

# Bose-Einstein condensation of a two-magnon bound state in a spin-one triangular lattice



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## Abstract

Bose-Einstein condensation (BEC) of the **two-magnon bound state** at the saturation field is found in  $\text{Na}_2\text{BaNi}(\text{PO}_4)_2$ :

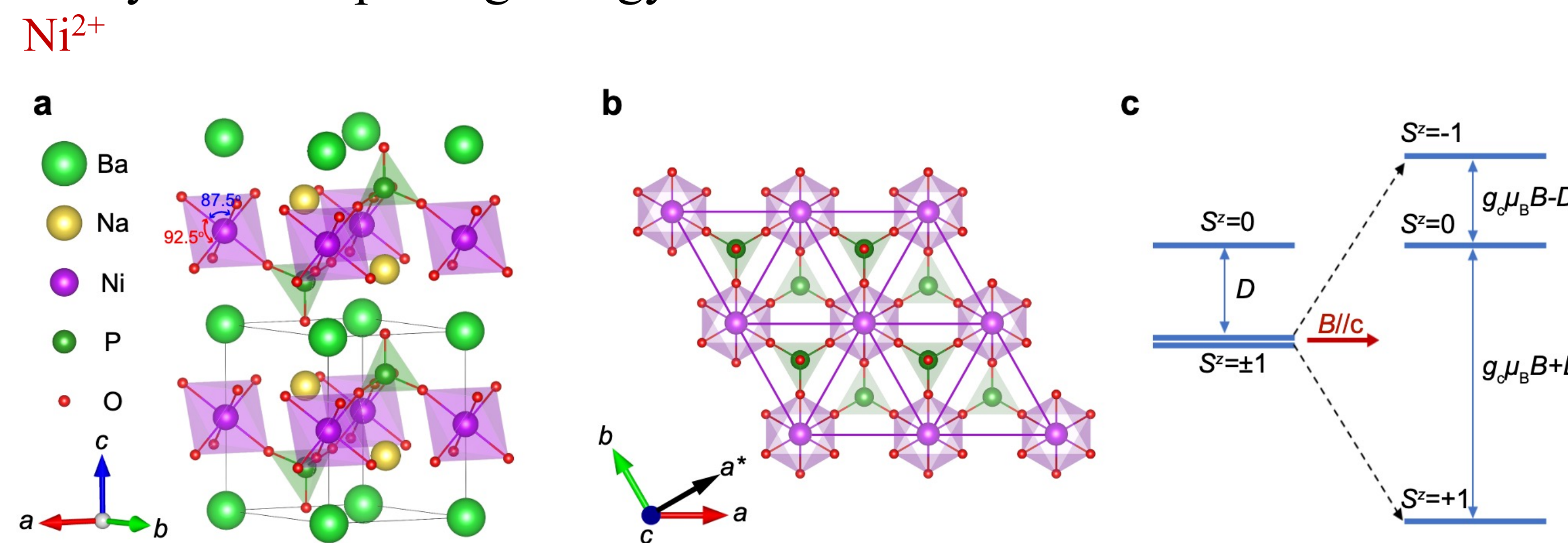
- The magnetic exchange model is established with inelastic neutron scattering experiments
- An exact solution of the model found stable two-magnon bound states and is further confirmed by an electron spin resonance (ESR) experiment
- Density Matrix Renormalized Group (DMRG) offer further evidence of the spin nematic (SN) phases below saturation field.

### Reference

J. Sheng, J.-W. Mei, L. Wang, W. Jiang, L. Xu, H. Ge, N. Zhao, T. Li, A. Candini, B. Xi, J. Zhao, Y. Fu, J. Yang, Y. Zhang, G. Biasiol, S. Wang, J. Zhu, P. Miao, X. Tong, D. Yu, R. Mole, L. Ma, Z. Zhang, Z. Ouyang, W. Tong, A. Podlesnyak, L. Wang, F. Ye, D. Yu, L. Wu and Z. Wang, Bose-Einstein condensation of a two-magnon bound state in a spin-one triangular lattice (2023), arXiv:2306.09695 [cond-mat].

## Structure and Model

### Crystal and splitting energy levels



### Hamiltonian

$$\mathcal{H}_{\text{TL}} = J \sum_{\langle ij \rangle ab} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z) - D \sum_i (S_i^z)^2$$

- Triangular lattice (TL)
- $S = 1$
- In-plane dominant
- XXZ model
- Easy-axis  $J/D \sim 1/4$

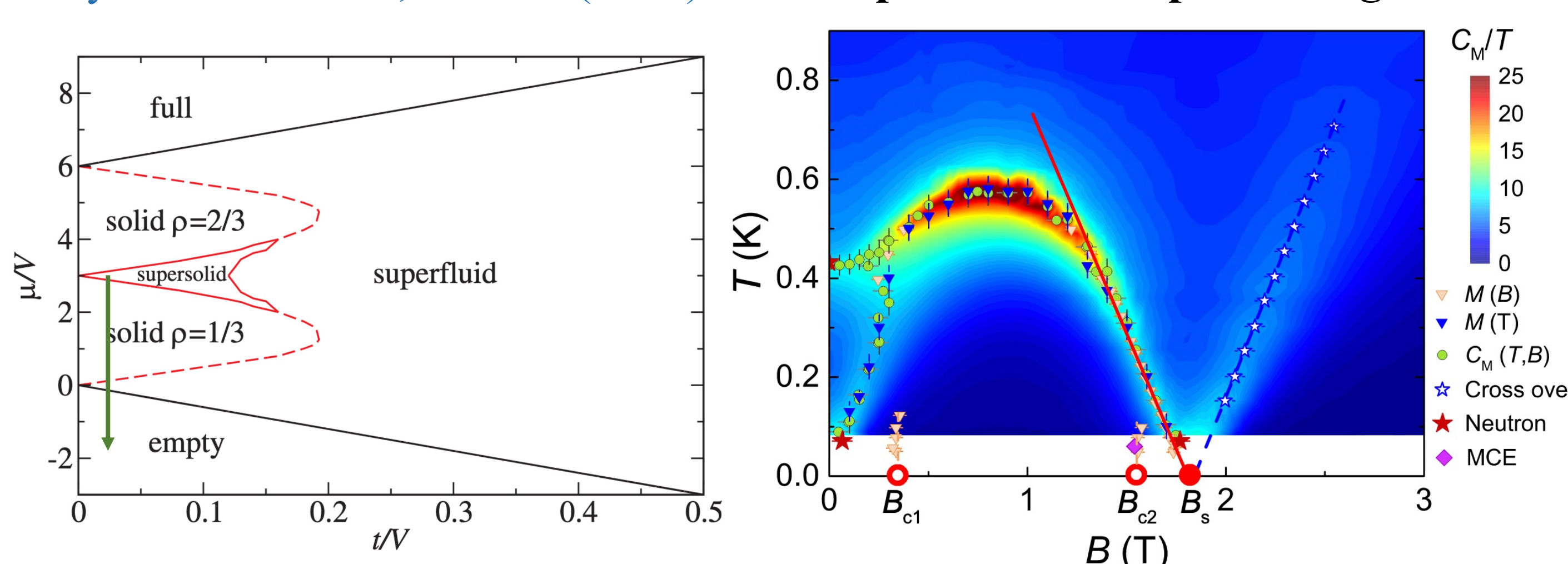
low-energy effective model,  
project to the hard-core Bose-Hubbard model

$$\mathcal{H}_{\text{eff}} = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + V \sum_{\langle ij \rangle} n_i n_j - \mu \sum_i n_i$$

$$t = \frac{J^2}{2D}, V = \frac{2J^2}{D} + 4\Delta J, \mu = \frac{6J^2}{D} + 12\Delta J - 2g_c \mu_B B$$

### Phase diagram

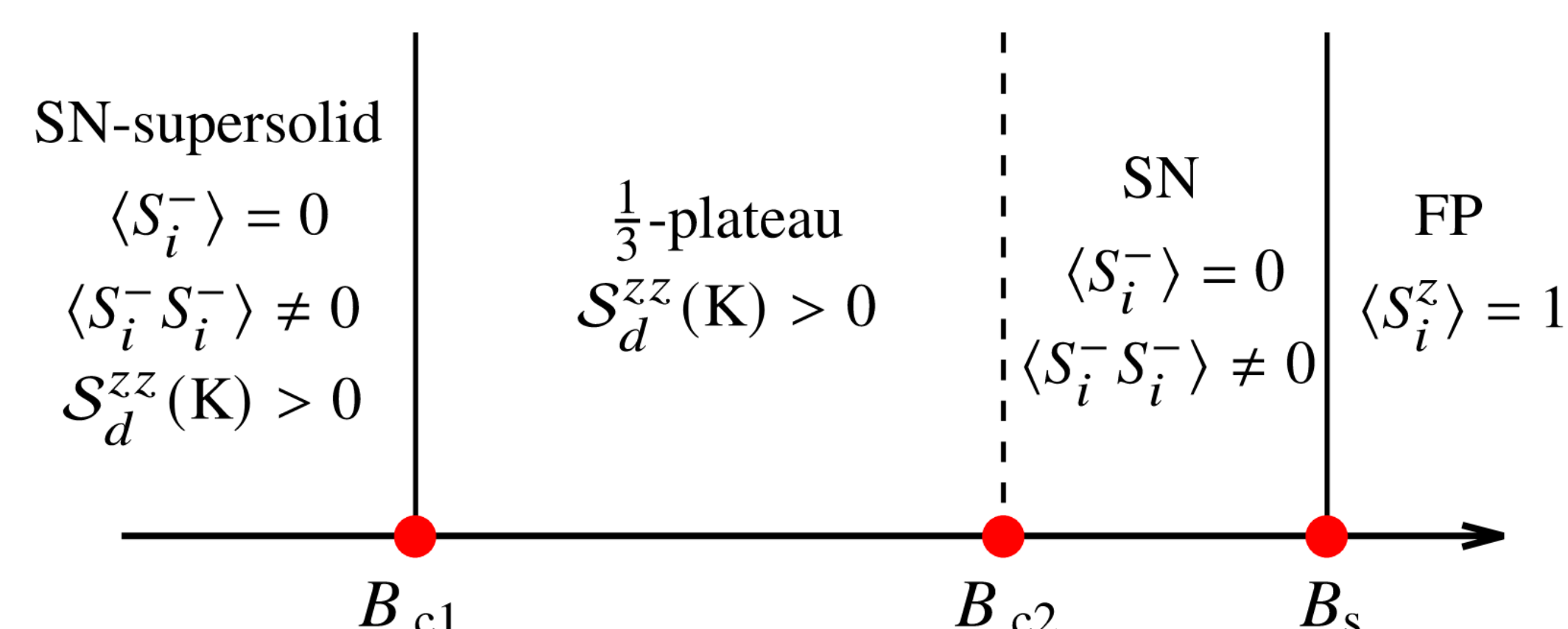
Phys. Rev. Lett. 95, 127205 (2005).



### Schematic 0 K phase diagram

- $B_{c1}, B_s$ : 2nd order transition
- $B_{c2}$ : 1st order transition

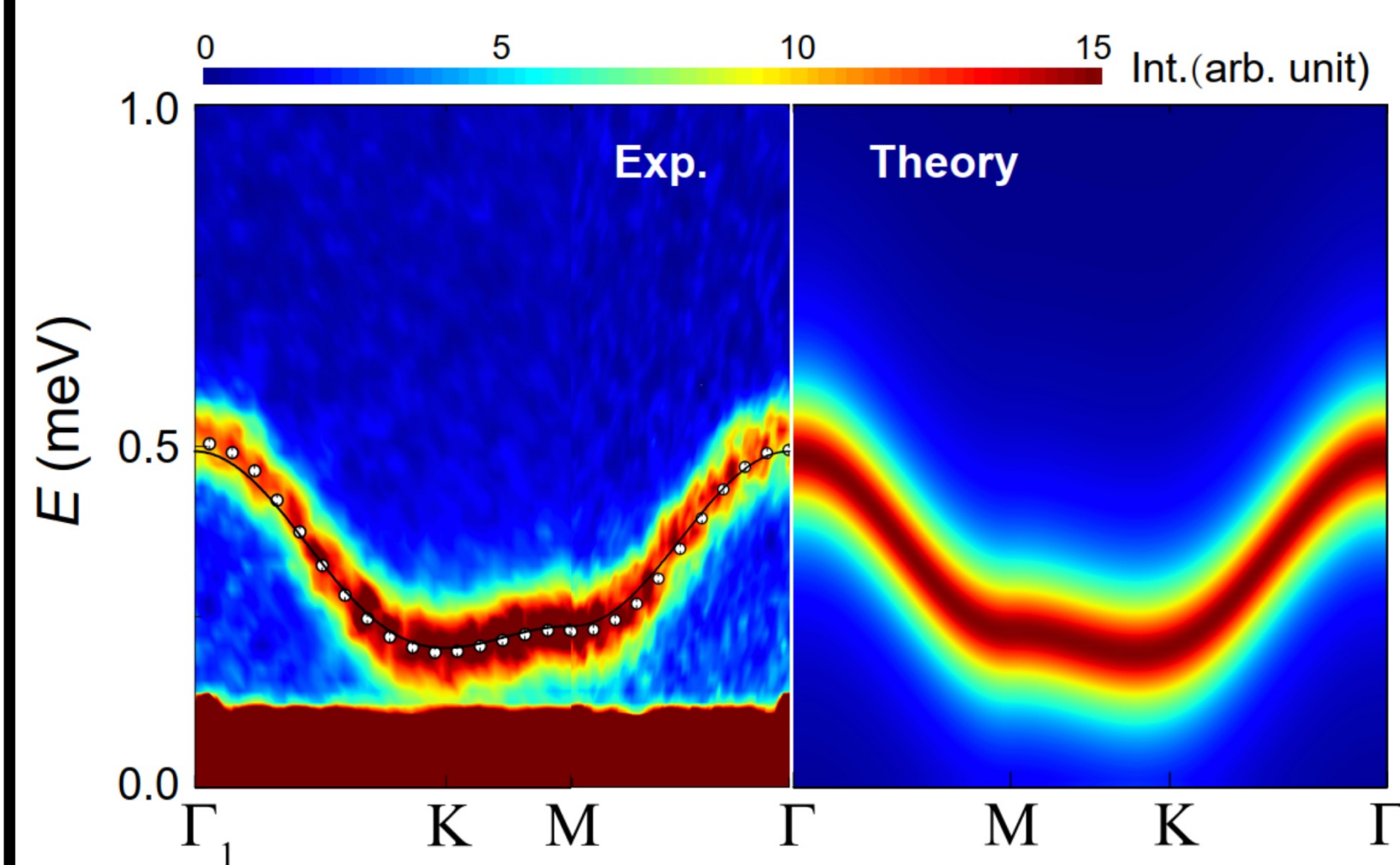
$$S_d^{zz}(\mathbf{k}) = \frac{1}{N} \sum_{i,j} e^{-i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle S_i^z S_j^z \rangle$$



## Results

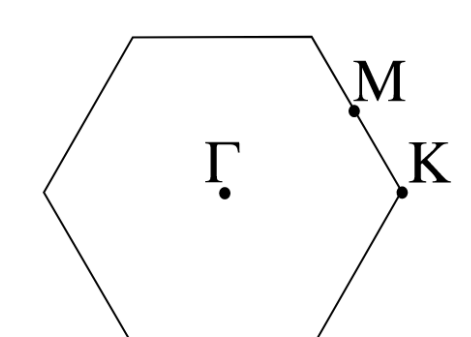
### One-magnon dispersion

$$E_1(\mathbf{k}) = 2J \left( \cos k_x + 2 \cos \frac{k_x}{2} \cos \frac{\sqrt{3}k_y}{2} \right) - 6\Delta J + D + g_c \mu_B B$$



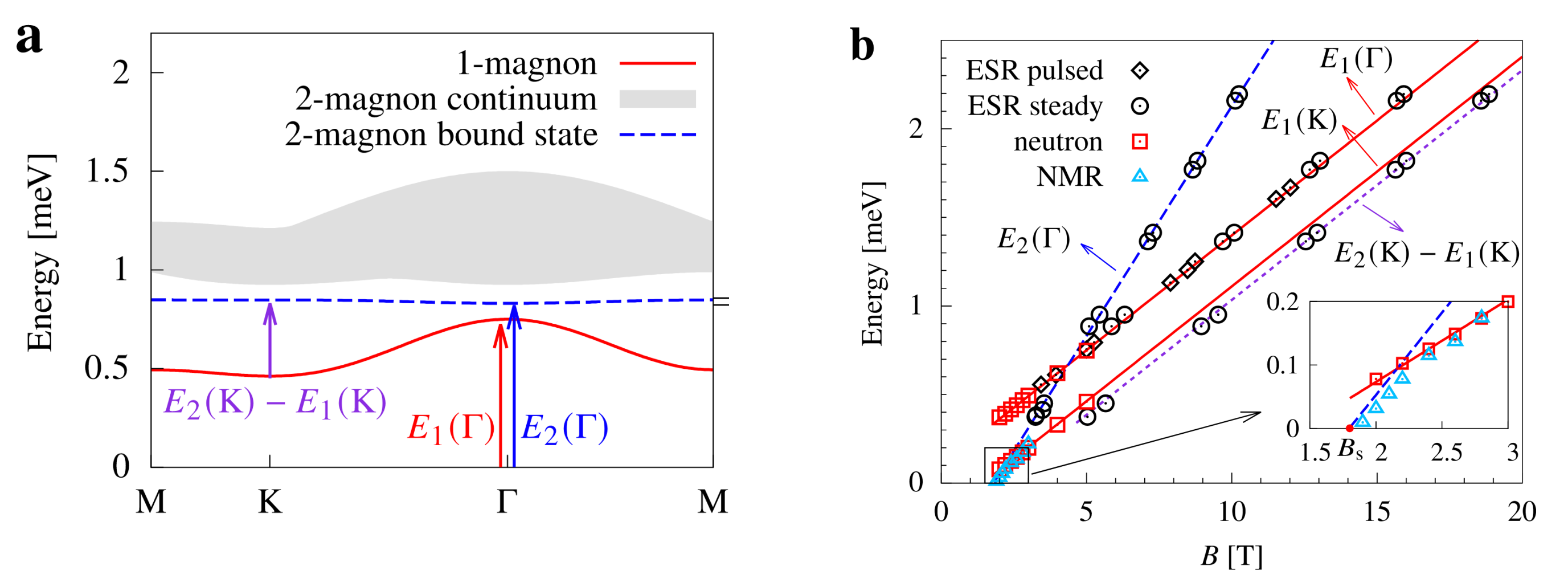
### INS intensity

- $T = 60$  mK,  $B = 5$  T
- one-magnon BEC is below the experimentally observed critical point ( $B_s \approx 1.8$  T)



### Lippmann-Schwinger equation

### Solution of the spin-1 model along with neutron, ESR and NMR data



Two-magnon state  $|\mathbf{r}_1, \mathbf{r}_2\rangle = 1/2 S_{\mathbf{r}_1}^- S_{\mathbf{r}_2}^- |FP\rangle$

- The first excitation agrees with  $E_2(\Gamma)$ .
- The second excitation along the line  $E_2(\mathbf{K}) - E_1(\mathbf{K})$ , corresponding to a transition from the 1-magnon band to the 2-magnon bound state.

### U(1) symmetric Density Matrix Renormalization Group (DMRG)

- in-plane dipolar structure factor

$$S_d^\perp(\mathbf{k}) = \frac{1}{N} \sum_{i,j} e^{-i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle S_i^x S_j^x + S_i^y S_j^y \rangle$$

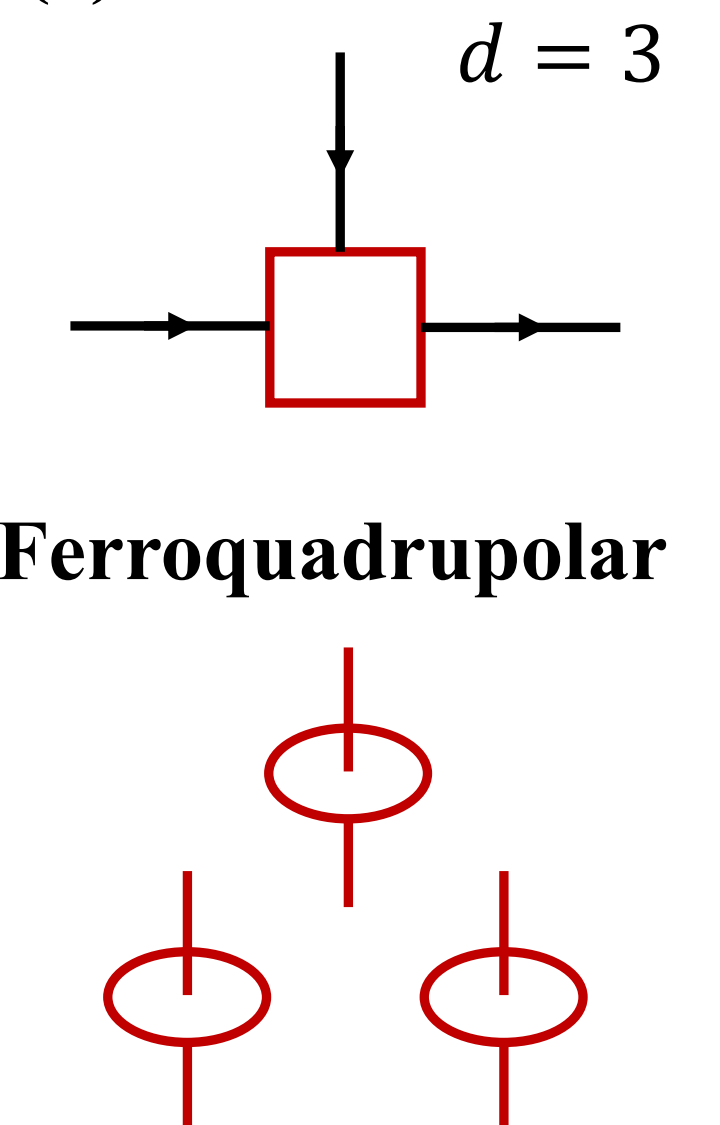
- in-plane quadrupolar structure factor

$$S_q^\perp(\mathbf{k}) = \frac{1}{N} \sum_{i,j} e^{-i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle Q_i^{x^2-y^2} Q_j^{x^2-y^2} + Q_i^{xy} Q_j^{xy} \rangle$$

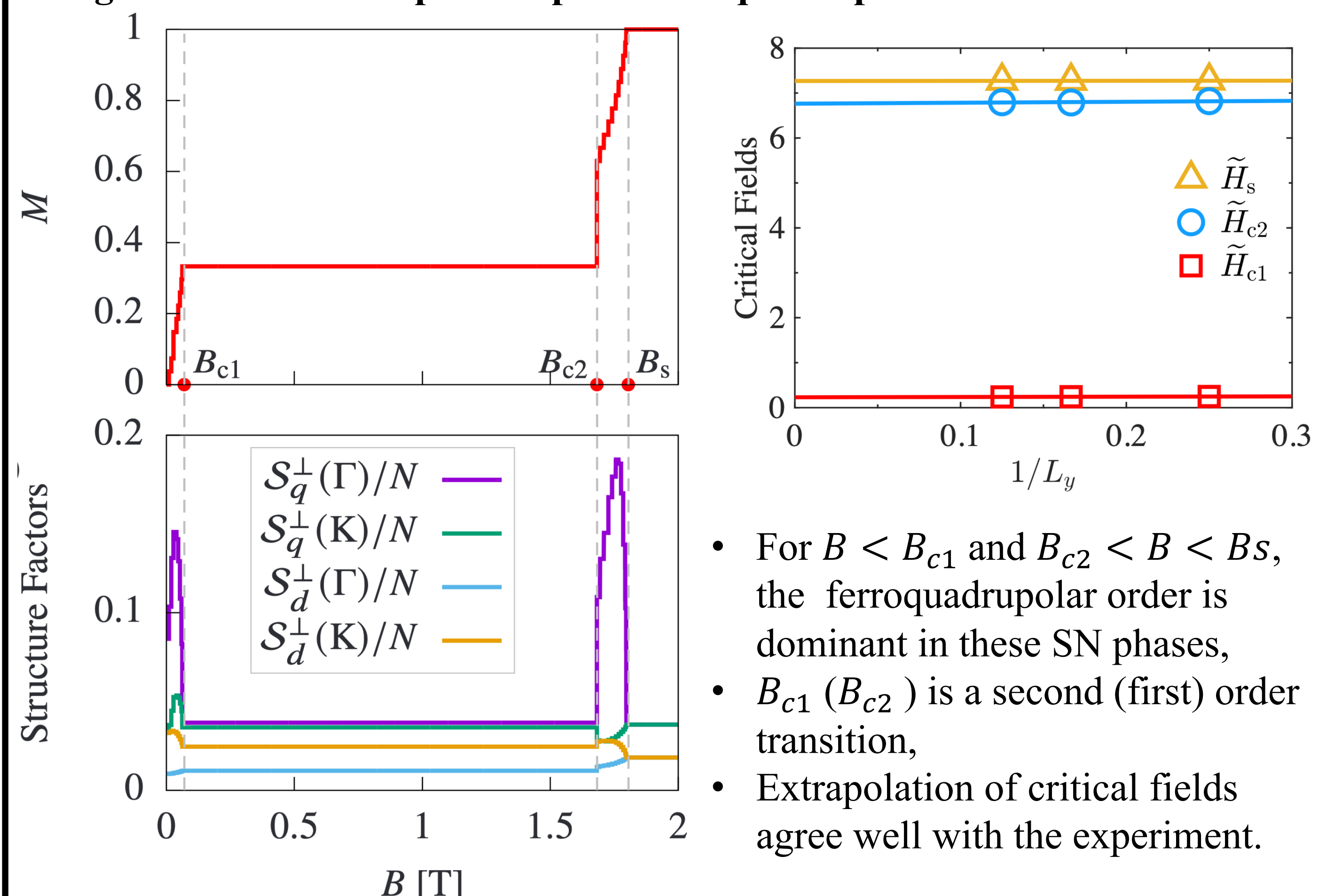
$$Q_i^{x^2-y^2} \equiv S_i^x S_i^x - S_i^y S_i^y, Q_i^{xy} \equiv S_i^x S_i^y + S_i^y S_i^x$$

- $9 \times 6$  TL torus for magnetization and structure factor,
- $6 \times 4, 9 \times 6,$  and  $12 \times 8$  TL cylinder for extrapolation to the thermodynamic limit,
- U(1) tensor is used to improve the performance of the simulation,
- $E(S_z, \tilde{H}) = E(S_z, 0) - \tilde{H} S_z, \tilde{H} = g_c \mu_B B$ .

### U(1) tensor



### Magnetization and in-plane dipolar and quadrupolar structure factors



- For  $B < B_{c1}$  and  $B_{c2} < B < B_s$ , the ferroquadrupolar order is dominant in these SN phases,
- $B_{c1}$  ( $B_{c2}$ ) is a second (first) order transition,
- Extrapolation of critical fields agree well with the experiment.