2nd JUNO Neutrino Summer School

Neutrino oscillation phenomenology

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Outline

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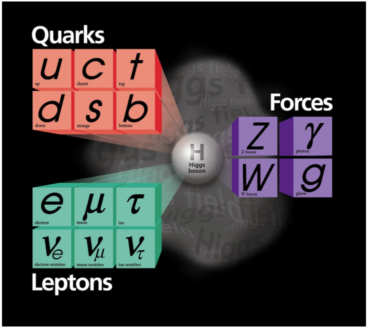
The Standard Model



Fermions in the Standard Model come in three generations ("Flavours")

Neutrinos are the "partners" of the charged leptons

more precisely: they form a doublet under the SU(2) gauge symmetry

Flavour neutrinos

A neutrino of flavour α is defined by the charged current interaction with the corresponding charged lepton:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \bar\nu_{\alpha L} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

for example

$$\pi^+ \to \mu^+ \nu_\mu$$

the muon neutrino ν_μ comes together with the charged muon μ^+

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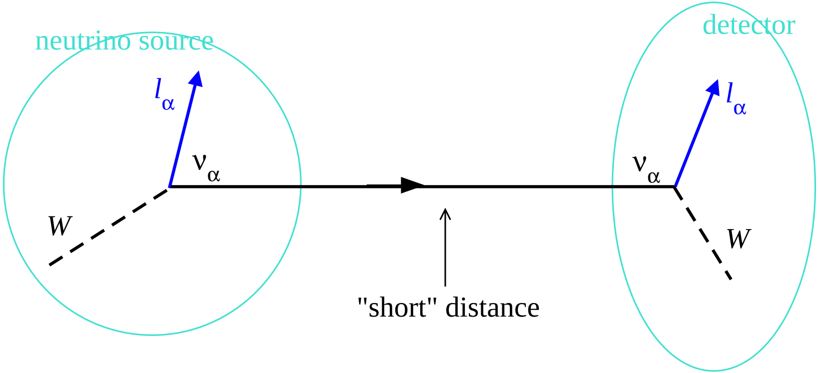

Let's give mass to the neutrinos

Majorana mass term:

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}^{T} C^{-1} \mathcal{M}_{\alpha\beta} \nu_{\beta L} + \mathrm{h.c.}$$

$\mathcal{M}$: symmetric mass matrix

In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

any complex symmetric matrix $\mathcal{M}$ can be diagonalised by a unitary matrix

$$U_\nu^T \mathcal{M} U_\nu = m \,, \qquad m : \text{ diagonal, } m_i \geq 0$$

Lepton mixing

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \bar{\nu}_{iL} U_{\alpha i}^* \gamma_\rho \ell_{\alpha L} + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} \sum_{i=1}^{3} \nu_{iL}^T C^{-1} \nu_{iL} m_i^\nu - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m_\alpha^\ell + \mathrm{h.c.}$$

Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\mathrm{PMNS}}$$

Lepton mixing

- Flavour neutrinos ν_α are superpositions of massive neutrinos ν_i:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i \qquad (\alpha = e, \mu, \tau)$$

- mismatch between mass and interaction basis
- Example for two neutrinos:

$$\begin{aligned} \nu_e &= \cos\theta\,\nu_1 + \sin\theta\,\nu_2 \\ \nu_\mu &= -\sin\theta\,\nu_1 + \cos\theta\,\nu_2 \end{aligned}$$

- The same phenomenon happens also for quarks (CKM matrix)

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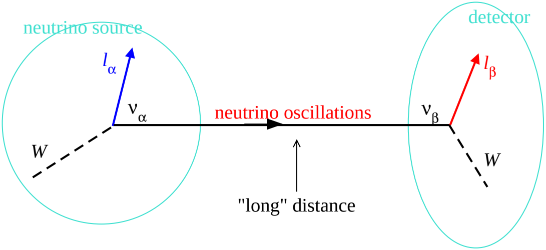
Lepton mixing

Neutrino oscillation fundamentals
- Oscillations in vacuum
- QFT approach to neutrino oscillations
- Coherence requirements for oscillations
- Oscillations in matter
- Varying matter density and MSW

Global data and 3-flavour oscillations
- Qualitative picture
- Global analysis
- $\delta_{\rm CP}$ and mass ordering

Summary and concluding comments

Neutrino oscillations



$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle \qquad e^{-i(E_i t - p_i x)} \qquad |\nu_\beta\rangle = U^*_{\beta i}|\nu_i\rangle$$

oscillation amplitude:

$$\begin{aligned}\mathcal{A}_{\nu_\alpha\to\nu_\beta} &= \langle\nu_\beta| \quad \text{propagation} \quad |\nu_\alpha\rangle \\ &= \sum_{i,j} U_{\beta j}\langle\nu_j| \quad e^{-i(E_i t - p_i x)} \quad |\nu_i\rangle U^*_{\alpha i} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)}\end{aligned}$$

Neutrino oscillations in vacuum

oscillation amplitude:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)} \quad \to \quad P_{\nu_\alpha \to \nu_\beta} = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta} \right|^2$$

need to calculate phase differences:

$$\phi_{ji} = (E_j - E_i)t - (p_j - p_i)x \quad \text{with} \quad E_i^2 = p_i^2 + m_i^2$$

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after some hand waving:

$$\phi_{ji} \approx \frac{\Delta m^2_{ji} L}{2\,E} \quad \text{with} \quad \Delta m^2_{ji} \equiv m_j^2 - m_i^2$$

A hand-waving derivation for two flavours

oscillation phase:

$$\phi = (E_2 - E_1)t - (p_2 - p_1)x \quad \text{with} \quad E_i^2 = p_i^2 + m_i^2$$

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define: $\Delta E = E_2 - E_1, \quad \Delta E^2 = E_2^2 - E_1^2, \quad \bar{E} = (E_1 + E_2)/2$

then: $\Delta E^2 = 2\bar{E}\Delta E$ (similar for p and m)

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use "average velocity" of the neutrino $v = \bar{p}/\bar{E}$ and $x \approx vt$:

$$\boxed{\phi \approx \frac{\Delta m^2}{2\bar{p}}x \approx \frac{\Delta m^2}{2\bar{E}}x}$$

Effective Schrödinger equation

The evolution of the flavour state can be described by an effective Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H_{\rm vac}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$\begin{aligned} H^{\nu}_{\rm vac} &= U\mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger \\ H^{\bar\nu}_{\rm vac} &= U^*\mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^T \end{aligned}$$

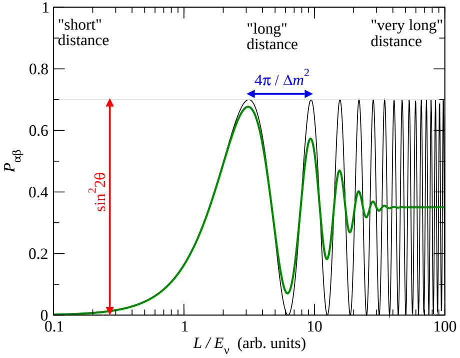
equivalent to $\mathcal{A}_{\nu_\alpha\to\nu_\beta} = \sum_i U_{\beta i}U^*_{\alpha i}e^{-i(E_i t - p_i x)}$ and $\phi_{ji} \approx \frac{\Delta m^2_{ji} L}{2\,E}$

2-neutrino oscillations

Two-flavour limit:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad P = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

oscillations are sensitive to mass differences (not absolute masses)



$$\frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2[\mathrm{eV}^2]\, L[\mathrm{km}]}{E_\nu[\mathrm{GeV}]}$$

Appearance vs. disappearance

- appearance experiments:

$$P_{\nu_\alpha \to \nu_\beta}\,, \qquad \alpha \neq \beta$$

"appearance" of a neutrino of a new flavour $\beta \neq \alpha$ in a beam of ν_α

- disappearance experiments:

$$P_{\nu_\alpha \to \nu_\alpha}$$

measure the "survival" probability of a neutrino of given flavour

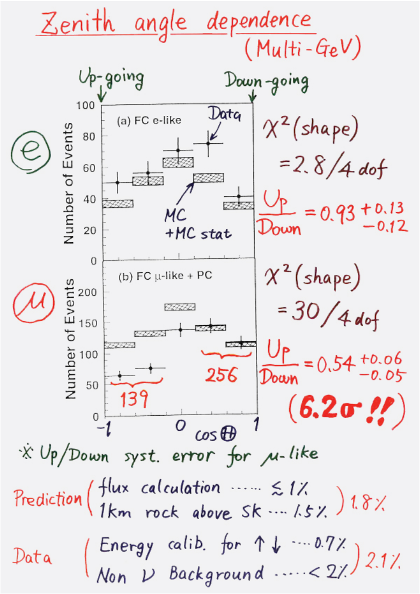
Neutrinos oscillate!

1998: SuperKamiokande atmospheric neutrinos

- zenith-angle dependent deficit of multi-GeV μ-like events
- consistent with $\nu_\mu \to \nu_\tau$ oscillations with

$\Delta m^2 \simeq 2.5 \times 10^{-3}$ eV^2
$\sin^2 2\theta \simeq 1$

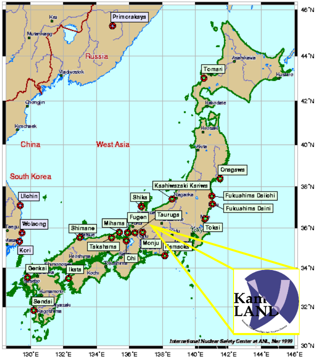
Nobel prize 2015
Takaaki Kajita



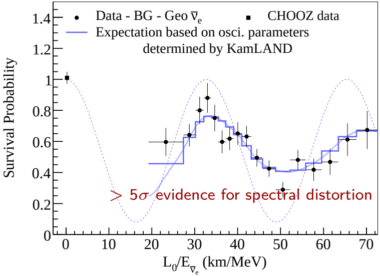
Neutrinos oscillate!

$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

KamLAND $\bar{\nu}_e \to \bar{\nu}_e$



$\langle L \rangle \sim 180$ km



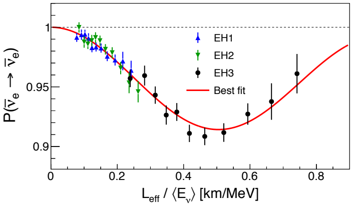
$> 5\sigma$ evidence for spectral distortion

Neutrinos oscillate!

$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

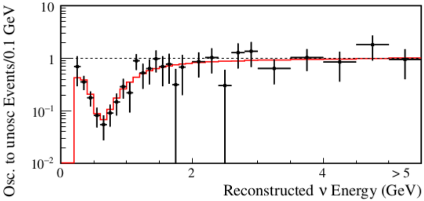
DayaBay, 2015

$\bar{\nu}_e \to \bar{\nu}_e$, $\langle L\rangle \sim 2$ km



T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L\rangle \sim 295$ km



the naive approach to calculate the oscillation probability is problematic at least for the following reasons:

- production and detection regions are localised in space $\rightarrow$ inconsistent with plane wave ansatz for neutrino propagation $\propto e^{-i(E_i t - p_i x)}$
- plane waves correspond to states with exact energy/momentum $\rightarrow$ neutrino mass states are distinguishable particles $\rightarrow$ why is the sum in the amplitude coherent (inside modulus)?

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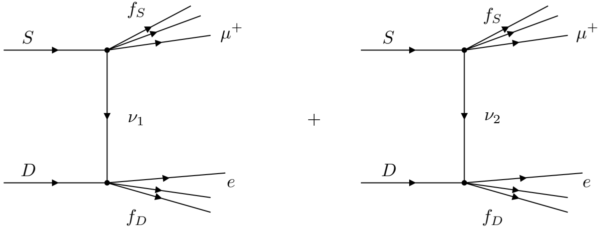
Two approaches:

- assume wave-packets for neutrinos
- QFT approach, neutrino as internal line, wave-packets for external particles

relation of the two approaches e.g., Akhmedov, Kopp, JHEP (2010) [1001.4815]

QFT approach to neutrino oscillations

joint process of neutrino production and detection



early papers:
Rich,1993; Giunti,Kim,Lee,Lee,1993; Grimus,Stockinger,1996; Kiers,Weiss,1998

review paper:
M. Beuthe, Oscillations of Neutrinos and Mesons in Quantum Field Theory, Phys. Rept. 375 (2003) 105 [hep-ph/0109119]

QFT approach to neutrino oscillations

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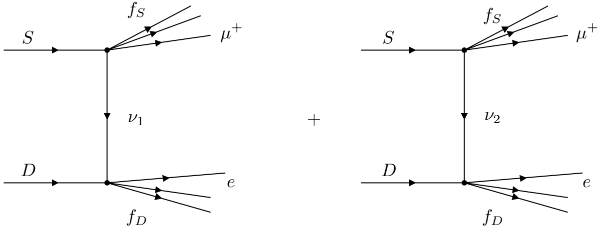

- neutrino corresponds to internal line, unobservable
- "standard" Feynman rules to calculate amplitude $\mathcal{A}$ of the whole process
- take into account that production and detection vertices are macroscopically separated in space and time
- coherence properties determined by localization (or momentum spread) of external particles

QFT approach Dobrev, Melnikov, TS, 2504.10600

Initial states at source and detector ($X = S, D$):
superpositions of momentum eigenstates:

$$|X\rangle = \int [\mathrm{d}\vec{k}_X]\, \phi_X(\vec{k}_X)\, |\vec{k}_X\rangle\,, \qquad [\mathrm{d}\vec{k}_X] = \frac{\mathrm{d}^3\vec{k}_X}{(2\pi)^{3/2}\sqrt{2E_X}}$$

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S-matrix element for the full process $i \to f$ (final state moment P_S, P_D fixed)

$$S_{if} = \int [\mathrm{d}\vec{k}_S][\mathrm{d}\vec{k}_D]\, \phi_S(\vec{k}_S)\phi_D(\vec{k}_D)\, (2\pi)^4\delta^{(4)}(k_S + k_D - P_S - P_D)\mathcal{M}_{if}$$

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and differential transition probability $i \to f$:

$$\mathrm{d}W_{if} = |S_{if}|^2\, \mathrm{d}\nu_f$$

with density of final states $\mathrm{d}\nu_f = \mathrm{d}\Phi_S\, \mathrm{d}\Phi_D$

QFT approach Dobrev, Melnikov, TS, 2504.10600

$$\mathrm{d}W_{if} = \int [\mathrm{d}\vec{k}_S][\mathrm{d}\vec{k}'_S][\mathrm{d}\vec{k}_D][\mathrm{d}\vec{k}'_D]\, \mathrm{d}\Phi_S \mathrm{d}\Phi_D\, \phi_S(\vec{k}_S)\phi_D(\vec{k}_D)\phi^*_S(\vec{k}'_S)\phi^*_D(\vec{k}'_D)\times$$
$$(2\pi)^8 \delta^{(4)}(k_S + k_D - P_S - P_D)\delta^{(4)}(k'_S + k'_D - P_S - P_D)\mathcal{M}_{if}\mathcal{M}'^*_{if}$$

k_X, k'_X: initial state momenta of the two interfering diagrams

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neutrino momenta: $q = k_S - P_S = P_D - k_D$, $q' = k'_S - P_S = P_D - k'_D$

momenta differences: $\kappa = q - q'$, $\kappa_X = k_X - k'_X$, $(X = S, D)$

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introduce source and detector "times" t_S, t_D:

$$(2\pi)^2 \delta(\kappa^0_S - \kappa^0)\delta(\kappa^0_D + \kappa^0) = \int \mathrm{d}t_S\, e^{-i(\kappa^0_S - \kappa^0)t_S} \int \mathrm{d}t_D\, e^{-i(\kappa^0_D + \kappa^0)t_D}$$

Crucial step 1

statistical averaging over quantum states described by ϕ_S and ϕ_D gives time-dependent momentum-space density matrices

$$\langle \phi_X(\vec{k}_X)\phi_X^*(\vec{k}'_X)e^{-i\kappa_X^0 t_X}\rangle = \rho_X(\vec{k}_X, \vec{k}'_X, t_X)$$

related to the Wigner function by

$$\rho_X(\vec{k}_X, \vec{k}'_X, t_X) = \int \mathrm{d}^3\vec{r}_X \, n_X(\vec{r}_X, \vec{l}_X, t_X)e^{-i\vec{\kappa}_X\vec{r}_X} \qquad (X = S, D)$$

the classical limit of a Wigner function is the phase-space distribution of particles

$$n_X(\vec{r}_X, \vec{l}_X, t_X)$$

with position $\vec{r}$, momentum $\vec{l}$, at the time t.

Dobrev, Melnikov, TS, 2504.10600; Ginzburg, Kotkin, Polityko, Serbo, Sov. J. Nucl. Phys. 55 (1992) 1847; Sov. J. Nucl. Phys. 55 (1992) 1855; Kotkin, Serbo, Schiller, Int. J. Mod. Phys. A 7 (1992) 4707

Crucial step 2

amplitude factorizes: $\mathcal{M}_{if} \approx -\sin\theta\cos\theta\, M_S \sum_{a=1,2} \frac{(-1)^a}{q^2 - m_a^2 + i\epsilon} M_D$

$$\mathcal{M}_{if}\mathcal{M}_{if}'^{*} = \sin^2\theta\cos^2\theta\, |M_S|^2 |M_D|^2 \sum_{a,b=1,2} \frac{(-1)^a(-1)^b}{(q^2 - m_a^2 + i\epsilon)(q'^2 - m_b^2 - i\epsilon)}$$

M_S, M_D smooth functions of neutrino energy (neglect neutrino masses)

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M_S, M_D smooth functions of neutrino energy (neglect neutrino masses)

$$I = \int \frac{\mathrm{d}Q^2}{2\pi} \frac{\mathrm{d}^4\kappa}{(2\pi)^4} e^{-i\kappa x} \sum_{a,b=1,2} \frac{(-1)^a(-1)^b}{(Q^2 + Q\kappa - m_a^2 + i\epsilon)(Q^2 - Q\kappa - m_b^2 - i\epsilon)}$$

$$= \ldots = \frac{2}{|\vec{Q}|}\delta(t_D - t_S - r_\parallel)\,\delta^{(2)}(\vec{r}_\perp)\,\Theta(r_{||})\sin^2\left(\frac{\Delta m^2 L}{4|\vec{Q}|}\right)$$

where $\vec{r} = \vec{r}_D - \vec{r}_S,\ L \equiv r_{||} = \vec{r}\cdot\vec{e}_\nu,\ \vec{r}_\perp\cdot\vec{e}_\nu = 0,\ |\vec{Q}| \approx E_\nu$

Result for transition probability

$$\mathrm{d}W_{if} = \int \mathrm{d}^3\vec{p}_\nu \int \mathrm{d}t_S \mathrm{d}^3\vec{l}_S \mathrm{d}^3\vec{r}_S \, n_S(\vec{r}_S, \vec{l}_S, t_S) \, \frac{\mathrm{d}\Gamma_S}{\mathrm{d}^3\vec{p}_\nu}$$
$$\times \int \mathrm{d}r \, \mathrm{d}^3\vec{l}_D \, n_D(\vec{r}_S + r\vec{e}_\nu, \vec{l}_D, t_S + r) \, \mathrm{d}\sigma_{D\nu} \, P_{\mathrm{osc}}(r)$$

$$P_{\mathrm{osc}}(r) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

convolution of standard oscillation probability with neutrino flux, detection cross section, and phase-space densities at source and detector

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$$\times \int \mathrm{d}r \, \mathrm{d}^3\vec{l}_D \, n_D(\vec{r}_S + r\vec{e}_\nu, \vec{l}_D, t_S + r) \, \mathrm{d}\sigma_{D\nu} \, P_{\mathrm{osc}}(r)$$

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convolution of standard oscillation probability with neutrino flux, detection cross section, and phase-space densities at source and detector

coherence properties encoded in phase-space densities n_S and n_D

Coherence properties

- assume isotropic Gaussian phase-space densities for $X = S, D$:

$$n_X(\vec{r}_X, \vec{l}_X, t_X) \propto \exp\left[-\frac{1}{2}\left(\frac{\vec{r}_X - \vec{L}_X}{\delta_X}\right)^2 - \frac{1}{2}\left(\frac{\vec{l}_X - \vec{\mathrm{l}}_X}{\sigma_X}\right)^2 - \frac{1}{2}\left(\frac{t_X - T_X}{\tau_X}\right)^2\right]$$

- mean locations $\vec{L}_{S,D}$, mean momenta $\vec{\mathrm{l}}_{S,D}$, and mean times $T_{S,D}$
- Quantum mechanical uncertainty relation: $\delta_X \sigma_X > 1/2$
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$$P_{\rm osc} = \exp\left[-\frac{(\vec{L}_D - \vec{L}_S)_\perp^2}{2\delta^2}\right] \exp\left[-\frac{(L-T)^2}{2\tau^2}\right] \frac{\sin^2 2\theta}{2}\left[1 - \xi_{\rm loc}\xi_{\rm en}\cos\left(\frac{\Delta m^2 L}{2E_\nu}\right)\right]$$

where

$$\tau^2 \equiv \tau_D^2 + \tau_S^2, \quad \delta^2 \equiv \delta_D^2 + \delta_S^2, \quad L \equiv (\vec{L}_D - \vec{L}_S)\cdot\vec{e}_\nu, \quad T \equiv T_D - T_S$$

Localization decoherence

$$\xi_{\rm loc} = \exp\left[-2\left(\pi\frac{\delta}{L_{\rm osc}}\right)^2\right] \quad \text{with} \quad L_{\rm osc} = 2\pi\frac{2E_\nu}{\Delta m^2}$$

production and detection regions have to be localised much better than the oscillation length: $\delta \ll L_{\rm osc}$ (note $\delta^2 = \delta_S^2 + \delta_D^2$)

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Energy decoherence

$$\xi_{\rm en} = \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2 L\sigma}{2E_\nu^2}\right)^2\right] = \exp\left[-2\pi^2\left(\frac{L}{L_{\rm osc}}\frac{\sigma}{E_\nu}\right)^2\right]$$

- for experiments at the oscillation maximum ($L \approx L_{\rm osc}$) the neutrino energy needs to be well defined: $\sigma \ll E_\nu$
- this term can be interpreted as decoherence due to neutrino wave packet separation, identifying $v_j \approx 1 - m_j^2/(2E_\nu^2)$

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Why are oscillations possible?

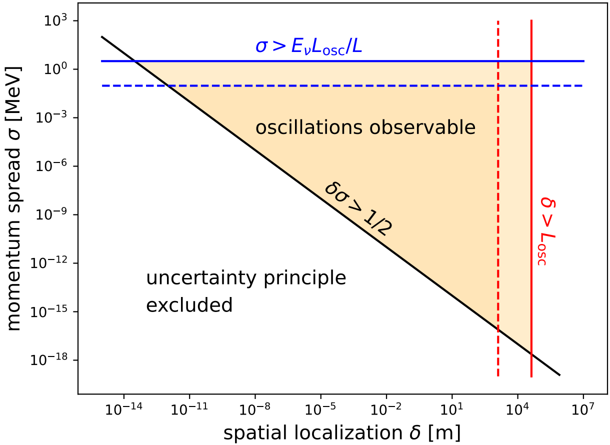
$\xi_{\rm loc}$ and $\xi_{\rm en}$ have opposite dependence on spreads:

- uncertainties have to be
  large enough that mass states can interfere: $\sigma_{\rm loc} \gg \Delta m^2/E_\nu$
  small enough that intereference is not damped: $\sigma \ll E_\nu L_{\rm osc}/L$
  but QM uncertainty implies: $\sigma_{\rm loc} = 1/(2\delta) < \sigma$
- there are many orders of magnitude available to fulfill these requirements, because

$$\Delta m^2/E_\nu^2 \ll 1 \qquad \text{or} \qquad E_\nu L_{\rm osc} \gg 1$$

Why are oscillations possible?

Ex.:JUNO: $E_\nu = 4\,\mathrm{MeV}$, $L = 53\,\mathrm{km}$, solid(dash) $\Delta m^2 = 7.5 \cdot 10^{-5}(2.5 \cdot 10^{-3})\,\mathrm{eV}^2$



Dobrev, Melnikov, TS, 2504.10600

Classical averaging

consider averaging of the event rates $R(L, E_\nu)$:

$$\int dL'\, R(L', E_\nu) \frac{1}{\sqrt{2\pi}\delta_{\rm clas}} \exp\left[-\frac{(L'-L)^2}{2\delta_{\rm clas}^2}\right]$$

$$\int dE'_\nu\, R(L, E'_\nu) \frac{1}{\sqrt{2\pi}\sigma_{\rm clas}} \exp\left[-\frac{(E'_\nu-E_\nu)^2}{2\sigma_{\rm clas}^2}\right]$$

same decoherence factors $\xi_{\rm loc}$ and $\xi_{\rm en}$ with (in the Gaussian case)

$$\delta^2 \to \delta^2 + \delta_{\rm clas}^2\,, \qquad \sigma^2 \to \sigma^2 + \sigma_{\rm clas}^2$$

Classical averaging

- quantum mechanical and classical decoherence have the same effect and are indistinguishable phenomenologically
  Kiers, Nussinov, Weiss, 1996; Stodolsky, 1998; Ohlsson, 2001
- classical averaging due to experimental reasons: size of production region, finite detector resolutions (in space and energy),...
- fundamental averaging effects due to experimental configuration and physics principles: phase space integrals of unobserved particles, Doppler broadening,...

Classical averaging

In the approach of Dobrev, Melnikov, TS, 2504.10600, the QM uncertainties of the initial state particles manifest themselves as classical convolution of the probability with Wigner distributions:

$$\mathrm{d}W_{if} = \int \mathrm{d}^3\vec{p}_\nu \int \mathrm{d}t_S \mathrm{d}^3\vec{l}_S \mathrm{d}^3\vec{r}_S \, n_S(\vec{r}_S, \vec{l}_S, t_S) \, \frac{\mathrm{d}\Gamma_S}{\mathrm{d}^3\vec{p}_\nu} \times \int \mathrm{d}r \, \mathrm{d}^3\vec{l}_D \, n_D(\vec{r}_S + r\vec{e}_\nu, \vec{l}_D, t_S + r) \, \mathrm{d}\sigma_{D\nu} \, P_{\mathrm{osc}}(r)$$

conditions of observability of oscillations imply that coherence assumptions of the derivation are satisfied

Decoherence parameters - numerical example

estimates for reactor oscillation experiments
Krueger, TS, 2303.15524, Akhmedov, Smirnov, 2208.03736

$$-\ln \xi_{\rm loc} = \frac{1}{2}\left(\frac{\Delta m^2}{4E_\nu \sigma_{\rm loc}}\right)^2 \approx 1.3 \times 10^{-19} \left(\frac{\Delta m^2}{1\,{\rm eV}^2}\right)^2 \left(\frac{1\,{\rm MeV}}{E_\nu}\right)^2 \left(\frac{500\,{\rm eV}}{\sigma_{\rm loc}}\right)^2$$

$$-\ln \xi_{\rm en} = 2\pi^2 \left(\frac{L}{L_{\rm osc}}\frac{\sigma_{\rm en}}{E_\nu}\right)^2 \approx 4.9 \times 10^{-12} \left(\frac{L}{L_{\rm osc}}\right)^2 \left(\frac{1\,{\rm MeV}}{E_\nu}\right)^2 \left(\frac{\sigma_{\rm en}}{0.5\,{\rm eV}}\right)^2$$

$\Rightarrow$ QM decoherence (incl. localization and “wave packet separation”) is irrelevant for all practical purposes

decoherence effects completely dominated by classical averaging
(e.g., typical energy resolution in reactor exps: $\sigma_{\rm clas} \simeq 0.1$ MeV)

Summary oscillations in vacuum

- "simple QM" derivation of oscillation probability is based on *wrong* assumptions but gives the *correct* result
- well founded derivation and understanding of (coherence) conditions for the observability of oscillations can be obtained by a QFT or wave packet treatment
- fundamental QM decoherence effects are *unobservable* for all practical purposes, coherence is dominated by classical averaging effects
  (or would require rather exotic BSM physics)

The matter effect

When neutrinos pass through matter the SM interactions with the particles in the background induce an effective potential for the neutrinos

Effective 4-point interaction Hamiltonian

$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}}\,\bar\nu_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha \underbrace{\sum_f \bar f\gamma^\mu(g_V^{\alpha,f} - g_A^{\alpha,f}\gamma_5)f}_{J^\mu_{\rm mat}}$$

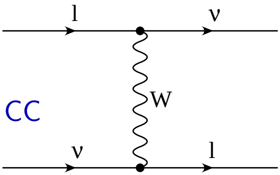
coherent forward scattering amplitude leads to an "index of refraction"
$\rightarrow$ proportional to G_F! (not G_F^2)

L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* D **20**, 2634 (1979)

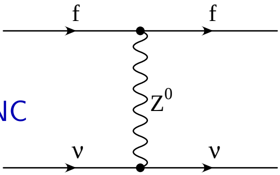
Effective matter potential

$$V_{\mathrm{mat}} = \sqrt{2} G_F \, \mathrm{diag}\left(N_e - N_n/2, -N_n/2, -N_n/2\right)$$

CC



NC



- only ν_e feel CC (there are no μ, τ in normal matter)
- NC is the same for all flavours ⇒ potential proportional to identiy has no effect on the evolution
- NC has no effect for 3-flavour active neutrinos, but is important in the presence of sterile neutrinos

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$H = \underbrace{U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger}_{\text{vaccum}}$$

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$N_e(x)$: electron density along the neutrino path

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for non-constant matter: $H(t) \rightarrow$ time-dependent Schrödinger eq.
"MSW resonance" Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)

Matter effect in QFT

- transition probability for neutrinos in matter can be derived also within the QFT approach to neutrino oscillations
  e.g., Cardall, Chung, hep-ph/9904291; Akhmedov, Wilhelm, 1205.6231

- consider the effect of background matter on the neutrino propagator appearing in the QFT Feynman diagram

- neutrino propagator has to be derived from a solution of the Dirac equation with matter-induced potential

Neutrino oscillations in constant matter

diagonalize the Hamiltonian in matter:

$$
\begin{aligned}
H^{\nu}_{\mathrm{mat}} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \mathrm{diag}(\sqrt{2} G_F N_e, 0, 0) \\
&= U_m \mathrm{diag}\left(\lambda_1, \lambda_2, \lambda_3\right) U_m^\dagger
\end{aligned}
$$

Same expression for oscillation probability, but replace "vacuum" parameters by "matter" parameters

2-neutrino oscillations in constant matter

Two-flavour case:

$$P_{\rm mat} = \sin^2 2\theta_{\rm mat} \, \sin^2 \frac{\Delta m^2_{\rm mat} L}{4E}$$

with

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$\Delta m^2_{\rm mat} = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

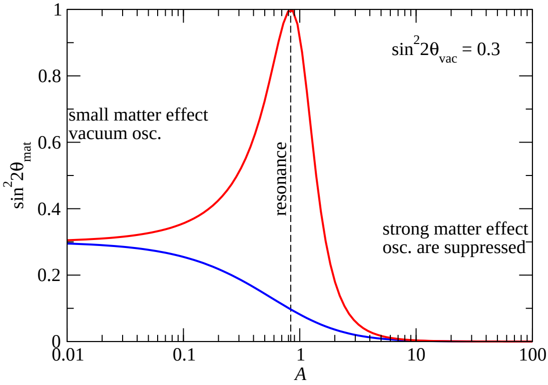
$$A \equiv \frac{2EV}{\Delta m^2}$$

2-neutrino oscillations in constant matter

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A \equiv \frac{2EV}{\Delta m^2}$$

resonance for $\cos 2\theta = A$: "MSW resonance"

Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)



resonance and mass ordering

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resonance for $\cos 2\theta = A$:

- the appearance of the resonance depends on the relative signs of $V, \Delta m^2, \cos 2\theta = \cos^2\theta - \sin^2\theta$
- for neutrinos $V > 0$, for antineutrinos $V < 0$
- e.g., if the resonance happens for neutrinos we must have

  $$\Delta m^2 \cos 2\theta = (m_2^2 - m_1^2)(\cos^2\theta - \sin^2\theta) > 0$$

  possible solution: $m_2^2 > m_1^2$ and $\theta < 45^\circ$
- in two flavours, there is a degeneracy between labeling the mass states and the allowed range of the mixing angle
  more involved parameter dependencies in three flavours

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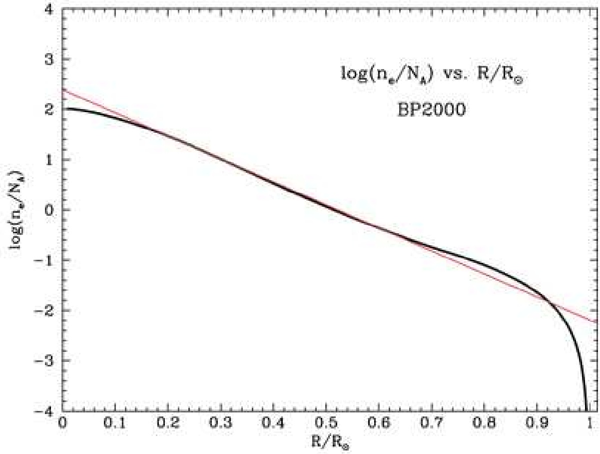
Varying matter density: example solar neutrinos

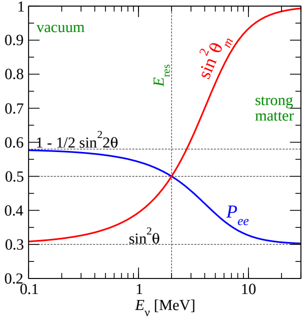
The electron density in the sun:



Adiabatic conversion of solar neutrinos

- matter density changes slowly on the scale of the oscillation length ⇒ adiabatic propagation of neutrino mass states in matter
- oscillations between sun and earth averaged out:

$$P_{ee} = \sum_i |U^m_{ei}|^2 |U_{ei}|^2$$
$$\to \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta$$



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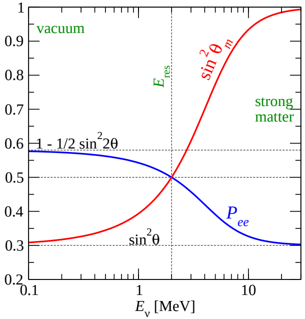

$$E \ll E_{\rm res} \approx 2\,\mathrm{MeV}: \quad \theta_m \approx \theta \Rightarrow P_{ee} = c^4 + s^4 = 1 - \frac{1}{2}\sin^2 2\theta$$
$$E \gg E_{\rm res} \approx 2\,\mathrm{MeV}: \quad \theta_m \approx \frac{\pi}{2} \Rightarrow P_{ee} = \sin^2\theta$$

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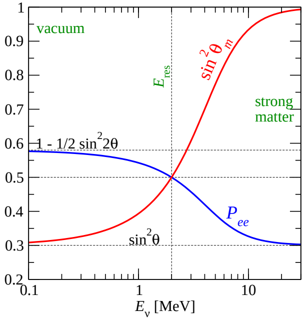

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additional correction from propagation through earth matter $\to$ day-night effect

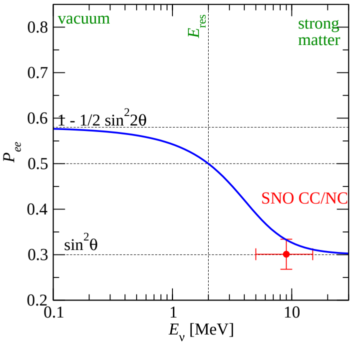
Solar neutrinos and the Sudbury Neutrino Observatory

2002: SNO: CC to NC ratio of solar neutrino flux

CC: $\nu_e + d \to p + p + e^-$
NC: $\nu_x + d \to p + n + \nu_x$

- evidence for $\nu_e \to \nu_\mu, \nu_\tau$ conversion
- MSW effect inside the sun adiabatic conversion through resonance

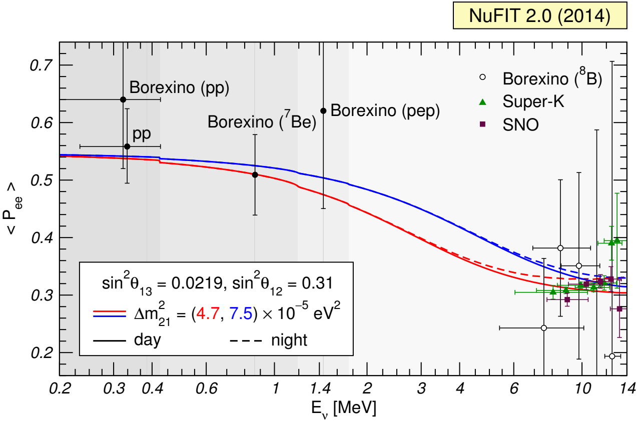
Nobel prize 2015
Art McDonald



$$P_{ee} = \frac{\phi_e}{\phi_e + \phi_\mu + \phi_\tau} = \frac{\phi_{\rm CC}}{\phi_{\rm NC}}$$

Evidence for LMA-MSW

solar neutrino experiments Homestake, SAGE+GNO, Super-K, SNO, Borexino



- $\sin^2\theta < 0.5$ is strong evidence for MSW conversion
- for energies above resonance: $P_{ee} \approx \sin^2\theta$ → best determination of θ_{12}

"Solar" mass ordering

- for the mass-squared difference relevant for solar neutrinos, we adopt the convention that $\Delta m^2 > 0$, i.e., $m_2 > m_1$
- observations show that in the sun the resonance happens for *neutrinos*
- $\Rightarrow \cos 2\theta > 0$: $\theta \sim 33^\circ < 45^\circ$
- physical statement:
  there is one mass state which is dominantely electron-neutrino like, and this mass state has to be the lighter one of the "solar pair"

$$\begin{aligned} \nu_e &= \cos\theta\,\nu_1 + \sin\theta\,\nu_2 \\ \nu_{\mu,\tau} &= -\sin\theta\,\nu_1 + \cos\theta\,\nu_2 \end{aligned}$$

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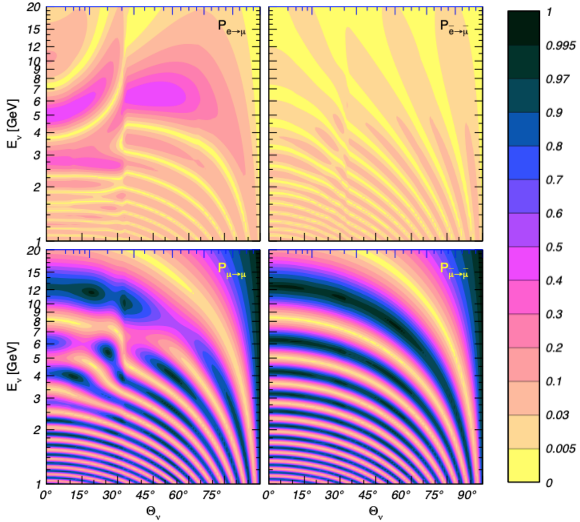
Matter effects in the ν_μ, ν_τ sector

- in the two-flavour approximation there is no matter effect in $\nu_\mu \to \nu_\mu$ and $\nu_\mu \to \nu_\tau$ transitions
- the ν_μ-disappearance channel in long-baseline experiments is described by vacuum oscillations to good approximation
  e.g., Denton, Parke, 2401.10326
- typically matter effects in these channels are suppressed by three-flavour effects (in particular θ_{13}), however, there can be resonance enhancement in atmospheric neutrinos

Matter effects in atmospheric neutrinos



Akhmedov, Maltoni, Smirnov, hep-ph/0612285

Final comments on oscillation fundamentals

Neutrino oscillations in vacuum and in matter

- are sensitive only to neutrino mass-squared *differences* (interference phenomenon)
- no sensitivity to absolute neutrino mass
  $\rightarrow$ beta-decay, neutrinoless double-beta decay, cosmology
- cannot distinguish between Dirac and Majorana neutrino mass (lepton number is conserved in oscillations) $\rightarrow$ neutrinoless double-beta decay

Outline

3-flavour neutrino parameters

- 3 masses: Δm^2_{21}, Δm^2_{31}, m_0
- 3 mixing angles: θ_{12}, θ_{13}, θ_{23}
- 3 phases: 1 Dirac (δ), 2 Majorana (α_1, α_2)

neutrino oscillations
absolute mass observables
lepton-number violation (neutrinoless double-beta decay)

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Δm^2_{31} Δm^2_{21}

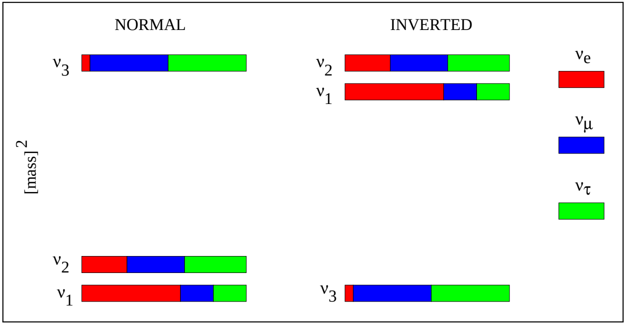
$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm+LBL(dis) react+LBL(app) solar+KamLAND

3-flavour effects are suppressed: $\Delta m^2_{21} \ll \Delta m^2_{31}$ and $\theta_{13} \ll 1$ ($U_{e3} = s_{13} e^{-i\delta}$)

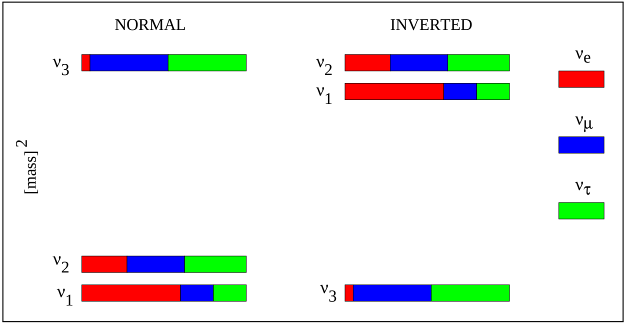
$\Rightarrow$ dominant oscillations are well described by effective two-flavour oscillations
$\Rightarrow$ present data is already sensitive to sub-leading effects
$\Rightarrow$ CP-violation is suppressed by θ_{13} and $\Delta m^2_{21}/\Delta m^2_{31}$

What we know – masses



- The two mass-squared differences are separated roughly by a factor 30: $\Delta m^2_{21} \approx 7 \times 10^{-5}\mathrm{eV}^2\,, \quad |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx 2.4 \times 10^{-3}\mathrm{eV}^2$
- at least two neutrinos are massive

Physical interpretation of mixing angles



$\sin\theta_{13} = |U_{e3}|$ (ν_e component in ν_3) = (ν_3 component in ν_e)

$\tan\theta_{12} = \frac{|U_{e2}|}{|U_{e1}|}$ ratio of ν_2 and ν_1 component in ν_e

$\tan\theta_{23} = \frac{|U_{\mu 3}|}{|U_{\tau 3}|}$ ratio of ν_μ and ν_τ component in ν_3

What we know – mixing

- approx. equal mixing of ν_μ and ν_τ in all mass states: $\theta_{23} \approx 45^\circ$ (with significant uncertainty)
- there is one mass state ("ν_1") which is dominantely ν_e ($\theta_{12} \approx 30^\circ$), and it is the lighter of the two states of the doublet with the small splitting (MSW in sun)
- there is a small ν_e component in the mass state ν_3: $\theta_{13} \approx 9^\circ$ we *do not know* whether this mass state is the heaviest (normal ordering) or the lightest (inverted ordering)

Complementarity of global oscillation data

param	experiment	comment
θ_{12}	SNO, SuperK, (KamLAND)	resonant matter effect in the Sun
θ_{23}	SuperK, T2K, NOvA	ν_μ disappearance atmospheric (accelerator) neutrinos
θ_{13}	DayaBay, RENO, D-Chooz (T2K, NOvA)	$\bar{\nu}_e$ disappearance reactor experiments @ ~ 1 km
Δm^2_{21}	KamLAND, (SNO, SuperK)	$\bar{\nu}_e$ disappearance reactor @ ~ 180 km (spectrum)
$\lvert\Delta m^2_{31}\rvert$	MINOS, T2K, NOvA, DayaBay	ν_μ and $\bar{\nu}_e$ disapp (spectrum)
δ	T2K, NOvA + DayaBay	combination of $(\nu_\mu \to \nu_e) + \bar{\nu}_e$ disap

- global data fits nicely with the 3 neutrinos from the SM
- a few "anomalies" $> 3\sigma$: LSND, MiniBooNE, Gallium anomaly

Global 3-flavour fit

- NuFit collaboration: www.nu-fit.org
  with M.C. Gonzalez-Garcia, M. Maltoni, et al.

- latest paper: NuFit 6.0 (as of Sept 2024)
  Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Pinheiro, Schwetz, JHEP(2024), 2410.05380

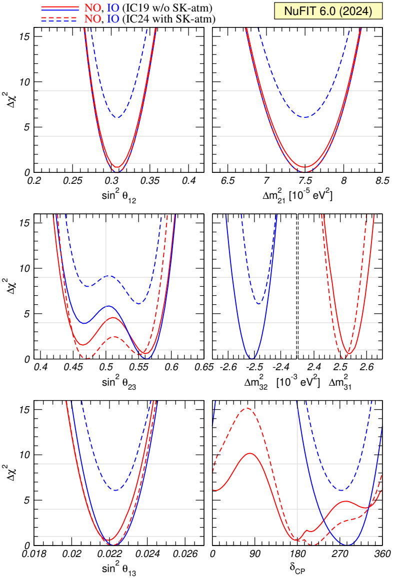
- provides updated global fit results
  tables & figures, χ^2 data for download

Global 3-flavour fit

- robust determination (relat. precision at 3σ):

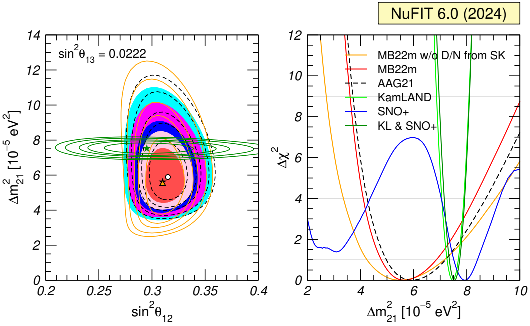
$$\theta_{12}\,(13\%)\quad,\quad \theta_{13}\,(8\%)$$
$$\Delta m^2_{21}\,(15\%)\,,\quad |\Delta m^2_{3\ell}|\,(5.1\%)$$

- broad allowed range for $\theta_{23}\,(20\%)$, non-significant indications for non-maximality/octant
- ambiguity in sign of $\Delta m^2_{3\ell}$ $\rightarrow$ mass ordering
- values of $\delta_{\rm CP} \simeq 90°$ disfavoured preference for $\delta_{\rm CP} \simeq 270°$ for IO

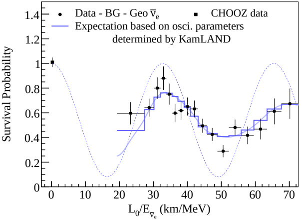


"Solar" sector: $\theta_{12}, \Delta m^2_{21}$

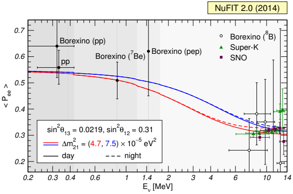
synergy between solar neutrinos and KamLAND reactor neutrinos



KamLAND



solar neutrinos

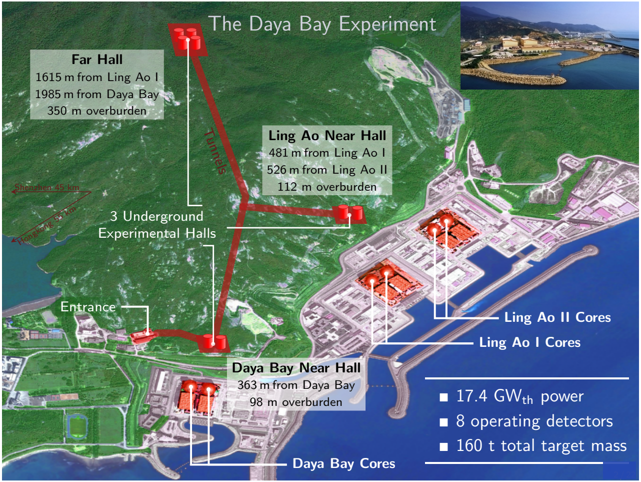


"Atmsopheric" sector: $\theta_{23}, \theta_{13}, \delta_{\rm CP}, \Delta m^2_{31}$

- atmospheric neutrino experiments (SuperK, IceCube, ORCA)
- accelerator experiments (T2K, NOvA)
- reactor experiments (DayaBay, RENO)

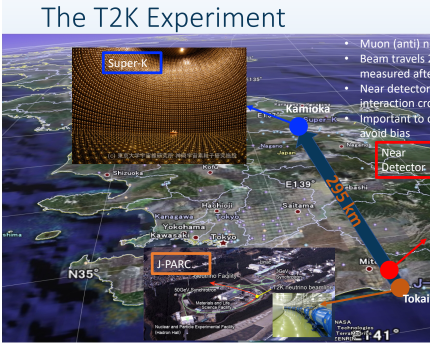
Daya Bay reactor experiment

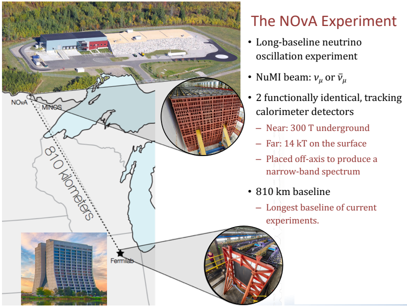
- $\bar{\nu}_e \to \bar{\nu}_e$ disappearance



T2K and NOvA accelerator experiments

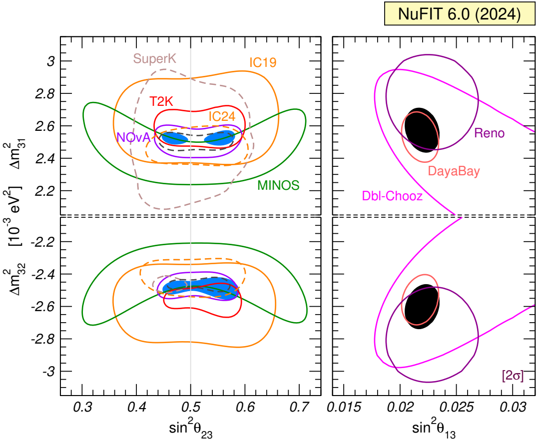
- $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ disappearance
- $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance





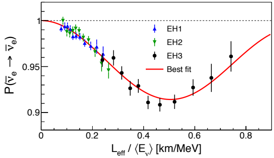
"Atmsopheric" sector: $\theta_{23}, \theta_{13}, \Delta m^2_{31}$

interplay of disappearance data at the Δm^2_{31}-scale



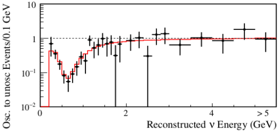
DayaBay, 2015

$\bar{\nu}_e \to \bar{\nu}_e$, $\langle L \rangle \sim 2$ km



T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L \rangle \sim 295$ km



Complementarity between beam and reactor experiments

- $\nu_\mu \to \nu_e$ appearance probability (T2K, NOvA):

$$P_{\mu e} \approx \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2(1-A)\Delta}{(1-A)^2} + \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\Delta + \delta_{\rm CP})$$

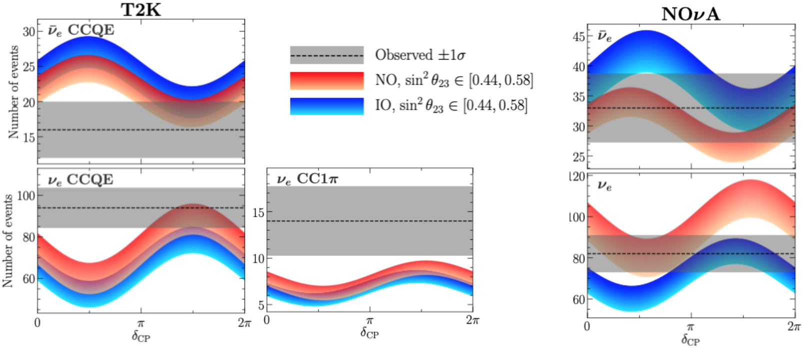
with $$\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu}, \quad \hat{\alpha} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\theta_{12}, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}}$$

- ν_e survival probability (reactor experiments, e.g. Daya Bay)

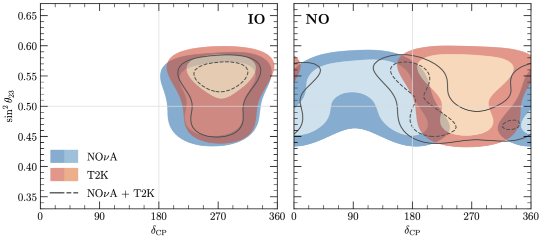
$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \Delta$$

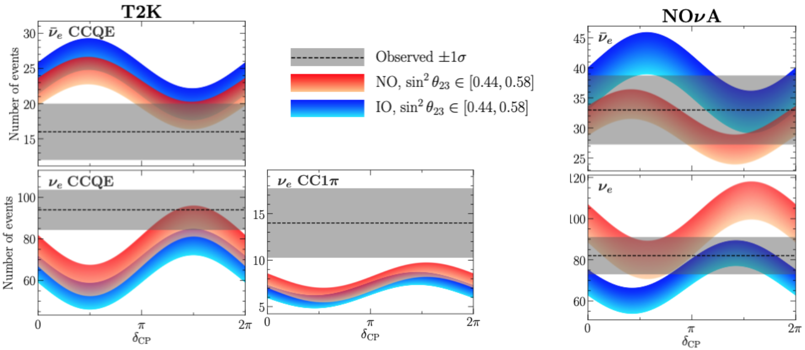
Appearance restults from T2K and NOvA

$$P_{\mu e} \approx \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2(1-A)\Delta}{(1-A)^2} + \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\Delta + \delta_{CP})$$



Appearance restults from T2K and NOvA

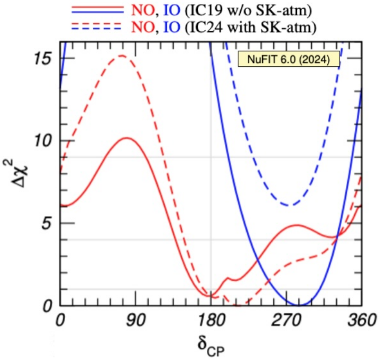




Status of $\delta_{\rm CP}$

some indications on the allowed range of $\delta_{\rm CP}$ due to the interplay of reactor (Daya Bay) and accelerator (T2K, NOvA) neutrino experiments

- values of $\delta_{\rm CP} \simeq 90^\circ$ disfavoured
- normal ordering:
  CP conservation ($\delta_{\rm CP} \simeq 180^\circ$) at 1σ
- inverted ordering:
  preference for $\delta_{\rm CP} \simeq 270^\circ$(maximal CPV)
  CP conservation disfavoured at $> 3.6\sigma$



CP violation in neutrino oscillations

Leptonic CP violation will manifest itself in a difference of the vacuum oscillation probabilities for neutrinos and anti-neutrinos
Cabibbo, 1977; Bilenky, Hosek, Petcov, 1980, Barger, Whisnant, Phillips, 1980

$$P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta} = -16\, J_{\alpha\beta} \sin\frac{\Delta m^2_{21} L}{4E_\nu} \sin\frac{\Delta m^2_{32} L}{4E_\nu} \sin\frac{\Delta m^2_{31} L}{4E_\nu}\,,$$

where

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J\,,$$

with $+(-)$ for (anti-)cyclic permutation of the indices e, μ, τ.

J: leptonic analogue to the Jarlskog-invariant in the quark sector
Jarlskog, 1985

CP violation

Jarlskog-invariant:

$$J = |\mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2})| = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta \equiv J^{\rm max} \sin\delta$$

neutrino oscillation data:

$$J^{\rm max} = 0.0333 \pm 0.0007 (\pm 0.0017) \qquad 1\sigma\ (3\sigma) \qquad \text{nu-fit 6.0}$$

in the quark sector:

$$J_{\rm CKM} = (3.12 \pm 0.13) \times 10^{-5} \qquad \text{PDG}$$

CP violation

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$$J = |\mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2})| = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta \equiv J^{\rm max} \sin\delta$$

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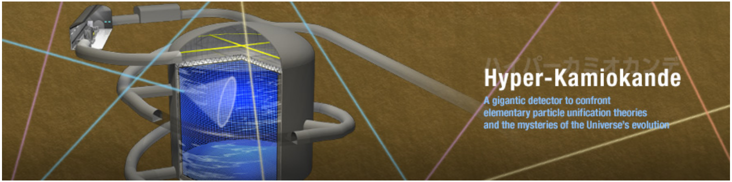
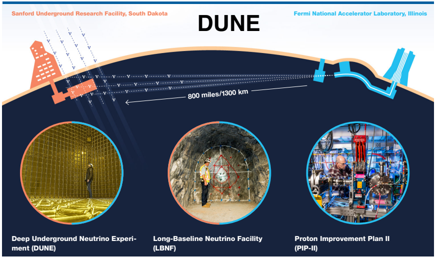
T2K: J-PARC → HyperK (285 km, WC detector)



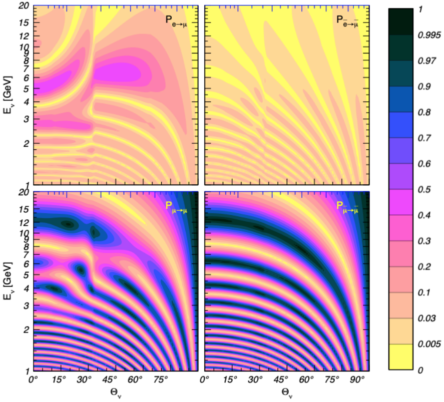
DUNE: Fermilab → Homestake
(1300 km, LAr detectors)



oscillation science goals:
determine mass ordering
and CP phase

Determining the mass ordering - 1

- Looking for the matter effect in transitions involving Δm^2_{31}
  - long-baseline accelerator experiments NOvA, DUNE
  - atmospheric neutrino experiments IceCube, ORCA, HyperK



Akhmedov, Maltoni, Smirnov, hep-ph/0612285

Determining the mass ordering - 2

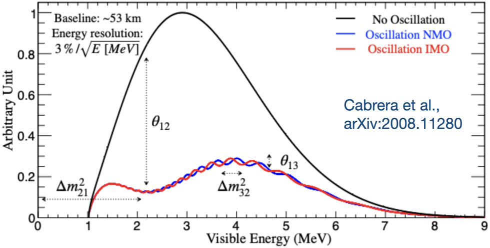
- Interference effect of oscillations with Δm^2_{31} and Δm^2_{21}
  Petcov, Piai, hep-ph/0112074
  - reactor experiment at 60 km JUNO

$$P_{ee}^{\rm reac} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \phi_{sol} - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \cos\phi_{\rm sol}\cos 2\phi_{\rm atm} + \cos 2\theta_{12}\sin\phi_{\rm sol}\sin 2\phi_{\rm atm}\right]$$

with

$$\phi_{\rm sol} = \frac{(m_2^2 - m_1^2)L}{4E}$$

$$\phi_{\rm atm} = \frac{[m_3^2 - (m_1^2 + m_2^2)/2]L}{4E}$$



Determining the mass ordering - 3

- Determination of $|\Delta m^2_{31}|$ from ν_e and ν_μ disappearance
  Nunokawa, Parke, Zukanovich Funchal, hep-ph/0503283

$$P_{\alpha\alpha} \approx 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2 \frac{\Delta m^2_{\alpha\alpha} L}{4E_\nu}\,, \qquad \alpha = \mu, e$$

with

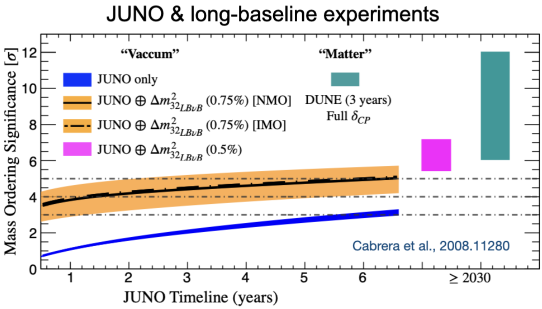
$$\begin{aligned}
\sin^2\theta_{\mu\mu} &= \cos^2\theta_{13}\sin^2\theta_{23} \\
\Delta m^2_{\mu\mu} &= \sin^2\theta_{12}\Delta m^2_{31} + \cos^2\theta_{12}\Delta m^2_{32} + \cos\delta_{\rm CP}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}\Delta m^2_{21} \\
\theta_{ee} &= \theta_{13} \\
\Delta m^2_{ee} &= \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}
\end{aligned}$$

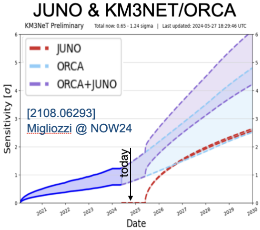
$$|\Delta m^2_{\mu\mu}| - |\Delta m^2_{ee}| = \mp\Delta m^2_{21}[\cos 2\theta_{12} - \cos\delta_{\rm CP}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}]$$

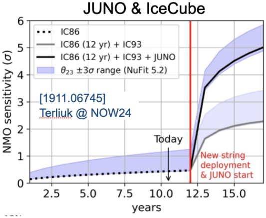
Determining the mass ordering - 3

- Determination of $|\Delta m^2_{31}|$ from ν_e and ν_μ disappearance
  Nunokawa, Parke, Zukanovich Funchal, hep-ph/0503283

- starts being relevant in current data
- will become more important with JUNO







Blennow, Schwetz, 1306.3988

Outline

Concluding comments

- global data on neutrino oscillations is (mostly) consistent with 3-flavour oscillations
- at least two neutrinos are massive
- typical mass scales

$$\sqrt{\Delta m^2_{21}} \sim 0.0086\,\mathrm{eV}$$
$$\sqrt{\Delta m^2_{31}} \sim 0.05\,\mathrm{eV}$$

  are much smaller than all other fermion masses
- all three mixing angles are measured with reasonable precision
- lepton mixing is VERY different from quark mixing

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- all three mixing angles are measured with reasonable precision
- lepton mixing is VERY different from quark mixing

The SM flavour puzzle

Lepton mixing:

$$\theta_{12} \approx 33^\circ \qquad \theta_{23} \approx 45^\circ \qquad \theta_{13} \approx 9^\circ$$

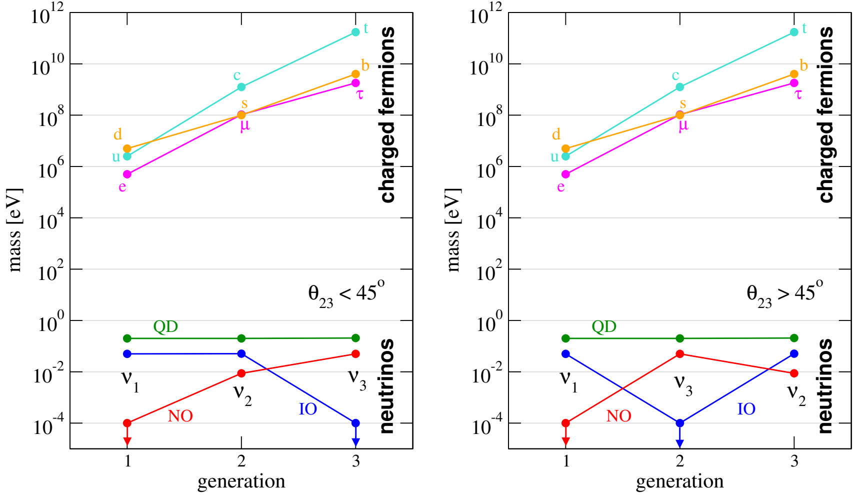
$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Quark mixing:

$$\theta_{12} \approx 13^\circ \qquad \theta_{23} \approx 2^\circ \qquad \theta_{13} \approx 0.2^\circ$$

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & {\scriptstyle\epsilon} \\ \epsilon & 1 & {\scriptstyle\epsilon} \\ {\scriptstyle\epsilon} & {\scriptstyle\epsilon} & 1 \end{pmatrix}$$

The SM flavour puzzle



Concluding comments

open questions for oscillation experiments:

- identify neutrino mass ordering
- establish leptonic CP violation
- precision measurments (e.g., $\theta_{23} \approx 45^\circ$?)
- over-constrain 3-flavour oscillations (search for non-standard properties, sterile neutrinos, exotic neutrino interactions,...)

questions which cannot be addressed by oscillations:

- absolute neutrino mass scale
- Dirac or Majorana nature

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