

重味物理理论方法综述



于福升
兰州大学



5th LHCb workshop @ Wuhan, 2025.4.26

2019第一届LHCb会

Photon Polarization of $b \rightarrow s \gamma$: Theory (I)



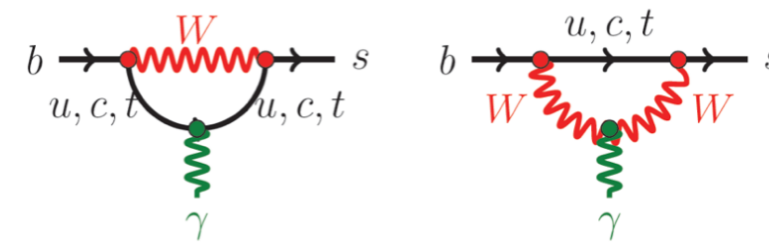
Fu-Sheng Yu
Lanzhou University

LHCb workshop @ PKU, 2019.12.15

堅守·奮鬥

FSY, Kou, C.D.Lu, JHEP (1305.3173)
Akar, Ben-Haim, Hebinger, Kou, FSY, JHEP (1802.09433)
W.Wang, FSY, Z.X.Zhao, 1909.13083

Photon Polarization in the SM



W boson
couples only to
left-handed
quarks in SM

$$\propto \bar{s} \sigma_{\mu\nu} q^\nu \left(\frac{m_b}{2} (1 + \gamma_5) + \frac{m_s}{2} (1 - \gamma_5) \right) b$$

chirality flip on external line

Photon is dominantly
left-handed in b
quark decay in SM

$$\frac{\mathcal{A}(b_L \rightarrow s_R \gamma)_S^{LO}}{\mathcal{A}(b_R \rightarrow s_L \gamma)_S^{LO}} = \frac{m_s}{m_b}$$

right-handed in anti-b quark decay

2023第三届LHCb研讨会

Baryon CP Violation by T-odd and T-even correlations



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兰州大学

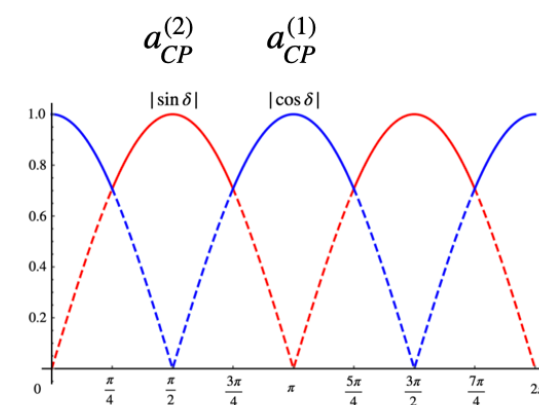


Based on [J.P.Wang, Q.Qin, FSY, arXiv:2211.07332]

LHCb workshop, 2023.4.15

Complementary: $\cos \delta_s$ vs $\sin \delta_s$

- Precise prediction on strong phases is far beyond control currently
- Complimentary CPV observables proportional to $\sin \delta$ or $\cos \delta$ cover all the $(0, 2\pi)$ region
- Whatever the strong phase is, either $|\sin \delta|$ or $|\cos \delta|$ would be larger than 0.7 which is large enough for measurements



- It might reduce the sensitivity of CPV on the strong phase, avoid the theoretical uncertainties on strong phases, and then increase the possibility of observation of baryon CPV

$$a_{CP}^{(1)} \propto \cos \delta_s \sin \phi_w$$
$$a_{CP}^{(2)} \propto \sin \delta_s \sin \phi_w$$

J.P.Wang, Q.Qin, FSY, arXiv:2211.07332

2024第四届LHCb研讨会

Everything of baryon CPV Why? What? When?



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4th LHCb workshop @ Yantai, 2024.7.30

Global CPV from $N\pi$ scatterings

$N\pi$ scatterings	decay processes	global CPV	CPV of $\cos \theta < 0$	CPV of $\cos \theta > 0$
$N\pi \rightarrow \Delta^{++}\pi^-$	$\Lambda_b^0 \rightarrow (\Delta^{++}\pi^-)K^-$	5.9%	8.0%	3.6%
$m_{N\pi} \in [1.2, 1.9]\text{GeV}$	$\Lambda_b^0 \rightarrow (\Delta^{++}\pi^-)\pi^-$	-4.1%	-5.4%	-2.4%
$N\pi \rightarrow p\pi^0$	$\Lambda_b^0 \rightarrow (p\pi^0)K^-$	5.8%	8.2%	2.7%
$m_{N\pi} \in [1.1, 2.5]\text{GeV}$	$\Lambda_b^0 \rightarrow (p\pi^0)\pi^-$	-3.9%	-3.9%	-3.7%

J.P.Wang, FSY, 2407.04110

2019第一届LHCb会

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of b->s gamma: Theory (I)



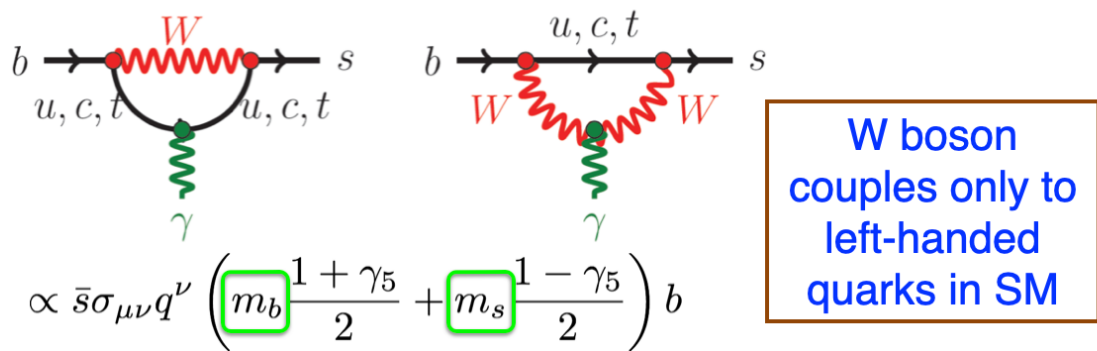
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Baryon CP Violation
by T-odd and T-even correlations



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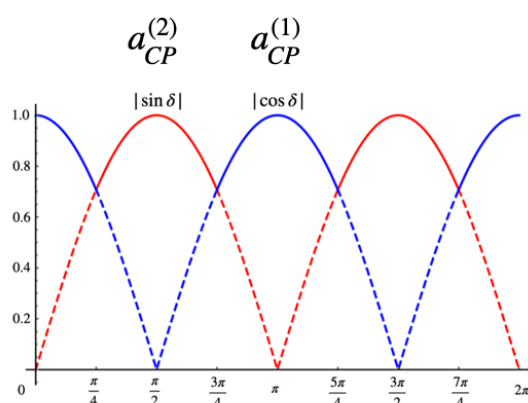


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J.P.Wang, FSY, 2407.04110

Outline

1. CP violation and non-leptonic decays
2. Topological diagrammatic approach
3. Generalized factorization, QCDF, PQCD
4. Final-state interaction, rescatterings

Introduction on CP violation

Definition:
$$A_{CP} = \frac{\Gamma(i \rightarrow f) - \Gamma(\bar{i} \rightarrow \bar{f})}{\Gamma(i \rightarrow f) + \Gamma(\bar{i} \rightarrow \bar{f})} = \frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2}$$

$$V_{CKM} \leftrightarrow V_{CKM}^*$$

$$A_f = |a_1|e^{i(\delta_1 + \phi_1)} + |a_2|e^{i(\delta_2 + \phi_2)}$$

$\phi_{1,2}$: weak phases, flip signs under $A_f \leftrightarrow \bar{A}_{\bar{f}}$

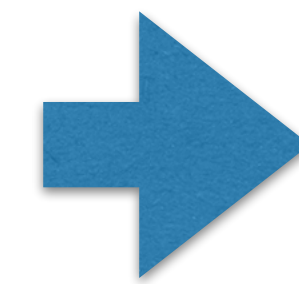
$$\bar{A}_{\bar{f}} = |a_1|e^{i(\delta_1 - \phi_1)} + |a_2|e^{i(\delta_2 - \phi_2)}$$

$\delta_{1,2}$: strong phases, keep signs under $A_f \leftrightarrow \bar{A}_{\bar{f}}$

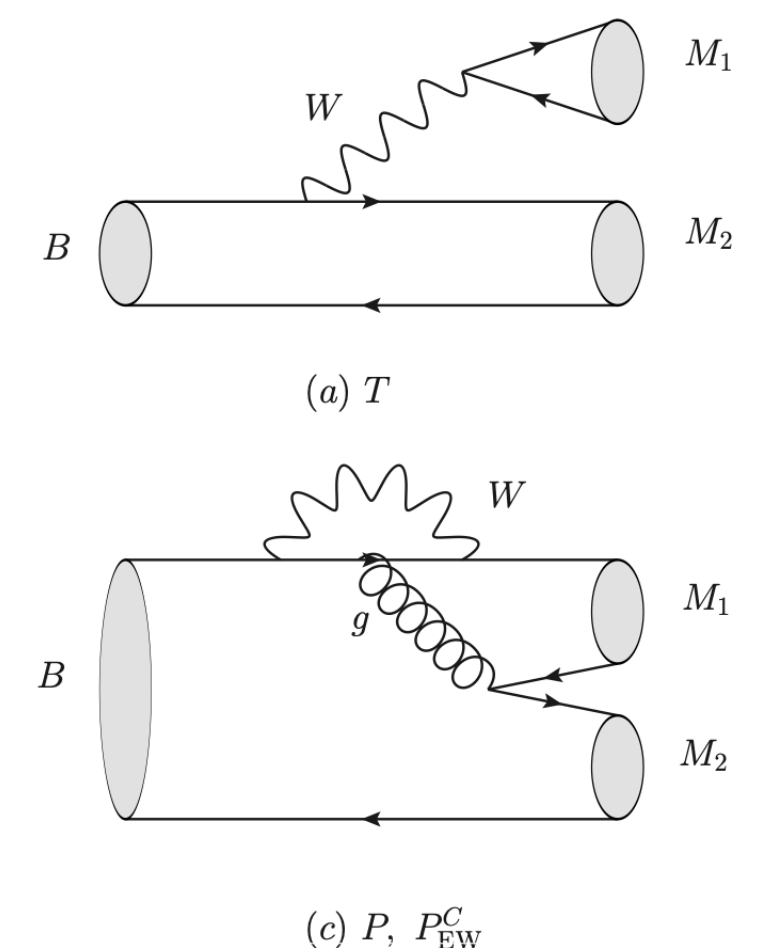
$$A_{CP} = - \frac{2|a_1 a_2| \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)}$$

CPV conditions:

1. At least two amplitudes
2. with different weak phases
3. with different strong phases



Tree + Penguin



SU(3) irreducible representation approach

- Zeppendfeld, 1981
- First SU(3) relations for B decays
- with reduced amplitudes
- Savage and Wise, 1989
- First **tensor contraction** formulae
- SU(3) irreducible representation

$$b(c) \rightarrow q_1 \bar{q}_2 q_3, \quad q_i = u, d, s$$

$$3 \otimes \bar{3} \otimes 3 = 3_p \oplus 3_t \oplus \bar{6} \oplus 15$$

$$\begin{aligned} \mathcal{A}_t^{\text{IRA}} = & A_3^T B_i (H_{\bar{3}})^i (M)_k^j (M)_j^k + C_3^T B_i (M)_j^i (M)_k^j (H_{\bar{3}})^k + B_3^T B_i (H_3)^i (M)_k^j (M)_j^k + D_3^T B_i (M)_j^i (H_{\bar{3}})^j (M)_k^j \\ & + A_6^T B_i (H_6)_k^{ij} (M)_j^l (M)_l^k + C_6^T B_i (M)_j^i (H_6)_k^{jl} (M)_l^k + B_6^T B_i (H_6)_k^{ij} (M)_j^k (M)_l^l \\ & + A_{15}^T B_i (H_{\bar{15}})_k^{ij} (M)_j^l (M)_l^k + C_{15}^T B_i (M)_j^i (H_{\bar{15}})_l^{jk} (M)_k^l + B_{15}^T B_i (H_{\bar{15}})_k^{ij} (M)_j^k (M)_l^l. \end{aligned}$$

- Recent applications in singly charmed baryons by Chao-Qiang Geng's group and in doubly heavy-flavor baryons in Wei Wang's group

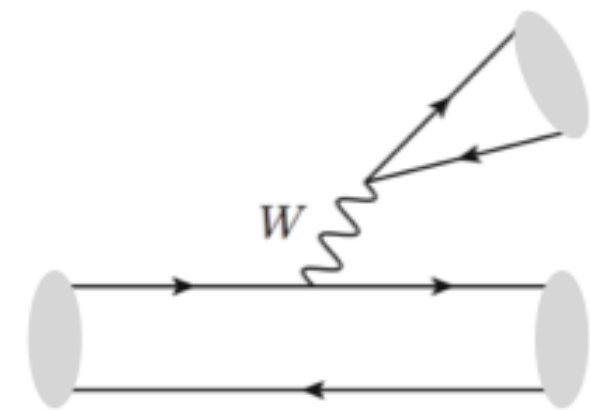
See Xin-Qiang Li's talk

Topological Diagrams

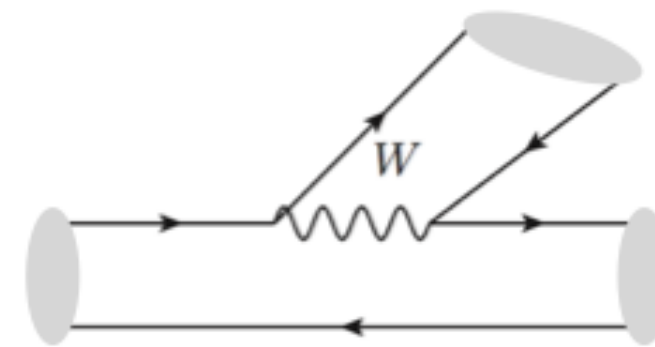
- Decaying amplitudes are classified according to the **weak flavour flows**
- All the strong interaction effects** are included. Therefore, non-perturbative contributions are all considered.

Chau,'86; Chau,Cheng,'87

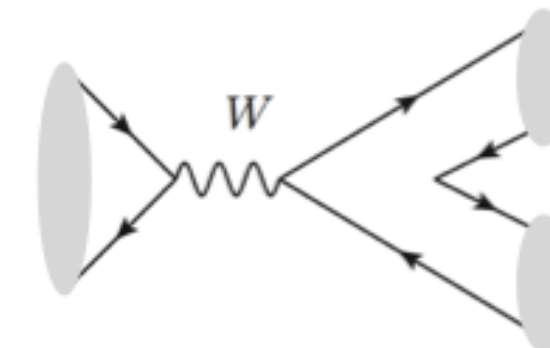
Tree:



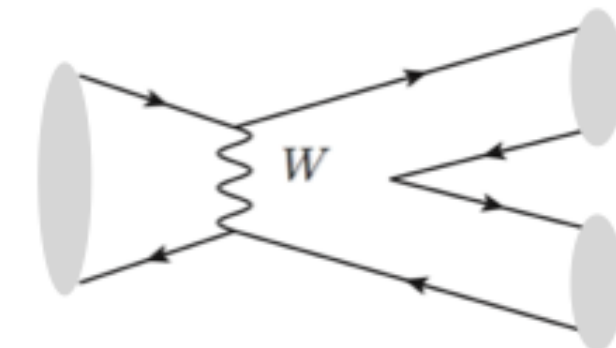
W外发射图



W内发射图

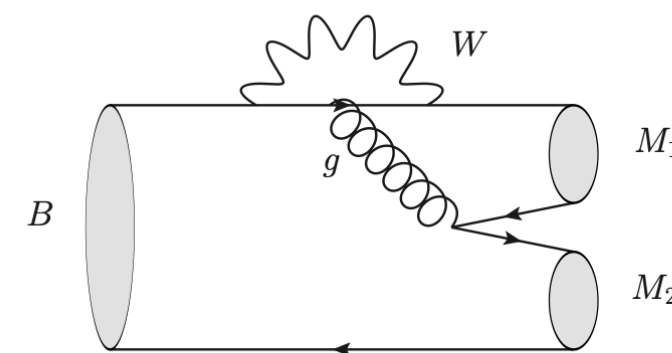


W湮灭图

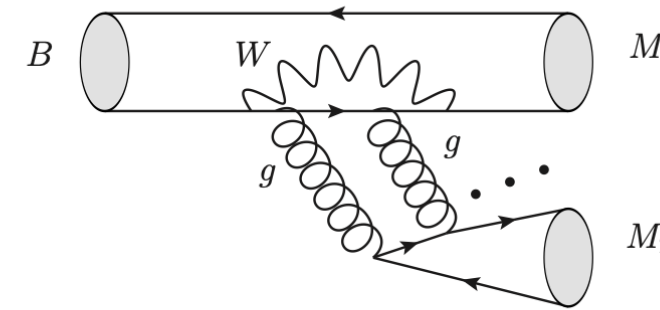


W交换图

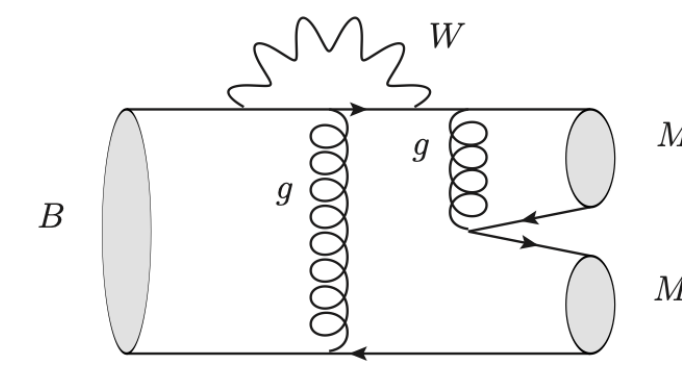
Penguin:



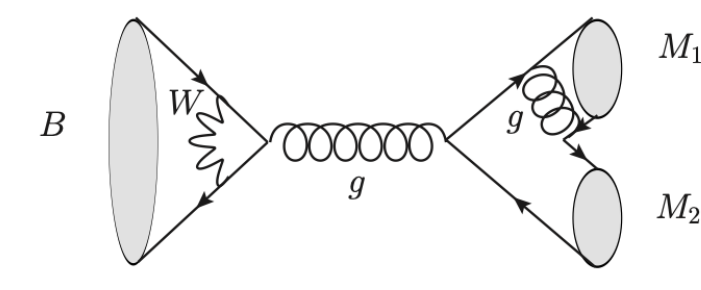
(c) P, P_{EW}^C



(d) S, P_{EW}



(g) PE, PE_{EW}



(h) PA, PA_{EW}

Topological Diagrams

方法一：直接用实验数据抽取，或者估计。包含非微扰效应，在粲物理中应用很成功

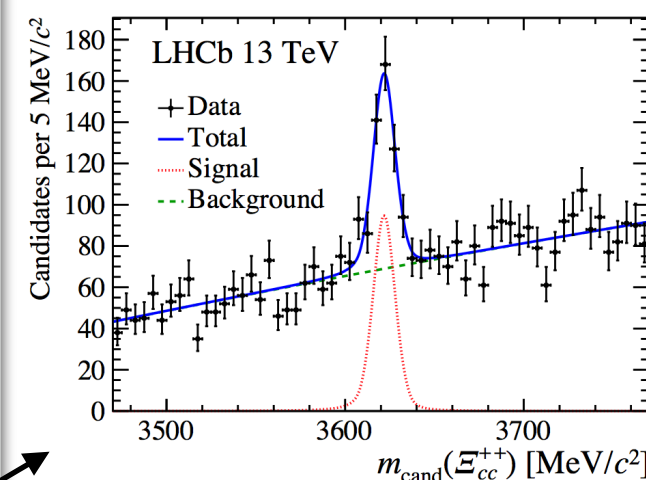
成功预言双粲重子发现道

$$|C/T| \sim |C'/T| \sim |E/T| \sim O(\Lambda_{\text{QCD}}/m_c) \sim 1$$

Non-leptonic decays of Ξ_{cc}^{++}

Modes		Br(first)($\times 10^{-3}$)	Br(final)($\times 10^{-3}$)	Representation
$p(D^+/D^0\pi^+)$	😄	8.	0.2	Ccm Vcd Vud
$p(D_s^+/D^0K^+)$		0.4	0.01	Ccm Vcd Vus
$(pK^-\pi^+/\Sigma^+)(D^+/D^0\pi^+)$	😄	80.	2.	Ccm Vcs Vud
$(pK^-\pi^+/\Sigma^+)(D_s^+/D^0K^+)$		3.	0.1	Ccm Vcs Vus
$(\Lambda_c^+\pi^+)(\pi^+\pi^-)$		3.	0.2	-(Ct Vcd Vud)
$(\Lambda_c^+\pi^+)(K^+\pi^-)$		0.2	0.008	Ct Vcd Vus
$(\Lambda_c^+\pi^+)(K^-\pi^+)$	😄	50.	3.	Ct Vcs Vud
$(\Lambda_c^+\pi^+)(K^+K^-)$		2.	0.08	Ct Vcs Vus
$\Lambda_c^+\pi^+$	😄	30.	1.	Ccb Vcd Vud + T Vcd Vud
$\Lambda_c^+K^+$		1.	0.06	Ccb Vcd Vus + T Vcd Vus
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)\pi^+$	😄	400.	20.	Ccb Vcs Vud + T Vcs Vud
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)K^+$		20.	0.9	Ccb Vcs Vus + T Vcs Vus

- discovery channels: Br=O($10^{-3\sim-4}$)

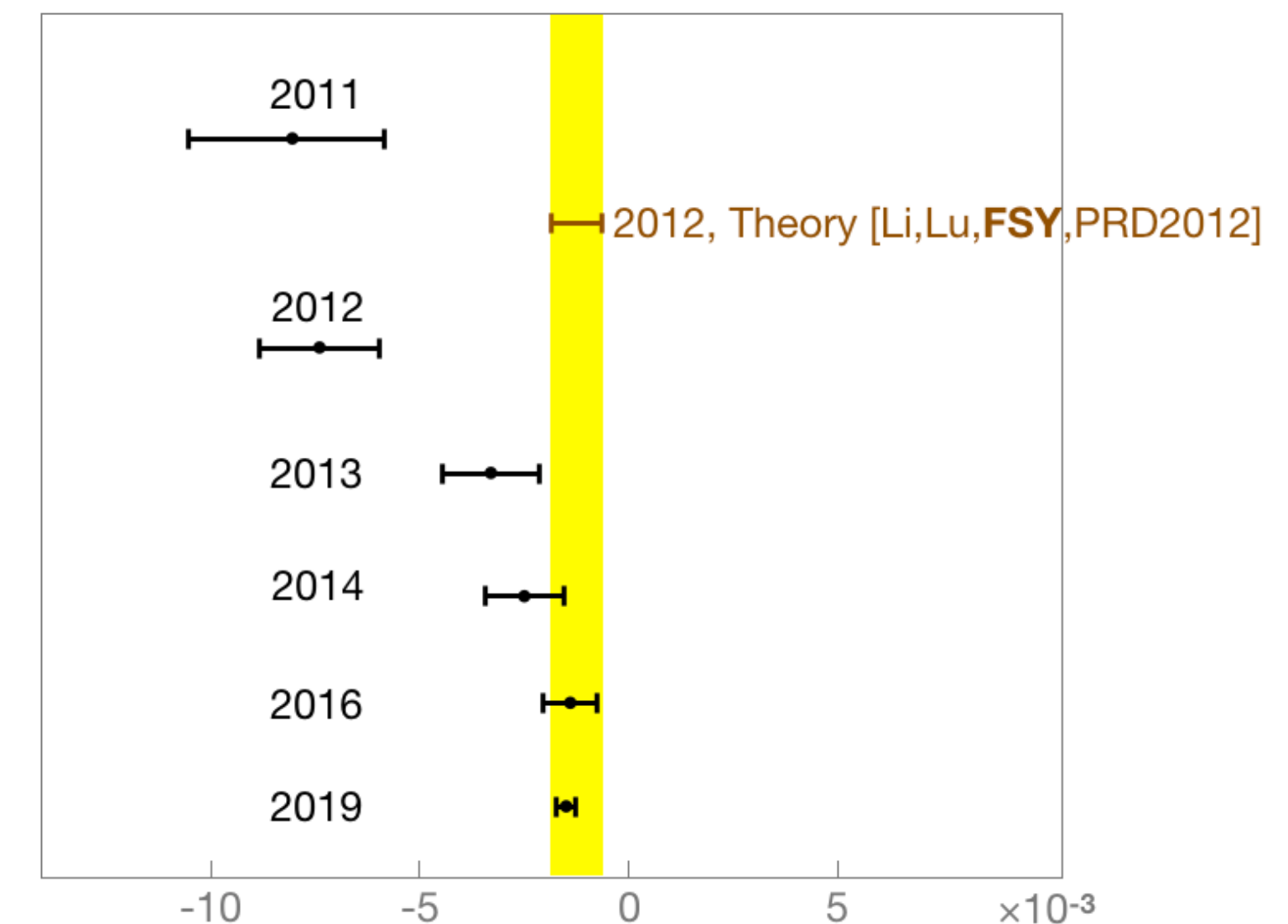


LHCb 2017.07

My talk at LHCb China group in 2016.12

成功预言粲介子CP破坏

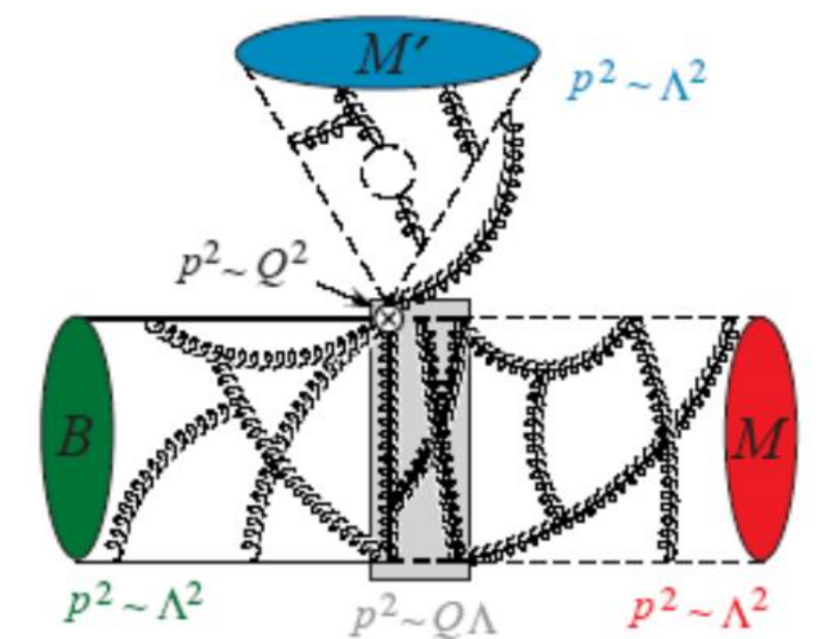
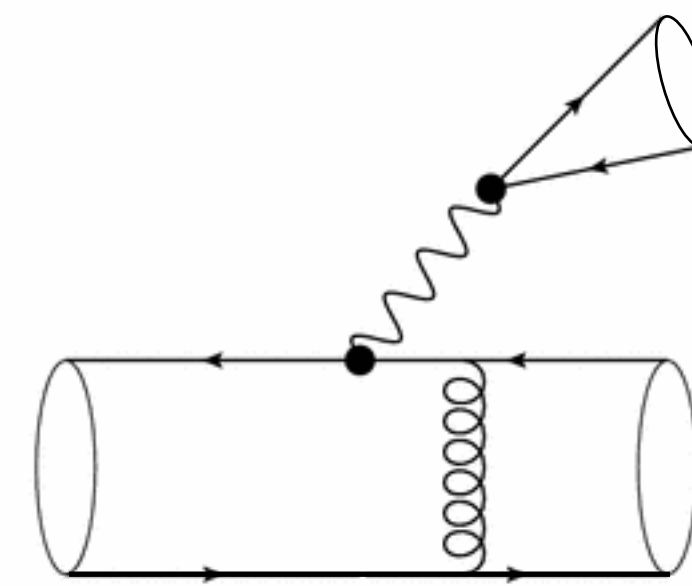
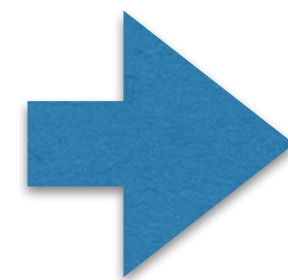
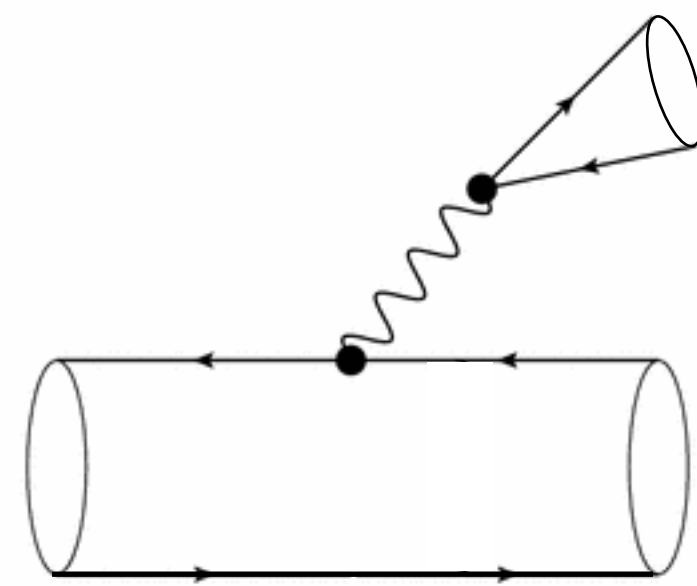
$$|P/T| \sim O(\Lambda_{\text{QCD}}/m_c) \sim 1$$



$$A_{CP}(D^0 \rightarrow K^+ K^-) - A_{CP}(D^0 \rightarrow \pi^+ \pi^-)$$

Topological Diagrams

方法二：用因子化等方法直接计算。画出对应的费曼图做微扰论计算，适用于底强子衰变



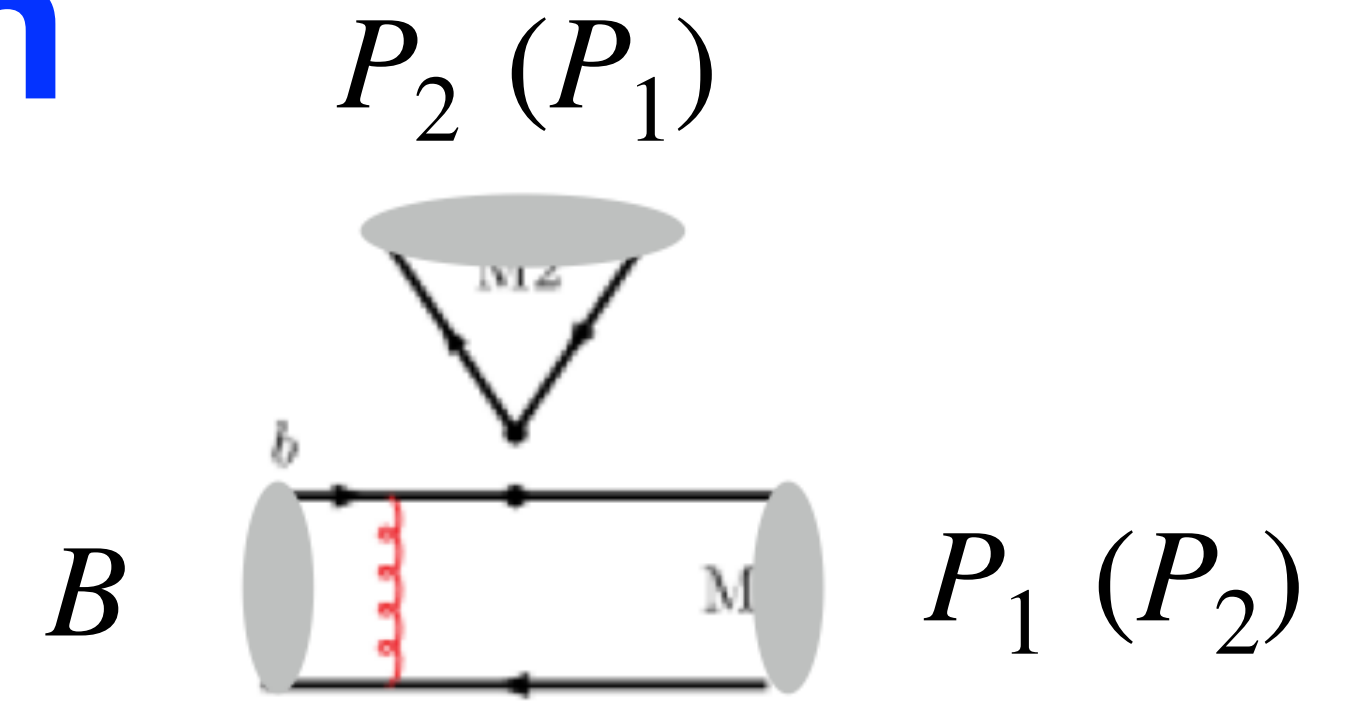
拓扑图：
没有费曼规则，无法直接计算

费曼图：
有传播子、顶点等费曼规则，
可以直接计算

如PQCD、QCDF、SCET方法

Naive Factorization

$$\begin{aligned} \langle P_1 P_2 | \mathcal{H}_{eff} | B \rangle &= Z_1 \langle P_1 | j^\mu | 0 \rangle \langle P_2 | j_\mu | B \rangle \\ &+ Z_2 \langle P_2 | j'^\mu | 0 \rangle \langle P_1 | j'_\mu | B \rangle \end{aligned}$$



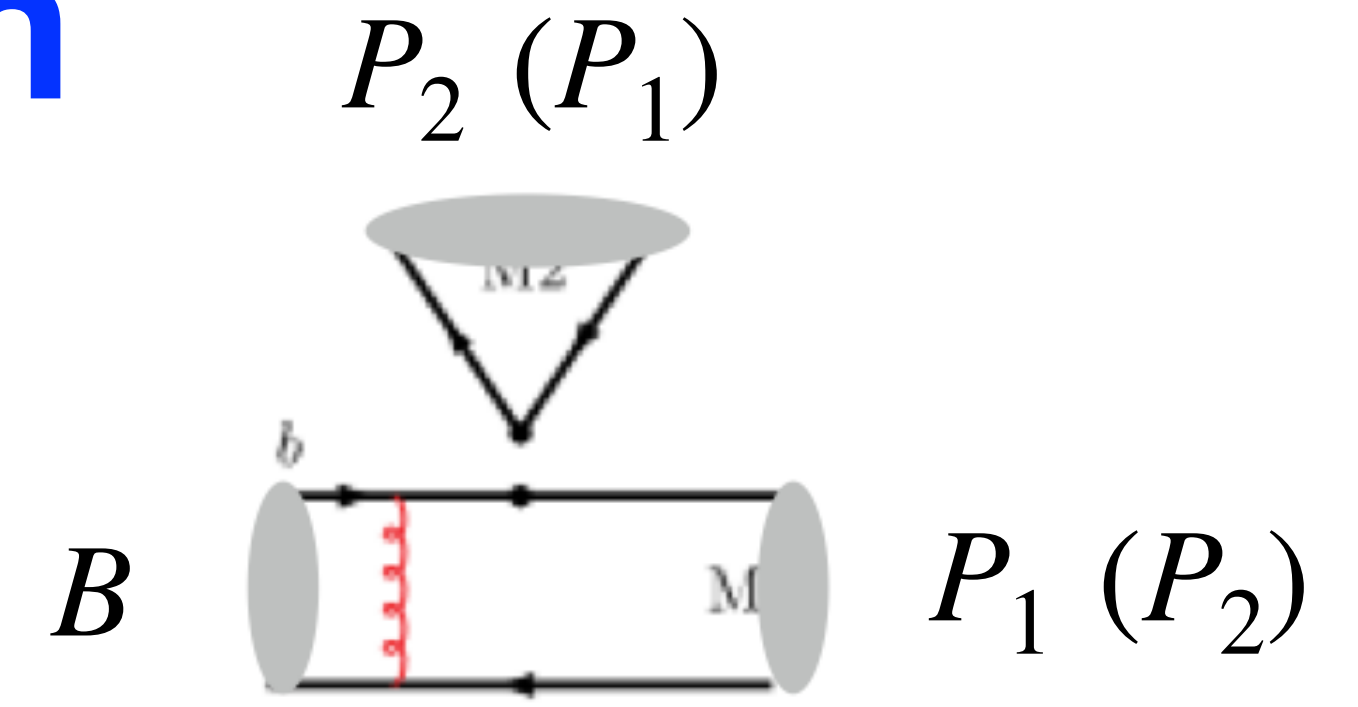
因子化假设 Bauer, Stech, Wirbel, 1987 (1900 citations)

色透明机制 [Bjorken, 1988]

$$\langle P_1 P_2 | \mathcal{H}_{eff} | B \rangle = i \frac{G_F}{\sqrt{2}} V_{qb} V_{qq'}^* \left(\frac{1}{N_c} C_i + C_j \right) f_{P_2} (m_B^2 - m_1^2) F_0^{B \rightarrow P_1} (m_2^2) + (1 \leftrightarrow 2)$$

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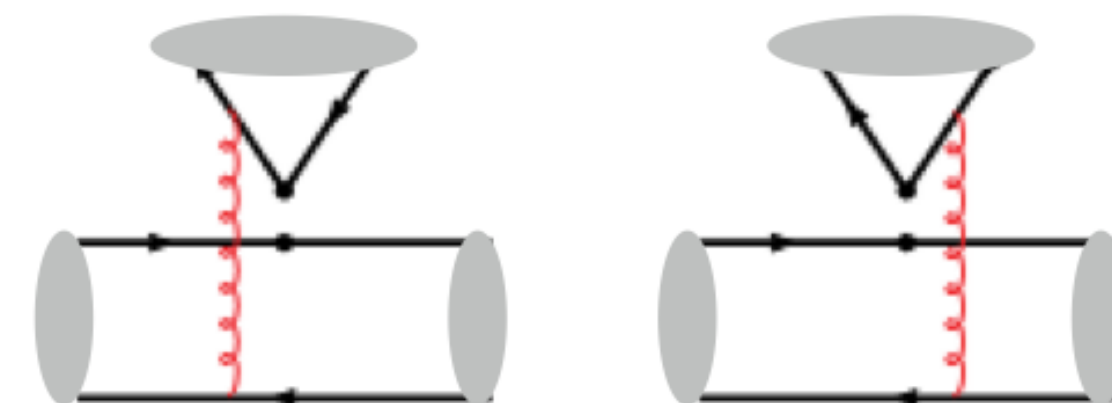
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问题1：实数，无强相位，无CPV

问题2：能标依赖

问题3：不能计算 “不可因子化图”

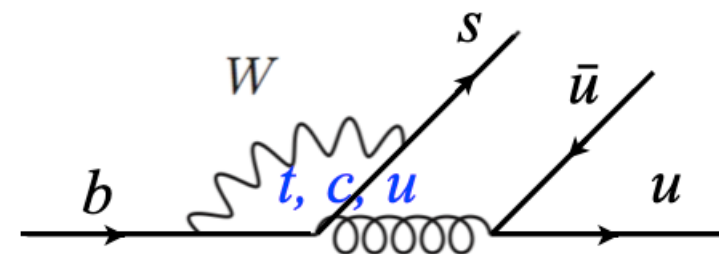


Generalized Factorization

$$\langle sq'\bar{q}'|\mathcal{H}_{eff}|b\rangle = \sum_{i,j} C_i^{eff}(\mu) \langle sq'\bar{q}'|O_j|b\rangle^{\text{tree}}$$

Ali, Kramer, C.D.Lu,
hep-ph/9804363 (600 citations)

$$C_3^{eff} = C_3 - \frac{1}{6} \frac{\alpha_s}{4\pi} (C_t + C_p + C_g) + \frac{\alpha_s}{4\pi} \left(r_V^T + \gamma_V^T \log \frac{m_b}{\mu} \right)_{3i} C_j + \dots,$$



$$C_t = - \sum_{q'=u,c} \frac{V_{q'b} V_{q'q}^*}{V_{tb} V_{tq}^*} \left[\frac{2}{3} + \frac{2}{3} \log \frac{m_{q'}^2}{\mu^2} - \Delta F_1 \left(\frac{k^2}{m_{q'}^2} \right) \right] C_1$$

2. 强子矩阵元的辐射修正,
标度依赖性降低

$$\Delta F_1(z) = -4 \int_0^1 dx x(1-x) \log [1 - z x(1-x) - i\epsilon] \quad \frac{m_b^2}{4} \lesssim k^2 \lesssim \frac{m_b^2}{2}$$

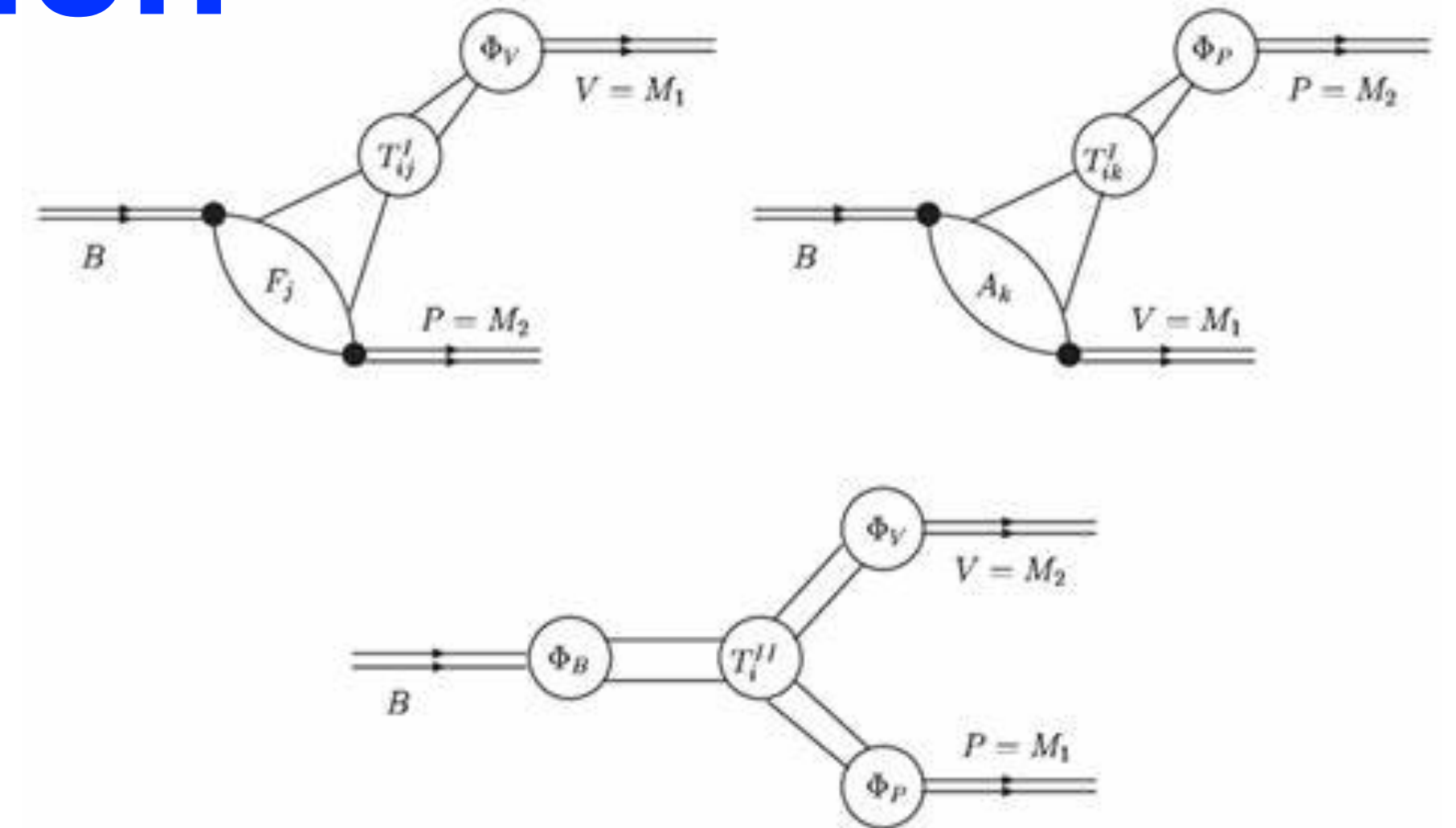
1. 夸克圈, BSS机制, 有强相位和CPV

问题: 外夸克在壳时有红外发散

应用广泛: 重子衰变[C.Q.Geng, Y.K.Hsiao, C.W.Liu, et al], 末态相互作用[FSY, et al]

QCD Factorization

$$\begin{aligned}\langle M_1 M_2 | O_i | B \rangle &= F^{B \rightarrow M_1}(0) \int dx \Phi_{M_2}(x) T^I(x) + (M_1 \leftrightarrow M_2) \\ &+ \int d\xi dx dy \Phi_B(\xi) \Phi_{M_2}(x) \Phi_{M_1}(y) T^{II}(\xi, x, y),\end{aligned}$$



- 基于共线因子化。 See Xin-Qiang Li's talk
- 因子化定理：有基于QCD的严格证明，形式漂亮。所有费曼图的红外发散，要么互相抵消，要么可以被非微扰的物理矩阵元吸收掉。
- 问题1：有端点奇异性，传播子 $\sim 1/x_1 x_2 Q^2 \rightarrow \infty$ when $x_{1,2} \rightarrow 0, 1$ 。
- 问题2：强相位来自辐射修正，CP破坏小。需参数化湮灭图并调节参数。唯象学受限。

Beneke, Buchalla, Neubert, Sachrajda, 1999 (1400 citations)

See Yue-Long Shen's talk

PQCD approach

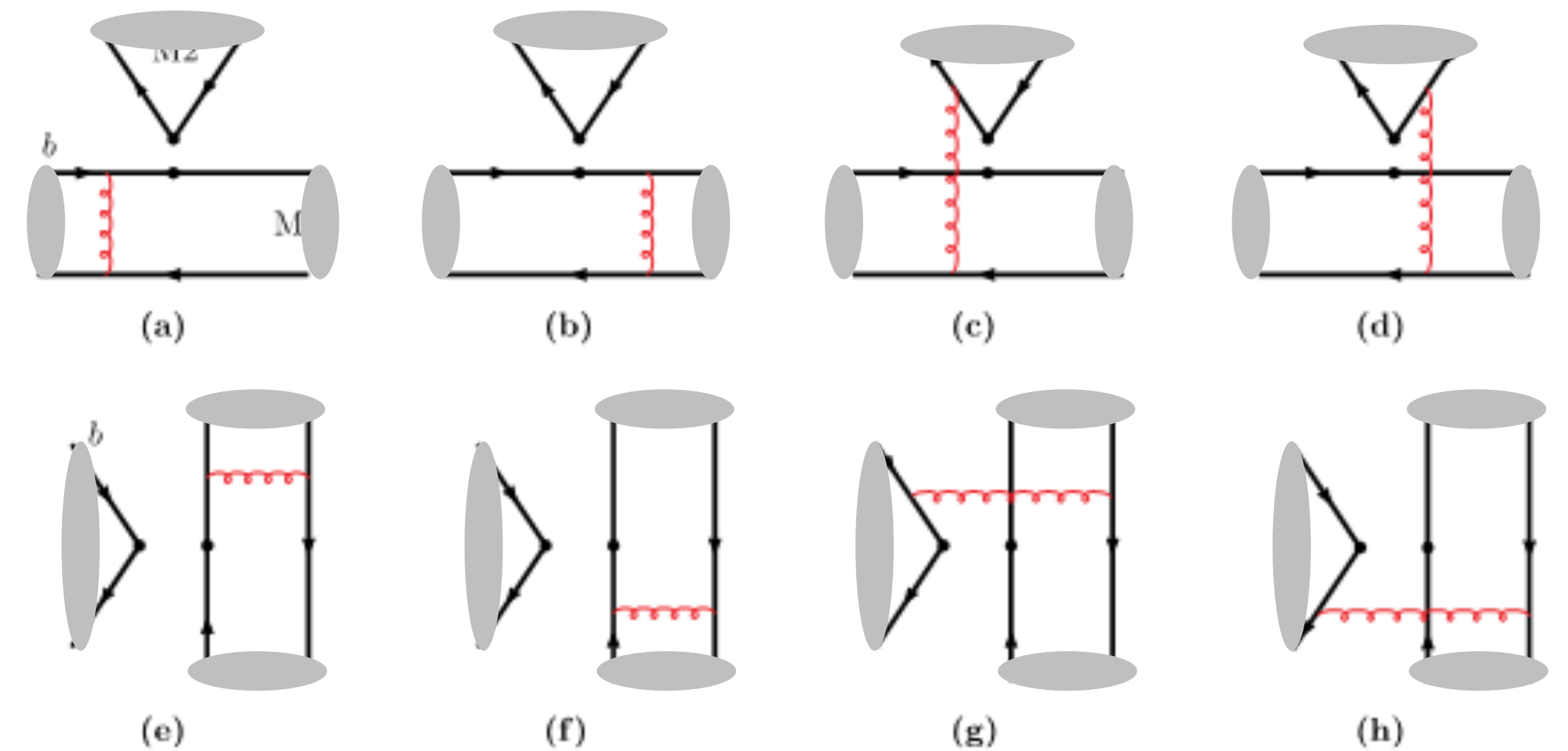
- PQCD approach (based on k_T factorization):
retain transverse momentum of parton k_T ,
 - propagator $\sim 1/(x_1 x_2 Q^2 + k_T^2)$

• 基于横动量因子化，部分贡献证明了因子化定理。

• 解决端点奇异性问题！

• 所有拓扑图都能计算！！

• CPV唯象学非常成功！！

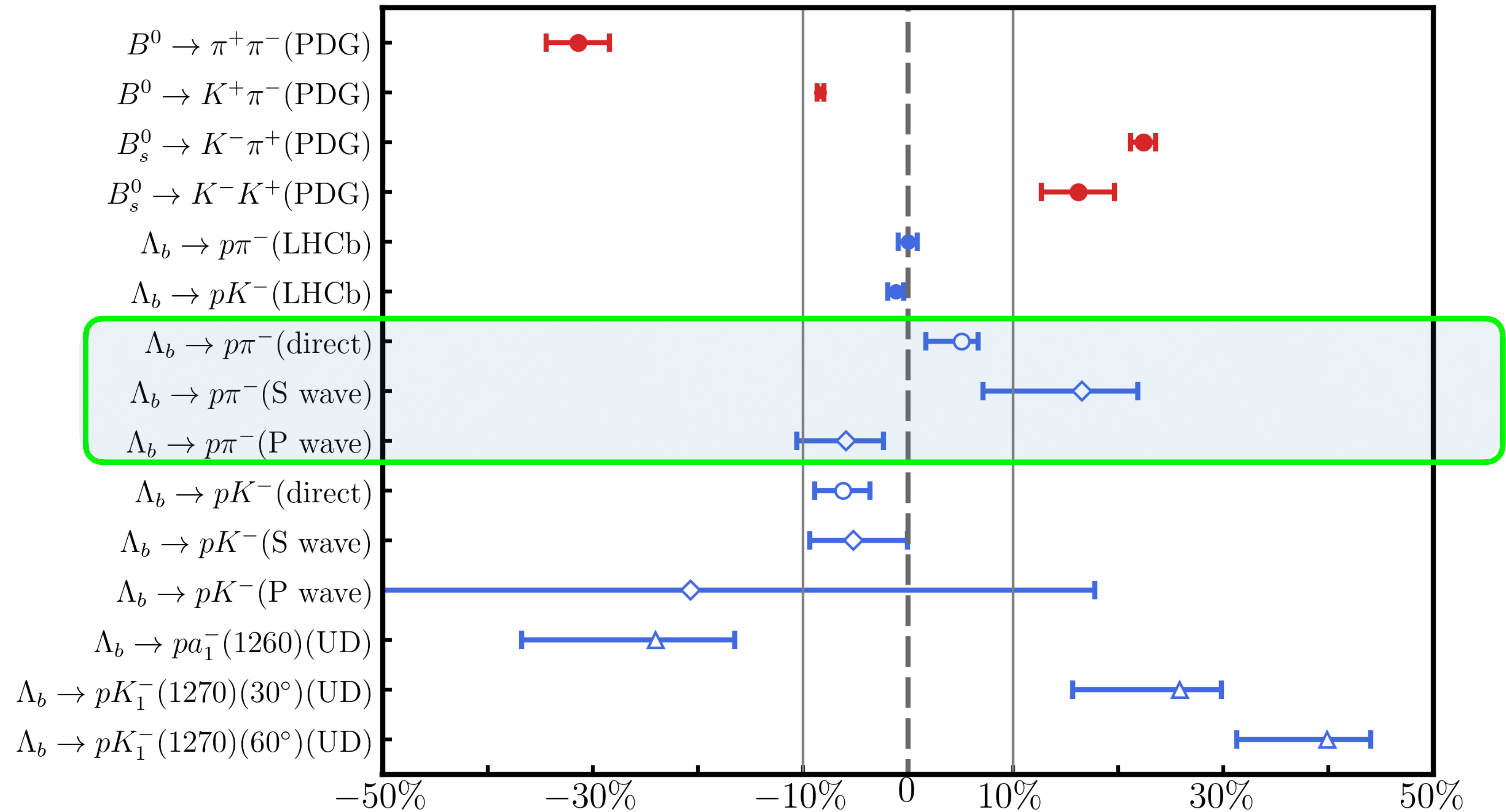


直接CP破坏(%)	GFA	QCDF	2000 PQCD	2004 exp.
$B \rightarrow \pi^+ \pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 20$	$+32 \pm 4$
$B \rightarrow K^+ \pi^-$	$+10 \pm 3$	$+5 \pm 9$	-17 ± 5	-8.3 ± 0.4

Keum, H.n.Li, Sanda, hep-ph/0004173 (800 citations); hep-ph/0004004 (700 citations)

C.D.Lu, Ukai, M.Z.Yang, hep-ph/0004213 (600 citations)

Λ_b decays in PQCD



分波CPV相消

See Ya Li's talk

J.X.Yu, J.J.Han, Y.Li, H.n.Li, J.P.Wang, Z.J.Xiao, FSY, 2409.02821

末态相互作用

- 描述长程非微扰贡献的物理图像

$$\mathcal{A} = \mathcal{S}^{1/2} \mathcal{A}_0$$

随时间演化的算符矩阵元
强相互作用效应出现
并贡献强相位

极短时间内的现象
短程的弱相互作用
无强相位

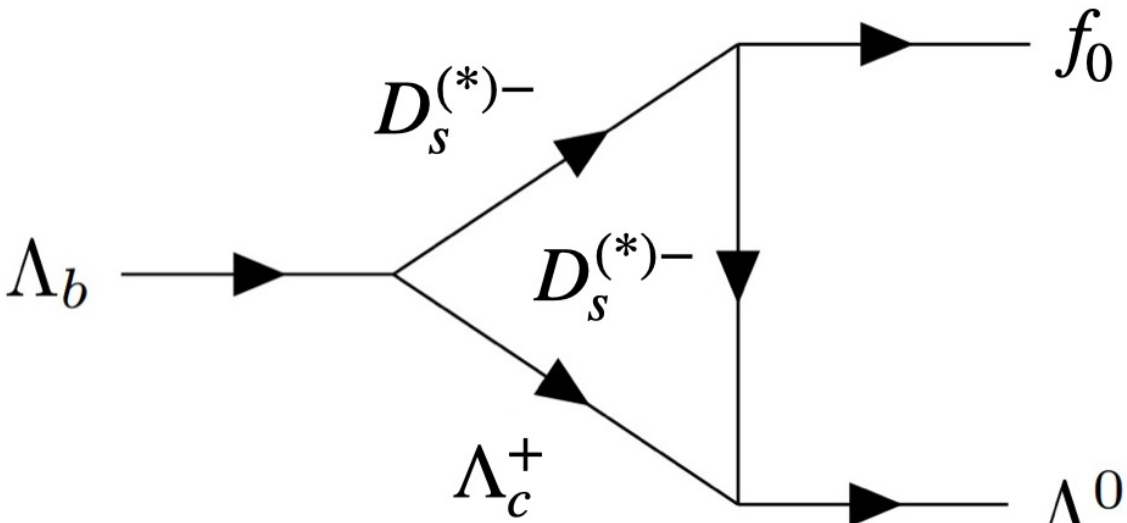
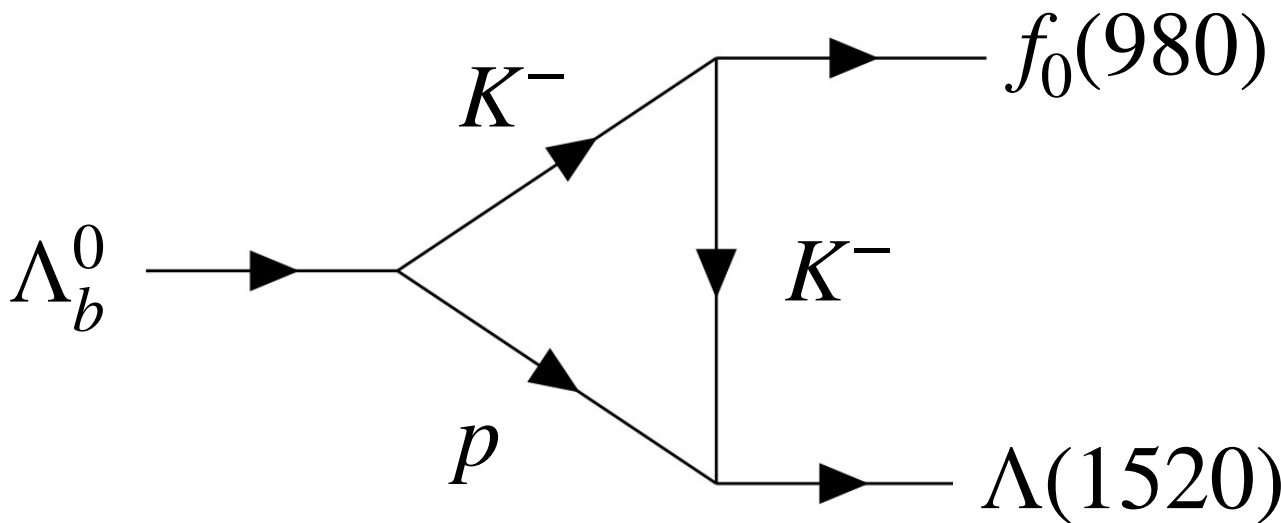
☞ L. Wolfenstein, *Phys. Rev. D* 43, 151 (1991).

- 应用广泛: Λ_c^+ and Λ_b^0 CPV

C.P.Jia, H.Y.Jiang, J.P.Wang, **FSY**, JHEP2024

Z.D.Duan, J.P.Wang, R.H.Li, C.D.Lu, **FSY**, 2412.20458

T.L.Feng, Q.Qin, H.Q.Shang, **FSY**, to appear

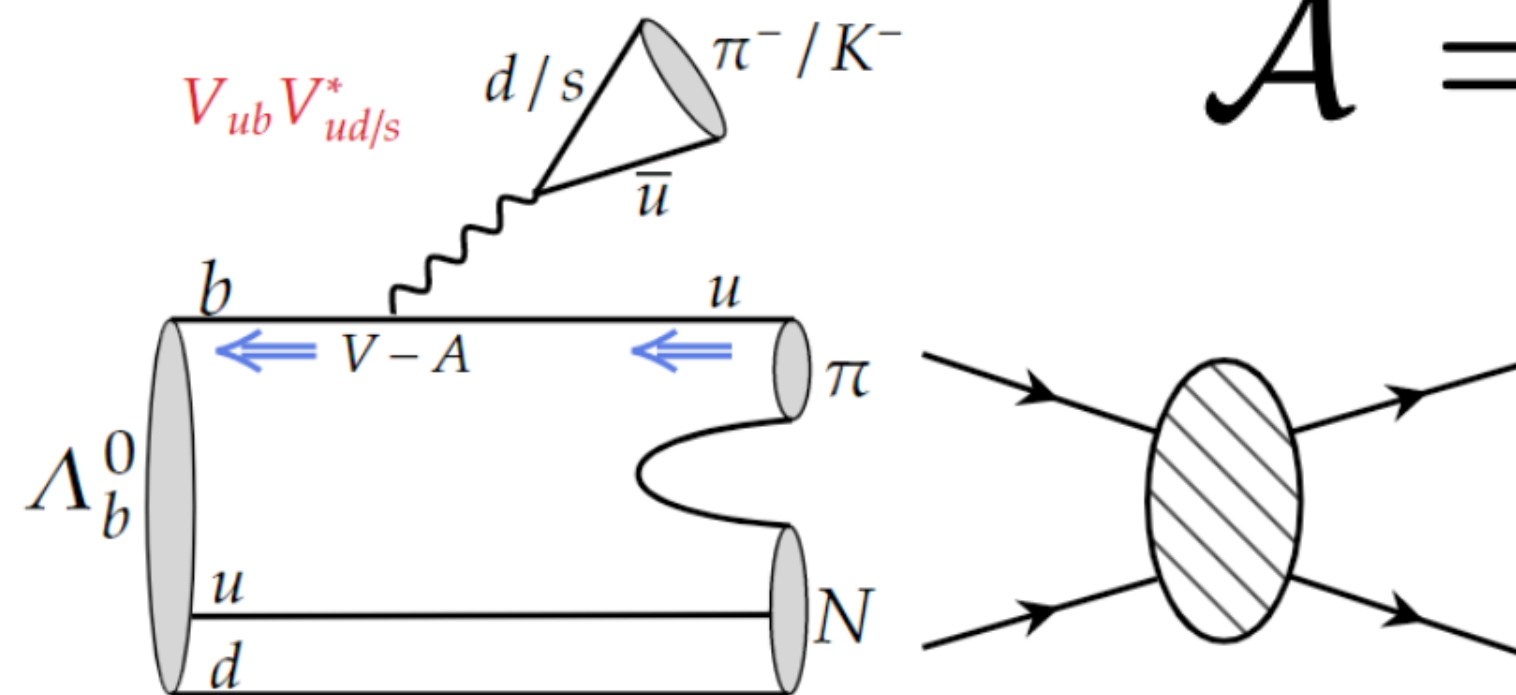


$$\Lambda_b^0 \rightarrow \Lambda(1520) f_0(980) \rightarrow (p K^-)(\pi^+ \pi^-)$$

See Qin Qin's talk

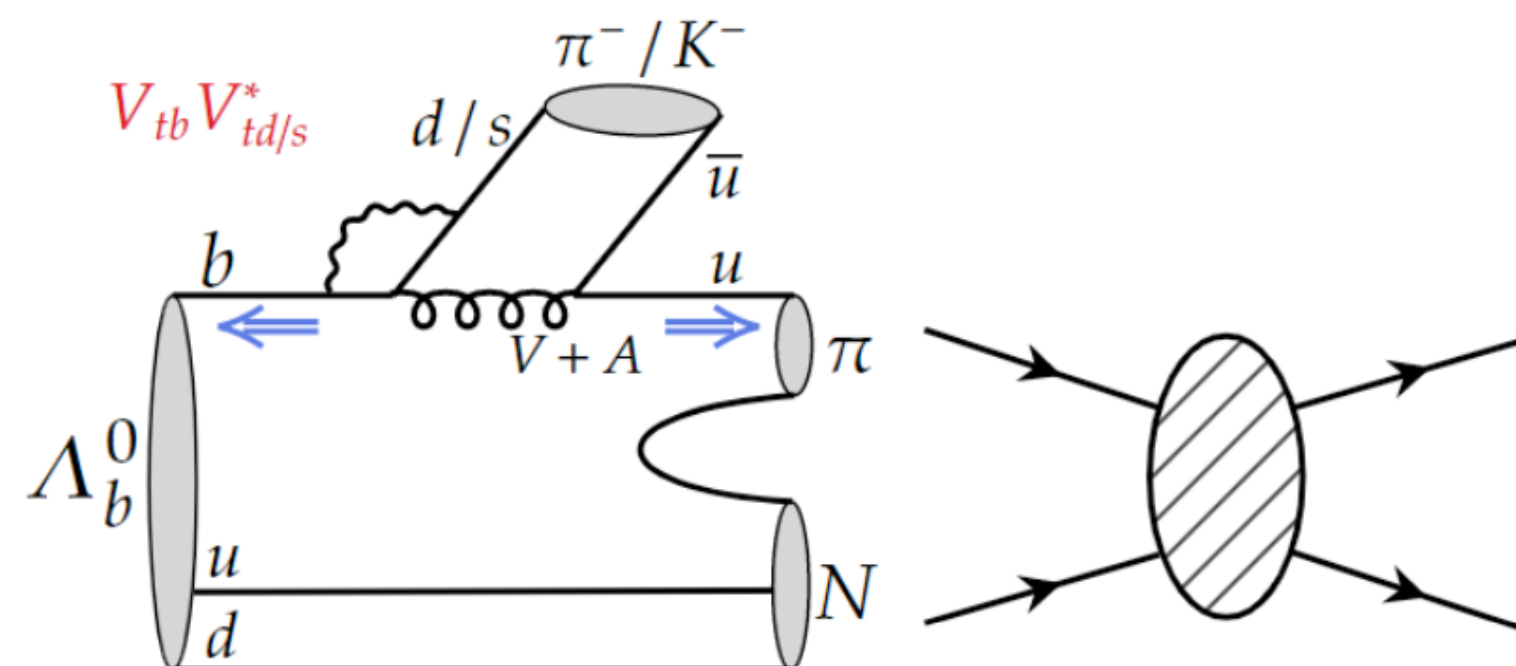
CPV via $N\pi$ rescatterings

•Tree:



$$\mathcal{A} = \mathcal{S}^{1/2} \mathcal{A}_0$$

•Penguin:



- Different chirality
- ➔ different helicity
- ➔ different partial waves
- ➔ PWA interference
- ➔ difference of strong phases
- ➔ **CPV**

- Short-distance weak decays
- Long-distance $N\pi \rightarrow N\pi, N\pi\pi$
- weak phases
- strong phases

J.P.Wang, **FSY**, 2407.04110

$N\pi$ scatterings

- N^* usually from $N\pi$ scatterings
- Data from SAID program

<https://gwdac.phys.gwu.edu/>



— Data Analysis Center —

Institute for Nuclear Studies

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

[INS DAC Home](#)
[INS DAC \[SAID\]](#)
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[Obituary R.A. Arndt](#)

Partial-Wave Analyses at GW
[\[See Instructions \]](#)
[Pion-Nucleon](#)
[Pi-Pi-N](#)
[Kaon\(+\)-Nucleon](#)
[Nucleon-Nucleon](#)
[Pion Photoproduction](#)
[Pion Electroproduction](#)
[Kaon Photoproduction](#)
[Eta Photoproduction](#)
[Eta-Prime Photoproduction](#)
[Pion-Deuteron \(elastic\)](#)
[Pion-Deuteron to Proton+Proton](#)

INS DAC Services [SAID Program]

- The SAID Partial-Wave Analysis Facility is based
- New features are being added and will first appear always welcome.

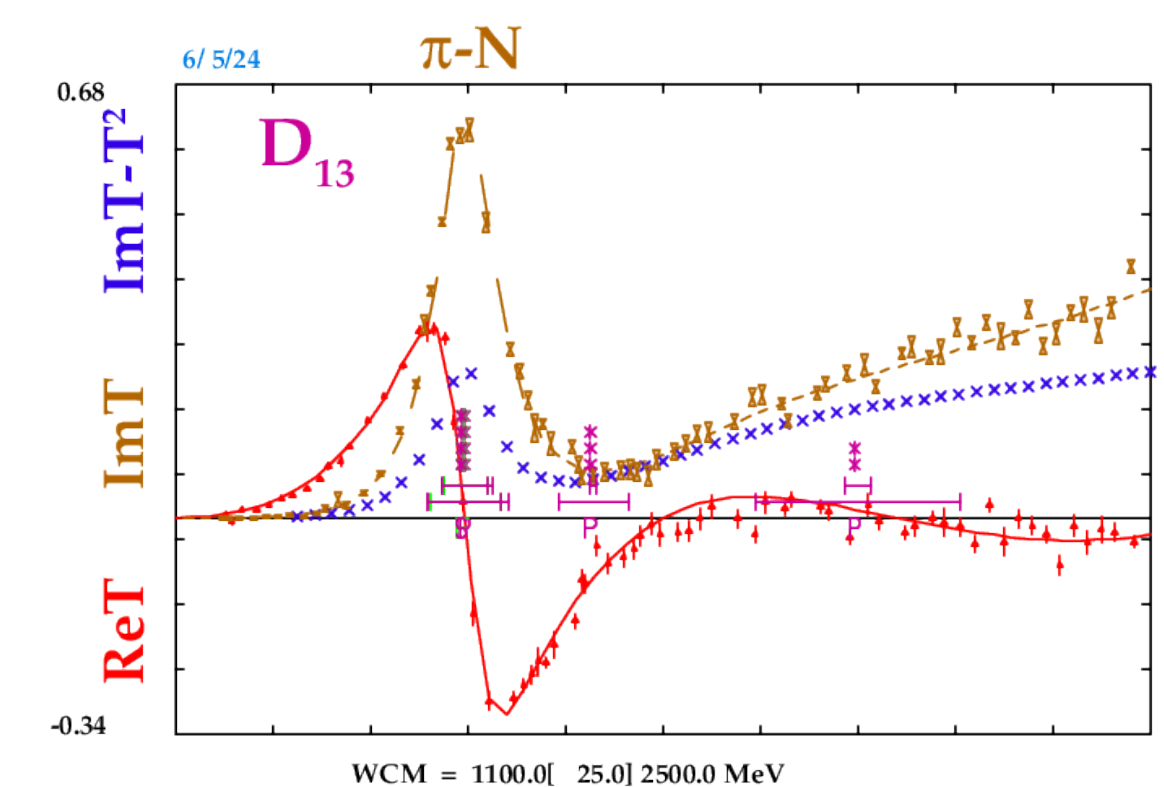
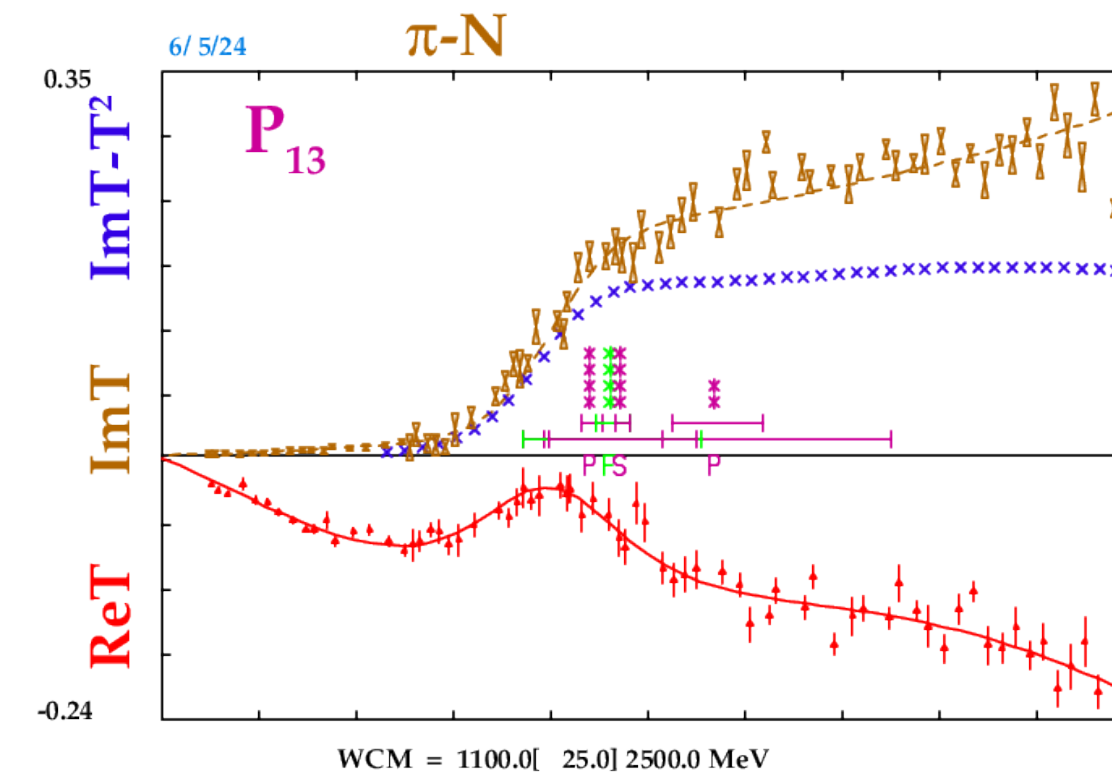
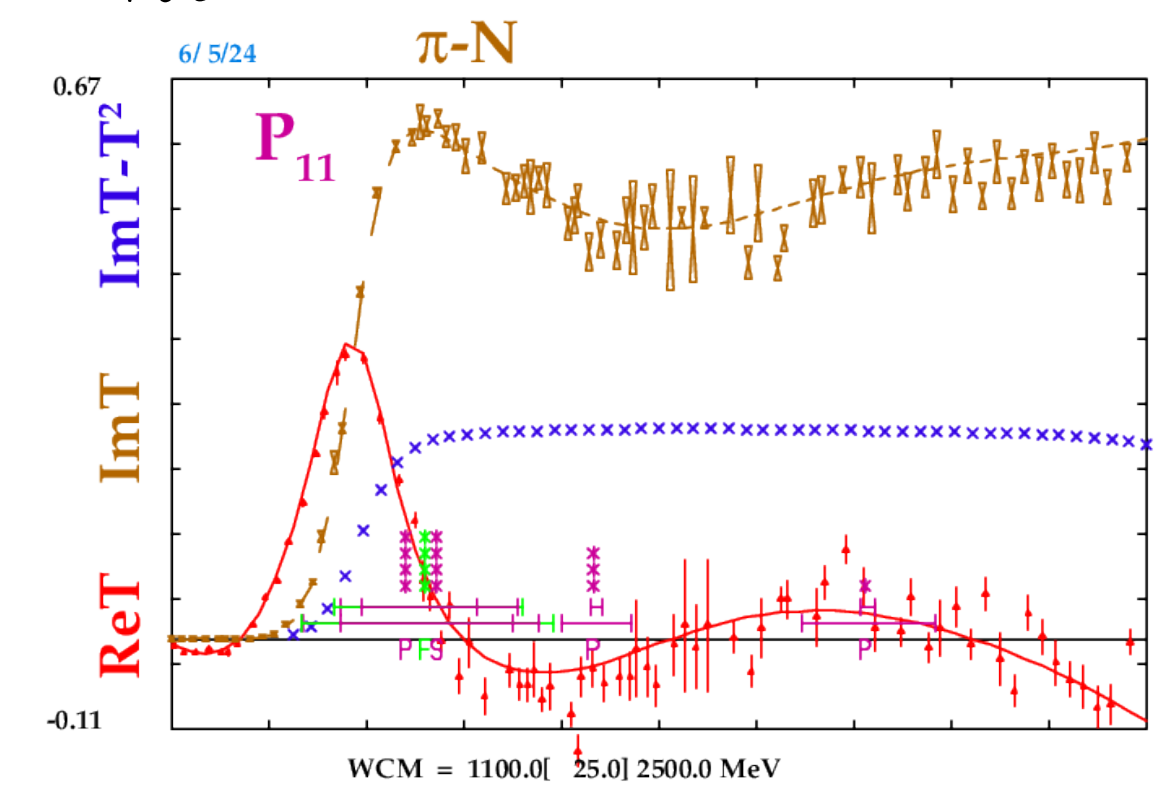
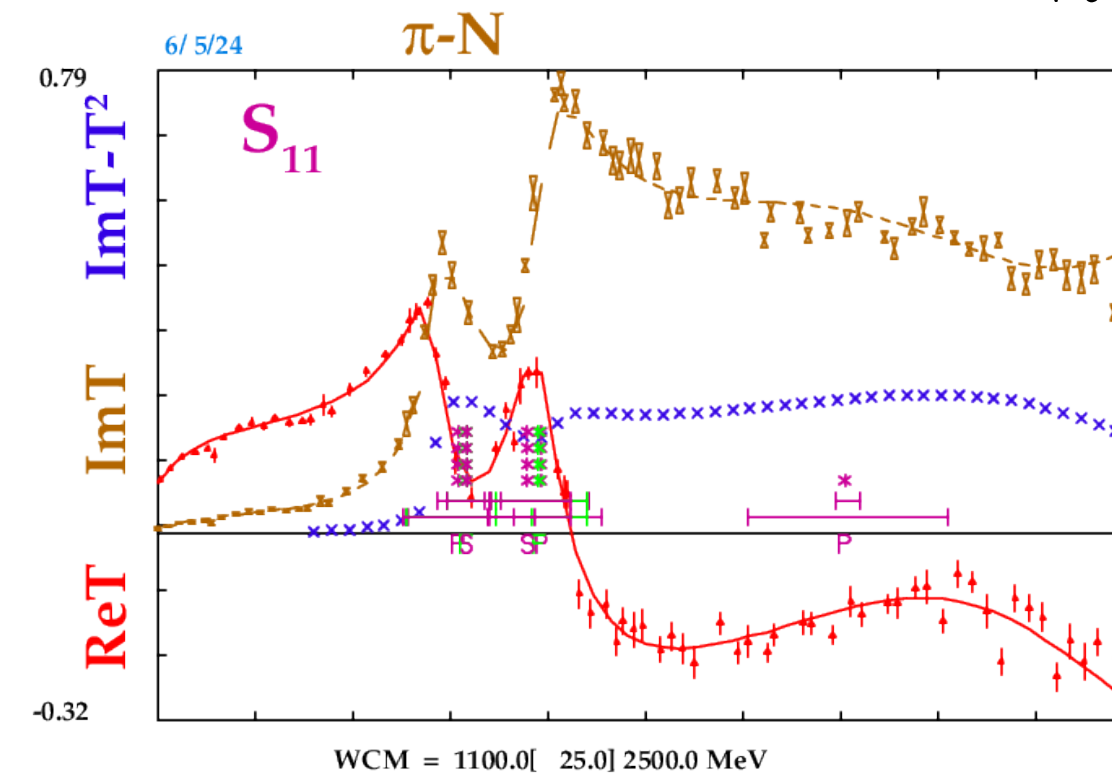
Instructions for Using the Partial-Wave Analyses

The programs accessible with the left-hand side navigation t available through the SAID program. Contact a member of c If you enter choices which are unphysical, you may still get garbage out' rule). Please report unexpected garbage-out to t

Note: These programs use HTML forms to run the SAID co setup first. The output is an (edited) echo of an interactive se SSH version. If the default example fails to clarify the speci mail message).

All programs expect energies in MeV units. All of the soluti Some are unstable beyond their upper energy limits. Extrapc **Increments:** The programs will not allow an arbitrary numb

$$N\pi \rightarrow N\pi$$



- Partial-wave amplitudes with strong phases!

- Data driven, **model independent**. Skip resonances, more precise strong phases.

CPV with $N\pi$ scatterings

$N\pi \rightarrow \Delta^{++}\pi^-$ $m_{N\pi} \in [1.2, 1.9]\text{GeV}$	decay processes	Scenarios	global CPV	CPV of $\cos\theta < 0$	CPV of $\cos\theta > 0$
	$\Lambda_b^0 \rightarrow (\Delta^{++}\pi^-)K^-$	S1	5.9%	8.0%	3.6%
		S2	5.8%	6.3%	5.3%
		S3	5.6%	4.3%	7.0%

J.P.Wang, **FSY**, 2407.04110 (CPC2024)

↓

LHCb: $(5.4 \pm 0.9 \pm 0.1) \%$

•LHCb:
2503.16954

a model-independent investigation of angular distributions [36] or utilising scattering data to extract the hadronic amplitude [28]. Applying this method using $\pi^+n \rightarrow p\pi^+\pi^-$ scattering data [37], an estimate of the CP asymmetry in $\Lambda_b^0 \rightarrow R(p\pi^+\pi^-)K^-$ decays aligns with the measurement in this work.

[28] J.-P. Wang and F.-S. Yu, *CP violation of baryon decays with $N\pi$ rescatterings*, [Chin. Phys. C48 \(2024\) 101002](#), [arXiv:2407.04110](#).

总结

- 不同的理论方法各有优缺点，在不同的衰变道研究中适用不同的理论方法
 - Λ_b^0 (准)两体非轻衰变（重子基态）： PQCD方法
 - Λ_b^0 准两体非轻衰变（一两个重子激发态）： 末态相互作用三角图方法
 - Λ_b^0 多体非轻衰变（多个重子激发态）： 末态重散射数据方法 See Qin Qin's talk
- 理论与实验紧密结合，共同推进重味物理发展

Thank you !

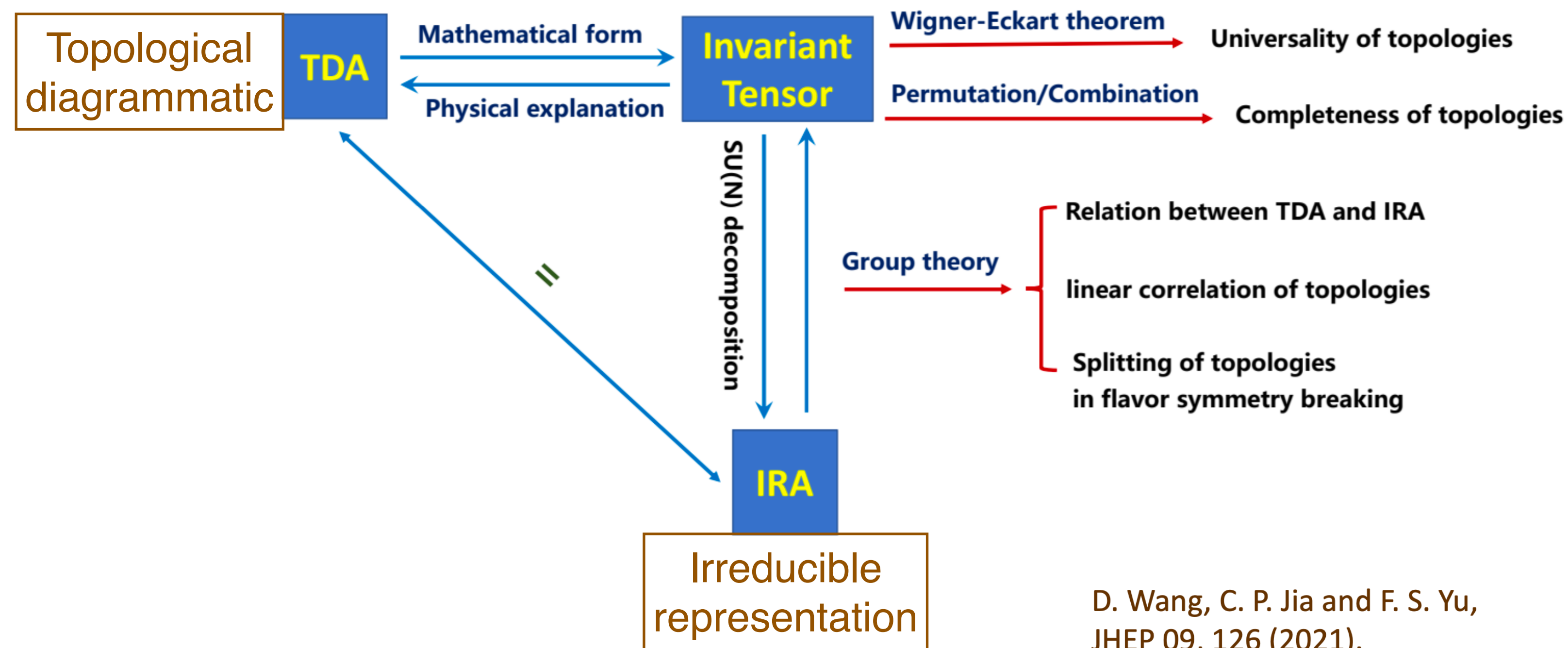
Backups

Theoretical methods for hadronic weak decays

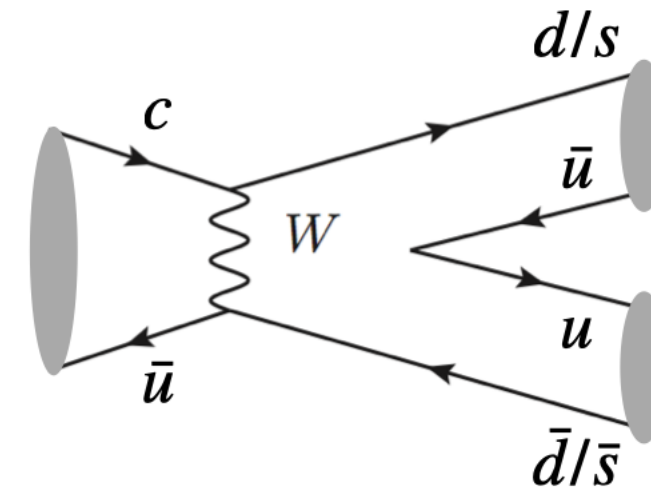
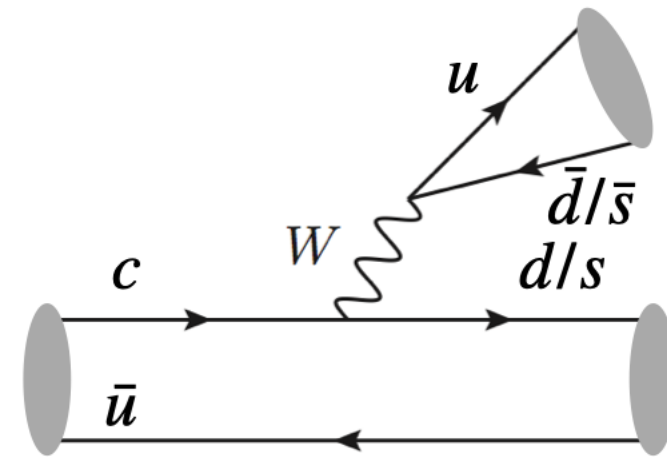
Theoretical approaches	Advantages	Disadvantages
QCD-inspired : QCDF, PQCD, SCET	(Almost) first-principle for dynamics, very predictive for B decays	Difficult for non-perturbative contributions, thus difficult for charm
Final-state interaction	Dynamics for non-perturbations	Suffer very large theoretical uncertainties
SU(3) irreducible representation	Based on approximate flavor symmetry, no additional assumptions	No link to dynamics
Topological diagrams	Include non-perturbations, successful for charm phenomenology	Mathematical foundation is not clear

Topological diagrams = Irreducible representations

- The Equivalence was firstly pointed out by [X.G.He, W.Wang, 2018]
- The invariant tensors are the bridge between the two approaches.
- One topological diagram is found independent.

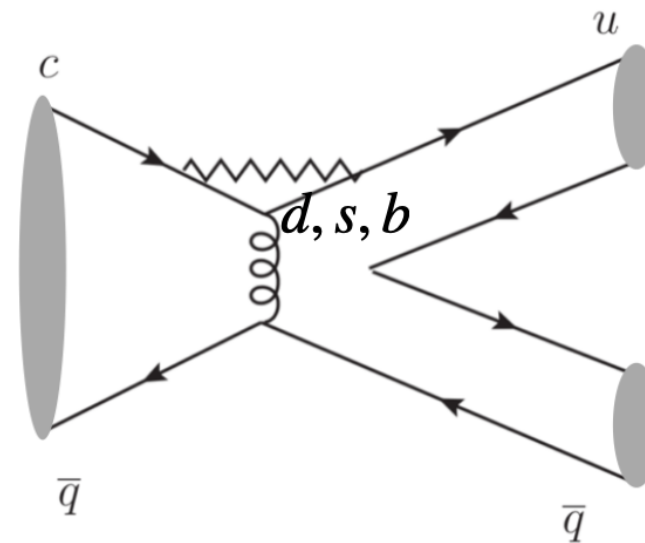
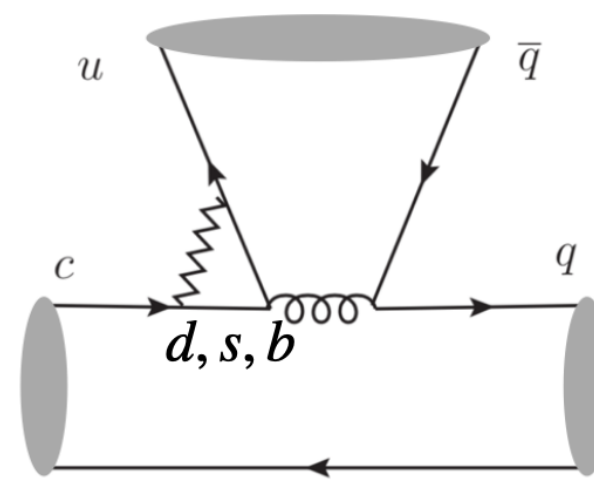


Tree



Tree diagrams are determined by data of branching fractions
Understand the dynamics at 1GeV

Penguin



Relate the penguins to the trees, with the known dynamics at 1GeV

CPV

$$\Delta A_{CP} = -2r \sin \gamma \left(\frac{|\mathcal{P}^{KK}|}{|\mathcal{T}^{KK}|} \sin \delta^{KK} + \frac{|\mathcal{P}^{\pi\pi}|}{|\mathcal{T}^{\pi\pi}|} \sin \delta^{\pi\pi} \right)$$

Then reliably predict charm CPV

Branching Fractions: topological diagrams

- Hierarchy of topological diagrams in heavy quark expansion

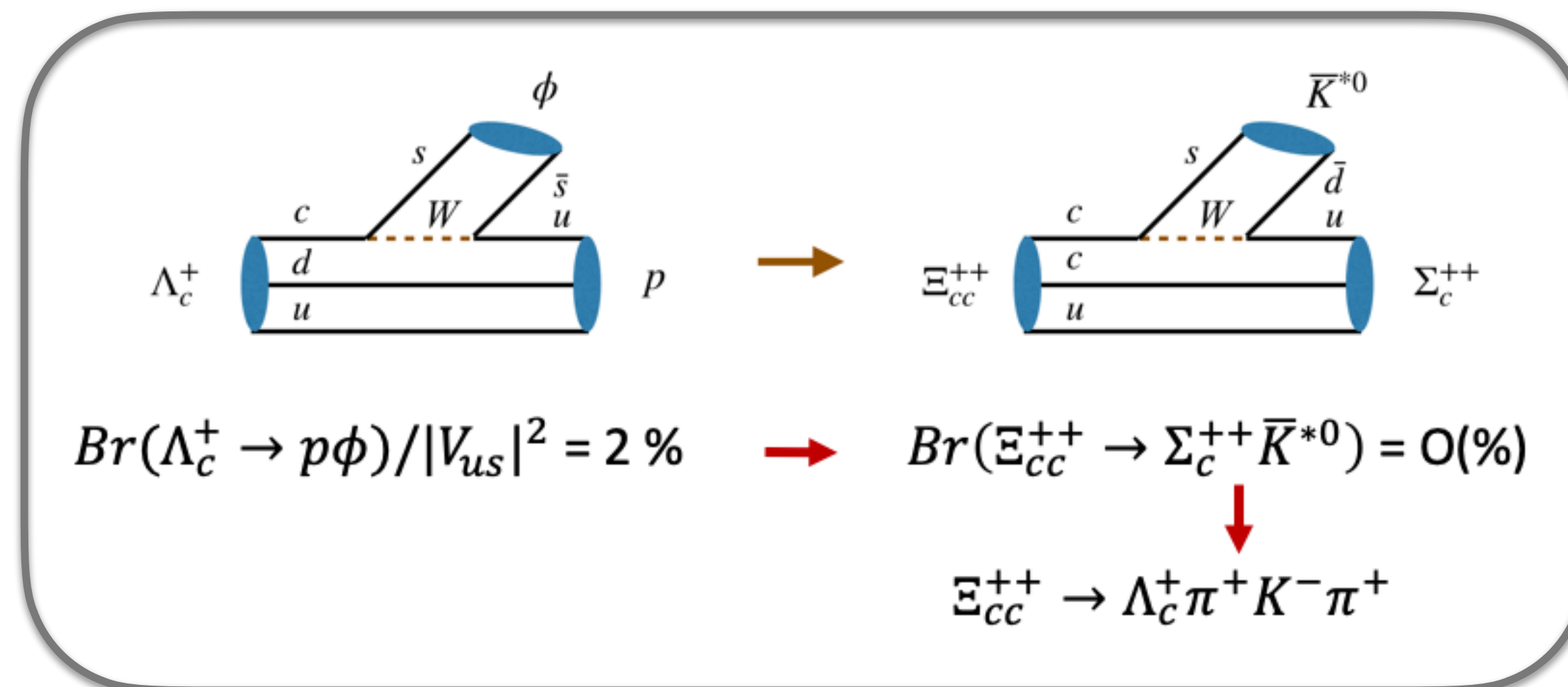
SCET: $|C/T| \sim |C'/T| \sim |E/T| \sim O(\Lambda_{\text{QCD}}/m_Q)$

Leibovich, Ligeti, Stewart, Wise, 2004

charm decay: $|C/T| \sim |C'/T| \sim |E/T| \sim O(\Lambda_{\text{QCD}}/m_c) \sim 1$

- BESIII measurements on Λ_c^+ decays are important

BESIII, 2016



$\Lambda_c^+ \rightarrow$

Modes	Representation	$\mathcal{B}_{\text{exp}}$
$p\bar{K}^0$	$\lambda_{sd}(C + E)$	$(3.04 \pm 0.17)\%$
$\Lambda^0\pi^+$	$\lambda_{sd}(T - C' + B - E)/\sqrt{2}$	$(1.24 \pm 0.08)\%$
$\Delta^{++}K^-$	$\lambda_{sd}E$	$(1.18 \pm 0.27)\%$

So far dynamics is almost ready

Effective Hamiltonian and Four-fermion operators

Current–Current:

$$Q_1 = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{s}_\beta c_\alpha)_{V-A} \quad Q_2 = (\bar{c}b)_{V-A} (\bar{s}c)_{V-A}$$

QCD–Penguins :

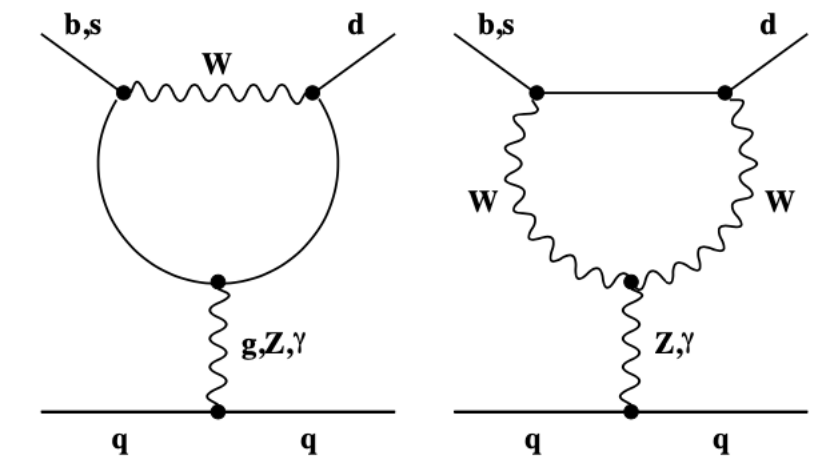
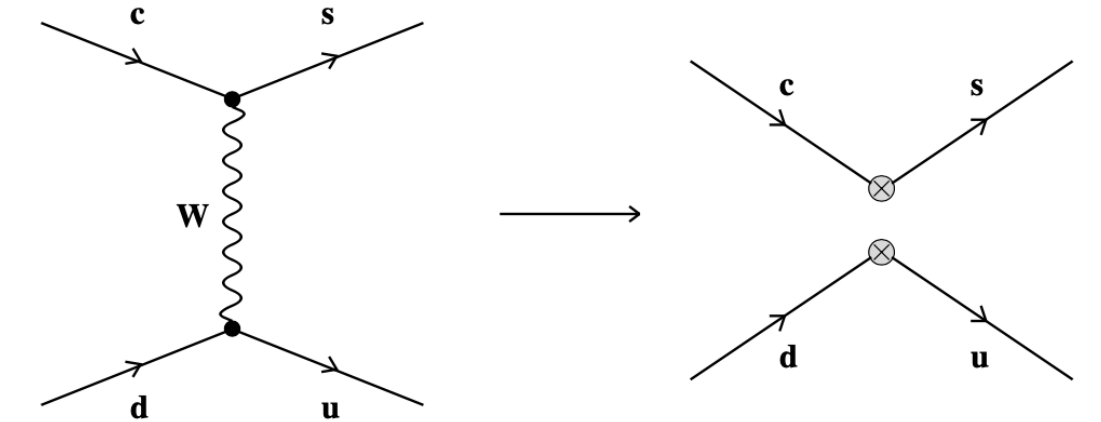
$$Q_3 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A} \quad Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A} \quad Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}$$

Electroweak-penguin:

$$Q_7 = \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}q)_{V+A} \quad Q_9 = \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}q)_{V-A}$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}_\beta q_\alpha)_{V+A} \quad Q_{10} = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q (\bar{q}_\beta q_\alpha)_{V-A}$$



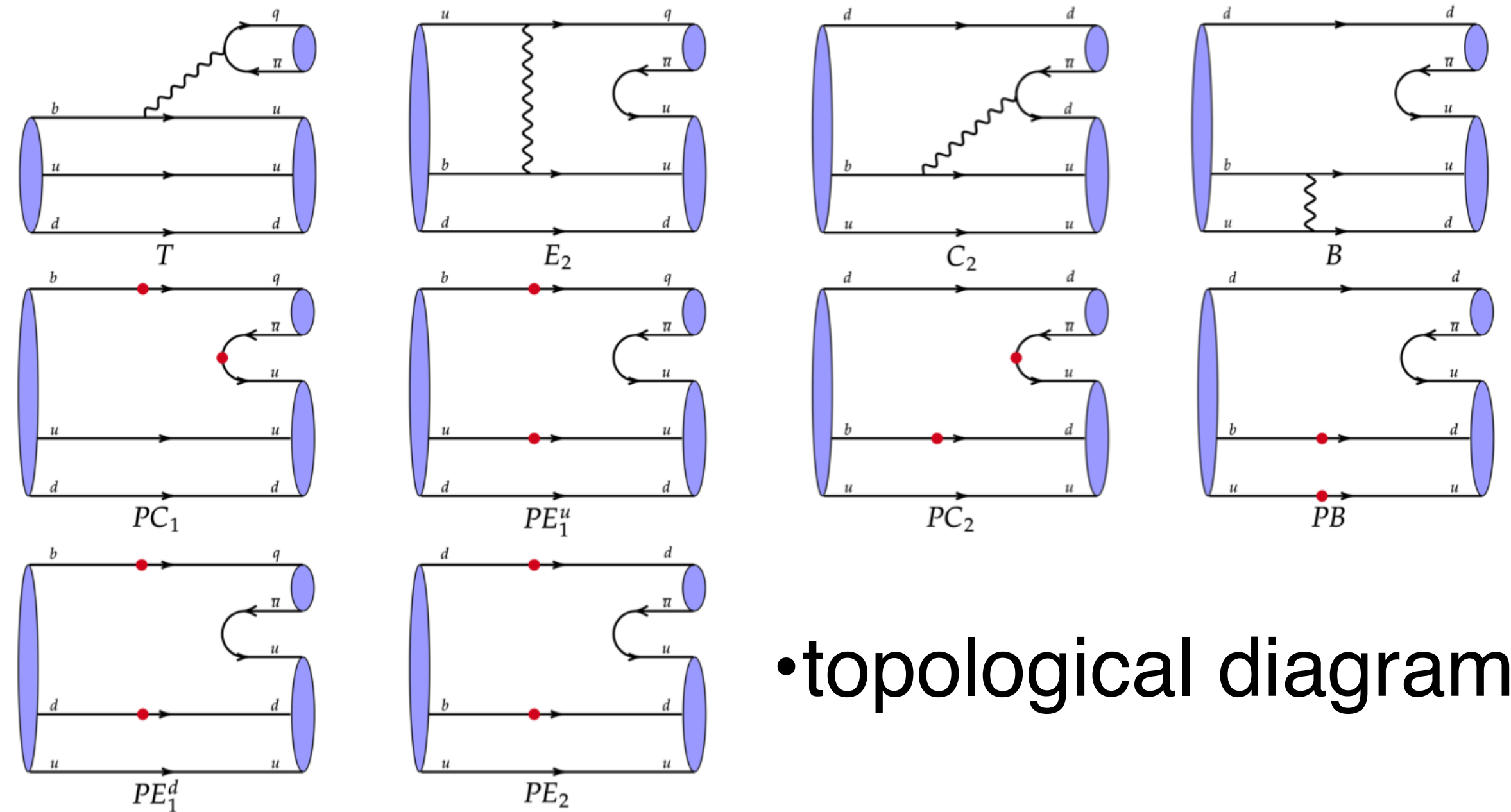
See Xin-Qiang Li's talk

Buras, hep-ph/9806471 (900 citations)

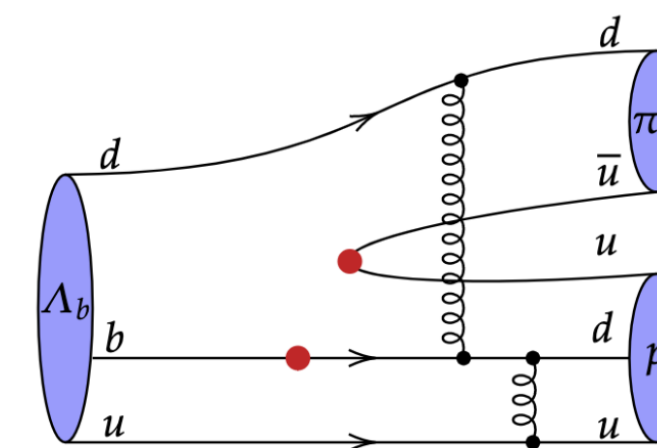
Buchalla, Buras, Lautenbacher, Rev.Mod.Phys 1996 (3000 citations)

Theoretical approach for dynamics

- The above crude argument needs to be justified by comprehensive QCD calculations
- There are more non-factorizable topological diagrams, such as PC2 and the exchange diagrams PE1, PE2
- They can be calculated by PQCD based on the k_T factorization



• topological diagrams



• Feynman diagram

$\Lambda_b \rightarrow p$ form factors in PQCD

- In 2009, form factors are two orders smaller than LatticeQCD/experiments, considering only the **leading twist** of LCDAs [C.D.Lu, Y.M.Wang, et al, 2009]
- In 2022, considering **high-twist** LCDAs, results are consistent with Lattice QCD [J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, **FSY**, 2022]. Consistent with power counting by SCET.

	Lattice/exp	PQCD(2009)	PQCD(2022)
$f_1^{\Lambda_b \rightarrow p}(0)$	0.22 ± 0.08	0.002 ± 0.001	0.27 ± 0.12

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3 ⁺⁻	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3 ⁻⁺	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	0.25	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

Direct CPV

$$\mathcal{M} = \bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$\Gamma = \frac{|\vec{p}|}{8\pi M^2} (|S|^2 + |P|^2), \quad \bar{\Gamma} = \frac{|\vec{p}|}{8\pi M^2} (|\bar{S}|^2 + |\bar{P}|^2)$$

$$S = |S_t|e^{i\delta_{s,t}}e^{i\phi_t} + |S_p|e^{i\delta_{s,p}}e^{i\phi_p}$$

$$P = |P_t|e^{i\delta_{p,t}}e^{i\phi_t} + |P_p|e^{i\delta_{p,p}}e^{i\phi_p}$$

$$\bar{S} = - \left\{ |S_t|e^{i\delta_{s,t}}e^{-i\phi_t} + |S_p|e^{i\delta_{s,p}}e^{-i\phi_p} \right\}$$

$$\bar{P} = |P_t|e^{i\delta_{p,t}}e^{-i\phi_t} + |P_p|e^{i\delta_{p,p}}e^{-i\phi_p}$$

- Four strong phases
- Two terms of CPV

Direct CPV

$$\mathcal{M} = \bar{u}_p(S + P\gamma_5)u_{\Lambda_b} \quad \Gamma = \frac{|\vec{p}|}{8\pi M^2} (|S|^2 + |P|^2), \quad \bar{\Gamma} = \frac{|\vec{p}|}{8\pi M^2} (|\bar{S}|^2 + |\bar{P}|^2)$$

$$S = |S_t|e^{i\delta_{s,t}}e^{i\phi_t} + |S_p|e^{i\delta_{s,p}}e^{i\phi_p}$$

$$P = |P_t|e^{i\delta_{p,t}}e^{i\phi_t} + |P_p|e^{i\delta_{p,p}}e^{i\phi_p}$$

$$\bar{S} = - \left\{ |S_t|e^{i\delta_{s,t}}e^{-i\phi_t} + |S_p|e^{i\delta_{s,p}}e^{-i\phi_p} \right\}$$

$$\bar{P} = |P_t|e^{i\delta_{p,t}}e^{-i\phi_t} + |P_p|e^{i\delta_{p,p}}e^{-i\phi_p}$$

- Four strong phases
- Two terms of CPV

$$\begin{aligned} a_{CP}^{dir} &= \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = \frac{|S|^2 + |P|^2 - |\bar{S}|^2 - |\bar{P}|^2}{|S|^2 + |P|^2 + |\bar{S}|^2 + |\bar{P}|^2} \\ &= - \frac{\sin(\delta_{s,t} - \delta_{s,p}) + r \sin(\delta_{p,t} - \delta_{p,p})}{K + [\cos(\delta_{s,t} - \delta_{s,p}) + r \cos(\delta_{p,t} - \delta_{p,p})] \cos \Delta\phi} \sin \Delta\phi \end{aligned}$$

Direct and partial-wave CPVs

$$\mathcal{A}(\Lambda_b \rightarrow ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$A_{CP}^{\text{dir}}(\Lambda_b \rightarrow ph) \equiv \frac{\Gamma(\Lambda_b \rightarrow ph) - \bar{\Gamma}(\bar{\Lambda}_b \rightarrow \bar{p}\bar{h})}{\Gamma(\Lambda_b \rightarrow ph) + \bar{\Gamma}(\bar{\Lambda}_b \rightarrow \bar{p}\bar{h})} \quad \Gamma \propto |S|^2 + \kappa|P|^2 \quad \kappa \approx 0.5$$

$$A_{CP}^{S\text{-wave}} \equiv \frac{|S|^2 - |\bar{S}|^2}{|S|^2 + |\bar{S}|^2}, \quad A_{CP}^{P\text{-wave}} \equiv \frac{|P|^2 - |\bar{P}|^2}{|P|^2 + |\bar{P}|^2}.$$

$$A_{CP}^{\text{dir}} \approx \kappa_S A_{CP}^{S\text{-wave}} + \kappa_P A_{CP}^{P\text{-wave}} \quad \kappa_S = \frac{|S|^2}{|S|^2 + \kappa|P|^2} \quad \kappa_P = \frac{\kappa|P|^2}{|S|^2 + \kappa|P|^2}$$

□ 末态相互作用

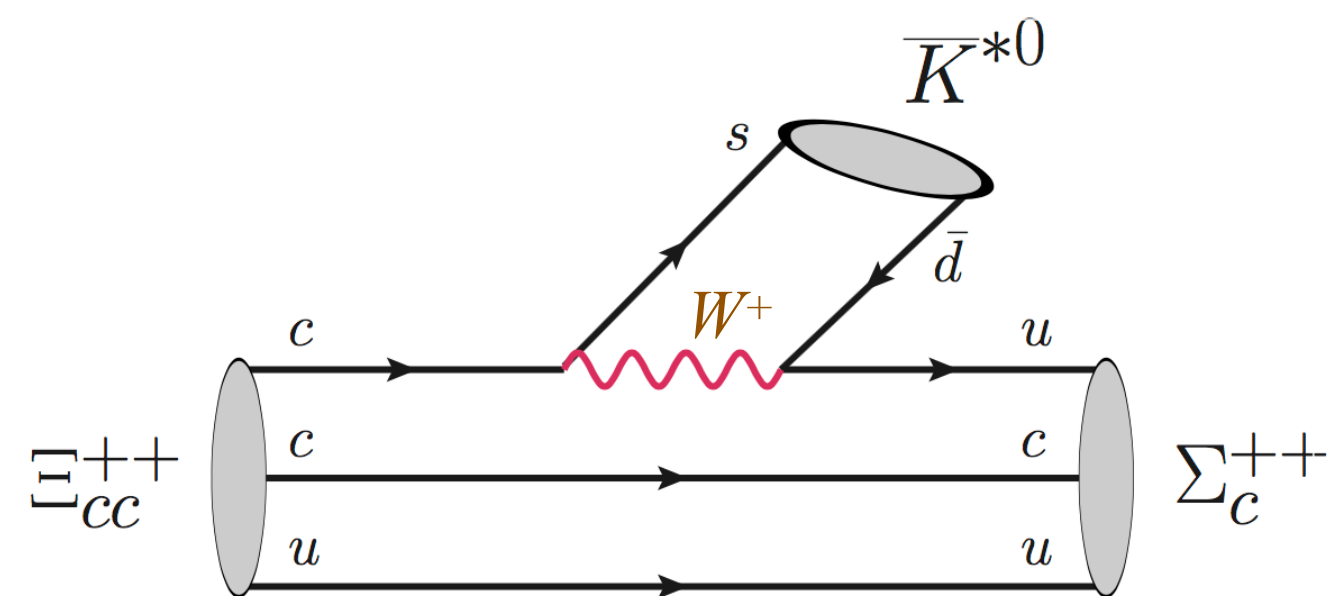
- 描述长程非微扰贡献的物理图像

$$\mathcal{A} = \mathcal{S}^{1/2} \mathcal{A}_0$$

随时间演化的算符矩阵元
强相互作用效应出现
并贡献强相位

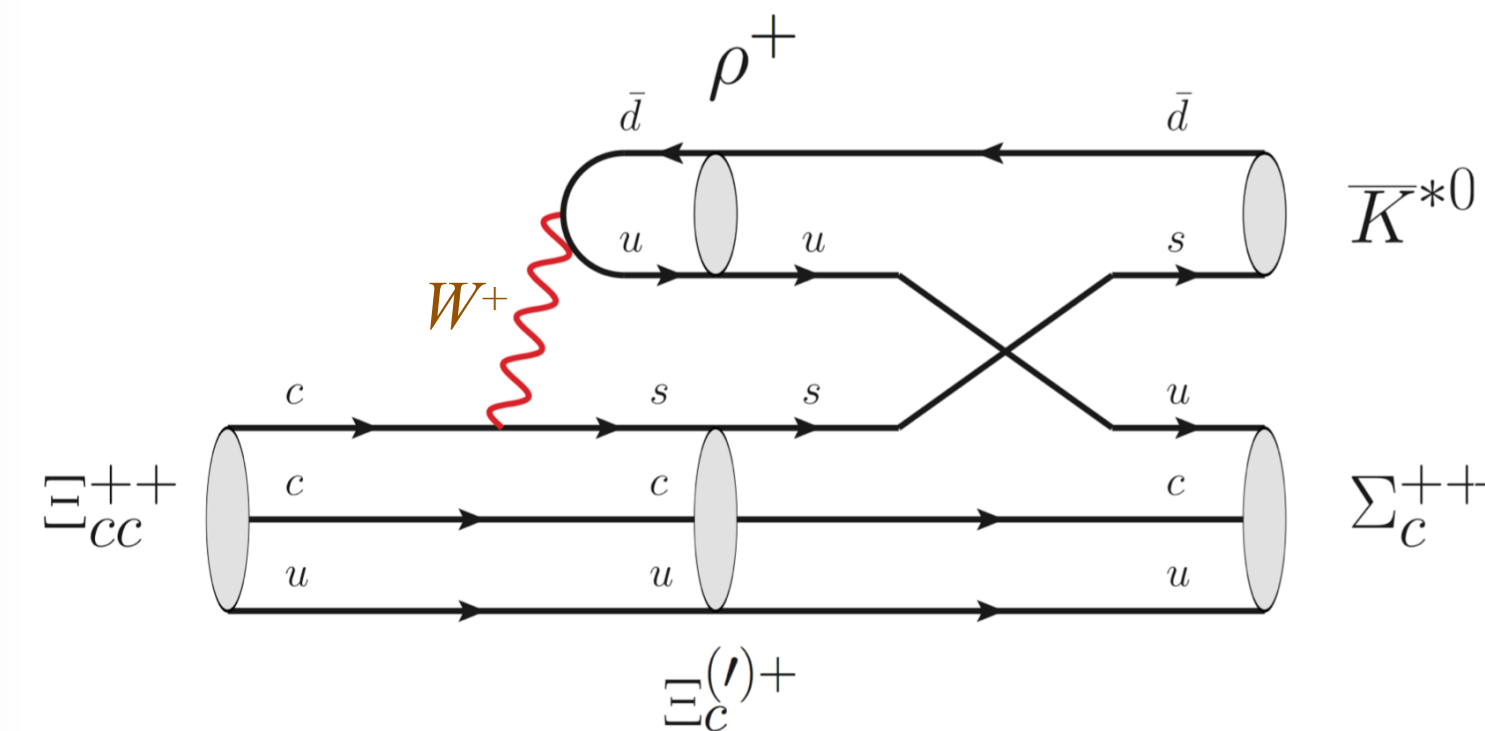
极短时间内的现象
短程的弱相互作用
无强相位

- 末态重散射机制



Topology: C

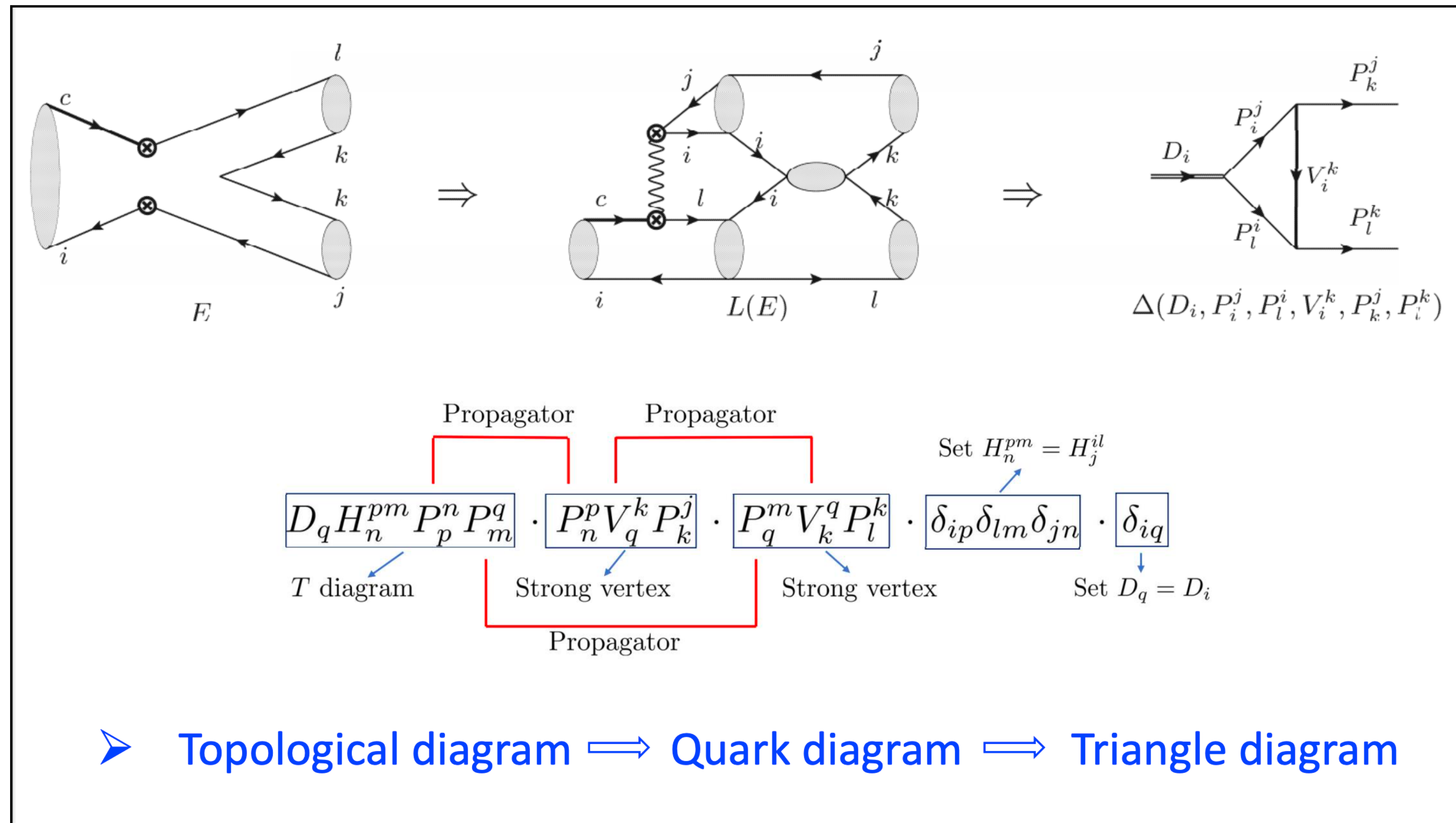
同一拓扑



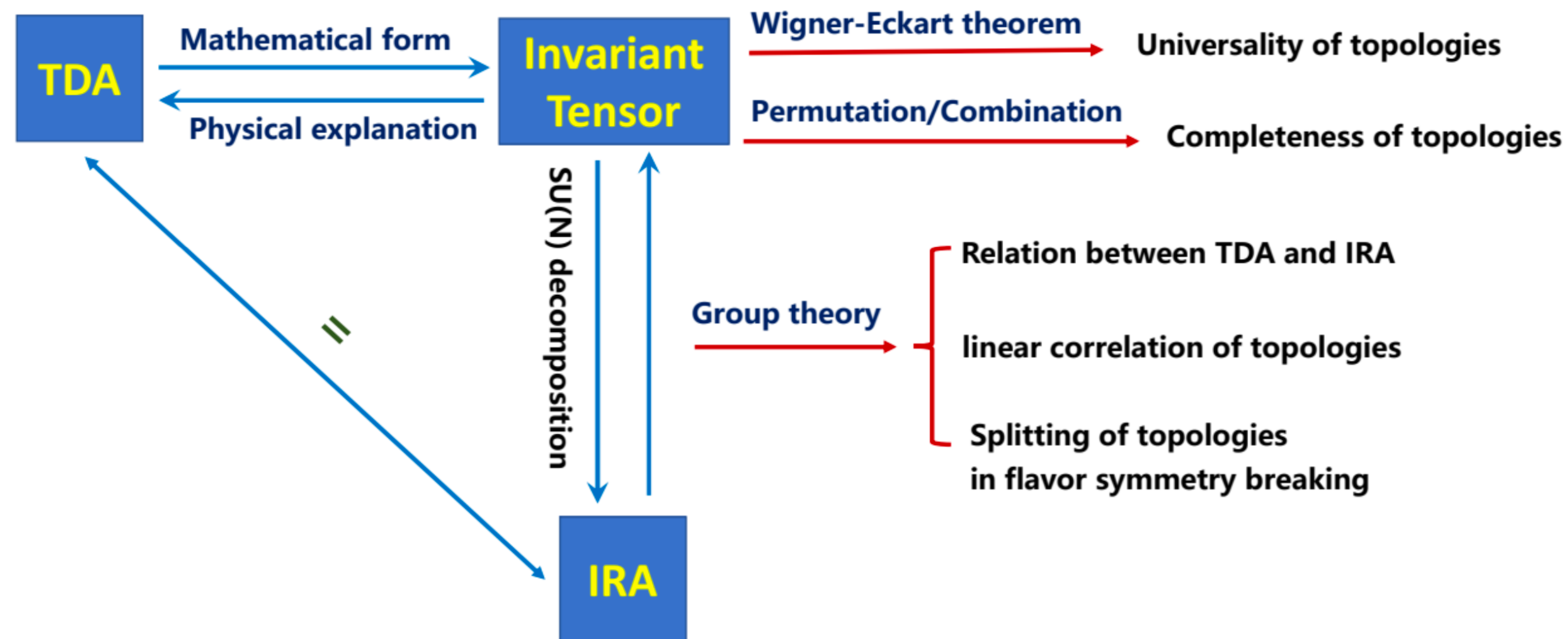
quark exchange

Final-state interaction = Topological diagrams

FSI = Long-distance contributions of topological diagrams [D.Wang, 2021]



FSI = QCD = Topological diagrams = SU(3) irreducible representations



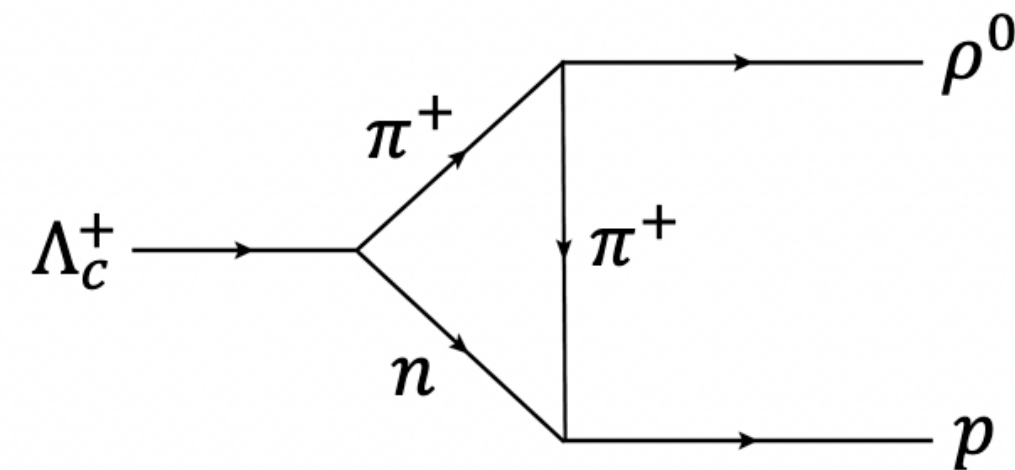
D. Wang, C. P. Jia and F. S. Yu,
JHEP 09, 126 (2021).

Triangle diagrams

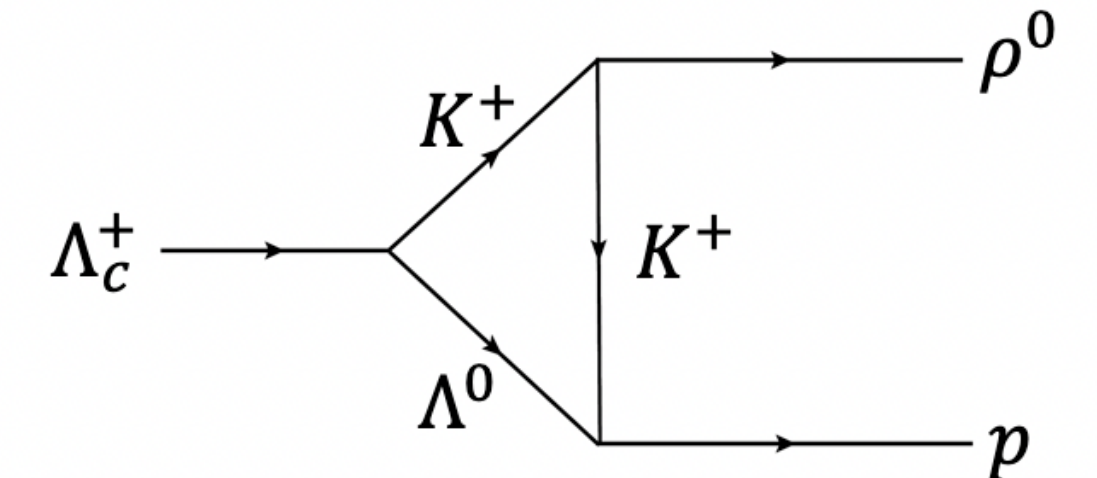
CPV can be easily obtained within the rescattering mechanism

weak phase

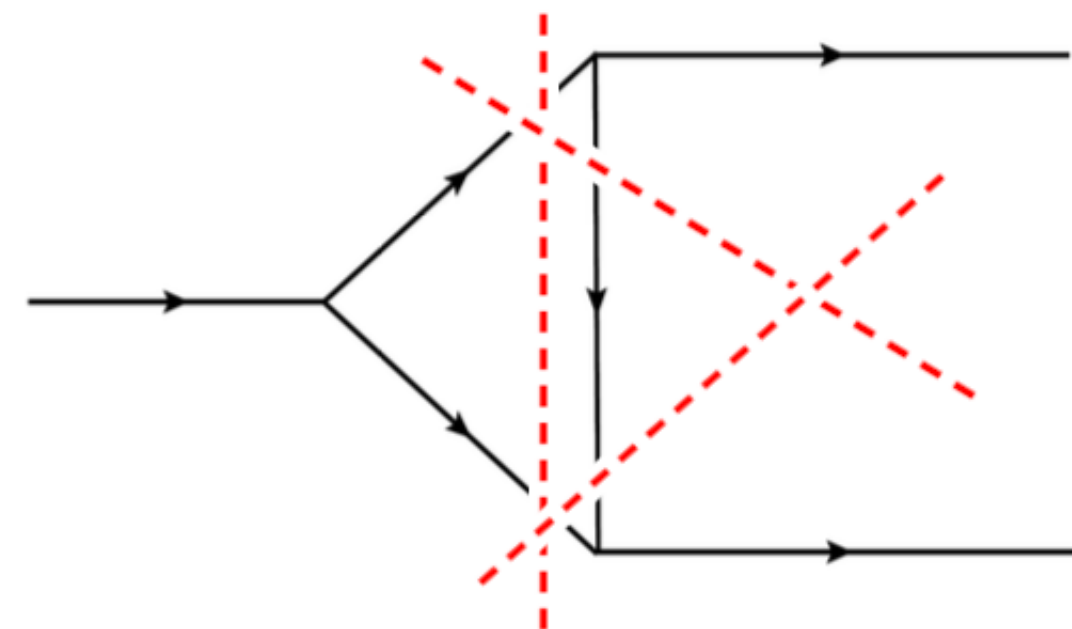
$$V_{ud}V_{cd}^*$$



$$V_{us}V_{cs}^*$$



strong phase from on-shell intermediate state

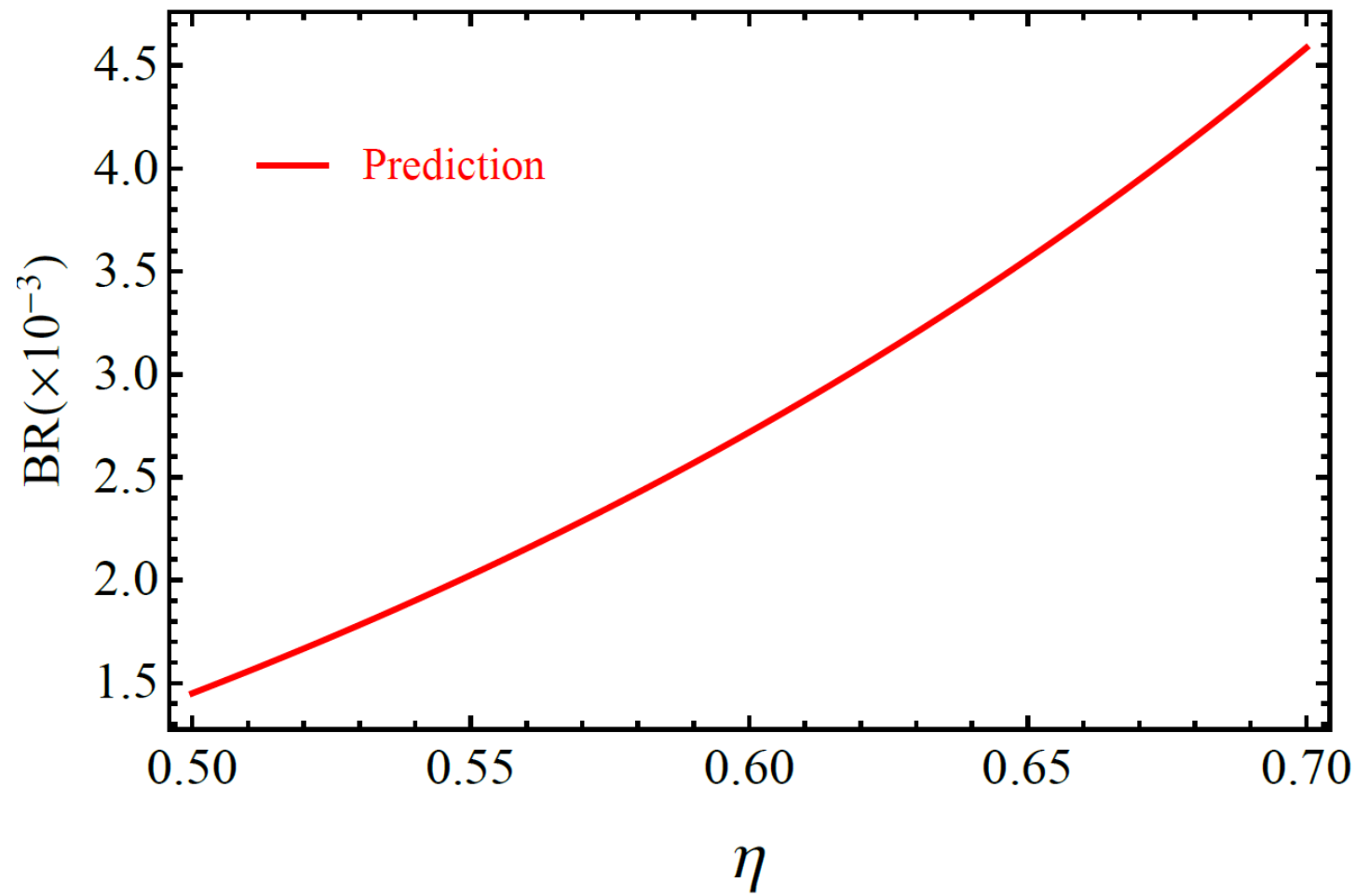


C.P.Jia, H.Y.Jiang, J.P.Wang, **FSY**, JHEP2024
Z.D.Duan, J.P.Wang, R.H.Li, C.D.Lu, **FSY**, 2412.20458

Dependence on η

Branching fractions

$$\Lambda_c^+ \rightarrow p\rho^0$$

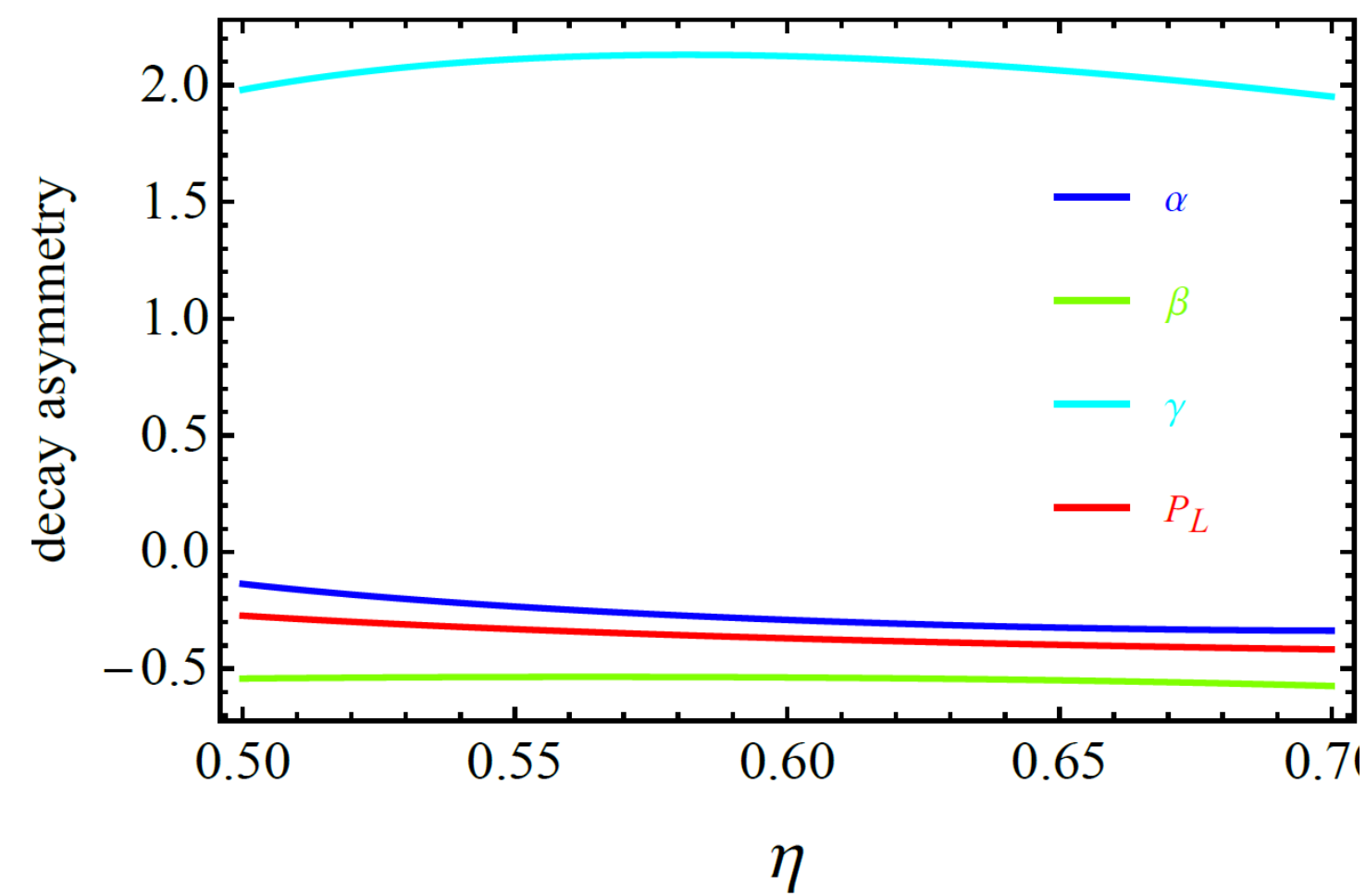


- Branching fractions are very sensitive to η

$$\Gamma \propto 2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2} (|S + D|^2 + |P_1|^2)$$

Decay asymmetries

$$\Lambda_c^+ \rightarrow p\rho^0$$



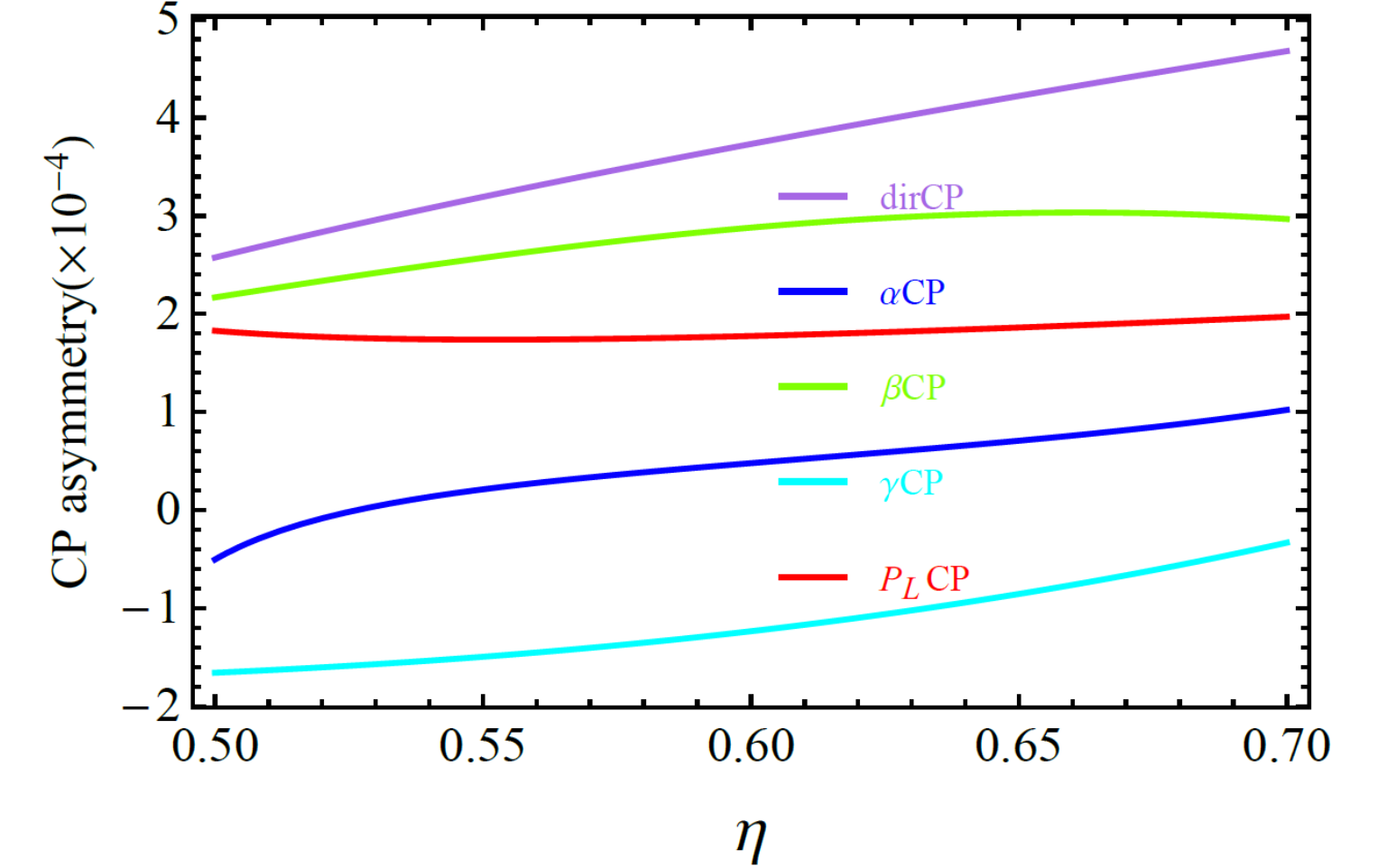
- The decay asymmetries and CPV are insensitive to η , whose dependences are mostly cancelled by the ratios

$$\alpha = \frac{\left| H_{1, \frac{1}{2}} \right|^2 - \left| H_{-1, -\frac{1}{2}} \right|^2}{\left| H_{1, \frac{1}{2}} \right|^2 + \left| H_{-1, -\frac{1}{2}} \right|^2}$$

$$A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

CP violations

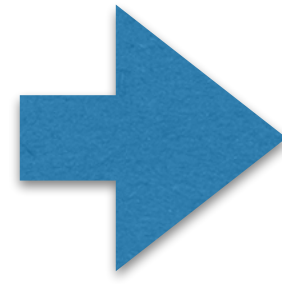
$$\Lambda_c^+ \rightarrow p\rho^0$$



CPV from $N\pi$ scatterings

$$\mathcal{A} = \mathcal{S}^{1/2} \mathcal{A}_0$$

$$\begin{aligned} \mathcal{A} = & \bar{u}_{N\pi,1/2^+} (A + B\gamma_5) u_{\Lambda_b} P_{11} \\ & + \bar{u}_{N\pi,1/2^-} (\tilde{A} + \tilde{B}\gamma_5) u_{\Lambda_b} S_{11} \end{aligned}$$



$$A = (\lambda_u a_1 - \lambda_t a_{46+}) f_1^{\frac{1}{2}+} m_-$$

$$B = (\lambda_u a_1 - \lambda_t a_{46-}) g_1^{\frac{1}{2}+} m_+$$

$$\tilde{A} = (-\lambda_u a_1 + \lambda_t a_{46-}) f_1^{\frac{1}{2}-} m_- \quad a_{46\pm} = a_4 \pm R_h a_6$$

$$\tilde{B} = (-\lambda_u a_1 + \lambda_t a_{46+}) g_1^{\frac{1}{2}-} m_+$$

$$\Lambda_b \rightarrow (N\pi)K^- : \quad \lambda_u = V_{ub} V_{us}^*, \quad \lambda_t = V_{tb} V_{ts}^*$$

$$\Lambda_b \rightarrow (N\pi)\pi^- : \quad \lambda_u = V_{ub} V_{ud}^*, \quad \lambda_t = V_{tb} V_{td}^*$$

$$m_{\pm} = m_{\Lambda_b} \pm m_{N\pi}$$

$$\mathcal{A}(\Lambda_b \rightarrow (\mathcal{B}M)h^-)$$

• Tree

$$= \lambda_u f_h \bar{u}_{N\pi} \left[a_1 \left(P_{11} f_1^{1/2+} - S_{11} f_1^{1/2-} + \dots \right) m_- + a_1 \left(P_{11} g_1^{1/2+} - S_{11} g_1^{1/2-} + \dots \right) m_+ \gamma_5 \right] u_{\Lambda_b}$$

• Penguin

$$+ \lambda_t f_h \bar{u}_{N\pi} \left[\left(a_{46+} P_{11} f_1^{1/2+} - a_{46-} S_{11} f_1^{1/2-} + \dots \right) m_- + \left(a_{46-} P_{11} g_1^{1/2+} - a_{46+} S_{11} g_1^{1/2-} + \dots \right) m_+ \gamma_5 \right] u_{\Lambda_b}$$

• weak phase difference

• strong phase difference

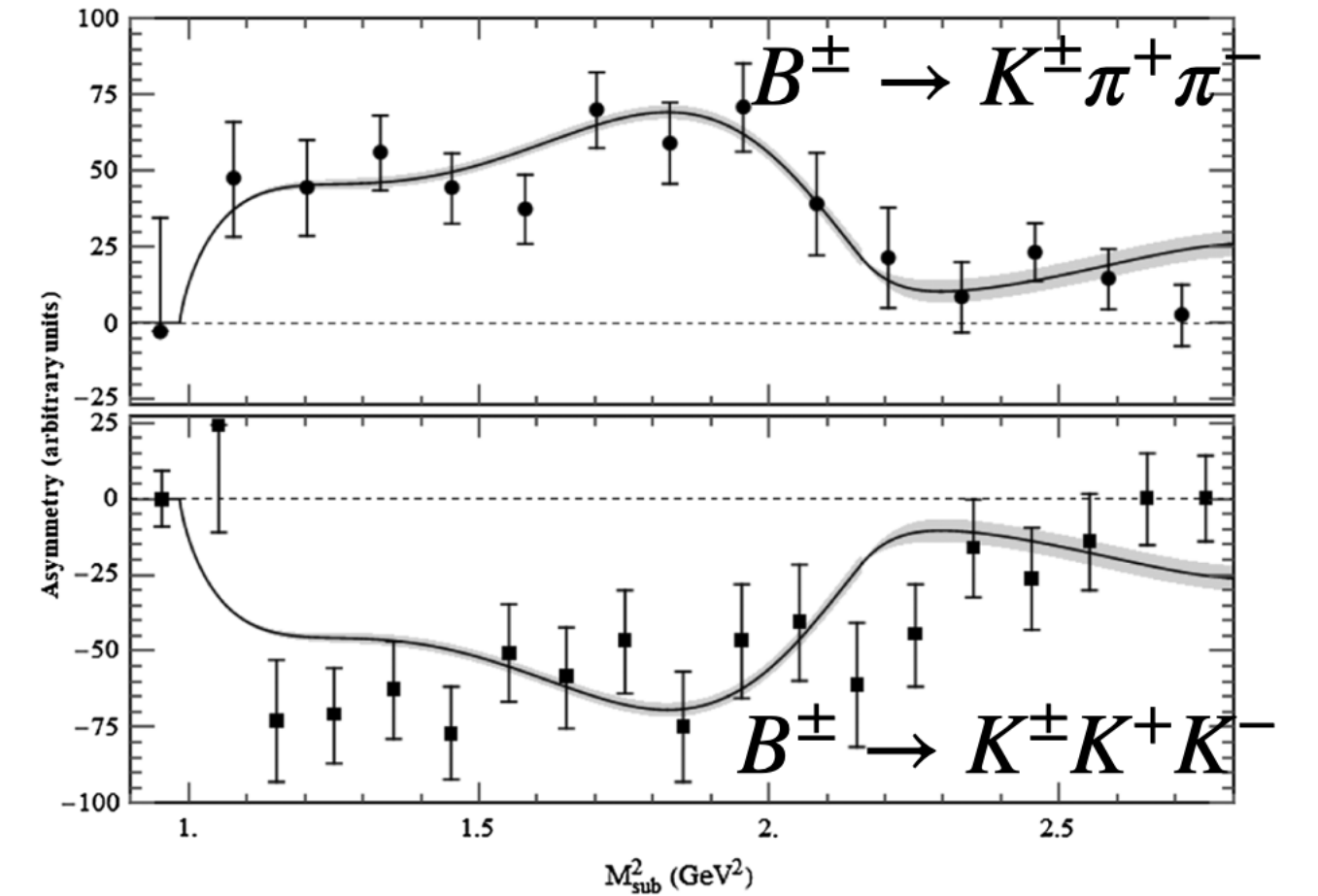
J.P.Wang, **FSY**, CPC48,101002(2024)

Rescatterings: Data driven

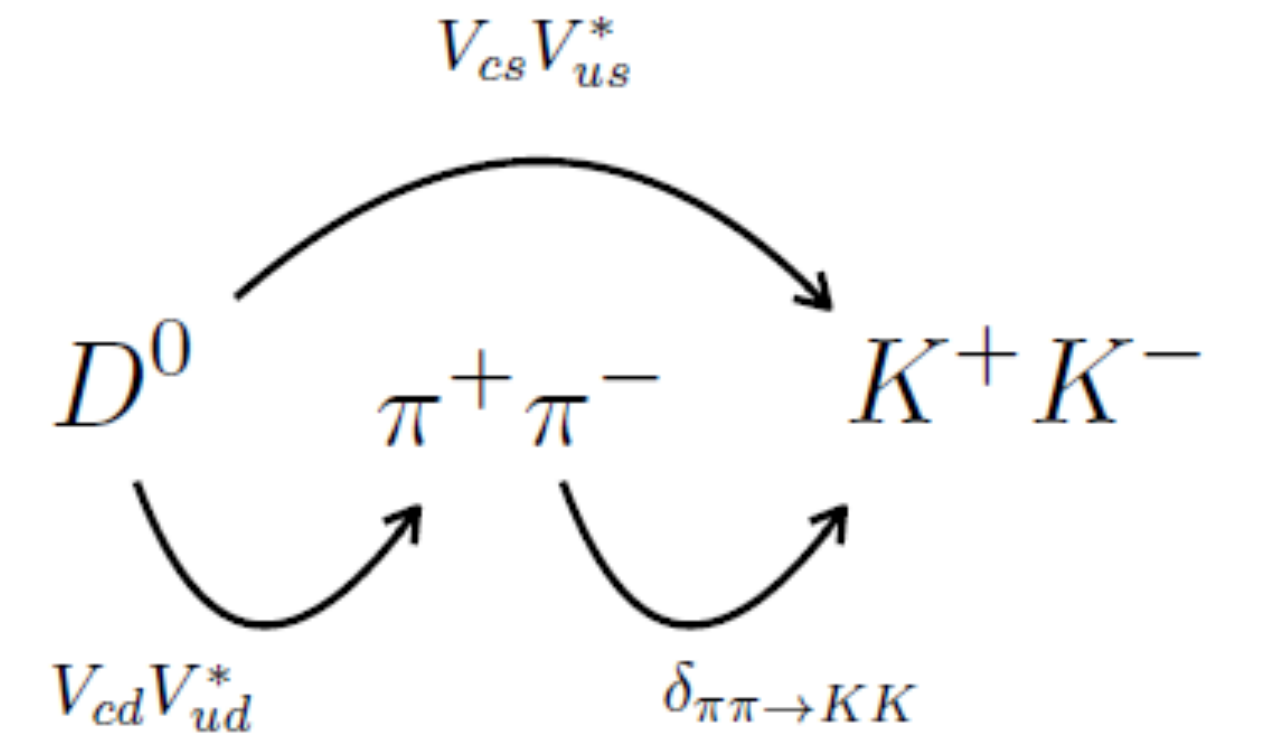
- Rescattering mechanism for CPV in $B^- \rightarrow (\pi^+ \pi^-) K^-$, $(K^+ K^-) K^-$.

Model-independent analysis of $\pi\pi \rightarrow K\bar{K}$ data [Bediaga, Frederico, Lourenco, 2013; H.Y.Cheng, C.K.Chua, 2020; Álvarez Garrote, Cuervo, Magalhães, Peláez, PRL2023]

$$\begin{pmatrix} A(B^- \rightarrow \pi^+ \pi^- P^-) \\ A(B^- \rightarrow K^+ K^- P^-) \end{pmatrix}_{\text{S-wave}}^{\text{FSI}} = S^{1/2} \begin{pmatrix} A(B^- \rightarrow \pi^+ \pi^- P^-) \\ A(B^- \rightarrow K^+ K^- P^-) \end{pmatrix}_{\text{S-wave}}$$



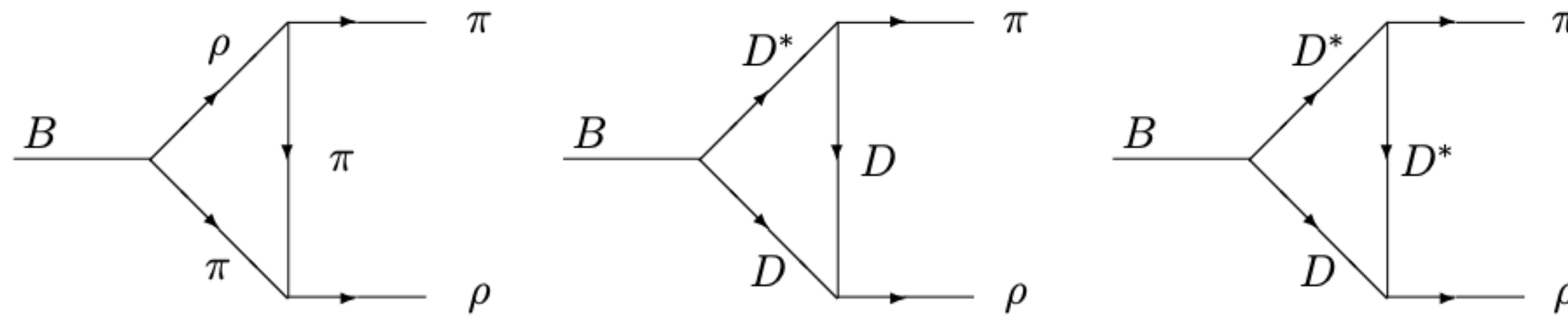
- Rescattering mechanism for charm CPV. Model-independent analysis of $\pi\pi \rightarrow K\bar{K}$ data [Bediaga, Frederico, Magalhaes, PRL2023; Pich, Solomonidi, Silva, PRD2023].



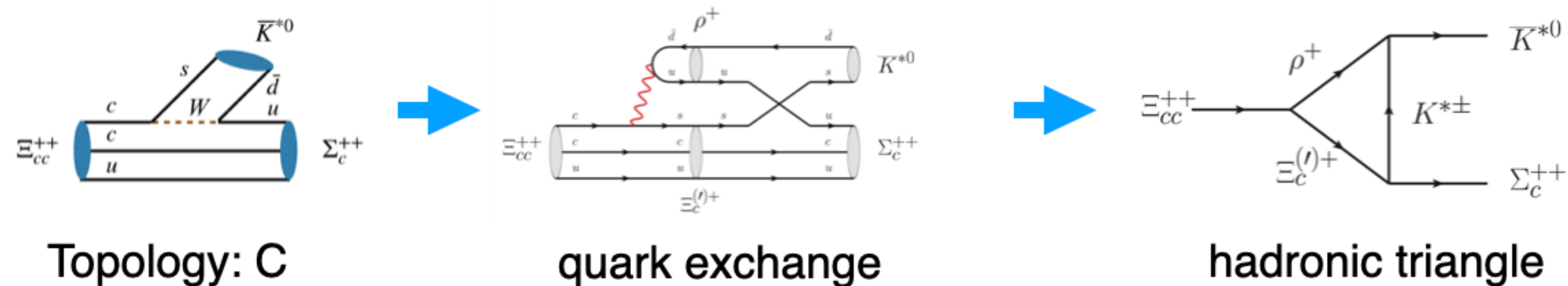
$$|\Delta A_{CP}^{\text{short-distance}}| < 2 \times 10^{-4} \quad \text{v.s.} \quad \Delta A_{CP}^{\text{FSI}} = -(6.4 \pm 1.8) \times 10^{-4}$$

Rescatterings: Hadronic loops

- CP violation can be enhanced by final-state interaction in B meson decays [Suzuki, Wolfenstein, 1999; H.Y.Cheng, C.K.Chua, Soni, 2005] and charmed baryon decays [X.G.He, C.W.Liu, 2024; C.P.Jia, H.Y.Jiang, J.P.Wang, **FSY**, 2024]



- Rescattering mechanism have been successfully used to predict the discovery channel of $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ [**FSY**, Jiang, Li, Lu, Wang, Zhao, 2017]



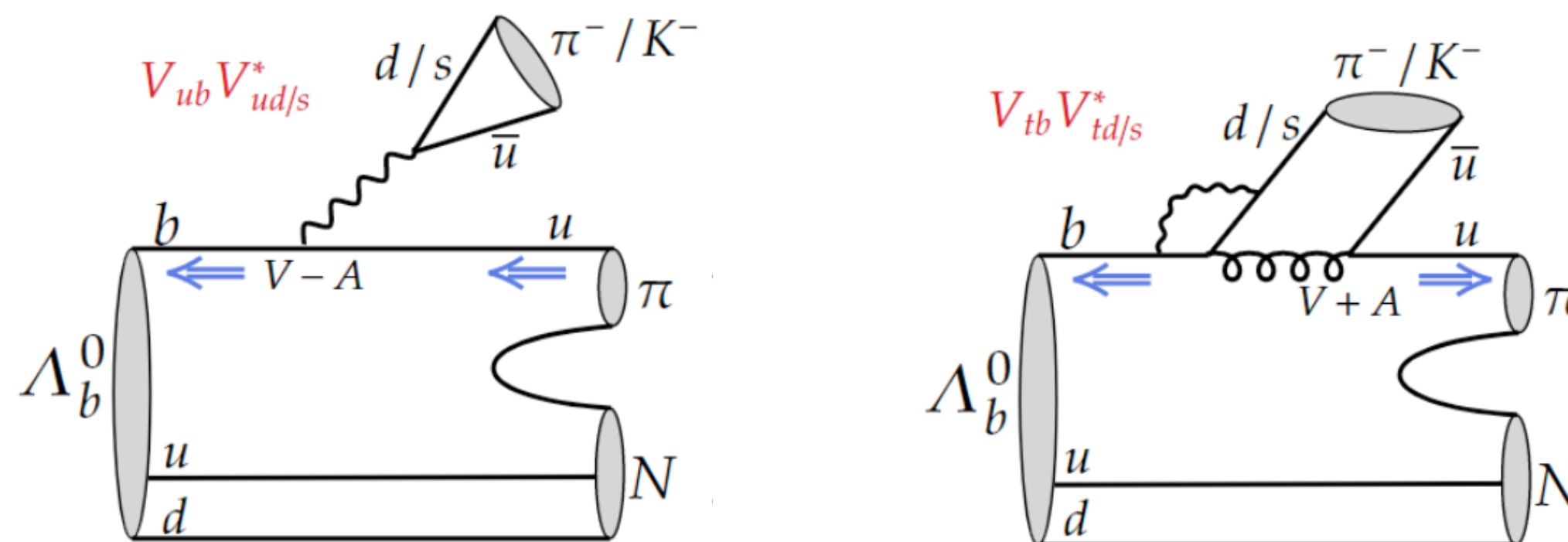
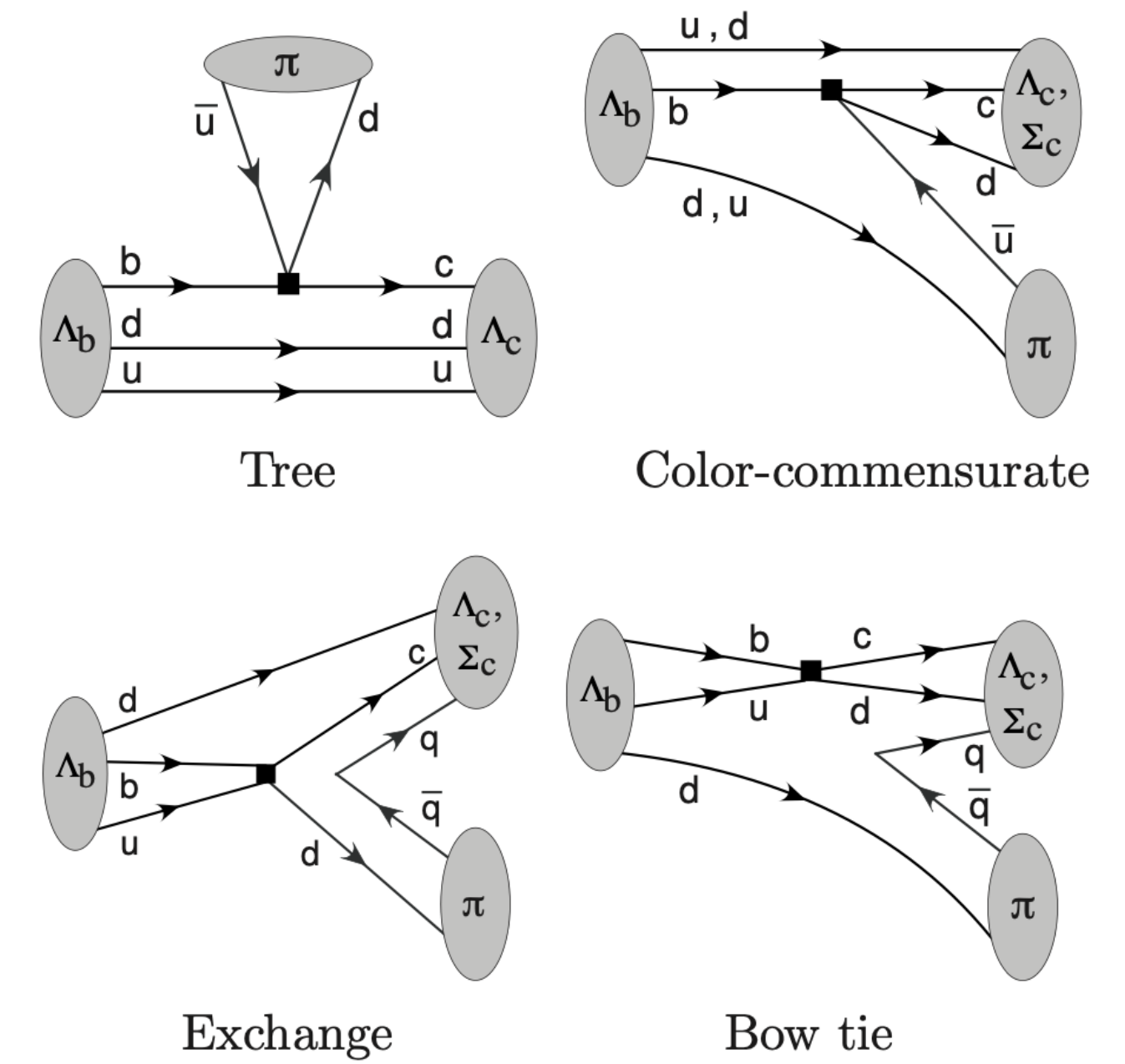
Hierarchy to topological diagrams

- In the heavy quark expansion,

$$\left| \frac{C}{T} \right| \sim \left| \frac{E}{T} \right| \sim O\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right) \quad \left| \frac{B}{C} \right| \sim O\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right)$$

Leibovich, Ligeti, Stewart, Wise, 2004

- So we only consider the color-favored emitted tree diagram and corresponding penguin diagram.



Multi-body decays of Λ_b

- Advantage: more resonances, more chances for large CPV
- Disadvantage: Too many resonances, and with large uncertainties

$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****

$N(1700)$ BREIT-WIGNER MASS	1650 to 1800 (≈ 1720) MeV
$N(1700)$ BREIT-WIGNER WIDTH	100 to 300 (≈ 200) MeV
$N(1710)$ BREIT-WIGNER MASS	1680 to 1740 (≈ 1710) MeV
$N(1710)$ BREIT-WIGNER WIDTH	80 to 200 (≈ 140) MeV
$N(1720)$ BREIT-WIGNER MASS	1680 to 1750 (≈ 1720) MeV
$N(1720)$ BREIT-WIGNER WIDTH	150 to 400 (≈ 250) MeV

- Close to each other, with large decay widths. No clear dominant one.