Study of $e^+e^- \to \bar{p}n\pi^+ + c.c.$ around J/ψ

Yaoyu Duan[1,2], Hailong Ma[1] and Xinping Xu[2]

[1]Soochow University
[2]IHEP

April 2, 2025

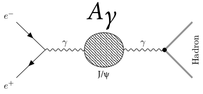
BESIII xx Group Meeting

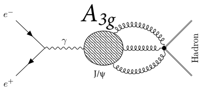
Outline

Motivation

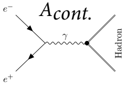
Relationship between the Strong and EM interactions.

- The lineshape of hadron cross section around the J/ψ is determined by the interference between:
- Resonance production:





- Non-res production:



- The total amplitude of $e^+e^- \to light\ hadrons$ can be written as

$$\sigma^f_{\text{Dress}} = \frac{4\pi\alpha^2}{3s}\left|\frac{1}{|1-\Pi_0(s)|} + (1+Ae^{i\phi_{\gamma,3g}})\frac{s}{M}\frac{3\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}/\alpha}{s-M^2+iM\Gamma}e^{i\phi_{\gamma,\text{cont}}}\right|^2 \cdot \mathcal{P}(s)\cdot(\frac{\mathcal{F}}{s^{n/2}})^2, \tag{1}$$

in which ϕ is the phase angle between the strong and Eletron-Magnetic amplitudes to be determined.

Motivation

Theortical predication:

- QCD: $\varphi = 0^\circ$ or 180°
- unsubtracted dispersion relations: $\varphi = \pm 90^\circ$

Experimental results:

Indirect results: $J/\psi \to$

- $1^-0^-, 0^-0^- : \varphi \sim \pm 90^\circ$ Phys. Rev. D41, 1389
- $1^-1^- : \varphi = (138 \pm 37)^\circ$ Phys. Rep. 174, 67
- $1^+0^- : \varphi \sim 90^\circ$ Phys. Rev. D38, 2695
- $N\bar{N} : \varphi \sim 91^\circ$ Phys. Lett.B444, 111

Indirect results:

- $\psi(2S) \to 1^-0^-, 1^+0^- : \varphi \sim 0^\circ$ Phys. Rev. D63, 054021
- $\psi(2S) \to N\bar{N} : \varphi = (-98 \pm 25)^\circ$ or $(135 \pm 25)^\circ$
- $\psi(3770) \to 1^-0^- : \varphi \sim -90^\circ$ Phys. Rev. D 58,111504
- $\psi(3770) \to N\bar{N} : \varphi = (-137.5 \pm 2.7)^\circ$

Motivation

- Direct experimental results through lineshape scan:
  - $J/\psi \to 5\pi : \varphi = (84.9 \pm 3.6)^\circ$ or $(-84.7 \pm 3.1)^\circ$ Phys.Lett. B 791, 375
  - $\psi(3770) \to p\bar{p} : \varphi = \left(255.8^{+39.0}_{-26.6} \pm 4.8\right)^\circ$ or $(266.9^{+6.1}_{-6.3} \pm 0.9)^\circ$ Phys. Lett. B 735, 101
- This work aims to study the phase difference in $J/\psi \to \bar{p}n\pi^+ +$ c.c. by analyzing the J/ψ scan data following Tianyou Li's and Yijia Zeng's analyses. BESIII Doc-1095, Tau&QCD Group Meeting

Data set and MC sample

- Data Set:
  - J/ψ: 224.0M(09), Boss 708 (background estimation)
  - J/ψ Scan: 86 pb^{-1}, Boss 713
  - R Scan: 159 pb^{-1}, Boss 713
  - τ Scan: 30 pb^{-1}, Boss 704
  - J/ψ off-res @3080: 167 pb^{-1}, Boss 708
- Inclusive MC
  - J/ψ: 224M(09), Boss 708
  - off-res @3080: 167 pb^{-1}, Boss 708
- Exclusive MC sample:
  - J/ψ, R, τ Scan @25 energy points: Boss 713/704
  - J/ψ off-res @3080: Boss 708
  - Numbers of event generated
    - Signal $e^+e^- \to \bar{p}n\pi^+$ PHSP @26 energy points
    - Signal $e^+e^- \to \bar{p}n\pi^+$ DIY @26 energy points

Data set and MC sample

Datasets	Run Number	BEMS MeV	Corrected MeV	$\mathcal{L}(\mathrm{pb}^{-1})$	Boss version
3080.0	27147-27233,28241-28266 54982-55053,59016-59141	3080.00 ± 0.20	-	167.06 ± 0.10	7.0.8
3096.9	$9947 \sim 10878$	3096.99 ± 0.20	-	79.63 ± 0.07	7.0.8
3000.0	$39680 \sim 39710$	3000.00 ± 0.20	-	15.85 ± 0.11	7.1.3
3020.0	$39711 \sim 39738$	3020.00 ± 0.20	-	17.32 ± 0.12	7.1.3
3080.0(R)	$39355 \sim 39618$	3080.00 ± 0.20	-	126.21 ± 0.90	7.1.3
3049.6	$28312 \sim 28346$	3050.21 ± 0.03	3049.64 ± 0.06	14.92 ± 0.16	7.1.3
3058.7	$28347 \sim 28381$	3059.25 ± 0.03	3058.69 ± 0.06	15.06 ± 0.16	7.1.3
3082.5	$28382 \sim 28387, 28466 \sim 28469$	3083.06 ± 0.02	3082.50 ± 0.06	4.77 ± 0.06	7.1.3
3088.9	$28388 \sim 28416, 28472 \sim 28475$	3089.42 ± 0.02	3088.85 ± 0.06	15.56 ± 0.17	7.1.3
3091.8	$28417 \sim 28453, 28476 \sim 28478$	3092.32 ± 0.02	3091.76 ± 0.06	14.91 ± 0.16	7.1.3
3094.7	$28479 \sim 28482$	3095.26 ± 0.08	3094.70 ± 0.10	2.14 ± 0.03	7.1.3
3095.4	$28487 \sim 28489$	3095.99 ± 0.08	3095.43 ± 0.10	1.82 ± 0.02	7.1.3
3095.8	$28490 \sim 28492$	3096.39 ± 0.08	3095.83 ± 0.09	2.14 ± 0.03	7.1.3
3097.2	$28493 \sim 28495$	3097.78 ± 0.08	3097.21 ± 0.09	2.07 ± 0.03	7.1.3
3098.3	$28496 \sim 28498$	3098.90 ± 0.08	3098.34 ± 0.09	2.20 ± 0.03	7.1.3
3099.0	$28499 \sim 28501$	3099.61 ± 0.09	3099.04 ± 0.11	0.76 ± 0.01	7.1.3
3101.4	$28504 \sim 28505$	3101.92 ± 0.11	3101.36 ± 0.12	1.61 ± 0.02	7.1.3
3105.6	$28506 \sim 28509$	3106.14 ± 0.09	3105.58 ± 0.10	2.11 ± 0.03	7.1.3
3112.1	$28510 \sim 28511$	3112.62 ± 0.09	3112.05 ± 0.11	1.72 ± 0.02	7.1.3
3119.9	$28512 \sim 28513$	3120.44 ± 0.12	3119.88 ± 0.13	1.26 ± 0.02	7.1.3
3087.6	$55060 \sim 55065$	3087.59 ± 0.13	-	2.47 ± 0.02	7.0.4
3095.7	$55066 \sim 55073$	3095.73 ± 0.08	-	2.92 ± 0.02	7.0.4
3096.2	$55074, 55079 \sim 55083$	3096.20 ± 0.07	-	4.98 ± 0.03	7.0.4
3097.0	$55084 \sim 55088$	3096.99 ± 0.08	-	3.10 ± 0.02	7.0.4
3097.3	$55089 \sim 55091$	3097.23 ± 0.10	-	1.68 ± 0.01	7.0.4
3097.7	$55092 \sim 55097$	3097.65 ± 0.08	-	4.66 ± 0.03	7.0.4
3098.7	$55098 \sim 55103$	3098.73 ± 0.08	-	5.64 ± 0.03	7.0.4
3104.0	$55104 \sim 55109$	3104.00 ± 0.08	-	5.72 ± 0.03	7.0.4

Good charged tracks and particle identification

Charged Track Selection

- $|V_{xy}| < 1$cm, $|V_z| < 10$cm, $|\cos\theta| < 0.93$.

Photon Selection (Good Photon Conditions)

- $E_{\gamma,\ \mathrm{EMC}}^{\mathrm{Barrel}} > 25$MeV $\quad |\cos\theta| < 0.80$
- $E_{\gamma,\ \mathrm{EMC}}^{\mathrm{Endcap}} > 50$MeV $\quad 0.86 < |\cos\theta| < 0.92$
- $\theta_{c\gamma} > 10^\circ$
- TDC$\in (0,\ 14) \times 50$ns

Selection of the minimum χ^2 in all

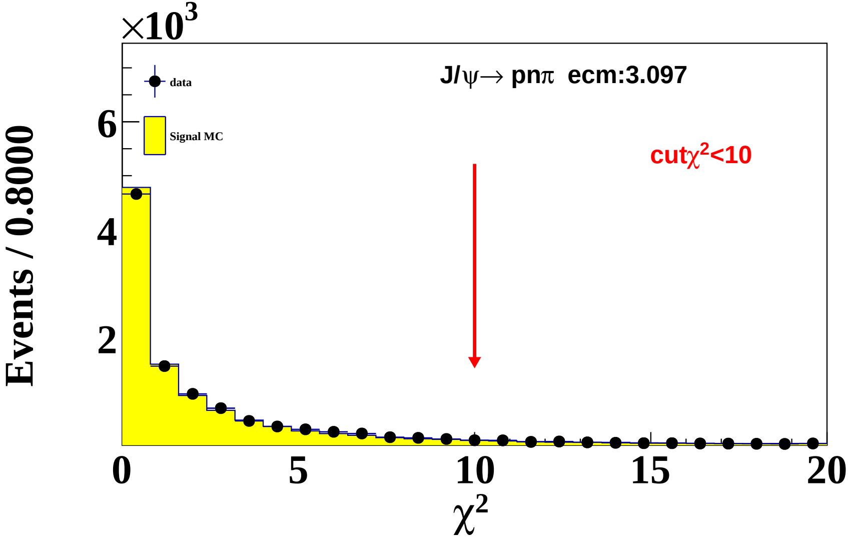
p selection

- CL$(p) >$ CL(K), CL$(p) >$ CL(π)

π selection

- CL$(\pi) >$ CL(K), CL$(\pi) > 0$

χ^2 between data and MC



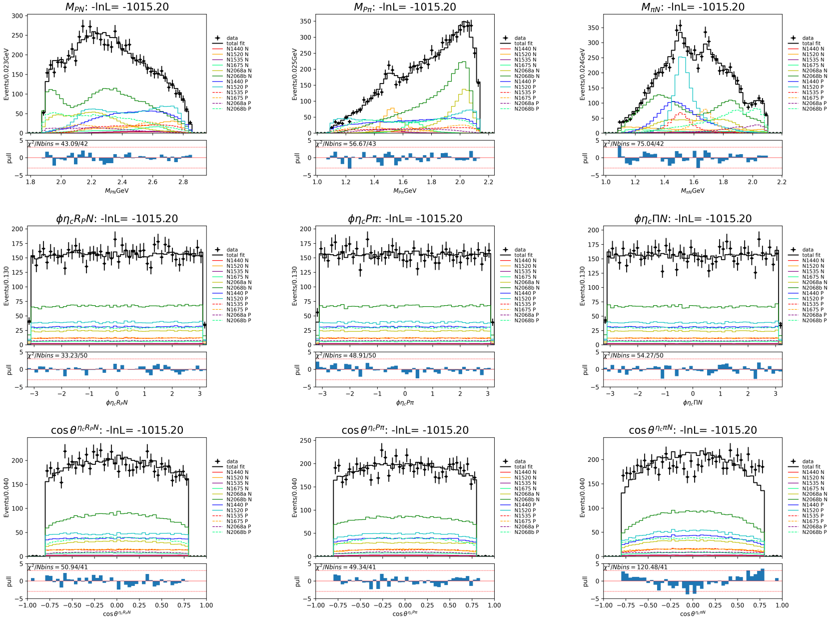
Background analysis

We peform the topology analysis by using the J/ψ inclusive MC

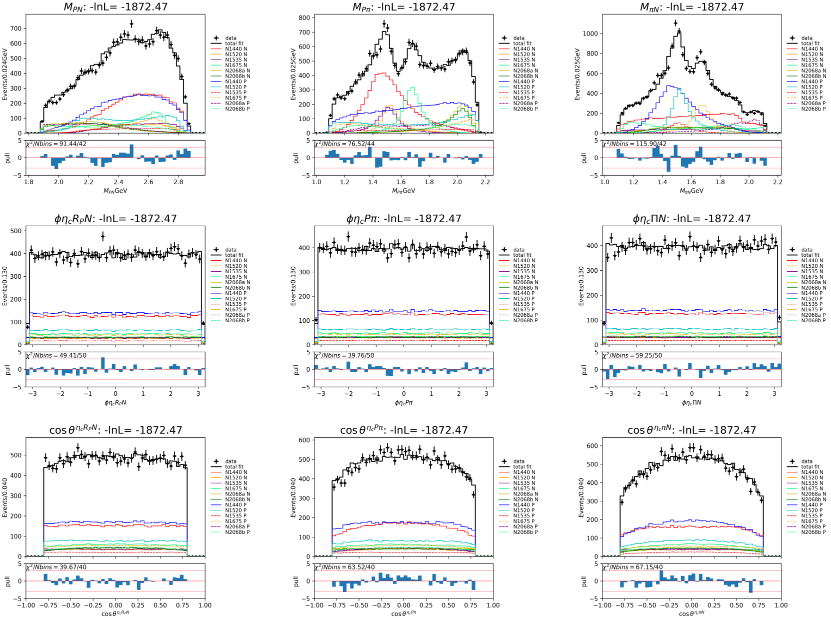
Table 1: Decay trees and their respective final states.

rowNo	decay tree	decay final state	iDcyTr	nEtr	nCEtr
1	$J/\psi \to \pi^+ n\bar{p}$	$\pi^+ n\bar{p}$	1	16314	16314
2	$J/\psi \to \pi^- \bar{n}p$	$\pi^- \bar{n}p$	0	15834	32148
3	$J/\psi \to \eta_c\gamma, \eta_c \to \pi^+ n\bar{p}$	$\pi^+ n\bar{p}\gamma$	15	484	32632
4	$J/\psi \to \eta_c\gamma, \eta_c \to \pi^- \bar{n}p$	$\pi^- \bar{n}p\gamma$	10	472	33104
5	$J/\psi \to \Lambda\bar{\Lambda}, \Lambda \to \pi^0 n, \bar{\Lambda} \to \pi^+\bar{p}$	$\pi^0\pi^+ n\bar{p}$	8	214	33318
6	$J/\psi \to \Lambda\bar{\Lambda}, \Lambda \to \pi^- p, \bar{\Lambda} \to \pi^0\bar{n}$	$\pi^0\pi^- \bar{n}p$	2	212	33530
7	$J/\psi \to \pi^0\pi^0\pi^0\pi^+\pi^-$	$\pi^0\pi^0\pi^0\pi^+\pi^-$	31	185	33715
8	$J/\psi \to \pi^+ n\bar{p}\gamma^f$	$\pi^+ n\bar{p}\gamma^f$	26	130	33845
9	$J/\psi \to \pi^- \bar{n}p\gamma^f$	$\pi^- \bar{n}p\gamma^f$	17	126	33971
10	$J/\psi \to \pi^- \bar{n}\Delta^+, \Delta^+ \to \pi^0 p$	$\pi^0\pi^- \bar{n}p$	19	108	34079
11	$J/\psi \to \pi^+\Delta^0\bar{p}, \Delta^0 \to \pi^0 n$	$\pi^0\pi^+ n\bar{p}$	24	106	34185
12	$J/\psi \to \pi^+ n\bar{\Delta}^+, \bar{\Delta}^+ \to \pi^0\bar{p}$	$\pi^0\pi^+ n\bar{p}$	6	103	34288
13	$J/\psi \to \pi^-\bar{\Delta}^0 p, \bar{\Delta}^0 \to \pi^0\bar{n}$	$\pi^0\pi^- \bar{n}p$	41	92	34380
14	$J/\psi \to e^+e^-$	e^+e^-	29	69	34449
15	$J/\psi \to \pi^0\pi^- \bar{n}p$	$\pi^0\pi^- \bar{n}p$	30	59	34508
16	$J/\psi \to \pi^0\pi^+ n\bar{p}$	$\pi^0\pi^+ n\bar{p}$	11	55	34563
17	$J/\psi \to e^+e^-\gamma^f$	$e^+e^-\gamma^f$	70	53	34616
18	$J/\psi \to \Sigma^+\bar{\Sigma}^-, \Sigma^+ \to \pi^+ n, \bar{\Sigma}^- \to \pi^0\bar{p}$	$\pi^0\pi^+ n\bar{p}$	58	50	34666
19	$J/\psi \to \pi^0\pi^0\pi^+\pi^-\gamma^F$	$\pi^0\pi^0\pi^+\pi^-\gamma^F$	127	49	34715
20	$J/\psi \to \Sigma^+\bar{\Sigma}^-, \Sigma^+ \to \pi^0 p, \bar{\Sigma}^- \to \pi^-\bar{n}$	$\pi^0\pi^- \bar{n}p$	3	44	34759
21	$J/\psi \to \mu^+\mu^-$	$\mu^+\mu^-$	21	38	34797
22	$J/\psi \to \pi^- \bar{n}p\gamma^F$	$\pi^- \bar{n}p\gamma^F$	117	37	34834
23	$J/\psi \to \pi^0 K^- K^{*+}, K^{*+} \to \pi^+ K^0, K^0 \to K^0_L$	$\pi^0 K^0_L \pi^+ K^-$	92	34	34868
24	$J/\psi \to \pi^+ n\bar{p}\gamma^F$	$\pi^+ n\bar{p}\gamma^F$	63	29	34897
25	$J/\psi \to \pi^+ K^* K^{*-}, K^* \to \pi^0 K^0, K^{*-} \to \pi^0 K^-, K^0 \to K^0_L$	$\pi^0\pi^0 K^0_L \pi^+ K^-$	78	27	34924
26	$J/\psi \to e^+e^-\gamma^f\gamma^f$	$e^+e^-\gamma^f\gamma^f$	71	27	34951
27	$J/\psi \to \pi^0\pi^+ K^0_S K^-, K^0_S \to \pi^0\pi^0$	$\pi^0\pi^0\pi^0\pi^+ K^-$	133	27	34978

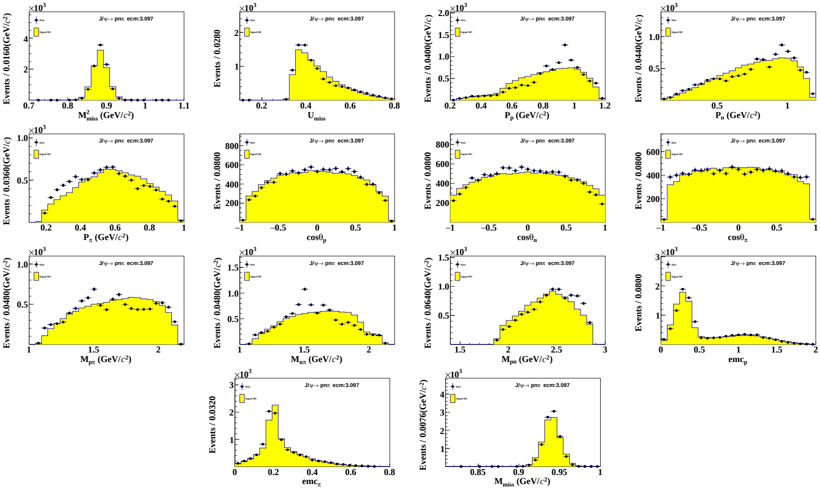
Amplitude analysis result @3080.0



Amplitude analysis result @3097.0

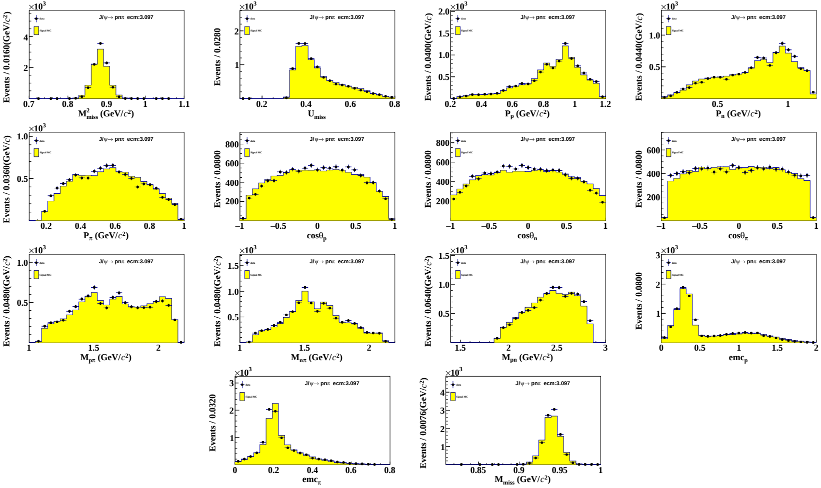


Data/MC Consistancy of PHSP



PHSP model doesn't have good consistency

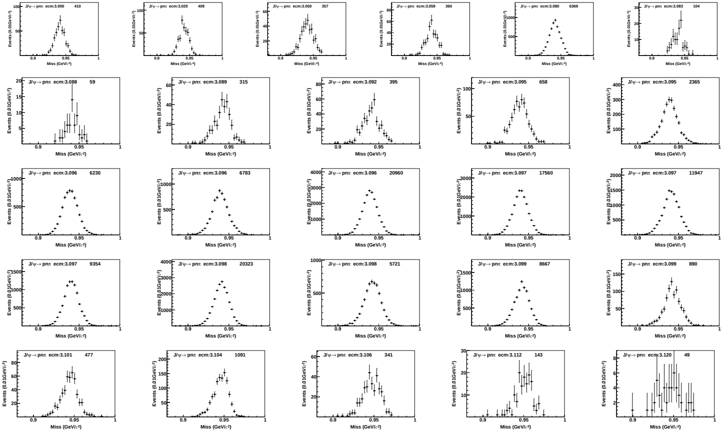
Data/MC Consistancy of TFPWA



Using TFPWA improves consistency

Signal yields

Get yields by count



Observed cross section

$$\sigma_{\text{obs}} = \frac{N_{\text{sig}}}{\epsilon \cdot \mathcal{L}}$$

Datasets	N_{sig}	ϵ	$\mathcal{L}$ (pb^{-1})	σ_{obs}(pb)
3000.0	412 ± 21	22.61 ± 0.11	15.85 ± 0.11	85.3 ± 5.4
3020.0	410 ± 21	23.39 ± 0.11	17.32 ± 0.12	77.7 ± 5.2
3049.6	360 ± 19	22.84 ± 0.11	14.92 ± 0.16	81.4 ± 5.4
3058.7	361 ± 19	22.12 ± 0.11	15.06 ± 0.16	69.8 ± 4.9
3080.0	6375 ± 80	22.52 ± 0.11	293.27 ± 0.91	74.7 ± 1.2
3082.5	104 ± 11	21.51 ± 0.10	4.77 ± 0.06	70.8 ± 8.9
3087.6	59 ± 8	22.89 ± 0.11	2.47 ± 0.02	77.9 ± 13.2
3088.9	315 ± 18	21.92 ± 0.10	15.56 ± 0.17	95.2 ± 5.5
3091.8	399 ± 21	22.08 ± 0.11	14.91 ± 0.16	114.3 ± 6.1
3094.7	659 ± 26	34.30 ± 0.13	2.14 ± 0.03	1035.8 ± 43.1
3095.4	2368 ± 49	35.74 ± 0.13	1.82 ± 0.02	4021.7 ± 92.2
3095.7	6233 ± 79	33.03 ± 0.13	2.92 ± 0.02	6839.2 ± 98.2
3095.8	6783 ± 83	35.91 ± 0.13	2.14 ± 0.03	10076.4 ± 134.6
3096.2	20964 ± 145	36.06 ± 0.13	4.98 ± 0.03	13086.9 ± 104.3
3097.0	17566 ± 133	35.93 ± 0.13	3.10 ± 0.02	17504.8 ± 153.6
3097.2	11966 ± 110	36.06 ± 0.13	2.07 ± 0.03	17423.1 ± 179.9
3097.3	9354 ± 97	35.96 ± 0.13	1.68 ± 0.01	17560.9 ± 207.9
3097.7	20324 ± 143	35.63 ± 0.13	4.66 ± 0.03	13610.1 ± 110.6
3098.3	5729 ± 77	35.87 ± 0.13	2.20 ± 0.03	8172.9 ± 120.5
3098.7	8671 ± 94	35.63 ± 0.13	5.64 ± 0.03	4606.7 ± 58.4
3099.0	892 ± 30	35.55 ± 0.13	0.76 ± 0.01	3102.0 ± 125.9
3101.4	478 ± 22	33.98 ± 0.13	1.61 ± 0.02	945.3 ± 48.3
3104.0	1091 ± 33	31.11 ± 0.12	5.72 ± 0.03	584.6 ± 21.0
3105.6	344 ± 19	31.14 ± 0.12	2.11 ± 0.03	516.0 ± 31.4
3112.1	146 ± 12	26.17 ± 0.11	1.72 ± 0.02	364.0 ± 29.6
3119.9	49 ± 7	16.36 ± 0.09	1.26 ± 0.02	267.9 ± 30.0

Fitting formula of Observed cross section

Fitting the observed cross section and taking into account the ISR effect and the beam energy spread:

$$\sigma_{\text{obs}}(\sqrt{s}) = \int_{\sqrt{s}-5S_E}^{\sqrt{s}+5S_E} G(\sqrt{s}' - \sqrt{s}, S_E) d\sqrt{s}' \int_0^{1-\frac{s_{\text{min}}}{s}} dx \cdot F_{ISR}(x, s) \cdot \sigma_{\text{Dress}}(s(1-x)) \quad (2)$$

- $\sigma_{\text{Dress}}(s)$ is the Dress cross section
- $G(\sqrt{s}' - \sqrt{s}, S_E)$ is the normal distribution to describe energy spread
- F_{ISR} is the ISR-function by Kuraev and Fadin
- s is the square of energy $\sqrt{s}$ in the center of mass system

Parameterization of the Dress cross section

$$\sigma^f_{\text{Dress}} = \frac{4\pi\alpha^2}{3s}\left|\frac{1}{|1-\Pi_0(s)|} + (1+Ae^{i\phi_{\gamma,3g}})\frac{s}{M}\frac{3\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}/\alpha}{s-M^2+iM\Gamma}e^{i\phi_{\gamma,\text{cont}}}\right|^2 \cdot \mathcal{P}(s)\cdot(\frac{\mathcal{F}}{s^{n/2}})^2, \tag{3}$$

- M and Γ are the mass and total width of J/ψ
- the form factor can be expressed as:

$$\Gamma_f = \mathcal{P}(M^2)(\frac{\mathcal{F}}{M^n})^2\Gamma_{\mu\mu}\left|1+Ae^{i\phi_{\gamma,3g}}\right|^2, \tag{4}$$

- $\mathcal{P}(s)$ is the phase space of the J/ψ decay final state, for this decay, we consider it as:

$$\mathcal{P}(s) = 1. \tag{5}$$

χ^2 minimization function

Let $\vec{S}$ be the vector of measured cross sections and $\sigma_{\text{obs}}(\sqrt{s})$ be the functional form of these cross sections with unknown parameters, define $\Delta\mathbf{X} = \vec{S} - \vec{\sigma}_{\text{obs}}$, we minimize:

$$\chi^2 = \Delta\mathbf{X}^{\mathsf{T}}\mathbf{M}^{-1}\Delta\mathbf{X}. \tag{6}$$

Here $\mathbf{M}$ is the covariance matrix of the measurements which can be written as the sum of three matrices:

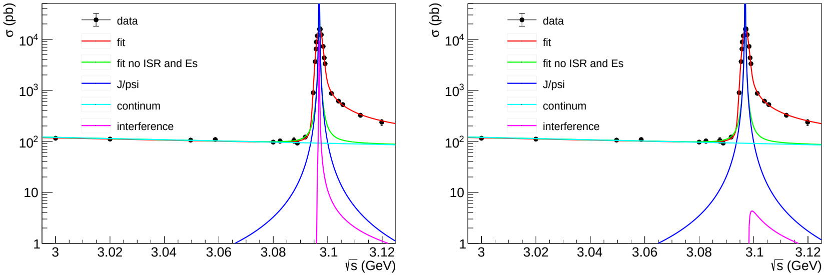
$$\mathbf{M} = \mathbf{M}_{\text{stat}} + \mathbf{M}_{\text{stat}}^{\sqrt{s}} + \mathbf{M}_{\text{syst}}. \tag{7}$$

- $\mathbf{M}_{\text{stat}}$ is a diagonal matrix consisting statistical uncertainties of the cross sections
- $\mathbf{M}_{\text{stat}}^{\sqrt{s}}$ is a diagonal matrix associated with uncertainties in energy measurement
- $\mathbf{M}_{\text{syst}}$ is the covariance matrix of systematic uncertainties contain correlated and independent (At present, not considered yet)

Observed cross section fitting results

The fit only includes statistical uncertainty.

Solution	$\varphi(^\circ)$	$\delta(\times 10^{-4} MeV)$	$\mathcal{B}(\times 10^{-3})$	$\mathcal{B}_{\mathrm{PDG}}(\times 10^{-3})$
Positive	108.2 ± 3.5	9.15 ± 0.23	4.65 ± 0.10	$\mathcal{B}_{p\pi^-\bar{n}} = 2.36 \pm 0.02 \pm 0.21$
Negative	-108.0 ± 3.9	9.16 ± 0.17	4.93 ± 0.08	$\mathcal{B}_{\bar{p}\pi^+n} = 2.47 \pm 0.02 \pm 0.24$



Summary

- The cross section of the $e^+e^- \to \bar{p}n\pi^+ + c.c.$ process is measured in the energy region around the J/ψ peak
- From the fit to this cross section line shape, two solutions of phase angles and branching fractions are obtained.

Solution	$\varphi(^\circ)$	$\mathcal{B}(\times 10^{-3})$	$\mathcal{B}_{\mathrm{PDG}}(\times 10^{-3})$
Positive	108.2 ± 3.5	4.65 ± 0.10	$\mathcal{B}_{p\pi^-\bar{n}} = 2.36 \pm 0.02 \pm 0.21$
Negative	-108.0 ± 3.9	4.93 ± 0.08	$\mathcal{B}_{\bar{p}\pi^+n} = 2.47 \pm 0.02 \pm 0.24$

- Next to do
  - Study of systematic uncertainties
  - prepare the MEMO

Thank you!

Back up

Gauss and ISR formula

The beam energy spread is described is described by convolving Gaussian function and the expression is:

$$G(\sqrt{s},\delta) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\sqrt{s}-\sqrt{s}'}{\delta}\right)^2} \tag{8}$$

The convolution ISR function is expressed as:

$$\begin{aligned} F(x,s) =& \Delta \cdot x^{\beta-1}\beta - \frac{\beta}{2}(2-x) \\ & + \frac{\beta^2}{8}\left[-4(2-x)\ln x - \frac{1+3(1-x)^2}{x}\ln(1-x) - 6 + x\right] \end{aligned} \tag{9}$$

- $\Delta = 1 + \frac{3}{4}\beta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \beta^2\left(\frac{9}{32} - \frac{\pi^2}{12}\right)$
- $\beta = \frac{2\alpha}{\pi}\left(2\ln\frac{\sqrt{s}}{m_e} - 1\right)$