粲强子衰变末态轻味强子态的理论研究

钟显辉

湖南师范大学

“第三届BESIII-Belle II-LHCb粲强子物理联合研讨会” 湖南长沙
2025年6月27-30日

CONTENTS

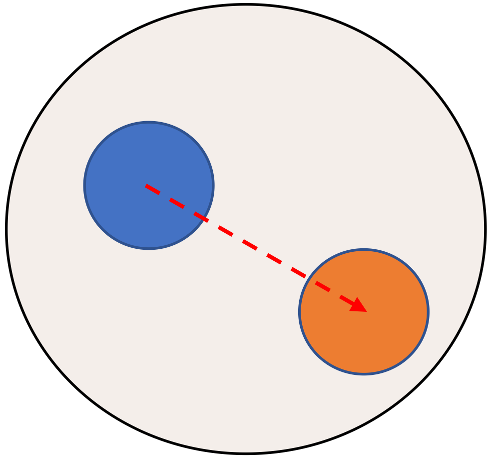
- Background
- Strange mesons
- Strange baryons
- Summary

Background

Quark model

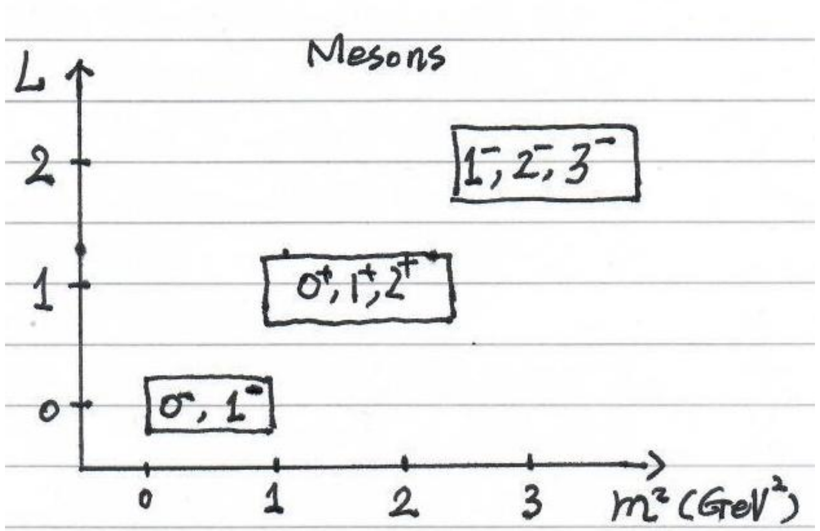
Symmetry: SU(6)⊗O(3)

Meson

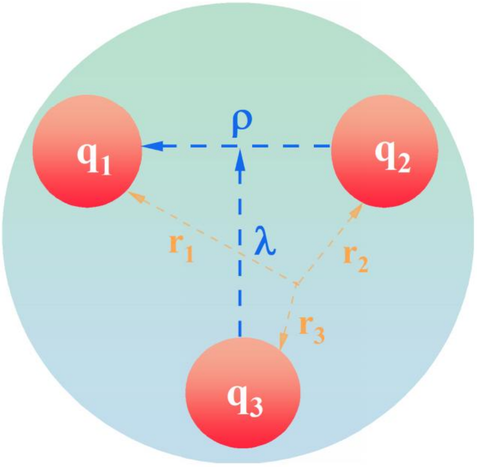


$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,$$

$$\mathbf{R}_{\text{c.m.}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}.$$



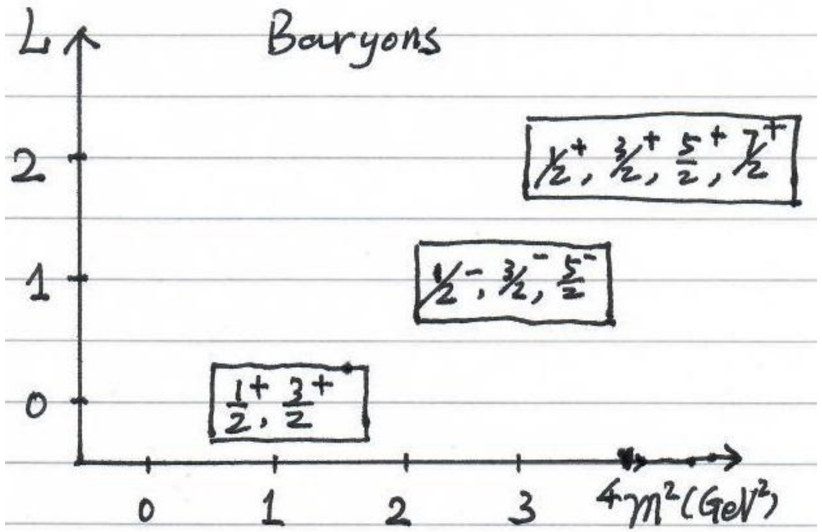
Baryon



$$\rho = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2),$$

$$\lambda = \sqrt{\frac{2}{3}}\left(\frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_1 + m_2} - \boldsymbol{r}_3\right),$$

$$\boldsymbol{R} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + m_3\boldsymbol{r}_3}{m_1 + m_2 + m_3}.$$



Light meson spectrum observed from exp.

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1370, 1500, 1710)$	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[c]	$\omega(1650)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[d]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$	$K_1(1650)$		
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$[e]	$f_2(1640)$
2^1D_2	2^{-+}	$\pi_2(1880)$			
3^1S_0	0^{-+}	$\pi(1800)$	$K(1830)$		$\eta(1760)$

Poorly understanding for the excited isoscalar states

- ✓ Tetraquark state?
- ✓ Gluon ball?
- ✓ How to understand the experimental findings
- ✓ Why were the 0++, 1++, 1-- states not well established or still missing?

Light baryon spectrum observed from exp.

J^P	(D, L_N^P)	S		Octet members			Singlets
$1/2^+$	$(56,0_0^+)$	$1/2$	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$	$1/2$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$		
$1/2^-$	$(70,1_1^-)$	$1/2$	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(1620)$	$\Lambda(1405)$
					$\Sigma(1560)^\dagger$		
$3/2^-$	$(70,1_1^-)$	$1/2$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$	$3/2$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(1690)$	
					$\Sigma(1620)^\dagger$		
$3/2^-$	$(70,1_1^-)$	$3/2$	$N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$	$3/2$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$	
$1/2^+$	$(70,0_2^+)$	$1/2$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(1810)^\dagger$
$3/2^+$	$(56,2_2^+)$	$1/2$	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$	$1/2$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$	$1/2$	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$	$3/2$	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$	$1/2$	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	
				Decuplet members			
$3/2^+$	$(56,0_0^+)$	$3/2$	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$	
$3/2^+$	$(56,0_2^+)$	$3/2$	$\Delta(1600)$	$\Sigma(1690)^\dagger$	$\Xi(?)$	$\Omega(?)$	
$1/2^-$	$(70,1_1^-)$	$1/2$	$\Delta(1620)$	$\Sigma(1750)^\dagger$	$\Xi(?)$	$\Omega(?)$	
$3/2^-$	$(70,1_1^-)$	$1/2$	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(2012)$	
$5/2^+$	$(56,2_2^+)$	$3/2$	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$7/2^+$	$(56,2_2^+)$	$3/2$	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$	
$11/2^+$	$(56,4_4^+)$	$3/2$	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	

Poorly understanding for the excited hyperon states, especially for Ω and Ξ

- ✓ **Missing resonance problem**
- ✓ **Low mass problem**
- ✓ **How to looking for the missing states**
- ✓ **Inner structure**
- ✓ **Meson-baryon components**
- ✓ **Dynamics**

Model for spectrum——potential model

Hamiltonian

$$H_0 = \sum_{i=1}^{3}\sqrt{\boldsymbol{p}_i^2+m_i^2}+\sum_{i<j}^{3}(V_{ij}+C_{ij}),$$

$$V_{ij} = V_{ij}^{Conf} + V_{ij}^{OGE}$$

$$V^{Conf} = \sum_{i<j}^{3} -\frac{3}{16}(\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j)b_{ij}r_{ij}.$$

$$V^{OGE} = \sum_{i<j}\left(V_{ij}^{Coul}+V_{ij}^{sd}\right),$$

$$V_{ij}^{Coul} = \sum_{i<j} -\frac{2\alpha_{ij}}{3}\frac{1}{r_{ij}},$$

Spin-dependent potential of OGE:

$$V_{ij}^{ss} = -\frac{\alpha_{ij}}{4}(\boldsymbol{\lambda_i}\cdot\boldsymbol{\lambda_j})\left\{\frac{\pi}{2}\cdot\frac{\sigma_{ij}^3 e^{-\sigma_{ij}^2 r_{ij}^2}}{\pi^{\frac{3}{2}}}\cdot\frac{16}{3m_im_j}(\boldsymbol{S_i}\cdot\boldsymbol{S_j})\right\},$$

$$V_{ij}^{T} = -\frac{\alpha_{ij}}{4}(\boldsymbol{\lambda_i}\cdot\boldsymbol{\lambda_j})\frac{1}{m_im_jr_{ij}^3}\left\{\frac{3(\boldsymbol{S_i}\cdot\boldsymbol{r_{ij}})(\boldsymbol{S_j}\cdot\boldsymbol{r_{ij}})}{r_{ij}^2}-\boldsymbol{S_i}\cdot\boldsymbol{S_j}\right\},$$

$$V_{ij}^{so(\nu)} = \sum_{i<j}\frac{1}{r_{ij}}\frac{dV_{ij}^{Coul}}{dr_{ij}}\left(\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_i}{2m_i^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_j}{2m_j^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_i-\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_j}{m_im_j}\right),$$

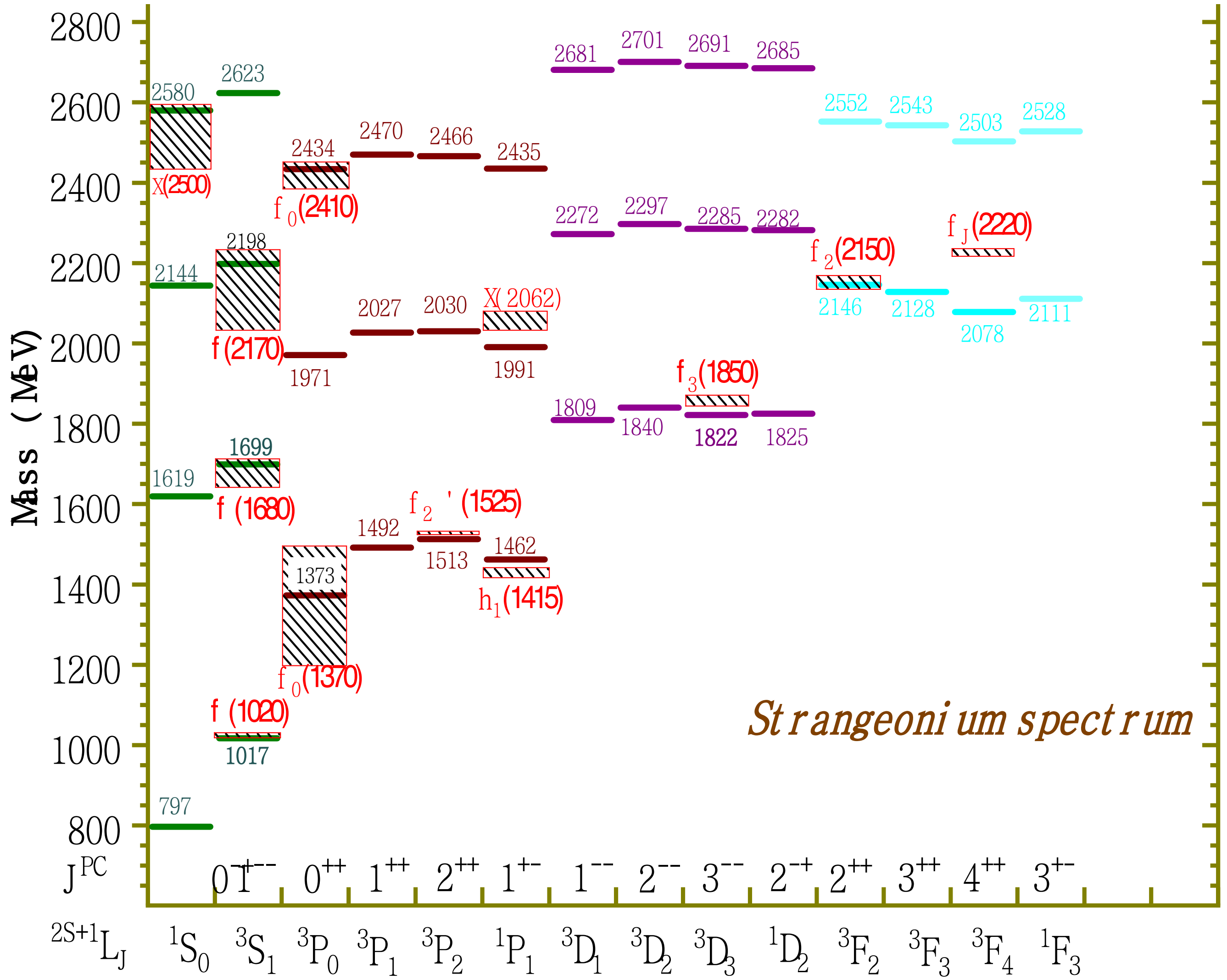
$$V_{ij}^{so(TP)} = \sum_{i<j}-\frac{1}{r_{ij}}\frac{\partial V_{ij}^{Conf}}{\partial r_{ij}}\left(\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_i}{2m_i^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_j}{2m_j^2}\right).$$

Strange mesons

Based on our works:

FX Liu, MS Liu, XH, Q Zhao, PRD 103, 016016 (2021);
Q Li, LC Gui, MS Liu, QF Lv, XH, Chin. Phys. C 45, 023116 (2021);
FX Liu et al, unpublished

Strangeonium spectrum



Many low-lying states are still missing!

- **How to understand $f_0(1500,1710)$?**

  $f_0(1370)$: 1250-1440, 200-500

  $1^3P_0(ss)$: 1373, 338

- **Could X(2500) be evidence of the 4^1S_0 state?** 【BESIII, PRD93,112011(2016)】

  Exp: 2428-2586, 162-350, ϕϕ

  Th: 2580, 409, KK*10% , ϕϕ1%

- **Could $f_0(2410)$ be the 3^3P_0 state?**

  【BESIII, PRD98,172003(2019)】

  Exp: 2411, 348, KK

  Th: 2434, 346, K*K* 10%,KK3%,

- **Could X(2062) be evidence of the 2^1P_1 state?** 【BESIII, PRD99,112008(2019)】

  Exp: 2062, 177, ϕη'

  Th: 1991, 279, KK*18% , ϕη' 2%

Strangeonium spectrum

- Could $\phi(2170)$ be the 3^3S_1 state?

  Exp: 2164, 88+26-21, not seen in K*K*,seen in $\phi\eta$,$\phi\eta'$,$\phi\pi\pi$

  Th: 2198, 276, K*K*8% , $\phi\eta$ 3%, $\phi\eta'$ 0.1%

  Predicted width is too broad, the K*K* decay mode is not seen!

- Could $\phi(2170)$ be the 2^3D_1 state?

  Exp: 2164, 88+26-21, not seen in K*K*,seen in $\phi\eta$,$\phi\eta'$,$\phi\pi\pi$

  Th: 2272, 283, K*K*8% , $\phi\eta$ 0.9%, $\phi\eta'$ 0.1%

  Predicted mass and width are too large, the K*K* decay mode is not seen!

- Could the newly observed state X(2300) be the 3^1P_1 state?

  [BESIII, PRL134,191901(2025)]

  Exp: 2316, 89+-41, seen in $\phi\eta$,$\phi\eta'$

  Th: 2435, 269, KK*5% , K*K*7%, $\phi\eta$ 2%, $\phi\eta'$ 0.6%

  Predicted mass and width are too large!

Could $\phi(2170)$ and X(2300) be tetraquark states?

Towards establishing the 1D states

◆ 1^3D_2 state
Th: 1840, 128, KK*70% , φη 20%
Worth observing!

◆ 1^1D_2 state
Th: 1825, 80, KK*90% , K*K* 10%
Worth observing!

◆ 1^3D_1 state
Th: 1809, 707, KK1(1270)88% , KK4%, KK*6%, Too broad!

Towards establishing the 2P states

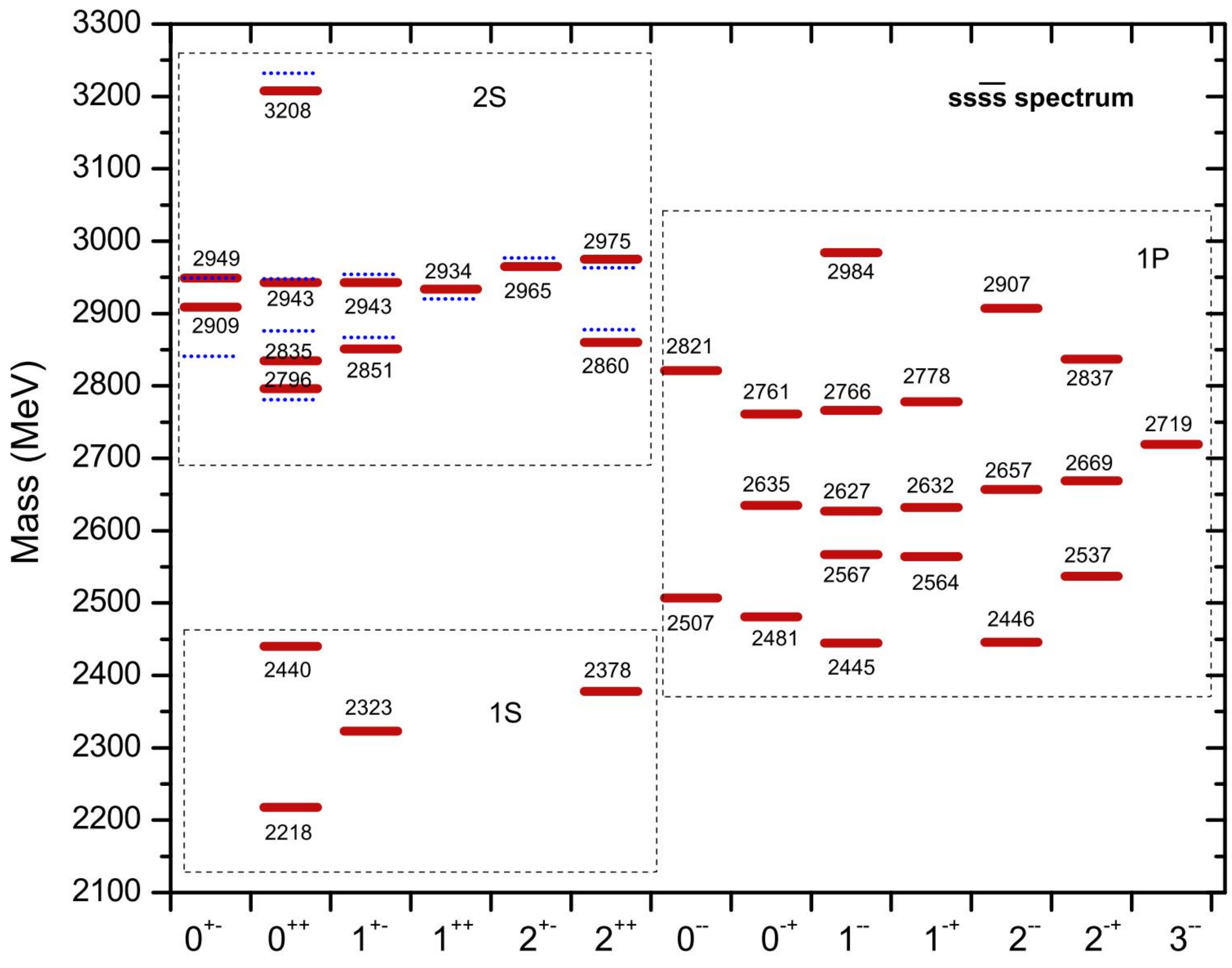
◆ 2^3P_1 state
Th: 2027, 315, KK*13%, K*K*16%
Worth observing!

◆ 2^3P_2 state
Th: 2030, 147, KK3%, K*K*17%
Worth observing!

◆ 2^1P_1 state
Th: 1991, 179, KK*18%, K*K*17%, φη 6%, φη' 2%
Worth observing!

◆ 2^3P_0 state
Th:1971, 849, $KK_1(1270)$50%, KK(1460) 35%
Too broad!

Fully strange tetraquarks



Could ϕ(2170) and X(2300) be tetraquark states?

- The mass of the lowest tetraquark state is about 2.2 GeV.
- The ϕ(2170) cannot be explained as a fully strange tetraquark state.
- From the point of view of mass, the X(2300) is a good candidate of a fully strange tetraquark state with $J^{PC}=1^{+-}$.

Fall-apart decays

State	$\eta\eta$	$\eta\eta'$	$\eta'\eta'$	$\phi\phi$	ηf_0	ηf_1	$\eta f_2'$	$\eta' f_0$	$\eta' f_1$	$\eta' f_2'$	ϕh_1	$\eta\eta(2S)$	$\eta'\eta(2S)$	$\phi\phi(2S)$
$M_{f_1}+M_{f_2}$	1096	1506	1916	2034	1921	2040	2061	2331	2450	2471	2479	2167	2577	2716
$T_{0^{++}(2218)(1S)}$	3.22	11.00	8.72	13.86	/	/	/	//	//	//	//	0.16	//	//
$T_{0^{++}(2440)(1S)}$	0.13	0.48	0.45	35.48	/	/	/	[illegible]	//	//	//	○	//	//
$T_{0^{++}(2798)(2S)}$	0.60	1.91	1.46	0.48	/	/	/	/	/	/	/	1.10	1.58	28.92
$T_{0^{++}(2876)(2S)}$	○	0.24	0.35	26.46	/	/	[illegible]	/	/	/	/	0.14	2.00	33.08
$T_{0^{++}(2954)(2S)}$	0.33	1.36	1.49	5.92	/	/	/	/	/	/	/	1.58	15.05	24.04
$T_{0^{++}(3155)(2S)}$	○	0.17	0.20	1.94	/	/	/	/	/	/	/	0.66	1.45	10.48
$T_{1^{++}(2943)(2S)}$	/	/	/	0.24	0.07	0.64	1.14	0.13	0.51	1.14	3.92	/	/	0.16
$T_{2^{++}(2378)(1S)}$	0.36	0.75	0.26	○	/	0.69	0.21	/	//	//	//	○	//	//
$T_{2^{++}(2854)(2S)}$	0.20	0.51	0.29	0.60	/	0.59	0.21	/	0.47	0.13	○	0.11	○	10.72
$T_{2^{++}(2981)(2S)}$	0.05	0.23	0.25	8.60	/	1.10	1.08	/	1.40	1.16	○	0.11	0.27	105.44
$T_{0^{-+}(2481)(1P)}$	/	/	/	7.34	4.82	/	0.73	3.14	/	//	//	/	//	//
$T_{0^{-+}(2635)(1P)}$	/	/	/	28.92	3.81	/	2.11	8.64	/	1.14	11.92	/	/	//
$T_{0^{-+}(2761)(1P)}$	/	/	/	4.88	13.12	/	8.89	29.54	/	9.17	1.24	/	/	○
$T_{1^{-+}(2564)(1P)}$	/	/	/	0.24	/	3.12	0.25	/	5.25	○	2.08	○	//	//
$T_{1^{-+}(2632)(1P)}$	/	/	/	5.14	/	○	0.07	/	0.11	○	38.80	○	○	//
$T_{1^{-+}(2778)(1P)}$	/	/	/	○	/	7.96	○	/	14.96	○	○	○	○	○
$T_{2^{-+}(2537)(1P)}$	/	/	/	○	0.80	1.51	1.88	0.22	0.31	0.80	0.24	/	//	//
$T_{2^{-+}(2669)(1P)}$	/	/	/	0.10	0.23	0.28	2.06	0.18	0.13	3.70	14.00	/	/	//
$T_{2^{-+}(2837)(1P)}$	/	/	/	0.16	0.09	0.12	1.06	0.11	0.09	2.36	15.00	/	/	○

Preliminary

Fall-apart decays

Preliminary

State $M_{f_1}+M_{f_2}$	$\eta\phi$ 1565	$\eta'\phi$ 1975	ηh_1 2010	$\eta' h_1$ 2420	$f_0\phi$ 2390	$f_1\phi$ 2509	$f_2'\phi$ 2530	$\eta\phi(2S)$ 2247	$\phi\eta(2S)$ 2636	$\eta'\phi(2S)$ 2657
$T_{0^{+-}(2891)(2S)}$	/	/	2.94	5.61	1.08	0.48	2.12	/	/	/
$T_{0^{+-}(2967)(2S)}$	/	/	1.29	0.89	1.16	5.52	9.04	/	/	/
$T_{1^{+-}(2323)(1S)}$	6.54	10.13	/	//	//	//	//	○	//	//
$T_{1^{+-}(2835)(2S)}$	1.42	2.00	/	/	1.68	1.32	1.88	6.62	13.84	5.43
$T_{1^{+-}(2950)(2S)}$	2.06	5.03	/	/	3.00	5.16	8.08	0.05	38.44	7.66
$T_{2^{+-}(2965)(2S)}$	○	0.11	0.34	0.29	0.32	0.80	0.08	0.05	○	0.09
$T_{0^{--}(2507)(1P)}$	4.66	6.01	/	/	/	//	//	0.12	//	//
$T_{0^{--}(2821)(1P)}$	1.37	3.47	/	/	/	69.96	3.20	0.60	○	0.16
$T_{1^{--}(2445)(1P)}$	0.07	○	○	○	24.80	//	//	○	//	//
$T_{1^{--}(2567)(1P)}$	4.65	6.77	1.17	0.53	0.80	32.72	13.20	○	//	//
$T_{1^{--}(2627)(1P)}$	○	0.07	2.68	3.65	1.64	3.20	5.64	○	//	//
$T_{1^{--}(2766)(1P)}$	0.41	0.31	3.05	3.96	51.40	0.48	2.12	0.20	○	0.09
$T_{1^{--}(2984)(1P)}$	0.87	1.31	13.76	25.76	1.68	16.84	40.76	○	0.08	○
$T_{2^{--}(2446)(1P)}$	1.15	1.40	0.11	0.09	0.20	//	//	○	//	//
$T_{2^{--}(2657)(1P)}$	0.14	0.24	0.18	○	○	23.76	21.28	○	○	//
$T_{2^{--}(2907)(1P)}$	0.07	0.09	○	○	1.20	22.08	30.48	0.11	○	0.09
$T_{3^{--}(2719)(1P)}$	0.20	0.18	0.09	0.07	1.48	3.04	27.20	○	○	○

Some states may be found in experiments

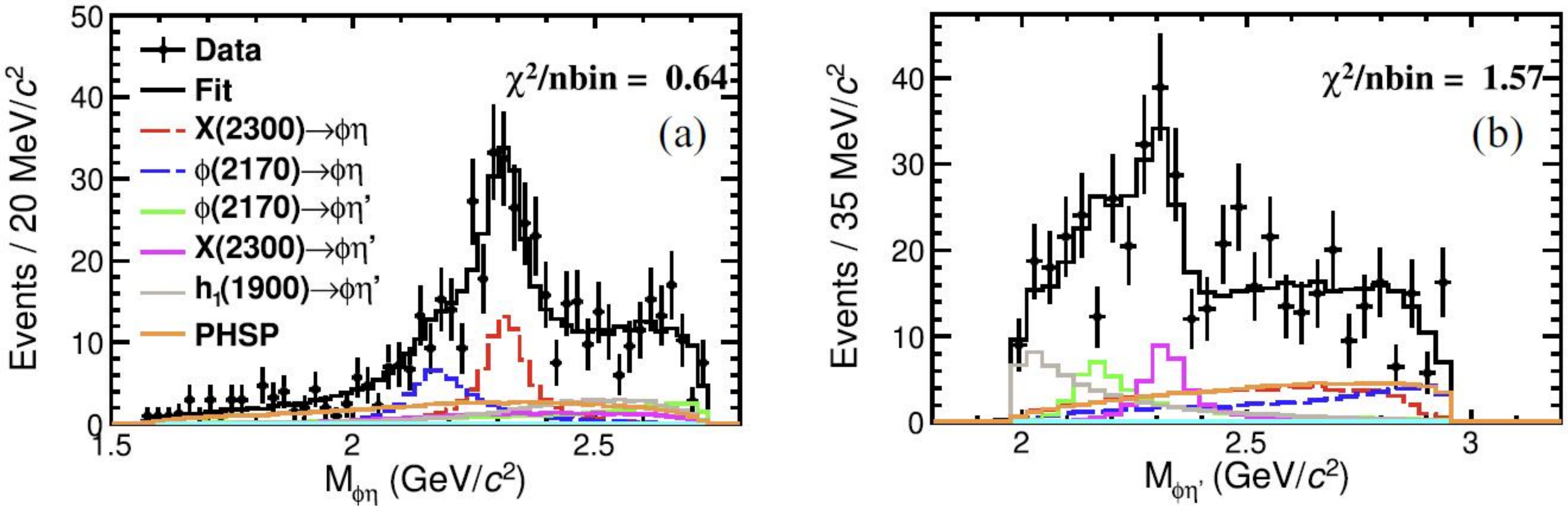
- Two 1S 0++ states (2218,2440) and one 2S 0++ state (2876) have large decay rates into ϕϕ.
- The 2S 0++ states have large decay rates into the ϕϕ(1680) channel.

State $M_{f_1}+M_{f_2}$	$\eta\eta$ 1096	$\eta\eta'$ 1506	$\eta'\eta'$ 1916	$\phi\phi$ 2034	ηf_0 1921	ηf_1 2040	$\eta f_2'$ 2061	$\eta' f_0$ 2331	$\eta' f_1$ 2450	$\eta' f_2'$ 2471	ϕh_1 2479	$\eta\eta(2S)$ 2167	$\eta'\eta(2S)$ 2577	$\phi\phi(2S)$ 2716
$T_{0^{++}(2218)(1S)}$	3.22	11.00	8.72	13.86	/	/	/	//	//	//	//	0.16	//	//
$T_{0^{++}(2440)(1S)}$	0.13	0.48	0.45	35.48	/	/	/	/	//	//	//	○	//	//
$T_{0^{++}(2798)(2S)}$	0.60	1.91	1.46	0.48	/	/	/	/	/	/	/	1.10	1.58	28.92
$T_{0^{++}(2876)(2S)}$	○	0.24	0.35	26.46	/	/	/	/	/	/	/	0.14	2.00	33.08
$T_{0^{++}(2954)(2S)}$	0.33	1.36	1.49	5.92	/	/	/	/	/	/	/	1.58	15.05	24.04
$T_{0^{++}(3155)(2S)}$	○	0.17	0.20	1.94	/	/	/	/	/	/	/	0.66	1.45	10.48
$T_{1^{++}(2943)(2S)}$	/	/	/	0.24	0.07	0.64	1.14	0.13	0.51	1.14	3.92	/	/	0.16
$T_{2^{++}(2378)(1S)}$	0.36	0.75	0.26	○	/	0.69	0.21	/	//	//	//	○	//	//
$T_{2^{++}(2854)(2S)}$	0.20	0.51	0.29	0.60	/	0.59	0.21	/	0.47	0.13	○	0.11	○	10.72
$T_{2^{++}(2981)(2S)}$	0.05	0.23	0.25	8.60	/	1.10	1.08	/	1.40	1.16	○	0.11	0.27	105.44

The newly observed X(2300) favors the 1+- T4s state

$\psi(3686)\to\phi\eta\eta'$ @ BESIII [PRL 134,191901 (25)]

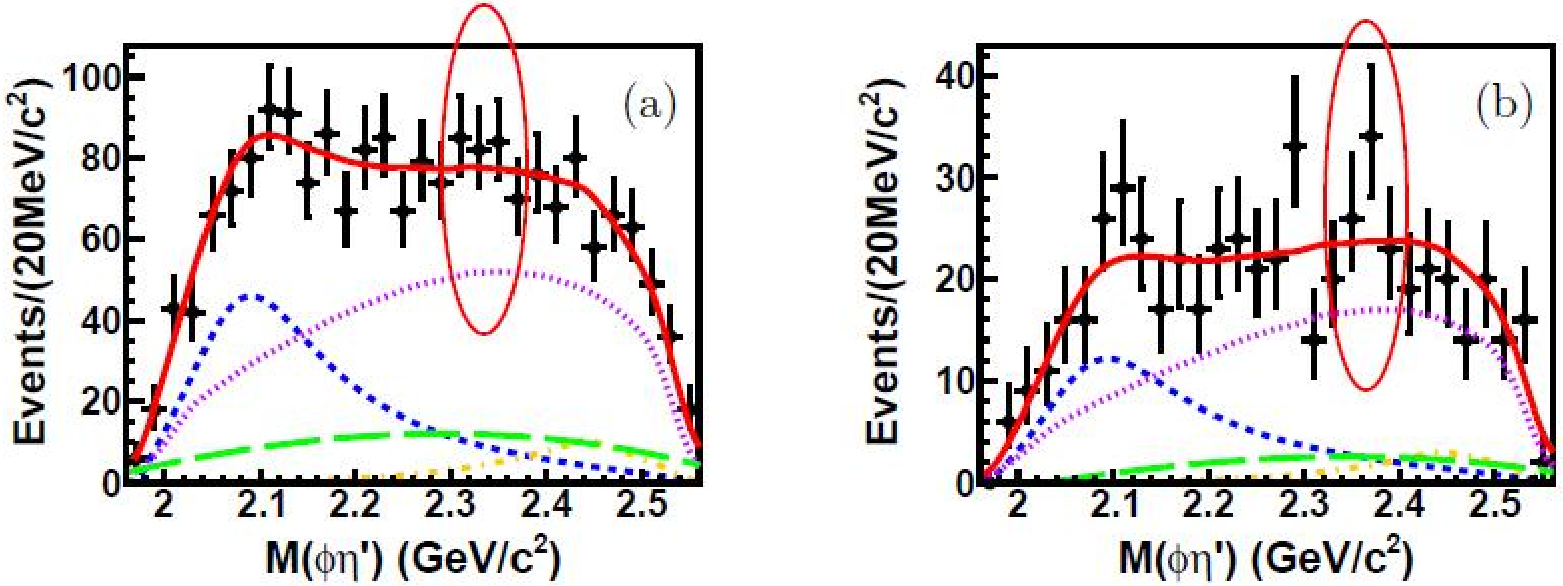
$M_{exp}= 2316^{+39}_{-39}$ MeV, $\Gamma_{exp}=89^{+41}_{-41}$ MeV



Both the mass and decay properties predicted in theory are consistent with the observations.

Some hints of 1+- T_{4s}(2323) may be seen in J/ψ decays as well

J/ψ→φηη’ @ BESIII [PRD 99,112008 (19)],



More precise observations in φη, φη’ may is needed!

Can we observe the 1+- T4c (6500) state in J/ψη_c channel?

Experimental evidence for 0++ tetraquarks

$J/\psi \to \gamma X \to \gamma \phi \phi$ @ BESIII [PRD 93,112011 (16)]

Two 0++ resonances with masses around 2.2 GeV and 2.4 GeV were extracted from the data.

$B_s^0 \to J/\psi \phi \phi$ @ LHCb [JHEP1603,040 (16)]

One 0++ resonance with mass around 2.2 GeV was extracted from the data.

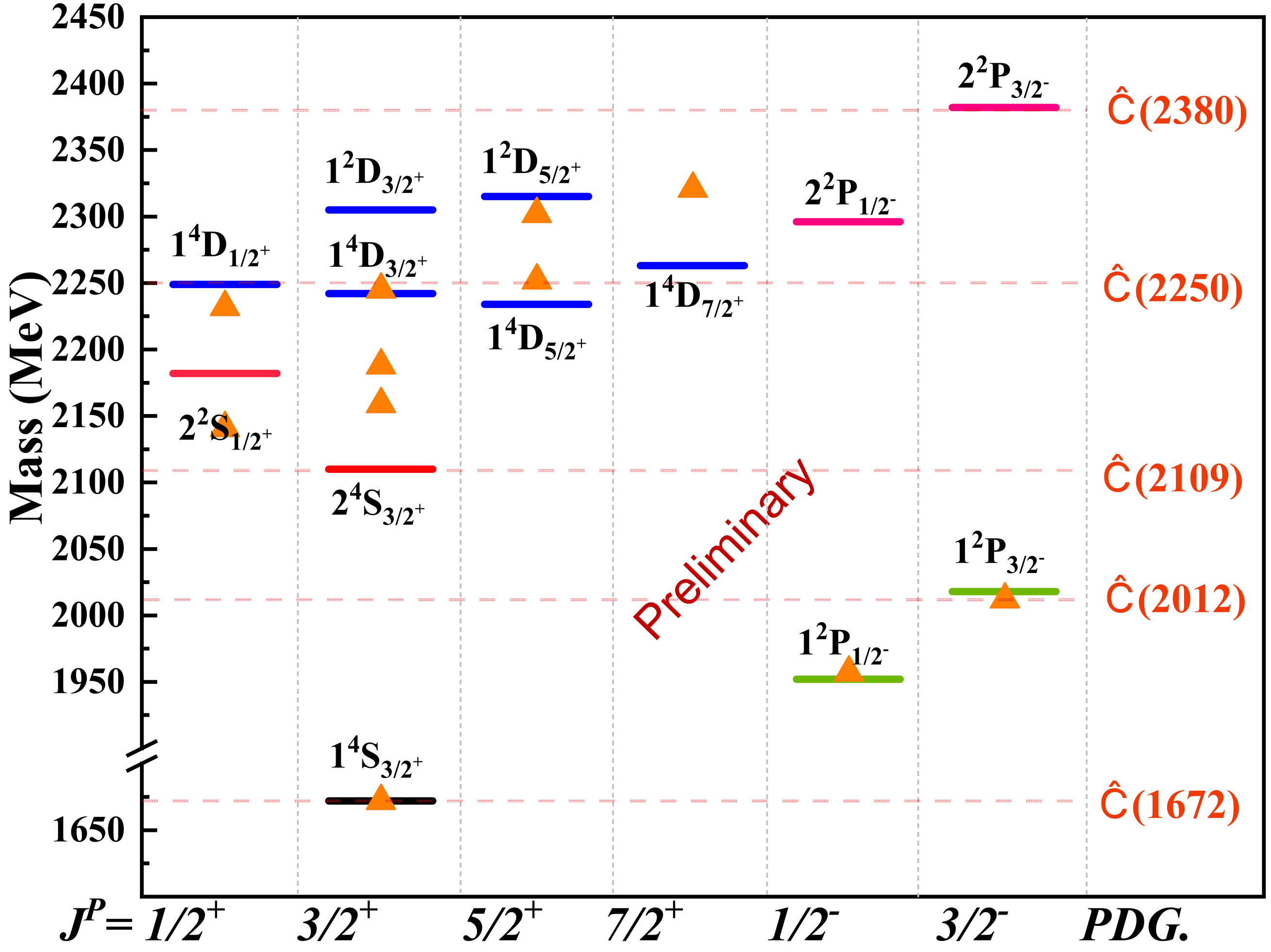
A. A. Kozhevnikov, PRD99, 014019 (19);PRD 95, 014005 (17).

Strange baryons

Based on our works:

Hui-Hua Zhong et al, unpublished;
MS Liu, KL Wang, QF Lv, XH, PRD101, 016002(2020);
LY Xiao, XH, PRD98, 034004 (2018)

Ω(sss) spectrum



Old results labeled by Triangles（唯象）：

$$V_{ij}^{LS} = \frac{\alpha_{SO}}{\rho^2+\lambda^2} \cdot \frac{\mathbf{L}\cdot\mathbf{S}}{3(m_1+m_2+m_3)^2}.$$

New results labeled by short lines (based on QCD):

$$V_{ij}^{so(v)} = \frac{1}{r_{ij}}\frac{dV^{Coul}(r_{ij})}{dr_{ij}}\left(\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_i}{2\tilde{m}_i^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_j}{2\tilde{m}_j^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_i-\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_j}{\tilde{m}_i\tilde{m}_j}\right),$$

$$V_{ij}^{so(s)} = -\frac{1}{r_{ij}}\frac{dV^{Conf}(r_{ij})}{dr_{ij}}\left(\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_i\cdot\boldsymbol{S}_i}{2\tilde{m}_i^2}-\frac{\boldsymbol{r}_{ij}\times\boldsymbol{p}_j\cdot\boldsymbol{S}_j}{2\tilde{m}_j^2}\right)$$

Obvious differences of two models can be seen!

Confirm the mass gap between the two 1P states: ~60 MeV! The mass of the 1P ½- state should be ~1950 MeV!

Three-body spin-orbit term $\boldsymbol{\rho} \times \boldsymbol{p}_\lambda$

Complete form from OGE:

$$V_{ij}^{so(\nu)} = \sum_{i<j} \frac{1}{r_{ij}} \frac{dV_{ij}^{Coul}}{dr_{ij}} \left(\frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_i \cdot \boldsymbol{S}_i}{2m_i^2} - \frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_j \cdot \boldsymbol{S}_j}{2m_j^2} - \frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_j \cdot \boldsymbol{S}_i - \boldsymbol{r}_{ij} \times \boldsymbol{p}_i \cdot \boldsymbol{S}_j}{m_i m_j} \right)$$ [Isgur]

A simple form with approximation

$$P_i = -P_j \Rightarrow \frac{\alpha_{s0}}{3r_{ij}^3} \left[\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j} \right) \boldsymbol{L}_{ij} \cdot \left(\boldsymbol{S}_i + \boldsymbol{S}_j \right) + \left(\frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \boldsymbol{L}_{ij} \cdot \left(\boldsymbol{S}_i - \boldsymbol{S}_j \right) \right]$$ [Hiyama]

However, for a baryon system $P_i \neq - P_j$

The complete form should be:

$$V_G^{LS}(r_{12}) = \frac{1}{2\sqrt{2}\tilde{m}_u^2} \left(\frac{\alpha_S}{\rho^3} - \frac{b}{2\rho} \right) \boldsymbol{L}_\rho \cdot (\boldsymbol{S}_1 + \boldsymbol{S}_2) - \frac{1}{4\sqrt{6}\tilde{m}_u^2} \left(\frac{2\alpha_S}{\sqrt{3}\rho^3} + \frac{b}{\rho} \right) \boldsymbol{\rho} \times \boldsymbol{p}_\lambda \cdot (\boldsymbol{S}_1 - \boldsymbol{S}_2)$$

There is an additional three body spin-orbit （TBso）term!!!

» T.Yoshida, E.Hiyama, PRD.92.114029 (2015); S. Capstick and N. Isgur, PRD 34, 2809 (1986)

Three-body spin-orbit term $\boldsymbol{\rho} \times \boldsymbol{p}_{\lambda}$ is crucial !

Mass splitting : Δ(1620)1/2-—Δ(1700)3/2-

Hui-Hua Zhong, MS Liu, RH Ni, MY Chen, XH,
PRD 110 (2024) 11, 116034

No Tbso term: $V_1^{LS} \propto \boldsymbol{L}_{\rho} \cdot (\boldsymbol{S}_1 + \boldsymbol{S}_2) = 0$ ⇒ **Degenerate states**

With Tbso term: $V_2^{LS} \propto \boldsymbol{\rho} \times \boldsymbol{p}_{\lambda} \cdot (\boldsymbol{S}_1 - \boldsymbol{S}_2) \neq 0$ ⇒ $\Delta M \simeq 60$ MeV

Mass splitting : Ω(2012)(1P)1/2-—Ω(1950)(1P)3/2-

No Tbso term: **Degenerate states**

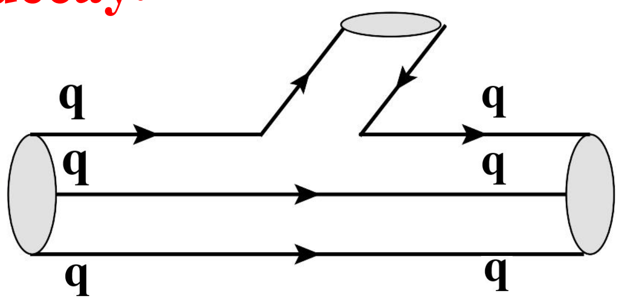
With Tbso term: $\Delta M \simeq 60$ MeV

Strong decays–chiral quark model

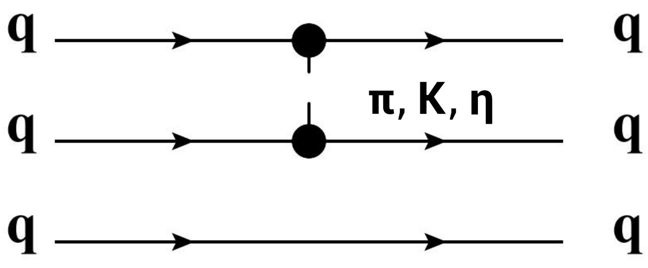
Chiral Lagrangian:

$$\mathcal{L}_{ps} = \frac{\delta}{\sqrt{2}f_{\mathbb{M}}}\bar{\psi}_j\gamma_\mu\gamma_5\psi_j\vec{I}\cdot\partial^\mu\vec{\phi}_{\mathbb{M}},$$

strong decay:



mass spectrum:



$$H_I = \mathcal{H}^{NR} + \mathcal{H}^{RC},$$

$$\mathcal{H}^{NR} = g\sum_j\left(\mathcal{G}\boldsymbol{\sigma}_j\cdot\boldsymbol{q} + \frac{\omega_{\mathbb{M}}}{2\mu_q}\boldsymbol{\sigma}_j\cdot\boldsymbol{p}_j\right)F(\boldsymbol{q}^2)I_j\varphi_{\mathbb{M}},$$

$$\mathcal{H}^{RC} = -\frac{g}{32\mu_q^2}\sum_j[m_{\mathbb{M}}^2(\boldsymbol{\sigma}_j\cdot\boldsymbol{q}) + 2\boldsymbol{\sigma}_j\cdot(\boldsymbol{q}-2\boldsymbol{p}_j)\times(\boldsymbol{q}\times\boldsymbol{p}_j)]F(\boldsymbol{q}^2)I_j\varphi_{\mathbb{M}}$$

$$V_{OBE}^C(r_{ij}) = V_\pi^C(r_{ij}) + V_K^C(r_{ij}) + V_\eta^C(r_{ij}).$$

$$V_{OBE}^T(r_{ij}) = V_\pi^T(r_{ij}) + V_K^T(r_{ij}) + V_\eta^T(r_{ij}),$$

Strong decay properties

	ΞK	$\Xi^* K$	$\Omega\eta$	Sum	
$1^2P_{\frac{1}{2}^-}$	24.70	-	-	24.70	NR
1952	34.71	-	-	34.71	NR+RC
$1^2P_{\frac{3}{2}^-}$				2.8 – 10.8	Ω(2012)
2018	7.10		-	7.10	NR
* * *	10.13	-	-	10.13	NR+RC
$2^2S_{\frac{1}{2}^+}$	0.69	0.39	-	1.08	NR
2162	19.09	2.02	-	31.26	NR+RC
$2^4S_{\frac{3}{2}^+}$				3.9 – 39.3	Ω(2109)
2110	1.69	2.65	-	4.34	NR
* * *	21.52	12.17	-	33.69	NR+RC

Preliminary

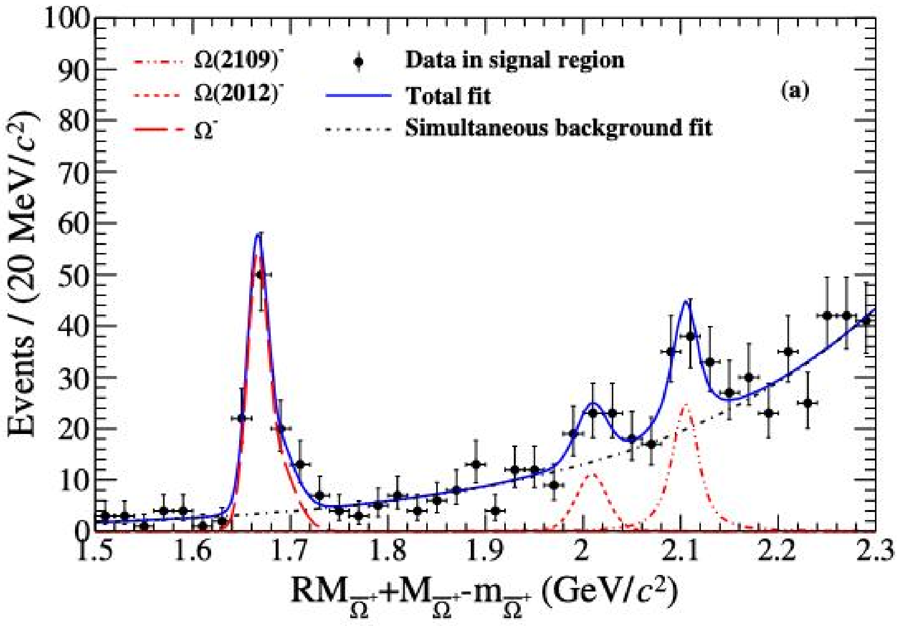
$1^4D_{\frac{3}{2}^+}$	17.59	12.73	0.25	30.57	NR
2236	6.18	3.26	0.03	9.47	NR+RC
	3.88	2.67	0.02	6.57	mix
$1^2D_{\frac{3}{2}^+}$	1.88	4.74	0.31	6.93	NR
2298	0.83	6.28	0.12	7.23	NR+RC
	0.14	5.43	0.07	5.64	mix
$1^4D_{\frac{7}{2}^+}$				37 – 73	Ω(2250)
2268	18.98	1.99	0.00	20.97	NR
	36.16	4.07	0.01	40.24	NR+RC
$1^4D_{\frac{5}{2}^+}$	3.55	16.26	0.33	20.14	NR
2237	6.73	5.21	0.06	12.00	NR+RC
	5.10	2.22	0.01	7.33	mix
$1^2D_{\frac{5}{2}^+}$	2.64	12.12	1.94	6.70	NR
2314	4.91	5.69	0.46	11.06	NR+RC
	6.92	4.43	0.09	11.44	mix

Preliminary

All of the excited states within N=2 shell have relatively narrow widths!

New Ω(2109) favors the 2S state with JP=3/2+

$e^+e^- \rightarrow \Omega(2109)^-\overline{\Omega}^+$, @ BESIII [PRL 134,131903
]



$$M_{\Omega(2109)^-} = [2108.5 \pm 5.2_{stat} \pm 0.9_{syst}]\ \mathrm{MeV}/c^2$$

$$\Gamma_{\Omega(2109)^-} = [18.3 \pm 16.4_{stat} \pm 5.7_{syst}]\ \mathrm{MeV}$$

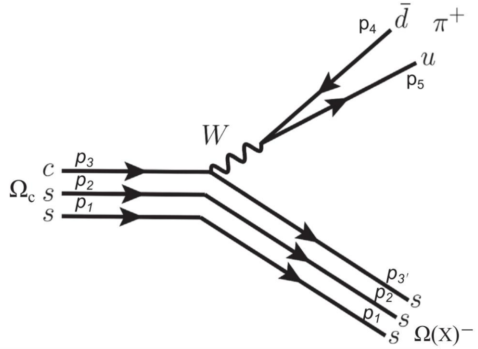
Both the mass and decay properties predicted in theory are consistent with the observations.

also suggested by the Lanzhou group, [arXiv:2504.14648]

Towards establishing the low-lying 1P state with JP=½-

◆ Mass ~1950 MeV, width ~ 30 MeV
Strong decay channel: $K^-\Xi^0$, $\overline{K}^0\,\Xi^-$.

◆ production



Final state	M_f (MeV)	RPF			NRPF			MHPF		
		Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0\to\Omega^{(*)}(X)^-\pi^+]}{\Gamma[\Omega_c^0\to\Omega^-\pi^+]}$	Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0\to\Omega^{(*)}(X)^-\pi^+]}{\Gamma[\Omega_c^0\to\Omega^-\pi^+]}$	Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0\to\Omega^{(*)}(X)^-\pi^+]}{\Gamma[\Omega_c^0\to\Omega^-\pi^+]}$
$\Omega(1^4S_{\frac{3}{2}^+})\pi^+$	1672	26	10.5	1.0	3.8	1.6	1.0	21	8.2	1
$\Omega(1^2P_{\frac{1}{2}^-})\pi^+$	1957	9.5	3.8	0.38	2.0	0.80	0.50	8.7	3.6	0.44
$\Omega(1^2P_{\frac{3}{2}^-})\pi^+$	2012	5.4	2.2	0.22	1.2	0.49	0.31	5.2	2.1	0.26
$\Omega(2^2S_{\frac{1}{2}^+})\pi^+$	2232	1.2	5.0×10^{-1}	0.05	3.9×10^{-1}	0.16	0.01	1.5	6.3×10^{-1}	0.08
$\Omega(2^4S_{\frac{3}{2}^+})\pi^+$	2159	3.0	1.2	0.12	0.8	0.34	0.21	3.3	1.4	0.17
$\Omega(1^2D_{\frac{3}{2}^+})\pi^+$	2245	2.1×10^{-1}	8.4×10^{-2}	0.008	6.7×10^{-2}	2.7×10^{-2}	0.002	2.6×10^{-1}	1.1×10^{-1}	0.01
$\Omega(1^2D_{\frac{5}{2}^+})\pi^+$	2303	1.3×10^{-2}	5.0×10^{-3}	5.0×10^{-4}	5.4×10^{-3}	2.0×10^{-3}	1×10^{-3}	1.9×10^{-2}	7.7×10^{-3}	9.4×10^{-4}
$\Omega(1^4D_{\frac{1}{2}^+})\pi^+$	2141	3.3	1.3	0.13	8.8×10^{-1}	0.36	0.23	3.6	1.5	0.18
$\Omega(1^4D_{\frac{3}{2}^+})\pi^+$	2188	2.3	0.95	0.09	6.8×10^{-1}	0.28	0.18	2.7	1.1	0.13
$\Omega(1^4D_{\frac{5}{2}^+})\pi^+$	2252	3.3×10^{-3}	1.3×10^{-3}	1.3×10^{-4}	1.2×10^{-3}	4.5×10^{-4}	2.8×10^{-4}	4.2×10^{-3}	1.7×10^{-3}	2.1×10^{-4}
$\Omega(1^4D_{\frac{7}{2}^+})\pi^+$	2321	3.2×10^{-3}	1.3×10^{-3}	1.3×10^{-4}	1.3×10^{-3}	5.1×10^{-4}	3.2×10^{-4}	4.7×10^{-3}	1.9×10^{-3}	2.3×10^{-4}

KL Wang, QF Lv, JJ Xie, XH, PRD107, 034015(2023)

Summary

» The study of $\bar{s}s$ is crucial, the first orbital states are not well established .

» A better understanding of the convectional $\bar{s}s$ meson states is useful for glueball searching at BESIII.

» Fully strange tetraquark states may have been observed at BESIII.

» The new Ω(2109) observed at BESIII favors the 2S state with JP=3/2+ .

» It is worth to looking for the missing orbital state Ω(1950)1/2-.

Thanks