A three-body form factor at sub-leading power in the high-energy limit: planar integral

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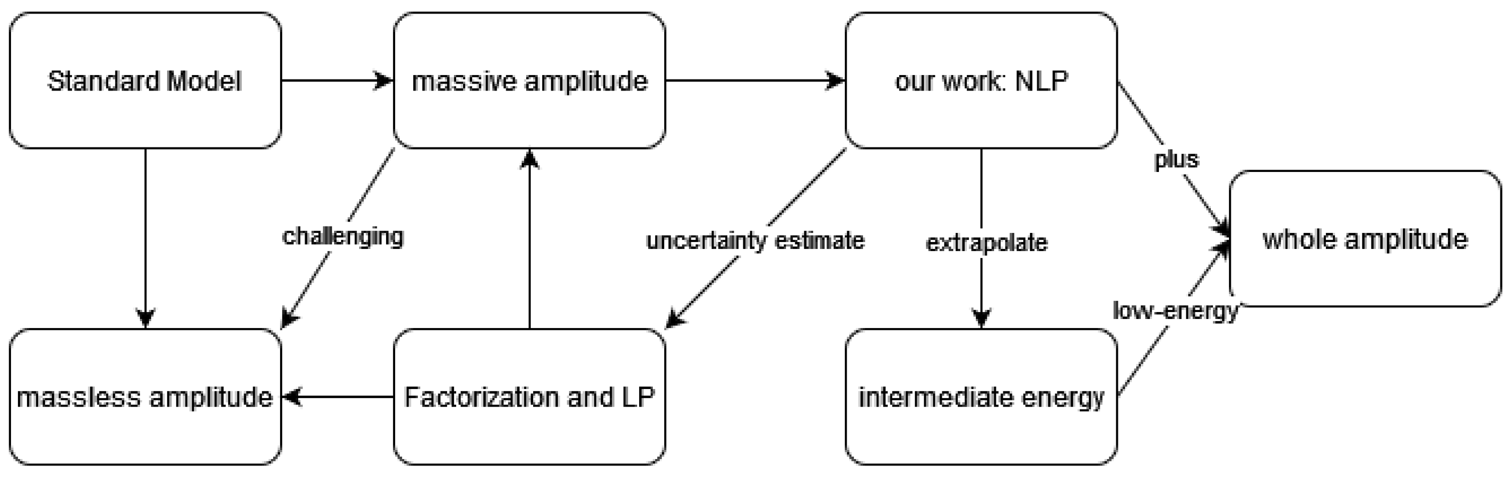
Motivation mind map



Challenging massive amplitude

To explore the Standard Model and physics BSM, precise theoretical calculations of scattering amplitudes involving massive particles are necessary, but remain challenging compared to massless particles. Fortunately at high-energy colliders like the Large Hadron Collider, particle masses are often much smaller than other kinematic invariants, and factorization has been developed as a standard technique for handling such scale hierarchies.

Factorization and LP

Organized into different order of ratio $\lambda \sim m^2/E^2$, and massive scattering amplitude in the high energy limit can be factorized into a massless amplitude, a soft function and several collinear functions at leading power(LP) corresponding to order λ^0. Such factorization formula can be used to predict the structure of the large logarithms of the form $\ln(m/E)$ at higher orders in perturbation theory, and also allows the resummation of these logarithms to all orders.

Why NLP

Investigating sub-leading power corrections is crucial for estimating uncertainties in leading-power calculations, extrapolating high-energy approximations to intermediate energiy range, and ultimately achieving a complete description of scattering amplitudes across whole phase space.

Our work

We consider the virtual corrections to the process $\gamma^*(p_4) \to e^+(p_1) + e^-(p_2) + \gamma(p_3)$ at NNLO in the high energy limit. We provide details on the evaluation of the planar master integrals. With the similar calculation methods, the complete NNLO amplitude will be obtained soon.

Calculation setup

Dimensionless kinematica variables

$$x = \frac{m^2}{-s_{123}}, \quad y = \frac{s_{12}}{s_{123}}, \quad z = \frac{s_{23}}{s_{123}}$$

with

$$s_{12} = (p_1+p_2)^2, s_{23} = (p_2+p_3)^2, s_{123} = (p_1+p_2+p_3)^2, p_1^2 = p_2^2 = m_e^2, p_3^2 = 0$$

MIs high-energy limit asymptotic expansion:

$$\mathcal{I}_i(x,y,z) = \sum_{n_1,n_2,n_3} \mathcal{C}_{i,n_1,n_2,n_3}(y,z)\, \epsilon^{n_1} x^{n_2} \log^{n_3}(x)$$

Applying asymptotic expansion into DE gives independent coefficients PDE with respect to y and z:

$$\partial \mathcal{C}_I(y,z) = \sum_J \mathcal{A}_{I,J}(y,z)\mathcal{C}_J(y,z)$$

where $\mathcal{A}$s are almost lower-triangular matirx with suitable choice.

Simple cases

First order non-homogeneous linear system of DE $\partial\mathcal{C} = \mathcal{A}\,\mathcal{C} + \mathcal{G}$ have general solution and relation

$$\mathcal{C}(y,z) = \frac{1}{\mathcal{M}}[\mathcal{F}(y,z) + c]$$

$$c_y = \frac{\mathcal{M}_z}{\mathcal{M}_y}(\mathcal{F}_y + c_z) - \mathcal{F}_z$$

with $1/\mathcal{M} = \exp\int A$ and $\mathcal{F} = \int \mathcal{M}\mathcal{G}$.

Take partial derivative of both sides respect to z, we have new DE

$$\frac{\partial c_z}{\partial z} = -\frac{\mathcal{M}_y}{\mathcal{M}_z}\frac{\partial}{\partial z}\left(\frac{\mathcal{M}_z}{\mathcal{M}_y}\right) c_z - \frac{\mathcal{M}_y}{\mathcal{M}_z}\frac{\partial}{\partial z}\left(\frac{\mathcal{M}_z}{\mathcal{M}_y}\mathcal{F}_y - \mathcal{F}_z\right)$$

which is easy to solve with suitable boundary condition.

With `AMFlow`, we use `PSLQ` algotithm to reconstruct boundary constants, whose transcendental basis are from $\{\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Li}_4(1/2), \zeta_5\}$ up to weight 5.

2×2 blocks

$$\partial \begin{bmatrix}\mathcal{C}_1\\ \mathcal{C}_2\end{bmatrix} = \begin{bmatrix}\mathcal{A}_{11} & \mathcal{A}_{12}\\ \mathcal{A}_{21} & \mathcal{A}_{22}\end{bmatrix}\begin{bmatrix}\mathcal{C}_1\\ \mathcal{C}_2\end{bmatrix} + \begin{bmatrix}\mathcal{G}_1\\ \mathcal{G}_2\end{bmatrix}.$$

Construct linear rational combination $\mathcal{C}' = b_1\mathcal{C}_1 + b_2\mathcal{C}_2$ such that derivatives of $\mathcal{C}'$ only depend on itself, and we get

$$\mathcal{A}' \equiv \frac{\partial b_1 + b_1\mathcal{A}_{11} + b_2\mathcal{A}_{21}}{b_1} = \frac{\partial b_2 + b_1\mathcal{A}_{12} + b_2\mathcal{A}_{22}}{b_2}$$

which can also be rewritten as Riccati equation for the ratio b_2/b_1

$$\partial\left(\frac{b_2}{b_1}\right) - \mathcal{A}_{21}\left(\frac{b_2}{b_1}\right)^2 + (\mathcal{A}_{22} - \mathcal{A}_{11})\frac{b_2}{b_1} + \mathcal{A}_{12} = 0$$

After finding a partical solution with `Mathemacica`, replace $\mathcal{C}_2$ with $\mathcal{C}'$, and 2×2 block becomes simple cases.

3×3 blocks and more

For linear rational combination $\mathcal{C}' = b_1\mathcal{C}_1 + b_2\mathcal{C}_2 + b_3\mathcal{C}_3$, we have similar relation. Since relations only constrain ratio, assume b_is are polynominals

$$b = \sum_{i=0}^{n}\sum_{j=0}^{n-i} c_{ij} y^i z^j$$

Substitute ansatz into relation, then increase n unless we find a non-trival solution. Finally replace $\mathcal{C}_3$ with $\mathcal{C}'$, 3×3 block becomes 2×2 block.

Summary

We expressed MIs in terms of simple GPLs and finally the planar topology amplitude is expressed in a compact and concise form, with size of 1.6MB.

Reference and acknowledgments

[1]. T. Gehrmann, E. Remiddi, Two loop master integrals for $\gamma^* \to 3jets$: the Planar topologies, Nucl. Phys. B [hep-ph/0008287]

[2]. The Two loop QCD matrix element for $e^+e^- \to 3jets$, Nucl. Phys. B [hep-ph/0112081]

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