粲介子半轻衰变的一点理论研究

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iii $D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

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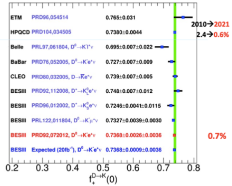
Why semileptotic $D_{(s)}$ decays

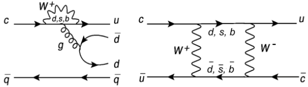
Semileptotic $D_{(s)}$ decays

play a crucial role in the precision era of particle physics

- fundamental parameters, like the CKM matrix element $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]
  - the result measured via $D \to Kl\nu$ and $D_s \to \mu\nu_\mu$ consist with each other ($\sim 1.5\sigma$ derivation)
  - $\sim 3\sigma$ tension three years ago [PDG 2020, 2021]
  - the improvement mainly due to the high precision of $D \to K$ form factor from lattice evaluation and the f_{D_s} from the BESIII



- new physical mechanism via the FCNC
  - anomalous measured in $B \to K^* \mu^+\mu^-$, 3.6σ derivation of $d\mathcal{B}/dq^2$ in $q^2 \in [1, 6]$ GeV^2, 1.9σ derivation of $p_5' = S_5/\sqrt{F_L(1 - F_L)}$ in $[4, 8]$ GeV^2
  - a plausible effect in up-type FCNC process $c \to ull$ [Bharucha 2011.12856] SM $\mathcal{B}(D \to \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
  - LCSRs prediction $\mathcal{B}(D \to \pi\mu\mu) = 1.33^{+0.17}_{-0.24} \times 10^{-8}$, at the same order of LHCb limit 6.7×10^{-8} [A. Bansal, A. Khodjamirian and T. Mannel 2505.21369[hep-ph]],
  - first measurement of $D^0 \to \pi^+\pi^- e^+e^-$ [LHCb 2412.09414] $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ



Semileptotic $D_{(s)}$ decays

New physics hunter $D \to \pi\mu^+\mu^-$

- my talk in the " 超级陶粲装置研讨会" at LZU, July 8th, 2024

- Experimentail potentials

Experiment	Measurement	Sensitivity
LHCb [talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]	Angular observables	$\sim 0.2\%$ with $50\,\mathrm{fb}^{-1}$, $\sim 0.08\%$ with $300\,\mathrm{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\,\mathrm{fb}^{-1}$, $\sim 3\times10^{-9}$ with $300\,\mathrm{fb}^{-1}$
Belle-II [BABAR Collaboration 1107.4465]	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)

Run 4 $\sim$ 2030

Run 5 $\sim$ 2038

$N(D\bar{D}) \sim 10^9/\mathrm{ab}^{-1}$ angular observables $\sim 0.2\%$

- BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
  $\mathcal{B}(D \to \pi^+\pi^-e^+e^-) < 0.7\times10^{-5}$ with $N(c\bar{c}) = 2\times10^7$ at 3.7 GeV

3.770	1	$D^0\bar{D}^0$ $D^+\bar{D}^-$ $D^0\bar{D}^0$ $D^+\bar{D}^-$	3.6 2.8	3.6×10^9 2.8×10^9 7.9×10^8 5.5×10^8	 Single Tag Single Tag

STCF $N(D\bar{D}) \sim 8\times10^9$ Branching ratio $\sim 10^{-8}$

- STCF is still competitive in hunting the NP via $D \to \pi\mu^+\mu^-, \pi\pi\mu^+\mu^-$

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Semileptotic $D_{(s)}$ decays

- a clean environment to study the scalar mesons

- ◦ $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
- ◦ $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another flavor nonet

  compact tetraquark and $K\bar{K}$ bound state in spectral analysis, $q\bar{q}$ is dominated in the B_s decay
- ◦ how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?

- The solution of the above questions deduces to not only the precise pertuabtive QCD, but also **the accurate nonperturbative prediction** of the form factors

- ◦ unstable particle are measured in the lineshape of $\pi\pi$, $K\pi$ invariant mass
- ◦ dynamics of B_{l4} is governed by $B \to \pi\pi$ form factors, a big task of the QCD methods [S. Faller, et.al., 1310.6660, X.W. Kang, et.al., 1312.1193]

- **Dipion LCDAs** are introduced in the LCSRs prediction of $B \to \pi\pi$ form factors

  - [**SC**, A. Khodjamirian and J. Virto, 1701.01633[hep-ph]] B-meson LCSRs
  - [C. Hambrock and A. Khodjamirian, 1511.02509[hep-ph]] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, 1709.00173[hep-ph]] timelike-helicity FF F_t and F_0
  - [**SC**, 1901.06071[hep-ph]] 2πDAs updates and $B \to [\pi\pi]_{\rm S,P}$ FFs
  - [**SC** and J.M Shen, 1907.08401[hep-ph], **SC** and S.L Zhang, 2307.02309[hep-ph]] Pheno
  - [**SC**, 2502.07333[hep-ph]] first study of twist-three 2πDAs and $|V_{ub}|$ extraction

D_s^* width

- $\alpha_s : \alpha : G_F \sim \mathcal{O}(1) : \mathcal{O}(1/137) : \mathcal{O}(10^{-5})$
- very hard to measure weak decay from strong and EM interactions
- the total widths of heavy-light vector mesons are still in lack [PDG 2022]
  - △ $\Gamma_{D^{*+}} = 84.3 \pm 1.8$ KeV ($\to D^0\pi^+, D^+\pi^0, D^+\gamma$)
  - △ $\Gamma_{D^{*0}} < 2.1$ MeV ($\to D^0\pi^0, D^0\gamma$), $\Gamma_{D_s^{*+}} < 1.9$ MeV ($\to D_s^+\gamma, D_s^+\pi^0, D_s^+e^+e^-$)
  - △ $\Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement
- but important to $g_{D_s^* D_s\gamma}$ and et.al., $\to$ non-perturbative approaches

	$g_{D^{*+}D^+\gamma}$ (GeV^{-1})	$g_{D^{*0}D^0\gamma}$ (GeV^{-1})	$g_{D_s^{*+}D_s^+\gamma}$ (GeV^{-1})
this work	$-0.15^{+0.11}_{-0.10}$	$1.48^{+0.29}_{-0.27}$	$-0.079^{+0.086}_{-0.078}$
HHχPT [24]	-0.27 ± 0.05	2.19 ± 0.11	0.041 ± 0.056
HQET+VMD [35]	$-0.29^{+0.19}_{-0.11}$	$1.60^{+0.35}_{-0.45}$	$-0.19^{+0.19}_{-0.08}$
HQET+CQM [71]	$-0.38^{+0.05}_{-0.06}$	1.91 ± 0.09	–
Lattice QCD [32]	-0.2 ± 0.3	2.0 ± 0.6	–
LCSR [21]	-0.50 ± 0.12	1.52 ± 0.25	–
QCDSR [20]	$-0.19^{+0.03}_{-0.02}$	0.62 ± 0.03	-0.20 ± 0.03
RQM [72]	-0.44 ± 0.06	2.15 ± 0.11	-0.19 ± 0.03
experiment [16–18]	-0.47 ± 0.06	1.77 ± 0.03	–

LCSRs, hadronic photon NLO [Li 2020]

LCSRs, LP NLO corrections [Pullin 2021]

$$g_{D_s^* D_s\gamma} = 0.60^{+0.19}_{-0.18}$$

very sensitive to different contributions
(radiative corrections, power corrections)
a benchmark to probe the involved dynamics

D_s^* width

- $\Gamma_{D^{*0}}, \Gamma_{D_s^*}$ plays important role in the $B_{(s)} \to D_{(s)} hh'$ decays

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Role of $D^*_{(s)}$ and their contributions in $B_{(s)} \to D_{(s)} hh'$ decays

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We demonstrate the role of $D^*_{(s)}$ and their contributions in the quasi-two-body decays $B_{(s)} \to D_{(s)} hh'$ $(h, h' = \{\pi, K\})$ in the perturbative QCD approach, stemming from the quark flavor changing $\bar{b} \to \bar{c} q_2 \bar{q}_1$ and $\bar{b} \to c \bar{q}_1 \bar{q}_2$ with $q_1, q_2 = \{s/d, u\}$. The main motivation of this study is the measurement of significant derivations from the simple phase-space model in the channels $B_{(s)} \to D_{(s)} hh'$ at B factories and LHC, which are now clarified as the Breit-Wigner-tail effects from the corresponding intermediate resonant states $D^*_{(s)}$. We confirm that these effects from D^* are small (∼5%) in the quasi-two-body $B_{(s)} \to D\pi\pi(K)$ decaying channels and predict the tiny (< 1%) contributions from D^* in the $B_{(s)} \to D_s K\pi(K)$ decaying channels. Our result that the $B_s \to DK\pi(K)$ decaying channel contributions were only from the Breit-Wigner-tail effect of D_s^* is in agreement with the current LHCb measurement. We recommend that the Belle-II and the LHCb Collaborations restudy the processes $B^+ \to \bar{D}^{*0}\pi^+(K^+) \to D^-\pi^+\pi^+(K^+)$ to reveal the structure of D^{*0} and the strong decay $D^{*0} \to D^+\pi^-$.

$\mathcal{B}_\nu(B^+ \to \bar{D}^{*0}\pi^+ \to D^-\pi^+\pi^+)$

$= (23.3^{+10.1}_{-7.70}) \times 10^{-5}|_{\rm PQCD}$

$(22.3 \pm 3.20) \times 10^{-5}$ [Belle 2004]

$\mathcal{B}_\nu(B_s^0 \to D_s^{*-}\pi^+ \to \bar{D}^0 K^-\pi^+)$

$= (1.90^{+1.01}_{-0.68}) \times 10^{-5}|_{\rm PQCD}$

$(4.70 \pm 4.38) \times 10^{-5}$ [LHCb 2014]

- **Impressive lattice QCD evaluation** [HPQCD 2013]

$$\Gamma^{\rm HPQCD}_{D_s^{*+}} = 70(28)\,\mathbf{eV}$$

△ the longest-lived charged vector meson ?

△ dominated by EM decay, strong decay is forbidden/suppressed by the phase space

△ encourage us to study D_s^* (ie., $\Gamma_{D_s^*}$) via the exclusive weak decay

△ $54.9 \pm 5.4\,\mathbf{eV}$ [Yu Meng, et. al., 2401.13475[hep-lat]]

Opportunities in $D_{(s)}^{(*)}$ decays
(private opinions)

i D_s^* weak decays

ii $D_s \to [f_0, \cdots \to] \pi\pi e^+\nu$

iii $D \to \rho l^+\nu$ and $D_{(s)} \to K^* l^+\nu$

D_s^* weak decays

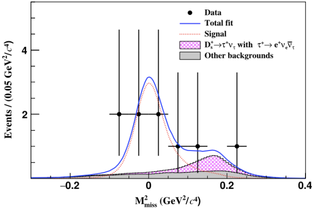
Leptonic D_s^* decay

- helicity enhanced rather than suppressed in D_s case
  [**SC**, Y.H. Ju, Q. Qin and F.S. Yu, 2203.06797[hep-ph]]

$$\Gamma_{D_s^* \to l\nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 \left(1 - \frac{m_l^2}{m_{D_{s^*}^2}}\right)\left(1 + \frac{m_l^2}{m_{D_{s^*}^2}}\right) = 2.44 \times 10^{-12}\ \mathbf{GeV}$$

- $\mathcal{B}(D_s^* \to \mu\nu) = \frac{\Gamma_{D_s^* \to \mu\nu}}{\Gamma_{D_s^*}} \sim \frac{\Gamma_{D_s \to \mu\nu}}{\Gamma_{D_s^*}} \frac{2m_{D_s^*}^2}{3m_\mu^2} \sim 2 \times 10^{-5}$

  close to $(3.4 \pm 1.4) \times 10^{-5}$ [HPQCD 2013]



[arXiv:2304.12159, BESIII]

- $\mathcal{B}(D_s^* \to e\nu_e) = (2.1^{+1.3}_{-0.9}) \times 10^{-5}$
- $\Gamma_{D_s^*} = 122^{+70}_{-52} \pm 12$ eV

  $\Uparrow \frac{f_{D_s^*}}{f_{D_s}} = 1.12 \pm 0.01$ [χQCD 2021]

  agree with the lattice evaluation within 1σ

- quantum number determination 1^- [arXiv:2305.14631, BESIII]

Semileptonic $D_s^* \to \phi l\nu$ decays

- heavy-to-light form factors (FFs) play the key role in weak decays
- both pert. and nonpert. physics enter into the game
- the measurement would reveal the inner structures of hadrons

- QCD-based approaches to calculate FFs, LCSRs, PQCD, LQCD, et al.
- implements of LCSRs in charm $D \to \pi, K, \eta^{(\prime)}, \phi$ [Khodjamirian 2000, Ball 2006, Offen 2013, Du 2003, Wu 2006] and also in strange-charm $D_s \to f_0, a_0$ [Cheng 2006, Lü 2007, Han 2013, **SC** 2019, Fu 2022]

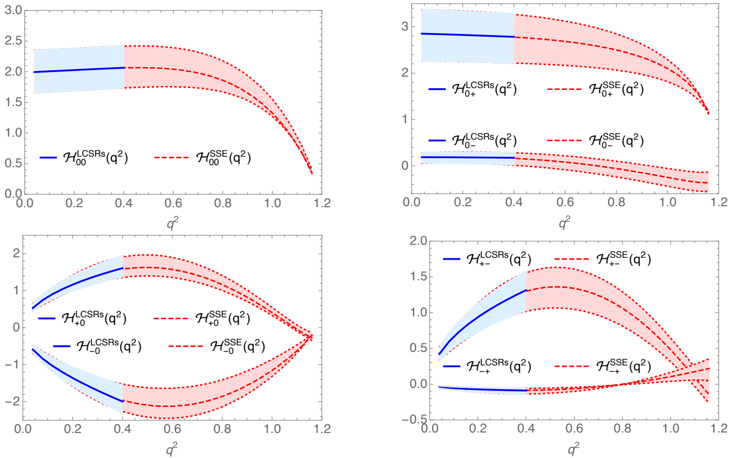
- $D_s^* \to \phi$ FFs considered here is the first attempt of $V \to V$ type transition from LCSRs [**SC**, Y.H Ju, Q Qin and F.S Yu, 2203.06797[hep-ph]]

  △ semileptonic decays with $m_l \sim 0$, consider seven helicity FFs: $\mathbf{00}, \mathbf{0}\pm, \pm\mathbf{0}, \pm\mp$

  △ LCSRs prediction is reliable in large recoiled region $[0, 0.4]$ GeV^2

  △ parameterisation to the full kinematical region $[0, 1.2]$ GeV^2

$D_s^* \to \phi$ form factors [**SC**, Y.H Ju, Q Qin and F.S Yu, 2203.06797[hep-ph]]



- dependence on the $q^2_{\rm LCSR,max}$ are checked by varying it in $[0.2, 0.4]\,{\rm GeV}^2$, $\mathcal{H}_{ij}(1.2\,{\rm GeV}^2)$ change no more than four percents.

$D_s^* \to \phi l \nu$ and $D_s^* \to \phi\pi, \rho$ decays

- **semileptonic decay** $\quad D_s^* \to \phi l \nu_l$

$$\frac{d\Gamma_{ij}(q^2)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s^*}^3} \lambda^{1/2}(m_{D_s^*}^2, m_\phi^2, q^2)\, q^2\, |H_{ij}(q^2)|^2$$

$$\Gamma_{D_s^* \to \phi l \nu_l} = \frac{1}{3}\int_0^{q_0^2} dq^2 \sum_{i,j=0,\pm} \frac{d\Gamma_{ij}(q^2)}{dq^2} = \left(3.28^{+0.82}_{-0.71}\right) \times 10^{-14}\ \mathbf{G}eV$$

- **hadronic decays** $\quad D_s^* \to \phi\rho, \phi\pi$, naive factorization assumption

$$\mathcal{A}(D_s^{*+} \to \phi\rho^+) = \frac{G_F}{\sqrt{2}} V_{cs}\, a_1\, m_\rho f_\rho^{\|(\perp)} \sum_{i,j} H_{ij}(m_\rho^2)$$

$$\Gamma_{D_s^{*+} \to \phi\pi^+} = \left(3.81^{+1.52}_{-1.33}\right) \times 10^{-14}\ \mathbf{G}eV, \quad \Gamma_{D_s^{*+} \to \phi\rho^+} = \left(1.16^{+0.42}_{-0.39}\right) \times 10^{-13}\ \mathbf{G}eV$$

△ $a_1(\mu) = 0.999$, $f_\pi = 0.130$ GeV, $f_\rho^{\|} = 0.210$ GeV

△ the result of $\phi\pi$ channel is marginally consistent with the PQCD [Yang 2022]

△ with the lattice evaluation $\Gamma_{D_s^*} = (0.70 \pm 0.28) \times 10^{-8}$ GeV [HPQCD 2013]

$$\mathcal{B}(D_s^* \to \phi l \nu) = (0.47\,^{+0.12}_{-0.10} \pm 0.19) \times 10^{-6},$$

$$\mathcal{B}(D_s^{*+} \to \phi\pi^+) = (0.54\,^{+0.22}_{-0.19} \pm 0.22) \times 10^{-6}, \quad \mathcal{B}(D_s^{*+} \to \phi\rho^+) = (1.65\,^{+0.61}_{-0.56} \pm 0.66) \times 10^{-6}.$$

Exclusive D_s^* weak decays

- **Belle II** clear background

  △ 2023, 428 fb^{-1}, reconstruct 2×10^5 data samples of $D_s^*(D_s)$ from $\phi\pi$ channel

  △ phase 3 running (2024-2026), 10 ab^{-1}, $\mathcal{O}(1 \times 10^7)$ data sample of $D_s^*(D_s)$

  △ the number of D_s^* production is $\mathcal{O}(10^9)$ $\Leftarrow \mathcal{B}(D_s \to \phi\pi) = (4.5 \pm 0.4)\,\%$

  △ excellent potential to study the D_s^* weak decays, 50 ab^{-1}

- **LHCb** excellent particle identification to distinguish K, π and μ

  △ the channel $D_s^* \to \phi(KK)\pi$ with the D_s^* producing by $B_s \to D_s^* \mu\nu$

- **BESIII** low background [arXiv:2304.12159, 2305.14631]

  △ directly produced from e^+e^- collision at the $D_s D_s^*$ threshold

  △ have collected $\sim 6 \times 10^6$ D_s^* mesons with the 3.2 fb^{-1} data at 4.178 GeV

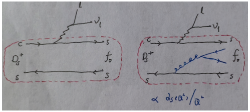
  △ provides the good chance for the leptonic decay $D_s^* \to l\nu$, Statistical error

- heavy quark symmetry (**HQS**) has been examined in $\bar{B} \to D^{(*)} l\bar{\nu}$, can also be tested in $D_s^{(*)} \to \phi l^+ \nu$
- lepton flavour university (**LFU**) in vector charm sector

$$D_s \to [f_0, \cdots \to] \pi\pi e^+ \nu$$

Semileptotic D_s decays

- a clean environment to study the scalar mesons, especially for $f_0(980)$



⋆ tetraquark contribution is suppressed doubly by strong coupling and power

⋆ FSI is weak too

$$|f_0(980)\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$
$$\psi^n_{f_0}(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i | f_0\rangle$$

⋆ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_n \int_0^1 dx_i \mathcal{H}^n(x_i, Q)\psi^n(x_i, \mu)$$

⋆ ψ^n is universal, however H^n is process dependent, hence different observables might highlike the contributions from different components

- **how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?**
- The solution of the above questions deduces to not only the precise pertuabtive QCD, but also **the accurate nonperturbative prediction** of the form factors
- large width of $f_0(980)$ gives a big theoretical pollution
  - unstable particle are measured in the lineshape of $\pi\pi$, $K\pi$, KK invariant masses

$D_s \to [f_0, \cdots \to] \pi\pi e\nu$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons

◦ $D_s \to f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \to a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII '19]

◦ $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical consideration

$$\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |\mathbf{V_{cs}}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \; D_s \to f_0 \text{ FF}$$

- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1 \beta_\pi(s)}{|m_{\rm S}^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))|^2}, \text{ BESIII}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}} q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2, \; D_s \to \pi\pi \text{ FF}$$

$D_s \to f_0$ form factor and $D_s^+ \to (f_0, [\pi\pi]_{\rm S}) e^+ \nu_e$ decay

- $M^2 = 5.0 \pm 0.5\,{\rm GeV}^2$ and $s_0 = 6.0 \pm 0.5\,{\rm GeV}^2$

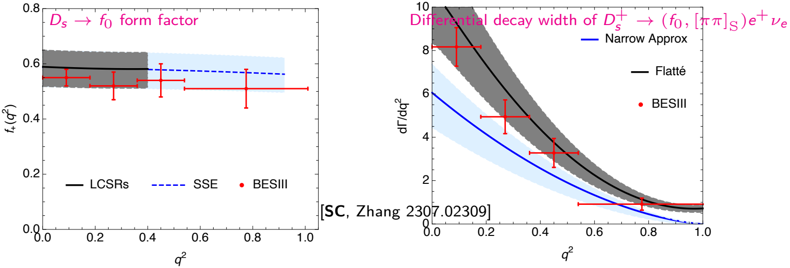
this wrok	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

◦ the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)

we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, Zhang 2307.02309]

◦ the uncertainty estimation is conservative, without NLO correction

◦ we need a model independent calculation, not only for the QCD understanding, but also for the future partial-wave measurement

$D_s \to [\pi\pi]_{\rm S}$ form factors

- Definition of $D_s \to [\pi\pi]_{\rm S}$ form factor

$$\langle [\pi(k_1)\pi(k_2)]_{\rm S}\, |\bar{s}\gamma_\mu(1-\gamma_5)c|D_s^+(p)\rangle = -iF_t k^t_\mu - iF_0(q^2,s,\zeta)k^0_\mu - iF_\parallel k^\parallel_\mu$$

$$k^t_\mu = \frac{q_\mu}{\sqrt{q^2}},\, k^0_\mu = \frac{2\sqrt{q^2}}{\sqrt{\lambda_{D_s}}}\left(k_\mu - \frac{k\cdot q}{q^2}q_\mu\right),\, k^\parallel_\mu = \frac{1}{\sqrt{k^2}}\left(\bar{k}_\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_{D_s}}k_\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_{D_s}}q_\mu\right)$$

- LCSRs calculations start with the correlation functions

$$\Pi^{ab}_\mu(q,k_1,k_2) = i\int d^4x e^{iq\cdot x}\langle\pi^a(k_1)\pi^b(k_2)|T\{j_{1,\mu}(x),j_2(0)\}|0\rangle$$

- Introduce a parameter angle to describe the mixing

$$[\pi\pi]_{\rm S} = |\bar{n}n\rangle\cos\theta + |\bar{s}s\rangle\sin\theta, \quad [KK]_{\rm S} = -|\bar{n}n\rangle\sin\theta + |\bar{s}s\rangle\cos\theta$$

- The chiral even two quark isoscalar 2πDAs **our knowledge of 2πDAs is still at leading twist**

$$\langle\left[\pi^a(k_1)\pi^b(k_2)\right]_{\rm S}|\bar{s}(xn)\gamma_\mu s(0)|0\rangle = 2\delta^{ab}k_\mu\sin\theta\int du e^{iux(k\cdot n)}\Phi^{l=0}_{\parallel,[\pi\pi]_{\rm S}}(u,\zeta,k^2)$$

$$\Phi^{l=0}_{\parallel,[\pi\pi]_{\rm S}} = 6u(1-u)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{odd}}^{n+1}B^{l=0}_{\parallel,nl}(k^2,\mu)C^{3/2}_n(2u-1)C^{1/2}_l(2\zeta-1)$$

- Do the similar LCSRs to $D_s \to f_0$ and consider the partial-wave expansion

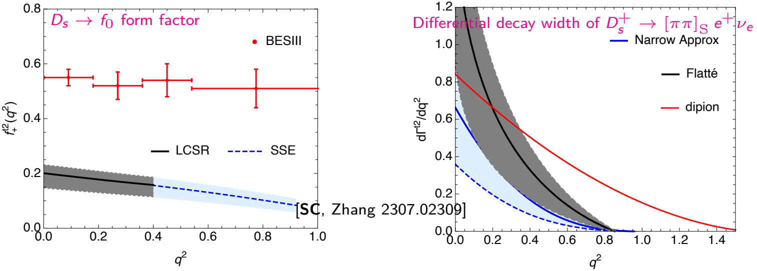
$$F_0(q^2,k^2,\zeta) = \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\,F^{(\ell)}_{0,t}(q^2,k^2)P^{(0)}_\ell(\cos\theta_\pi)$$

$D_s \to [\pi\pi]_{\rm S}$ form factor and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_{\rm S}$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$F_0^{(\ell')}(q^2,k^2) = \frac{m_c(m_c+m_s)\sin\theta}{m_{D_s}^2 f_{D_s}\sqrt{\lambda_{Ds}}\sqrt{q^2}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2,k^2,M^2,s_0)\, B_{n\ell',\parallel}^{I=0}(k^2)$$

- Leading twist $D_s \to f_0, [\pi\pi]_{\rm S}$ form factors and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay



[SC, Zhang 2307.02309]

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_{\rm S}$ transitions

does not indicate a breakdown of the twist expansion, the asymptotic term in the leading twist LCDAs is zero due to the charge conjugate invariance

- Further measurements, high twist DiPion LCDAs [SC, 2502.07333[hep-ph]]

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

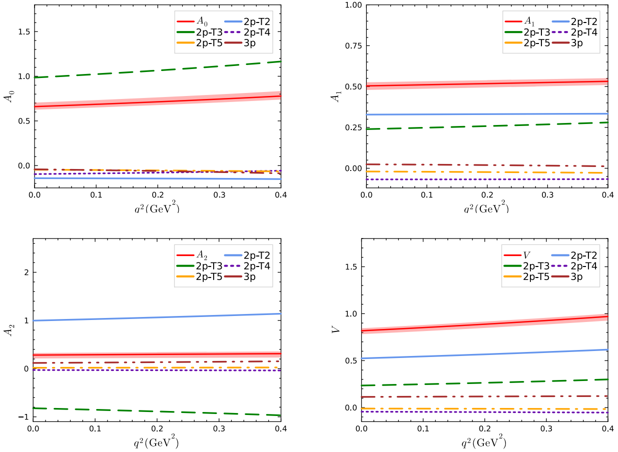
- motived by the recent BESIII measurements

Decay Mode	Collaboration	Year	Reference
$D^0 \to K^{*-} \mu^+ \nu_\mu$	BESIII	2025	PRL 134(2025)1,011803
$D^0 \to \rho^- e^+ \nu_e$	BESIII	2024	PRD 110(2024)11,112018
$D_s^+ \to K^{*0} e^+ \nu_e$	BESIII	2019	PRL 122 (2019) 6, 061801
$D^+ \to \rho^0 e^+ \nu_e$	CLEO	2013	PRL 110(2013)13,131802
$D^+ \to K^{*0} \mu^+ \nu_\mu$	FOCUS	2006	PLB 637(2006)32-38

- state-of-the-art LCSRs calculation of $D \to V$ form factors with high twist LCDAs
- to examine the width effects and the non-resonant QCD backgrounds in the D_{l4} decays ($D_{(s)} \to V[\to P_1 P_2] l\nu$)

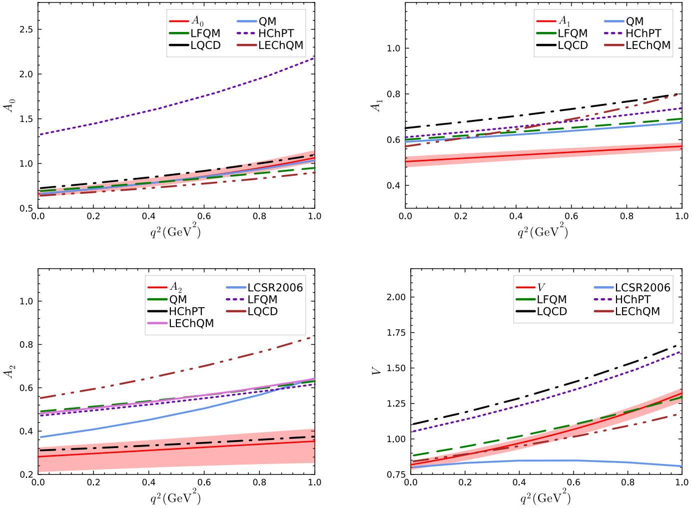
$D^0 \to \rho^-$ form factors from different twist (power) contributions

[W. Lin, X.E Huang, **SC** and D.L Yao, 2505.01329]



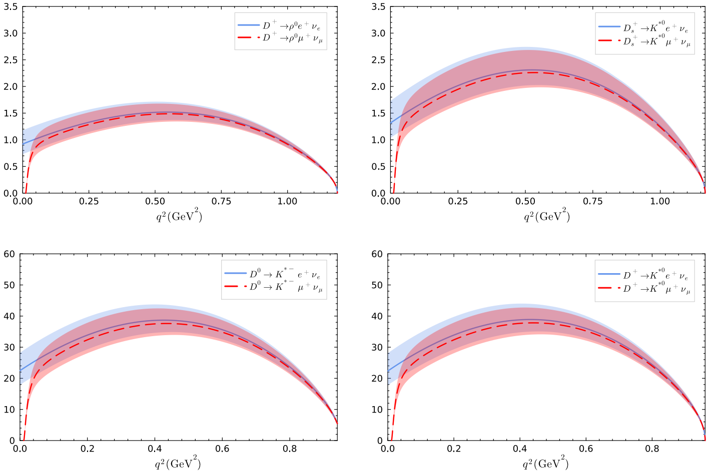
- both the three-particle LCDAs and the power correction from the heavy quark expansion at $\mathcal{O}(1/m_c)$ contribute significantly to the form factors A_1 and A_2.
- The OPE expansion exhibits good convergence: while two-particle twist-three LCDAs contribute sizeably or even dominantly in the axial-vector transition, the higher-twist effects remain well under control

$D^0 \to \rho^-$ form factors



- LCSRs predictions in comparing to the results obtained from other approaches

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$



- Differential decay widths $d\Gamma/dq^2$ of $D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$ decays
- the lepton mass effect is mostly prominent in the large-recoil region of the semileptonic charm to vector meson decays

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

- branching ratios of semileptonic charm decays (in unit of 10^{-6})

	$D^+ \to \rho^0 \ell^+ \nu_\ell$	$D_s^+ \to K^{*0} \ell^+ \nu_\ell$	$D^0 \to K^{*-} \ell^+ \nu_\ell$	$D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$
This work	$2.30^{+0.32}_{-0.25}$	$1.55^{+0.30}_{-0.20}$	$17.6^{+2.4}_{-1.9}$	$45.2^{+6.2}_{-5.0}$
	$2.20^{+0.30}_{-0.23}$	$1.48^{+0.29}_{-0.19}$	$16.6^{+2.2}_{-1.8}$	$42.7^{+5.7}_{-4.5}$
LCSR2006 [21]	$2.29^{+0.23}_{-0.16}$	$2.33^{+0.29}_{-0.30}$	21.2 ± 0.9	$53.7^{+2.4}_{-2.3}$
	$2.20^{+0.21}_{-0.16}$	$2.24^{+0.27}_{-0.29}$	20.1 ± 0.9	$51.0^{+2.3}_{-2.1}$
CLFQM [51]	2.32	1.90	—	73.2
	2.22	1.82		69.3
HChPT [32]	2.50	2.20	22.0	56.0
PDG [2]	1.90 ± 0.10	2.15 ± 0.28	21.5 ± 1.60	54.0 ± 1.00
	2.40 ± 0.40	—	18.9 ± 2.40	52.7 ± 1.50

- 10%-20% discrepancy when confronting our LCSR predictions with experimental measurements
- a sizable $SU(3)$ flavor-breaking effect is observed in $D_{(s)} \to K^* l\nu$ decays
- the necessity to further implement the effects of vector meson widths and non-resonant QCD background

Conclusions and Prospects

- **The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{l4} processes**

◦ a new booster on the accurate calculation in flavor physics

◦ improvement study in the CKM determinations and the flavor anomalies

- The studied of DiPion LCDAs and H_{l4} decays are mostly at leading twist

◦ determine the parameters by low energy effective theory and data constraints

◦ evolution of k^2 from the threshold to large scale $\mathcal{O}(m_c^2, m_b\lambda_{QCD})$

◦ universal phase shift in $\pi\pi$ scattering and heavy decay ?

- Go further to high twist LCDAs [**SC** 2502.07333[hep-ph]]

◦ $B \to \pi\pi l\nu, B \to [\rho\rho \to] \to 4\pi,\ D_s \to \pi\pi l\nu, D \to K\pi\mu\nu,\ D \to \pi\pi e^+e^-$ et al.

Thank you for your patience.