Theoretical Review on B Physics

- selected topics -

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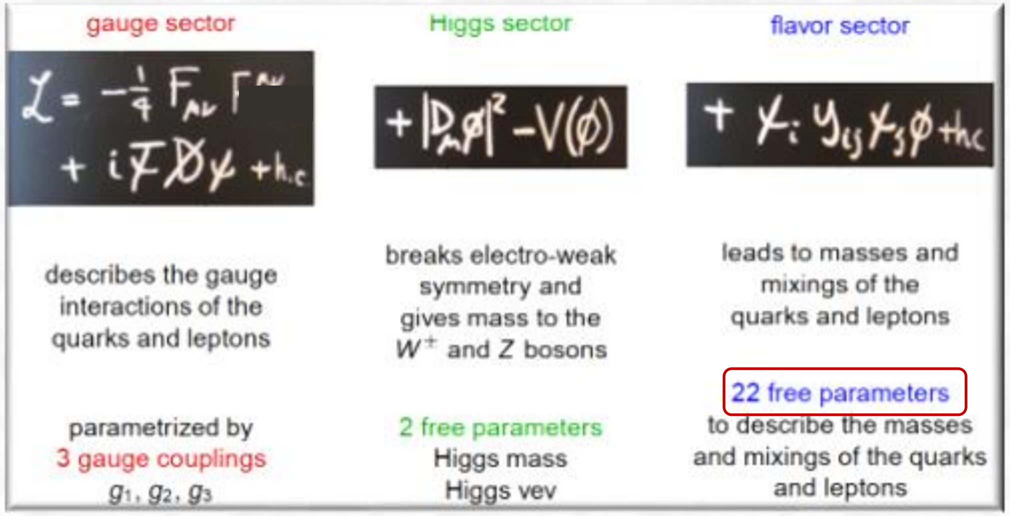
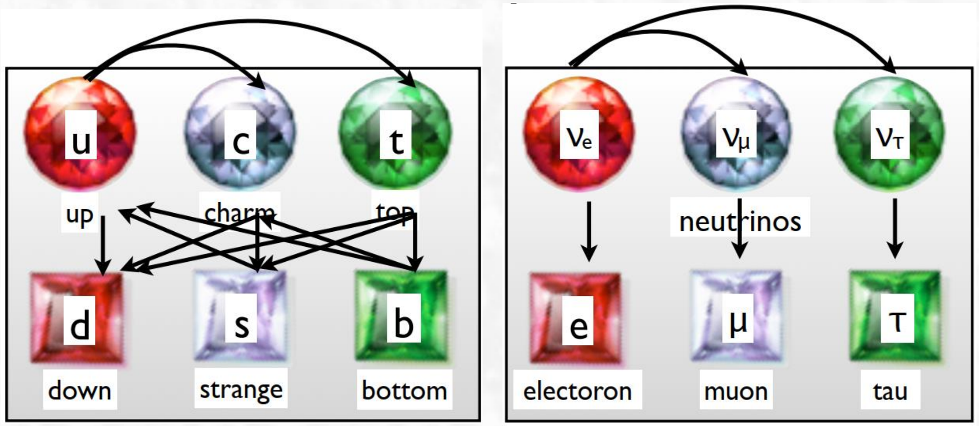
Outline

- Introduction
- Theoretical framework: why effective Hamiltonian & how to calculate hadronic matrix elements
- Purely leptonic decays, lifetimes, neutral B-meson mixings, semi-leptonic FCCC decays, rare FCNC decays, ……
- Two-body hadronic B decays & QCD factorization & $SU(3)_F$
- Summary

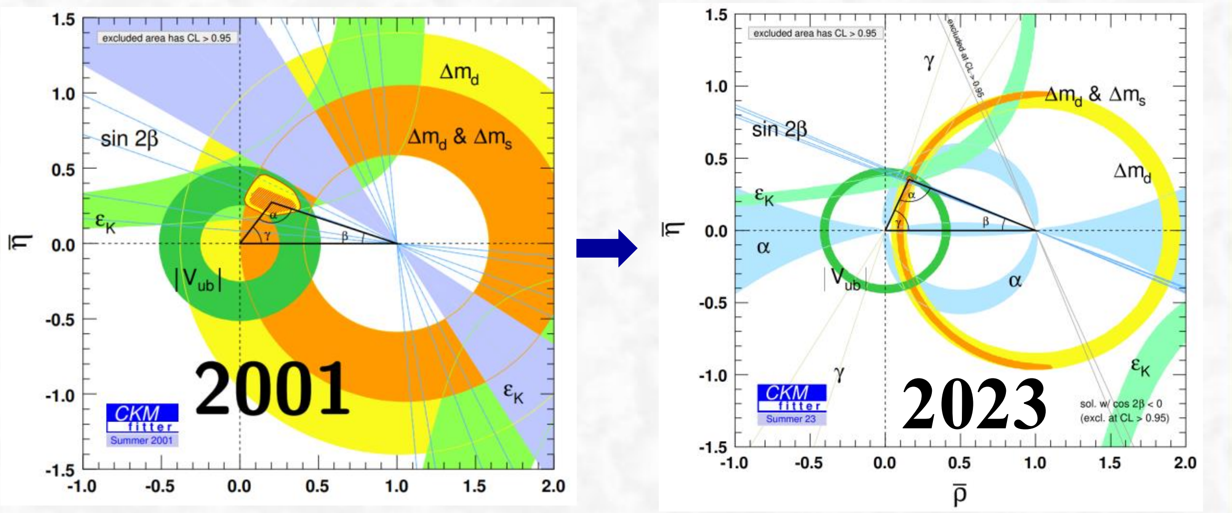
Heavy Flavor Physics

- **An important branch of particle physics: most free parameters from flavor sector**





- **Much exp. & theo. progress achieved, now entering a *precision flavor era!***

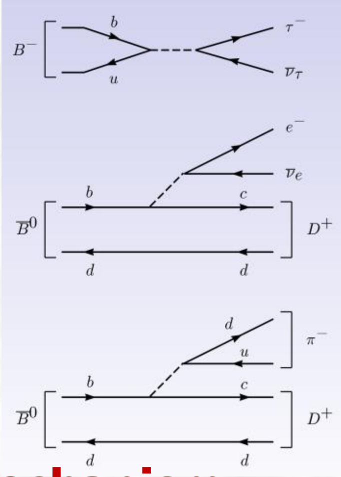


- **With the current LHCb, Belle-II & future STCF, CEPC, FCC, ...: bright prospects expected**

Why B physics

- ☐ **b-quark massive enough to have many decay modes & various observables available**

$$b \to \begin{cases} c \\ u \end{cases} + W^{*-} \to \begin{cases} c \\ u \end{cases} + \begin{cases} \bar{u} + d \\ \bar{c} + s \\ \bar{u} + s \\ \bar{c} + d \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{cases}$$



purely leptonic

semi-leptonic

hadronic

Branching ratios,

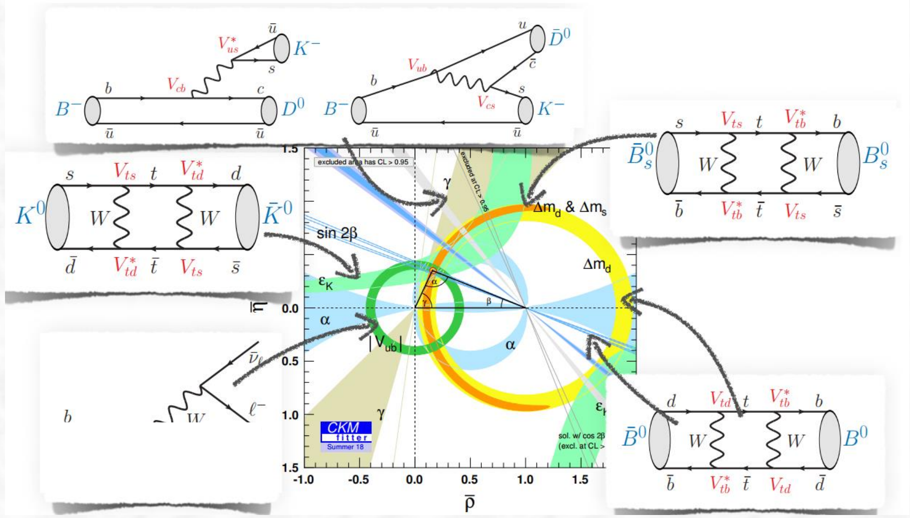
CP asymmetries,

Polarizations,

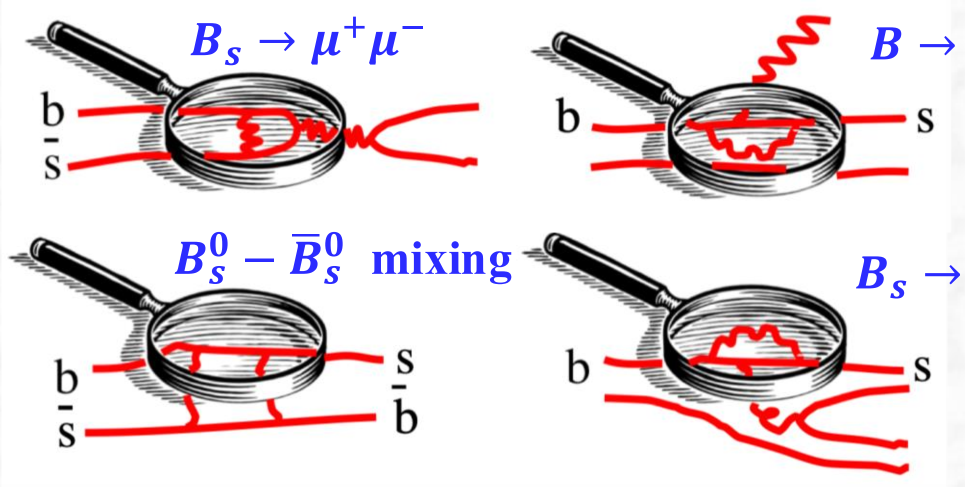
Dalitz distributions,

. . .

- ☐ **Testing SM: quark mixing & KM mechanism**

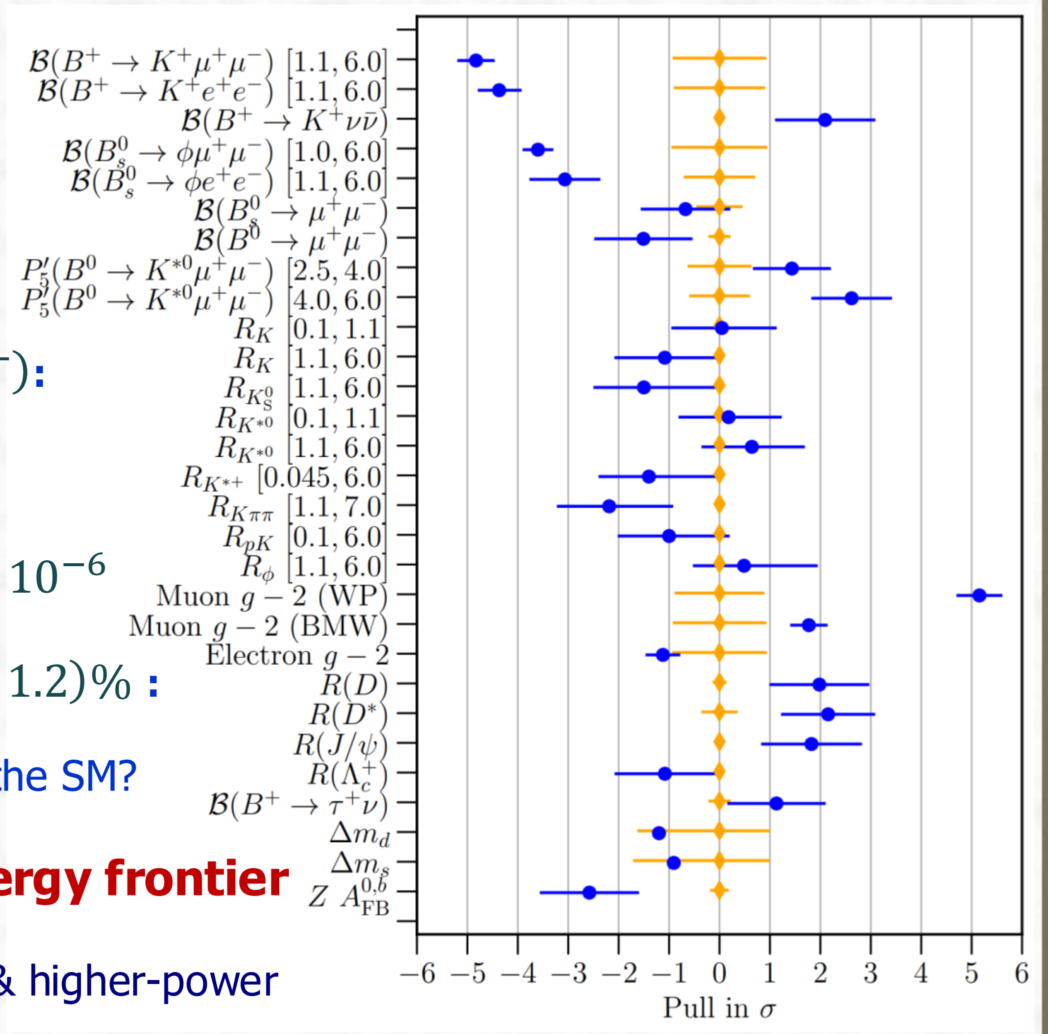


- ☐ **Efficient indirect probe of NP**



B Anomalies

https://www.nikhef.nl/~pkoppenb/anomalies.html



- **Several discrepancies observed in B physics:**

  - $R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}l\nu_l)}$: lepton flavor university violation?

  - $\mathcal{B}(B^+ \to K^{(*)+}\mu^+\mu^-), \mathcal{B}(B^+ \to K^+\nu\bar{\nu}), P_5'(B^0 \to K^{*0}\mu^+\mu^-)$:

    insufficient QCD estimates or NP in rare FCNC $b \to s$ decays?

  - $\mathcal{B}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9) \times 10^{-6}$ vs $(1.55 \pm 0.16) \times 10^{-6}$

    $\Delta A_{CP}(B \to \pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-) = (11.0 \pm 1.2)\%$:

    pert. vs non-pert. QCD effects before discussing NP beyond the SM?

- **However, no clear evidence of NP from high-energy frontier**

  - **Hadronic matrix elements:** further higher-order & higher-power QCD/QED corrections, or introduce some new mechanisms?

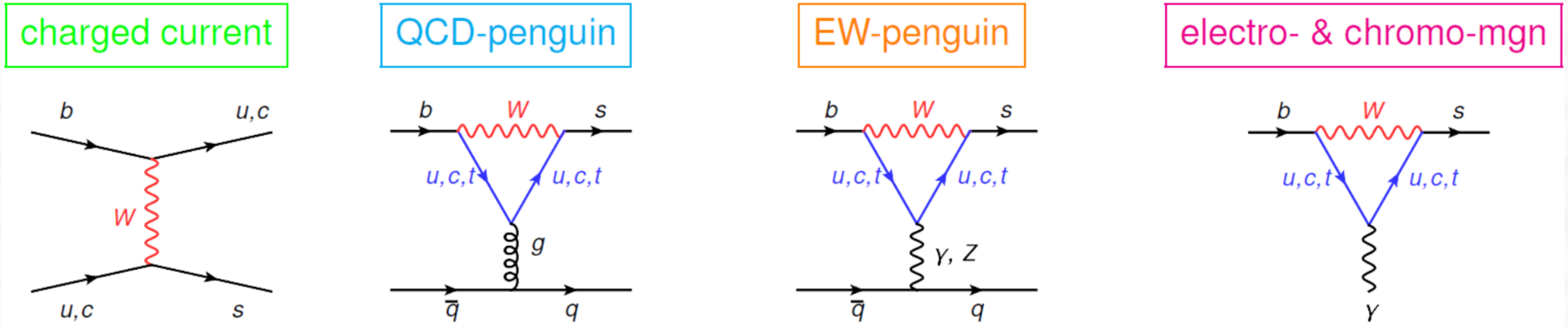
  - **NP implications:** if existed, must have specific flavor structures, any new flavor- & CP-violating sources?

B physics in full SM theory

□ At the quark level, flavor-changing charged-current processes mediated by W bosons

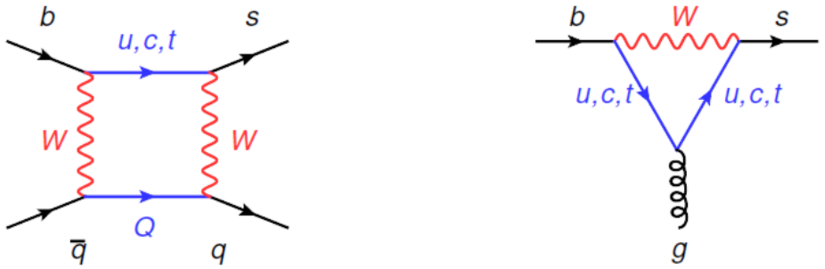
$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} J^{\mu}_{\mathrm{CC}} W^{\dagger}_{\mu} + \mathrm{h.c.}$$

$$J^{\mu}_{\mathrm{CC}} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\gamma^{\mu}\begin{pmatrix} e_{\mathrm{L}} \\ \mu_{\mathrm{L}} \\ \tau_{\mathrm{L}} \end{pmatrix} + (\bar{u}_{\mathrm{L}}, \bar{c}_{\mathrm{L}}, \bar{t}_{\mathrm{L}})\gamma^{\mu} V_{\mathrm{CKM}} \begin{pmatrix} d_{\mathrm{L}} \\ s_{\mathrm{L}} \\ b_{\mathrm{L}} \end{pmatrix}$$



□ Multi-scale involved with large hierarchy

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\mathrm{QCD}} \sim 1$ GeV



$$\boxed{P(M_W, m_b) = 1 + \alpha_s\left(\# \ln\frac{M_W}{m_b} + *\right) + \alpha_s^2\left(\# \ln^2\frac{M_W}{m_b} + *\right) + \ldots}$$

Problem: due to large logs, uncontrolled perturbative series, thus spoiling perturbative convergence

RG-improved perturbative theory

- **Solution:** re-organize the perturbative series and make all large logs $(\alpha_s \ln\frac{M_W}{m_b})^n$ re-summed

- step1: through matching to achieve a separation of scales, sometimes also called "factorization"

$$\left[1+\alpha_s\left(\#\ln\frac{M_W}{\mu}+*\right)+\ldots\right]\cdot\left[1+\alpha_s\left(\#\ln\frac{\mu}{m_b}+*\right)+\ldots\right]$$

$$P(M_W,m_b)=C(M_W,\mu)D(m_b,\mu)$$

at the cost of introducing a "factorization scale" μ.

- step2a: $P(M_W,m_b)$ is formally μ-indep., but $C(M_W,\mu)$ and $D(m_b,\mu)$ by themselves are μ-dep. and obey

$$\text{RGEs:}\left\{\begin{array}{rcl}\mu\frac{d}{d\mu}C(M_W,\mu) & = & \gamma(\mu)\,C(M_W,\mu)\\ \mu\frac{d}{d\mu}D(M_W,\mu) & = & -\gamma(\mu)\,D(M_W,\mu)\end{array}\right\}\Rightarrow\mu\frac{d}{d\mu}(CD)=0$$

["C and D run with μ."]

- step2b: solve the RGEs and then evolve

$\boxed{\mu_{\text{high}}\sim M_W}$

$$\begin{array}{rcl}C(M_W,\mu) & = & \underline{C(M_W,\mu_{\text{high}})}\,U(\mu_{\text{high}},\mu)\\ D(m_b,\mu) & = & \underline{D(m_b,\mu_{\text{low}})}\,U(\mu,\mu_{\text{low}})\end{array}$$

$\boxed{\mu \text{ arbitrary}}$

$\boxed{\mu_{\text{low}}\sim m_b}$

- **Final result for $P(M_W,m_b)$:**

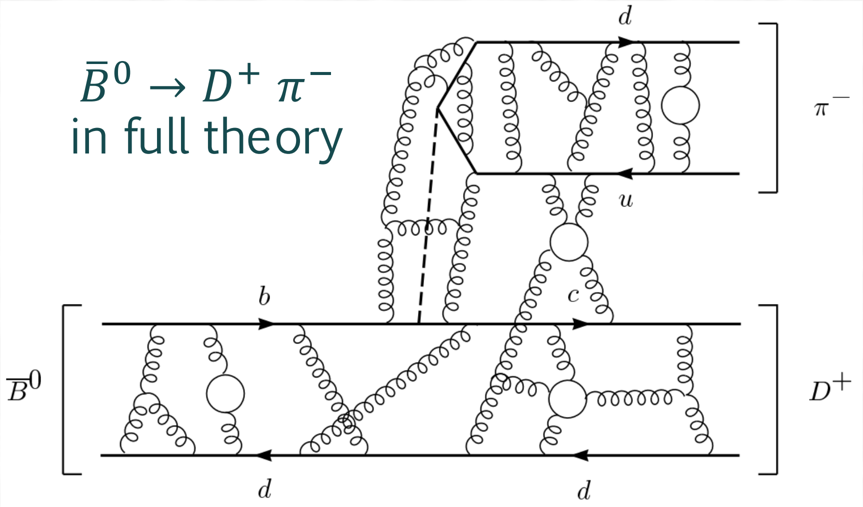
$$\boxed{P(M_W,m_b)=\underbrace{C(M_W,\mu_{\text{high}})U(\mu_{\text{high}},\mu_{\text{low}})}_{C_{\text{RGimproved}}(M_W,\mu_{\text{low}})}D(m_b,\mu_{\text{low}})}$$

$U(\mu_{\text{high}},\mu_{\text{low}})$ takes an exponential form, and thus re-sums large logs $(\alpha_s \ln\frac{\mu_{\text{high}}}{\mu_{\text{low}}})^n$

Effective Hamiltonian for $b \to c\bar{u}d$ decay

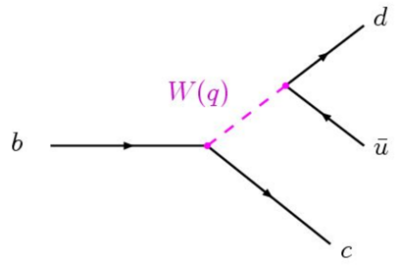
- **In full theory: described in terms of current-current interaction weighted by g & CKM elements**

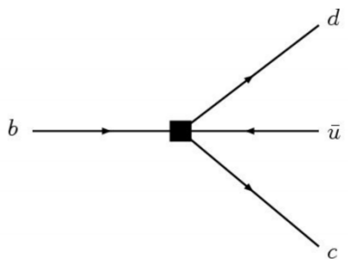
$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar{c}\,\gamma_\alpha(1-\gamma_5)\,b]\,, \qquad \bar{J}_\beta^{(d\to u)} = V_{ud}^*\,[\bar{d}\,\gamma_\beta(1-\gamma_5)\,u]$$



- **At tree level in full theory: $|q| \le m_b \ll M_W$**

Full theory (SM) $\rightarrow$ Fermi model





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)}\,\frac{-g^{\alpha\beta}+\frac{q^\alpha q^\beta}{M_W^2}}{q^2-M_W^2}\,\bar{J}_\beta^{(d\to u)} \xrightarrow{|q|\ll M_W} \frac{G_F}{\sqrt{2}}\,J_\alpha^{(b\to c)}\,g^{\alpha\beta}\,\bar{J}_\beta^{(d\to u)}$$

q limited by the mass of decaying b-quark

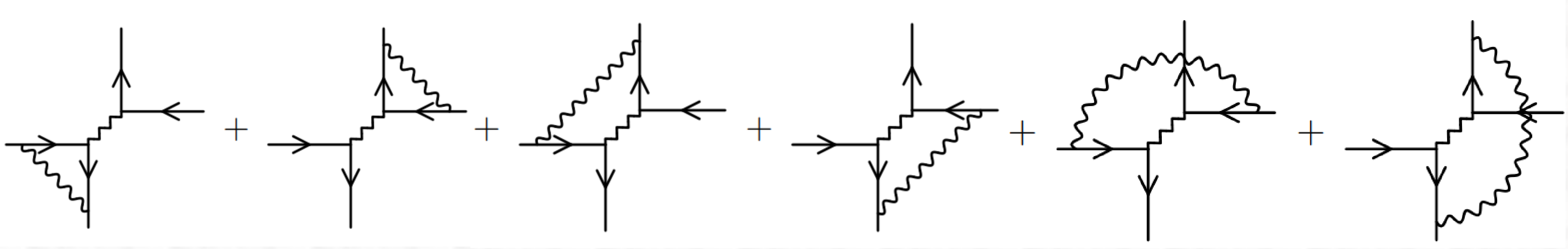
- **At tree level in EFT: described by local four-fermion operators with only light fields**

- **High-scale effects encoded in**

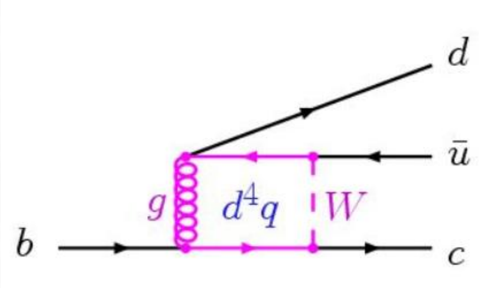
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **One-loop QCD corrections to $b \to c\bar{u}d$ decays in full theory**



- **key point: compared to the tree-level term, the W-boson momentum $|q|$ is an internal loop momentum that should be integrated between 0 and ∞**



$$\int d^4q \frac{1}{D_1 D_2 D_3 D_4}$$

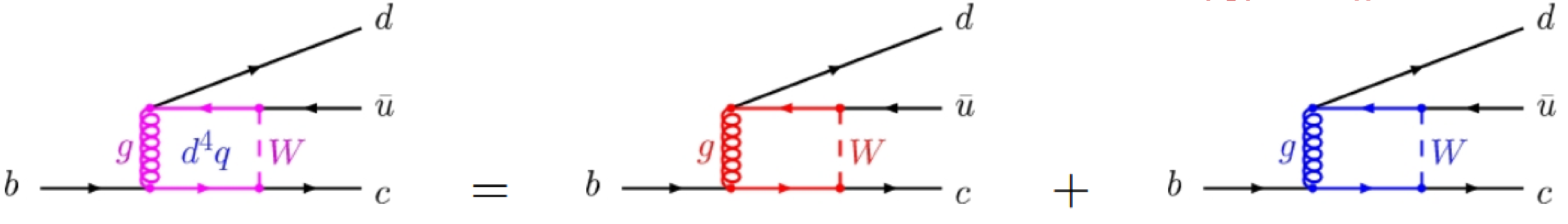
- **we cannot now simply expand in terms of $|q|/M_W$**
- **separate into the cases $|q| \geq M_W$ and $|q| \ll M_W$**

Effective Hamiltonian for $b \to c\bar{u}d$ decay

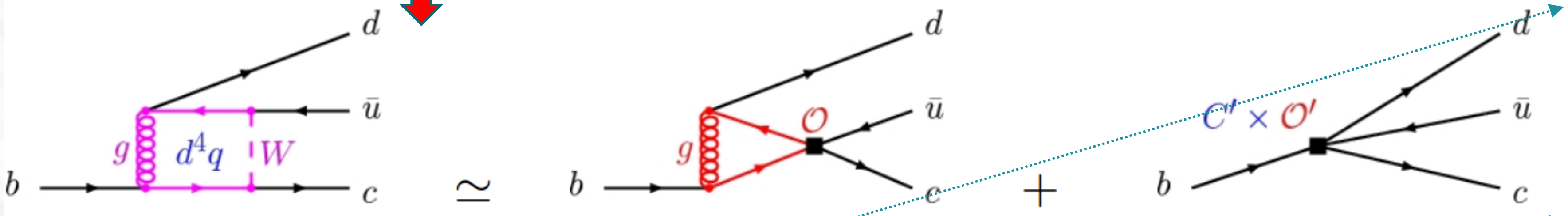
☐ **Factorization idea: expansion by regions** [Beneke and Smirnov '97]

full theory = IR region $(M_W \to \infty)$ + UV region $(m_{b,c} \to 0)$

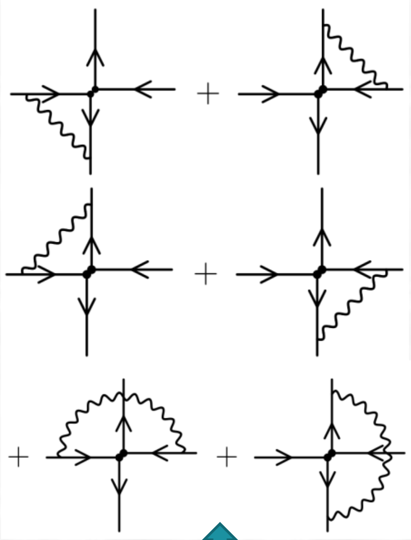
$|q| \ll M_W$ $|q| \geq M_W$



$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq I_{IR}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + I_{UV}(\alpha_s;\ \frac{\mu}{m_W})$$



$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq \langle\mathcal{O}\rangle^{\text{loop}}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + C'(\alpha_s;\ \frac{\mu}{m_W}) \times \langle\mathcal{O}'\rangle^{\text{tree}}$$



1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.

- independent of M_W
- UV divergent $\to \mu$

1-loop coefficient for new operator $\mathcal{O}'$ in EFT

- independent of $m_{b,c}$
- IR divergent $\to \mu$

Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **Two important observations for QCD corrections:**

  - short-distance QCD corrections preserve chirality, so both $(V-A)\otimes(V-A)$
  - quark-gluon vertices contain T^a, and thus induce a second color structure

- **Effective Hamiltonian for $b \to c\bar{u}d$ decay:**

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i + \text{h.c.}$$

  - **Effective operators:** there are now two dim-6 local current-current operators

$$\mathcal{O}_1 = (\bar{d}_L^i \gamma_\alpha u_L^j)(\bar{c}_L^j \gamma^\alpha b_L^i)$$
$$\mathcal{O}_2 = (\bar{d}_L^i \gamma_\alpha u_L^i)(\bar{c}_L^j \gamma^\alpha b_L^j)$$

$$C_i(\mu) = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} + \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{\mu^2}{M_W^2} + \frac{11}{6} \right) \left\{ \begin{array}{c} 3 \\ -1 \end{array} \right\} + \mathcal{O}(\alpha_s^2)$$

  - **Wilson coefficients $C_i(\mu)$:** to ensure absence of large log terms, $\mu \sim M_W$ and thus $C_i(\mu)$ calculated reliably in fixed-order perturbation theory

    - $\mu \sim M_W$: **matching scale**
    - $C_i(\mu)$: **matching condition**

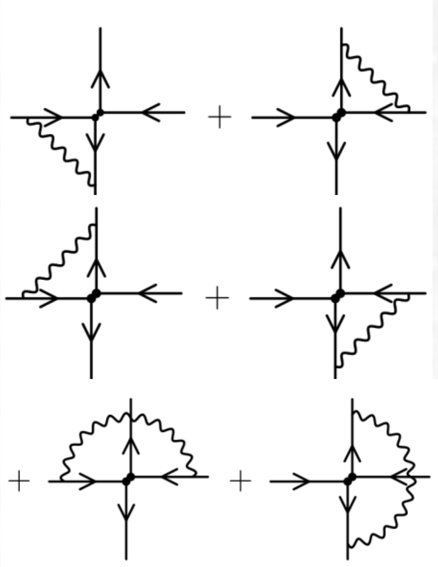
Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **Reality:** the hadronic matrix element $\langle D^+\pi^-|O_i|\bar{B}^0\rangle$ calculated at the low-energy scale $\mu \sim m_b$

- **Remember:** only the combination $C_i(\mu)\,\langle O_i\rangle(\mu)$ are scale & scheme independent

  we need evolve the Wilson coefficients from $\mu \sim M_W$ at a low scale $\mu \sim m_b$!

- **The scale dependence of $C_i(\mu)$ available from ADM of operators and RGE**



Renormalization constant matrix and ADM of the EFT operators

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix} \qquad \hat{\gamma} = \hat{Z}^{-1}\frac{d\hat{Z}}{d\ln\mu}$$

- **RGE of $C_i(\mu)$ governed by the anomalous dimension matrix**

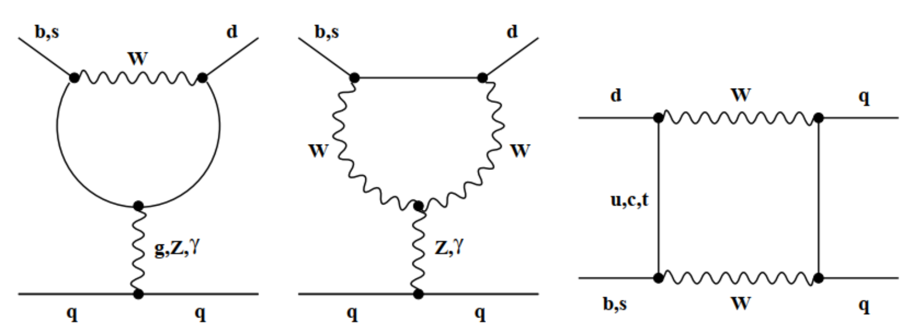
$$\frac{\partial}{\partial \ln\mu} C_i(\mu) \equiv \gamma_{ji}(\mu)\, C_j(\mu) = \left(\frac{\alpha_s(\mu)}{4\pi}\gamma_{ji}^{(1)} + \dots\right) C_j(\mu)$$

$$\boxed{\vec{C}(\mu) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}\hat{J}\right)\hat{U}^{(0)}(\mu,\mu_W)\left(\vec{A}^{(0)} + \frac{\alpha_s(\mu_W)}{4\pi}\left[\vec{A}^{(1)} - (\hat{r}^T + \hat{J})\,\vec{A}^{(0)}\right]\right).}$$

$\boxed{\langle D^+\pi^-|O_i|\bar{B}^0\rangle ?}$

Effective Hamiltonian for other processes

◻ Feynman diagrams in full and effective theory



→ QCD Penguin Operators & Chromomagnetic Operator

$$\mathcal{O}_3 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^b), \qquad \mathcal{O}_4 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_L^b \gamma^\mu q_L^a),$$

$$\mathcal{O}_5 = (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^b), \qquad \mathcal{O}_6 = (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} (\bar{q}_R^b \gamma^\mu q_R^a),$$

$$\mathcal{O}_8^g = \frac{g_s}{8\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^A b_R) G^A_{\mu\nu}.$$

◻ H_{eff} for $b \to s(d)$ FCNC decays

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(V_{ub} V_{us}^* \mathcal{O}_i^{(u)} + V_{cb} V_{cs}^* \mathcal{O}_i^{(c)} \right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=3}^{10} C_i(\mu, x_t) \mathcal{O}_i + C_8(\mu, x_t) \mathcal{O}_8^g \right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=9}^{10} C_i(\mu, x_t) \mathcal{O}_i^{\ell\ell} + C_7(\mu, x_t) \mathcal{O}_7^\gamma \right)$$

→ Electroweak Penguin Operators $\mathcal{O}_{7-10}$

$$\mathcal{O}_7 = \frac{2}{3} (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} e_q (\bar{q}_L^b \gamma^\mu q_L^b), \quad \mathcal{O}_8 = \frac{2}{3} (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} e_q (\bar{q}_L^b \gamma^\mu q_L^a),$$

$$\mathcal{O}_9 = \frac{2}{3} (\bar{s}_L^a \gamma_\mu b_L^a) \sum_{q \neq t} e_q (\bar{q}_R^b \gamma^\mu q_R^b), \quad \mathcal{O}_{10} = \frac{2}{3} (\bar{s}_L^a \gamma_\mu b_L^b) \sum_{q \neq t} e_q (\bar{q}_R^b \gamma^\mu q_R^a).$$

depend on electromagnetic charge of final state quarks !

→ Electromagnetic operators $\mathcal{O}_7^\gamma$

$$\mathcal{O}_7^\gamma = \frac{e}{8\pi^2} m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}$$

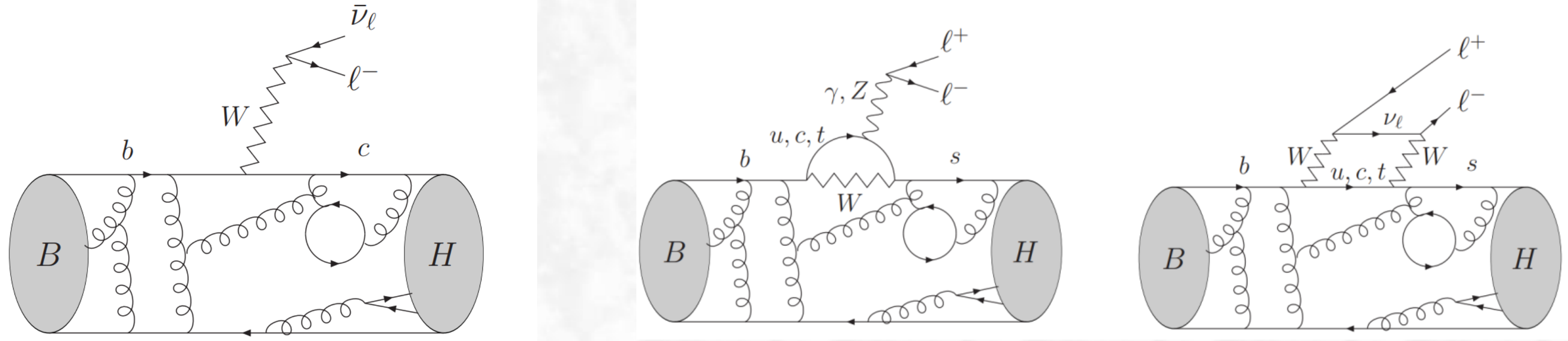
main contribution to $b \to s(d)\gamma$ and $b \to s(d)\ell^+\ell^-$ decays.

how to calculate matrix elements of $\mathcal{O}_i$?

Effective Hamiltonian for $b \to s(d)$ decays

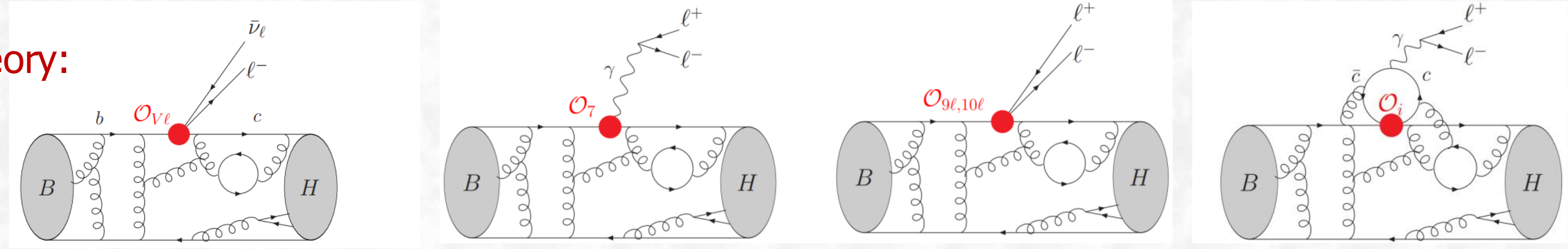
- **Different languages used by experimentalists vs by theorists**

Full theory:



vs

Effective theory:



- **Decay amplitude:**

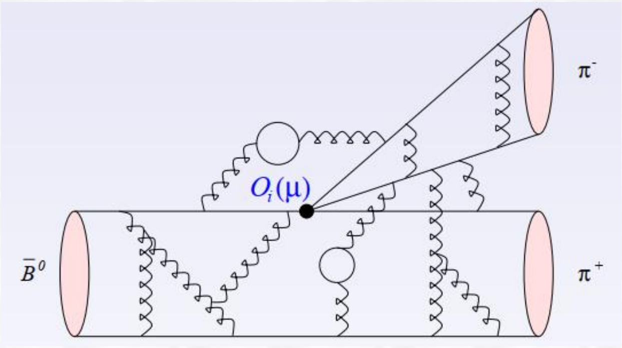
$$\boxed{\mathcal{A}(\bar{B} \to f) = \sum_i [\lambda_{\text{CKM}} \cdot C_i \cdot \langle f|\mathcal{O}_i|\bar{B}\rangle]}$$

how to calculate $\langle f|\mathcal{O}_i|\bar{B}\rangle_{\text{QCD,QED}}$

Hadronic matrix elements

- $\langle f|\mathcal{O}_i|\bar{B}\rangle_{\mathrm{QCD,QED}}$: **depending on the specific processes**
  - **Exclusive vs. (semi-)inclusive modes?**
  - **Hadronic decays** $\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle$: depends on spin & parity of $M_{1,2}$ and FSI introduces strong phases, and hence direct CPV

a difficult, multi-scale, QCD & QED problem!



$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle = \langle M_1|\bar{u}\ldots b|\bar{B}\rangle\langle M_2|\bar{d}\ldots u|0\rangle$$

naïve fact. approach [Bauer, Stech, Wirbel '87]

$\underbrace{\text{HQE/OPE, lattice, (QCD sum rules)}}$ — QCD factorization, (flavour symmetries)

$\langle 0|\mathcal{O}|B\rangle$, $\langle B|\mathcal{O}|B\rangle$ → $\langle M|\mathcal{O}|B\rangle$ → $\langle M_1 M_2|\mathcal{O}|B\rangle$

Increasingly difficult

$B \to X_c \ell\nu, X_s\gamma, X_s\ell\ell$

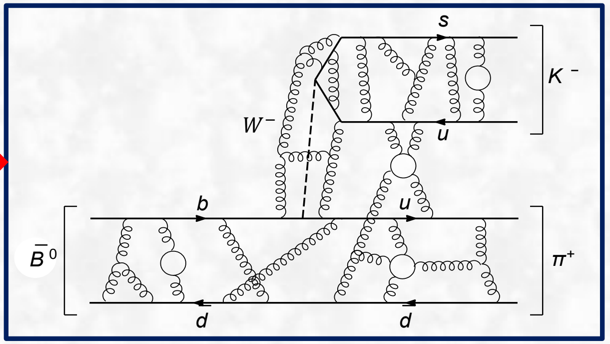
$B \to \tau\nu_\tau$	$B \to D\tau\nu_\tau$	$B \to \rho\gamma$	Direct CP asym
$B_s \to \mu^+\mu^-$	$\lvert V_{ub}\rvert$	$B \to K^{(*)}\ell\ell$	$B_s \to \pi K, KK, \ldots$
$\Delta M_{B_d,B_s}$	$B \to K\nu\bar{\nu}$		$B_s \to \pi\pi$
$\Delta\Gamma_{B_s}$			$B_s \to \phi\phi, K^{*0}\bar{K}^{*0}$

- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, $\cdots$
  [Keum, Li, Sanda, Lü, Yang '00; Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
- Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, $\cdots$
  [Zeppenfeld, '81; London, Gronau, Rosner, He, Chiang, Cheng *et al.*]
- Combination of dynamical approaches with flavor symmetries [FAT (Li, Lü et al.)…]

Example

- **With $\langle f|\mathcal{O}_i|\bar{B}\rangle_{\text{QCD,QED}}$ at hand, we can then do what we want to do**

$\bar{B}^0 \to \pi^+ K^-$



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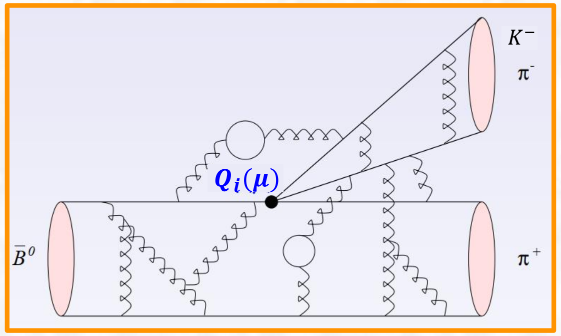
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\text{QCD}} \sim 1$ GeV

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V^*_{pD} V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right)$$



electroweak parameters

WCs due to NP

non-perp. parameters

$$\mathcal{A}(\bar{B}^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}} \sum_{ij} V_{\text{CKM}} \left(\mathcal{C}_i^{\text{SM}} + \mathcal{C}_i^{\text{NP}}\right) \Big[F_j^{B\to\pi}(m_K^2) \int_0^1 du\, T_{ij}^{\text{I}}(u) \Phi_K(u) + (\pi \leftrightarrow K)$$
$$+ \int_0^1 d\xi\, du\, dv\, T_i^{\text{II}}(\xi, u, v) \Phi_B(\xi) \Phi_\pi(v) \Phi_K(u) \Big]$$

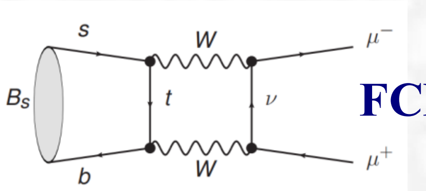
related to exp. Br & CPV

WCs from SM, also perp. calculable

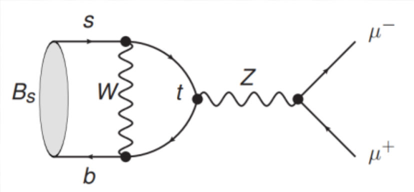
perp. calculable in QCD & QED

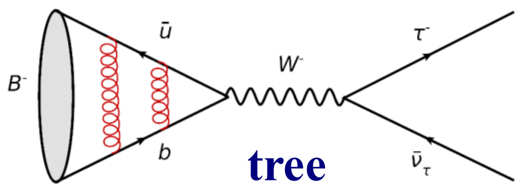
Purely leptonic decays

- **Have simplest hadronic structure; all QCD dynamics encoded in** $f_{B_s} = (230.3 \pm 1.3)\,\mathrm{MeV}$



FCNC

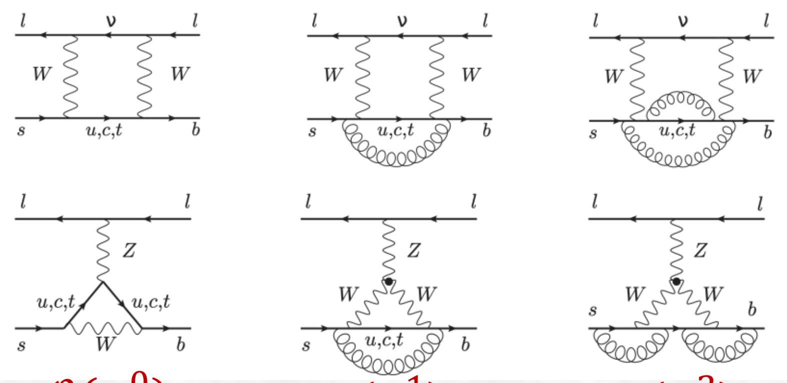




tree

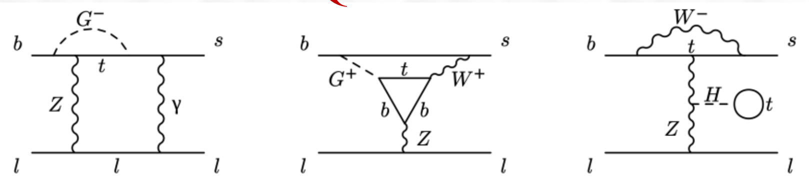
$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 b(0)|\bar{B}(p)\rangle = i f_B p^\mu$$

- **Much progress achieved due to multi-loop techniques, EFTs , & LQCD, …**



$\mathcal{O}(\alpha_s^0)$ $\mathcal{O}(\alpha_s^1)$ $\mathcal{O}(\alpha_s^2)$

NNLO QCD correction



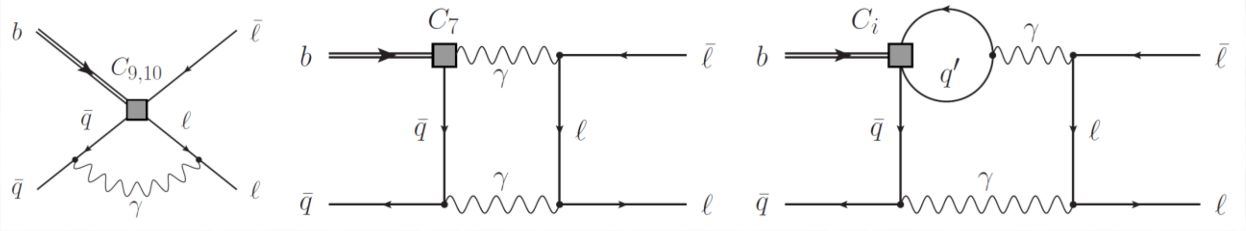
$\mathcal{O}(\alpha_e)$ NLO EW correction

$$\mathcal{L}_{\Delta B=1} = \mathcal{N}_{\Delta B=1}\left[\sum_{i=1}^{10} C_i(\mu_b)\,Q_i + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*}\sum_{i=1}^{2} C_i(\mu_b)\left(Q_i^u - Q_i^c\right)\right]$$

$$\begin{aligned} Q_9 &= \frac{\alpha_{em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell) \\ Q_{10} &= \frac{\alpha_{em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell) \\ Q_7 &= \frac{e}{16\pi^2} m_b\,(\bar{q}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \end{aligned}$$

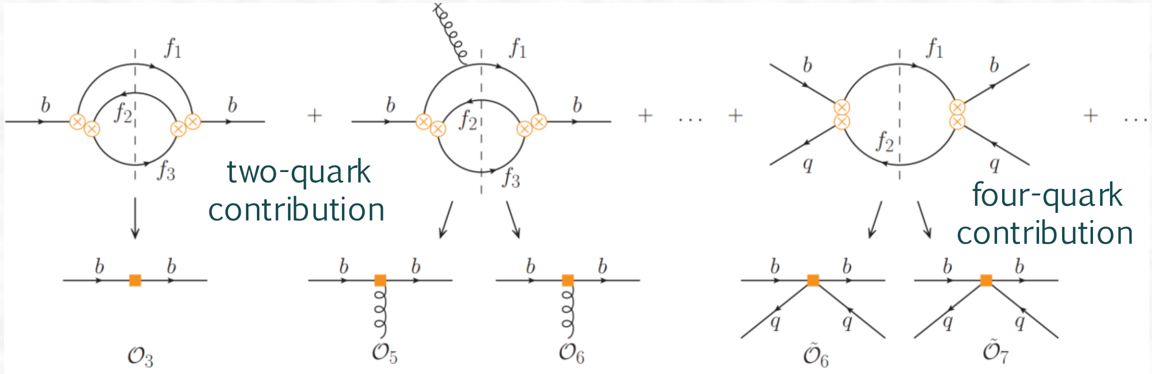
$\overline{\mathcal{B}}_{s\mu}^{\,\exp} = (3.34 \pm 0.27)\times 10^{-9}$ vs $\overline{\mathcal{B}}_{s\mu}^{\,SM} = (3.64 \pm 0.12)\times 10^{-9}$

- **QED effects below m_b needed to match exp. precision**



↳ when struc.-dep. QED effects included, need inner structure of B

Lifetime of b-hadrons



$$\Gamma(\mathcal{B}) = \frac{1}{2m_{\mathcal{B}}}\sum_X \int_{\rm PS} (2\pi)^4\delta^{(4)}(p_{\mathcal{B}}-p_X)\ |\langle X(p_X)|\mathcal{H}_{\rm eff}|\mathcal{B}(p_{\mathcal{B}})\rangle|^2$$

Optical Theorem

$$= \frac{1}{2m_{\mathcal{B}}}\,{\rm Im}\,\langle\mathcal{B}(p_{\mathcal{B}})|\, i\int d^4x\, T\left\{\mathcal{H}_{\rm eff}(x),\mathcal{H}_{\rm eff}(0)\right\}|\mathcal{B}(p_{\mathcal{B}})\rangle$$

$$\Gamma(\mathcal{B}) = \Gamma_3 + \Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_b^2} + \Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_b^3} + \ldots + 16\pi^2\left[\tilde{\Gamma}_6\frac{\langle\tilde{\mathcal{O}}_6\rangle}{m_b^3} + \tilde{\Gamma}_7\frac{\langle\tilde{\mathcal{O}}_7\rangle}{m_b^4} + \ldots\right]$$

☐ Lifetime based on HQE in $1/m_b$: [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, 2402.04224]

☐ status of SD coefficients:

	Semi-leptonic				Non-leptonic		
	LO	NLO	N^{2}LO	N^{3}LO	LO	NLO	N^{2}LO
Γ_3	✓	✓	✓	✓⋆	✓	✓	(✓)
Γ_5	✓	✓			✓	(✓)•	
Γ_6	✓	✓			✓◇	🕒	
Γ_7	✓				🕒		
Γ_8	✓						
$\tilde{\Gamma}_6$	✓	✓	🕒		✓	✓	🕒
$\tilde{\Gamma}_7$	✓	🕒			✓	🕒	

◇ [Lenz, Piscopo, AR, 2004.09527], [Mannel, Moreno, Pivovarov, 2004.09485]

⋆ [Fael, Schönwald, Steinhauser, 2011.13654] • [Mannel, Moreno, Pivovarov, 2304.08964 (for $m_c = 0$)]

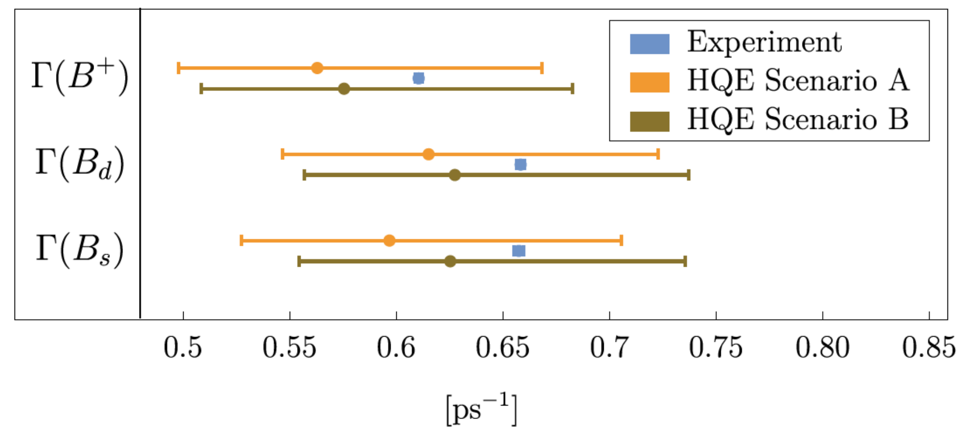
✓ - known (✓) - partly known 🕒 - in progress or planned [Karlsruhe, Siegen]

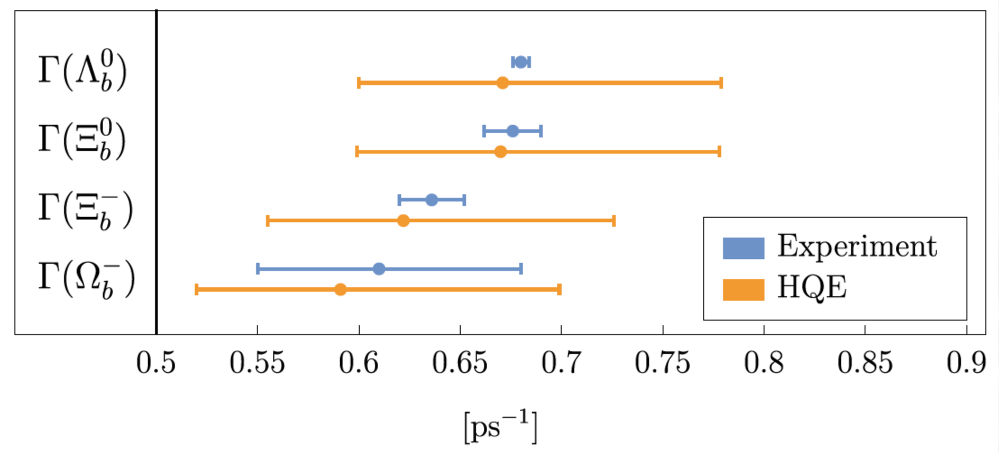
☐ status of hadronic matrix elements:

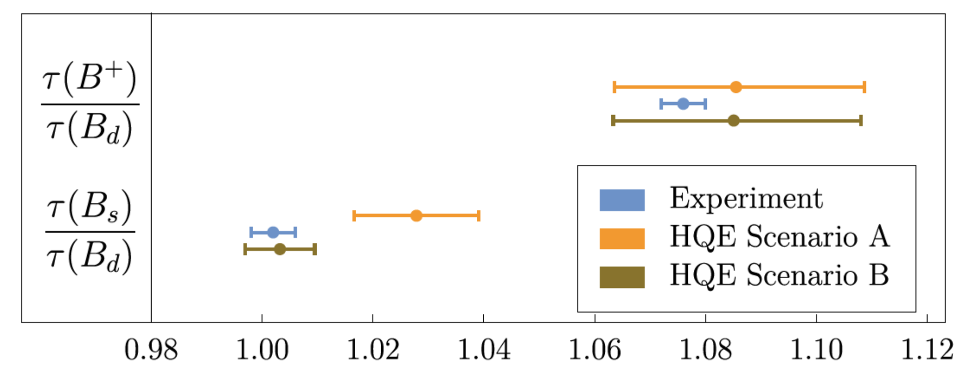
$\langle Q_5\rangle_{B_d}$	1993/96 2013-2023 2017/18	QCD sum rule [234, 235] Fit of inclusive data [236–241] Lattice QCD [242, 243]
$\langle Q_5\rangle_{B_s}$	2011	Spectroscopy relations [244]
$\langle Q_5\rangle_{\mathcal{B}}$	2023	Spectroscopy relations [34]
$\langle Q_6\rangle_{B_d}$	1994/2022 2013-2023	EOM relation [31, 245] Fit of inclusive data [236–241]
$\langle Q_6\rangle_{B_s}$	1994/2022 2011	EOM relation [31, 245] Sum rule [244]
$\langle Q_6\rangle_{\mathcal{B}}$	2023	EOM relation [34]
$\langle\tilde{Q}_6\rangle_{B_d}$	2017	HQET sum rule [246]
$\langle\tilde{Q}_6\rangle_{B_s}$	2022	HQET sum rule [247]
$\langle\tilde{Q}_6\rangle_{\Lambda_b}$	1996	QCD sum rule [248]
$\langle\tilde{Q}_6\rangle_{\mathcal{B}}$	2023	NRCQM [34]
$\langle\tilde{Q}_7\rangle$		VIA

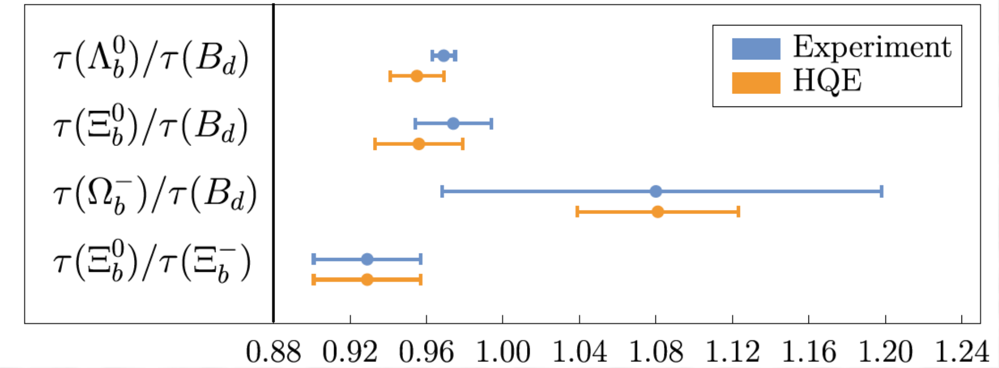
Lifetime of b-hadrons

□ Exp. data & SM predictions: [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, 2402.04224]









- ➢ excellent agreement between theory & data
- ➢ no indication of sizeable quark-hadron duality violation

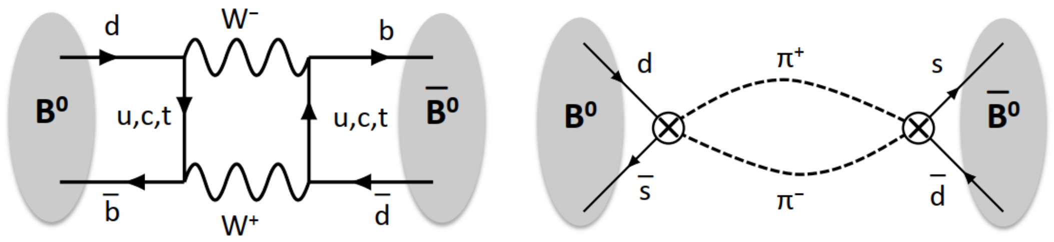
- ➢ HQE also works for c-hadron lifetimes

[H. Y. Cheng, C. W. Liu, 2305.00665]

Neutral B-meson mixings

- **For B_q^0 meson:** flavor eigenstates $\neq$ mass eigenstates $\Rightarrow$ mix with each other via box diagrams

$$i\frac{d}{dt}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$$



"short-distance" (=virtual particle exchange)

"long-distance" (=real particle exchange)

- **Three observables for B mixings**

  - **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
    $|M_{12}|$: heavy internal particles: t, SUSY, ...
  - **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi$ **(on-shell)**
    $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!
  - **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to X l \nu$ (*semi-leptonic*)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\overline{B}_q(t)\to f) - \Gamma(B_q(t)\to \bar{f})}{\Gamma(\overline{B}_q(t)\to f) + \Gamma(B_q(t)\to \bar{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi$$

$$M_{12} = \frac{G_F^2}{12\pi^2}\left(V_{tq}^* V_{tb}\right)^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

† 1-loop calculation $S_0\left(x_t = m_t^2/M_W^2\right)$

† 2-loop perturbative QCD corrections $\hat{\eta}_B$

† $\frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \left\langle \overline{B}_q \middle| (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \middle| B_q \right\rangle$

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4\left(\Gamma_4^{(0)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^5\left(\Gamma_5^{(0)} + \ldots\right) + \ldots$$

HQE

- ✓ M_{12}: dispersive (off-shell) part of the box diagram
- ✓ Γ_{12}: absorptive (on-shell) part of the box diagram
- ✓ $\phi = \arg(-M12/\Gamma12)$: relative phase between them

➢ **indirect searches for BSM effects**

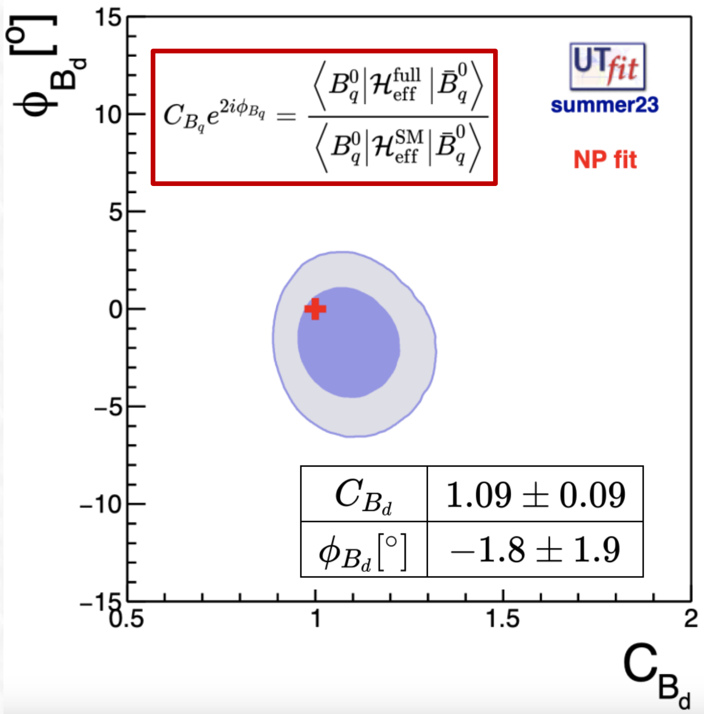
NP constraints from neutral B mixings

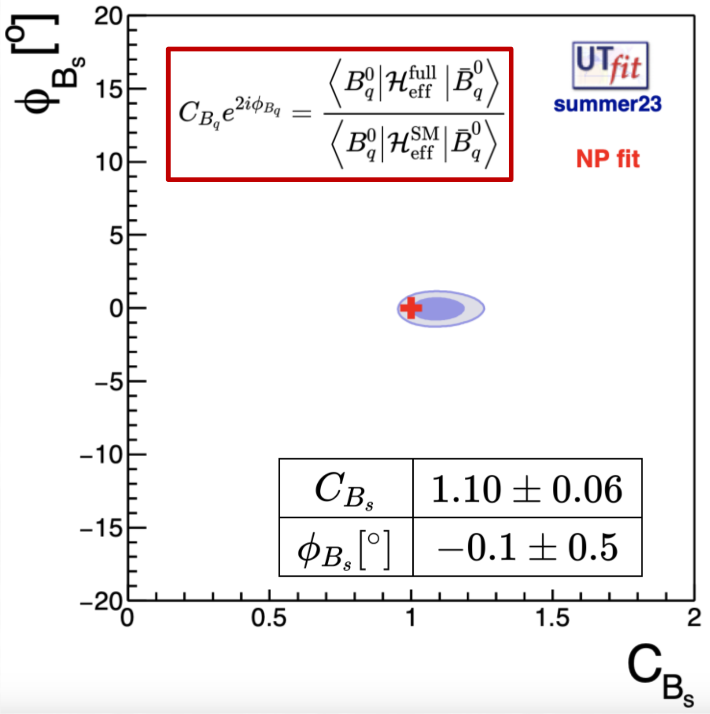
- **Exp. observables are related to the SM and NP parameters:**

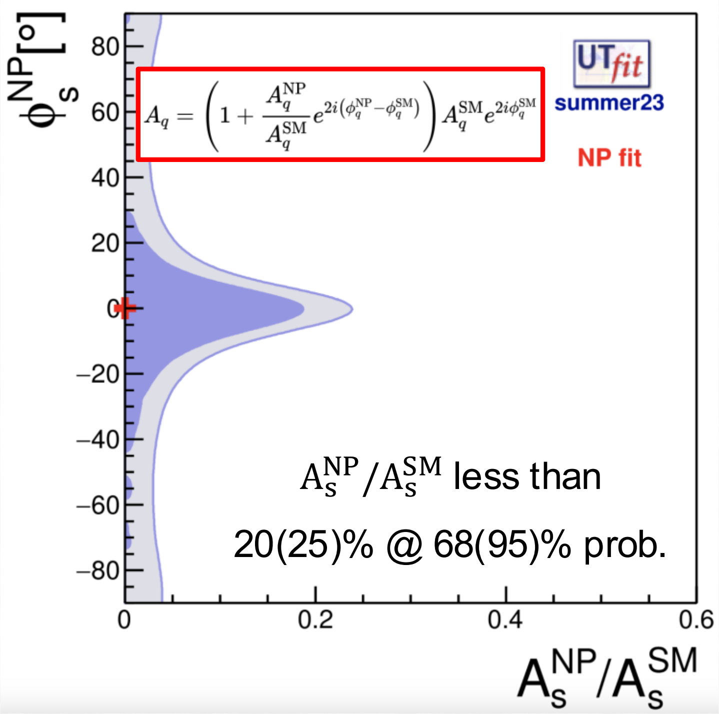
$$\Delta M_d^{\text{exp}} = C_{B_d}\Delta M_d^{\text{SM}}, \qquad \sin 2\beta^{\text{exp}} = \sin\left(2\beta^{\text{SM}} + 2\phi_{B_d}\right)$$

$$\Delta M_s^{\text{exp}} = C_{B_s}\Delta M_s^{\text{SM}}, \qquad \phi_s^{\text{exp}} = \left(\beta_s^{\text{SM}} - \phi_{B_s}\right)$$

- **Latest fit results by UTfit group:**



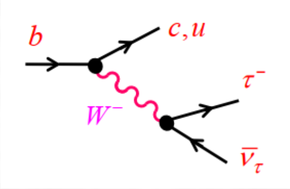




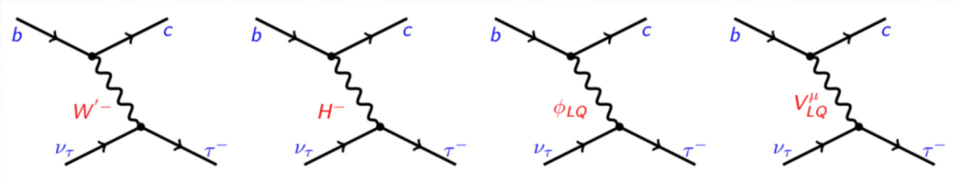
→ **consistency between data & SM of B mixing observables puts stringent constraint on NP**

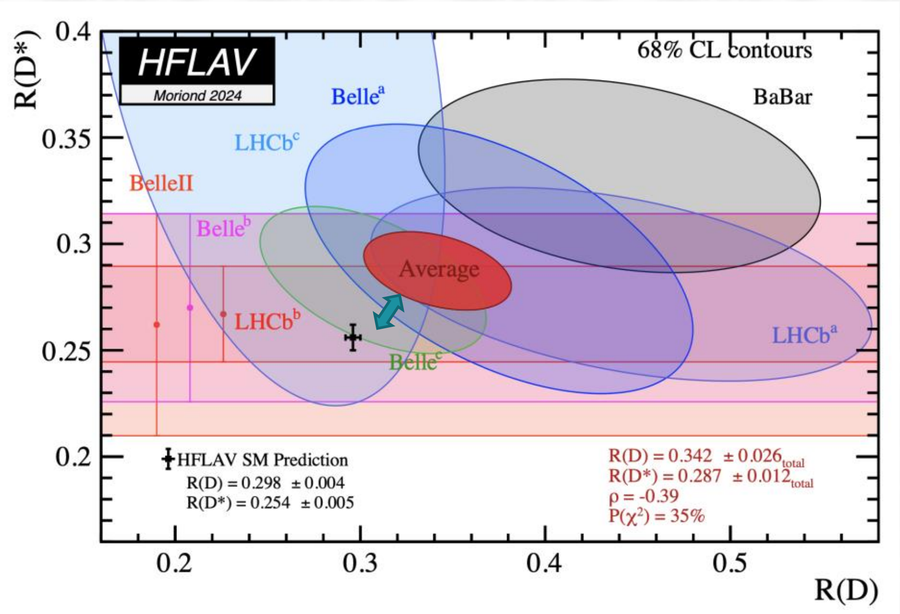
Semi-leptonic B decays

- $R(D^{(*)})$ **anomalies: first observed by BaBar in 2012; currently still having ~3.31σ deviation**



$$R\left(D^{(*)}\right) = \frac{Br(B\to D^{(*)}\tau\nu_\tau)}{Br(B\to D^{(*)}l\nu_l)}$$





- **Model-indep. result for $R(D)$, $R(D^*)$ & $R(\Lambda_c)$:** [Iguro, Kitahar, Watanabe, 2405.06062; Duan, Iguro, Li, Watanabe, Yang, 2410.21384]

$$\mathcal{H}_{\rm eff} = 2\sqrt{2}G_F V_{cb}\Big[(1+C_{V_L})O_{V_L} + C_{V_R}O_{V_R} + C_{S_L}O_{S_L} + C_{S_R}O_{S_R} + C_T O_T\Big]$$

with

$$O_{V_L} = (\bar c\gamma^\mu P_L b)(\bar\tau\gamma_\mu P_L\nu_\tau)\,,\qquad O_{V_R} = (\bar c\gamma^\mu P_R b)(\bar\tau\gamma_\mu P_L\nu_\tau)\,,$$
$$O_{S_L} = (\bar c P_L b)(\bar\tau P_L\nu_\tau)\,,\qquad O_{S_R} = (\bar c P_R b)(\bar\tau P_L\nu_\tau)\,,$$
$$O_T = (\bar c\sigma^{\mu\nu} P_L b)(\bar\tau\sigma_{\mu\nu} P_L\nu_\tau)\,,$$

$$\begin{aligned}
\frac{R_P}{R_P^{\rm SM}} =\ & |1+C_{V_L}^{q\tau}+C_{V_R}^{q\tau}|^2 + a_P^{SS}|C_{S_L}^{q\tau}+C_{S_R}^{q\tau}|^2 + a_P^{TT}|C_T^{q\tau}|^2 \\
& + a_P^{VS}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}+C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*}+C_{S_R}^{q\tau*}\right)\right] + a_P^{VT}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}+C_{V_R}^{q\tau}\right)C_T^{q\tau*}\right]\,,\\
\frac{R_V}{R_V^{\rm SM}} =\ & |1+C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_V^{SS}|C_{S_L}^{q\tau}-C_{S_R}^{q\tau}|^2 + a_V^{TT}|C_T^{q\tau}|^2 \\
& + a_V^{V_LV_R}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] + a_V^{VS}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}-C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*}-C_{S_R}^{q\tau*}\right)\right] \\
& + a_V^{V_LT}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_V^{V_RT}\mathrm{Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right]\,,\\
\frac{R_H}{R_H^{\rm SM}} =\ & |1+C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_H^{SS}\left[|C_{S_L}^{q\tau}|^2+|C_{S_R}^{q\tau}|^2\right] + a_H^{TT}|C_T^{q\tau}|^2 + a_H^{V_LV_R}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] \\
& + a_H^{VS_1}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_{S_L}^{q\tau*}+C_{V_R}^{q\tau}C_{S_R}^{q\tau*}\right] + a_H^{VS_2}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_{S_R}^{q\tau*}+C_{V_R}^{q\tau}C_{S_L}^{q\tau*}\right] \\
& + a_H^{S_LS_R}\mathrm{Re}\left[C_{S_L}^{q\tau}C_{S_R}^{q\tau*}\right] + a_H^{V_LT}\mathrm{Re}\left[\left(1+C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_H^{V_RT}\mathrm{Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right]\,.
\end{aligned}$$

Sum rule for $b \to c$ sector

$\cancel{|1 + C_{V_L}^{q\tau}|^2}$

$\cancel{\mathrm{Re}[(1 + C_{V_L}^{q\tau})C_{S_L}^{q\tau *}]}$

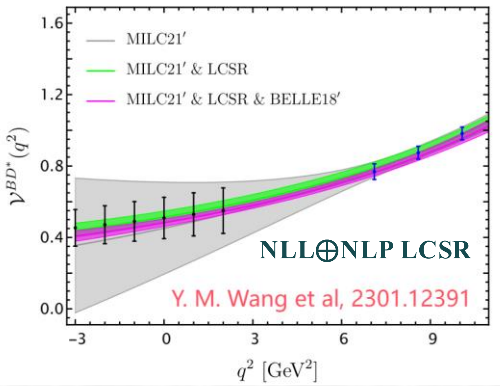
□ **Sum rule for $R(D)$, $R(D^*)$ & $R(\Lambda_c) = Br(\Lambda_b \to \Lambda_c \tau \nu_\tau)/Br(\Lambda_b \to \Lambda_c \ell \nu_\ell)$:**

$$\boxed{\frac{R_H}{R_H^{\rm SM}} = b\frac{R_P}{R_P^{\rm SM}} + c\frac{R_V}{R_V^{\rm SMM}} + \delta_H(C_i)}$$

➡ $b + c = 1$ & $a_P^{VS} b + a_V^{VS} c = a_H^{VS_1}$, so that $\delta_H(C_i)$ small

➡ model-indep. & holds for any tau-philic NP!

□ **State-of-the-art prediction:** [Duan, Iguro, Li, Watanabe, Yang, 2410.21384]



	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM
$B \to D$	Refs. [85, 86]	no data	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$B \to D^*$	Refs. [87–89]	no data (*)	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$\Lambda_b \to \Lambda_c$	Ref. [80]	Ref. [92]	no data	no data	–

- important to properly consider the correlations among FF parameters

➡ $$\boxed{\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}} = (0.272 \pm 0.015)\frac{R_D}{R_D^{\rm SM}} + (0.728 \mp 0.015)\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{\Lambda_c}}$$

- provide a unique prediction of $R(\Lambda_c)$ model-indep.ly

$$\begin{aligned}
\delta_{\Lambda_c} = {} & (-0.001 \pm 0.005)\ \left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) + (-0.007 \pm 0.005)\,\mathrm{Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau *}\right) \\
& + \underline{(-2.681 \pm 6.907)\,|C_T^{c\tau}|^2} + (-0.561 \pm 1.439)\,\mathrm{Re}\left(C_{V_R}^{c\tau} C_T^{c\tau *}\right) \\
& + \mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{(0.041 \pm 0.034) C_{V_R}^{c\tau *} + (0.594 \pm 1.274) C_T^{c\tau *}\right\}\right] \\
& + (-0.002 \pm 0.009)\mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right) C_{S_R}^{c\tau *} + C_{S_L}^{c\tau} C_{V_R}^{c\tau *}\right]
\end{aligned}$$

$$R_{\Lambda_c}^{\rm SR} = 0.370 \pm 0.017|_{R_X^{\rm SM,exp}} \pm (< 0.001)|_{\rm SR}$$

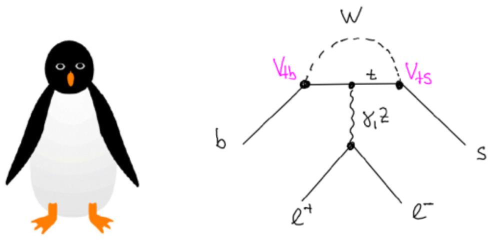
$\Updownarrow$

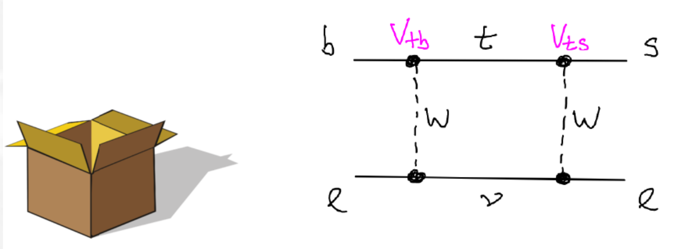
$$R_{\Lambda_c}^{\rm LHCb} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

Rare FCNC decays

- occur firstly at 1-loop; suppressed by **loop factor**
- proportional to $|V_{tb}V_{ts}^*|$; $\mathrm{Br}(b \to s\,\ell\ell) \sim 10^{-6}$
- sensitive to various NP beyond the SM

□ Why $b \to s\ell^+\ell^-$ processes:

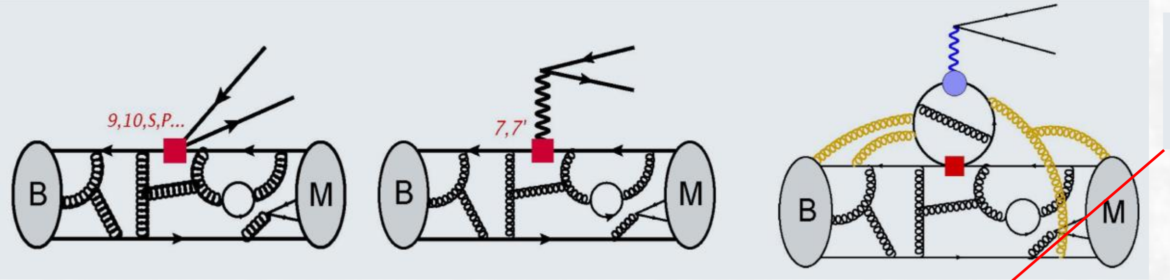




$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

$$\begin{aligned} Q_9 &= \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell) \\ Q_{10} &= \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell) \\ Q_7 &= \frac{e}{16\pi^2} m_b (\bar{q}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \end{aligned}$$

□ Local & non-local hadronic matrix elements:



Non-local form-factors:

$$\mathcal{H}_\lambda(k,q) = i\int d^4x\, e^{iq\cdot x}\mathcal{P}_\lambda^\mu \langle \bar{M}(k)|T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

$$\mathcal{A}_\lambda^{L,R}(B\to M_\lambda \ell\ell) = \mathcal{N}_\lambda\left\{(C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7\mathcal{F}_\lambda^T(q^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

- $B \to K^{(*)}\mu\mu$
- $B_s \to \varphi\,\mu\mu$, ...

Local form-factors,
involves e.g. $\mathcal{F}_\lambda(k,q) = \mathcal{P}_\lambda^\mu \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$

BSZ parametrization
LQCD & LCSR

- dominated by charming loops from $\mathcal{O}_{1,2}^c$
- they can easily mimic NP by a shift of C_9
- with a simple analytic structure, charming loops are small [Gubernari 2022; LHCb 2024]

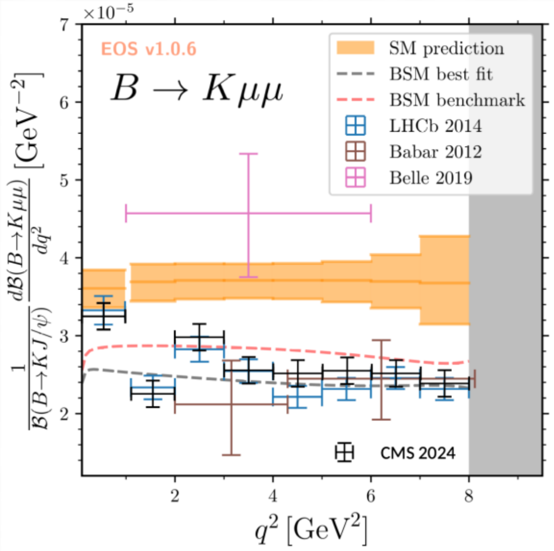
Rare FCNC decays

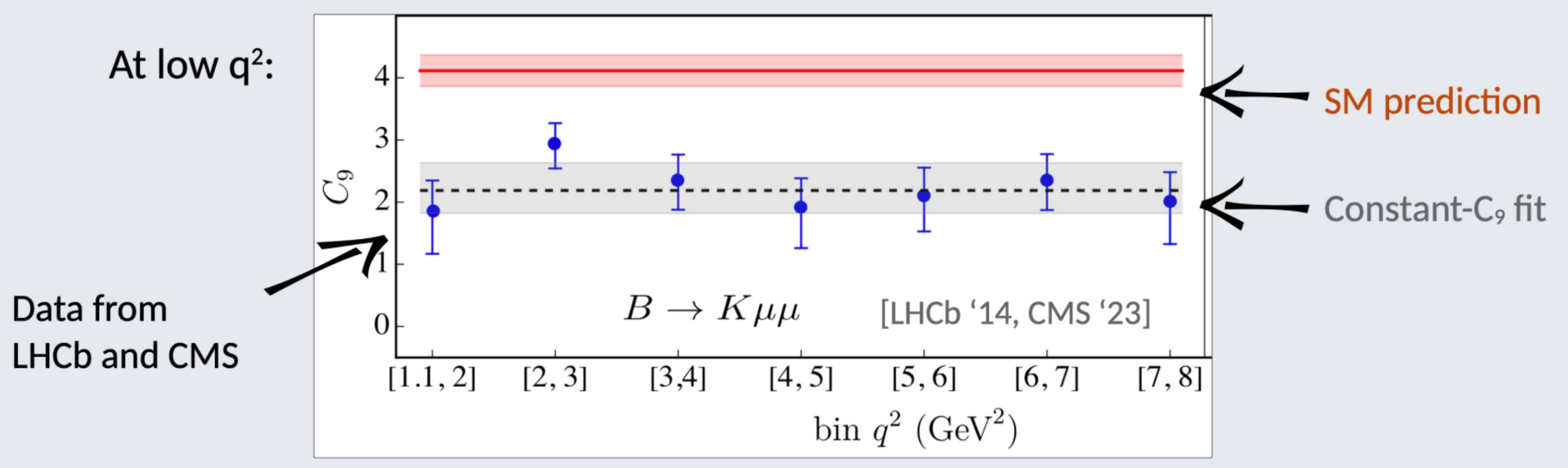
$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

dispersive bound

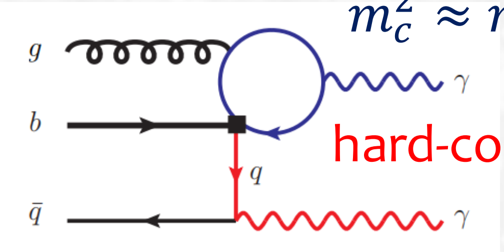
Gubernari, van Dyk, Virto '20

☐ No visible q^2 dep. for NP [Bordone etal 2024]





☐ The size of charming loops still under investigation, and further detailed studies required



hard-collinear

$m_c^2 \approx m_b \Lambda_{QCD}$:

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n) S_n^\dagger S_{\bar{n}}(0) S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}}(\tau_2 \bar{n}) \bar{n}^\nu n \cdot \gamma \gamma_\perp^\mu \gamma_5 S_{\bar{n}}^\dagger h_v(0)|\bar{B}_v\rangle$$

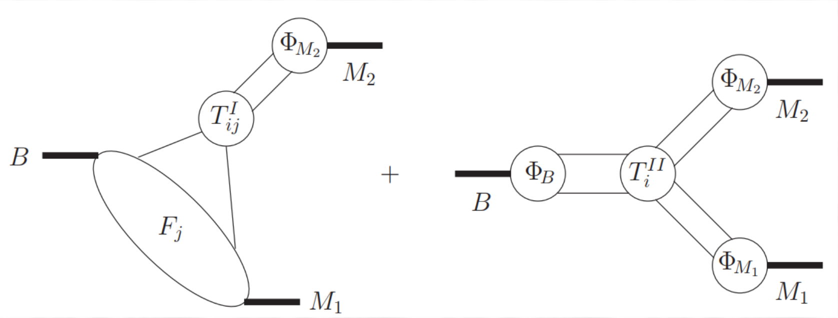
$$= 2F_B(\mu) m_B \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \ e^{-i(\omega_1\tau_1 + \omega_2\tau_2)} \Phi_G(\omega_1, \omega_2, \mu)$$

➢ Novel soft-function needed? [Qin, Shen, Wang, Wang, 2023; Huang, Ji, Shen, Wang, Wang, Zhao,2023]

QCD factorization for charmless B decays

◻ QCDF formulae for two-body charmless hadronic B decays [BBNS 99`]

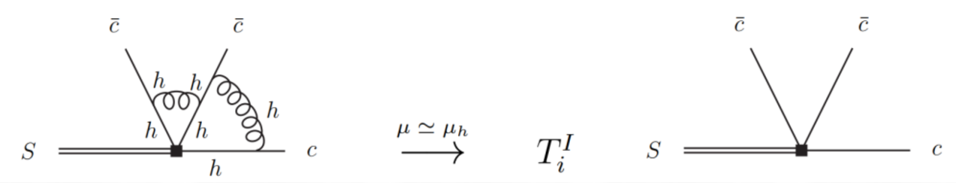
$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h,\mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2}\Phi_{M_2}(\mu_s)$$
$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h,\mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I,\mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1}\Phi_{M_1}(\mu_s) \star f_{M_2}\Phi_{M_2}(\mu_s) \Big\}$$

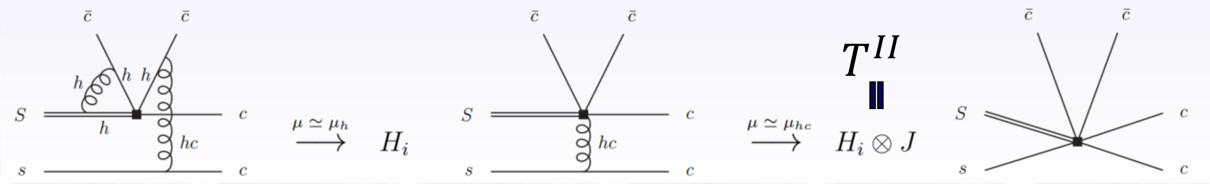


- systematically method based on QCD, and higher-order pert. corrections calculable systematically
- factorization valid at leading power in heavy-quark limit, and limited only by power corrections

◻ SCET formalism reproduces exact QCDF result, but more apparent & efficient [Beneke, 1501.07374]

- **for T^I:** only hard scale involved, one-step matching from QCD $\to$ $\mathrm{SCET_I}(\mathrm{hc, c, s})$!





- **for T^{II}:** two scales involved, two-step matching from QCD $\to$ $\mathrm{SCET_I}(\mathrm{hc, c, s})$ $\to$ $\mathrm{SCET_{II}}(\mathrm{c, s})$!

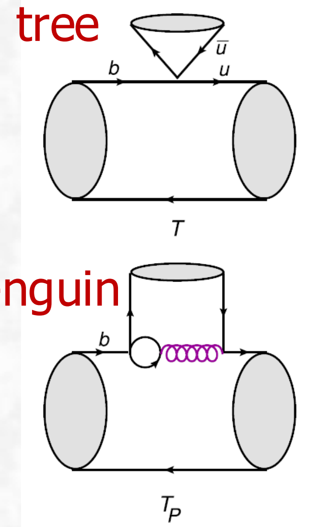
Status of the NNLO calculation of T^I & T^{II}

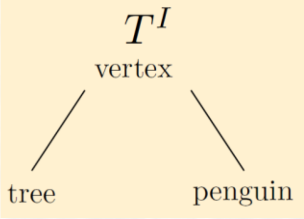
- **For each Q_i insertion, both tree & penguin topologies relevant for charmless decays**

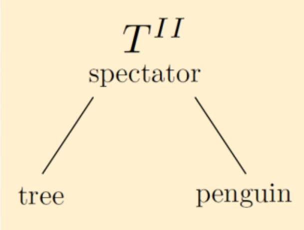
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

tree

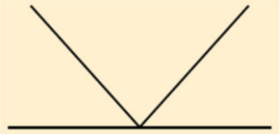
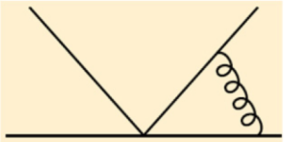
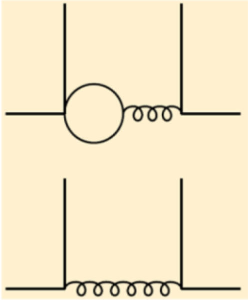
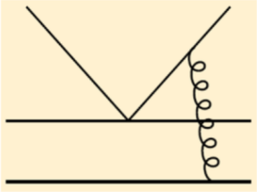
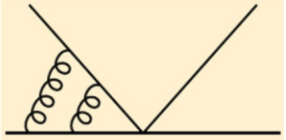
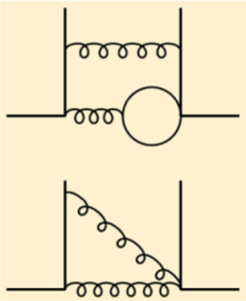
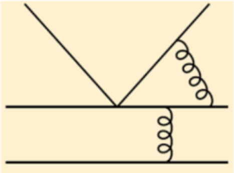
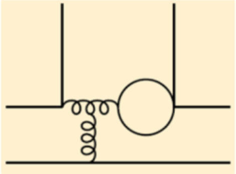
penguin



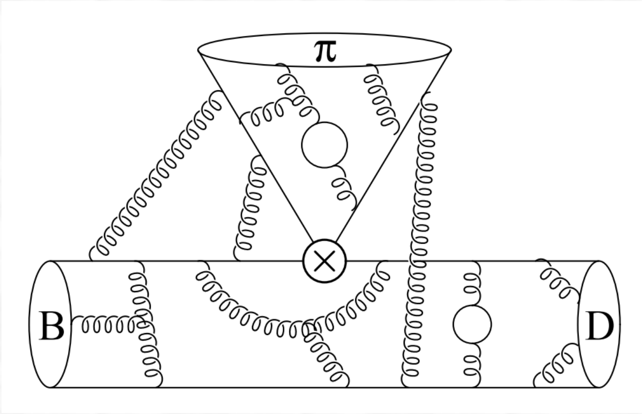




- For tree & penguin topologies, both contribute to T^I & T^{II}

	T_i^I, tree	T_i^I, penguin	T_i^{II}, tree	T_i^{II}, penguin
LO: $\mathcal{O}(1)$		$T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$		
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'03				$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07, '09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11 Bell, Beneke, Huber, Li '15, '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

$B_q^0 \to D_q^{(*)-} L^+$ class-I decays



- **At the quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

  all four flavors different from each other,

  no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- **For class-I decays:** QCDF formula much simpler;

  only the form-factor term at leading power

  [Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

i) only color-allowed tree topology $T = a_1$

ii) spectator & annihilation power-suppressed

iii) annihilation absent in $B^0_{d(s)} \to D^-_{d(s)} K(\pi)^+$ etc.

iv) they are theoretically simpler and cleaner

these decays used to test factorization theorems

- **Hard kernel T:** both NLO and NNLO results known;

  [Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Non-leptonic/semi-leptonic ratios

Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

$$R_{(s)L}^{(*)} \equiv \frac{\Gamma(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} L^-)}{d\Gamma(\bar{B}_{(s)}^0 \to D_{(s)}^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 \, f_L^2 \,|a_1(D_{(s)}^{(*)+} L^-)|^2 \, X_L^{(*)}$$

free from the uncertainties from

V_{cb} & $B_{d,s} \to D_{d,s}^{(*)}$ form factors

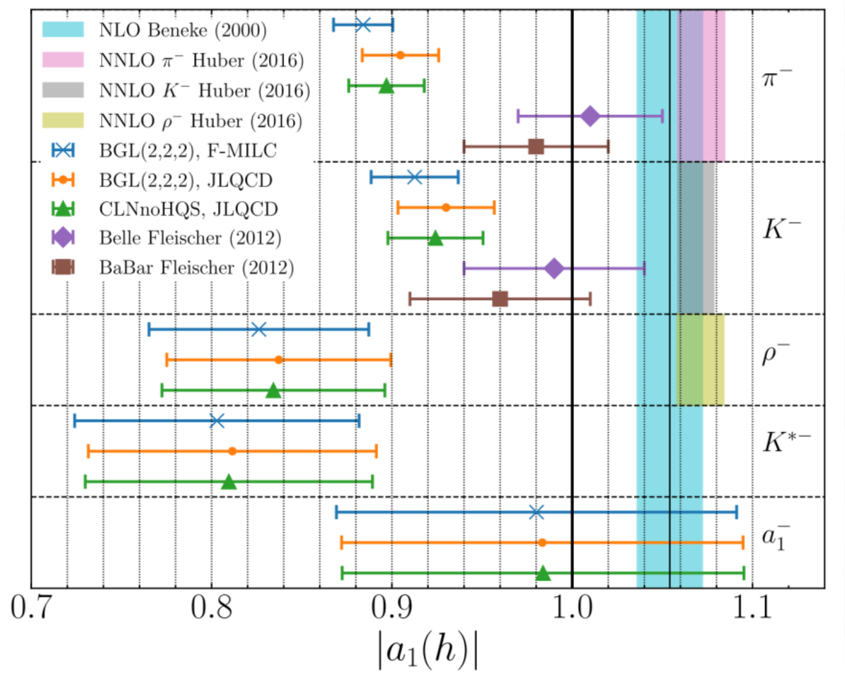
Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R_{(s)L}^{(*)}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07_{-0.04}^{+0.04}$	$1.10_{-0.03}^{+0.03}$	0.74 ± 0.06	5.4
R_π^*	1.00	$1.06_{-0.04}^{+0.04}$	$1.10_{-0.03}^{+0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94_{-0.19}^{+0.19}$	$3.02_{-0.18}^{+0.17}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83_{-0.03}^{+0.03}$	$0.85_{-0.02}^{+0.01}$	0.62 ± 0.05	4.4
R_K^*	0.72	$0.76_{-0.03}^{+0.03}$	$0.79_{-0.02}^{+0.01}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50_{-0.11}^{+0.11}$	$1.53_{-0.10}^{+0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07_{-0.04}^{+0.04}$	$1.10_{-0.03}^{+0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83_{-0.03}^{+0.03}$	$0.85_{-0.02}^{+0.01}$	0.46 ± 0.06	6.3

$\bar{B} \to D^+ K^-$

$\bar{B}_s \to D_s^+ \pi^-$

Confirmed by Belle: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016 \; [1.071_{-0.016}^{+0.020}]$

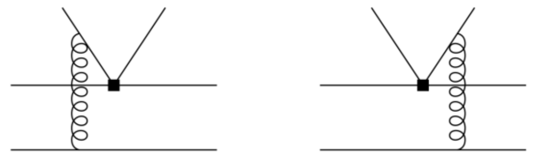
15% lower than SM

$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013 \; [1.069_{-0.016}^{+0.020}]$

Large power corrections?

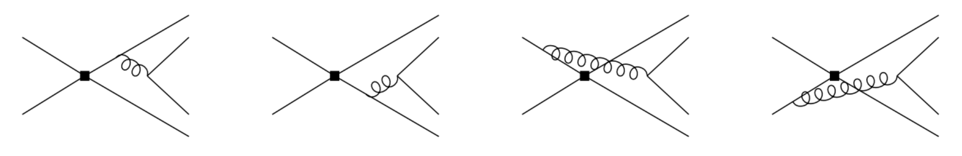
$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]
  - non-factorizable spectator interactions

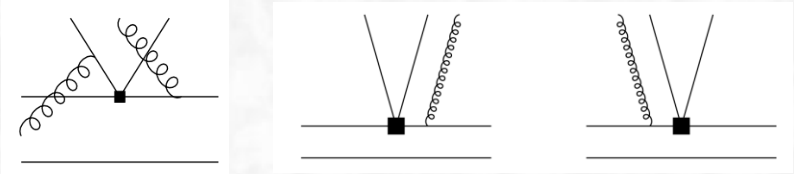


$$\frac{\Lambda_{\rm QCD}}{m_b}$$

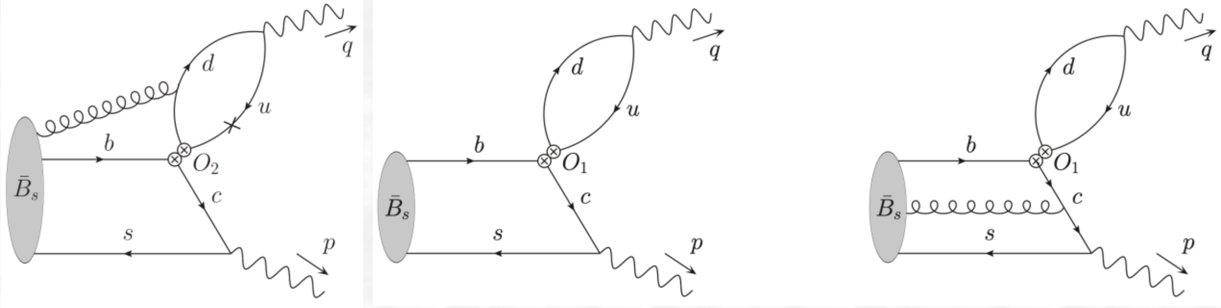
  - annihilation topologies



  - non-leading higher Fock-state contributions



- all are estimated to be power-suppressed, and no chirality-enhancement due to $(V-A) \otimes (V-A)$ structure
- very difficult to explain why the measured values of $|a_1(h)|$ several σ smaller than the SM predictions
- *must consider sub-leading power corrections more carefully*

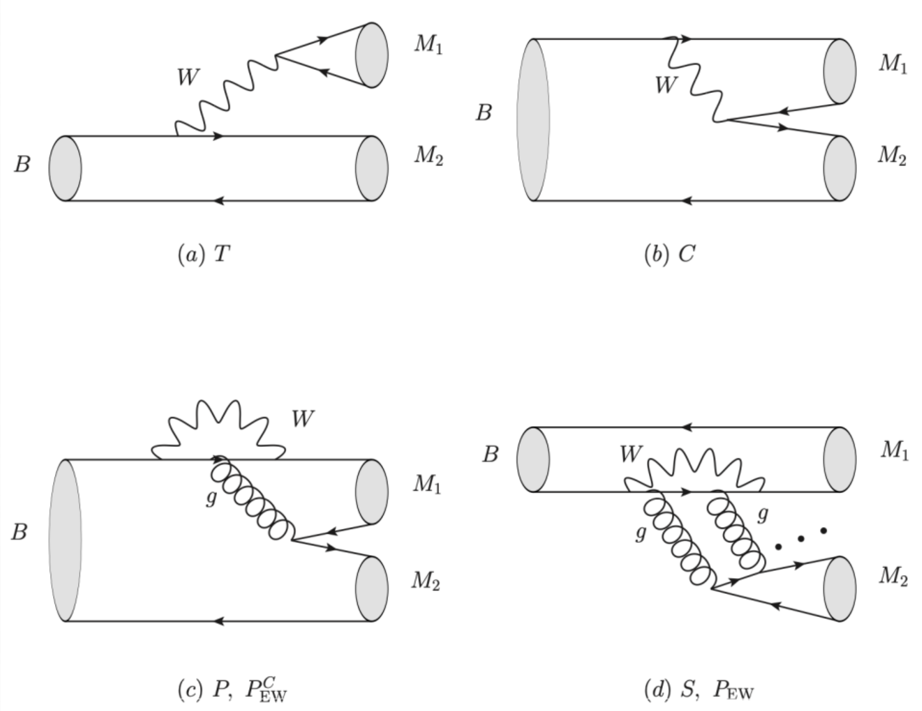


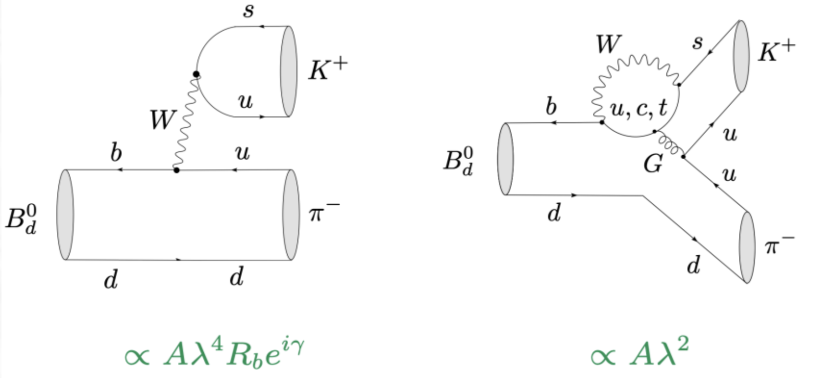
- **Non-fact. soft-gluon contributions in LCSR with B-meson LCDA:** [Maria Laura Piscopo, Aleksey V. Rusov, '23]

$$\mathrm{Br}(\bar{B}_s^0 \to D_s^+ \pi^-) = \left(2.15^{+2.14}_{-1.35}\right)\ [2.98 \pm 0.14] \times 10^{-3}$$
$$\mathrm{Br}(\bar{B}^0 \to D^+ K^-) = \left(2.04^{+2.39}_{-1.20}\right)\ [2.05 \pm 0.08] \times 10^{-4}$$

$B \to \pi K$ puzzle

- $B \to \pi K$ **decays dominated by QCD penguin diagrams**
- **For direct CPV, tree & EW penguin also crucial**





$\lambda^2 R_b = \mathcal{O}(0.02) \;\Rightarrow\;$ QCD penguins *dominate*

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0 K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\bar{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\big],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+ K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

$$A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\left(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\rm EW})\right) + \ldots$$

$$\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$$
$$= (11.3 \pm 1.2)\%$$ **differs from 0 by ~9σ**

$\boxed{\Delta A_{CP}(\pi K)\ \text{puzzle}}$

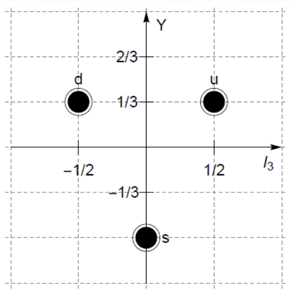
some mechanism or sub-leading power corrections

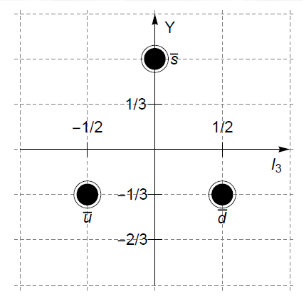
or even NP to enhance $C = \alpha_2$ or $P_{\rm EW} = \alpha_{3,\rm EW}^p$?

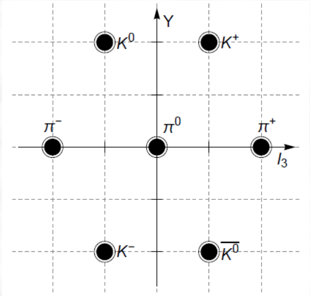
$B \to PP$ based on $SU(3)_F$ symmetry

Final-state $SU(3)_F$ decomposition:

- 3 light quarks, u, d, s, much lighter than b quark
- u, d, s = SU(3)$_F$ triplet; State $\to |\mathbf{irrep}, Y, I, I_3\rangle$
- $|u\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$, $|d\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}\right\rangle$, $|s\rangle = \left|\mathbf{3}, -\frac{2}{3}, 0, 0\right\rangle$
- $|\bar{d}\rangle = \left|\mathbf{3}^*, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$; Y = hypercharge, I = Isospin







- $\mathbf{3} \times \mathbf{3}^* = \mathbf{1} + \mathbf{8}$: *These are the 3 pions, 4 kaons, η, η'*
- $|\pi^+\rangle = |u\bar{d}\rangle = |\mathbf{8}, 0, 1, 1\rangle$ Similarly other pions and kaons are also octets

- Apply to two-body final states

$$|PP\rangle_{\rm sym} = (\mathbf{8} \times \mathbf{8})_{\rm sym} = \mathbf{1} + \mathbf{8} + \mathbf{27} = 36$$

$$|\pi^+\pi^-\rangle = \frac{1}{2}|\mathbf{1}\rangle_{0,0,0} - \sqrt{\frac{2}{5}}|\mathbf{8}\rangle_{0,0,0} - \frac{1}{2\sqrt{15}}|\mathbf{27}\rangle_{0,0,0} + \frac{1}{\sqrt{3}}|\mathbf{27}\rangle_{2,0,0}$$

Initial B states:

$$\bar{B}^0 = |\bar{d}b\rangle = |\bar{\mathbf{3}}\rangle_{\frac{1}{2},\frac{1}{2},-\frac{1}{3}}, \quad \bar{B}_s = |\bar{s}b\rangle = |\bar{\mathbf{3}}\rangle_{0,0,\frac{2}{3}}, \quad B^- = -|\bar{u}b\rangle = |\bar{\mathbf{3}}\rangle_{\frac{1}{2},-\frac{1}{2},-\frac{1}{3}}$$

Effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\lambda_u^{(s)}(C_1Q_1^{(u)} + C_2Q_2^{(u)}) + \lambda_c^{(s)}(C_1Q_1^{(c)} + C_2Q_2^{(c)}) - \lambda_t^{(s)}\sum_{i=3}^{10} C_iQ_i\right]$$

	$\overline{15}_{I=1}$	$\overline{15}_{I=0}$	$6_{I=1}$	$\overline{3}^{(a)}_{I=0}$	$\overline{3}^{(s)}_{I=0}$	$\overline{15}_{I=3/2}$	$\overline{15}_{I=1/2}$	$6_{I=1/2}$	$\overline{3}^{(a)}_{I=1/2}$	$\overline{3}^{(s)}_{I=1/2}$
$\bar{d}\,\bar{d}\,d$						$\sqrt{1/3}$	$-\sqrt{1/6}$			$\sqrt{1/2}$
$\bar{d}\,\bar{u}\,u$						$-\sqrt{1/3}$	$-\sqrt{1/24}$	$-1/2$	$1/2$	$\sqrt{1/8}$
$\bar{d}\,\bar{s}\,d$	$1/2$	$-\sqrt{1/8}$	$1/2$	$-1/2$	$\sqrt{1/8}$					

$B \to PP$ based on $SU(3)$ flavor symmetry

- **Physical amplitudes:**

$$\langle PP|\mathcal{H}_{\text{eff}}|B\rangle = \langle \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}|\mathbf{3}^* \oplus \mathbf{6} \oplus \mathbf{15}^*|\mathbf{3}\rangle = \sum_i C_i \langle \mathbf{1}, \mathbf{8}, \mathbf{27}|\mathbf{3}^*, \mathbf{6}, \mathbf{15}^*|\mathbf{3}\rangle_i$$

- $B \to PP$ decay amplitudes expressed in terms of $SU(3)_F$ RMEs & C-G coefficients, and then fit to all the data
- **Key point:** no any theoretical assumptions on RMEs $\Rightarrow$ completely rigorous on group-theoretical side
- Indep. RMEs: $V_{ub}V_{us}^* \to 5$, $V_{tb}V_{ts}^* \to 2$; ➡ 7 indep. RMEs = 13 real parameters

- **Enough data for the fit with only 7 RMEs in exact $SU(3)_F$**

$\Delta S = 0$ decays:

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to K^+\overline{K}^0$	1.31 ± 0.14	0.04 ± 0.14	
$B^+ \to \pi^+\pi^0$	5.59 ± 0.31	0.008 ± 0.035	
$B^0 \to K^0\overline{K}^0$	1.21 ± 0.16	0.06 ± 0.26	-1.08 ± 0.49
$B^0 \to \pi^+\pi^-$	5.15 ± 0.19	0.311 ± 0.030	-0.666 ± 0.029
$B^0 \to \pi^0\pi^0$	1.55 ± 0.16	0.30 ± 0.20	
$B^0 \to K^+K^-$	0.080 ± 0.015	??	??
$B_s^0 \to \pi^+K^-$	$5.90^{+0.87}_{-0.76}$	0.225 ± 0.012	
$B_s^0 \to \pi^0\overline{K}^0$	??	??	??

$\Delta S = 1$ decays:

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to \pi^+K^0$	23.52 ± 0.72	-0.016 ± 0.015	
$B^+ \to \pi^0K^+$	13.20 ± 0.46	0.029 ± 0.012	
$B^0 \to \pi^-K^+$	19.46 ± 0.46	-0.0836 ± 0.0032	
$B^0 \to \pi^0K^0$	10.06 ± 0.43	-0.01 ± 0.10	0.57 ± 0.17
$B_s^0 \to K^+K^-$	$26.6^{+3.2}_{-2.7}$	-0.17 ± 0.03	0.14 ± 0.03
$B_s^0 \to K^0\overline{K}^0$	17.4 ± 3.1	??	??
$B_s^0 \to \pi^+\pi^-$	$0.72^{+0.11}_{-0.10}$	??	??
$B_s^0 \to \pi^0\pi^0$	2.8 ± 2.8		

$B \to PP$ based on $SU(3)$ flavor symmetry

☐ State-of-the-art $SU(3)_F$ fit [Huber, Li, Malami, Tetlalmatzi-Xolocotzi, w.i.p; D. London et al., 2311.18011]

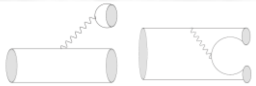
$$\begin{aligned} \lambda_u^{(q)} &: A_1 = \langle \mathbf{1} \| \mathbf{3}_1^* \| \mathbf{3} \rangle , \; A_8 = \langle \mathbf{8} \| \mathbf{3}_1^* \| \mathbf{3} \rangle , \\ \lambda_t^{(q)} &: B_1 = \langle \mathbf{1} \| \mathbf{3}_2^* \| \mathbf{3} \rangle , \; B_8 = \langle \mathbf{8} \| \mathbf{3}_2^* \| \mathbf{3} \rangle , \\ \lambda_u^{(q)} \;\&\; \lambda_t^{(q)} &: R_8 = \langle \mathbf{8} \| \mathbf{6} \| \mathbf{3} \rangle , \; P_8 = \langle \mathbf{8} \| \mathbf{15}^* \| \mathbf{3} \rangle , \\ & \quad P_{27} = \langle \mathbf{27} \| \mathbf{15}^* \| \mathbf{3} \rangle . \end{aligned}$$

$$B_1 = -\frac{4}{\sqrt{3}} \left(\frac{3}{2} PA_{tc} + P_{tc} \right) , \quad B_8 = -\sqrt{\frac{5}{3}} P_{tc} .$$

$$\begin{aligned} A_1 &= \frac{1}{2\sqrt{3}} \left(-3\tilde{T} + \tilde{C} - 8\tilde{P}_{uc} - 12\widetilde{PA}_{uc} \right) , \\ A_8 &= \frac{1}{8}\sqrt{\frac{5}{3}} \left(-3\tilde{T} + \tilde{C} - 8\tilde{P}_{uc} - 3\tilde{A} \right) , \\ R_8 &= \frac{\sqrt{5}}{4} \left(\tilde{T} - \tilde{C} - \tilde{A} \right) , \\ P_8 &= \frac{1}{8\sqrt{3}} \left(\tilde{T} + \tilde{C} + 5\tilde{A} \right) , \\ P_{27} &= -\frac{1}{2\sqrt{3}} \left(\tilde{T} + \tilde{C} \right) . \end{aligned}$$

$\vert\tilde{T}'/\tilde{T}\vert$	$\vert\tilde{C}'/\tilde{C}\vert$	$\vert\tilde{P}'_{uc}/\tilde{P}_{uc}\vert$	$\vert\tilde{A}'/\tilde{A}\vert$	$\vert P\tilde{A}'_{uc}/P\tilde{A}_{uc}\vert$
12±4	6.6±2.2	16±22	14±13	10±13

$\Delta S = 0$ fit:



$\vert\tilde{T}\vert$	$\vert\tilde{C}\vert$	$\vert\tilde{P}_{uc}\vert$	$\vert\tilde{A}\vert$	$\vert P_{tc}\vert$
4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5	0.8 ± 0.4

$\Delta S = 1$ fit:

$\vert\tilde{T}'\vert$	$\vert\tilde{C}'\vert$	$\vert\tilde{P}'_{uc}\vert$	$\vert\tilde{A}'\vert$	$\vert P'_{tc}\vert$
48 ± 14	41 ± 14	48 ± 15	81 ± 28	0.78 ± 0.16

- ✓ $\left|\frac{\tilde{C}}{\tilde{T}}\right| = 1.65$ (ΔS=0), 0.85 (ΔS = 1), 1.23 ($SU(3)_F$) vs $0.13 \leq \left|\frac{\tilde{C}}{\tilde{T}}\right| = 0.23 \leq 0.43$ based on QCDF
- ✓ for combined ΔS = 0 & ΔS = 1 decays: very poor fit, with 3.6σ disagreement with the $SU(3)_F$ limit
- ✓ a 1000% $SU(3)_F$-breaking effect required, much large than naive expectation of $f_K/f_\pi - 1 \sim 20\%$

☐ More precise measurements, especially of the missing observables (e.g. $B_s^0 \to K^0\bar{K}^0$ and $B_s^0 \to \pi^0\bar{K}^0$) may help to figure out true dynamical mechanism behind charmless B decays

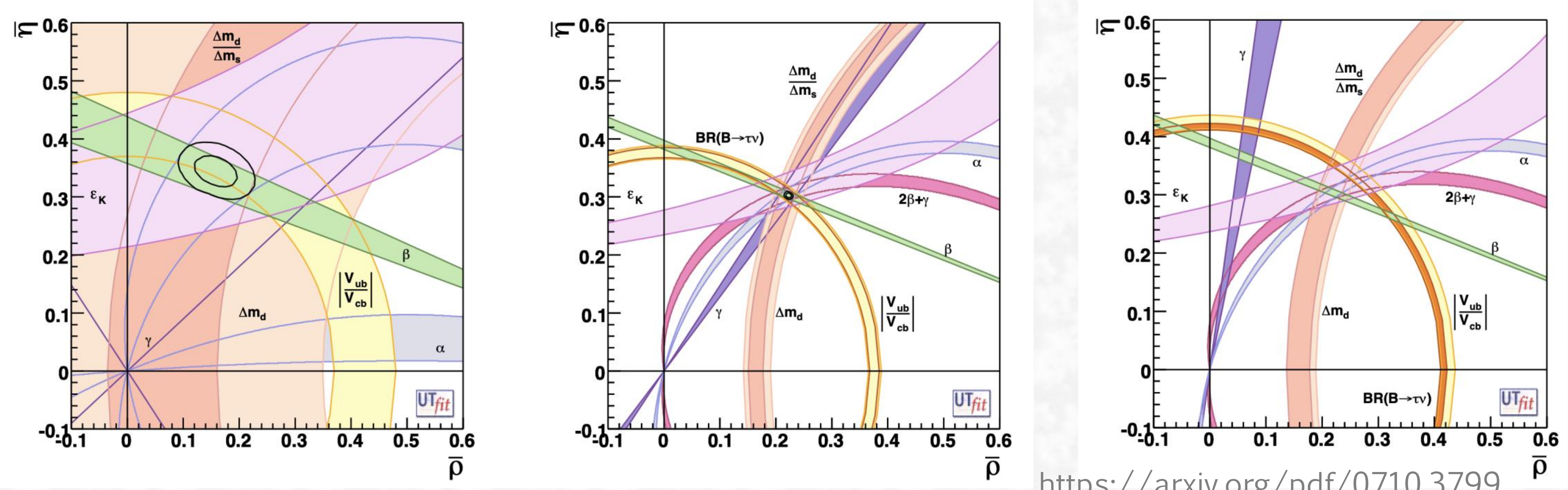
Summary

- With exp. and theor. progress, we are now entering a precision era for flavor physics

- Most consistent with SM but also several deviations observed → NP signals?

- More precise exp. data, & more precise theor. predictions, & LQCD inputs needed

many opportunities to explore SM & BSM physics in heavy flavor physics

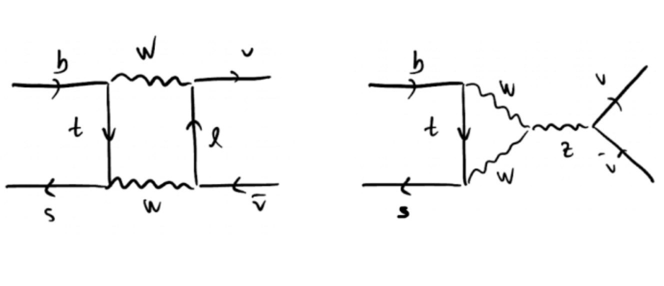


https://arxiv.org/pdf/0710.3799

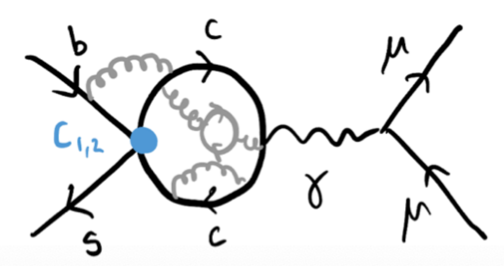
back-up

Rare FCNC B decays

☐ Another interesting FCNC decays: $B \to K^{(*)}\nu\bar{\nu}$



- there are no **photon-penguin diagrams**
- theoretically cleaner than $b \to s\ell\ell$ decays due to absence of **LD $c\bar{c}$-loop contributions**



☐ State-of-the-art SM prediction:

- **Effective Hamiltonian** in the SM:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi}\sum_i C_L^{\rm SM}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Li}) + \text{h.c.}\,,$$

$\lambda_t = V_{tb}V_{ts}^*$

- **Short-distance** contributions known to **good precision**:

$$C_L^{\rm SM} = -X_t/\sin^2\theta_W$$
$$= -6.32(7)$$

Including NLO QCD and two-loop EW contributions:

$$X_t = 1.462(17)(2)$$

[Buchala et al. '93, '99], [Misiak et al. '99], [Brod et al. '10]

$$\langle K^{(*)}|\bar{s}_L\gamma^\mu b_L|B\rangle = \sum_a K_a^\mu\,\mathcal{F}_a(q^2)$$

Form-factors (e.g., LQCD)

$$|V_{tb}V_{ts}^*| = |V_{cb}|\,(1+\mathcal{O}(\lambda^2))$$

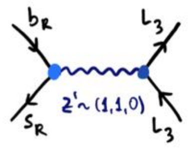
$$\mathcal{B}(B\to K\nu\bar{\nu})^{\rm SM}/|\lambda_t|^2 = \begin{cases}(1.33\pm0.04)_{K_S}\times10^{-3}\\(2.87\pm0.10)_{K^+}\times10^{-3}\end{cases}$$

≃ 3% uncertainty

$$\mathcal{B}(B\to K^*\nu\bar{\nu})^{\rm SM}/|\lambda_t|^2 = \begin{cases}(5.9\pm0.8)_{K^{*0}}\times10^{-3}\\(6.4\pm0.9)_{K^{*+}}\times10^{-3}\end{cases}$$

≃ 15% uncertainty

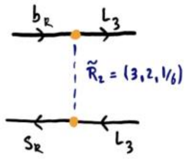
- Z':



$\mathcal{L}_{Z'} \supset g_{ij}^{\psi}(\bar{\psi}_i\gamma^\mu\psi_j)Z'_\mu$

sensitive to NP

- LQs:



$\mathcal{L}_{\tilde{R}_2} \supset y_{ij}^R(\bar{d}_{Ri}\tilde{R}_2 i\tau_2 L_j) + \text{h.c.}$