Finite-rate quench in disordered Chern and Z_2 topological insulators

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ABSTRACT

We study quantum quench of finite rate through topological quantum transitions in two-dimensional Chern and Z_2 topological insulators. We choose the representative Haldane model and the Kane-Mele model to investigate the behavior of excitation density generated by the quench and the impact of disorder. For the Haldane model, as long as the spectral gap is not closed by strong disorder, we find the excitation density at the end of the quench obeys the power-law relation with the quench rate, consistent with the prediction of the Kibble-Zurek mechanism. However, anti-Kibble-Zurek behavior is observed in disordered Kane-Mele model, which we attribute to the existence of a disorder-induced gapless region. Moreover, the dependence of particle's onsite occupation on the quench rate exhibits a similar behavior as the excitation density, which facilitates the experimental examination of KZ prediction in these models.

MODEL

- The disordered Haldane model [1] with additional NNNN hopping is given as

$$H = -t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle i,j \rangle\rangle} e^{i v_{ij}\phi} c_i^\dagger c_j + t_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} c_i^\dagger c_j + \sum_i (M_i + w_i) c_i^\dagger c_i$$

where t_1, t_2, and t_3 denote the hopping between the nearest neighboring (NN), the next NN, and the next next NN sites. The last term is the staggered potential and onsite disorder potential uniformly distributed between [-W, W].

- The disordered Kane-Mele model [2] is given as

$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,\sigma} (M_i + w_i)\, c_{i\sigma}^\dagger c_{i\sigma} + i\lambda_{\mathrm{S}} \sum_{\langle\langle i,j \rangle\rangle} \sum_{\sigma_1,\sigma_2} v_{ij} c_{i\sigma_1}^\dagger s^z_{\sigma_1\sigma_2} c_{j\sigma_2} + i\lambda_{\mathrm{R}} \sum_{\langle i,j \rangle} \sum_{\sigma_1,\sigma_2} c_{i\sigma_1}^\dagger \left(\boldsymbol{s}_{\sigma_1\sigma_2} \times \hat{\boldsymbol{d}}_{ij} \right)_z c_{j\sigma_2},$$

- the NN hopping, the staggered sublattice potential and disorder potential
- the intrinsic spin-orbit coupling (SOC)
- the Rashba SOC

METHOD

- The lattice is initially half filled. We then quench the system across the topological quantum phase transition of the model. We expect excitation density are analogous to the topological defects in the case of symmetry-breaking phase transitions. According to the KZ theory, excitation density obey a power-law relation with the quench rate $n_{\mathrm{ex}} \sim \tau^{-d\nu/(1+z\nu)}$, where z is the dynamical critical exponent and v is the critical exponent of the correlation length. They can be extracted from the relations $\Delta_{\mathrm{s}}(\lambda_c, L) \sim L^{-z}$ and $L^z \Delta_{\mathrm{s}} \sim f[(L^{1/\nu}(\lambda - \lambda_c))]$.
- We discretize the whole duration of the quench into N steps, the time evolution corresponding to the mth single-particle eigenstate can be $|\varphi_m(n\Delta t)\rangle = \mathrm{e}^{-i\Delta t H[\lambda((n-1)\Delta t)]} |\varphi_m((n-1)\Delta t)\rangle$ with time interval $\Delta t = \tau/N$. Initially $|\varphi_m(t=0)\rangle = |\psi_m(\lambda_i)\rangle$ where $|\psi_m(\lambda_i)\rangle$ is the mth instantaneous single-particle eigenstate of the Hamiltonian at $\lambda = \lambda_i$. The excitation density at time t can then be defined as

$$n_{\mathrm{ex}}(t) = 1 - \frac{1}{N_s/2} \sum_{m,n=0}^{\frac{N_s}{2}-1} |\langle \psi_m(\lambda(t)) | \varphi_n(t) \rangle|^2$$

- We examine whether the site resolved occupation of particles[3] can be a measurable quantity to detect the breakdown of adiabatic evolution. We compute the difference between the actual occupation at the end of the quench, and the static occupation for a specific lattice site i

$$\Delta\rho_i = \frac{1}{N_s/2} \sum_{m=0}^{N_s/2-1} \left(\langle n_i \rangle_{\varphi_m(t=\tau)} - \langle n_i \rangle_{\psi_m(\lambda_f)} \right)$$

where n_i is the occupation number operator on site i. Without loss of generality, we assume i belongs to one of the two sublattice of honeycomb lattice (say, the A lattice).

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- **References:**

[1] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).

[2] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005), Phys. Rev. Lett. **95**, 226801 (2005) .

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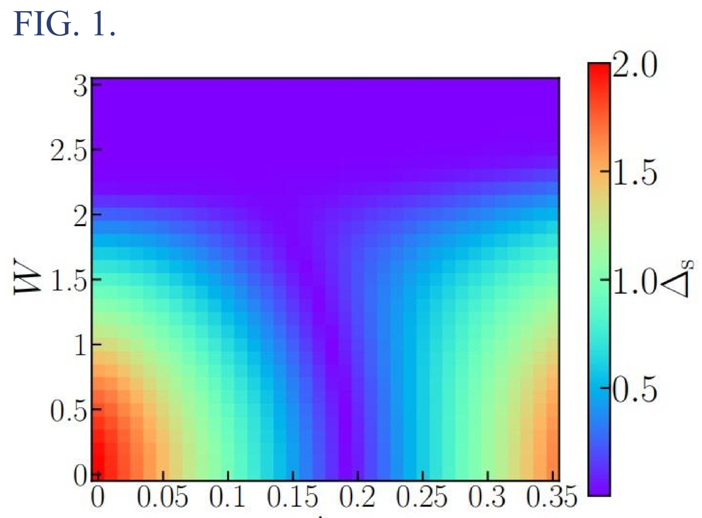
NUMERICAL RESULTS

- **Disorder effect on the scaling of excitation density**

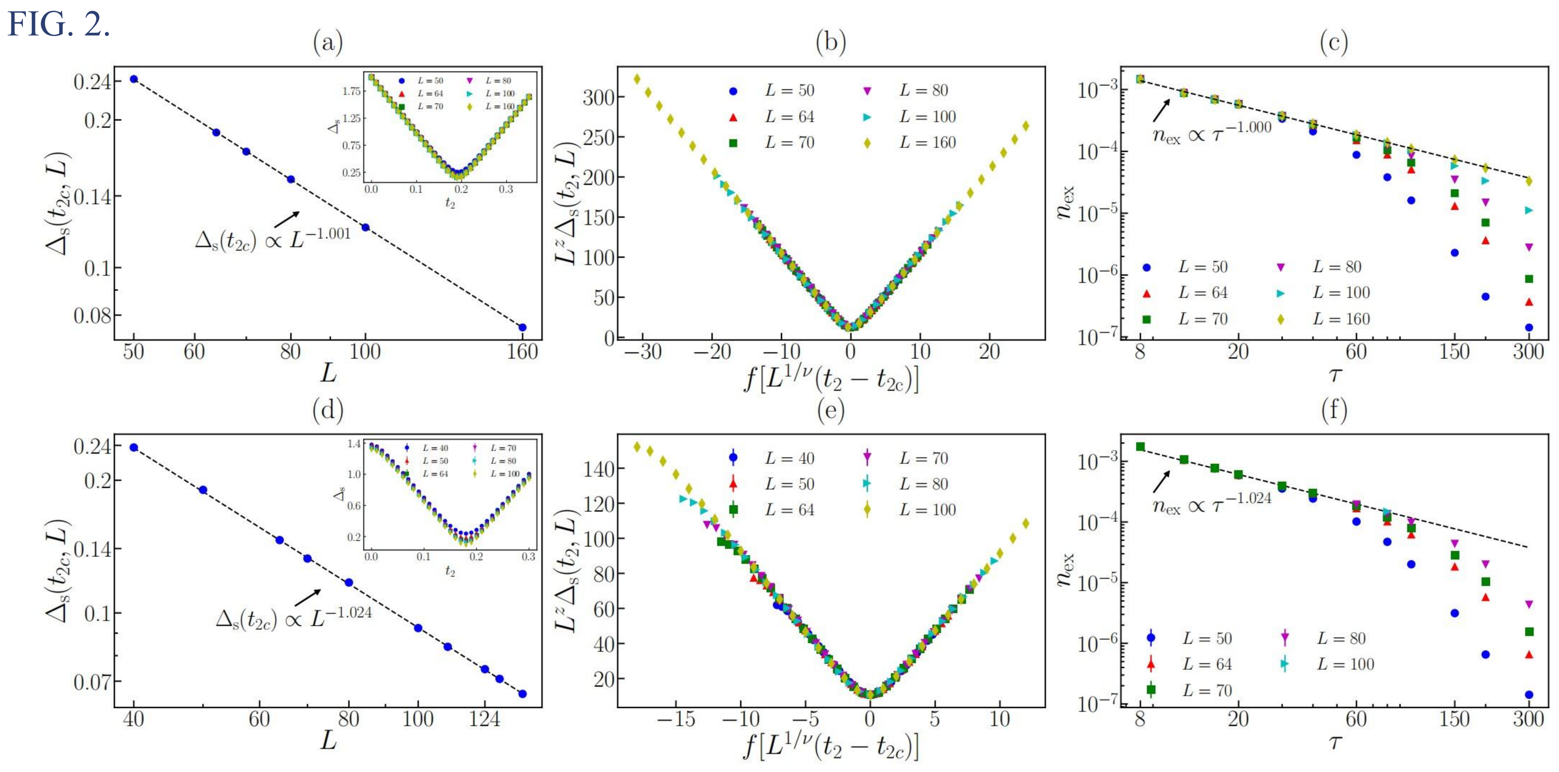
-Haldane model

FIG. 1.



- Spectral gap for Haldane model with system size L=60. Parameters of Haldane model are chosen as $M=1, t_1=-1, t_3=1/3, \phi=\pi/2$.
- The spectral gap develops a minimum at a critical point for W<2.
- With these parameters, the numerical extraction of z and v for clean Haldane model in Figs. 2(a) and (b) matches the theoretical known $z=\nu=1$ for Dirac fermions. With these values, the KZ theory predicts $n_{\mathrm{ex}} \sim \tau^{-1}$.

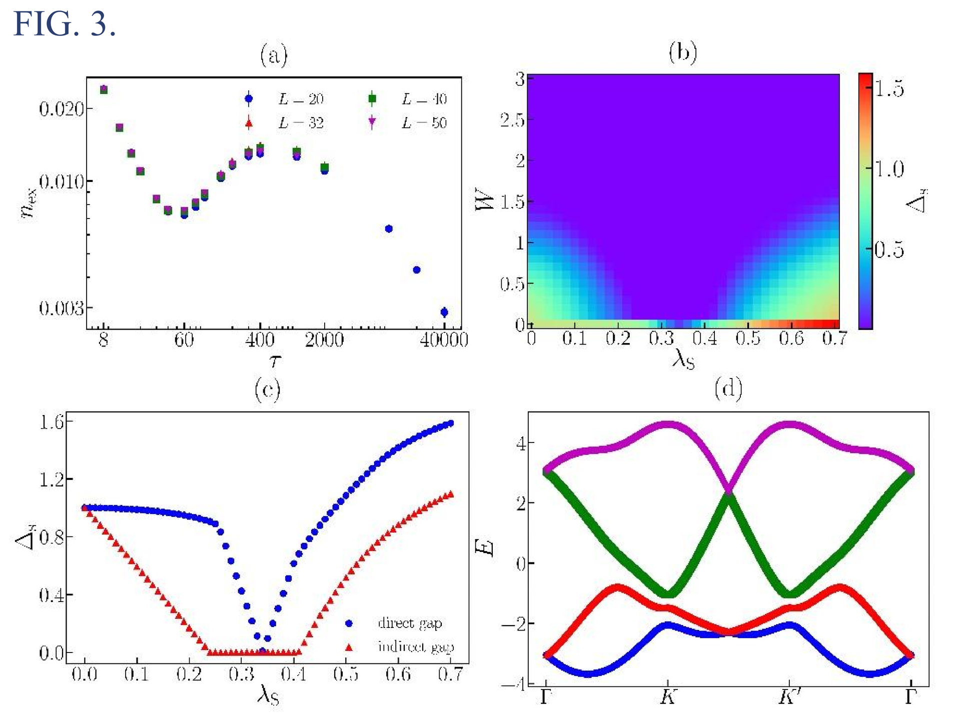
FIG. 2.



- The typical quench result is displayed in Fig. 2(c), where t_2 linearly varies with time from 0.1 to 0.3, scaling as $n_{\mathrm{ex}} \sim \tau^{-1}$ for sufficiently large system.
- The results for W=1 are shown in Figs. 2(d)-(f). We further extract $z \approx 1.024$ and $\nu \approx 1$, which together give KZ prediction $n_{\mathrm{ex}} \sim \tau^{-0.988}$.
- The numerical simulation of the quench dynamics suggests $n_{\mathrm{ex}} \sim \tau^{-1.024}$, which is in good agreement with the KZ prediction. Model parameters and quench protocol are the same as those in clean case.

-Kane-Mele model

FIG. 3.

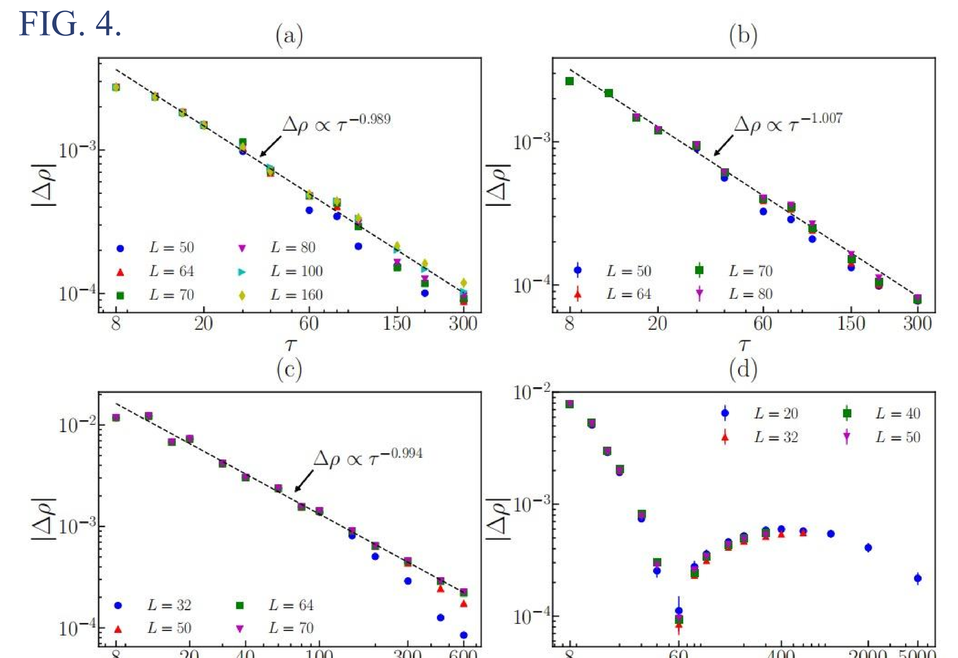


- Parameters of clean Kane-Mele model are chosen as $M=0.5$ and $\lambda_{\mathrm{R}}=1$.
- The dependence of the final excitation density on the quench rate for clean Kane-Mele model exhibits similar behavior as clean and disordered Haldane model, consisting with KZ prediction $n_{\mathrm{ex}} \sim \tau^{-1}$, when λ_{S} linearly varies from 0.1 to 0.7.
- For disordered Kane-Mele model, the behavior of the excitation density is more complicated. Typical data are shown in Fig. 3(a) for W=0.3. n_{ex} first decays with increasing quench rate for $8 \le \tau \le 60$. Then it starts to grow with τ and reaches a maximum at $\tau \approx 400$. At last, n_{ex} decays for sufficiently slow quench.
- Such non-monotonic behavior of excitation density will be attributed to the existence of disordered-induced gapless region [Fig. 3(b)].
- The origin of this gapless region can be understood by tracking the indirect band gap in the clean limit [Fig. 3(c)].
- The band structure in Fig. 3(d) of the clean Kane-Mele model along the trajectory $\Gamma \to K \to K'$ with $\lambda_{\mathrm{S}}=0.3$, from which the closing of the indirect band gap can be clearly seen.

- **Detect the breakdown of adiabatic evolution using particle's occupation**

-Periodical boundary conditions

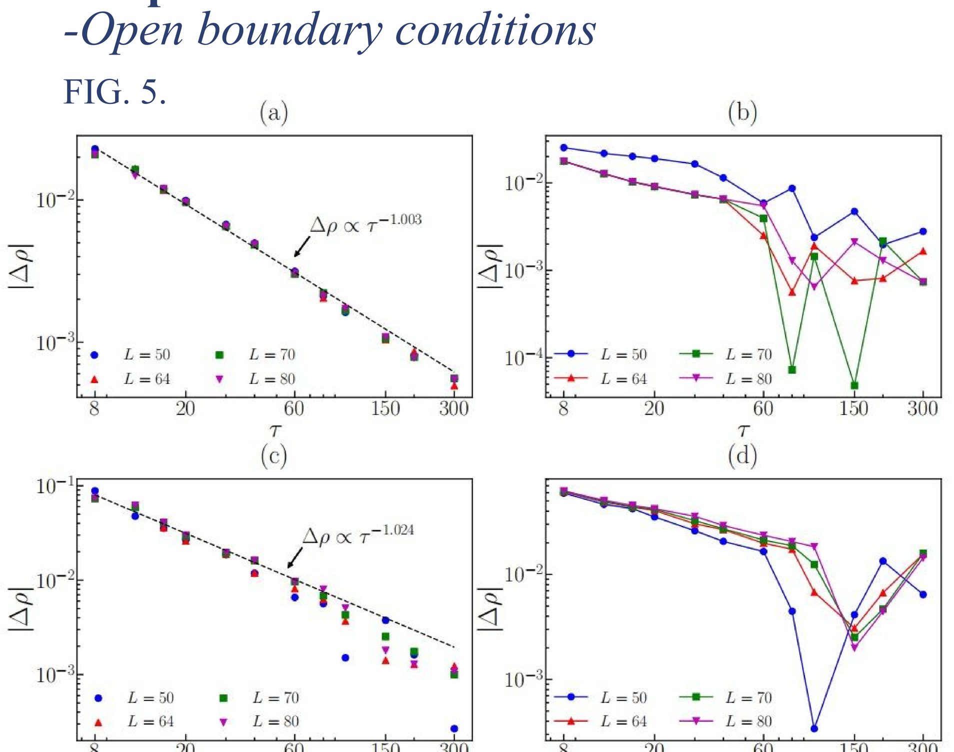
FIG. 4.



- The particle's occupation difference $\Delta\rho$ for (a) clean Haldane model, (b) disordered Haldane model with W=1, (c) clean Kane-Mele model and (d) disordered Kane-Mele model with W=1. Quench protocols are the same in Figs. 2 and 3. Parameters are the same for Haldane model while we choose $M=1$ for Kane-Mele model here.
- In (a)-(c), $\Delta\rho$ shows a power-law decay with exponents close to 1, while in (d), the behavior unveils the non-monotonic relation.

-Open boundary conditions

FIG. 5.



- The particle's occupation difference $\Delta\rho$ as for clean Haldane model [(a) and (b)], and the clean Kane-Mele model [(c) and (d)] under open boundary conditions.
- In (a) and (c), we display $\Delta\rho$ on the A site of the central unit cell, which show a power-law relation.
- In (c) and (d), we display $\Delta\rho$ averaged over the A sites in all unit cells on the boundary, which show non-monotonic dependence on τ .

CONCLUSION AND OUTLOOK

- **We study the dynamics of finite-rate quench in the Haldane and Kane-Mele model. For both clean and disordered Haldane model, we find the power-law decaying of the excitation density with quench rate consistent with Kibble-Zurek prediction. For the Kane-Mele model involving the Z_2 topological phase, while the KZ behavior of excitation density is also observed in the clean limit, disorder can induce the anti-KZ feature by extending the single critical point to a gapless region.**
- **We propose using electron's onsite occupation to experimentally detect the breakdown of the adiabatic evolution and capture the KZ and anti-KZ behavior of excitation density. Furthermore, the locality of the occupation could allow us to isolate the excitations in the bulk from those on the boundary of the system.**
- **There are a couple of open questions which deserve future investigation, such as understanding anti-KZ behavior of the excitation density in disordered Kane-Mele model and proceed to examine the KZ mechanism in interacting systems, especially those involving fractional topological orders.**