



第5届LHCb前沿物理研讨会

Study of multi-quark states with strangeness

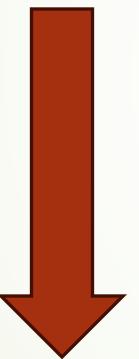
- ▶ Chu-Wen Xiao (肖褚文)
- ▶ Guangxi Normal University (广西师范大学)
- ▶ Collaborators: Eulogio Oset , Juan Nieves , Albert Feijoo
Bing-Song Zou (邹冰松), Jia-Jun Wu (吴佳俊), Wei-Fei Wang (王文飞)



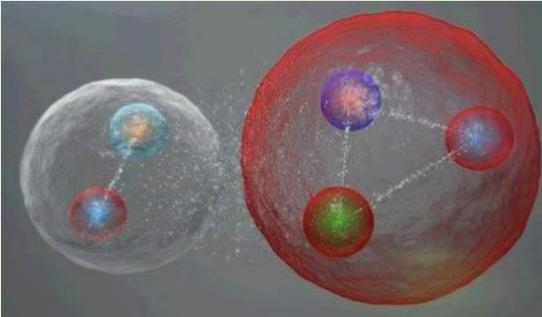
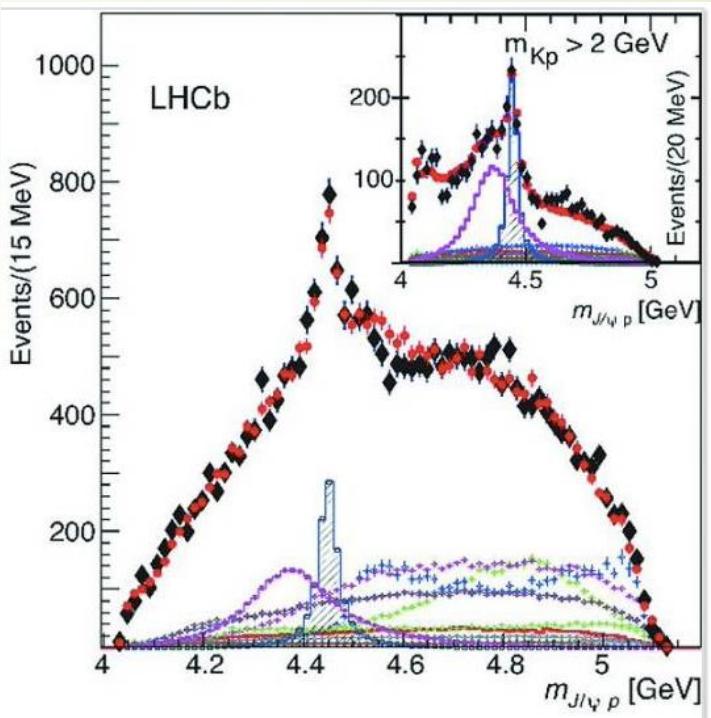
祝贺LHCb中国组！

2015年，两个
Pc态的发现

十年再磨一剑



2025年，重子
CPV的发现



$$\Lambda_b^0 \rightarrow p K^- \pi^+ \pi^-$$

$$\mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

5.2σ



Outline

1. Introduction
2. Formalism
3. Pentaquark states P_{cs}
4. Tetraquark states
5. Summary



§1. Introduction

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

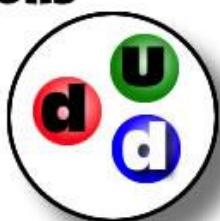
anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations $(q\bar{q}q)$, $(q\bar{q}qq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(q\bar{q}\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\bar{q}q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**.

Quark model

baryons



proton
up, up, down



neutron
up, down, down

mesons



pion
up & anti-down



kaon 0
down & anti-strange

Exotic States

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

CERN LIBRARIES, GENEVA

G. Zweig *)

CERN - Geneva

In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from \overline{AAAAA} , $\overline{AAAAAAA}$, etc., where \overline{A} denotes an anti-ace. Similarly, mesons could be formed from \overline{AA} , \overline{AAA} etc. For the low mass mesons and baryons we will assume the simplest possibilities, \overline{AA} and AAA , that is, "deuces and treys".

Pentaquark



H-dibaryon

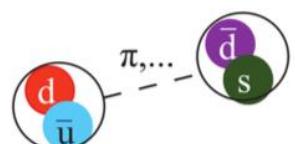


Tetraquark



diquark-diquark- antiquark

Molecule

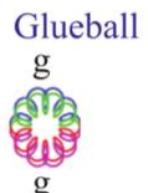


diquark-diquark- diquark

Hybrid



diquark-diantiquark



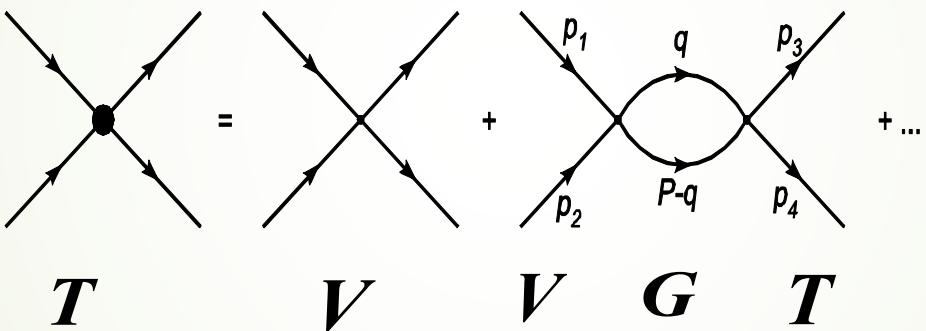


- H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, Phys. Rept. 639, 1 (2016)
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- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
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- Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019)
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§2. Formalism

► **Coupled Channel Unitary Approach**: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V G T, \quad T = [1 - V G]^{-1} V$$



D. L. Yao, L. Y. Dai, H. Q.
Zheng and Z. Y. Zhou, Rept.
Prog. Phys. 84, 076201 (2021)

where **V** matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438

E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99

J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263



G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

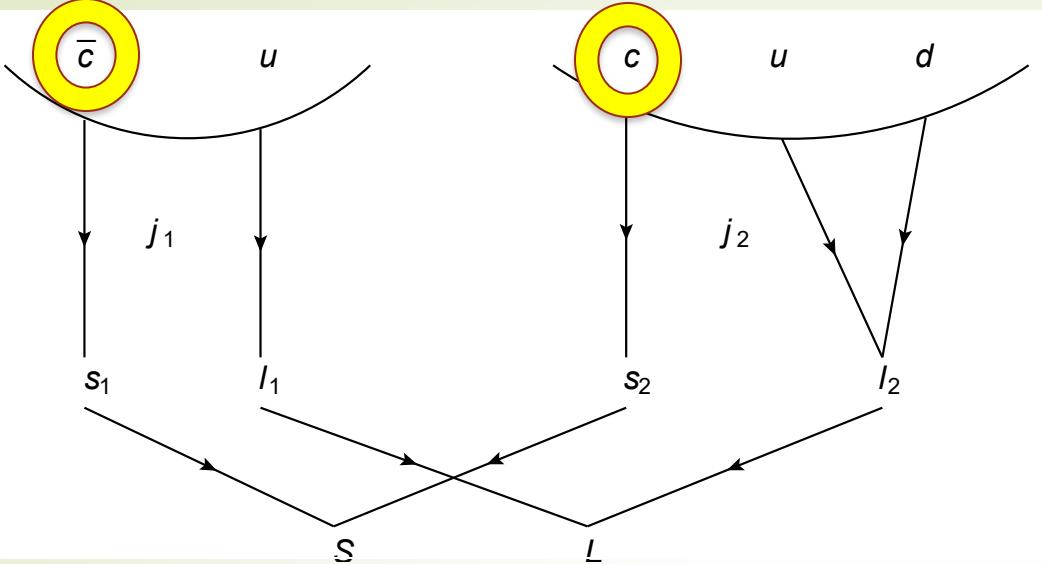
The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\text{Im } T_{ij} = T_{in} \sigma_{nn} T_{nj}^*$$

$$\sigma_{nn} \equiv \text{Im } G_{nn} = - \frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^I(s) + i \frac{q_{cm}}{4\pi\sqrt{s}}$$



Considering the heavy quark spin symmetry

$$\bar{D} \xrightarrow{\text{red arrow}} \bar{D}^* \quad \Sigma_c \xrightarrow{\text{red arrow}} \Sigma_c^*$$

$$\Xi_c \xrightarrow{\text{red arrow}} \Xi_c^*$$

$$(\ell'_M \ell'_B S') \langle S'_{c\bar{c}} \mathcal{L}'; J' | H^{QCD} | S_{c\bar{c}} \mathcal{L}; J, \rangle_{(\ell_M \ell_B S)} = \delta_{JJ'} \delta_{S'_{c\bar{c}} S_{c\bar{c}}} \delta_{\mathcal{L}\mathcal{L}'} \langle \ell'_M \ell'_B S' || H^{QCD} || \ell_M \ell_B S \rangle_{\mathcal{L}}$$

$$|\ell_M s_M j_M S; \ell_B s_B j_B; J\rangle = \sum_{\mathcal{L}, S_{c\bar{c}}} [(2S_{c\bar{c}} + 1)(2\mathcal{L} + 1)(2j_M + 1)(2j_B + 1)]^{\frac{1}{2}} \times \left\{ \begin{array}{ccc} \ell_M & \ell_B & \mathcal{L} \\ s_M & s_B & S_{c\bar{c}} \\ j_M & j_B & J \end{array} \right\} |\mathcal{L} S_{c\bar{c}}; J\rangle_{(\ell_M \ell_B S)}$$



§3. Pentaquark states

Pcs states

PRL 105, 232001 (2010)

PHYSICAL REVIEW LETTERS

week ending
3 DECEMBER 2010

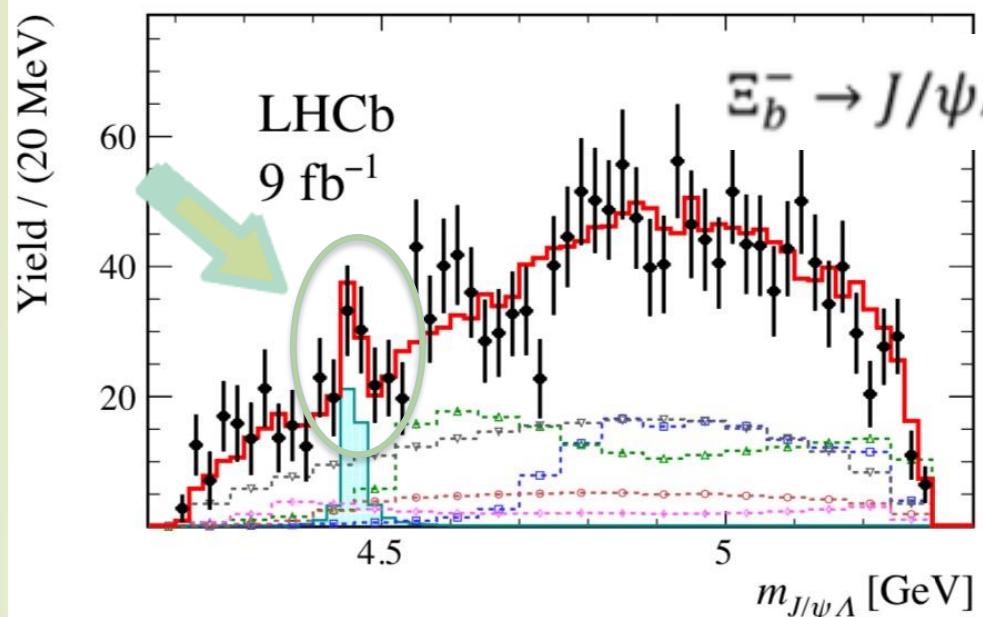
Prediction of Narrow N^* and Λ^* Resonances with Hidden Charm above 4 GeV

Jia-Jun Wu,^{1,2} R. Molina,^{2,3} E. Oset,^{2,3} and B. S. Zou^{1,3}

- J. J. Wu, R. Molina, E. Oset and B. S. Zou, **Phys. Rev. Lett.** **105** (2010) 232001.
- J. J. Wu, R. Molina, E. Oset and B. S. Zou, **Phys. Rev. C** **84** (2011) 015202.
- H. X. Chen, L. S. Geng, W. H. Liang, E. Oset, E. Wang and J. J. Xie, **Phys. Rev. C** **93** (2016) 065203.
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- E. Santopinto and A. Giachino, **Phys. Rev. D** **96** (2017) 014014.
- C. W. Shen, J. J. Wu and B. S. Zou, **Phys. Rev. D** **100** (2019) 056006.
- CWX, J. Nieves and E. Oset, Phys. Lett. B** **799** (2019) 135051.
- B. Wang, L. Meng and S. L. Zhu, **Phys. Rev. D** **101** (2020) 034018.

Findings for the Pcs state

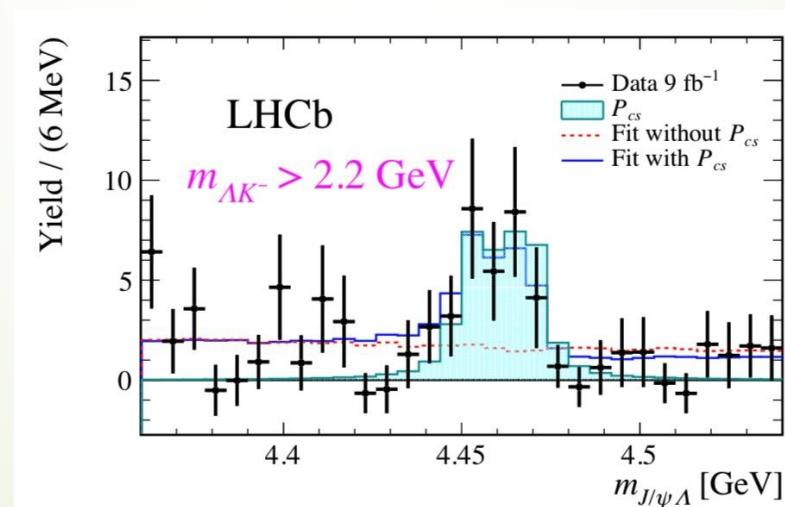
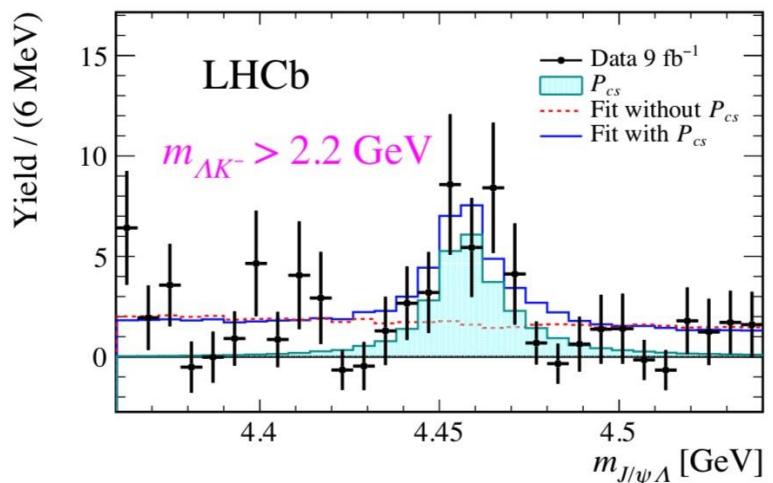
H.-X. Chen, L.-S. Geng, W.-H. Liang, E. Oset, E. Wang, J.-J. Xie,
Phys. Rev. C 93 (2016) 065203



R. Aaij et al. (LHCb Collaboration),
Sci. Bull. 66 (2021) 1278-1287

State	M_0 [MeV]	Γ_0 [MeV]	J^P ?
$P_{cs}(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	

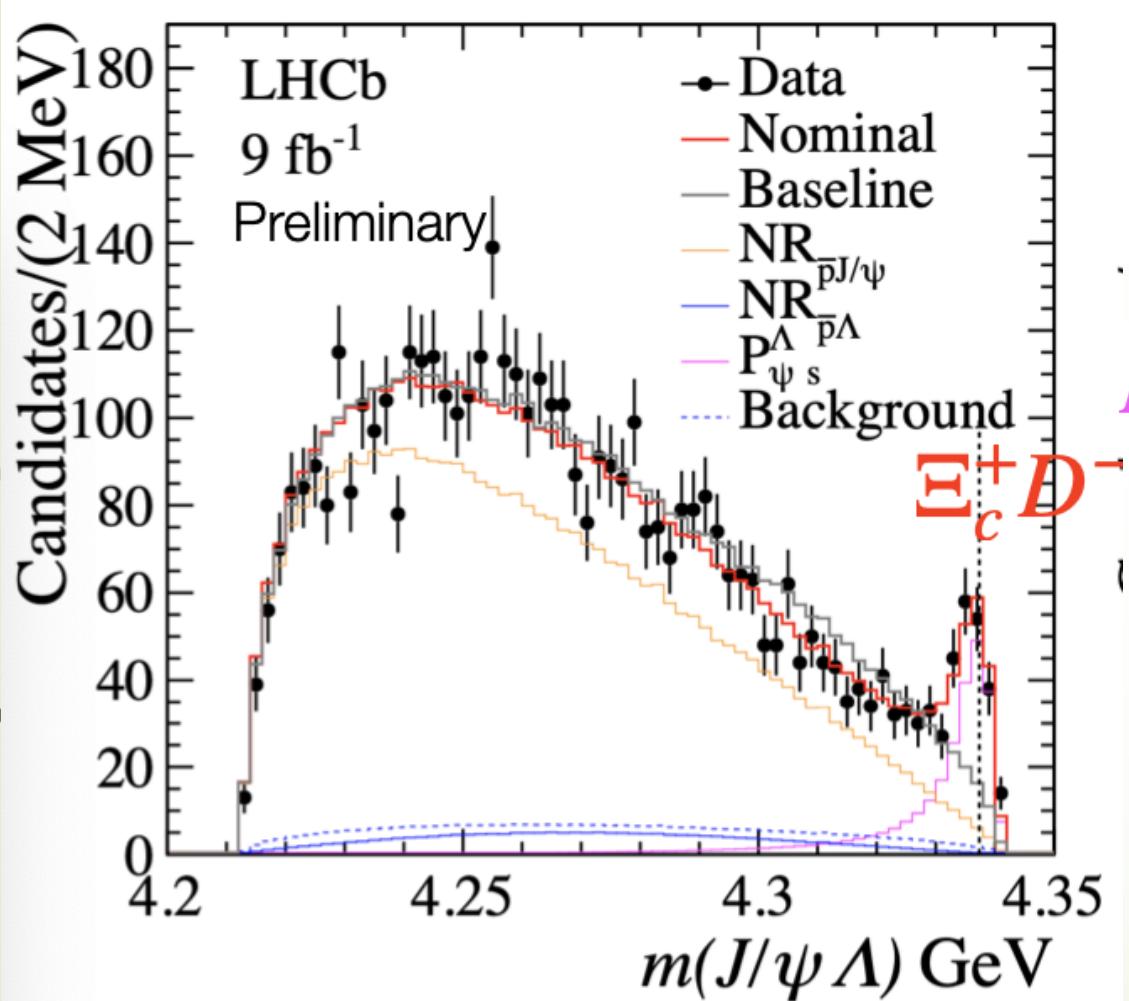
mass difference of about 6 MeV



B. Wang, L. Meng and
S. L. Zhu, Phys. Rev. D
101 (2020) 034018.

The data cannot confirm or refute the two-peak hypothesis.

$$B^- \rightarrow (J/\psi \Lambda) \bar{p}$$



$$m(P_{\psi s}^{\Lambda}) = 4338.3 \pm 0.7 \pm 0.4 \text{ MeV}$$

$$\Gamma(P_{\psi s}^{\Lambda}) = 7.0 \pm 1.2 \pm 1.3 \text{ MeV}$$

$P_{\psi s}^{\Lambda}(4338)^0$

Significance: $> 10\sigma$

LHCb, Phys. Rev. Lett. 131,
031901 (2023)

Recalling our prediction for Pcs

i) $J = 1/2, I = 0$

$\eta_c \Lambda, J/\psi \Lambda, \bar{D} \Xi_c, \bar{D}_s \Lambda_c, \bar{D} \Xi'_c, \bar{D}^* \Xi_c, \bar{D}_s^* \Lambda_c, \bar{D}^* \Xi'_c, \bar{D}^* \Xi_c^*$.

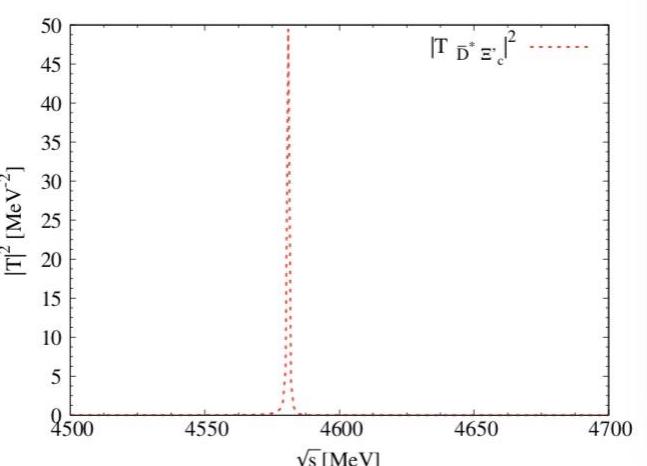
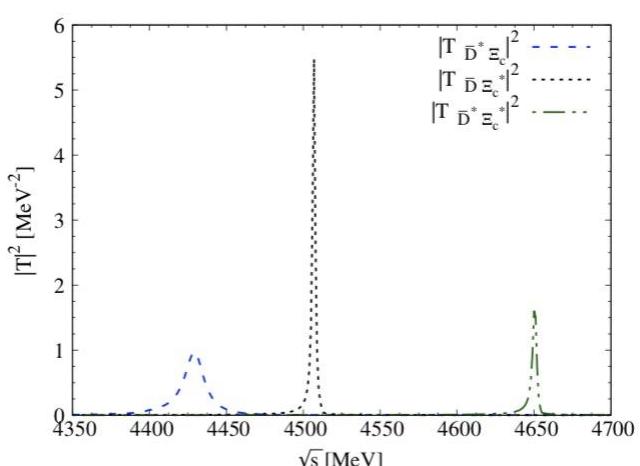
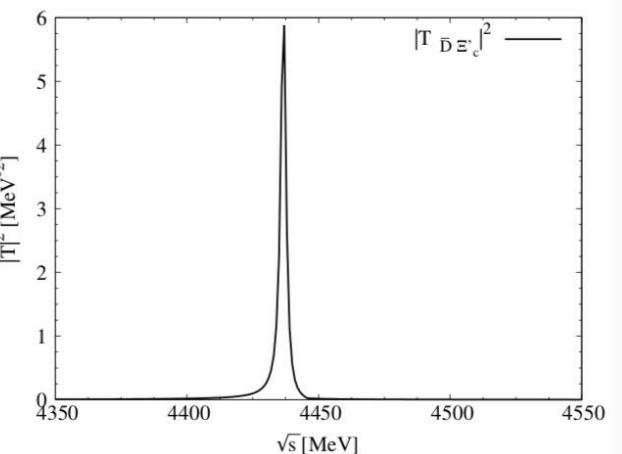
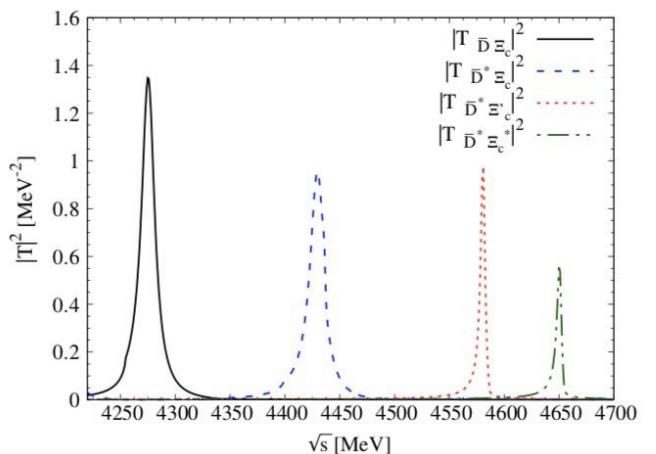


ii) $J = 3/2, I = 0$

$J/\psi \Lambda, \bar{D}^* \Xi_c, \bar{D}_s \Lambda_c, \bar{D}^* \Xi'_c, \bar{D} \Xi_c^*, \bar{D}^* \Xi_c^*$.

$$a(\mu = 1 \text{ GeV}) = -2.09$$

CWX, J. Nieves and
E. Oset, Phys. Rev. D
100 (2019) 014021



CWX, J. Nieves and E. Oset,
Phys. Lett. B 799 (2019) 135051.

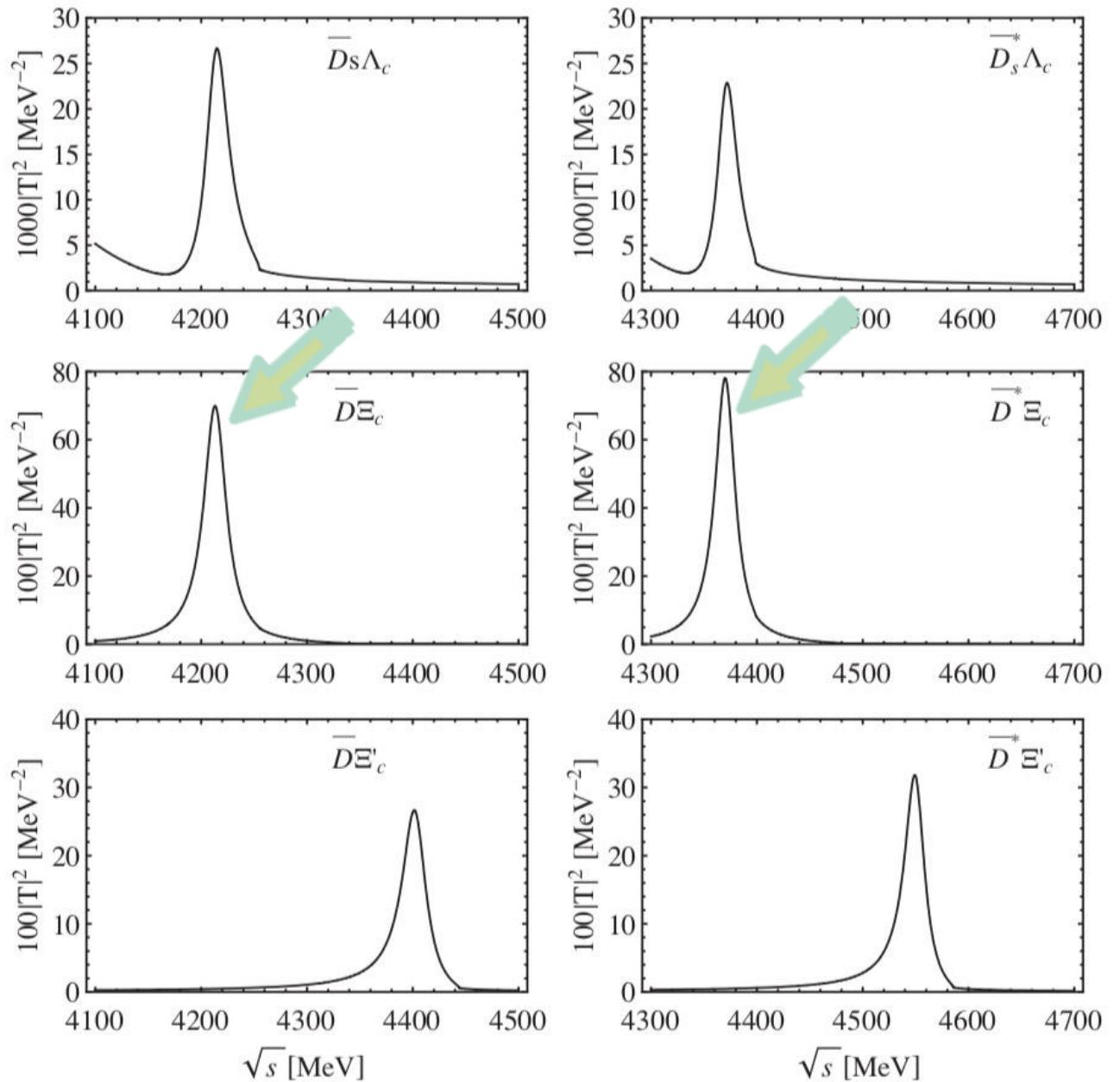
- $J = 1/2, I = 0$

$$a_\mu(\mu = 1 \text{ GeV}) = -1.94$$



Chan.	$\eta_c\Lambda$	$J/\psi\Lambda$	$\bar{D}\Xi_c$	$\bar{D}_s\Lambda_c$	$\bar{D}\Xi'_c$	$\bar{D}^*\Xi_c$	$\bar{D}_s^*\Lambda_c$	$\bar{D}^*\Xi'_c$	$\bar{D}^*\Xi_c^*$
Thres.	4099.58	4212.58	4336.61	4254.80	4445.34	4477.92	4398.66	4586.66	4654.48
	$4310.53 + i8.23$	$P_{\psi s}^\Lambda(4338)^0$							
$ g_i $	0.15	0.27	2.33	0.69	0.00	0.04	0.09	0.01	0.02
Γ_i	0.57	1.18	—	13.86	—	—	—	—	—
Br.	3.47%	7.16%	—	84.21%	—	—	—	—	—
	$4445.12 + i0.19$								
$ g_i $	0.10	0.06	0.00	0.00	0.72	0.08	0.04	0.01	0.01
Γ_i	0.29	0.08	0.00	0.00	—	—	0.04	—	—
Br.	74.74%	21.22%	0.01%	0.01%	—	—	10.62%	—	—
	$4459.07 + i6.89$	$P_{cs}(4459)$							
$ g_i $	0.22	0.13	0.00	0.00	0.07	2.16	0.61	0.03	0.02
Γ_i	1.59	0.46	0.00	0.00	0.01	—	11.14	—	—
Br.	11.57%	3.31%	0.00%	0.00%	0.70%	—	80.86%	—	—
	4586.66?								
$ g_i $	—	—	—	—	—	—	—	—	—
	4654.48?								
$ g_i $	—	—	—	—	—	—	—	—	—

- $J = 3/2, I = 0$

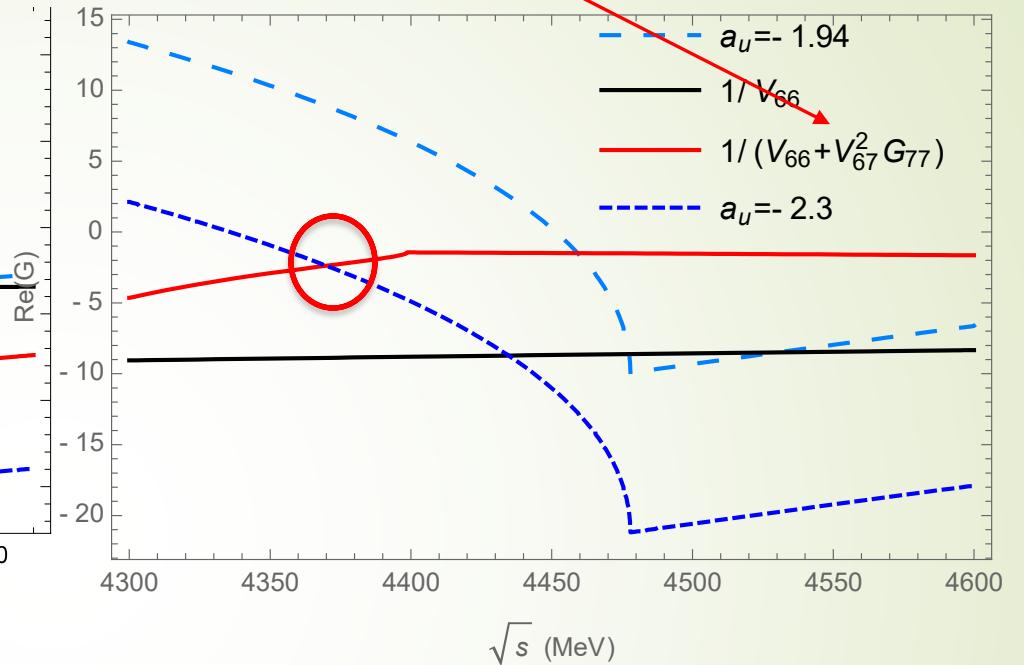
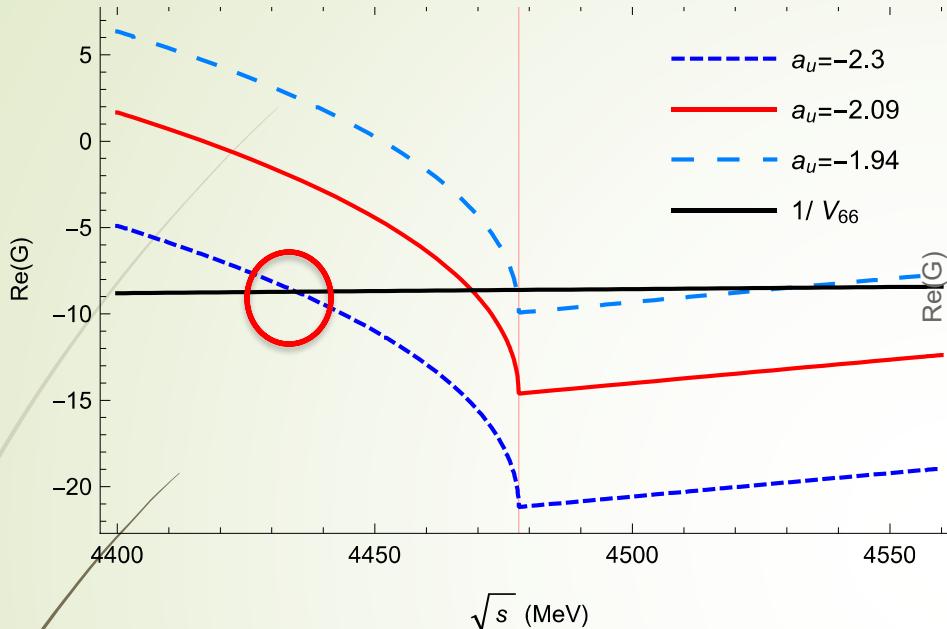


$\mu = 1000 \text{ MeV}$

$a(\mu) = -2.3$

for the $\bar{D}^* \Xi_c$ channel

$\bar{D}_s^* \Lambda_c$



	$J/\psi \Lambda$	$\bar{D}_s^* \Lambda_c$	$\bar{D}^* \Xi_c$	$\bar{D}^* \Xi'_c$
$J/\psi \Lambda$	0	$-\frac{1}{\sqrt{3}}\lambda$	$\frac{1}{\sqrt{6}}\lambda$	$-\frac{1}{\sqrt{2}}\lambda$
$\bar{D}_s^* \Lambda_c$		0	$-\sqrt{2}$	0
$\bar{D}^* \Xi_c$			-1	0
$\bar{D}^* \Xi'_c$				-1

$\bar{D}\Xi_c$ $\bar{D}_s\Lambda_c$



Noting the results for P_c

$\eta_c N$	$J/\psi N$	$\bar{D}\Lambda_c$	$\bar{D}\Sigma_c$	$\bar{D}^*\Lambda_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Sigma_c^*$
μ_1	0	$\frac{\mu_{12}}{2}$	$\frac{\mu_{13}}{2}$	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{13}$
0	μ_1	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	$-\frac{\mu_{12}}{2}$	$\frac{5\mu_{13}}{6}$	$\frac{\sqrt{2}\mu_{13}}{3}$
$\frac{\mu_{12}}{2}$	$\frac{\sqrt{3}\mu_{12}}{2}$	μ_2	0	0	$\frac{\mu_{23}}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{23}$
$\frac{\mu_{13}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	0	$\frac{1}{3}(2\lambda_2 + \mu_3)$	$\frac{\mu_{23}}{\sqrt{3}}$	$\frac{2(\lambda_2 - \mu_3)}{3\sqrt{3}}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_3 - \lambda_2)$
$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{12}}{2}$	0	$\frac{\mu_{23}}{\sqrt{3}}$	μ_2	$-\frac{2\mu_{23}}{3}$	$\frac{\sqrt{2}\mu_{23}}{3}$
$-\frac{\mu_{13}}{2\sqrt{3}}$	$\frac{5\mu_{13}}{6}$	$\frac{\mu_{23}}{\sqrt{3}}$	$\frac{2(\lambda_2 - \mu_3)}{3\sqrt{3}}$	$-\frac{2\mu_{23}}{3}$	$\frac{1}{9}(2\lambda_2 + 7\mu_3)$	$\frac{1}{9}\sqrt{2}(\mu_3 - \lambda_2)$
$\sqrt{\frac{2}{3}}\mu_{13}$	$\frac{\sqrt{2}\mu_{13}}{3}$	$\sqrt{\frac{2}{3}}\mu_{23}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_3 - \lambda_2)$	$\frac{\sqrt{2}\mu_{23}}{3}$	$\frac{1}{9}\sqrt{2}(\mu_3 - \lambda_2)$	$\frac{1}{9}(\lambda_2 + 8\mu_3)$

$$\mu_1 = 0, \quad \mu_{23} = 0, \quad \lambda_2 = \mu_3, \quad \mu_{13} = -\mu_{12},$$

$$\mu_2 = \frac{1}{4f^2}(k^0 + k'^0), \quad \mu_3 = -\frac{1}{4f^2}(k^0 + k'^0),$$

CWX, J. Nieves and E. Oset, Phys. Rev. D 100 (2019) 014021



Recalling the results in P_c states

states [MeV]	Widths [MeV]	Main channel	J^P	Experimental state
4306.4	15.2	$\bar{D}\Sigma_c$	$1/2^-$	$P_c(4312)$
4452.9	23.4	$\bar{D}^*\Sigma_c$	$1/2^-$	$P_c(4440)$
4452.5	3.0	$\bar{D}^*\Sigma_c$	$3/2^-$	$P_c(4457)$

$$a(\mu = 1 \text{ GeV}) = -2.09$$

M. Z. Liu, Y. W. Pan, F. Z. Peng, M. S. Sanchez, L. S. Geng, A. Hosaka, and M. P. Valderrama, Phys. Rev. Lett. 122 (2019) 242001

$$\begin{aligned} M_{P_{c1}} &= 4311.9 \pm 0.7^{+6.8}_{-0.6}, & \Gamma_{P_{c1}} &= 9.8 \pm 2.7^{+3.7}_{-4.5}, \\ M_{P_{c2}} &= 4440.3 \pm 1.3^{+4.1}_{-4.7}, & P_c : \bar{D}\Sigma_c, \bar{D}\Lambda_c \text{ and } \bar{D}^*\Sigma_c, \bar{D}^*\Lambda_c \\ M_{P_{c3}} &= 4457.3 \pm 0.6^{+4.1}_{-1.7}, & \Gamma_{P_{c3}} &= 6.4 \pm 2.0^{+5.7}_{-1.9}. \end{aligned}$$

CWX, J. Nieves and E. Oset, Phys. Rev. D 100 (2019) 014021

Updated investigation



PB($1/2^+$) sector with $J^P = \frac{1}{2}^-$

Poles		$\eta_c \Lambda$	$\bar{D}_s \Lambda_c$	$\bar{D} \Xi_c$	$\bar{D} \Xi'_c$
$4198.94 + i0.11$	g_i	$0.12 - i0.00$	$3.01 - i0.01$	$4.85 + i0.01$	$0.01 - i0.03$
	$g_i G_i^{II}$	$-0.35 + i1.01$	$-19.24 + i0.05$	$-20.35 - i0.07$	$-0.03 + i0.09$
$\rightarrow 4422.79 + i7.75$	g_i	$0.71 - i0.08$	$0.05 + i0.00$	$-0.06 + i0.04$	$2.79 - i0.35$
	$g_i G_i^{II}$	$1.14 + i10.71$	$0.76 + i1.82$	$-0.43 - i1.53$	$-26.54 + i0.64$

$P_{cs}(4459)$

VB($1/2^+$) sector with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$

Poles		$J/\psi \Lambda$	$\bar{D}_s^* \Lambda_c$	$\bar{D}^* \Xi_c$	$\bar{D}^* \Xi'_c$
$\rightarrow 4337.98 + i0.12$	g_i	$0.11 - i0.00$	$3.17 - i0.01$	$5.07 + i0.01$	$0.00 - i0.04$
	$g_i G_i^{II}$	$-0.13 + i1.07$	$-18.57 + i0.06$	$-19.94 - i0.07$	$-0.01 + i0.10$
$4565.73 + i15.58$	g_i	$0.70 - i0.16$	$0.09 - i0.03$	$-0.10 + i0.09$	$2.84 - i0.72$
	$g_i G_i^{II}$	$6.24 + i21.04$	$2.07 + i3.40$	$-1.37 - i2.81$	$-26.31 + i1.15$

$P_c : \bar{D} \Sigma_c, \bar{D} \Lambda_c$ and $\bar{D}^* \Sigma_c, \bar{D}^* \Lambda_c$

$P_{cs} : \bar{D} \Xi_c, \bar{D}_s \Lambda_c$ and $\bar{D}^* \Sigma_c, \bar{D}_s^* \Lambda_c$

A. Feijoo, W. F. Wang, **CWX**, J. J. Wu, E. Oset, J. Nieves, B. S. Zou, Phys. Lett. B 839 (2023) 137760

§4. Tetraquark states

PHYSICAL REVIEW LETTERS 127, 082001 (2021)

Editors' Suggestion

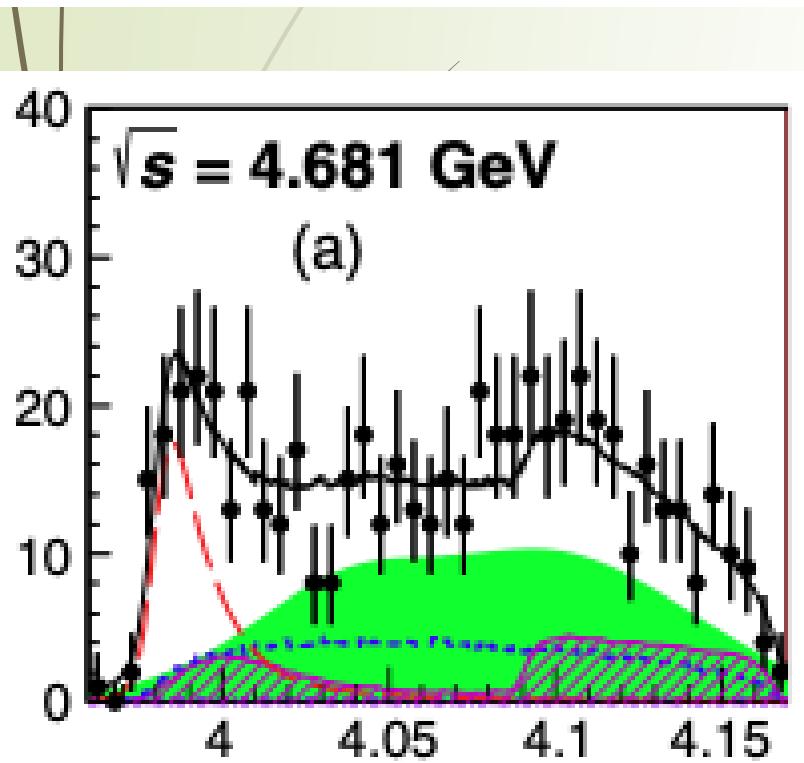
BESIII Collaboration

PHYSICAL REVIEW LETTERS 126, 102001 (2021)

Editors' Suggestion

Featured in Physics

Observation of a Near-Threshold Structure in the K^+ Recoil-Mass Spectra in $e^+e^- \rightarrow K^+(D_s^- D^{*0} + D_s^{*-} D^0)$

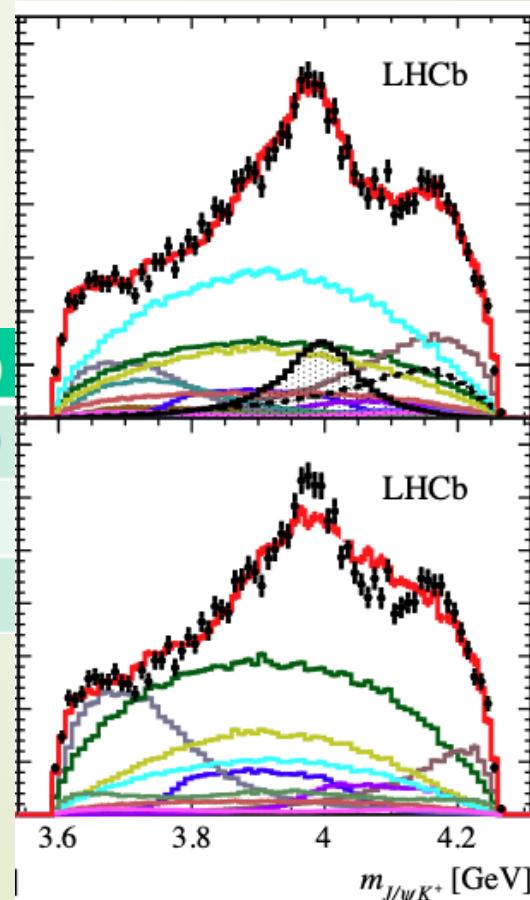


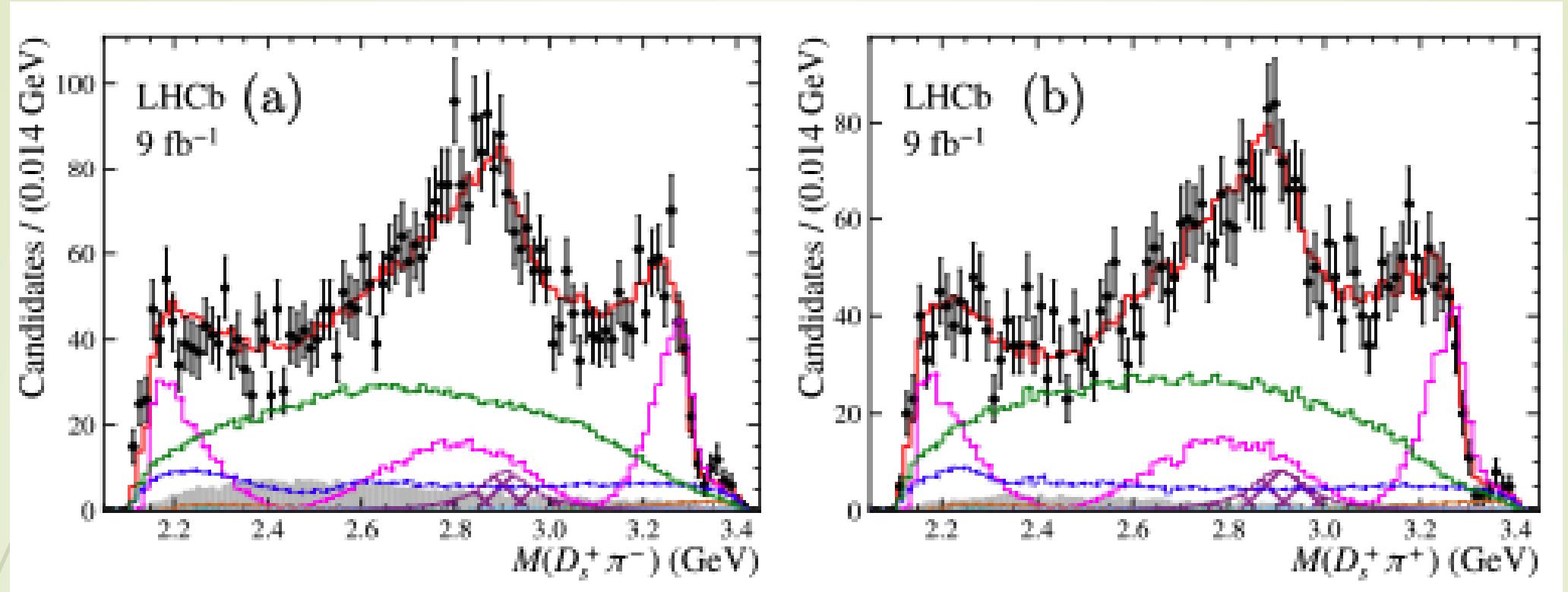
State	Signif.	JP	Mass (MeV)	Width (MeV)
$Z_{cs}(3985)$	5.3σ	??	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$
$Z_{cs}(4000)$	15σ	$1+$	$4003 \pm 6^{+4}_{-14}$	$131 \pm 15 \pm 26$
$Z_{cs}(4220)$	5.9σ	$1+$	$4216 \pm 24^{+43}_{-30}$	$233 \pm 52^{+97}_{-73}$

Observation of New Resonances Decaying to $J/\psi K^+$ and $J/\psi \phi$

R. Aaij *et al.**

(LHCb Collaboration)





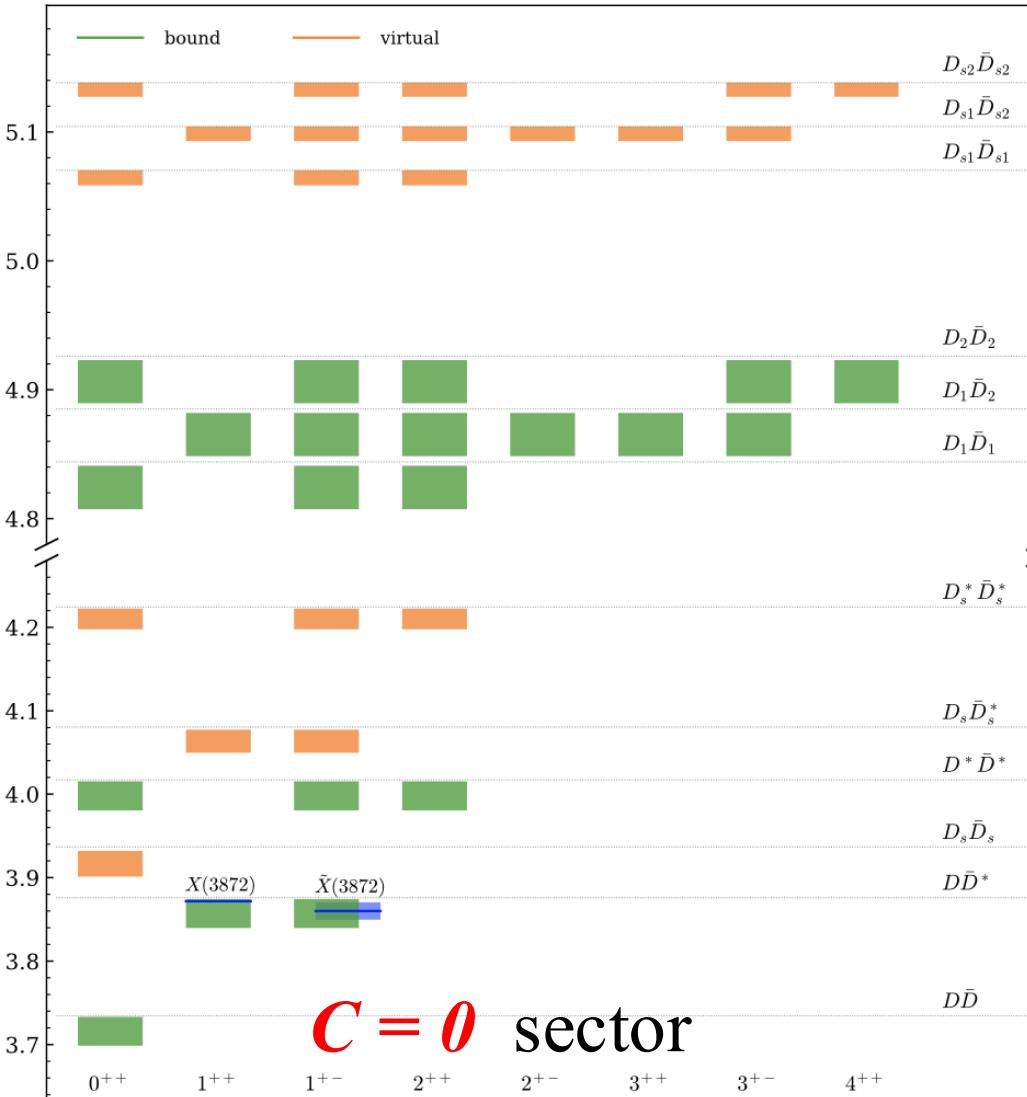
$T_{c\bar{s}0}^a(2900)^0 : M = 2.892 \pm 0.014 \pm 0.015 \text{ GeV},$
 $\Gamma = 0.119 \pm 0.026 \pm 0.013 \text{ GeV},$
 $T_{c\bar{s}0}^a(2900)^{++} : M = 2.921 \pm 0.017 \pm 0.020 \text{ GeV},$
 $\Gamma = 0.137 \pm 0.032 \pm 0.017 \text{ GeV},$

Xiang-Kun Dong^{1,2,*}, Feng-Kun Guo^{1,2,†} and Bing-Song Zou^{1,2,3‡}

A survey of heavy-antiheavy hadronic molecules

Dong Xiang-Kun^{1,2*}, Guo Feng-Kun^{1,2†}, Zou Bing-Song^{1,2,3‡}

System	E_{th}	J^{PC}	Pole (0.5)	Pole (1.0)
$D\bar{D}$	3734	0^{++}	(1, 1.31)	(1, 35.8)
$D\bar{D}^*$	3876	$1^{+\pm}$	(1, 1.56)	(1, 36.2)
$D_s\bar{D}_s$	3937	0^{++}	(2, 35.5)	(2, 4.72)
$D^*\bar{D}^*$	4017	$(0, 2)^{++}, 1^{+-}$	(1, 1.82)	(1, 36.6)
$D_s\bar{D}_s^*$	4081	$1^{+\pm}$	(2, 31.0)	(2, 3.15)
$D_s^*\bar{D}_s^*$	4224	$(0, 2)^{++}, 1^{+-}$	(2, 26.7)	(2, 1.92)
$D\bar{D}_2$	4330	$2^{-\pm}$	(1, 2.2)	(1, 36.7)
$D_s\bar{D}_{s2}$	4537	$2^{-\pm}$	(2, 21.3)	(2, 0.713)
$D_1\bar{D}_1$	4844	$(0, 2)^{++}, 1^{+-}$	(1, 3.01)	(1, 36.7)
$D_1\bar{D}_2$	4885	$(1, 2, 3)^{+\pm}$	(1, 3.06)	(1, 36.6)
$D_2\bar{D}_2$	4926	$(0, 2, 4)^{++}, (1, 3)^{+-}$	(1, 3.1)	(1, 36.6)
$D_{s1}\bar{D}_{s1}$	5070	$(0, 2)^{++}, 1^{+-}$	(2, 11.7)	(1, 0.074)
$D_{s1}\bar{D}_{s2}$	5104	$(1, 2, 3)^{+\pm}$	(2, 11.3)	(1, 0.104)
$D_{s2}\bar{D}_{s2}$	5138	$(0, 2, 4)^{++}, (1, 3)^{+-}$	(2, 10.9)	(1, 0.139)





PHYSICAL REVIEW D **105**, L051103 (2022)

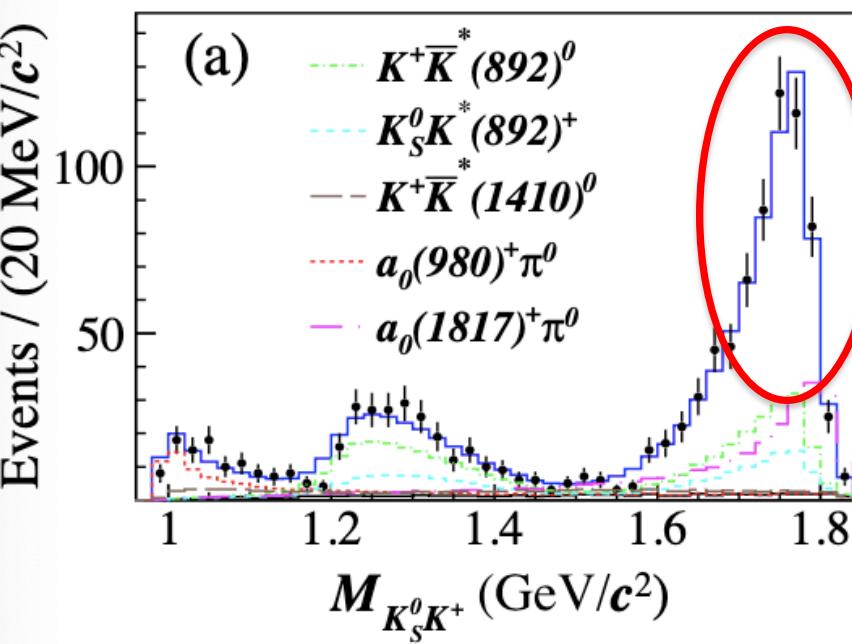
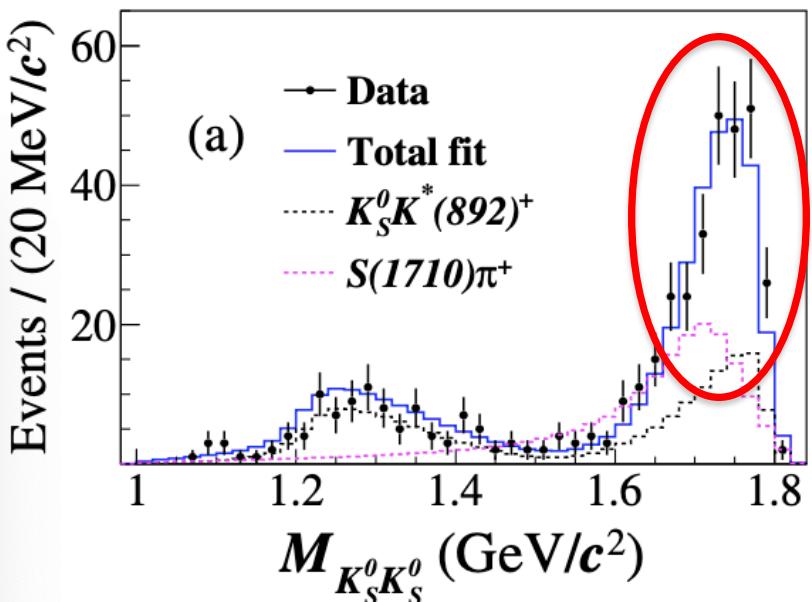
Letter

Study of the decay $D_s^+ \rightarrow K_S^0 K_S^0 \pi^+$ and observation of an isovector partner to $f_0(1710)$

existence of an isospin one partner of the $f_0(1710)$.

PHYSICAL REVIEW LETTERS **129**, 182001 (2022)

Observation of an a_0 -like State with Mass of 1.817 GeV in the Study of $D_s^+ \rightarrow K_S^0 K^+ \pi^0$ Decays





PHYSICAL REVIEW LETTERS 126, 152001 (2021)

Explaining the Many Threshold Structures in the Heavy-Quark Hadron Spectrum

Xiang-Kun Dong^{1,2}, Feng-Kun Guo^{1,2,*} and Bing-Song Zou^{1,2,3}

F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
 H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, Phys. Rept. 639, 1 (2016)
 N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, Phys. Rept. 873, 1 (2020)

Nuclear Physics A 620 (1997) 438–456

Chiral symmetry amplitudes
 in the S-wave isoscalar and isovector channels
 and the σ , $f_0(980)$, $a_0(980)$ scalar mesons

J.A. Oller, E. Oset

$K\bar{K}$ molecular state



$K^*\bar{K}^*$ molecular state

$f_0(1710)$ and $a_0(1710)$ states



L. S. Geng and E. Oset, Phys. Rev. D 79, 074009
 (2009): Coupled channel approach

$S = 0$ sector ?

We try to investigate.....

$S = 2, C = 0$

$S = -1, C = 1$

$S = 1, C = 1$

Systems	Coefficients $I = 0$	Coes. $I = 1, (\frac{1}{2})$	Exp.
KK	$\frac{1}{2}(-3P_t^\rho + P_t^\omega + 2V_t^\phi)g^2$ $-\frac{1}{2}(-3V_u^\rho + V_u^\omega + 2V_u^\phi)g^2$	$\frac{1}{2}(V_t^\rho + V_t^\omega + 2V_t^\phi)g^2$ $+\frac{1}{2}(V_u^\rho + V_u^\omega + 2V_u^\phi)g^2$	
KK^*	$\frac{1}{2}(-3V_t^\rho + V_t^\omega + 2V_t^\phi)g^2$	$\frac{1}{2}(V_t^\rho + V_t^\omega + 2V_t^\phi)g^2$	
K^*K^*	$\frac{1}{2}(-3V_t^\rho + V_t^\omega + 2V_t^\phi)g^2$ $-\frac{1}{2}(-3V_u^\rho + V_u^\omega + 2V_u^\phi)g^2$	$\frac{1}{2}(V_t^\rho + V_t^\omega + 2V_t^\phi)g^2$ $+\frac{1}{2}(V_u^\rho + V_u^\omega + 2V_u^\phi)g^2$	
$\bar{K}D$	$\frac{1}{2}(-3V_t^\rho + V_t^\omega - 2V_u^{D_s^*})g^2$	$\frac{1}{2}(V_t^\rho + V_t^\omega + 2V_u^{D_s^*})g^2$	$X(2900)$
$\bar{K}D^*$	$\frac{1}{2}(-3V_t^\rho + V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	
\bar{K}^*D	$\frac{1}{2}(-3V_t^\rho + V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	
\bar{K}^*D^*	$\frac{1}{2}(-3V_t^\rho + V_t^\omega - 2V_u^{D_s^*})g^2$	$\frac{1}{2}(V_t^\rho + V_t^\omega + 2V_u^{D_s^*})g^2$	$X_0(2866)$
KD	$\frac{1}{2}(-3V_t^\rho - V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	$D_{s0}^*(2317)$
KD^*	$\frac{1}{2}(-3V_t^\rho - V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	$D_{s1}(2460), D_{s1}(2536)$
K^*D	$\frac{1}{2}(-3V_t^\rho - V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	
K^*D^*	$\frac{1}{2}(-3V_t^\rho - V_t^\omega)g^2$	$\frac{1}{2}(V_t^\rho - V_t^\omega)g^2$	$D_{s2}^*(2573)$

$S = 2, C = 1$

KD_s	$(V_t^\phi + V_u^{D^*})g^2$	
KD_s^*	$V_t^\phi g^2$	
K^*D_s	$V_t^\phi g^2$	
$K^*D_s^*$	$(V_t^\phi + V_u^{D^*})g^2$	
$\bar{D}D_s$	$-V_t^{J/\psi} g^2$	
$\bar{D}D_s^*$	$-V_t^{J/\psi} g^2$	$Z_{cs}(3985)$
\bar{D}^*D_s	$-V_t^{J/\psi} g^2$	
$\bar{D}^*D_s^*$	$-V_t^{J/\psi} g^2$	$Z_{cs}(4000)$

$S = 1, C = 0$

$S = 1, C = 2$

DD_s	$(V_t^{J/\psi} + V_u^{K^*})g^2$	
DD_s^*	$V_t^{J/\psi} g^2$	
D^*D_s	$V_t^{J/\psi} g^2$	
$D^*D_s^*$	$(V_t^{J/\psi} + V_u^{K^*})g^2$	
$D_s D_s$	$(V_t^{J/\psi} + V_t^\phi)g^2$	
	$+(V_t^{J/\psi} + V_u^\phi)g^2$	
$D_s D_s^*$	$(V_t^{J/\psi} + V_t^\phi)g^2$	
$D_s^* D_s^*$	$(V_t^{J/\psi} + V_t^\phi)g^2$	
	$+(V_u^{J/\psi} + V_u^\phi)g^2$	

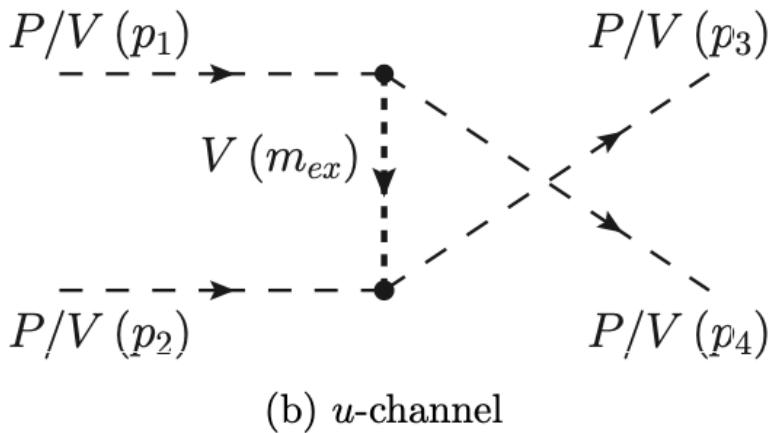
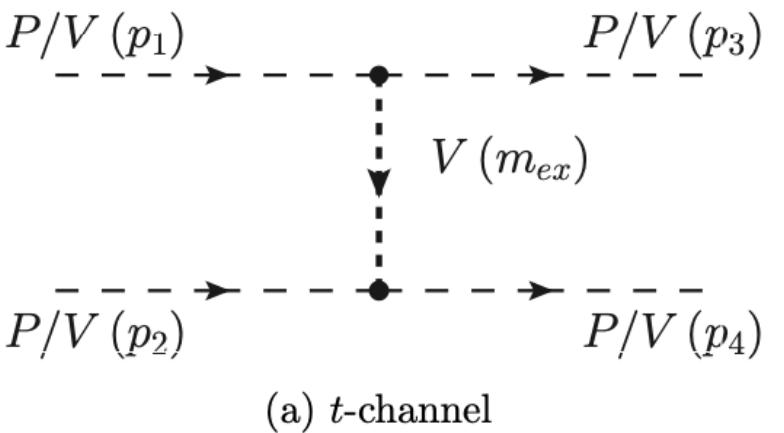
$S = 2, C = 2$



$$V_{t(u)}^{ex} = \frac{-V_{t(u)}}{t(u) - m_{ex}^2},$$

$$V_t = (k_1 + k_3) \cdot (k_2 + k_4)$$

$$V_u = (k_1 + k_4) \cdot (k_2 + k_3)$$



$$\begin{aligned} & \frac{1}{2} \int_{-1}^{+1} d \cos \theta \frac{-V_t}{t - m_{ex}^2} \\ &= - \frac{s}{\sqrt{\lambda(s, m_1^2, m_2^2) \lambda(s, m_3^2, m_4^2)}} (m_1^2 + m_2^2 + m_3^2 + m_4^2 - 2s - m_{ex}^2) \\ &\quad \times \log \frac{m_1^2 + m_3^2 - \frac{(s+m_1^2-m_2^2)(s+m_3^2-m_4^2)}{2s} - \frac{\sqrt{\lambda(s, m_1^2, m_2^2) \lambda(s, m_3^2, m_4^2)}}{2s} - m_{ex}^2}{m_1^2 + m_3^2 - \frac{(s+m_1^2-m_2^2)(s+m_3^2-m_4^2)}{2s} + \frac{\sqrt{\lambda(s, m_1^2, m_2^2) \lambda(s, m_3^2, m_4^2)}}{2s} - m_{ex}^2} - 1, \end{aligned}$$



Systems	Ours-new ($q_{max} = 883$)	Oset's ($q_{max} = 1100$)	Ours-new2 ($q_{max} = 760$)	Oset's-2 ($q_{max} = 843$)	Exp.
$\bar{K}D$ (2362.89)	2354.10	2346.74	2362.40	2362.82	
$\bar{K}D^*$ (2504.20)	—	2502.68	—	—	
\bar{K}^*D (2760.86)	$2694.17 + i42.94$	2689.01	2747.40	2741.24	
$\bar{K}^*D^*, J = 0$ (2902.17)	2866.14	2866.38	2897.82	2897.02	$X_0(2866)$
$\bar{K}^*D^*, J = 1$ (2902.17)	2894.66	2859.98	2902.06	2895.50	
$\bar{K}^*D^*, J = 2$ (2902.17)	$2813.41 - i135.96$	2745.76	$2807.55 \pm i66.51$	2837.49	
KD (2362.89)	2258.22	2262.15	2317.36	2317.14	$D_{s0}^*(2317)$
KD^* (2504.20)	2391.78	2400.22	24555.06	2456.29	$D_{s1}(2460)$
K^*D (2760.86)	$2698.42 - i237.63$	2504.39	$2684.60 \pm i170.50$	2620.27	
$K^*D^*, J = 0$ (2902.17)	$2874.93 + i192.31$	2713.15	$2849.62 + i135.30$	2804.40	
$K^*D^*, J = 1$ (2902.17)	$2866.91 + i164.04$	2748.72	$2857.40 \pm i112.55$	2826.76	
$K^*D^*, J = 2$ (2902.17)	$2824.87 + i259.85$	2600.70	$2817.27 + i189.09$	2732.68	$D_{s2}^*(2573)$



§5. Summary

- Are there also existing two Pcs states $P_{cs}(4459)$?
- From the view of molecular state, what components are for $P_{cs}(4459)$ $P_{\psi s}^{\Lambda}(4338)^0$
- We dynamically generate the resonances:
 $X_0(2866)$, $D_{s0}^*(2317)$ and $D_{s1}(2460)$
- We find some attractive interactions, which would be more molecular (tetraquark) states with strangeness as our predictions?

Hope future experiments and theories bring more clarifications on these issues.....



第八届强子谱和强子结构研讨会

Jul 11 – 15, 2025

Asia/Shanghai timezone

会议计划于2025年7月11日至15日在桂林市桂林宾馆召开，其中11日报到注册。会议统一安排食宿，费用自理。会议收取注册费，教师及博士后1500元/人，学生1000元/人。

会议网址：<https://indico.ihep.ac.cn/event/24044/>，注册截止时间为6月20日。



Thanks for your attention!

感谢大家的聆听！