Two-body nonleptonic decays of charmed baryons

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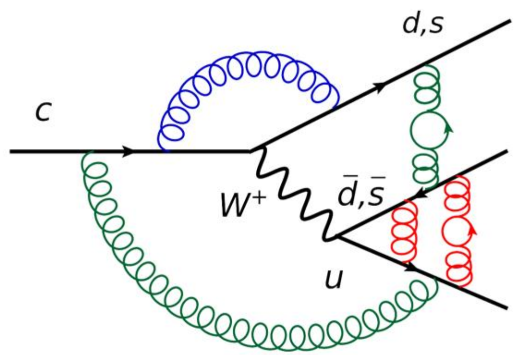
Contents

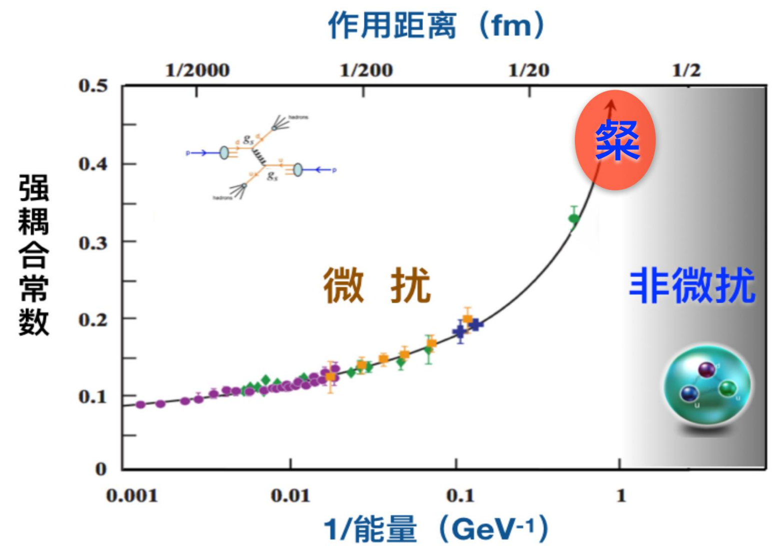
Contents

Charmed hadron decays

➢ **Non-perturbative**



$m_c \approx 1.3 GeV$



➢ **Up-type quark decay in hadron, search for new physics**

- Complementary to the decay of the down-type quarks *d, s, b*
- CP violation in the SM is suppressed

Charmed baryon decays

理论方法	优 势	劣 势
1.QCD方法	基于第一性原理	基于微扰论，非微扰贡献是难题
2.群表示方法	对称性分析，无其它近似	不与衰变动力学直接联系
3.拓扑图方法	包含非微扰效应	用实验数据抽取拓扑图的信息
4.动力学模型	非微扰的动力学	较大的理论不确定性

Cai-Ping Jia’ talk

Lu:2016ogy, Wang:2017azm, Shi:2017dto, Geng:2017esc, Geng:2017mxn, Wang:2017gxe, He:2018php, Wang:2018utj, He:2018joe, Geng:2018plk, Geng:2018bow, Jia:2019zxi, Geng:2019bfz, Wang:2020gmn, Li:2021rfj, He:2021qnc, Wang:2022wrb …

Contents

Flavor sum rules

➢ Isospin sum rules

$$-\sqrt{2}\,\mathcal{A}(\Xi_c^0 \to \pi^0\Xi^0) - \mathcal{A}(\Xi_c^0 \to \pi^+\Xi^- - \mathcal{A}(\Xi_c^+ \to \pi^+\Xi^0) = 0$$

$$2\,\mathcal{A}(\Xi_c^0 \to \pi^0\Sigma^0) - \mathcal{A}(\Xi_c^0 \to \pi^-\Sigma^+) - \mathcal{A}(\Xi_c^0 \to \pi^+\Sigma^-) \\ + \sqrt{2}\,\big(\mathcal{A}(\Xi_c^+ \to \pi^0\Sigma^+) + \mathcal{A}(\Xi_c^+ \to \pi^+\Sigma^0)\big) = 0$$

- Test flavor symmetry
- Estimate branching fraction

➢ How to get the flavor sum rules?

Charmed baryon decays

➢ Effective Hamiltonian

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V_{cq}^* V_{uq}\left(\sum_{q=1}^{2} C_i(\mu)O_i(\mu)\right) - V_{cb}^* V_{ub}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)\right)\right]$$

$$\mathcal{H}_{\rm eff} = \sum_p \sum_{i,j,k=1}^{3} (H^{(p)})^k_{ij} O^{(p)k}_{ij} \qquad O^{(p)k}_{ij} = \frac{G_F}{\sqrt{2}} \sum_{\rm color} \sum_{\rm current} C_p(\bar{q}_i q_k)(\bar{q}_j c)$$

CKM **Operator**

➢ Hadrons

$$|M^\alpha\rangle = (M^\alpha)^i_j |M^i_j\rangle \qquad M^{\pi^0} = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 \\ 0 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \cdots$$

D. Wang, C. P. Jia and F. S. Yu, JHEP 09, 126 (2021).

Decay amplitudes and Flavor relation

$$\mathcal{A}(\mathcal{B}_{c\bar{3}}^{\gamma} \to \mathcal{B}_8^{\alpha} M^{\beta}) = \langle \mathcal{B}_8^{\alpha} M^{\beta} | \mathcal{H}_{\rm eff} | \mathcal{B}_{c\bar{3}}^{\gamma} \rangle$$
$$= \sum_{p} \sum_{\rm Per.} \langle \mathcal{B}_8^{ijk} M_m^l | \mathcal{O}_{np}^q | [\mathcal{B}_{c\bar{3}}]_{rs} \rangle \times (\mathcal{B}_8^{\alpha})^{ijk} (M^{\beta})_m^l H_{np}^q (\mathcal{B}_{c\bar{3}}^{\gamma})_{rs}$$
$$= \sum_{\omega} X_{\omega}^{(p)} (C_{\omega}^{(p)})_{\alpha\beta\gamma}$$

➢ If there exists an operator T, under which the Hamiltonian is invariant, ***TH = 0***. The invariance of Hamiltonian under T is related to the generation of sum rules.

$$\langle M_{\alpha} \mathcal{B}_{\beta} | T \mathcal{H}_{\rm eff} | [\mathcal{B}_{c\bar{3}}]_{\gamma} \rangle = \sum_{\omega} \langle M_m^n \mathcal{B}_r^s | O_l^{jk} | [\mathcal{B}_{c\bar{3}}]_i \rangle \times (M_{\alpha})_m^n (\mathcal{B}_{\beta})_r^s (TH)_l^{jk} ([\mathcal{B}_{c\bar{3}}]_{\gamma})_i = 0$$

Isospin sum rule

- No operator T constructed by I_3, I_-, I_+ which $T\,H_c=0$.

$$\mathcal{H}_{\rm eff} = \sum_{i,j,k=1}^{3} H_k^{ij}(\bar{q}^i q_k)(\bar{q}^j c) \quad \longrightarrow \quad 3\otimes\bar{3}\otimes 3 = 3_p\oplus 3_t\oplus\bar{6}\oplus 15$$

$$[H(\bar{6})]^i_j = [H(\bar{6})]_{ij},$$

$$\{[H(15)]_i\}^k_j = [H(15)]^{ij}_k$$

$$I_-[H(\bar{6})] = I_-\cdot[H(\bar{6})] + I_-\cdot[H(\bar{6})]^T = 0,$$

$$I_-[H(3_{t,p})] = [H(3_{t,p})]\cdot I_- = 0,$$

$$\{I_-[H^{(0)}(15)]_i\}^k_j = 2\,\{[H^{(0)}(15)]_{(i}\cdot I_-\}^k_{j)} - \{I_-\cdot[H^{(0)}(15)]_i\}^k_j$$

$$I_-[H(15)]_{2,3} = 0, \qquad I_-[H(15)]_1 \neq 0$$

Isospin sum rule ?

- $I_-^2[H(15)]_1 = I_-\{I_-[H(15)]_1\} = 0$

Isospin sum rule

$$\sum_{\omega} \langle M^n_m \mathcal{B}^s_r | O^{jk}_l | [\mathcal{B}_{c\bar{3}}]_i \rangle \times (M_\alpha)^n_m (\mathcal{B}_\beta)^s_r (TH)^{jk}_l ([\mathcal{B}_{c\bar{3}}]_\gamma)_i = 0$$

- To obtain isospin sum rules, we apply operator I_- to the initial/final states and compute coefficients expanded by the initial/final states

$$\langle [M_8]_\alpha| = (\langle \pi^+|,\ \langle \pi^0|,\ \langle \pi^-|,\ \langle K^+|,\ \langle K^0|,\ \langle \overline{K}^0|,\ \langle K^-|,\ \langle \eta_8|)$$

Basis

$$[I_-]_{M_8} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Isospin sum rule

$$SumI_-[\gamma,\alpha,\beta] = \sum_\mu \left[([I_-]_{M_8})^\mu_\alpha \mathcal{A}_{\gamma\to\mu\beta} + ([I_-]_{\mathcal{B}_8})^\mu_\beta \mathcal{A}_{\gamma\to\alpha\mu} + ([I_-]_{\mathcal{B}_{c\bar{3}}})^\mu_\gamma \mathcal{A}_{\mu\to\alpha\beta}\right]$$

An isospin sum rule is generated if appropriate α, β, γ are selected

Mode	**CF**	**SCS**	**DCS**
$I_-^n H_c = 0$	$I_- H_{\rm CF} = 0$	$I_-^2 H_{\rm SCS} = 0$	$I_-^2 H_{\rm DCS} = 0$
n_{min}	$n \geq 1$	$n \geq 2$	$n \geq 2$
ΔS	$\Delta S = -1$	$\Delta S = 0$	$\Delta S = 1$
ΔQ	$\Delta Q = 1$	$\Delta Q = 2$	$\Delta Q = 2$

arXiv:2301.06104 [hep-ph]

Isospin sum rules

- CF mode

$$SumI_-\,[\Xi_c^0,\pi^+,\Xi^0] = -\sqrt{2}\,\mathcal{A}(\Xi_c^0\to\pi^0\Xi^0) - \mathcal{A}(\Xi_c^0\to\pi^+\Xi^-) - \mathcal{A}(\Xi_c^+\to\pi^+\Xi^0) = 0$$

- SCS/DCS mode

$$\begin{aligned} SumI_-^2\,[\Xi_c^0,\pi^+,\Sigma^+] &= -\sqrt{2}\,SumI_-\,[\Xi_c^0,\pi^+,\Sigma^0] - \sqrt{2}\,SumI_-\,[\Xi_c^0,\pi^0,\Sigma^+] \\ &\quad - SumI_-\,[\Xi_c^+,\pi^+,\Sigma^+] \\ &= 2\left[\,2\,\mathcal{A}(\Xi_c^0\to\pi^0\Sigma^0) - \mathcal{A}(\Xi_c^0\to\pi^-\Sigma^+)\right. \\ &\quad - \mathcal{A}(\Xi_c^0\to\pi^+\Sigma^-) + \sqrt{2}\,\big(\mathcal{A}(\Xi_c^+\to\pi^0\Sigma^+) \\ &\quad \left.+ \mathcal{A}(\Xi_c^+\to\pi^+\Sigma^0)\big)\right] = 0 \end{aligned}$$

Other decay modes: $[I_-]_{M_8,\mathcal{B}_8,\mathcal{B}_{c\bar{3}}}$ ➡ $[I_-]_{X_1,X_2,X_3}$

U-spin sum rules

U-spin sum rules: $[I_-] \longrightarrow [U_-]$

- The U-spin sum rules generated by U_- involve only the CF, SCS, or DCS transition.

- U-spin sum rules involve two or three decay types

$$S = -\lambda U_3 - \lambda^2 U_- + U_+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -\lambda^2 & \lambda \end{pmatrix}, \qquad SH = 0$$

$$\sum_\mu \left[(S_{M_8})^\mu_\alpha \mathcal{A}_{\gamma\to\mu\beta} + (S_{\mathcal{B}_8})^\mu_\beta \mathcal{A}_{\gamma\to\alpha\mu} + (S_{\mathcal{B}_{c\bar{3}}})^\mu_\gamma \mathcal{A}_{\mu\to\alpha\beta} \right] = 0$$

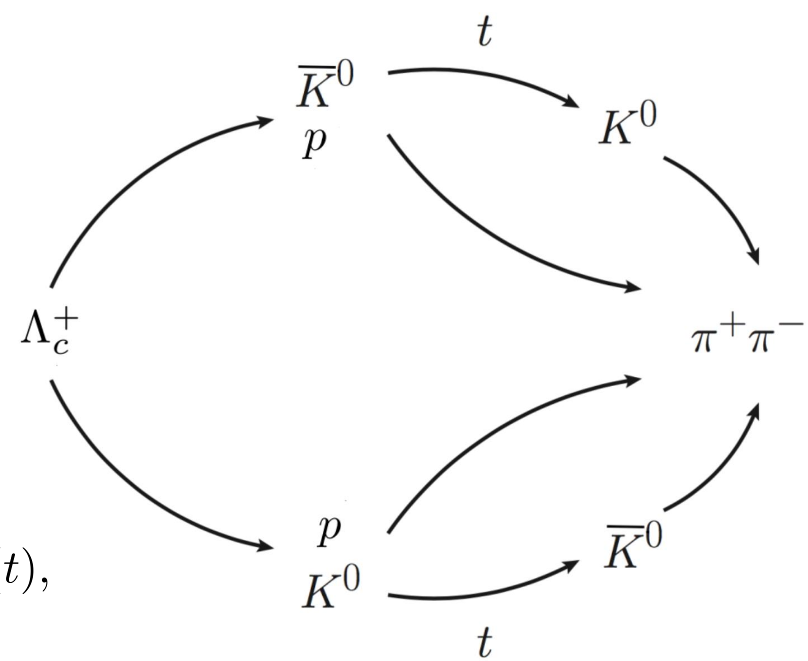
CPV in charm baryon decays into neutral kaons

$$A_{CP}(t) \equiv \frac{\Gamma_{\pi\pi}(t) - \overline{\Gamma}_{\pi\pi}(t)}{\Gamma_{\pi\pi}(t) + \overline{\Gamma}_{\pi\pi}(t)},$$

$$\Gamma_{\pi\pi}(t) \equiv \Gamma(\mathcal{B}_{c\overline{3}} \to \mathcal{B}K(t)(\to \pi^+\pi^-)),$$
$$\overline{\Gamma}_{\pi\pi}(t) \equiv \Gamma(\overline{\mathcal{B}}_{c\overline{3}} \to \overline{\mathcal{B}}K(t)(\to \pi^+\pi^-)).$$

$$A_{CP}(t) \simeq \left(A_{CP}^{\overline{K}^0}(t) + A_{CP}^{\rm dir}(t) + A_{CP}^{\rm int}(t)\right)/D(t),$$

$$A_{CP}(t_1 \ll \tau_S \ll t_2 \ll \tau_L)$$
$$\simeq \left[A_{CP}^{\overline{K}^0} + A_{CP}^{\rm dir} + A_{CP}^{\rm int}\right]/D$$
$$= \frac{-2\mathcal{R}e(\epsilon) + 2r_{\mathcal{B}}\sin\delta_{\mathcal{B}}\sin\phi - 4r_{\mathcal{B}}\mathcal{I}m(\epsilon)\cos\phi\sin\delta_{\mathcal{B}}}{1 - 2r_{\mathcal{B}}\cos\phi\cos\delta_{\mathcal{B}}}$$



[arXiv:1707.09297 [hep-ph]],
[arXiv:1709.09873 [hep-ph]]

U-spin analysis

$$r = |\mathcal{A}_1/\mathcal{A}_2|$$

- U-spin sum rules

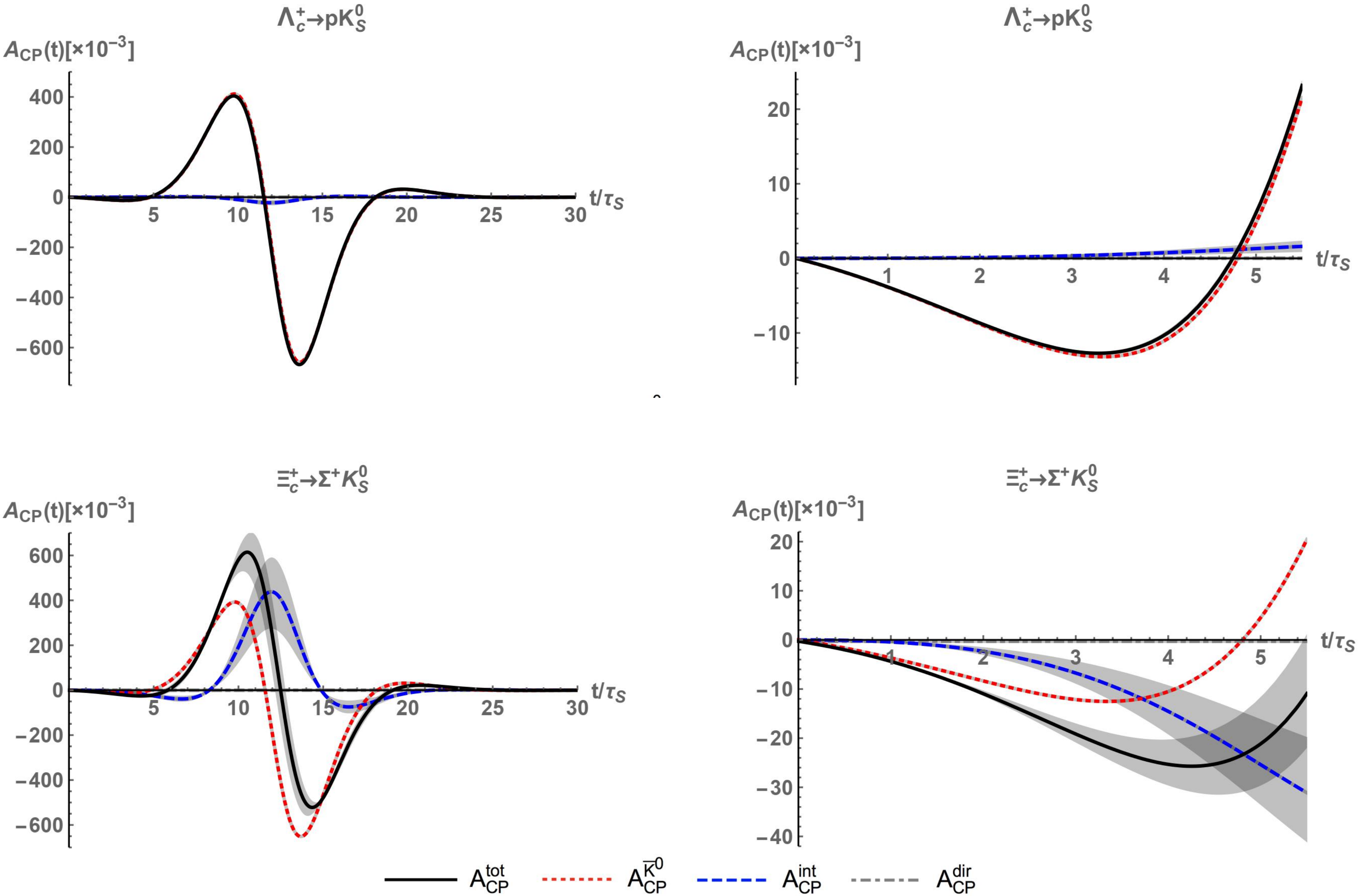
$$\frac{\mathcal{A}(\Lambda_c^+ \to pK^0)}{\mathcal{A}(\Xi_c^+ \to \Sigma^+\overline{K}^0)} = \frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}}, \qquad \frac{\mathcal{A}(\Xi_c^+ \to \Sigma^+ K^0)}{\mathcal{A}(\Lambda_c^+ \to p\overline{K}^0)} = \frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}}$$

$$r_p = -\left|\frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}}\right| \times r, \qquad r_{\Sigma^+} = -\left|\frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}}\right| \times \frac{1}{r}, \qquad \delta_p = -\delta_{\Sigma^+}$$

$$\mathcal{B}r(\Lambda_c^+ \to pK_S^0) = (16.1 \pm 0.7) \times 10^{-3}$$
$$\mathcal{B}r(\Xi_c^+ \to \Sigma^+ K_S^0) = (1.94 \pm 0.90) \times 10^{-3}$$

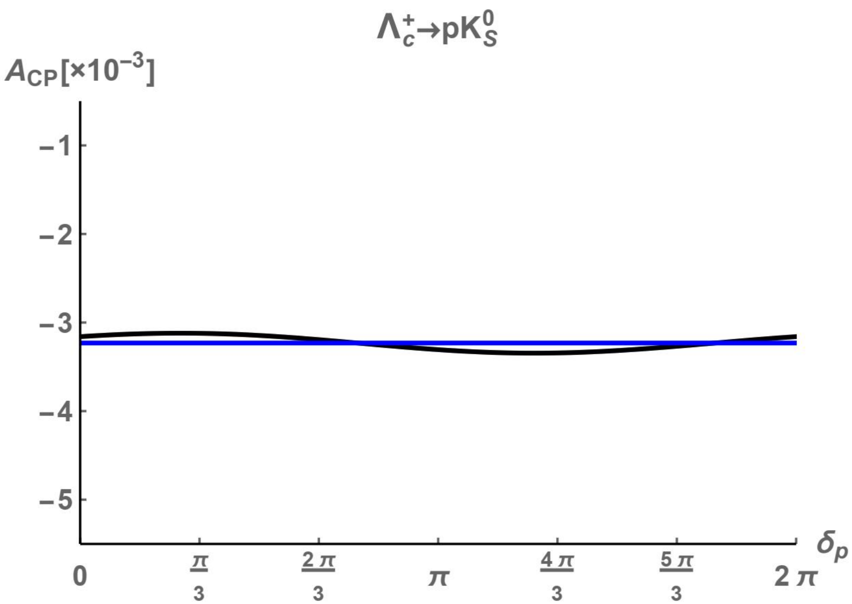
I. Adachi *et al*. [Belle-II and Belle], [arXiv:2503.17643 [hep-ex]].

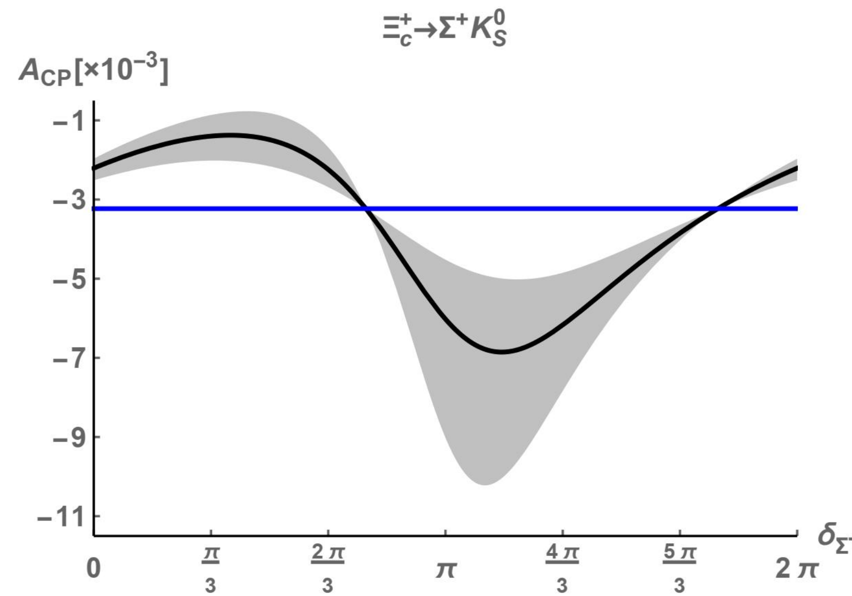
$$\rightarrow \quad r_p = (-1.14 \pm 0.26 \pm 0.32 \pm 0.34) \times 10^{-2}$$
$$r_{\Sigma^+} = (-23.2 \pm 5.4 \pm 1.5 \pm 7.0) \times 10^{-2}$$



$\Lambda_c^+ \to pK_S^0$
$A_{CP}(t)[\times 10^{-3}]$
400
200
0
−200
−400
−600
5
10
15
20
25
30
t/τ_S
$\Lambda_c^+ \to pK_S^0$
$A_{CP}(t)[\times 10^{-3}]$
20
10
0
−10
1
2
3
4
5
t/τ_S
$\Xi_c^+ \to \Sigma^+ K_S^0$
$A_{CP}(t)[\times 10^{-3}]$
600
400
200
0
−200
−400
−600
5
10
15
20
25
30
t/τ_S
$\Xi_c^+ \to \Sigma^+ K_S^0$
$A_{CP}(t)[\times 10^{-3}]$
20
10
0
−10
−20
−30
−40
2
3
4
5
t/τ_S
A_{CP}^{tot}
$A_{CP}^{\overline{K}^0}$
A_{CP}^{int}
A_{CP}^{dir}

CPV





- $\Xi_c^+ \to \Sigma^+ K_S^0$: verify interference between the hadron decay and neutral final-state kaon mixing

- $\Lambda_c^+ \to pK_S^0$: search for new physics beyond the SM

Contents

Topology ⟷ Invariant Tensor

$$\mathcal{A}(\mathcal{B}^{\gamma}_{c\overline{3}} \to \mathcal{B}^{\alpha}_{8} M^{\beta}) = \langle \mathcal{B}^{\alpha}_{8} M^{\beta} | \mathcal{H}_{\rm eff} | \mathcal{B}^{\gamma}_{c\overline{3}} \rangle$$
$$= \sum_{p} \sum_{\rm Per.} \langle \mathcal{B}^{ijk}_{8} M^{l}_{m} | \mathcal{O}^{q}_{np} | [\mathcal{B}_{c\overline{3}}]_{rs} \rangle \times (\mathcal{B}^{\alpha}_{8})^{ijk} (M^{\beta})^{l}_{m} H^{q}_{np} (\mathcal{B}^{\gamma}_{c\overline{3}})_{rs}$$
$$= \sum_{\omega} X^{(p)}_{\omega} (C^{(p)}_{\omega})_{\alpha\beta\gamma}$$

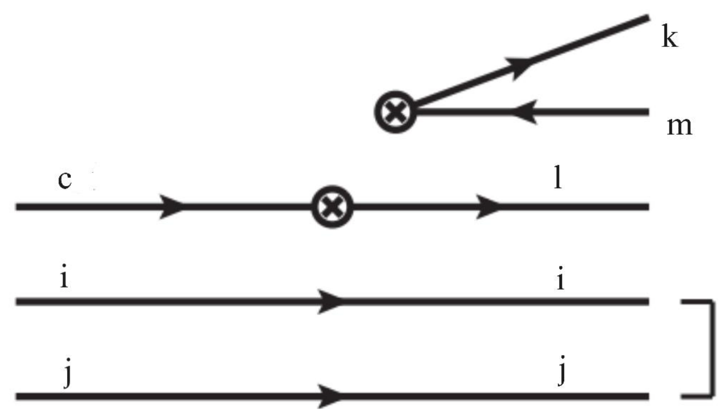
CG coefficient, mode-dependent

Reduced matrix element, mode-independent

Wigner-Eckart theorem

$$\langle jm | T^{(k)}_{q} | j'm' \rangle = \langle j'm'kq | jm \rangle \langle j | T^{(k)} | j' \rangle$$

- Index-contraction ⟷ Quark flowing



$$(\mathcal{B}_{c\overline{3}})_{ij} H^{m}_{kl} M^{k}_{m} (\mathcal{B}^{A}_{8})^{ijl}$$

arXiv:2406.14061

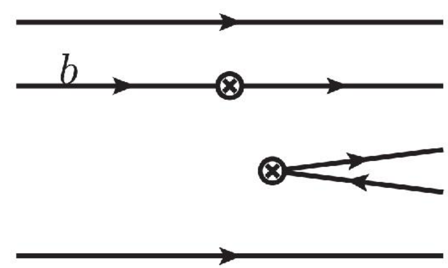
Meson .vs. Baryon

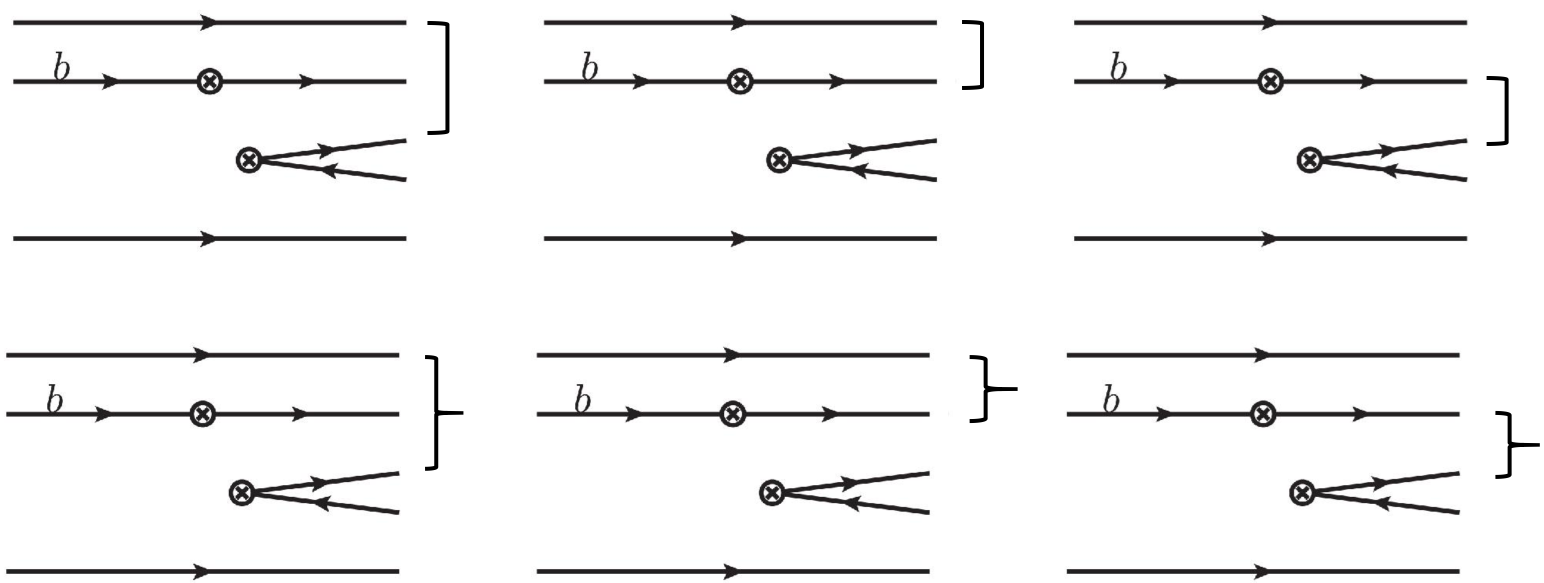
$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

- Wavefunction $\Psi = \phi_{\text{flavor}} \chi_{\text{spin}} \xi_{\text{color}} \eta_{\text{space}}$

- Pauli Exclusion Principle

$$\psi = \frac{1}{\sqrt{2}} (\phi_S \chi_S + \phi_A \chi_A)$$





Completeness of topologies

- Decay amplitude

$$\mathcal{A}(\mathcal{B}_{c\bar{3}} \to \mathcal{B}_8 M) = \frac{1}{\sqrt{2}}\left[\mathcal{A}^S(\mathcal{B}_{c\bar{3}} \to \mathcal{B}_8^S M) + \mathcal{A}^A(\mathcal{B}_{c\bar{3}} \to \mathcal{B}_8^A M)\right]$$

- Completeness of topologies

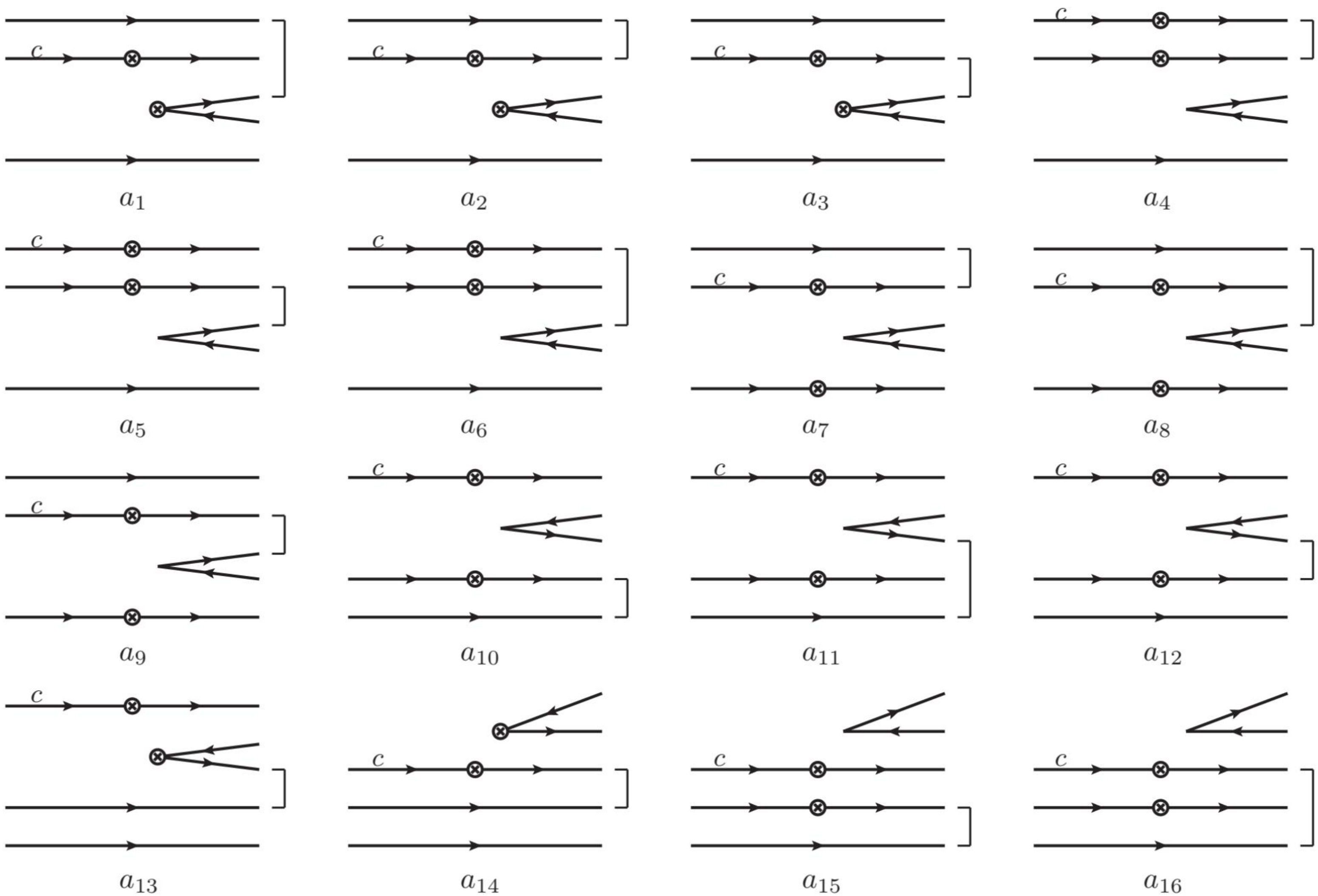
$$\langle \mathcal{B}_8^{ijk} M_m^l | \mathcal{O}_{np}^q | [\mathcal{B}_{c\bar{3}}]_{rs} \rangle$$

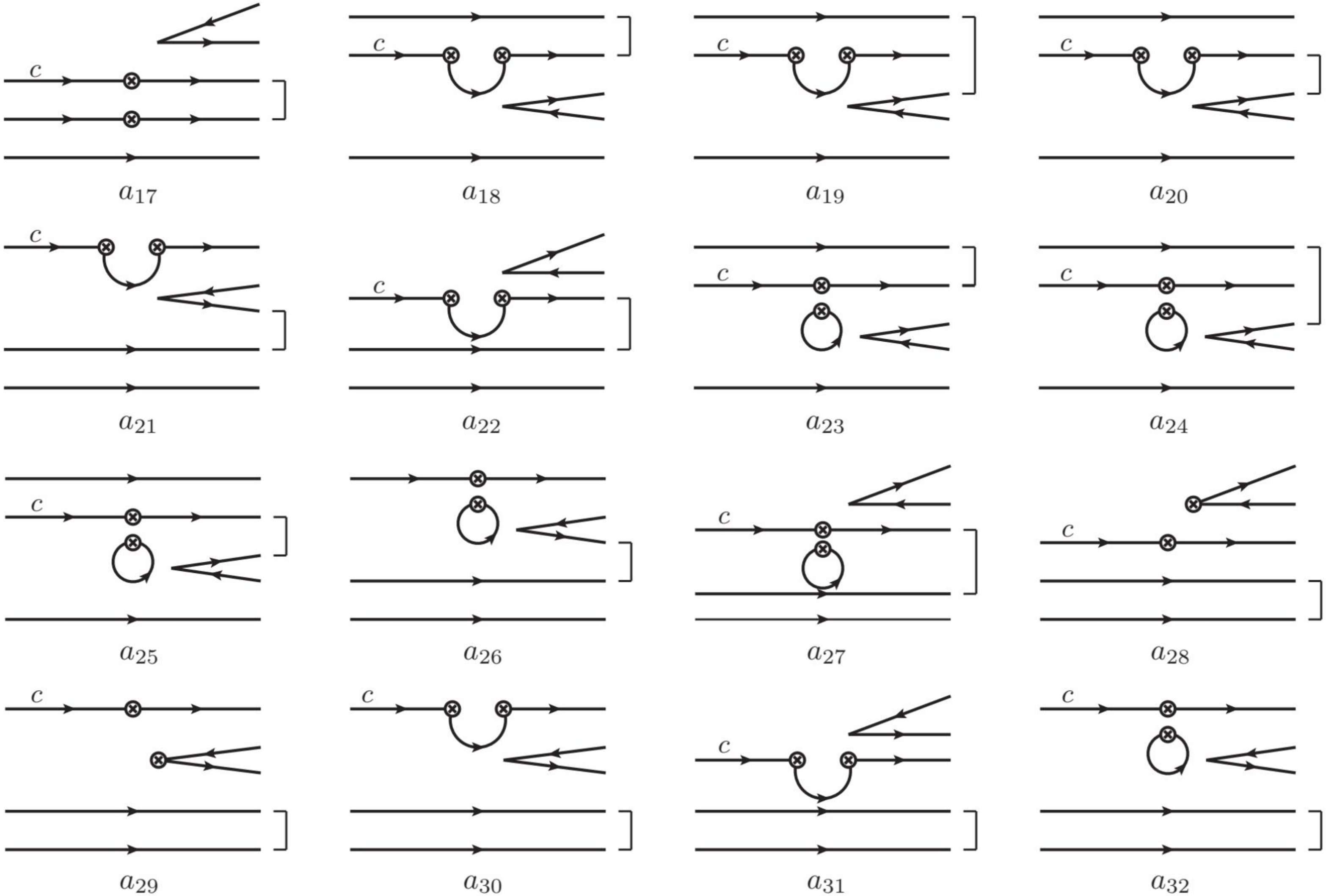
Invariant tensor: all indices are contracted

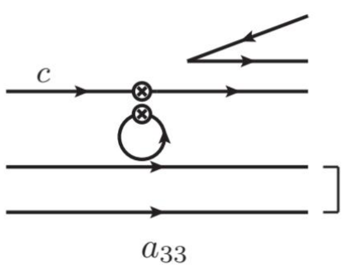
$$N_S + N_A = A_5^5/2 = 60,$$
$$N_A - N_S = A_3^3 = 6,$$
$$N_S = 27, \qquad N_A = 33.$$

Computed by permutation!

Topologies in charmed baryon decays (A)







- ✓ 36 diagrams: without quark loop
- ✓ 24 diagrams: with quark loop

Three indices ⟶ Two indices

- Relation between topologies and SU(3) irreducible amplitudes

$$(\mathcal{B}_8^S)^{ijk}/(\mathcal{B}_8^A)^{ijk}, \qquad (\mathcal{B}_8)^i_j \qquad 3\otimes 3\otimes 3 = 10\oplus 8\oplus 8\oplus 1$$

- Reducing indices $(\mathcal{B}_8)^{ijk} = \epsilon^{ijl}(\mathcal{B}_8)^k_l$ **?**

$$(\mathcal{B}_8^A)^{ijk} = \frac{1}{\sqrt{2}}\epsilon^{ijl}(\mathcal{B}_8)^k_l, \qquad (\mathcal{B}_8^S)^{ijk} = \frac{1}{\sqrt{6}}[\epsilon^{kil}(\mathcal{B}_8)^j_l + \epsilon^{kjl}(\mathcal{B}_8)^i_l]$$

$$(B)^1_3 \longrightarrow \begin{aligned} p^S &= \frac{1}{\sqrt{6}}(-2\mathcal{B}_8^{112} + \mathcal{B}_8^{211} + \mathcal{B}_8^{121}), \\ p^A &= \frac{1}{\sqrt{2}}(\mathcal{B}_8^{121} - \mathcal{B}_8^{211}). \end{aligned}$$

arXiv:2406.14061

(1,1)-rank topological amplitudes

$$
\begin{aligned}
&\mathcal{A}(\mathcal{B}_{c\bar{3}} \to \mathcal{B}_8 M) \\
&= A_1(\mathcal{B}_{c\bar{3}})^i H^j_{kl} M^l_i (\mathcal{B}_8)^k_j + A_2(\mathcal{B}_{c\bar{3}})^i H^j_{lk} M^l_j (\mathcal{B}_8)^k_i + A_3(\mathcal{B}_{c\bar{3}})^i H^j_{lk} M^l_i (\mathcal{B}_8)^k_j \\
&+ A_4(\mathcal{B}_{c\bar{3}})^i H^j_{kl} M^l_j (\mathcal{B}_8)^k_i + A_5(\mathcal{B}_{c\bar{3}})^i H^j_{ik} M^l_j (\mathcal{B}_8)^k_l + A_6(\mathcal{B}_{c\bar{3}})^i H^j_{il} M^l_k (\mathcal{B}_8)^k_j \\
&+ A_7(\mathcal{B}_{c\bar{3}})^i H^j_{ki} M^l_j (\mathcal{B}_8)^k_l + A_8(\mathcal{B}_{c\bar{3}})^i H^j_{li} M^l_k (\mathcal{B}_8)^k_j + +A_9(\mathcal{B}_{c\bar{3}})^i H^j_{ik} M^l_l (\mathcal{B}_8)^k_j \\
&+ A_{10}(\mathcal{B}_{c\bar{3}})^i H^j_{ki} M^l_l (\mathcal{B}_8)^k_j + A_{11}(\mathcal{B}_{c\bar{3}})^i H^j_{kj} M^l_i (\mathcal{B}_8)^k_l + A_{12}(\mathcal{B}_{c\bar{3}})^i H^j_{lj} M^l_k (\mathcal{B}_8)^k_i \\
&+ A_{13}(\mathcal{B}_{c\bar{3}})^i H^j_{kj} M^l_l (\mathcal{B}_8)^k_i + A_{14}(\mathcal{B}_{c\bar{3}})^i H^j_{ij} M^l_k (\mathcal{B}_8)^k_l + A_{15}(\mathcal{B}_{c\bar{3}})^i H^j_{jl} M^l_k (\mathcal{B}_8)^k_i \\
&+ A_{16}(\mathcal{B}_{c\bar{3}})^i H^j_{jk} M^l_l (\mathcal{B}_8)^k_i + A_{17}(\mathcal{B}_{c\bar{3}})^i H^j_{jk} M^l_i (\mathcal{B}_8)^k_l + A_{18}(\mathcal{B}_{c\bar{3}})^i H^j_{ji} M^l_k (\mathcal{B}_8)^k_l .
\end{aligned}
$$

18 terms

Linear correlation of topologies

$$(\mathcal{B}_{b\bar{3}})_{ij} = \epsilon_{ijk}(\mathcal{B}_{b\bar{3}})^k,$$

$$(\mathcal{B}_8^S)^{ijk} = \frac{1}{\sqrt{6}}[\epsilon^{kil}(\mathcal{B}_8)^j_l + \epsilon^{kjl}(\mathcal{B}_8)^i_l],$$

$$(\mathcal{B}_8^A)^{ijk} = \frac{1}{\sqrt{2}}\epsilon^{ijl}(\mathcal{B}_8)^k_l$$

- Relation between three- and (1,1)-rank octets
  - Example

$$\begin{aligned}
& a_1^S(\mathcal{B}_{b\bar{3}})_{ij}H^m_{kl}M^i_m(\mathcal{B}_8^S)^{jkl} \\
&= \frac{1}{\sqrt{6}}a_1^S\epsilon_{ijp}(\mathcal{B}_{b\bar{3}})^p H^m_{kl}M^i_m\epsilon^{ljq}(\mathcal{B}_8)^k_q + \frac{1}{\sqrt{6}}a_1^S\epsilon_{ijp}(\mathcal{B}_{b\bar{3}})^p H^m_{kl}M^i_m\epsilon^{lkq}(\mathcal{B}_8)^j_q \\
&= -\frac{1}{\sqrt{6}}a_1^S\,(\mathcal{B}_{b\bar{3}})^i H^j_{lk}M^l_j(\mathcal{B}_8)^k_i + \frac{2}{\sqrt{6}}a_1^S\,(\mathcal{B}_{b\bar{3}})^i H^j_{kl}M^l_j(\mathcal{B}_8)^k_i \\
&\quad + \frac{1}{\sqrt{6}}a_1^S\,(\mathcal{B}_{b\bar{3}})^i H^j_{ik}M^l_j(\mathcal{B}_8)^k_l - \frac{2}{\sqrt{6}}a_1^S\,(\mathcal{B}_{b\bar{3}})^i H^j_{ki}M^l_j(\mathcal{B}_8)^k_l.
\end{aligned}$$

$\Longrightarrow$

$$A_2 = -\frac{1}{\sqrt{6}}\,a_1^S + ..., \qquad A_4 = \frac{2}{\sqrt{6}}\,a_1^S + ...,$$

$$A_5 = \frac{1}{\sqrt{6}}\,a_1^S + ..., \qquad A_7 = -\frac{2}{\sqrt{6}}\,a_1^S + ...$$

SU(3) irreducible amplitude

$$H_{ij}^k = \frac{1}{8}H(15)_{ij}^k + \frac{1}{4}\epsilon_{ijl}H(\overline{6})^{lk} + \delta_j^k\Big(\frac{3}{8}H(3_t)_i - \frac{1}{8}H(3_p)_i\Big) + \delta_i^k\Big(\frac{3}{8}H(3_p)_j - \frac{1}{8}H(3_t)_j\Big)$$

$$3 \otimes \overline{3} \otimes 3 = 15 \oplus \overline{6} \oplus 3_p \oplus 3_t$$

2-rank TDA = IRA

$$b_1 = \frac{A_6 - A_8}{4}, \qquad b_2 = \frac{A_5 - A_7}{4}, \qquad \dots$$

3-rank TDA ⟶ (1,1)-rank TDA ⟶ (1,1)- rank IRA

$$b_1 = -\sqrt{2}(2a_4^A + a_7^A - a_9^A - a_{10}^A + a_{12}^A)/8$$
$$+\sqrt{6}(2a_5^S - 2a_6^S - a_7^S + 2a_8^S - a_9^S + a_{10}^S - 2a_{11}^S + a_{12}^S)/24,$$

$$b_2 = \sqrt{2}(a_1^A - a_2^A - 2a_3^A + a_5^A - a_6^A)/8 + \sqrt{6}(3a_1^S - 3a_2^S - a_5^S + a_6^S)/24,$$

$$b_3 = -\sqrt{2}(a_5^A - a_6^A - a_{15}^A + a_{16}^A + 2a_{17}^A)/8 + \sqrt{6}(a_5^S - a_6^S + 3a_{15}^S - 3a_{16}^S)/24,$$

……

➢ Property of SU(3) group

$$3_i \otimes 3_j \otimes \overline{3}^k = \ldots \epsilon_{ijl} \overline{6}^{lk} \ldots$$

$$b_1 (\mathcal{B}_{b\overline{3}})^i H(\overline{6})^k_{ij} M^j_l (\mathcal{B}_8)^l_k = b_1 (\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^j_l (\mathcal{B}_8)^l_k,$$

$$b_2 (\mathcal{B}_{b\overline{3}})^i H(\overline{6})^k_{ij} M^l_k (\mathcal{B}_8)^j_l = b_2 (\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^l_k (\mathcal{B}_8)^j_l,$$

$$b_3 (\mathcal{B}_{b\overline{3}})^i H(\overline{6})^k_{ij} M^l_l (\mathcal{B}_8)^j_k = b_3 (\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^l_l (\mathcal{B}_8)^j_k$$

$$b_4 (\mathcal{B}_{b\overline{3}})^i H(\overline{6})^k_{jl} M^j_i (\mathcal{B}_8)^l_k = b_4 \left[(\mathcal{B}_{b\overline{3}})_{jl} H(\overline{6})^{ki} M^j_i (\mathcal{B}_8)^l_k - (\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^j_l (\mathcal{B}_8)^l_k + (\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^l_l (\mathcal{B}_8)^j_k \right],$$

$$b_5 (\mathcal{B}_{b\overline{3}})^i H(\overline{6})^k_{jl} M^l_k (\mathcal{B}_8)^j_i = -b_5 \left[(\mathcal{B}_{b\overline{3}})_{ji} H(\overline{6})^{ki} M^l_k (\mathcal{B}_8)^j_l + (\mathcal{B}_{b\overline{3}})_{jl} H(\overline{6})^{ki} M^j_i (\mathcal{B}_8)^l_k \right]$$

➡

$$b'_1 = b_1 - b_4, \qquad b'_2 = b_2 - b_5,$$
$$b'_3 = b_3 + b_4, \qquad b'_4 = b_4 - b_5$$

Topologies in the Standard Model

- Effective Hamiltonian of charm decay

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V^*_{cq}V_{uq}\left(\sum_{q=1}^{2} C_i(\mu)O_i(\mu)\right) - V^*_{cb}V_{ub}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)\right)\right]$$

- Non-zero H^k_{ij}

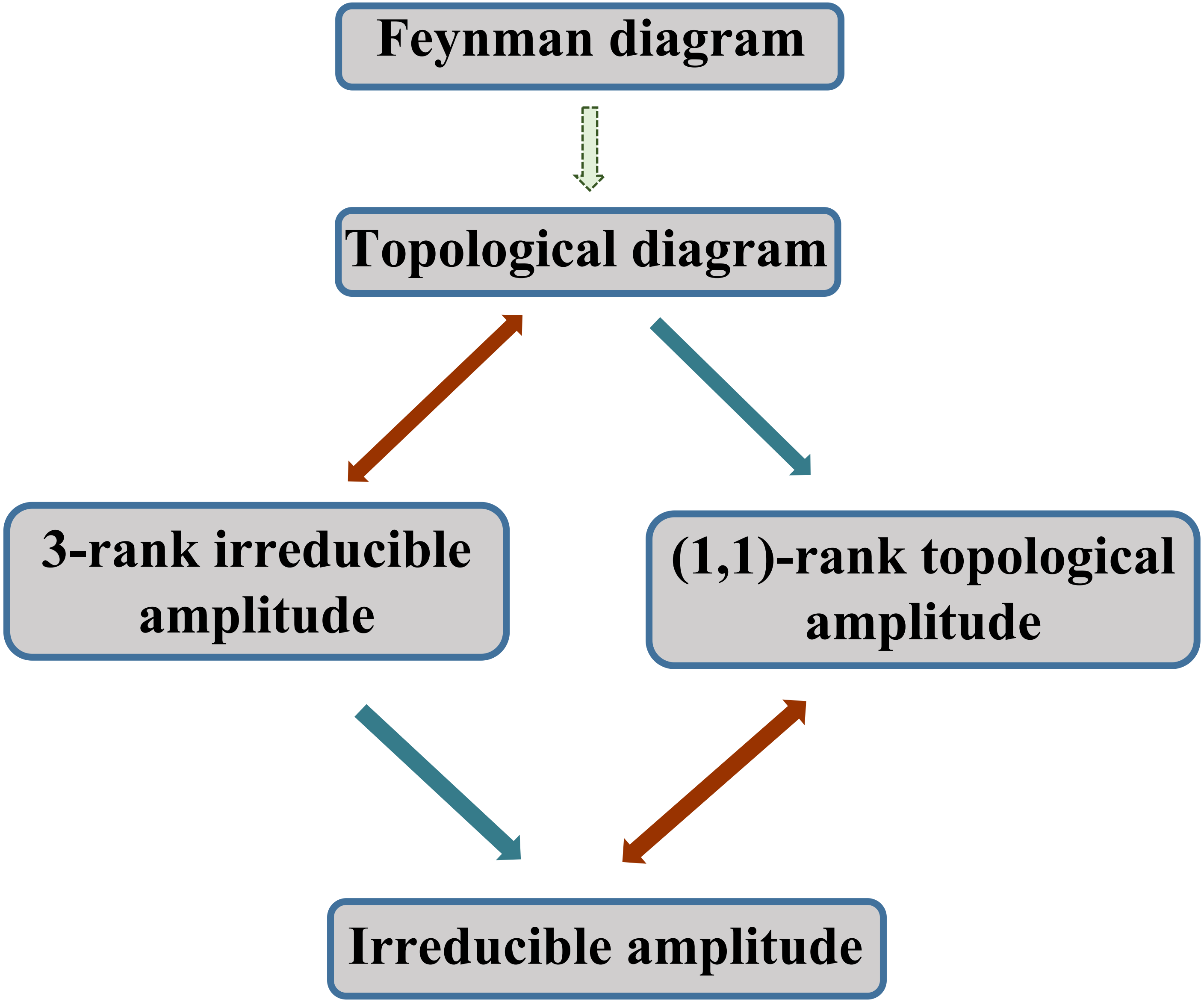
Tree

$$(H^{(0)})^2_{13} = V^*_{cs}V_{ud}, \qquad (H^{(0)})^2_{12} = V^*_{cd}V_{ud},$$
$$(H^{(0)})^3_{13} = V^*_{cs}V_{us}, \qquad (H^{(0)})^3_{12} = V^*_{cd}V_{us}$$

Penguin

$$(H^{(1)})^1_{11} = -V^*_{cb}V_{ub}, \qquad (H^{(1)})^2_{21} = -V^*_{cb}V_{ub},$$
$$(H^{(1)})^3_{31} = -V^*_{cb}V_{ub}$$

- 13 independent amplitudes in the SU(3)$_F$ limit: 9+4=13


Feynman diagram
Topological diagram
3-rank irreducible amplitude
(1,1)-rank topological amplitude
Irreducible amplitude

The Körner-Pati-Woo theorem

- Four decay amplitudes are very small due to small λ_b.

- **The Körner-Pati-Woo theorem**: If two quarks produced by weak operators enter one baryon, they are anti-symmetric in flavor.

  J. G. Korner, Nucl. Phys. B 25, 282-290 (1971).
  J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920-2922 (1971).

- Decay amplitudes: **9** ⟶ **5**

  - Parameters: $9 \times 4 - 1 = 35$ ⟶ $5 \times 4 - 1 = 19$

Test the Körner-Pati-Woo theorem

- SU(3) symmetry + KPW theorem

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+) = \mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+) + 3\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+) - \left|\frac{V_{ud}}{V_{cd}}\right|^2 \mathcal{B}(\Lambda_c^+ \to n\pi^+)$$

C. Q. Geng, X. G. He, X. N. Jin, C. W. Liu and C. Yang, Phys. Rev. D 109, no.7, L071302 (2024) .

	PDG	$SU(3)_F$ conserved	$SU(3)_F$ broken
$10^2\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+)$	1.43 ± 0.32	2.72 ± 0.09	2.9 ± 0.1

刘佳伟’ talk

- Isospin symmetry + KPW theorem

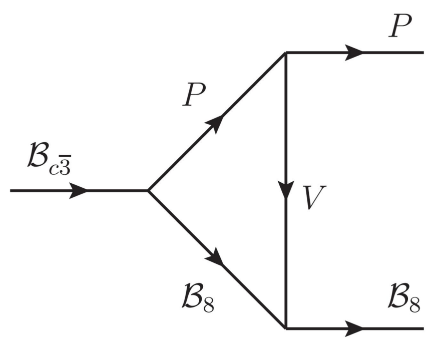
$$\mathcal{A}(\Lambda_c^+ \to \Sigma^+ K^0) = \sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^0 K^+)$$

C. Q. Geng, C. W. Liu and T. H. Tsai, Phys. Lett. B **794**, 19-28 (2019).

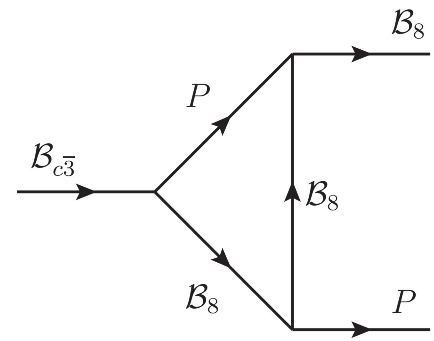
Final-state rescattering

- Final-state rescattering

C. P. Jia, H. Y. Jiang, J. P. Wang and F. S. Yu, JHEP **11**, 072 (2024).



$\Delta(\mathcal{B}_{c\bar{3}}, P, \mathcal{B}_8, V, P, \mathcal{B}_8)$



$\Delta(\mathcal{B}_{c\bar{3}}, P, \mathcal{B}_8, \mathcal{B}_8, \mathcal{B}_8, P)$

$\Lambda_c^+ \to \Sigma^0 K^{*+}$	C', E_2, B	$-$	$-$	$1.60^{+0.89}_{-0.62}$
$\Lambda_c^+ \to \Sigma^+ K^{*0}$	C', E_1	$-$	$-$	$2.10^{+1.37}_{-0.86}$

$\Lambda_c^+ \to \Sigma^0 K^+$	0.0393 ± 0.0022	-0.62 ± 0.07
	0.0393 ± 0.0021	-0.62 ± 0.07
$\Lambda_c^+ \to \Sigma^+ K_S$	0.0394 ± 0.0022	-0.62 ± 0.07
	0.0394 ± 0.0021	-0.62 ± 0.07

H. Y. Cheng, F. Xu and H. Zhong, [arXiv:2505.07150 [hep-ph]].

$\Lambda_c^+ \to \Sigma^+ K_S$	$0.37(3)$	$-0.47(14)$	$-0.87(8)$	$0.11(19)$
$\Lambda_c^+ \to \Sigma^0 K^+$	$0.37(3)$	$-0.47(14)$	$-0.87(8)$	$0.11(19)$

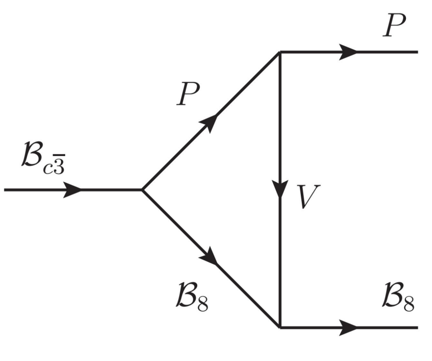
M. W. Li, X. G. He, A. Cheek and X. Chu [arXiv:2506.05511 [hep-ph]]

Final-state rescattering

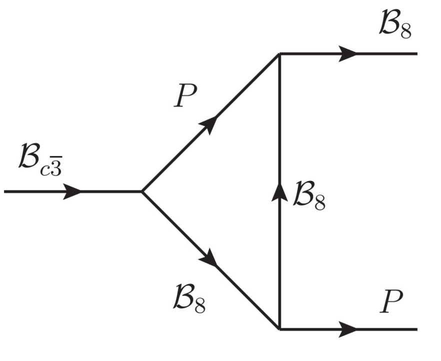
$$\mathcal{A}_L(\Lambda_c^+ \to \Sigma^0 K^+) = \frac{1}{\sqrt{2}}\Delta(\Lambda_c^+, \pi^+, n, \overline{K}^{*0}, K^+, \Sigma^0) + \frac{1}{\sqrt{2}}\Delta(\Lambda_c^+, \pi^+, n, \Sigma^-, \Sigma^0, K^+)$$
$$+ \frac{1}{\sqrt{2}}\Delta(\Lambda_c^+, K^+, \Lambda^0, \rho^0, K^+, \Sigma^0) + \frac{1}{\sqrt{2}}\Delta(\Lambda_c^+, K^+, \Lambda^0, \Xi^-, \Sigma^0, K^+)$$

$$\mathcal{A}_L(\Lambda_c^+ \to \Sigma^+ K^0) = \Delta(\Lambda_c^+, \pi^+, n, \Lambda^0, \Sigma^+, K^0) + \Delta(\Lambda_c^+, \pi^+, n, \Sigma^0, \Sigma^+, K^0)$$
$$+ \Delta(\Lambda_c^+, K^+, \Lambda^0, \rho^+, K^0, \Sigma^+) + \Delta(\Lambda_c^+, K^+, \Lambda^0, \Xi^0, \Sigma^+, K^0)$$

C. P. Jia, H. Y. Jiang, J. P. Wang and F. S. Yu, JHEP **11**, 072 (2024).

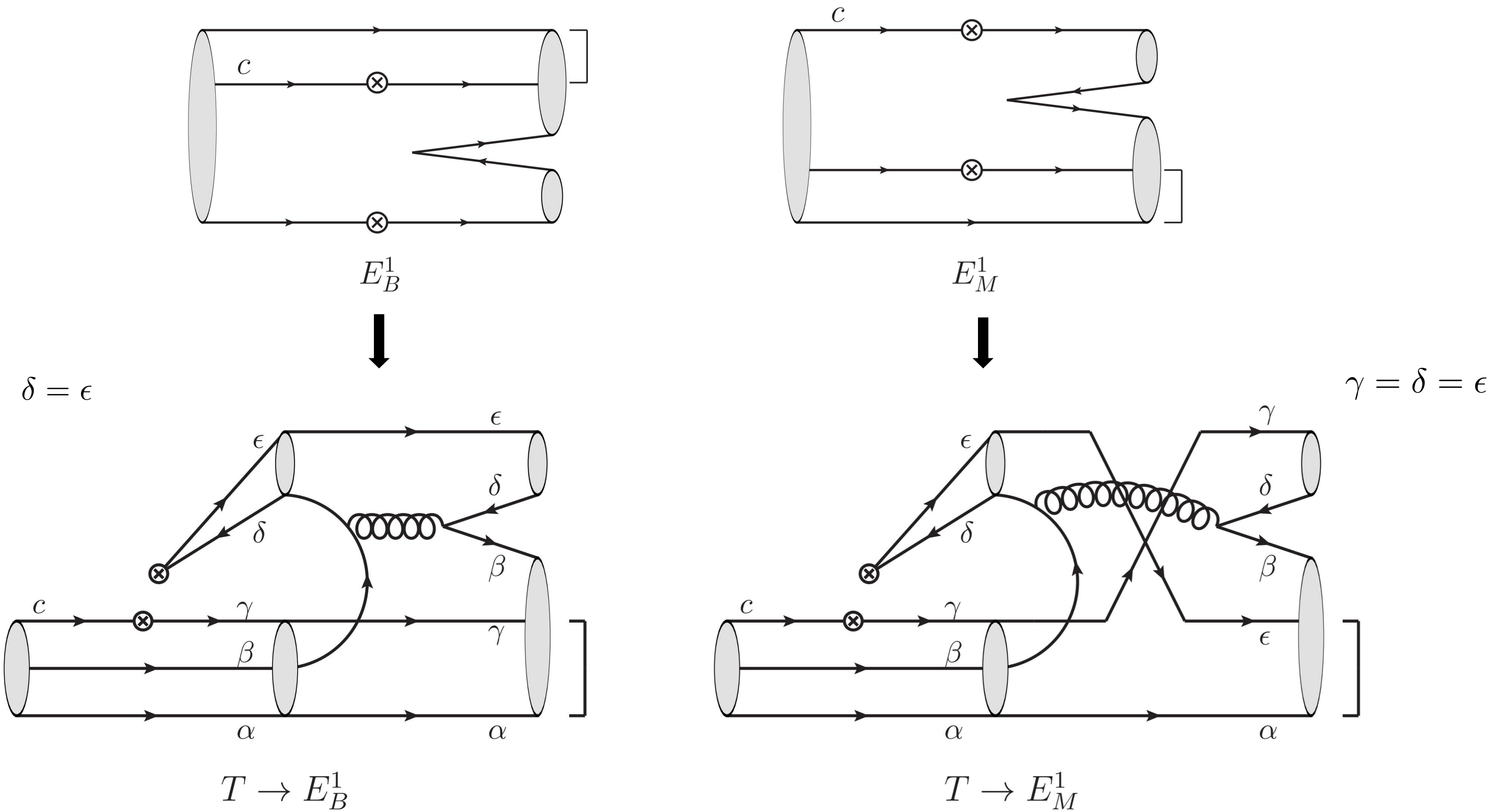


$\Delta(\mathcal{B}_{c\bar{3}}, P, \mathcal{B}_8, V, P, \mathcal{B}_8)$



$\Delta(\mathcal{B}_{c\bar{3}}, P, \mathcal{B}_8, \mathcal{B}_8, \mathcal{B}_8, P)$

Final-state rescattering



The Körner-Pati-Woo theorem

$$\mathcal{O}(x) = \{\bar{q}_{\alpha,i}(x)(1-\gamma_5)q_{\beta,j}(x)\}\{\bar{q}_{\gamma,k}(x)(1-\gamma_5)q_{\delta,l}(x)\}$$

$\alpha,\ \beta,\ \gamma,\ \delta$: color, $i,\ j,\ k,\ l$: flavor

Fierz transformation

$$\mathcal{O}(x) = \{\bar{q}_{\gamma,k}(x)(1-\gamma_5)q_{\beta,j}(x)\}\{\bar{q}_{\alpha,i}(x)(1-\gamma_5)q_{\delta,l}(x)\}$$

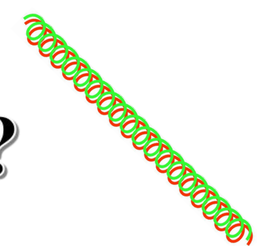
$(\alpha, i) \leftrightarrow (\gamma, k)$ symmetric

If two quarks enter baryon

SU(3)c singlet

$\alpha \leftrightarrow \gamma$ antisymmetric

?



$i \leftrightarrow k$ antisymmetric

J. G. Korner, Nucl. Phys. B 25, 282-290 (1971).
J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920-2922 (1971).

Test the Körner-Pati-Woo theorem

- Isospin symmetry + KPW theorem

$$\mathcal{B}r(\Lambda_c^+ \to \Sigma^+ K_S^0) = \mathcal{B}r(\Lambda_c^+ \to \Sigma^0 K^+)$$?

$$\mathcal{B}r(\Lambda_c^+ \to \Sigma^0 K^+) = (4.7 \pm 0.9 \pm 0.1 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}r(\Lambda_c^+ \to \Sigma^+ K_S^0) = (4.8 \pm 1.4 \pm 0.2 \pm 0.3) \times 10^{-4}$$

M. Ablikim et al. [BESIII], Phys. Rev. D **106**, no.5, 052003 (2022).

$$\mathcal{B}r(\Lambda_c^+ \to \Sigma^0 K^+) = (3.58 \pm 0.19 \pm 0.06 \pm 0.19) \times 10^{-4}$$

L. K. Li et al. [Belle] Sci. Bull. **68**, 583-592 (2023).

$\mathcal{B}r(\Lambda_c^+ \to \Sigma^+ K_S^0)$ Belle (II)

$\mathcal{B}r(\Lambda_c^+ \to \Sigma^0 \pi^+) = (1.29 \pm 0.05)\%$ PDG

$\mathcal{B}r(\Lambda_c^0 \to \Sigma^+ \pi^0) = (1.26 \pm 0.10)\%$ Isospin symmetry only

Test the Körner-Pati-Woo theorem

- Isospin symmetry + KPW theorem

$$\sqrt{\frac{3}{2}}\mathcal{A}(\Lambda_c^+ \to \Delta^+\pi^0) = \sqrt{3}\mathcal{A}(\Lambda_c^+ \to \Delta^0\pi^+) = -\mathcal{A}(\Lambda_c^+ \to \Delta^{++}\pi^-),$$

$$\mathcal{A}(\Lambda_c^+ \to \Sigma^{*+}K^0) = -\sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^{*0}K^+),$$

$$\mathcal{A}(\Xi_c^+ \to \Delta^{++}K^-) = \sqrt{3}\mathcal{A}(\Xi_c^+ \to \Delta^+\overline{K}^0)$$
$$= \sqrt{3}\mathcal{A}(\Xi_c^0 \to \Delta^0\overline{K}^0) = \sqrt{3}\mathcal{A}(\Xi_c^0 \to \Delta^+K^-),$$

$$\mathcal{A}(\Xi_c^+ \to \Sigma^{*0}\pi^+) = \mathcal{A}(\Xi_c^+ \to \Sigma^{*+}\pi^0),$$

$$\mathcal{A}(\Lambda_b^0 \to \Sigma^-K^+) = 2\,\mathcal{A}(\Lambda_b^0 \to \Sigma^0K_S^0)$$

Contents

Summary

- CP asymmetries in the $\Lambda_c^+ \to pK_S^0$ and $\Xi_c^+ \to \Sigma^+ K_S^0$ decays
- Test the KPW theorem: $\mathcal{B}r(\Lambda_c^+ \to \Sigma^+ K_S^0)$

Thanks for your attention!