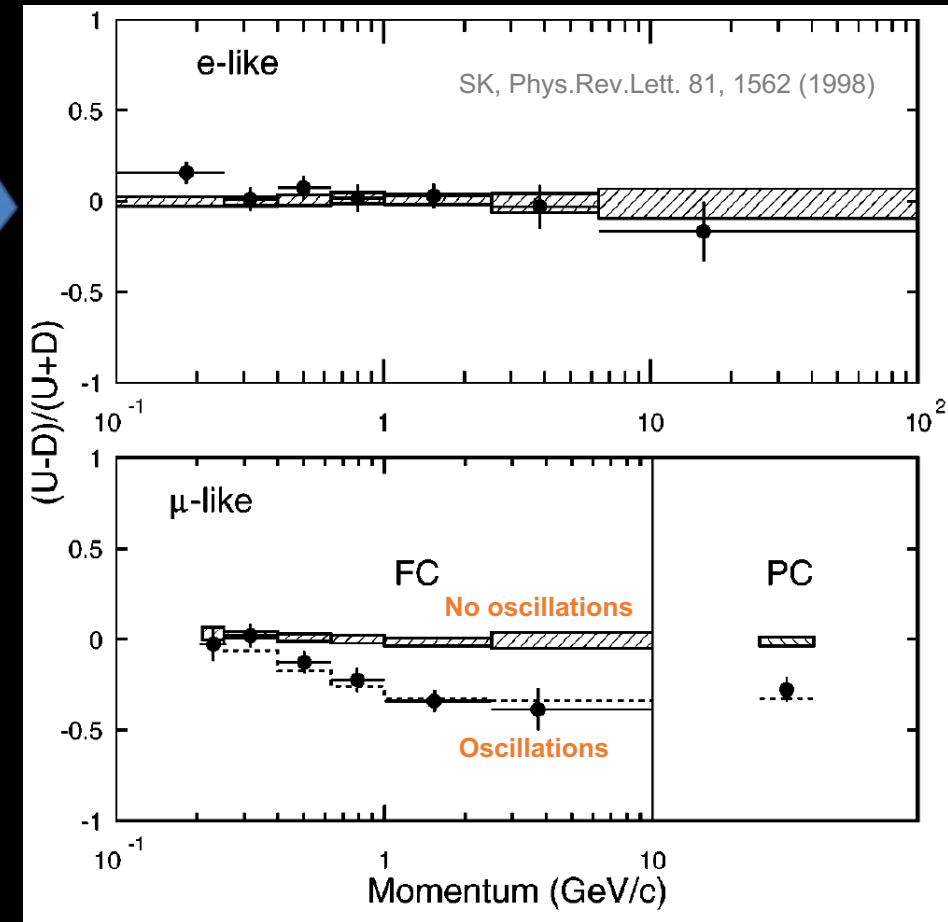


Atmospheric Neutrinos

Discovery of neutrino oscillations



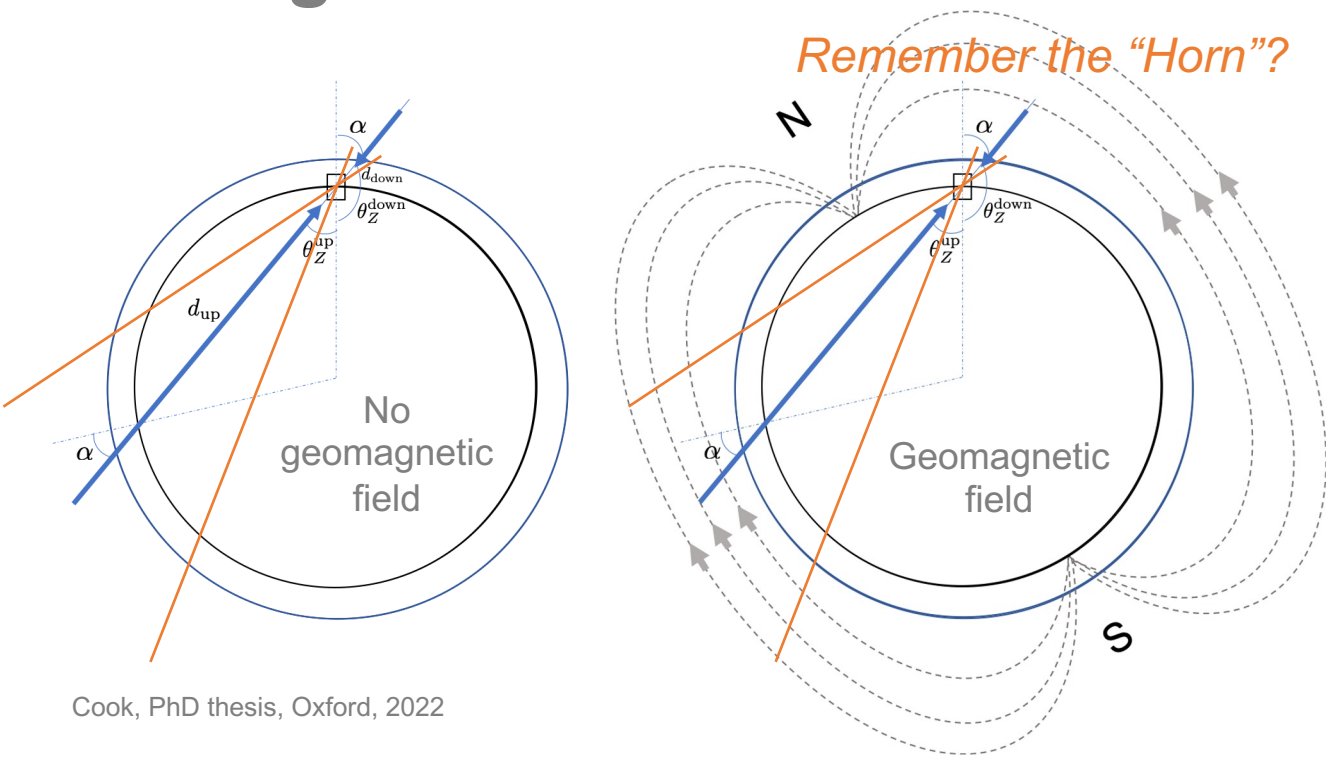
Kajita, Nobel Lecture



- L -dependence: zenith angle variation
- Many systematics cancelled in up-down asymmetry

© David Fierstein, originally published in Scientific American, August 1999

Geomagnetic effect



B (G)	Down (local sky)	Up (over much larger sky region)
SK	0.30	-
JUNO	0.38	-

- ❑ Intrinsic up-down flux asymmetry can mimic oscillation
 - Local B in flux calculation is crucial
 - Worse at low energy

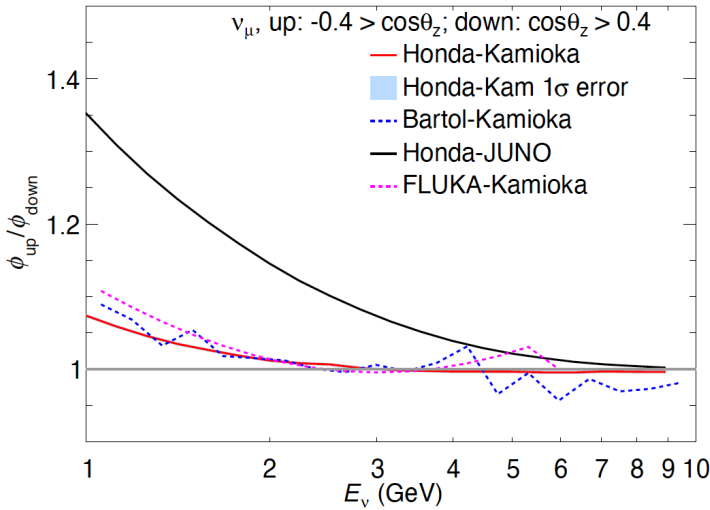
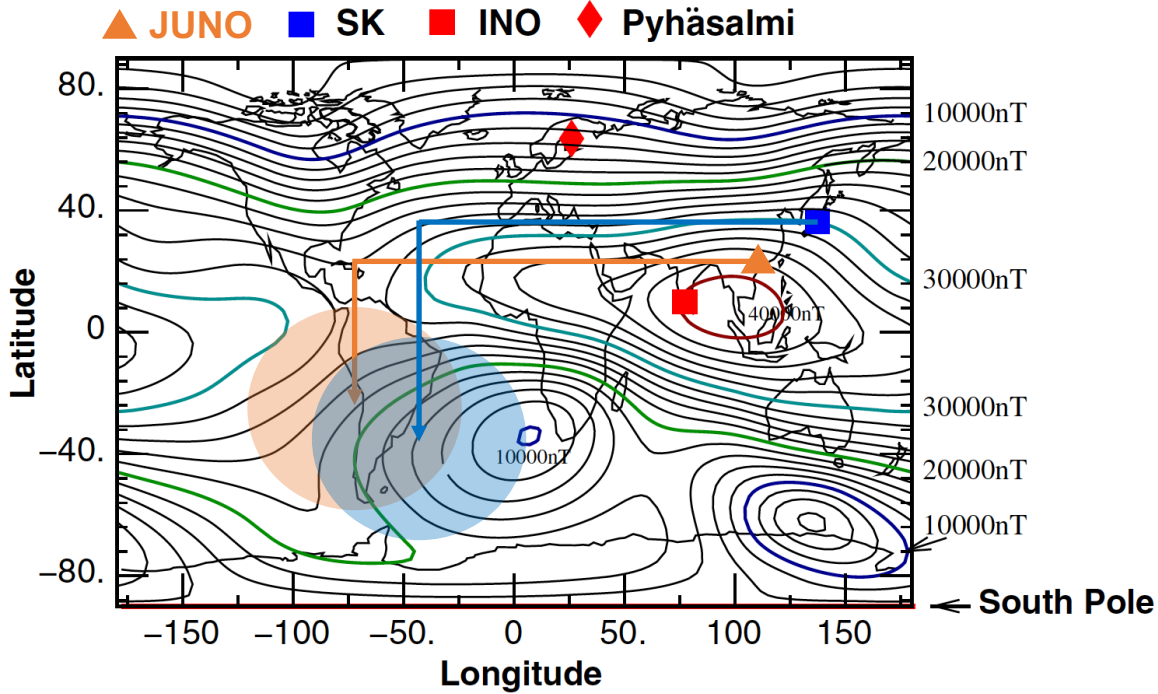


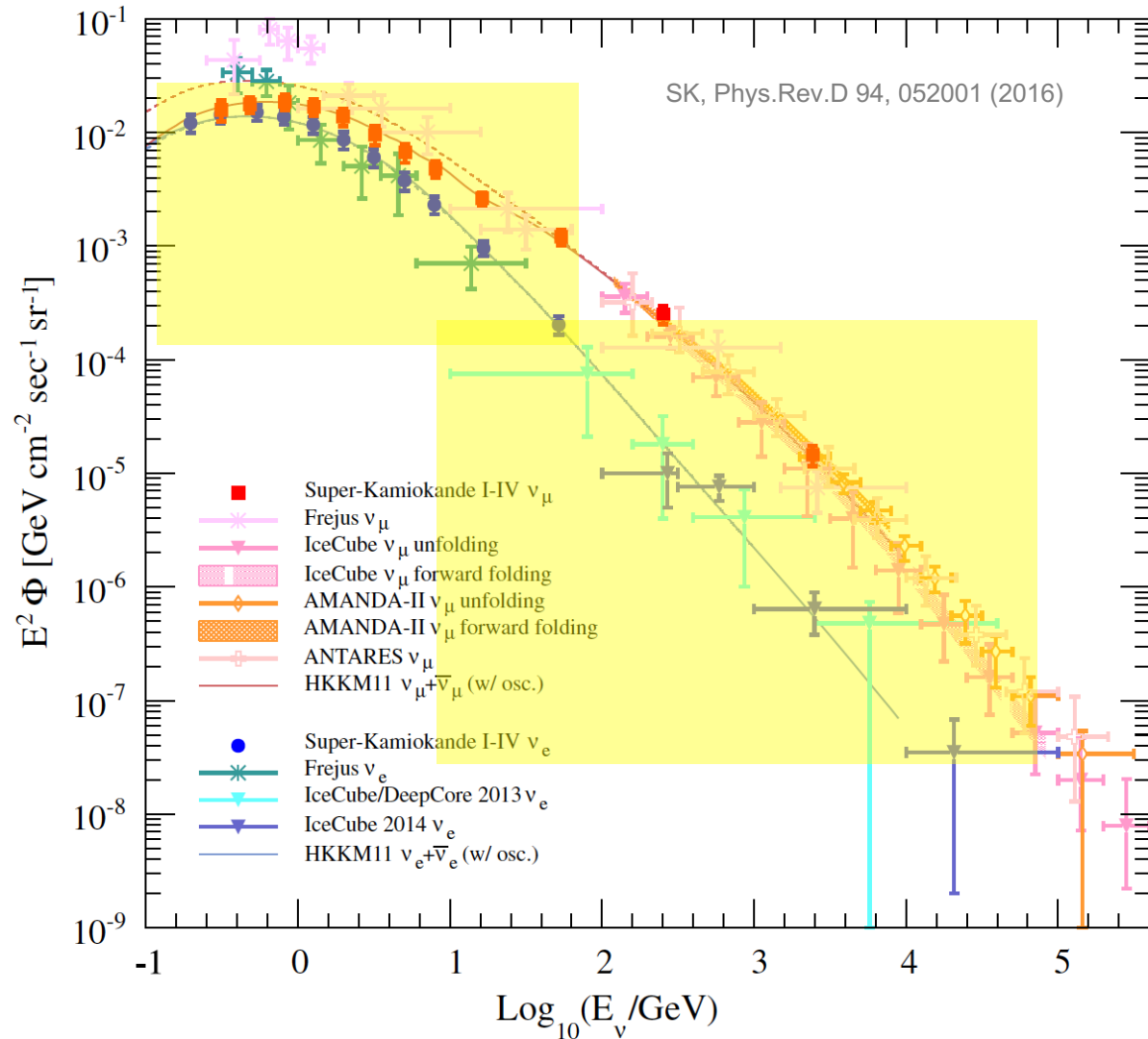
Figure courtesy:
Z.-O. He, Warwick

Table A1. Systematic error used in this analysis. The second column shows the best-fit value of the systematic error parameter in % and the third column shows the estimated 1σ error size in %. The fit result is within the

Systematic error			Fit value (%)	σ (%)
Flux normalization	$E_\nu < 1 \text{ GeV}^1$		7.3	25
	$E_\nu > 1 \text{ GeV}^2$		9.6	15
$(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$	$E_\nu < 1 \text{ GeV}$		1.3	2
	$1 < E_\nu < 10 \text{ GeV}$		2.7	3
	$E_\nu > 10 \text{ GeV}^3$		6.6	5
$\bar{\nu}_e/\nu_e$	$E_\nu < 1 \text{ GeV}$		−1.1	5
	$1 < E_\nu < 10 \text{ GeV}$		−2.7	5
	$E_\nu > 10 \text{ GeV}^4$		−0.34	8
$\bar{\nu}_\mu/\nu_\mu$	$E_\nu < 1 \text{ GeV}$		0.36	2
	$1 < E_\nu < 10 \text{ GeV}$		3.24	6
	$E_\nu > 10 \text{ GeV}^5$		9.2	15
Up/down ratio	< 400 MeV	e -like	0.079	0.1
		μ -like	0.24	0.3
		0-decay μ -like	0.87	1.1
	> 400 MeV	e -like	0.63	0.8
		μ -like	0.40	0.5
		0-decay μ -like	1.34	1.7
	Multi-GeV	e -like	0.55	0.7
		μ -like	0.16	0.2
	Multi-ring sub-GeV	e -like	0.32	0.4
		μ -like	0.16	0.2
	Multi-ring multi-GeV	e -like	0.24	0.3
		μ -like	0.16	0.2
	PC		0.16	0.2

SK flux error in a typical oscillation analysis

SK Measurement of Atmospheric Neutrino Fluxes

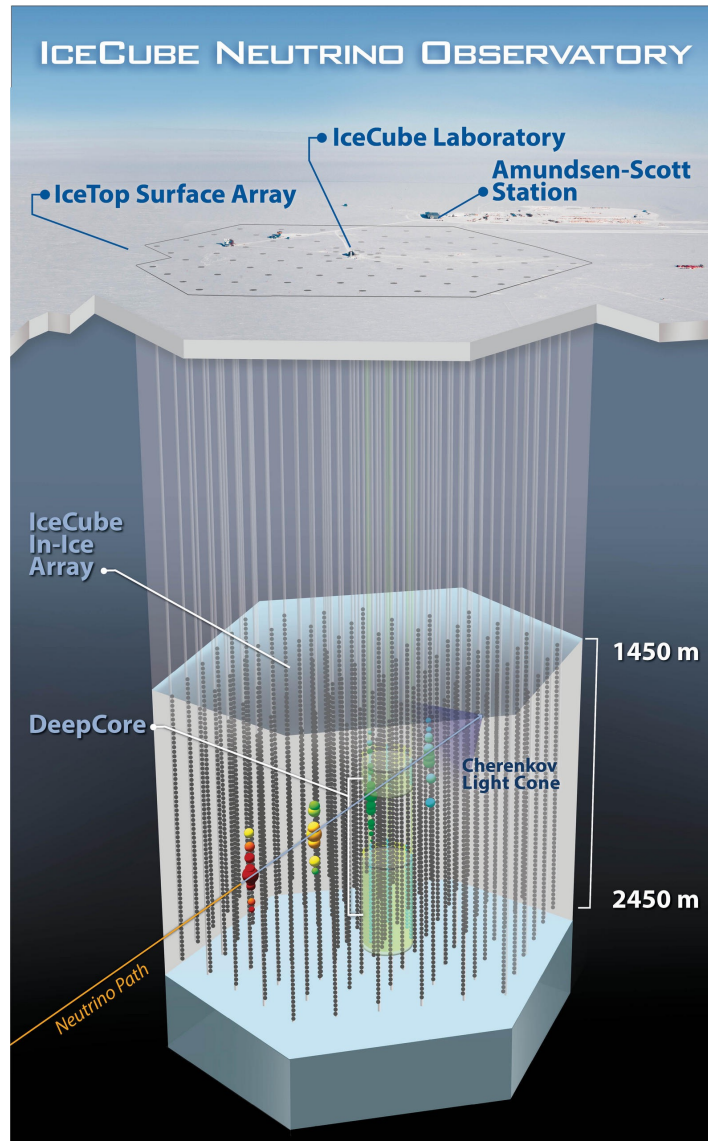


Coming back to IceCube...

SK: low energy
IceCube (DeepCore): high energy

Introducing IceCube

Slide courtesy: Donglian Xu



SK NMO Measurement with Atmospheric Neutrinos

Coming back to the MSW resonance...

Vacuum

$$P(\nu_e \rightarrow \nu_e) \cong 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{1.27 \Delta m_{32}^2 L}{E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\mu) \cong 1 - 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \times \sin^2 \left(\frac{1.27 \Delta m_{32}^2 L}{E} \right)$$

$$P(\nu_\mu \leftrightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{1.27 \Delta m_{32}^2 L}{E} \right)$$

Effective mixing and mass gap in matter

$$\sin^2 2\theta_{13,M} = \frac{\sin^2 2\theta_{13}}{(\cos 2\theta_{13} - A_{CC}/\Delta m_{32}^2)^2 + \sin^2 2\theta_{13}}$$

Normal / Inverted MO

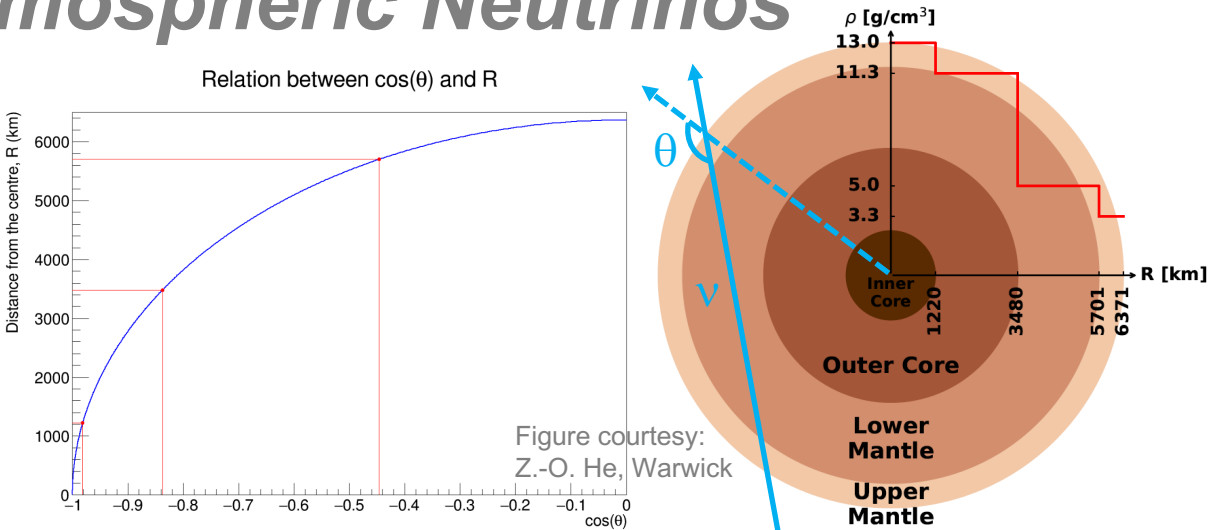
$\Delta m_{32,M}^2 = \Delta m_{32}^2 \sqrt{(\cos 2\theta_{13} - A_{CC}/\Delta m_{32}^2)^2 + \sin^2 2\theta_{13}}$

e.g. SK, PTEP 053F01 (2019)

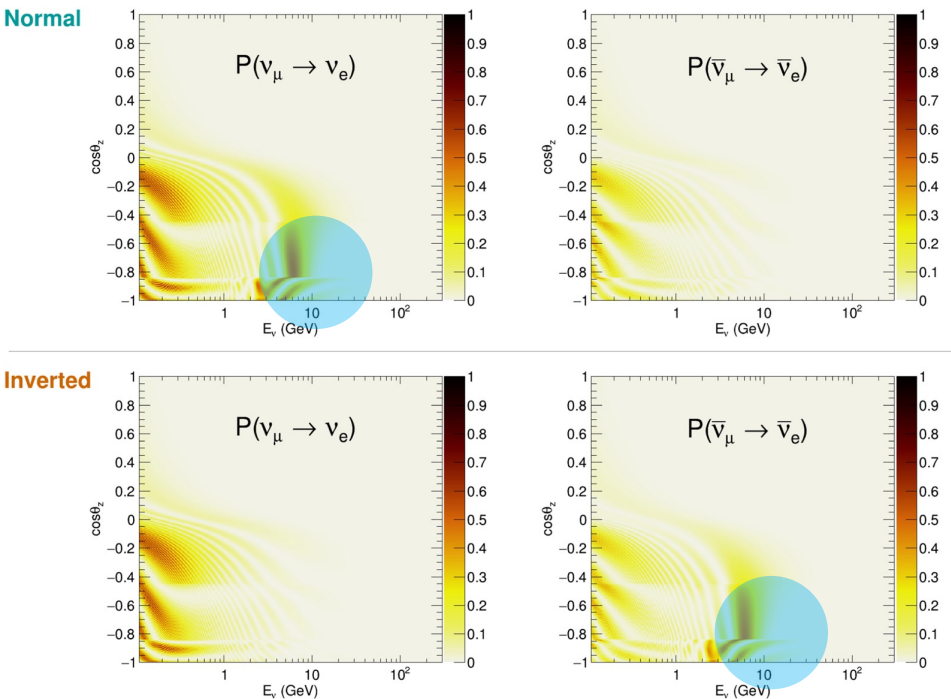
Small θ_{13} , MSW resonance at $A_{CC} \approx \Delta m_{32}^2$

NO: $A_{CC} > 0$, resonance with ν

IO: $A_{CC} < 0$, resonance with $\bar{\nu}$



Matter effect seen as probability distortion



$A_{CC} = \Delta m^2$

$E[\text{GeV}] n_e \left[\frac{N_A}{\text{cm}^3} \right] = \frac{\Delta m^2 [10^{-5} \text{eV}^2]}{2 \times 7.63 \times 10^{-14} \left[\frac{\text{eV cm}^3}{N_A} \right]}$

$E n_e \approx \frac{\Delta m^2}{15}, E \text{ in GeV}, n_e \text{ in } \frac{N_A}{\text{cm}^3}, \Delta m^2 \text{ in } 10^{-5} \text{eV}^2$

Water $n_e = \frac{5}{9} \frac{N_A}{\text{cm}^3}$

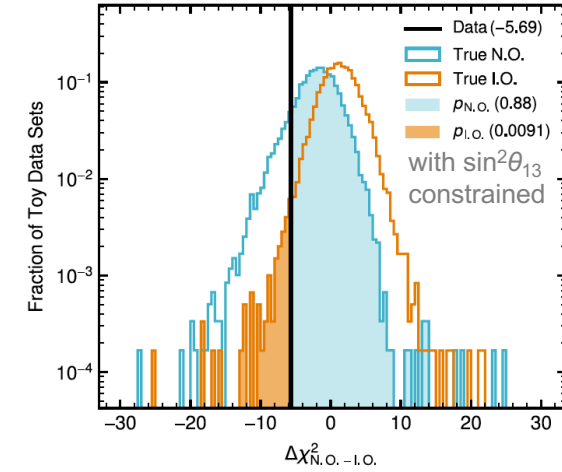
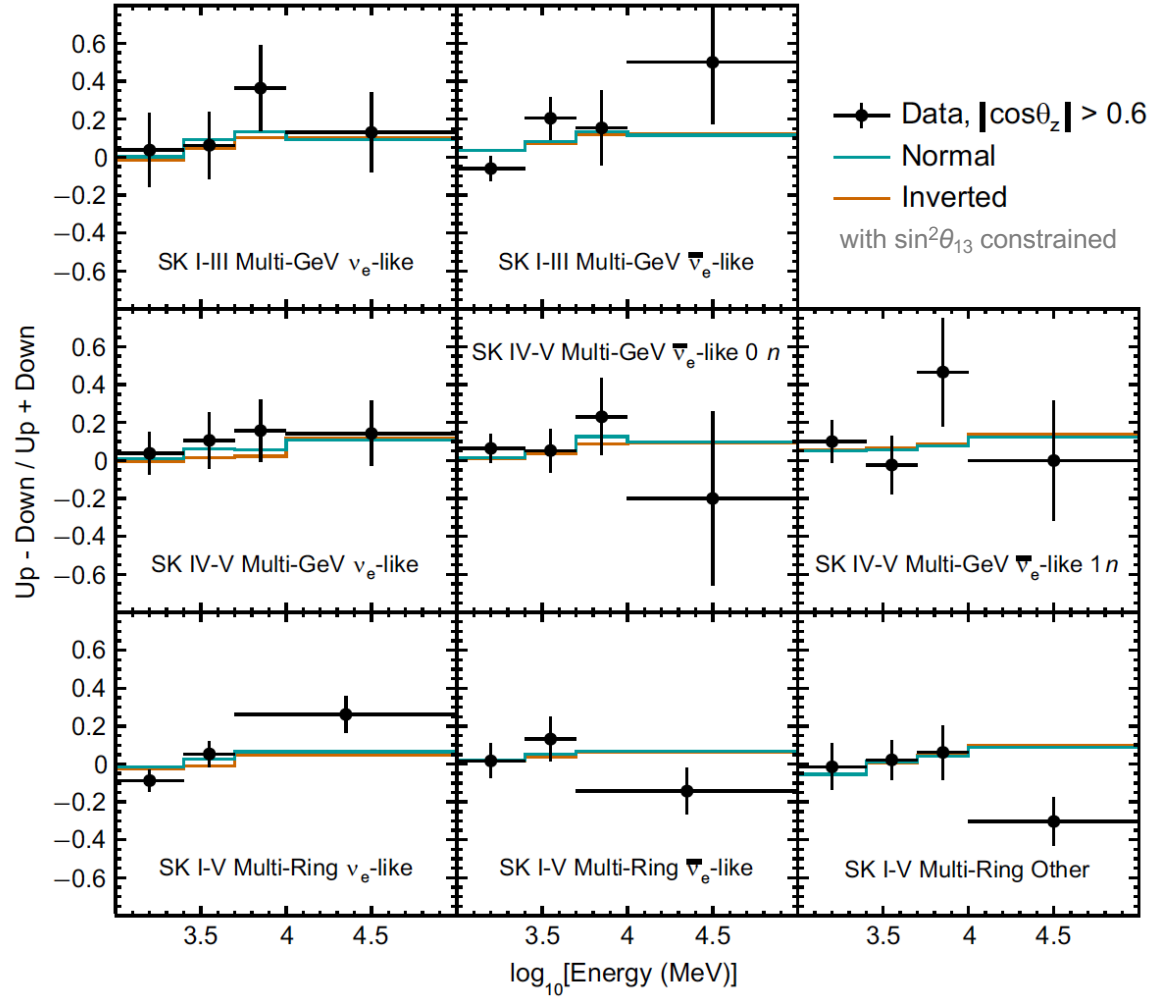
For $\Delta m^2 = 10^{-5} \text{eV}^2, E = 0.12 \text{ GeV}$

For $|\Delta m_{23}^2| = 2 \times 10^{-3} \text{eV}^2, E = 24 \text{ GeV}$

SK, Phys.Rev.D 109, 072014 (2024)

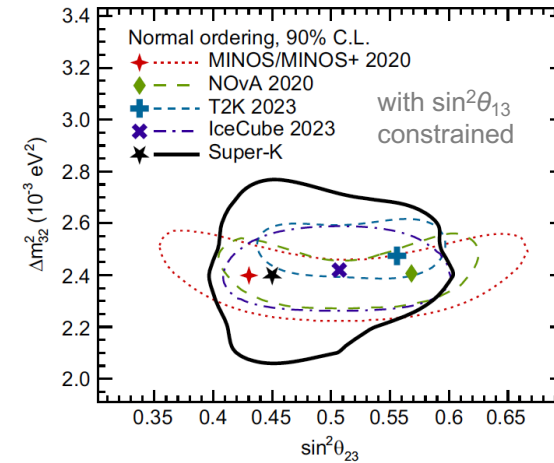
SK NMO Measurement with Atmospheric Neutrinos

SK, Phys.Rev.D 109, 072014 (2024)



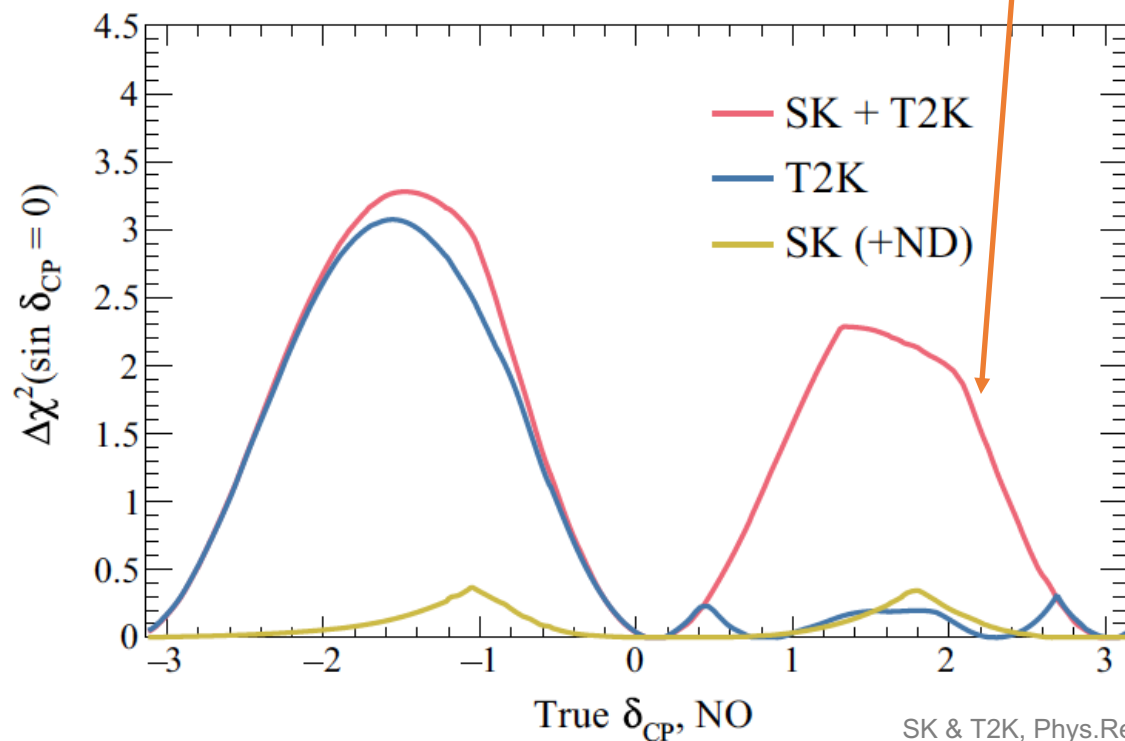
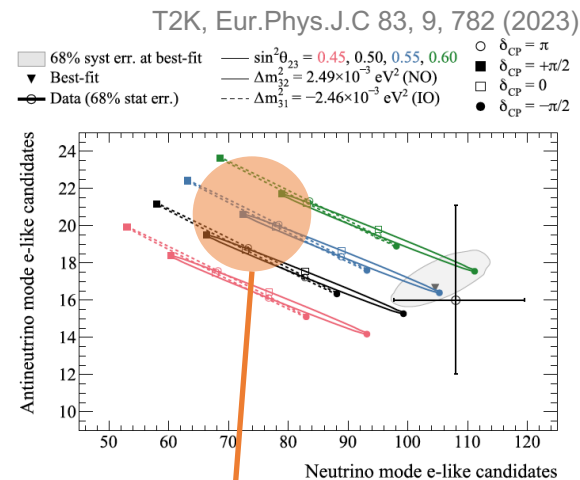
$$\text{CL}_s = \frac{p_{\text{I.O.}}}{1 - p_{\text{N.O.}}} = 0.077, \text{ rejecting IO at 92.3\% } (\sim 1.4\sigma)$$

Posiadala-Zezula, NEUTRINO 2024



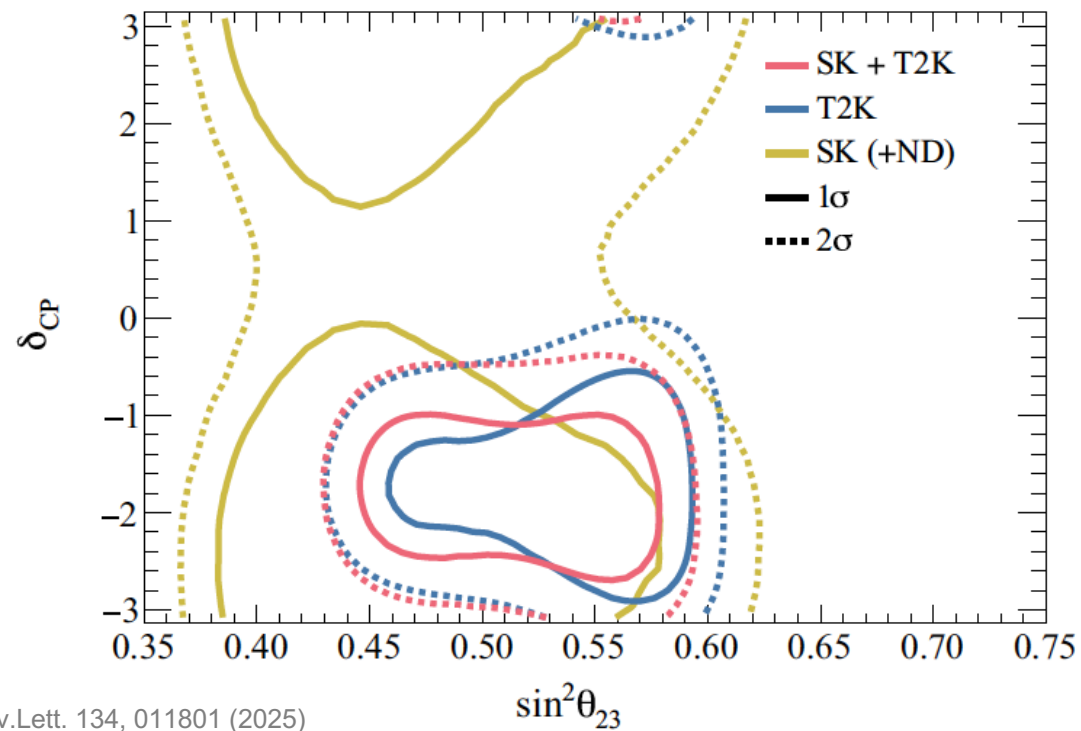
T2K+SK Joint Analysis

Coming back to NMO constraints...



Sensitivity

SK & T2K, Phys.Rev.Lett. 134, 011801 (2025)

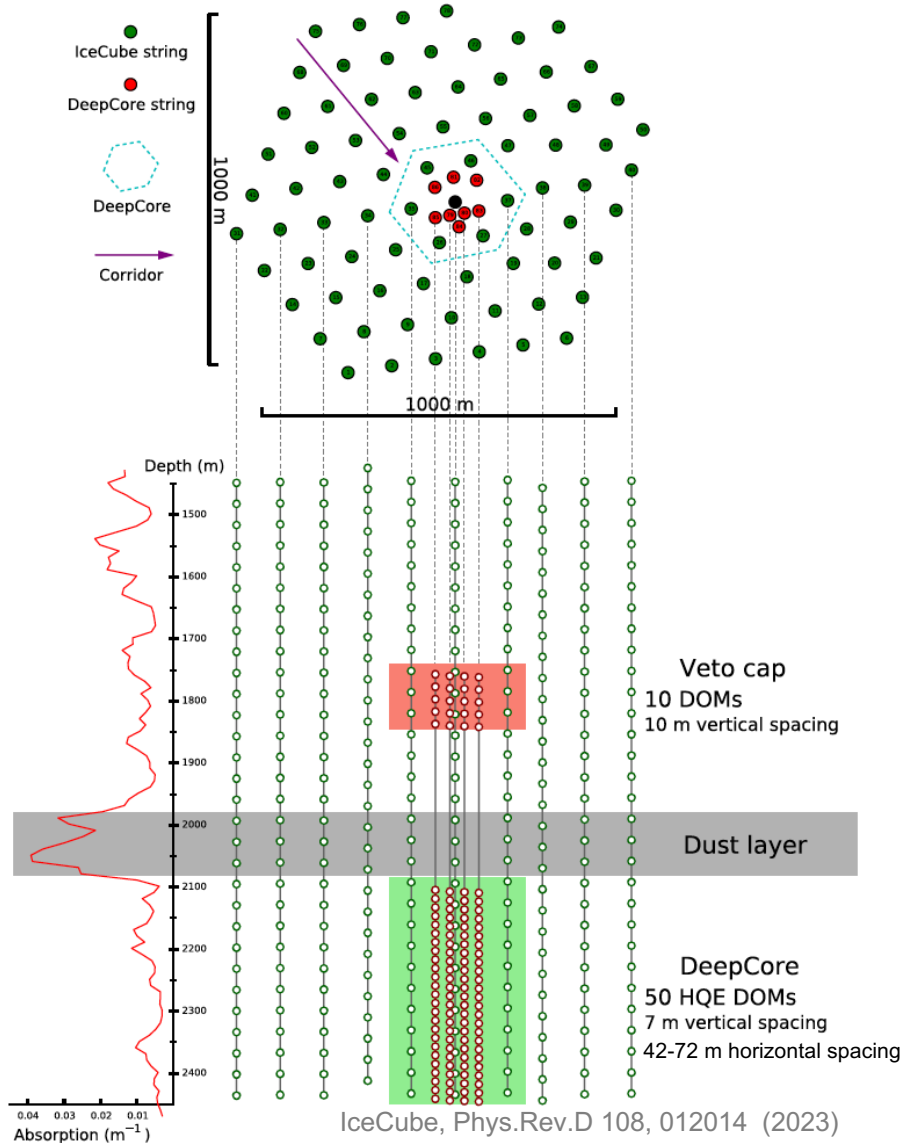


Data results

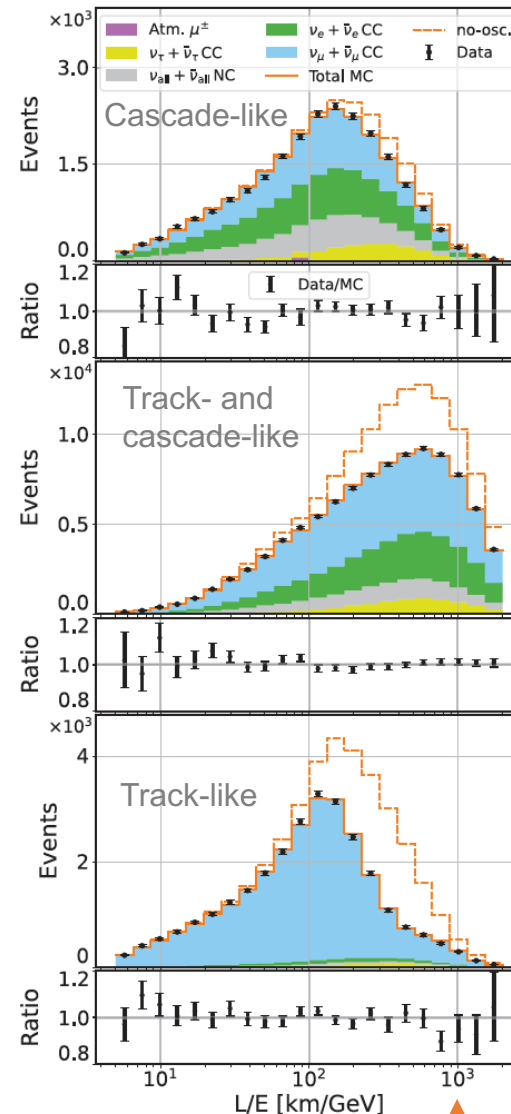
IceCube DeepCore Atmospheric Oscillation Measurement

IceCube, Phys.Rev.Lett. 134, 091801 (2025)

Quiz: What are cascade-like events?

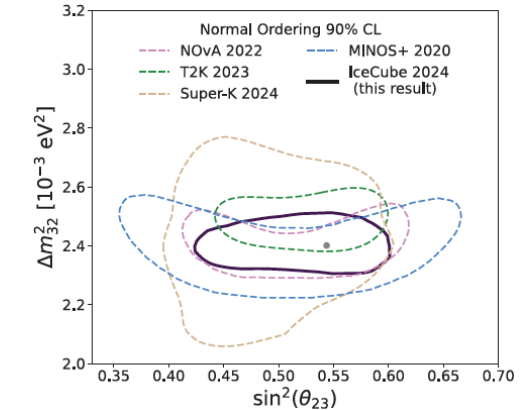


IceCube, Phys.Rev.D 108, 012014 (2023)



Larger E ← 10k km/10 GeV → Larger L

IceCube, Phys.Rev.Lett. 134, 091801 (2025)



Competitive with other experiments!

1. Huge statistics: km³-scale detector → far more multi-GeV
2. Best L/E coverage: optimised for 5–50 GeV range
3. Lower flux uncertainty: multi-GeV flux is better modelled
4. Cross section uncertainty: multi-GeV xsec dominated by DIS
5. In-situ flux calibration: high-energy unoscillated sample

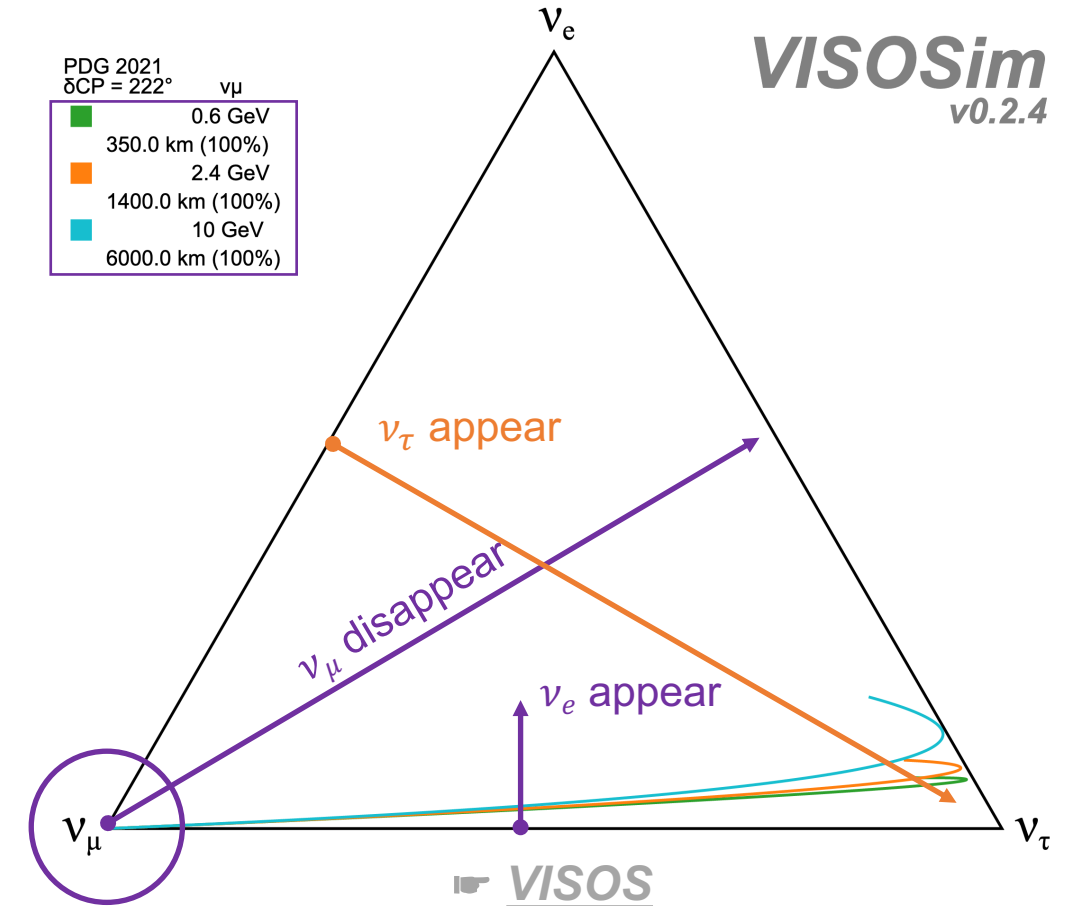
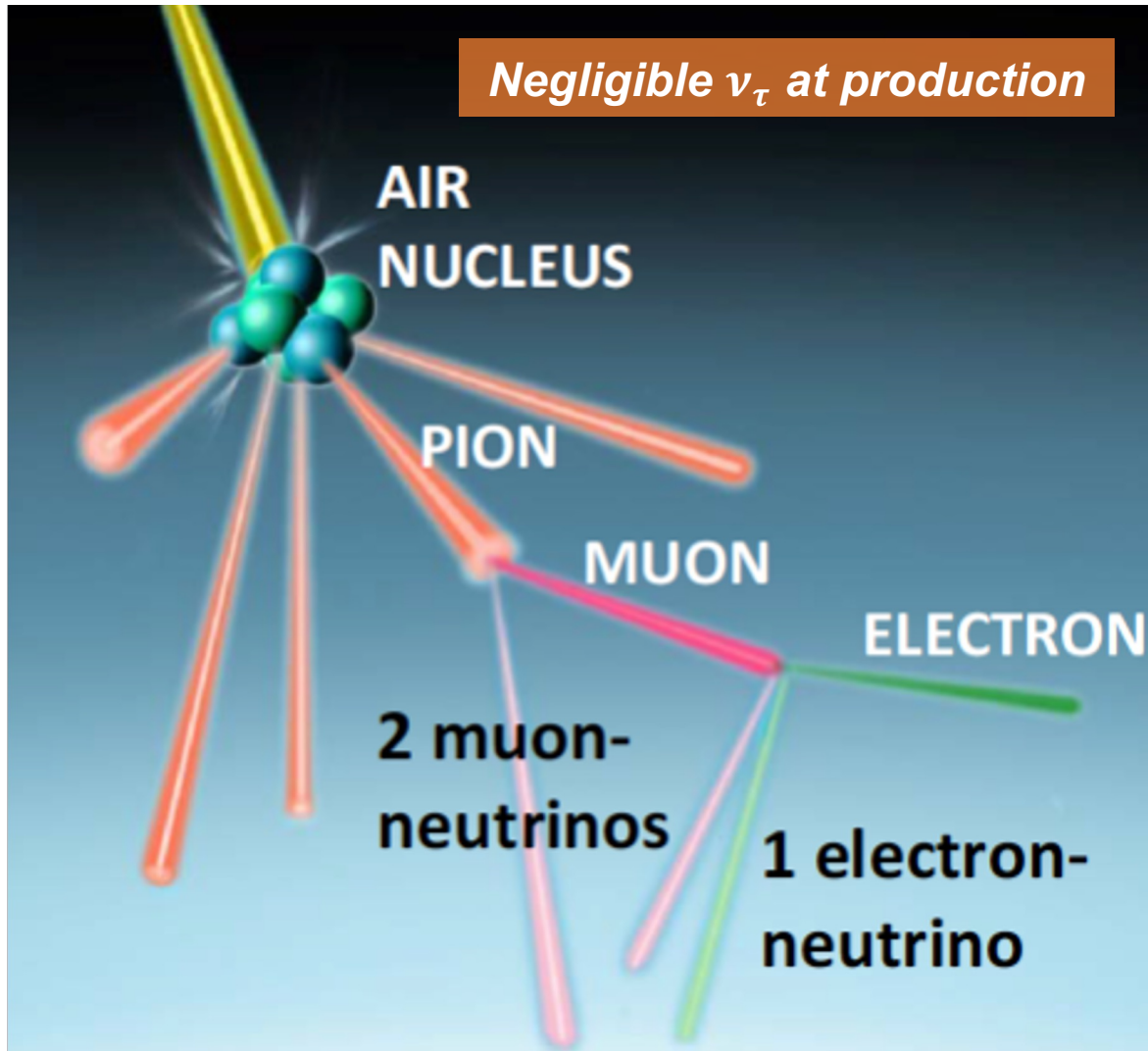
But not for NMO:

- Miss the MSW resonance at lower energy
- PINGU (or ORCA) will lower the energy threshold
 - ❖ But then all the low-energy systematics (flux, xsec) are like SK—no way around for NMO
 - ❖ One can play with stat. and detector technology.

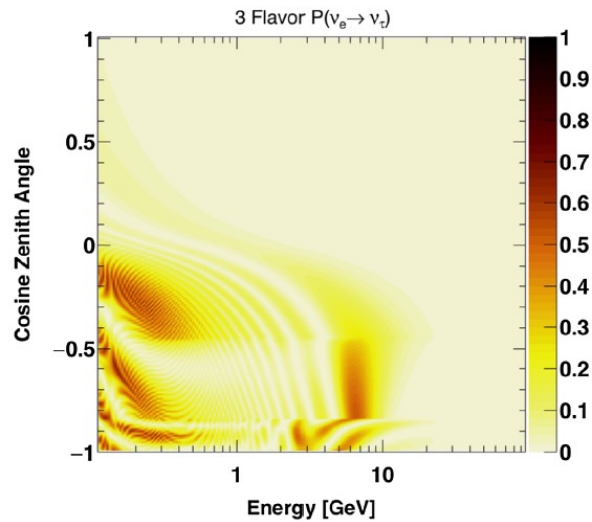
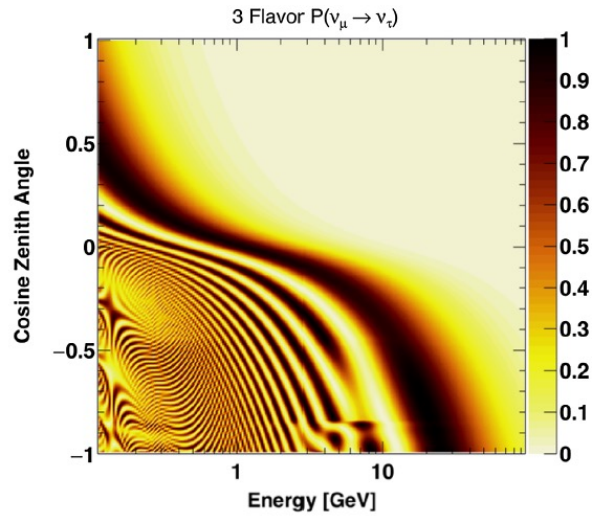
ν_τ Appearance with Atmospheric Neutrinos

Completing the 3-flavour oscillation picture

Quiz: Is there matter effect in $\nu_\mu \rightarrow \nu_\tau$?

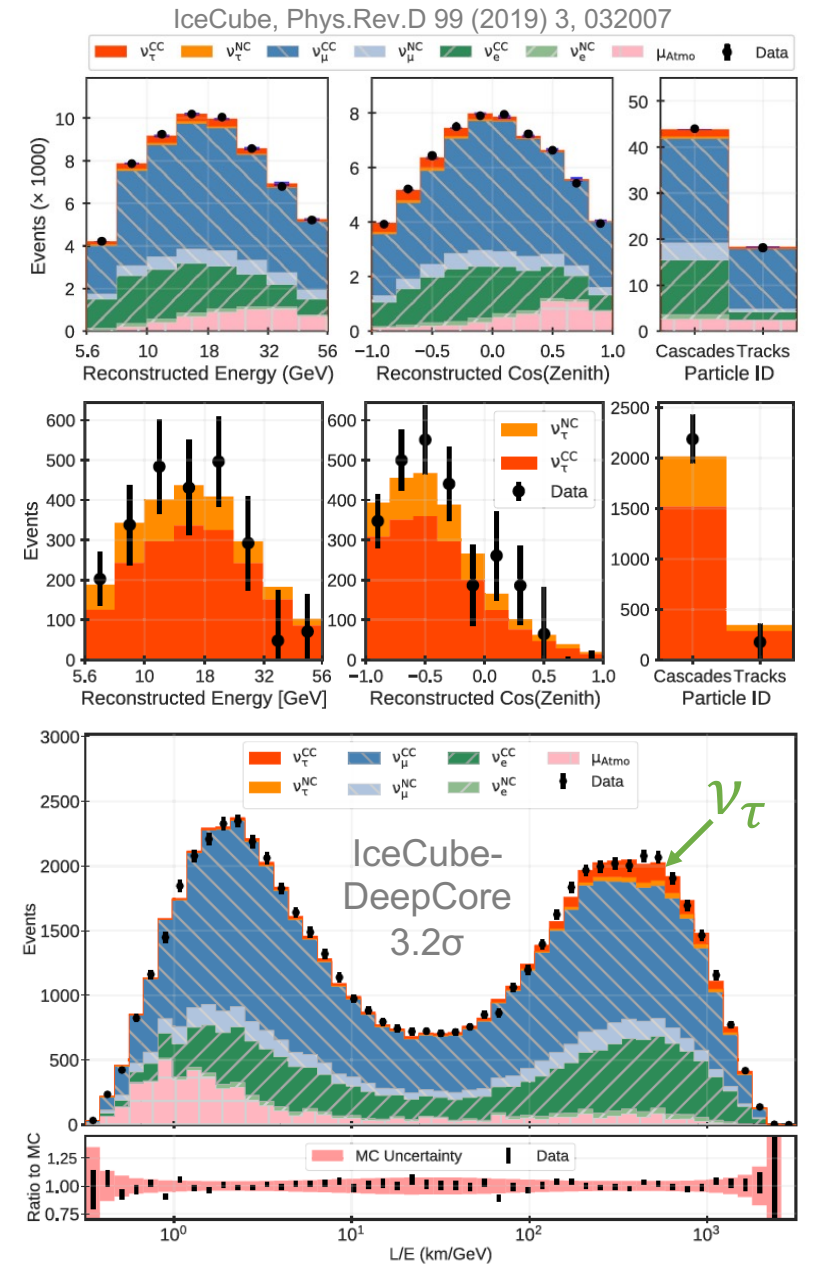
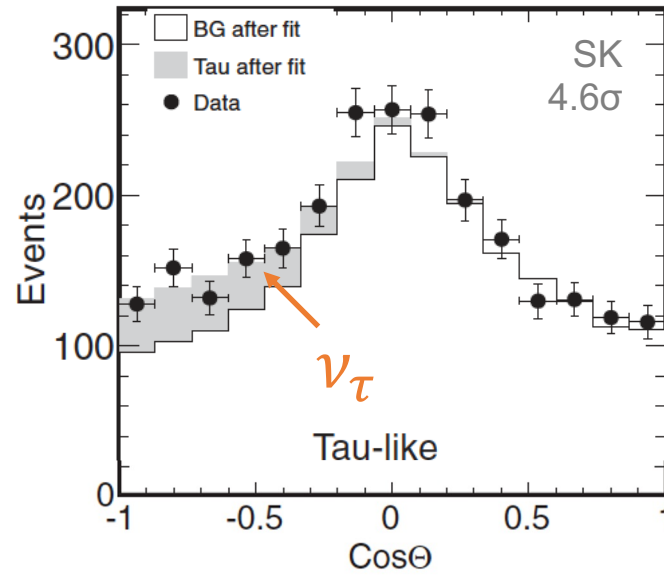
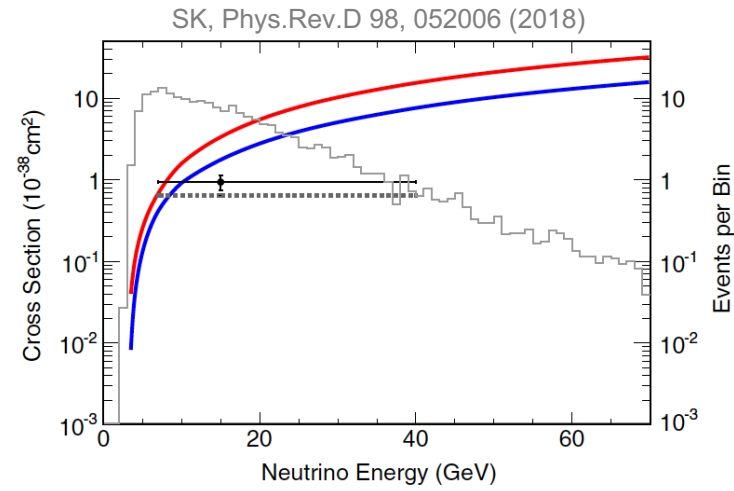


Measurements of ν_τ Appearance



SK, Phys.Rev.D 98, 052006 (2018)

Expected appearance from both ν_μ and ν_e fluxes



Accelerator and Atmospheric Neutrinos

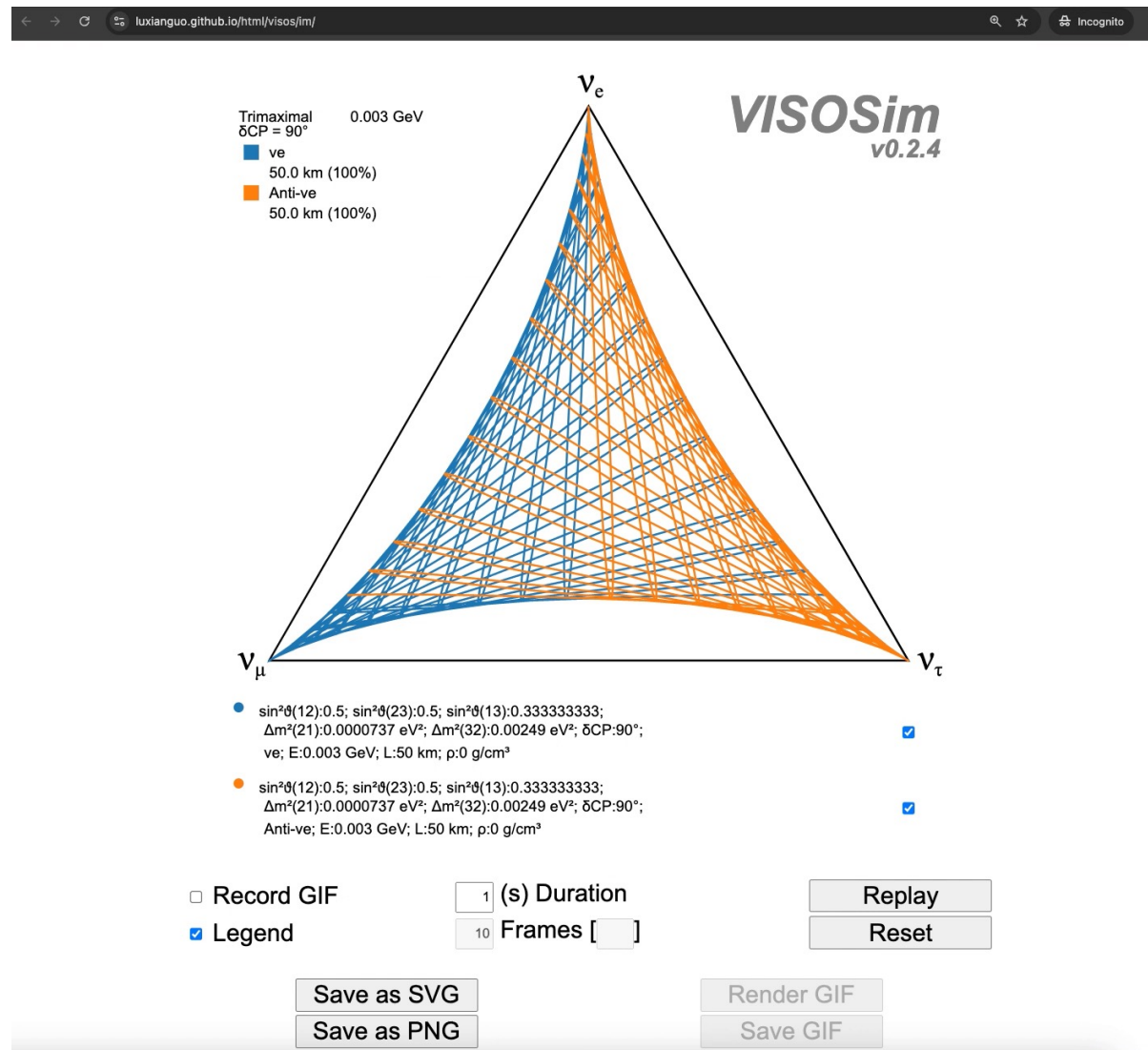
Neutrino mass gap and mixing $\rightarrow$ oscillation

Three masses $\rightarrow$ NMO

Three-flavor mixing $\rightarrow \delta_{\text{CP}}$

1. Neutrino Oscillations
 - a. Visualisation: **spin rotation analogy, VISOSim**
2. Accelerator Neutrinos
 - a. Techniques: **hadron decay beams, near-far measurements**
 - b. Key results: **T2K and NOvA $\rightarrow \delta_{\text{CP}}$ precision needs NMO constraints**
 - c. Future vision: **nuSTORM (maybe!)—futuristic, unique ν_e and $\bar{\nu}_e$ machine**
3. Atmospheric Neutrinos
 - a. MSW matter effect: **start with no mixing / no gap cases**
 - b. Flux: **systematics**
 - c. Key results: **SK and IceCube $\rightarrow$ NMO determination still pending**

A Short Demo for VISOSim



Quiz

1. Which experiment has a 50 km baseline observing few-MeV neutrinos?
2. Is the oscillation driven by Δm_{21}^2 fast or slow? Estimate $|\Delta m_{32}^2|/\Delta m_{21}^2$.
3. Name a few experiments in the “wasteland”? What are they for?
4. Is it driven by Δm_{21}^2 or Δm_{32}^2 ? (What is $|\Delta m_{32}^2|/\Delta m_{21}^2$ now? Is it reasonable?)
5. Is there oscillation when the mixing angle is $\pi/2$?
6. What causes $\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} = 2$ to fail?
7. What are cascade-like events?
8. Is there matter effect in $\nu_\mu \rightarrow \nu_\tau$?

Homework

1. **Show that** $|S_x\rangle_t = \cos \frac{\Delta E}{2} t |S_x\rangle_0 - \sin \frac{\Delta E}{2} t |S_y\rangle_0$
2. **Estimate Δm_{21}^2 and compare your result with PDG.**
3. **Write down the Feynman diagrams of ν_e and $\bar{\nu}_e$ CC interactions with electrons.**
4. **Recover the previous no-mixing and small gap cases with the general effective mass gap and mixing angle.**

BACKUP

$$P(\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)) = P_1 + P_2 + P_3 + P_4$$

$$P_1 = \sin^2 \theta_{23} \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_{-+}} \right)^2 \sin^2 \left(\frac{B_{-+}}{2} L \right)$$

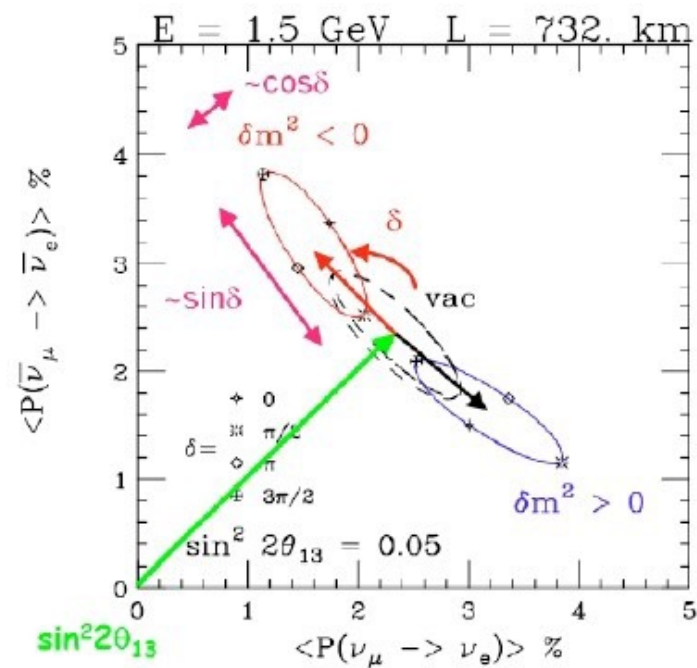
$$P_2 = \cos^2 \theta_{23} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \left(\frac{A}{2} L \right)$$

$$P_3 = \underbrace{J}_{\text{red}} \underbrace{\cos \delta}_{\text{magenta}} \underbrace{\cos \left(\frac{\Delta_{23}}{2} L \right) \left(\frac{\Delta_{12}}{A} \frac{\Delta_{13}}{B_{-+}} \right)}_{\text{blue}} \sin \left(\frac{A}{2} L \right) \sin \left(\frac{B_{-+}}{2} L \right)$$

$$P_4 = \pm \underbrace{J}_{\text{red}} \underbrace{\sin \delta}_{\text{magenta}} \underbrace{\sin \left(\frac{\Delta_{23}}{2} L \right) \left(\frac{\Delta_{12}}{A} \frac{\Delta_{13}}{B_{-+}} \right)}_{\text{blue}} \sin \left(\frac{A}{2} L \right) \sin \left(\frac{B_{-+}}{2} L \right)$$

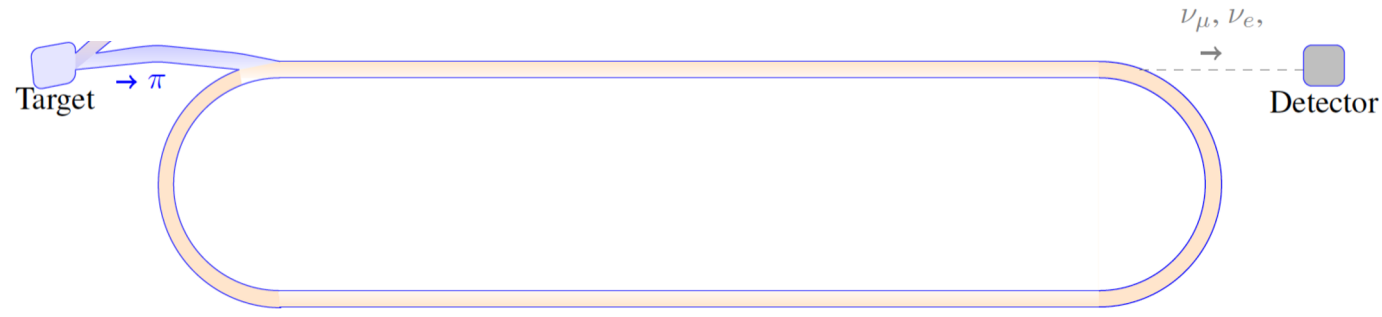
$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E} \quad A = \sqrt{2} G_F N_e \quad B_{-+} = |\Delta_{13} \mp A| \quad J = \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13}$$

- θ_{13}
- $\theta_{23} > 45$ or $\theta_{23} < 45$
- $\text{Sign}(\Delta m_{23}^2)$
- δ_{MNSP}



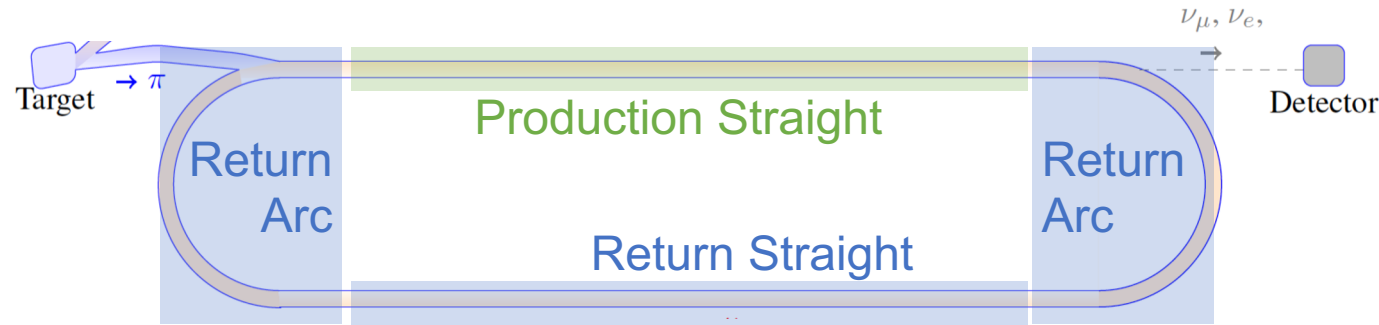
e.g. Boyd, [Warwick Week 2025 lecture](#)

ν from STORed Muons (nuSTORM)



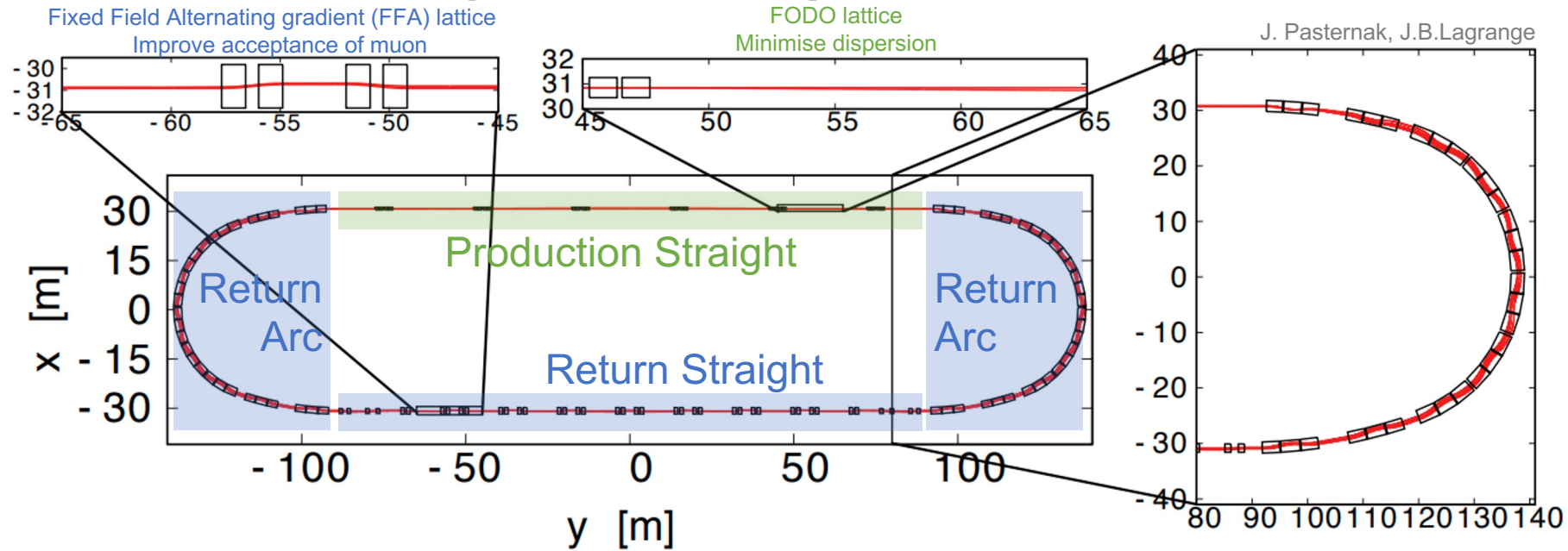
- ❑ **1st ν beam facility & highest ever beam power based on stored muons**
- ❑ Fine tune neutrino fluxes via μ acceptance

ν from STORed Muons (nuSTORM)



- ❑ **1st ν beam facility & highest ever beam power based on stored muons**
- ❑ Fine tune neutrino fluxes via μ acceptance

ν from *STORed* Muons (*nuSTORM*)



❑ **1st ν beam facility & highest ever beam power based on stored muons**

❑ Fine tune neutrino fluxes via μ acceptance

❑ Production Straight (example w/ π^+ injection)

❖ ν_μ flux from $\pi^+ \rightarrow \mu^+ \nu_\mu$ (“pion flash”)

❖ $\nu_e + \bar{\nu}_\mu$ flux from $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

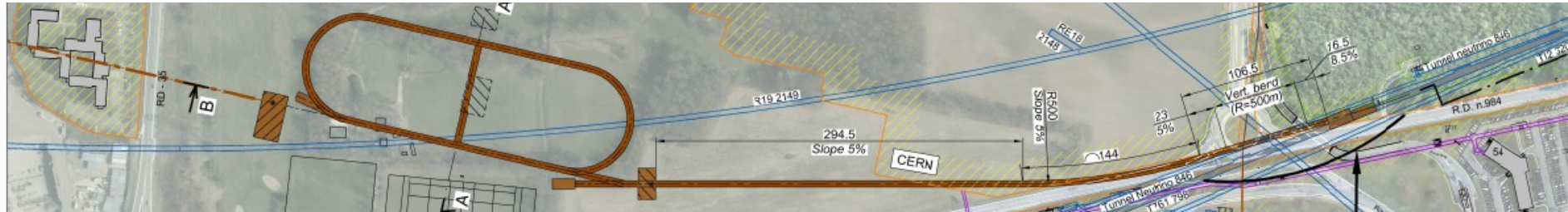
❖ Maximise μ capture efficiency

❑ Return Arcs and Straight

❖ p_μ tunable between 1 and 6 GeV/ c , spread $\pm 16\%$

A Brief History of nuSTORM

CERN-PBC-REPORT-2019-003
DOI:10.17181/CERN.FQTB.O8QN



2012-13 Lol and Proposal to FNAL PAC [arXiv:1206.0294, arXiv:1308.6822], EoI to CERN [arXiv:1305.1419]

- Sterile neutrinos
- Neutrino-nucleus scattering
- Technology test bed for muon accelerators

2014 Steriles sensitivity [Phys.Rev.D 89, 071301 (2014)]

- nuSTORM at FNAL

2019 Feasibility of nuSTORM at CERN [CERN-PBC-REPORT-2019-003]

- SPS 100 GeV proton beam
- Optimised for neutrino-nucleus scattering, maintaining sensitivity to BSM (steriles + non-unitarity, NSI, Lorentz-invariance/CPT violation)

2022 Snowmass 2021 [arXiv:2203.07545]

- Advocating synergy with ENUBET and Muon Collider Demonstrator
 - ✓ Muon Collider demonstrator
 - ❖ 6-D cooling
 - ✓ nuSTORM as test bed for muon storage ring
 - ❖ Complete implementation for large acceptance
 - ❖ R&D for very precise determination of stored-muon energy and spread

2025 On-going: European Strategy Update for Particle Physics

- Submission: <https://indico.cern.ch/event/1439855/abstracts/191113/>

□ Why do we need nuSTORM?

✓ Wishlist

1. ν_e and $\bar{\nu}_e$ beams with the relevant energy for appearance
2. Well-understood fluxes
3. High statistics
4. Low ν_μ background

□ What other physics can we do with it?