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# Measurement of Strong Coupling Constant from Hadronic Tau Decay at BESIII

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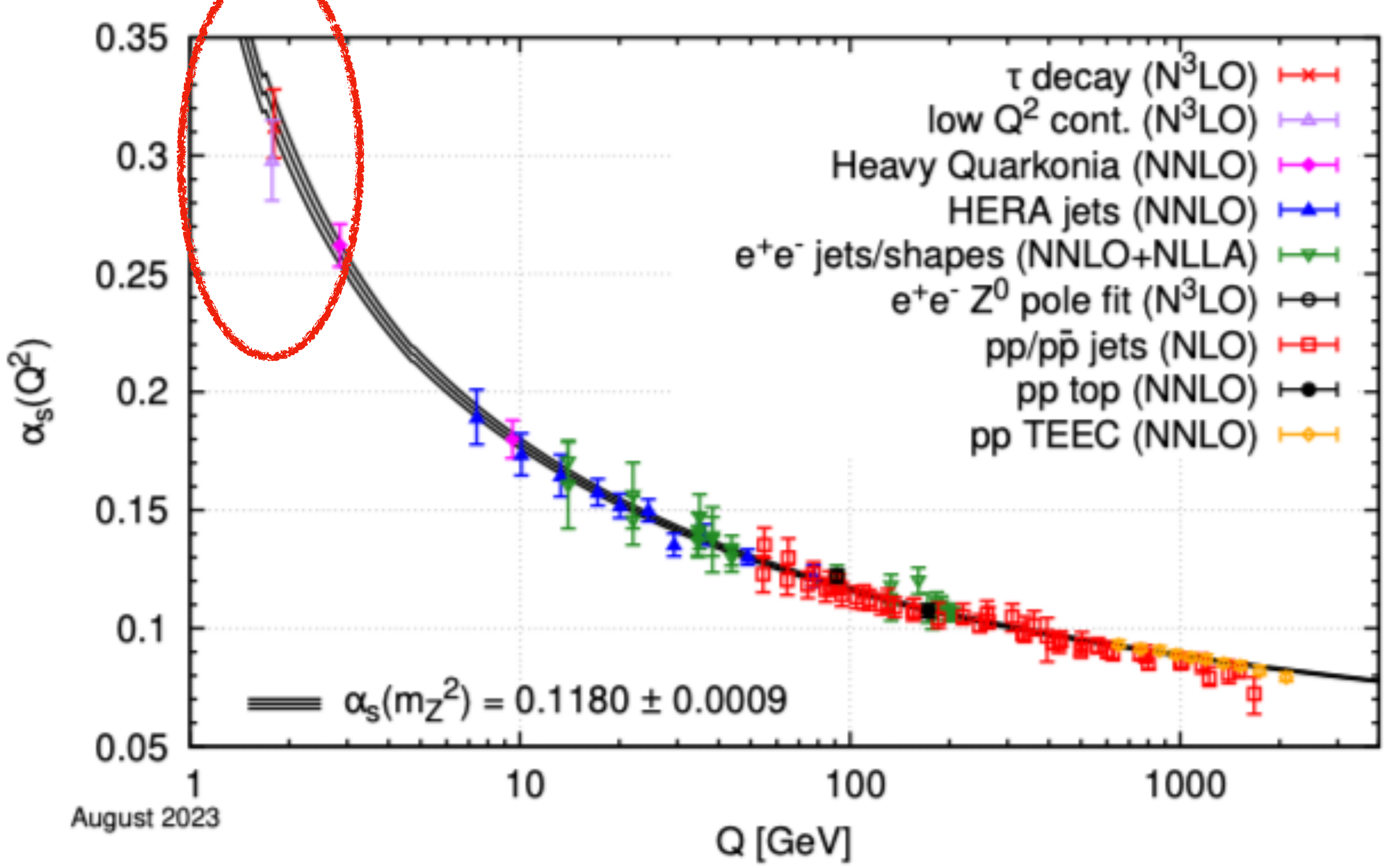
https://indico.ihep.ac.cn/event/21419/#5-measurement-of-the-strong-co
https://indico.ihep.ac.cn/event/22305/contributions/159139/

***Tau&QCD Group Meeting***

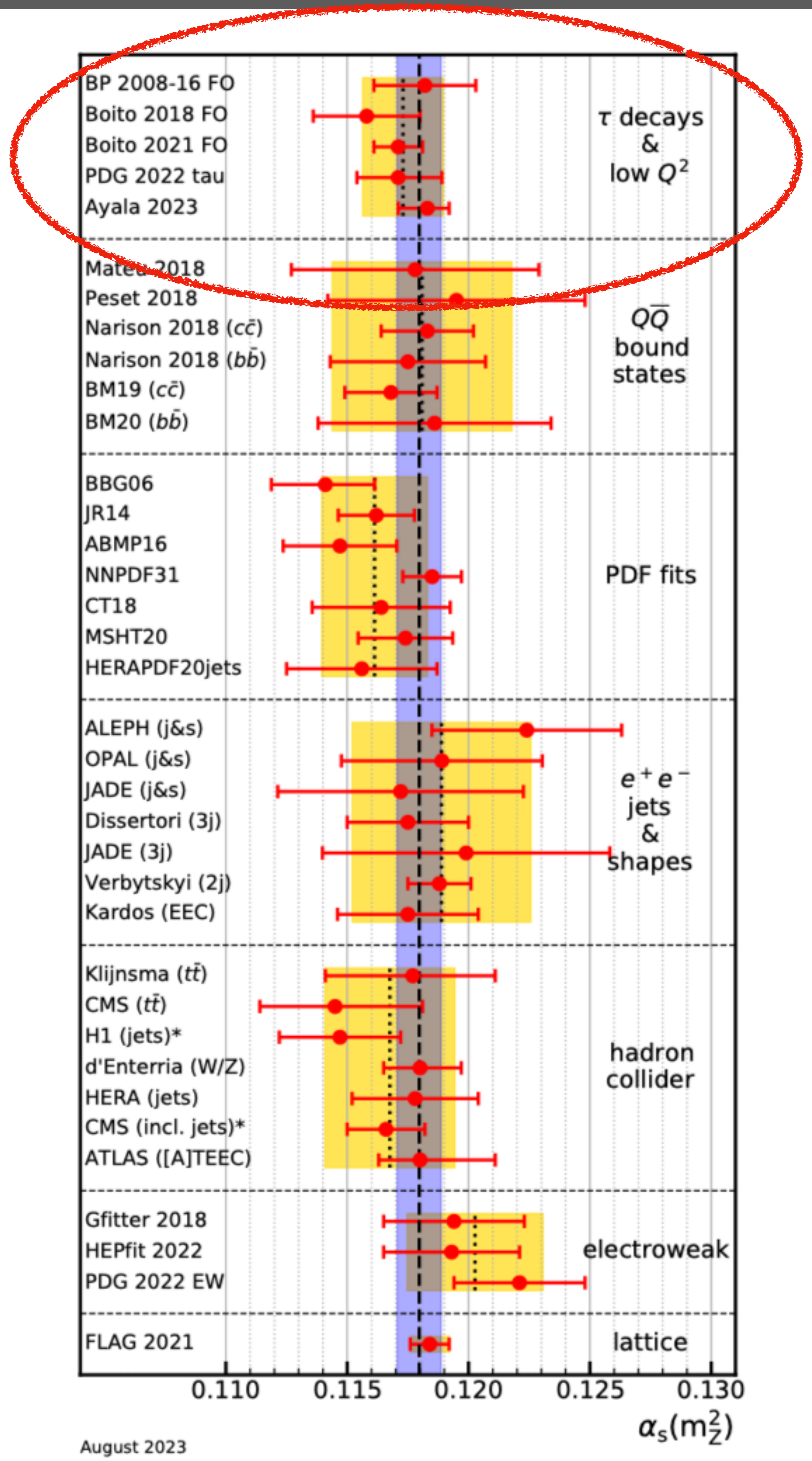
Apr. 16th, 2025

# The Strong Coupling Constant $\alpha_s$

- **The strong coupling constant $\alpha_s$ is**
  - a **fundamental parameter** of the Standard Model (SM) of particle physics and Quantum Chromodynamics (QCD).
  - the **least understood** among the three coupling constants.
- The $\alpha_s$ evolves versus the energy scale of QCD under control of the Renormalization Group Equation (RGE).




- The $\alpha_s$ can be determined from various processes:
  - R value, $\tau/Z/W$ decays, jet production, event shapes, etc.
- Among these, the most accurate results in PDG are from hadronic $\tau$ decays.



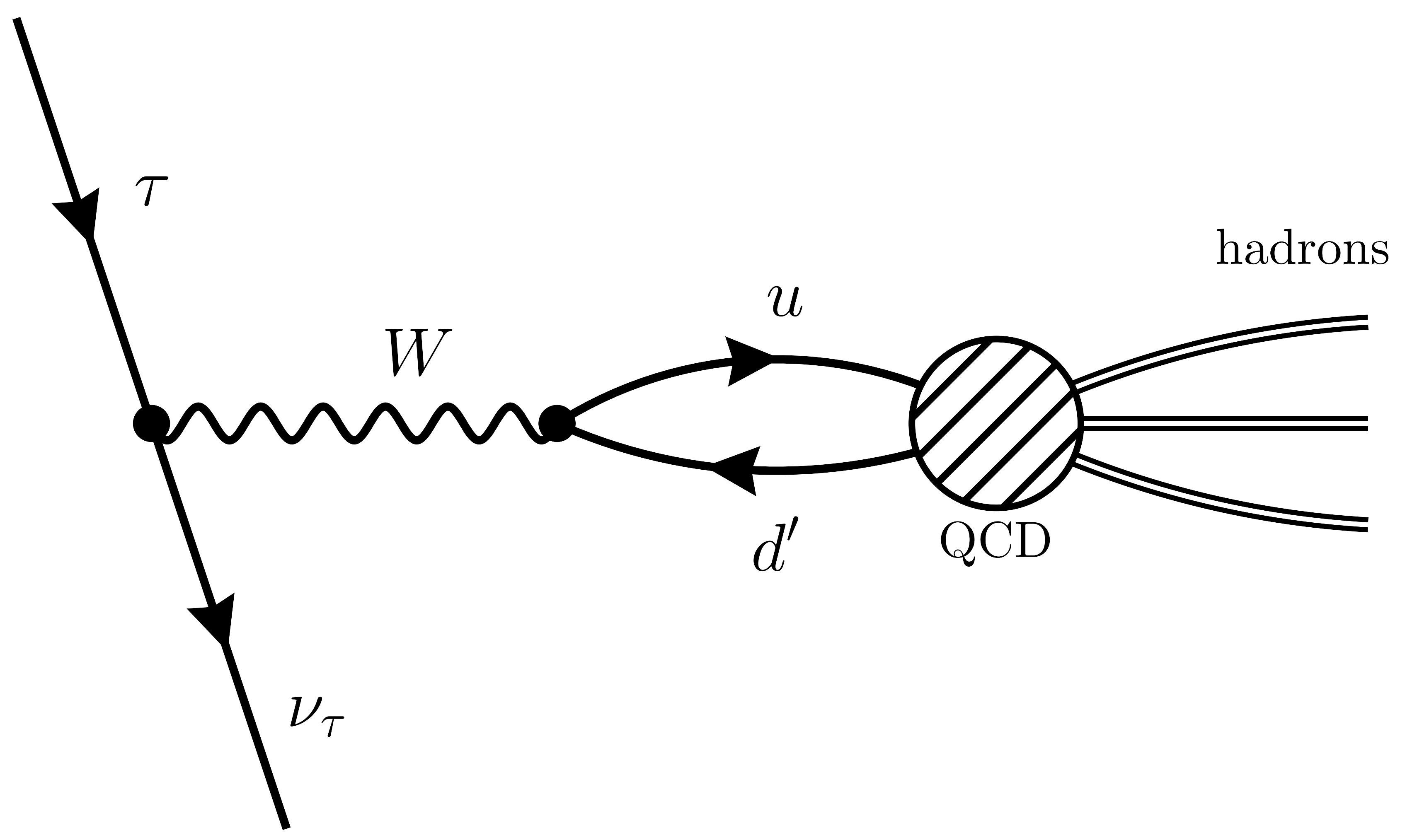
# Theory model of hadronic $\tau$ decay spectra

- **Observable**: $R_\tau \equiv \dfrac{\Gamma\left(\tau \to \nu_\tau \text{ hadrons}\right)}{\Gamma\left(\tau \to l\nu_\tau\nu_l\right)} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$

  - $R_{\tau,V+A} \equiv \dfrac{\Gamma\left(\tau \to \nu_\tau \text{ pions}\right)}{\Gamma\left(\tau \to l\nu_\tau\nu_l\right)}$

  - $R^{kl}_{\tau,V/A}(s_0) = \displaystyle\int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}$

  - $D^{kl}_\tau = \dfrac{R^{kl}_\tau}{R^{00}_\tau}$




- **Parameterization**:

  - $R^{kl}_{\tau,V/A}\left(s_0\right) = N_c S_{EW}\left|V_{\text{ud}}\right|^2 \left(r^{kl}\left(1+\delta^{(0),kl}\left(s_0;\alpha_S(s_0)\right)\right) + \displaystyle\sum_{D=2,4\ldots} \delta^{kl}_{\text{ud,V/A}}\left(s_0, D\right)\right)$

$$\delta_{\mathrm{P}} = \underbrace{\sum_{n=1} K_n A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=1} r_n a_\tau^n}_{\text{FOPT}} \qquad a_\tau \equiv \frac{\alpha_s(m_\tau^2)}{\pi}$$

$$\delta_{\mathrm{NP}}\left(\left\langle \frac{\alpha_s}{\pi}\mathrm{GG}\right\rangle, \mathcal{O}(6), \mathcal{O}(8)\right)$$

- $\alpha_s\left(M_\tau^2\right)$ **can be derived from a fit with** $R_\tau$, $D^{10}$, $D^{11}$ $D^{12}$, $D^{13}$.

V/A denotes vector/axial-vector components of non-strange hadronic $\tau$ decays, different from the number of pions.

# Observables of $\alpha_s(m_\tau^2)$ determination from $\tau$ decays

- $$R_{\tau,V+A} \equiv \frac{\Gamma\left(\tau \to \nu_\tau \text{ pions}\right)}{\Gamma\left(\tau \to l \nu_\tau \nu_l\right)} = \frac{\Gamma_h}{\Gamma_l}$$

can be derived from ***branching ratios***:

$$R_{\tau,V+A} = \frac{1 - \mathscr{B}_e - \mathscr{B}_\mu - \mathscr{B}_S}{\mathscr{B}_e}$$

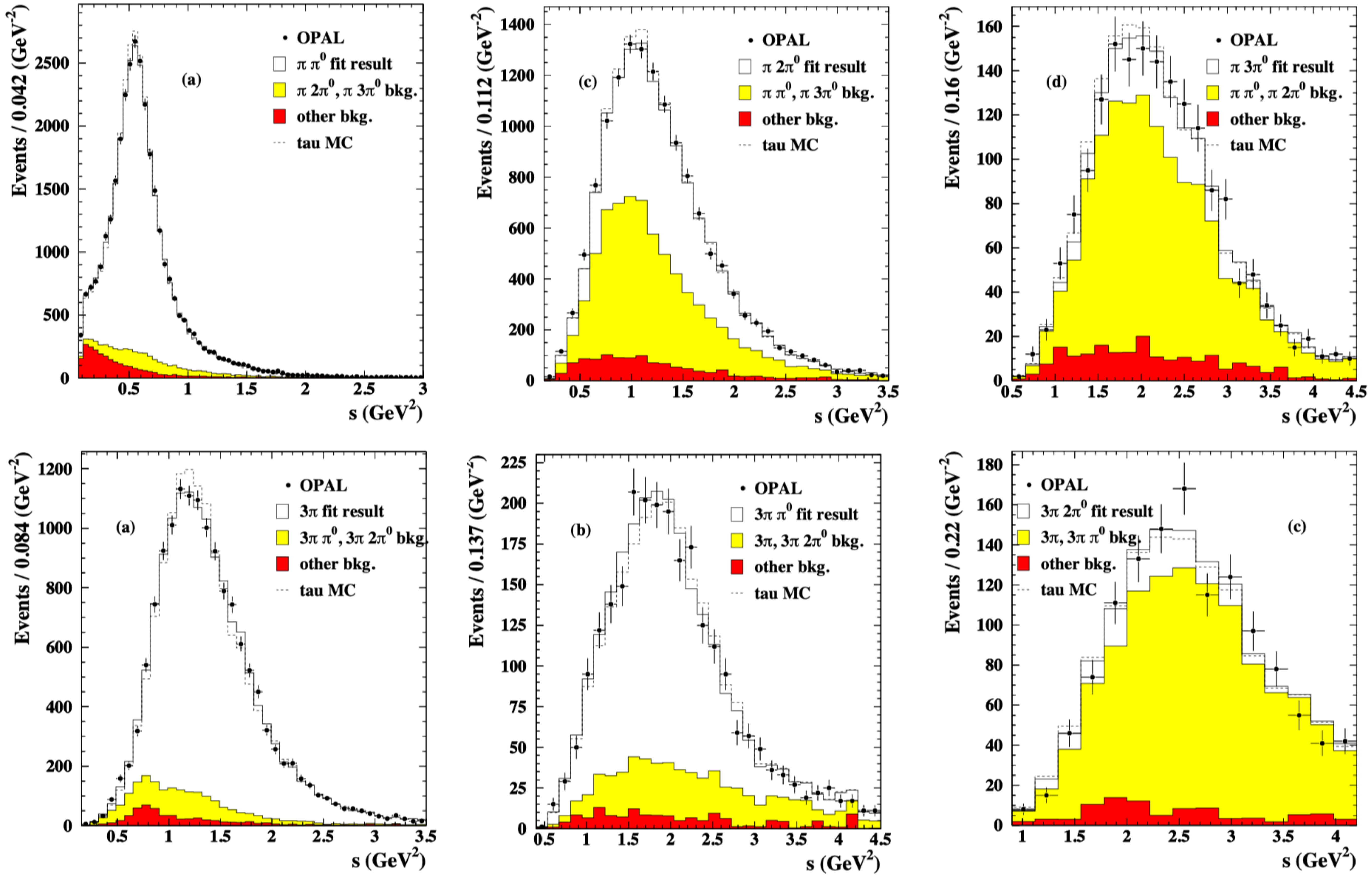
| Decay Mode | Branching Ratio (%) |
|---|---|
| $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ | $17.39 \pm 0.04$ |
| $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ | $17.82 \pm 0.04$ |

| Decay Mode | Branching Ratio |
|---|---|
| $\tau^- \to K^- \nu_\tau$ | $(6.96 \pm 0.10) \times 10^{-3}$ |
| $\tau^- \to K^- \pi^0 \nu_\tau$ | $(4.33 \pm 0.15) \times 10^{-3}$ |
| $\tau^- \to \pi^- \overline{K}^0 \nu_\tau$ | $(8.38 \pm 0.14) \times 10^{-3}$ |
| $\tau^- \to h^- \overline{K}^0 \pi^0 \nu_\tau$ | $(5.32 \pm 0.13) \times 10^{-3}$ |
| $\tau^- \to K^- \pi^- \pi^{\geq 0} h^{0-} \nu_\tau$ | $(4.77 \pm 0.14) \times 10^{-3}$ |

- $$R^{kl}_{\tau,V/A}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}, \quad D^{kl} = R^{kl}_{\tau,V+A} \big/ R_{\tau,V+A}$$

are derived from $M^2_{\text{had}}$ ***distributions***, $\dfrac{dR_{\tau,V/A}}{ds}$ :

arXiv: hep-ex/9808019




The spectrum for $\tau^- \to \pi^- \nu_\tau$ channel is not shown, as it corresponds to a $\delta(m_\pi^2)$,

# BESIII Detector & Data

# BESIII data set and MC samples

- **Data set**: $\sqrt{s} = 4.26\,\mathrm{GeV}$ $825.7\ \mathrm{pb}^{-1}$
- **BOSS version**: v7.0.3
- **Signal topology**: $\tau\tau \to \left(e^{\pm}\nu\nu\right)_{\mathrm{tag}}\left(n\pi^{\pm}m\pi^{0}\nu\right)_{\mathrm{signal}}, n = 1,3; m = 0,1,2,3,4,\ldots$
- **Signal MC: KKMC + TAUOLA**
  - Exclusive channels are extracted from MC truth.
- All the MC sample are normalized to data.

BESIII $\sqrt{s} = 4.26\,\mathrm{GeV}$ $e^+e^- \to \tau^+\tau^-$ event display

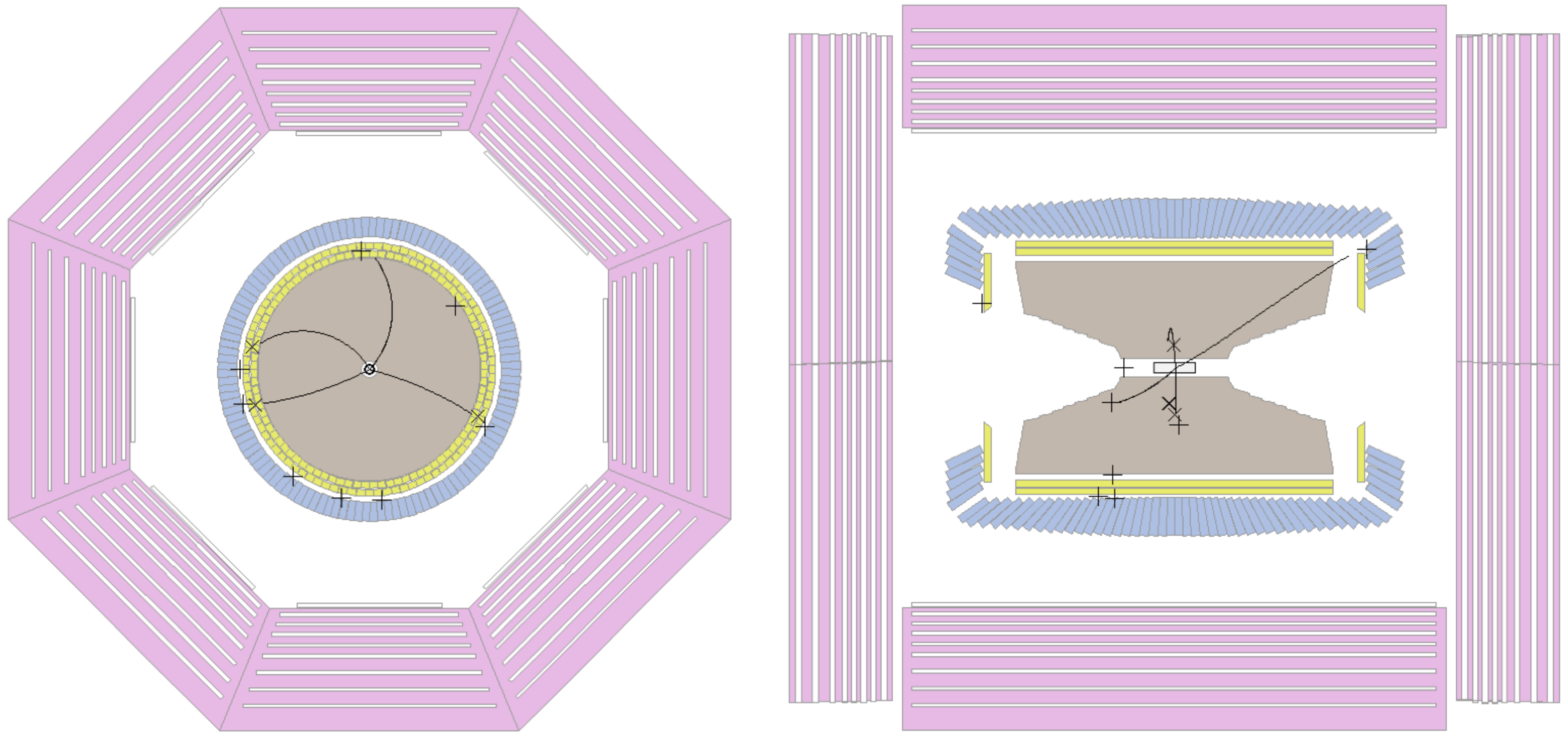


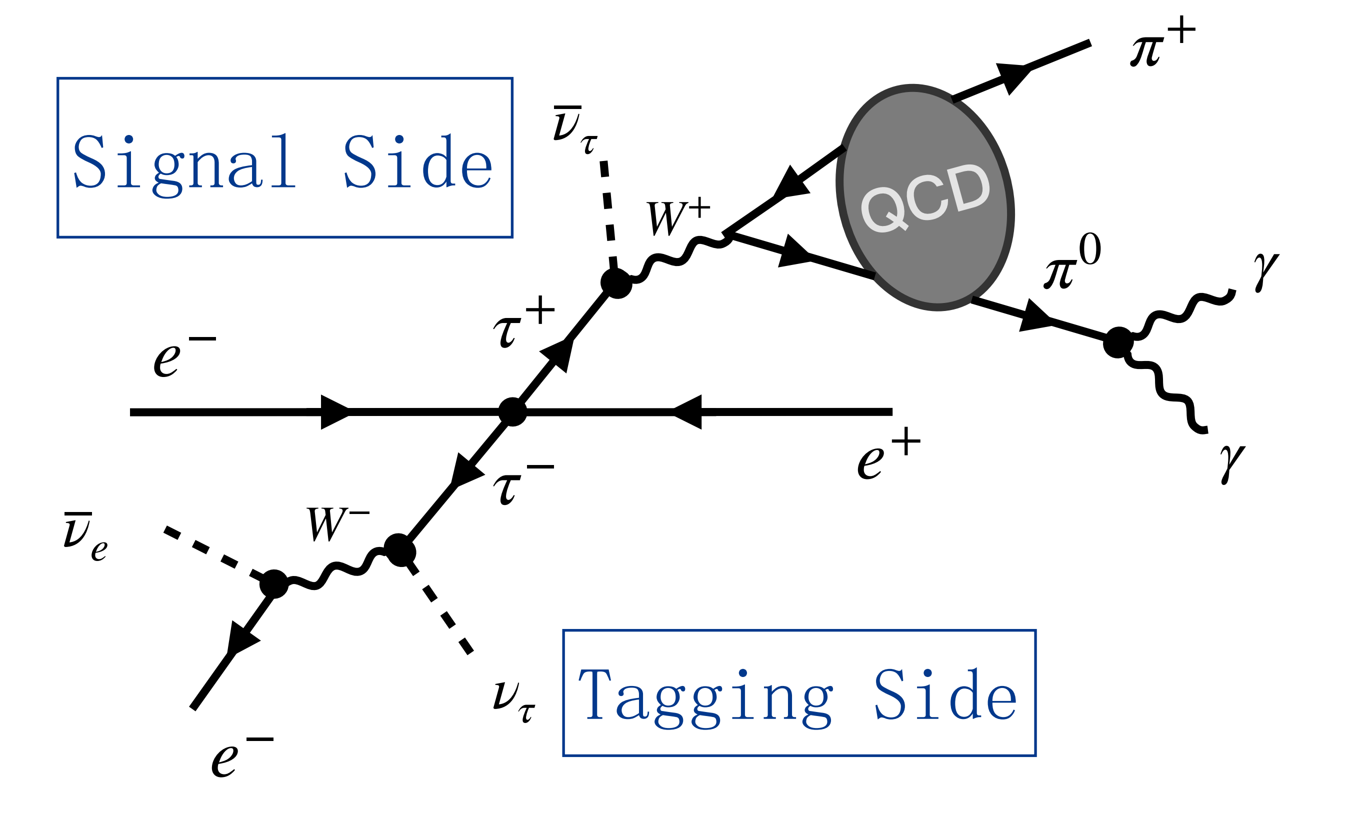
Table 3: Generator packages, cross-sections, statistics of the involved MC samples.

| | Cross section (nb) | Generator | #.Events ($\times 10^6$) | Expected #.Events ($\times 10^6$) |
|---|---|---|---|---|
| $e^+e^- \to$ hadrons | 18.9 | HYBRID | 49.0 | 15.6 |
| $e^+e^- \to \tau^+\tau^-$ | 3.8 | TAUOLA | 39.9 | 3.1 |
| $e^+e^- \to e^+e^-$ | 419 | BABAYAGA | 50 | 346 |
| $e^+e^- \to \mu^+\mu^-$ | 5.0 | BABAYAGA | 4.0 | 4.1 |
| $e^+e^- \to e^+e^-e^+e^-$ | 0.12 | DIAG36 | 0.2 | 0.1 |
| $e^+e^- \to e^+e^-\mu^+\mu^-$ | 34.7 | DIAG36 | 29.5 | 28.7 |
| $e^+e^- \to e^+e^-\eta$ | 0.011 | EKHARA | 0.1 | 0.01 |
| $e^+e^- \to e^+e^-\eta'$ | 0.018 | EKHARA | 0.1 | 0.01 |
| $e^+e^- \to e^+e^-K^+K^-$ | 0.24 | GALUGA | 0.04 | 0.2 |
| $e^+e^- \to e^+e^-\pi^+\pi^-$ | 2.67 | GALUGA | 2.2 | 2.2 |

# Event Selection

# $\tau\tau \to \left(e^{\pm}\nu\nu\right)_{\text{tag}} \left(n\pi^{\pm}m\pi^{0}\nu\right)_{\text{signal}}$ : Event selection and classification

**Event Topology：** $\tau\tau \to \left(e^{\pm}\nu\nu\right)_{\text{tag}} \left(n\pi^{\pm}m\pi^{0}\nu\right)_{\text{signal}}, n = 1,3; m = 0,1,2,3,4,\ldots$




**Inclusive selection：**

- Exactly 2 or 4 charged tracks，with zero net charges, and:
  $\left|V_r\right| < 1$ cm, $\left|V_z\right| < 10$ cm, $|\cos\theta| < 0.93$
- **PID**：$e^{\pm} + \pi^{\pm}$ 或 $e^{\pm} + 3\left(\mu^{\pm}/\pi^{\pm}\right)$
  - $e^{\pm}$:
    - $E_{\text{EMC}}/p_{\text{MDC}} > 0.8$;
    - $dE/dx$**+TOF PID**：$CL_e > CL_\pi$ and $CL_e > 0.001$
  - $\mu^{\pm}$:
    - $E_{\text{EMC}}/p_{\text{MDC}} < 0.7; E_{\text{EMC}} < 0.3$ GeV;
    - **Signal Depth in MuC**：
      $D_{\text{MuC}} > \min\left\{80(p/\text{GeV} - 0.65)\text{ cm}, 40\text{ cm}\right\}$;
    - $dE/dx$ **+ TOF PID**：$CL_\mu > CL_K$ **and** $CL_\mu > 0.001$
  - $\pi^{\pm}$:
    - $\mu^{\pm}$ veto (only in 2-track events);
    - $E_{\text{EMC}}/p_{\text{MDC}} < 0.6$;
    - $dE/dx$ **+ TOF PID**：$CL_\pi > CL_{K,e}$ and $CL_\pi > 0.001$

**$\pi^0$ Recosntruction：**

**Good photon：**

- **TDC:** $0 \le t \le 700$ ns
- **Cluster Energy：**
  - Barrel region ($|\cos\theta| < 0.8$)：$E > 25$ MeV
  - Endcap region ($0.86 < |\cos\theta| < 0.92$)：$E > 50$ MeV
- **Isolation：** Open angle with the nearest charged track $\theta_{\gamma f} > 10^{\circ}$

Consider all $\gamma\gamma$ combination in events (with no reused photons)，optimize the summation of the 1-C kinetic fit $\chi^2$.

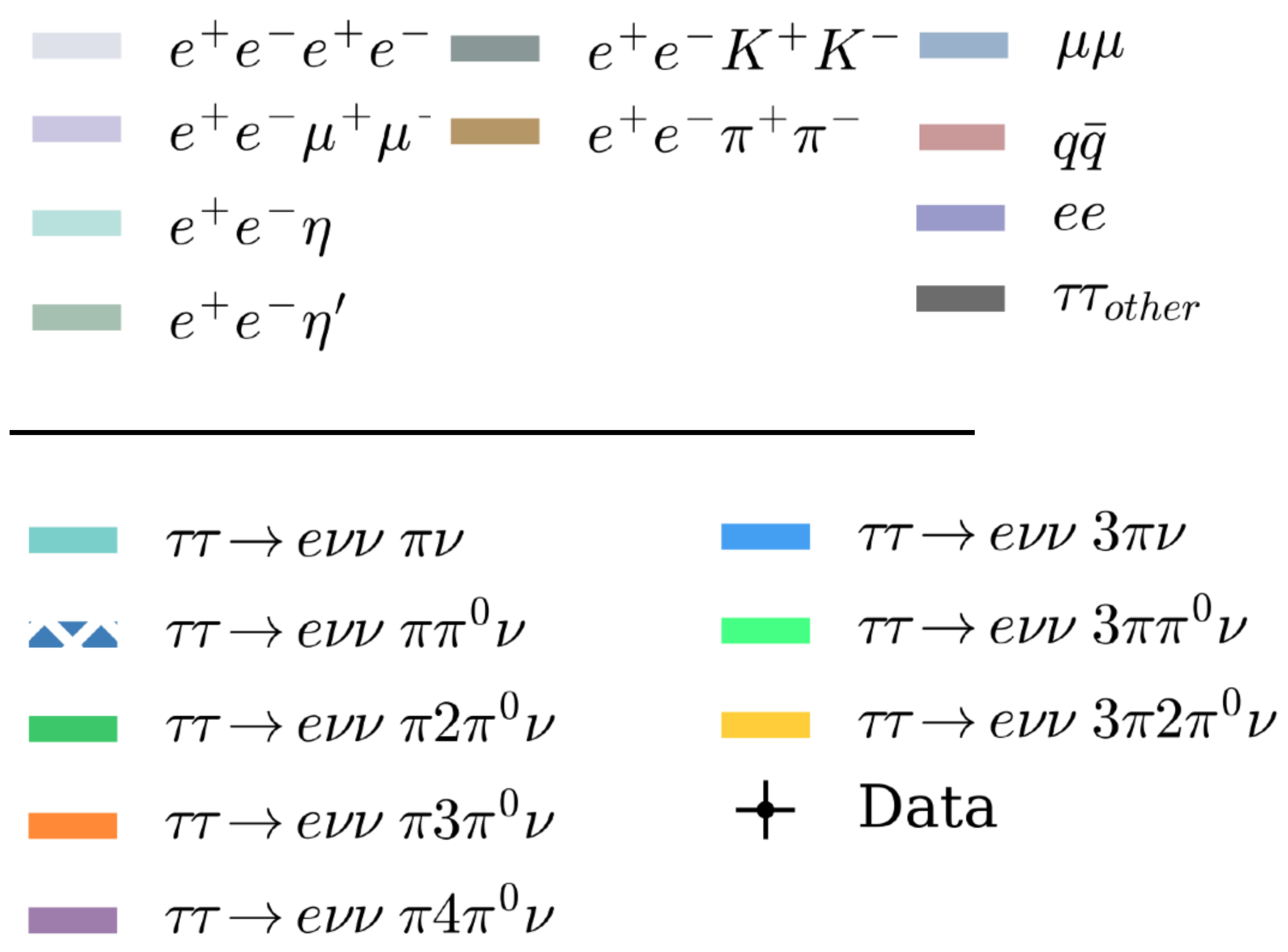
**Exclusive channel classification：** according to number of $\pi^{\pm}$ and $\pi^0$.

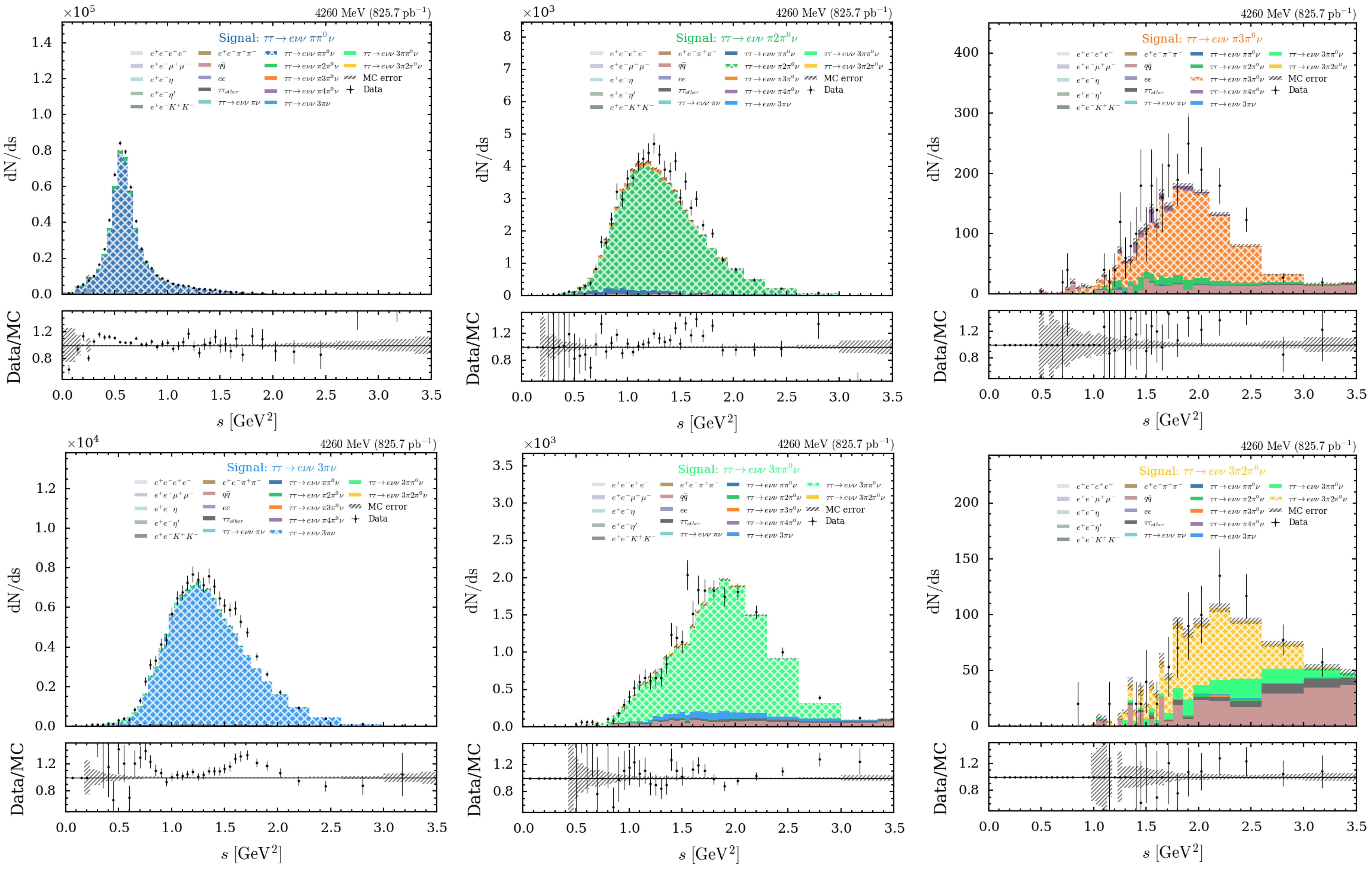
- $\tau \to \pi\nu, 3\pi\nu$: $10^{\circ} < \theta_{\text{acop}} < 160^{\circ}$, $\text{PTEM} = \dfrac{\left(c\vec{P}_1 + c\vec{P}_2\right)_T}{E_{c.m.} - \left|c\vec{P}_1\right| - \left|c\vec{P}_2\right|} > 0.2$,
- $\tau^- \to \pi^-\pi^0\nu_\tau$、$\pi^-2\pi^0\nu_\tau$、$\pi^-3\pi^0\nu_\tau$、$2\pi^-\pi^+\nu_\tau$、$2\pi^-\pi^+\pi^0\nu_\tau$ 和 $2\pi^-\pi^+2\pi^0\nu_\tau$: all good photons should be reconstructed as $\pi^0$.

# Invariant mass spectra after event selection

$s \equiv m^2_{\text{had}}$: invariant mass of multi-pion system.

| Decay Mode | #.Selected Events | #.Signal Events | $\epsilon$ [%] | $f_B(\tau)$ [%] | $f_B$(non-$\tau$) [%] |
|---|---|---|---|---|---|
| $\tau\tau \to e\nu\nu\ \pi\nu$ | 82,893 | 37,198 | 33.6 | 43.6 | 11.5 |
| $\tau\tau \to e\nu\nu\ \pi\pi^0\nu$ | 27,367 | 25,760 | 9.9 | 4.2 | 1.7 |
| $\tau\tau \to e\nu\nu\ \pi 2\pi^0\nu$ | 3,862 | 3,521 | 3.7 | 6.2 | 2.6 |
| $\tau\tau \to e\nu\nu\ \pi 3\pi^0\nu$ | 230 | 138 | 1.3 | 15.4 | 24.4 |
| $\tau\tau \to e\nu\nu\ 3\pi\nu$ | 6,640 | 6,332 | 6.6 | 3.5 | 1.1 |
| $\tau\tau \to e\nu\nu\ 3\pi\pi^0\nu$ | 2,466 | 1,806 | 3.8 | 10.9 | 15.9 |
| $\tau\tau \to e\nu\nu\ 3\pi 2\pi^0\nu$ | 425 | 62 | 1.2 | 16.1 | 69.3 |
| Sum | 123,883 | 74,818 | 25.2 | 30.8 | 8.8 |



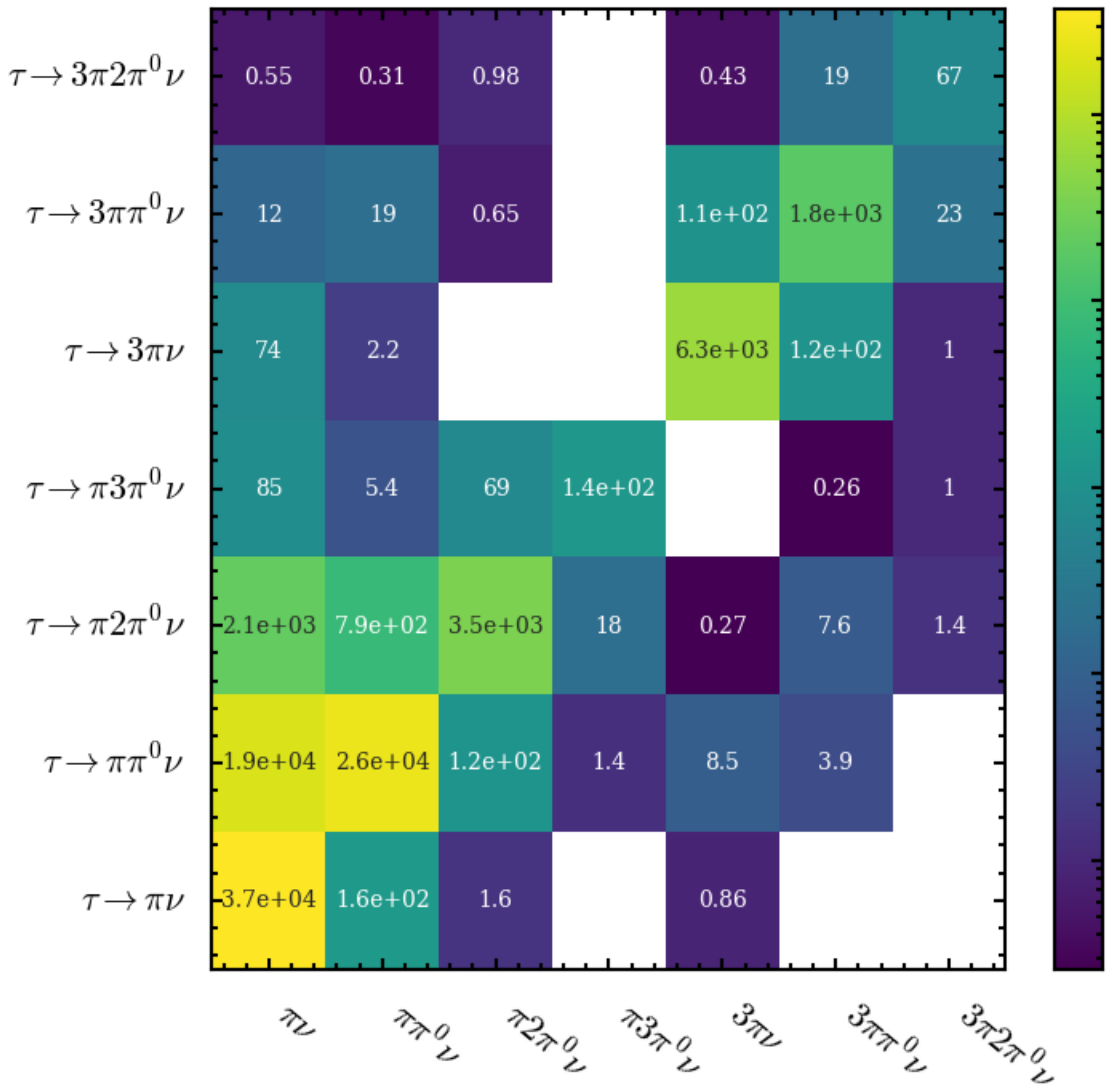
# Data Correction

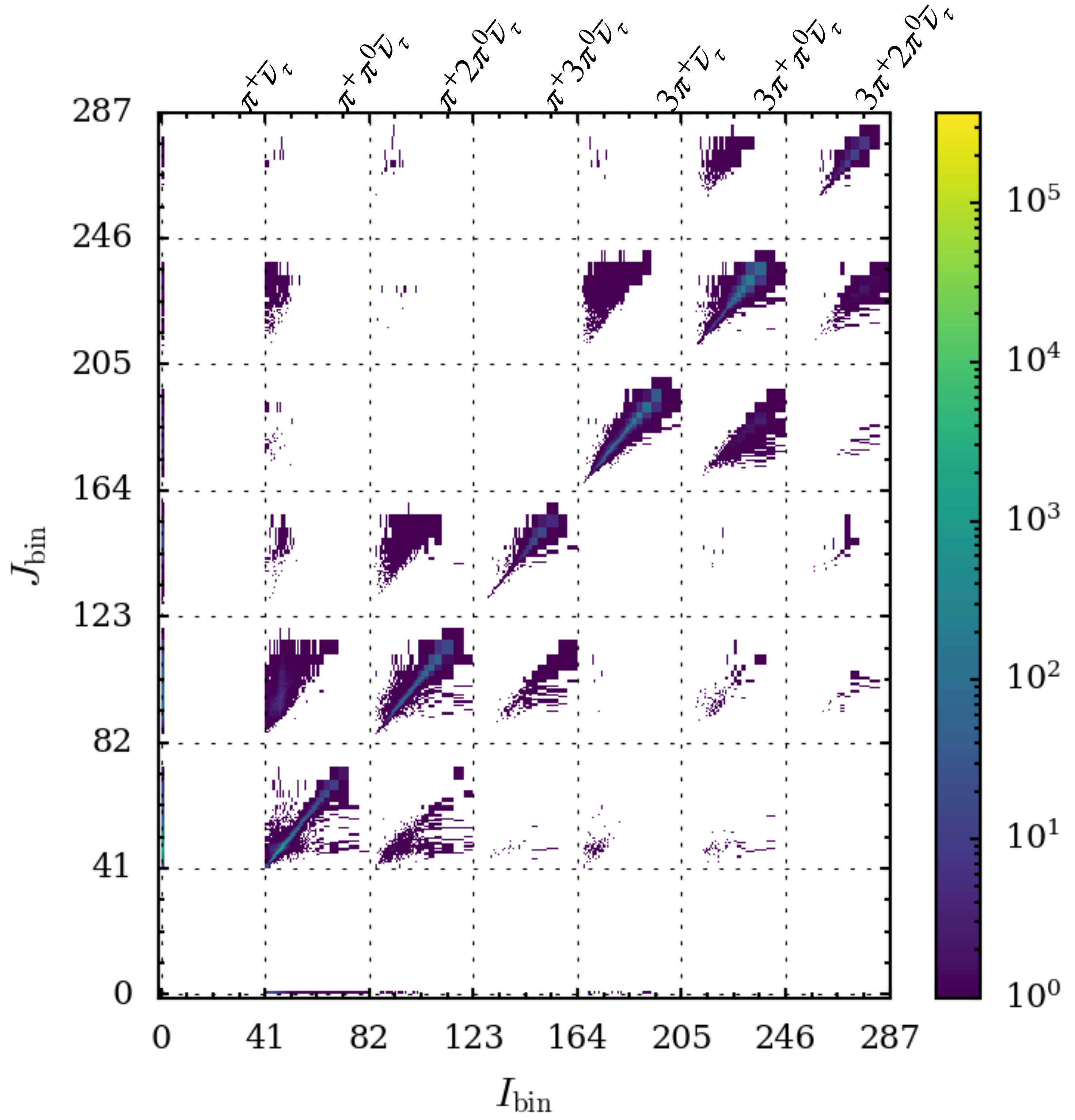
Background subtraction
Unfolding
Efficiency correction

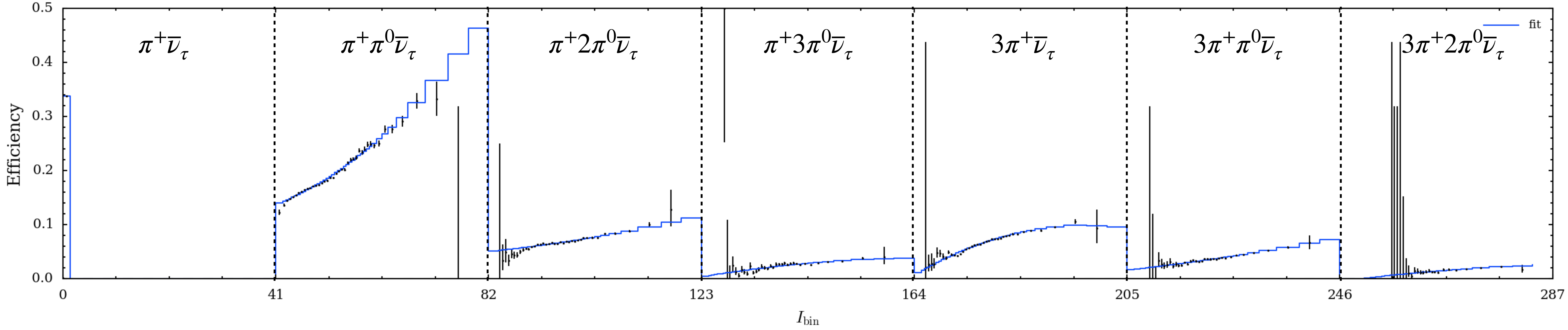
# Data Correction: Efficiency and Response Matrix

**Base on the MC simulation,**

- **Background subtraction:** Subtract MC histograms of background from data.
- **Unfolding:**
  - Joint event channels in to a large one, and unfold based on the response matrix from MC.
  - RooUnfold::RooUnfoldBayes, with iteration number of 4.
- **Efficiency correction:**
  - Fit the efficiency curve with second order Chebyshev polynomial.
  - Divide data with the efficiency.



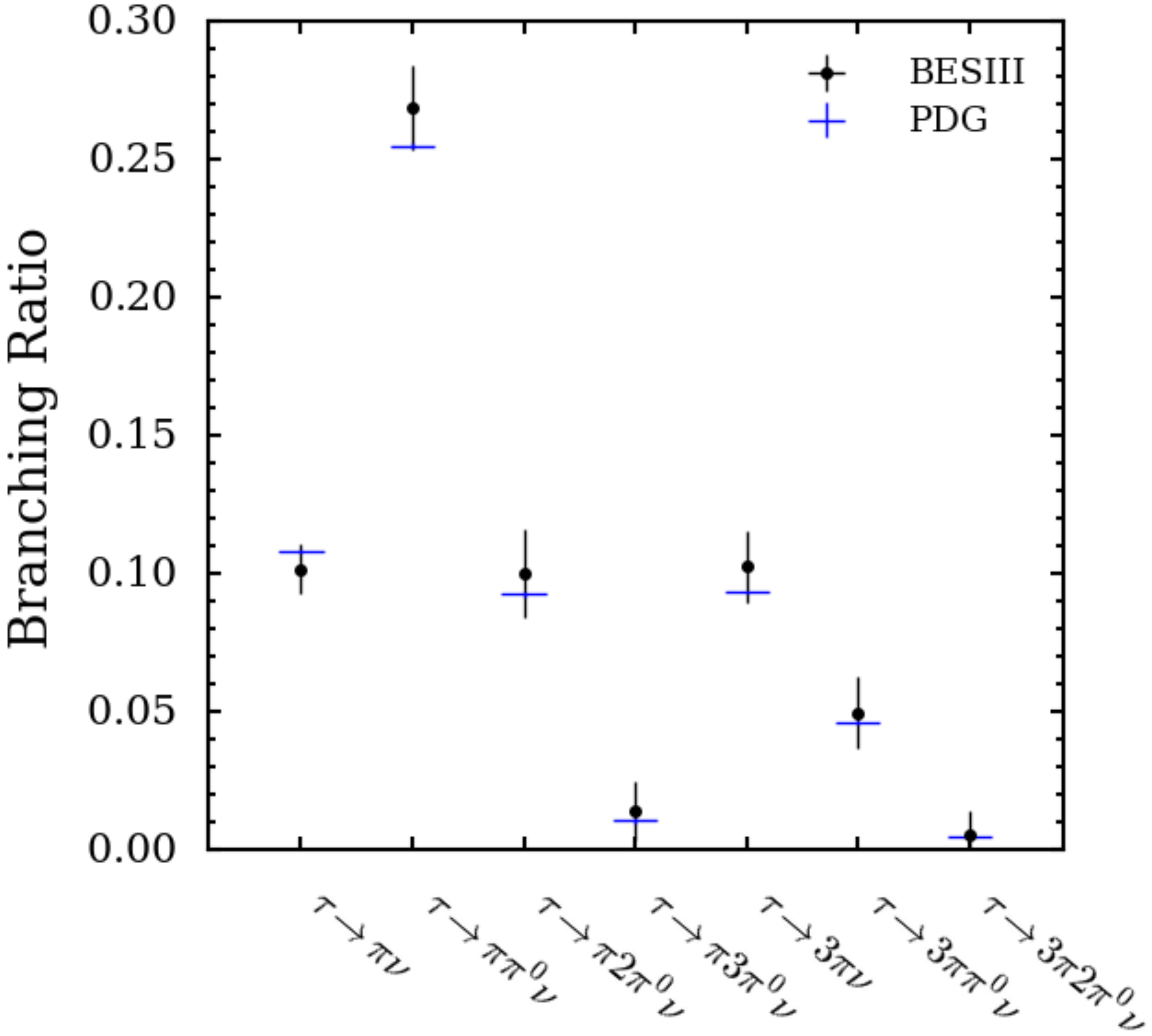



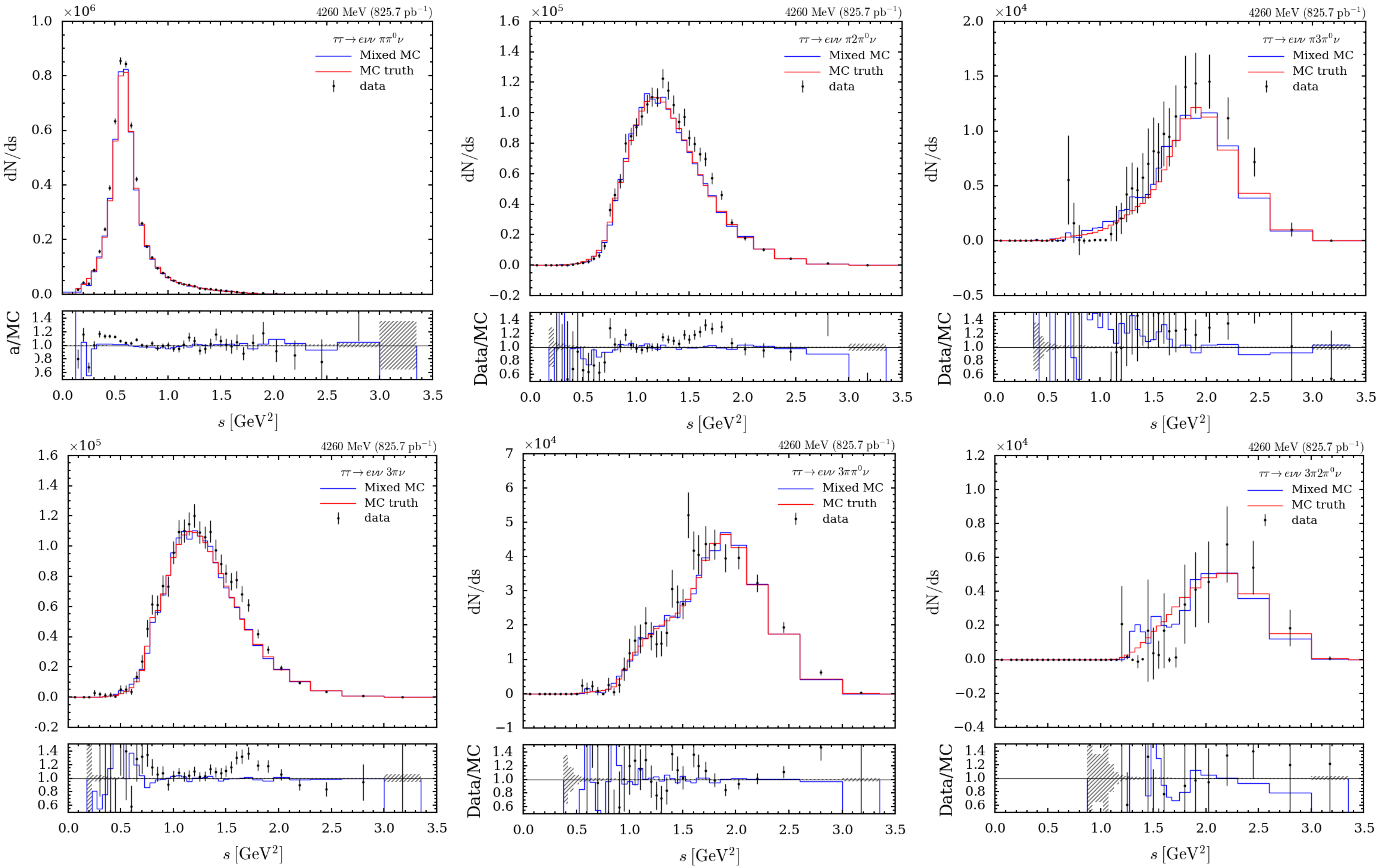



# Invariant mass distribution after correction

**Result of data correction：**

- The branching ratios of $\tau$ decays are estimated using the corrected spectra, as a validation.
- The obtained branching ratios agree with PDG world averages.



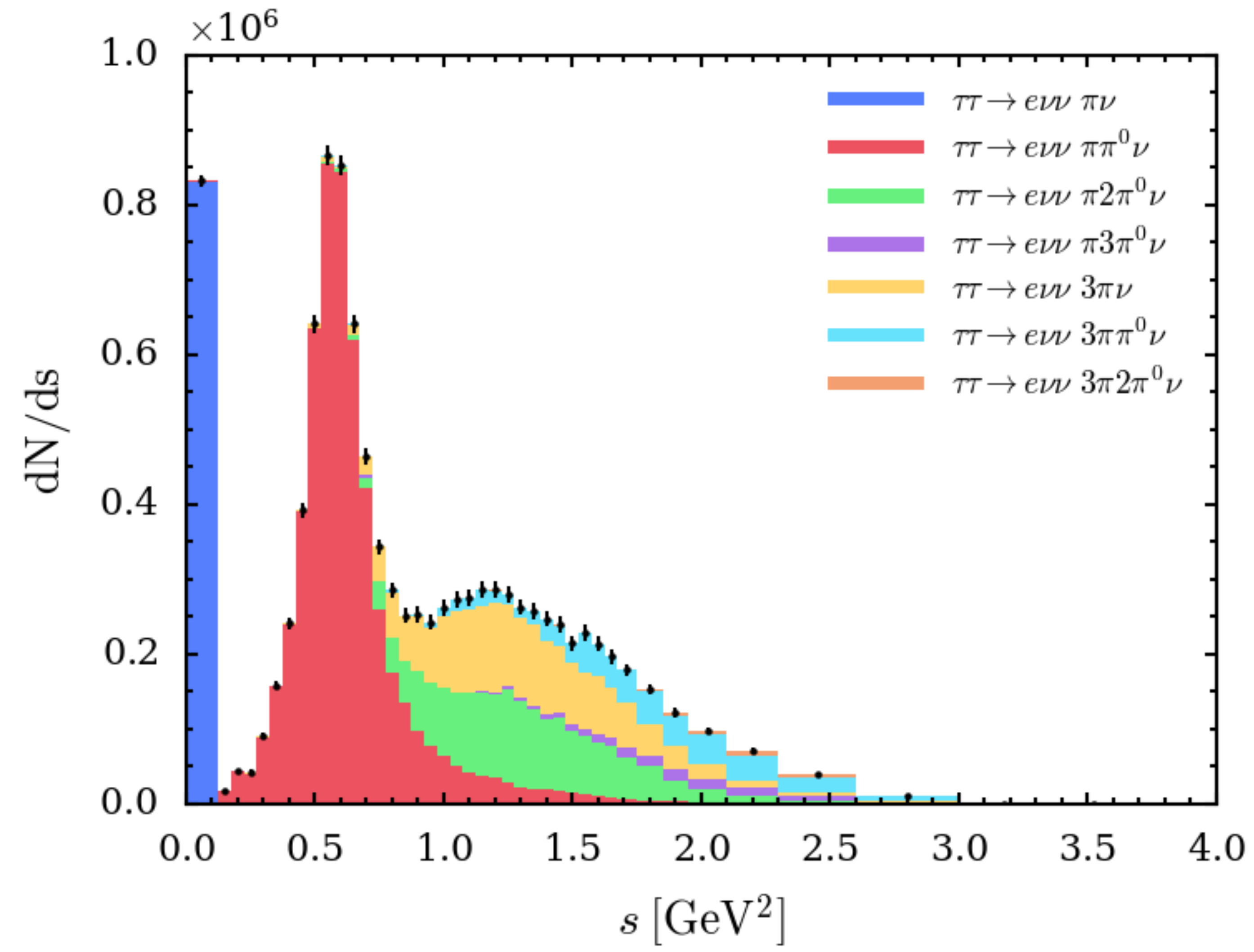



# Spectral moments and $\alpha_s(m_\tau^2)$ fit

- Fit input: $R_\tau = 3.475 \pm 0.011$ [*]

- $\chi^2$ fit:
  - $\chi^2 = \mathbf{A}\mathbf{V}^{-1}\mathbf{A}^T,\ \mathbf{A} = (\Delta R_\tau,\ \Delta D^{10},\ \Delta D^{11},\ \Delta D^{12},\ \Delta D^{13})$
  - $\mathbf{V} = V_{stat} + V_{theo}$
  - The theoretical uncertainty mainly arises from the truncation error of the perturbation expansion.
  - $V_{theo}$ is estimated from [*Eur. Phys. J. C* **4**, 409–431 (1998)] and scaled by a factor of 0.35 considering the one-order higher perturbative calculation [*Eur. Phys. J. C* **74**, 2803 (2014)].

- Based on bootstrap method, the statistic uncertainty are separately estimated as:
  $\alpha_s(m_\tau^2) = 0.3390 \pm 0.0010_{\text{stat}} \pm 0.0078_{\text{theo}}$




Inclusive s spectrum

| | $R_\tau$ | $D^{10}$ | $D^{11}$ | $D^{12}$ | $D^{13}$ |
|---|---|---|---|---|---|
| Measurement | 3.47500 | 0.71686 | 0.16190 | 0.05879 | 0.02606 |
| Exp. Error | 0.01100 | 0.00127 | 0.00039 | 0.00028 | 0.00020 |
| Theory Prediction | 3.47665 | 0.71485 | 0.16104 | 0.05886 | 0.02614 |
| Theory Error | 0.03300 | 0.00350 | 0.00280 | 0.00040 | 0.00020 |
| $\delta^{(0)}$ | 0.21884 | 0.24763 | 0.16603 | 0.14327 | 0.13244 |
| $\delta^{(2)}$ | -0.00041 | -0.00041 | -0.00000 | -0.00000 | -0.00000 |
| $\delta^{(4)}$ | -0.00275 | -0.00785 | 0.00507 | 0.00002 | 0.00000 |
| $\delta^{(6)}$ | 0.00175 | 0.00175 | 0.00058 | -0.00058 | -0.00000 |
| $\delta^{(8)}$ | -0.00204 | -0.00511 | 0.00306 | 0.00102 | -0.00102 |
| $\delta_{NP}$ | -0.00345 | -0.01161 | 0.00872 | 0.00046 | -0.00102 |

[*]: 10.1140/epjc/s10052-014-2803-9
[1]: 10.1007/s100529800895.

# Systematic Uncertainty

# Systematic Uncertainties

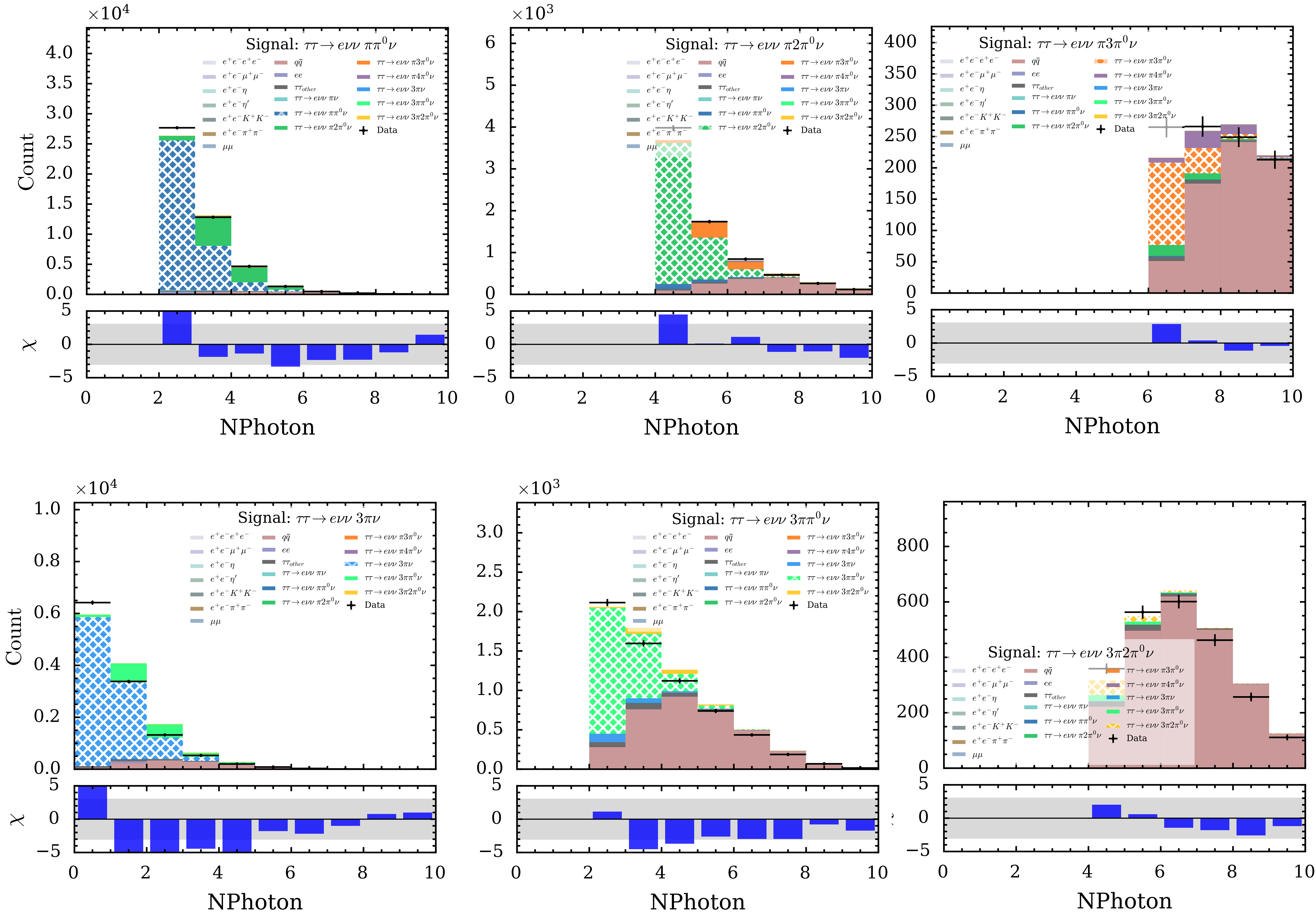
| Contribution | Estimating Method |
|---|---|
| **Cross section of hadronic background** | ***Floating*** *the cross-section of hadronic background in all exclusive channels by* $\pm 1\sigma$ |
| **Generator of hadronic background** | ***Comparing*** *the results derived from LUNDA and Hybrid generators.* |
| **Unfolding method & param.** | ***Comparing*** *results derived from Mixed MC after correction and from MC truth.* |
| **pi0 reconstruction** | ***Re-weighting*** *the MC samples according to the systematic study in BESIII DocDB-doc-510-v9. Comparing the resulting difference.* |
| **Tracking efficiency** | ***Re-weighting*** *the MC samples according to the systematic study K/π Tracking Efficiency @4178 in BOSS7.0.3. Comparing the resulting difference.* |
| **Isolated Good Photon** | ***Comparing*** *the result derived by requiring a)* ***all good photon*** *are reconstructed as* $\pi^0$*; and b)* ***all good photon with energy higher than 80 MeV*** *are reconstructed as* $\pi^0$. |
| **Unexpected Background** | *Repeat the nominal analysis using 3773, 4260 and 4680 MeV data, and compare the results.* |

# Systematic: isolated photons

- Data tends to have less good photons per event than MC samples, which affects the efficiency estimation through:
  - $\pi^0$ **reconstruction**
  - **zero isolated photon requirement:** no remained good photons in events after $\pi^0$ reconstruction.
- For the latter issue, the corresponding systematic uncertainty is estimated by comparing the two cuts:

  **A.** No remained **good photons** in events after $\pi^0$ reconstruction (nominal).

  **B.** No remained **good photons with energy larger than 80 MeV** in events after $\pi^0$ reconstruction.




MC and data comparison on the number of all good photons.
All the cuts and classification are implemented, except from the zero isolated photon condition.

# Summary of Systematic Uncertainties

- The systematic uncertainties on the spectral moments are dominated by:
  - $e^+e^- \to$ had generator
  - Cut on iso-photons
  - Unexpected background (fluctuation versus $E_{cm}$)
- The $\alpha_s(m_\tau^2)$ accuracy is dominated by the external input value of $R_\tau$.

Table 9: The nominal value, statistical and systematic uncertainties on the spectral moments.

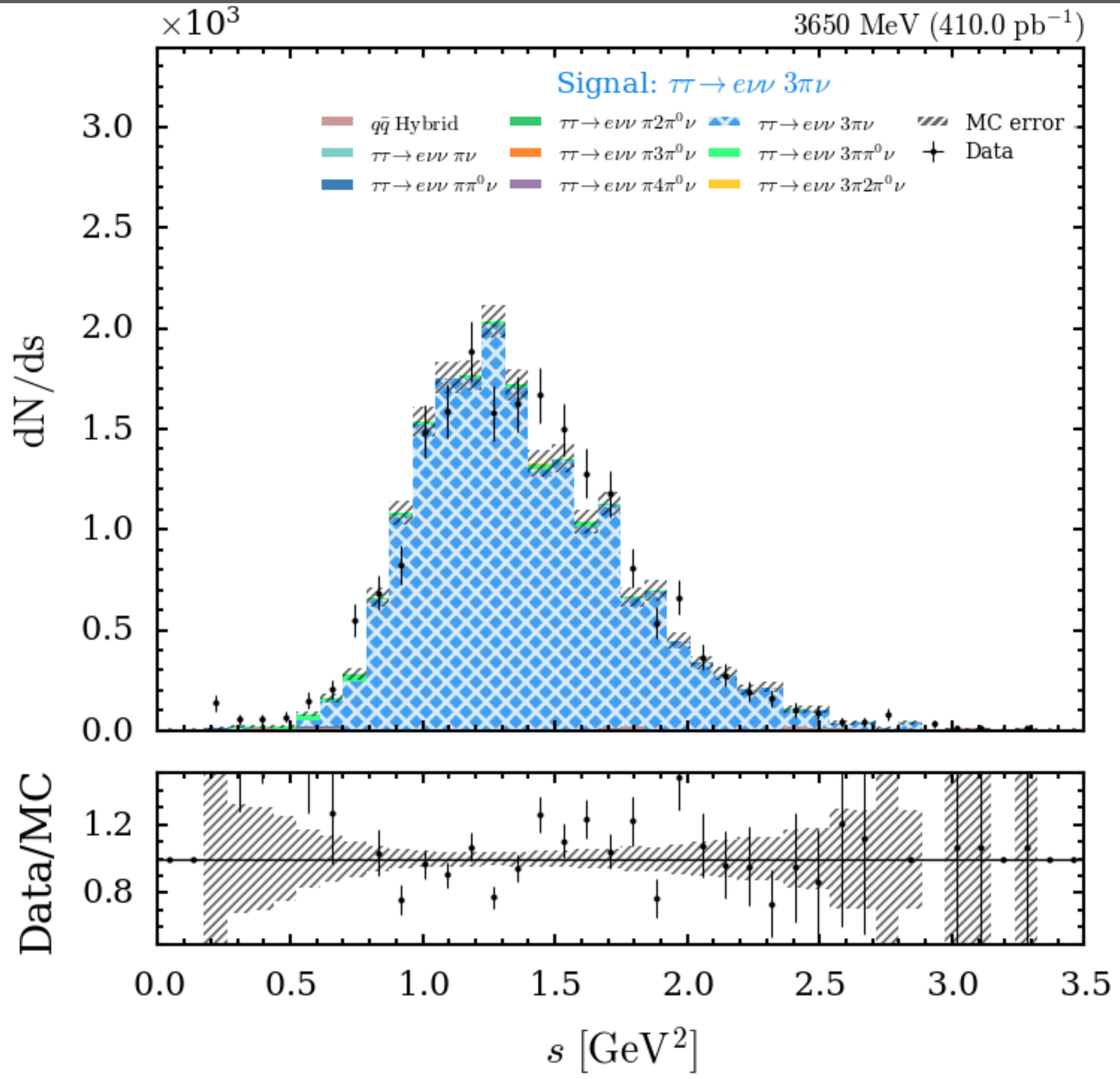
| BESIII 4260 MeV | $D^{10}$ | $D^{11}$ | $D^{12}$ | $D^{13}$ | $\alpha_s(m_\tau^2)$ |
|---|---|---|---|---|---|
| Nominal | 0.71686 | 0.16190 | 0.05879 | 0.02606 | 0.3390 |
| Statistic | 0.00121 | 0.00037 | 0.00028 | 0.00020 | 0.0014 |
| Systematic | 0.00393 | 0.00149 | 0.00071 | 0.00049 | 0.0052 |
| Cross-sections of the backgrounds | 0.00007 | 0.00001 | 0.00001 | 0.00001 | 0.00003 |
| Modeling of the $e^+e^- \to$ hadrons | 0.00153 | 0.00010 | 0.00020 | 0.00018 | 0.0006 |
| Unfolding parameter | 0.00052 | 0.00030 | 0.00011 | 0.00005 | 0.0007 |
| Photon simulation | 0.00199 | 0.00101 | 0.00050 | 0.00020 | 0.0016 |
| Unexpected background | 0.00296 | 0.00104 | 0.00044 | 0.00040 | 0.0002 |
| $\pi^0$ Reconstruction | 0.00023 | 0.00011 | 0.00004 | 0.00003 | 0.0002 |
| $\pi^\pm$ Tracking efficiency | 0.00021 | 0.00011 | 0.00005 | 0.00002 | 0.0002 |
| $R_\tau = 0.475 \pm 0.011$ | | | | | 0.0049 |
| Total uncertainty | 0.00411 | 0.00154 | 0.00076 | 0.00053 | 0.0054 |

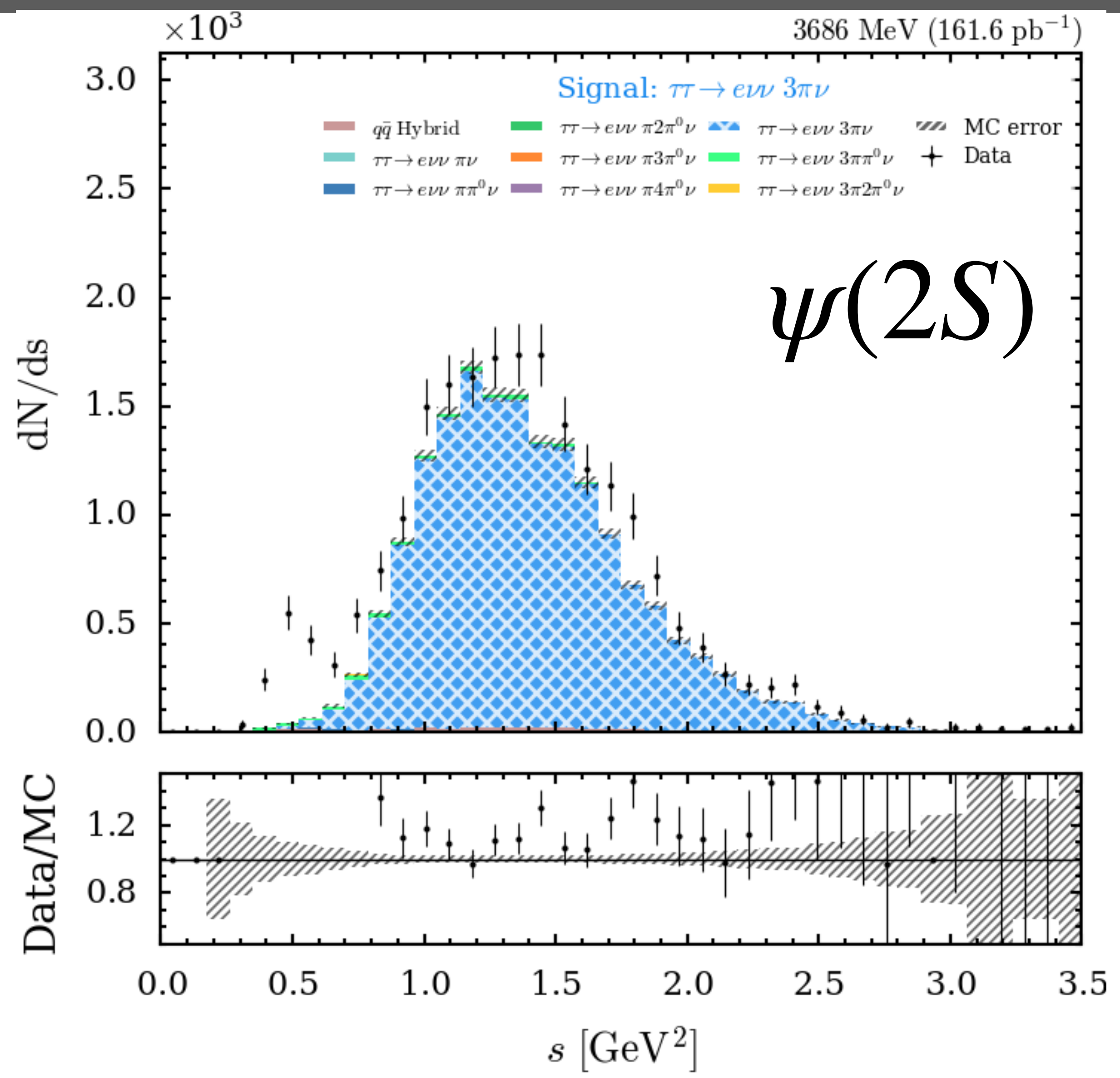
# Raw Spectrum of $2\pi^-\pi^+\nu_\tau$ channel at different $E_{cm}$

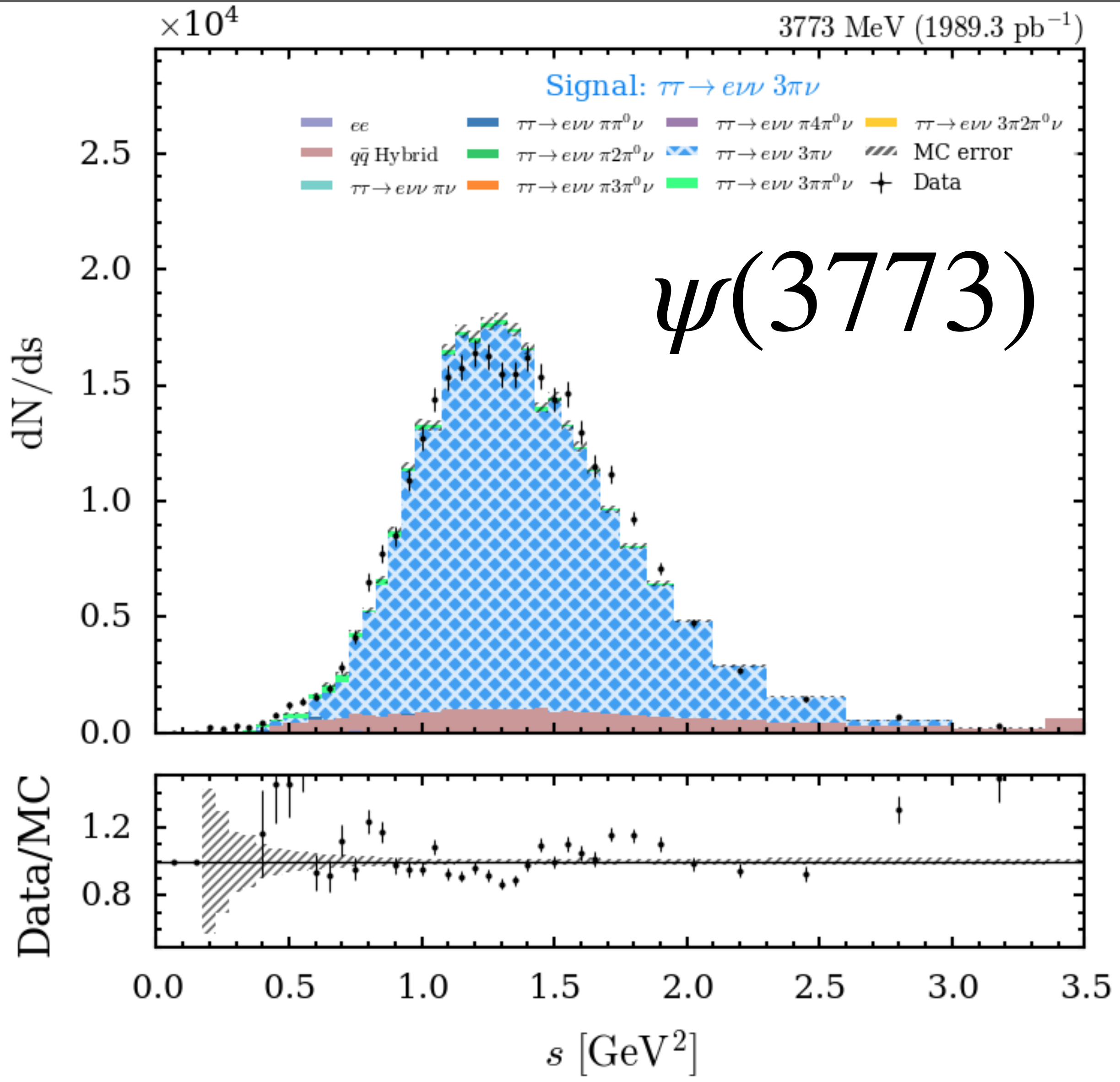
**Below the open charm threshold**, the 3650 MeV data aligns well with MC simulation.

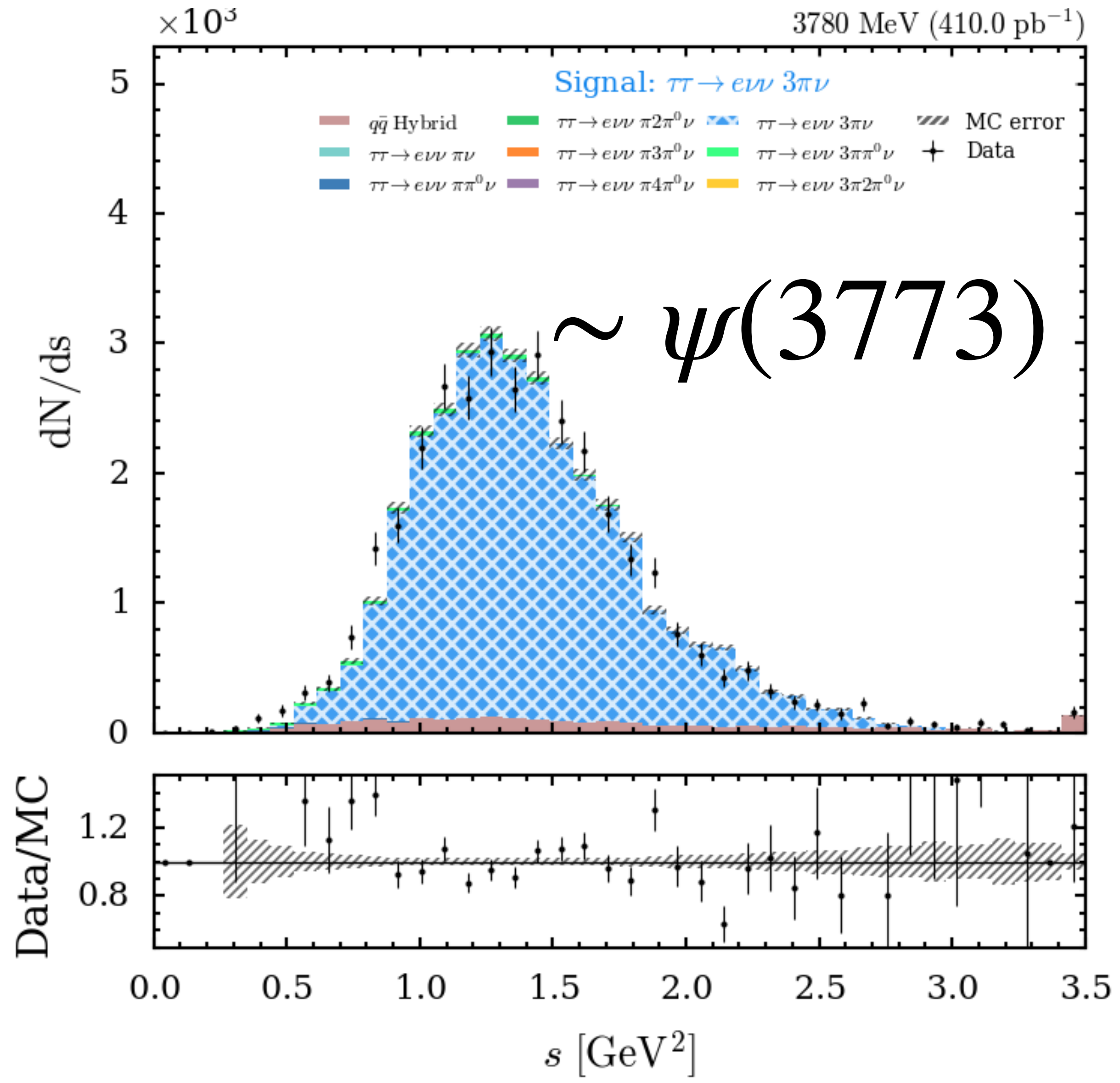
Meanwhile, data at $\psi(2S)$ peak shows a peak around the lower bound.

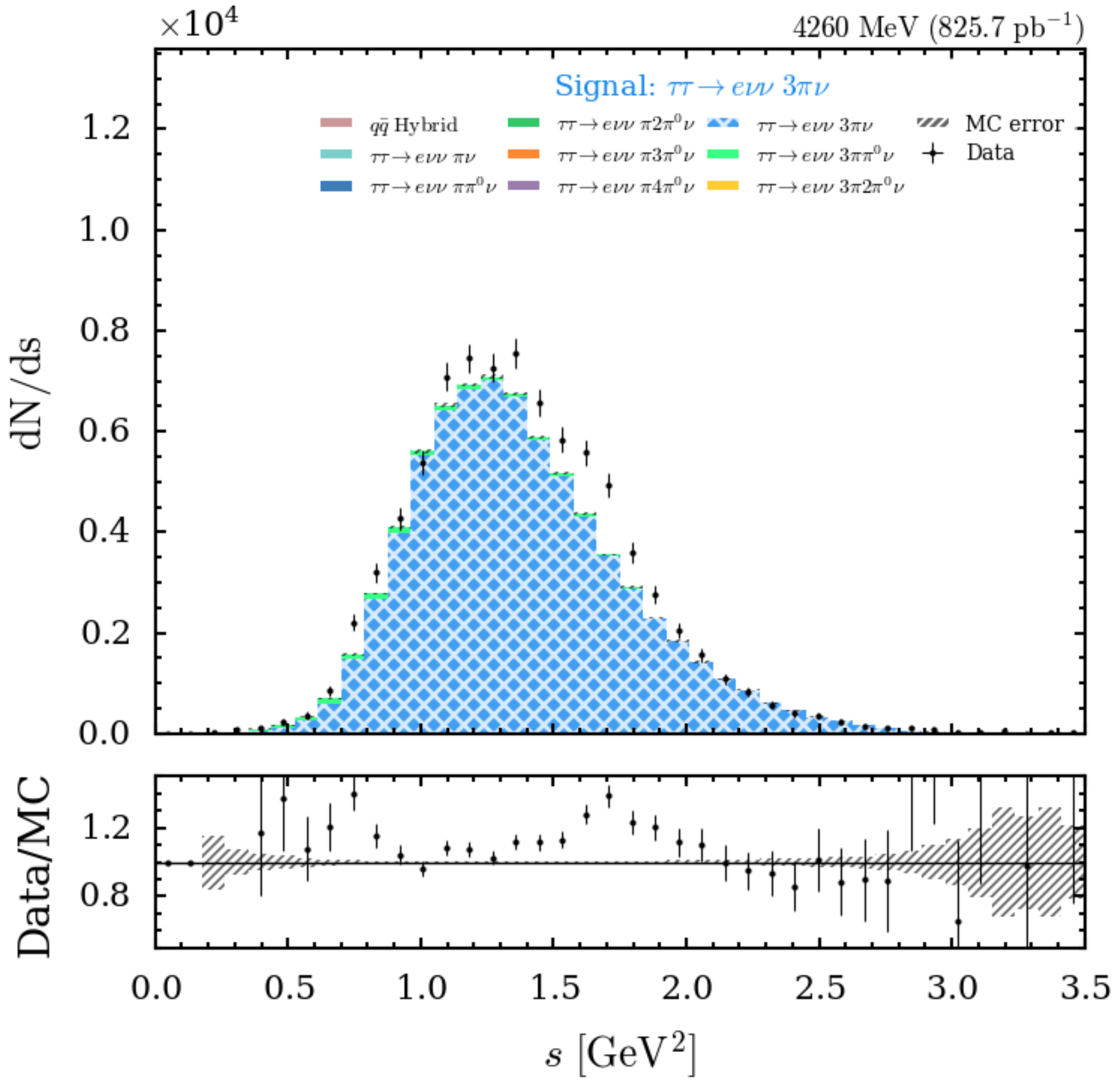
When $E_{cm}$ is **higher than the open charm threshold and lower than excited charm threshold**, the data shows **Good consistency with MC**.

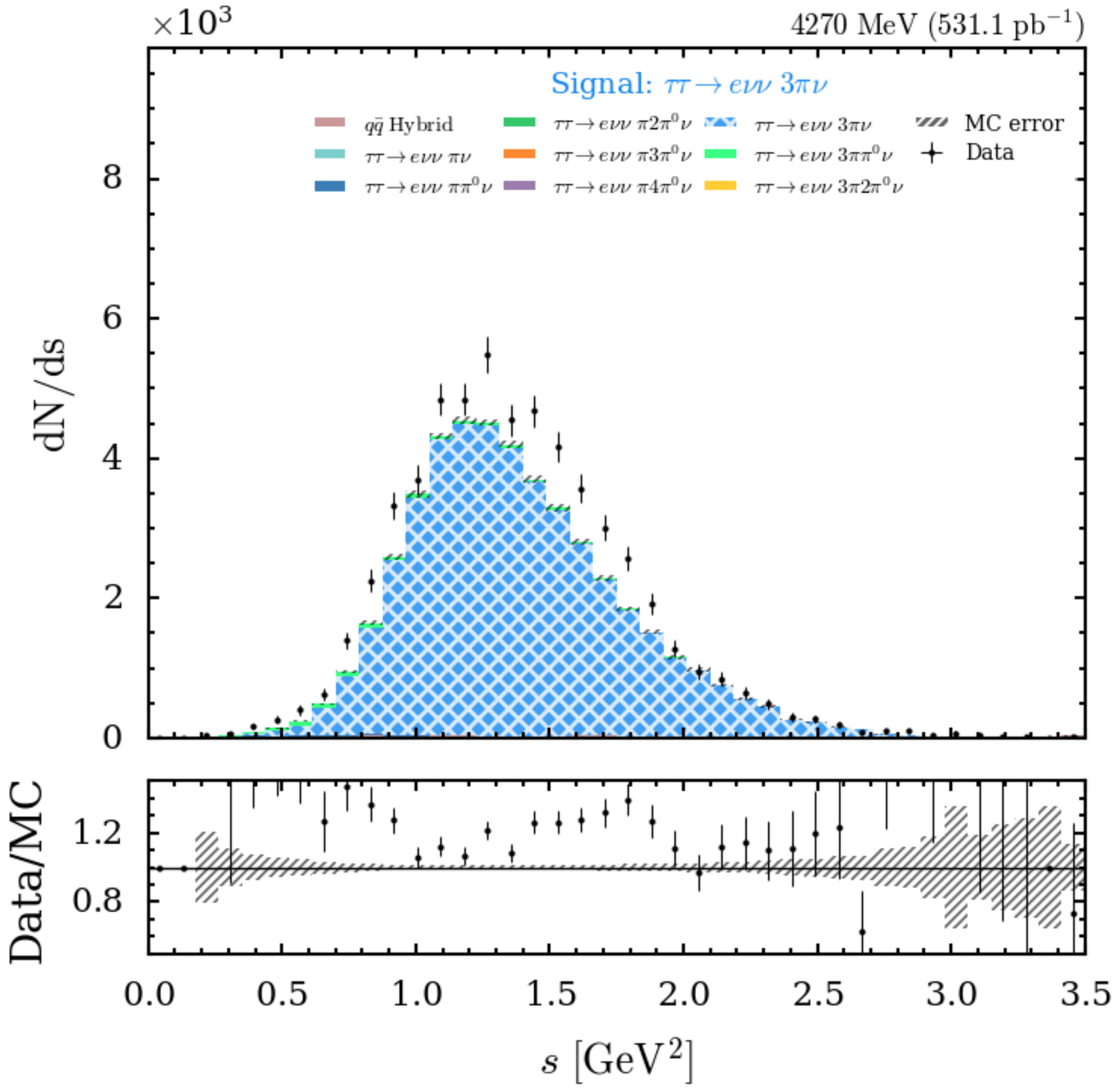


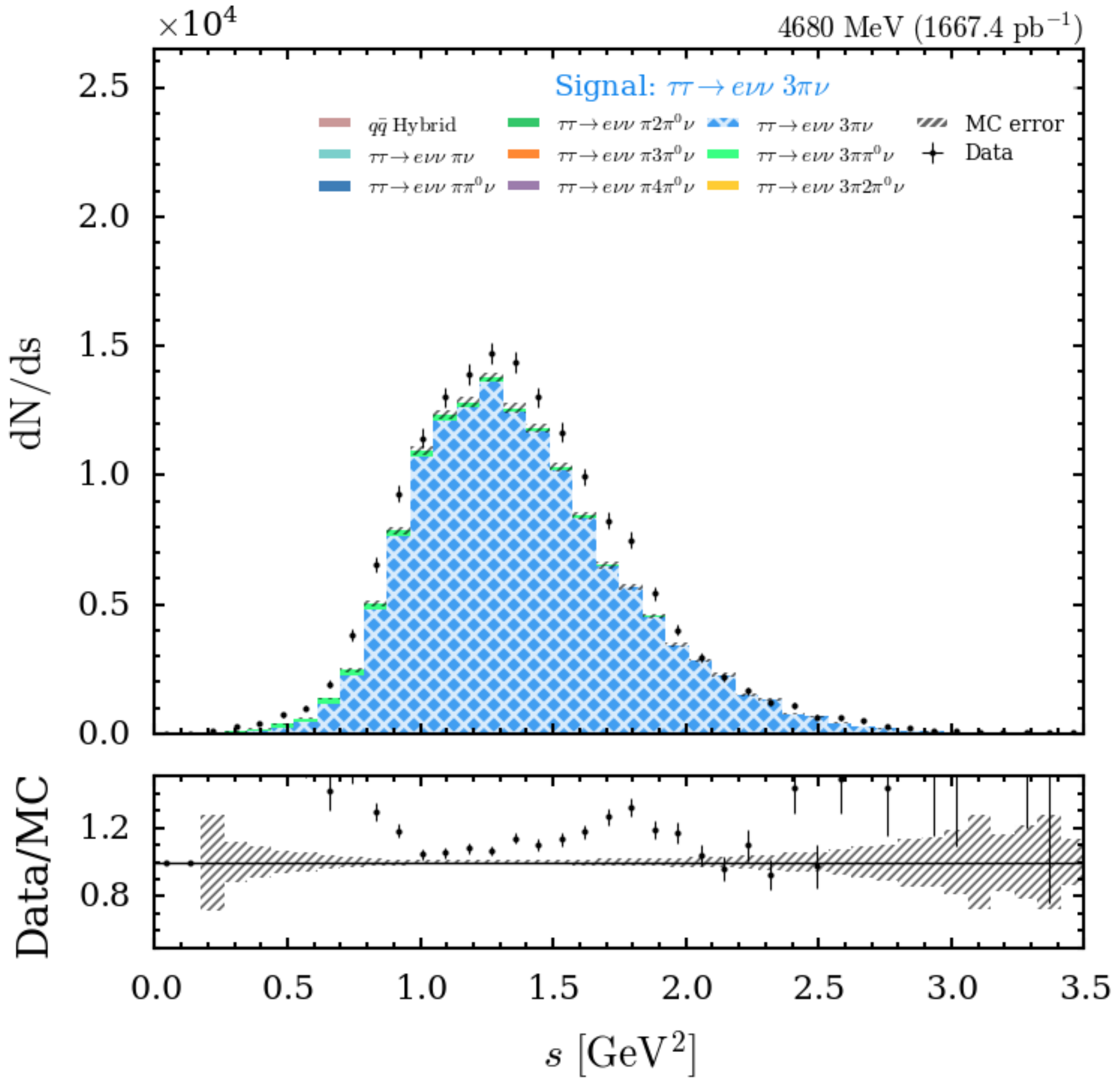



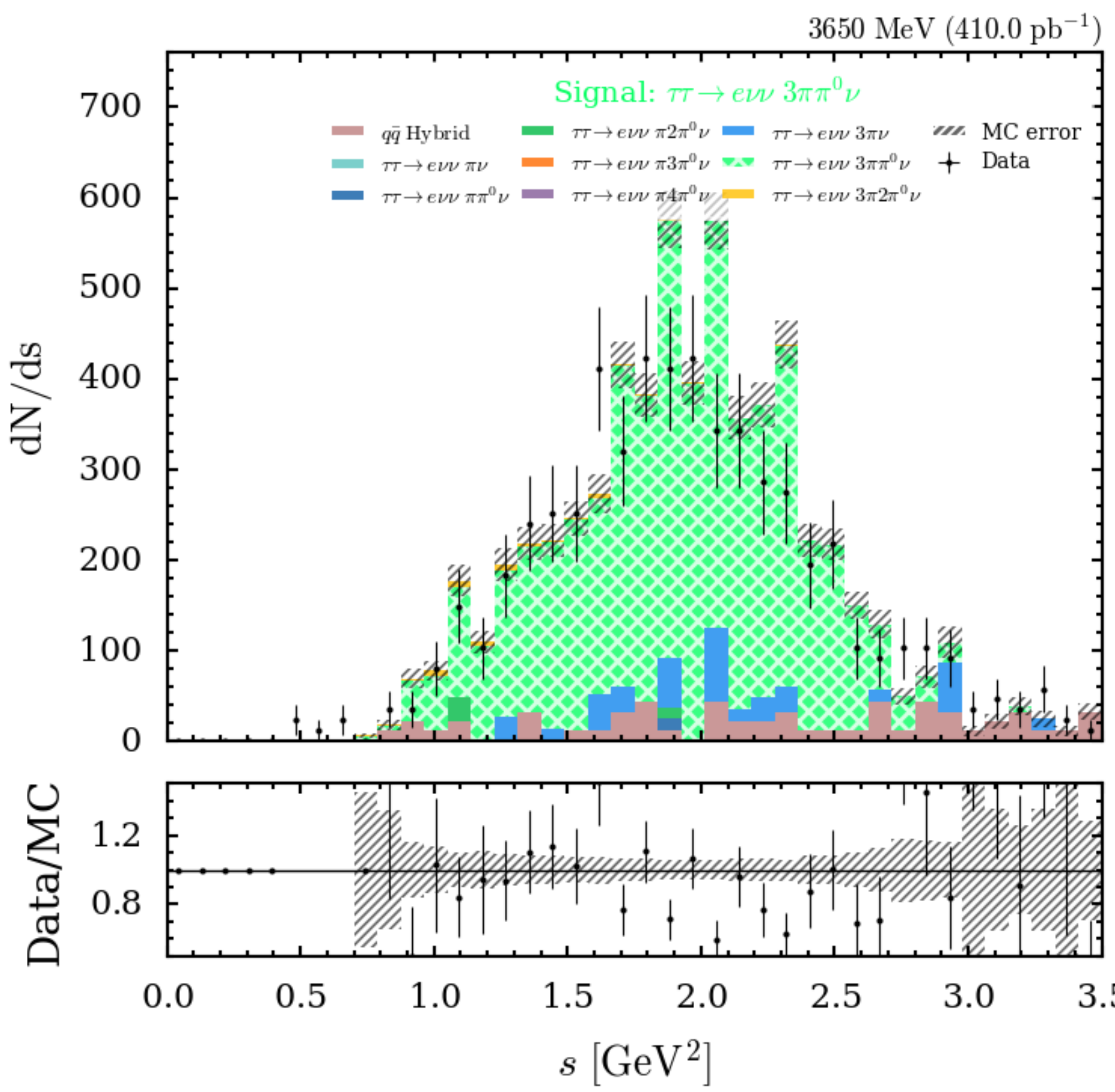
# Raw Spectrum of $2\pi^-\pi^+\pi^0\nu_\tau$ channel at different $E_{cm}$

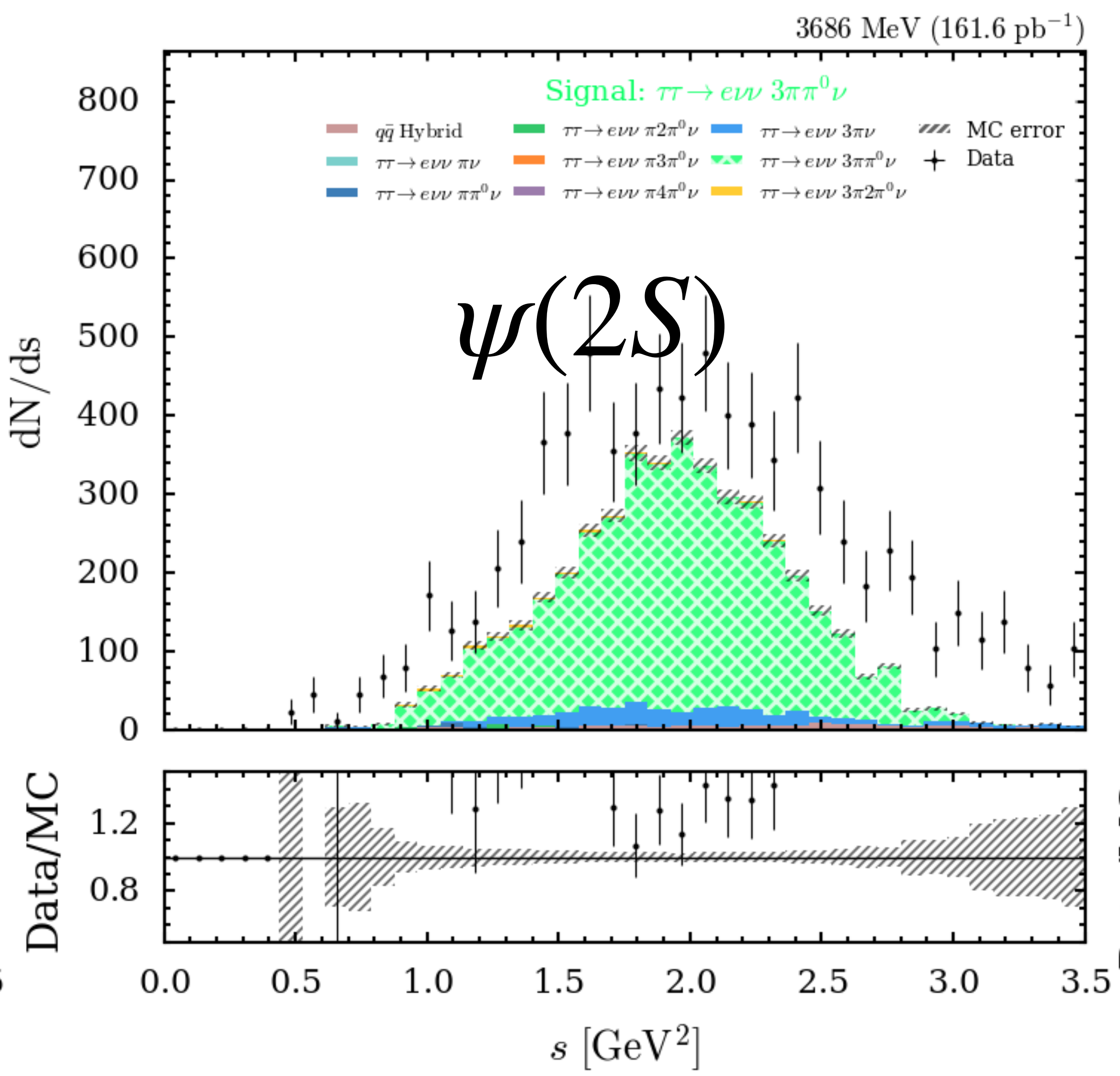
**Below the open charm threshold**, the 3650 MeV data has large statistical fluctuation.

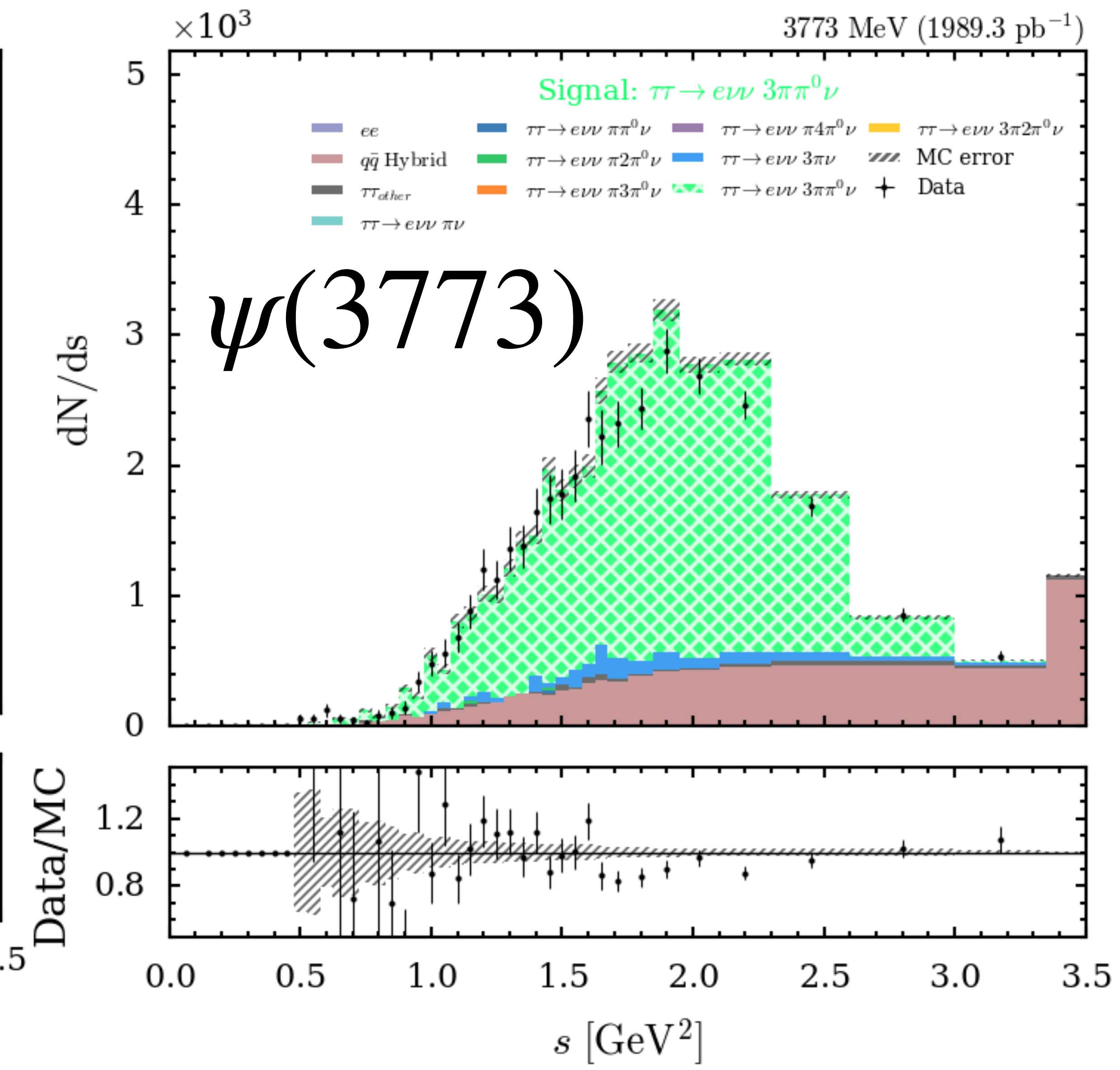
Meanwhile, data at $\psi(2S)$ peak shows different but larger MC/data discrepancy.

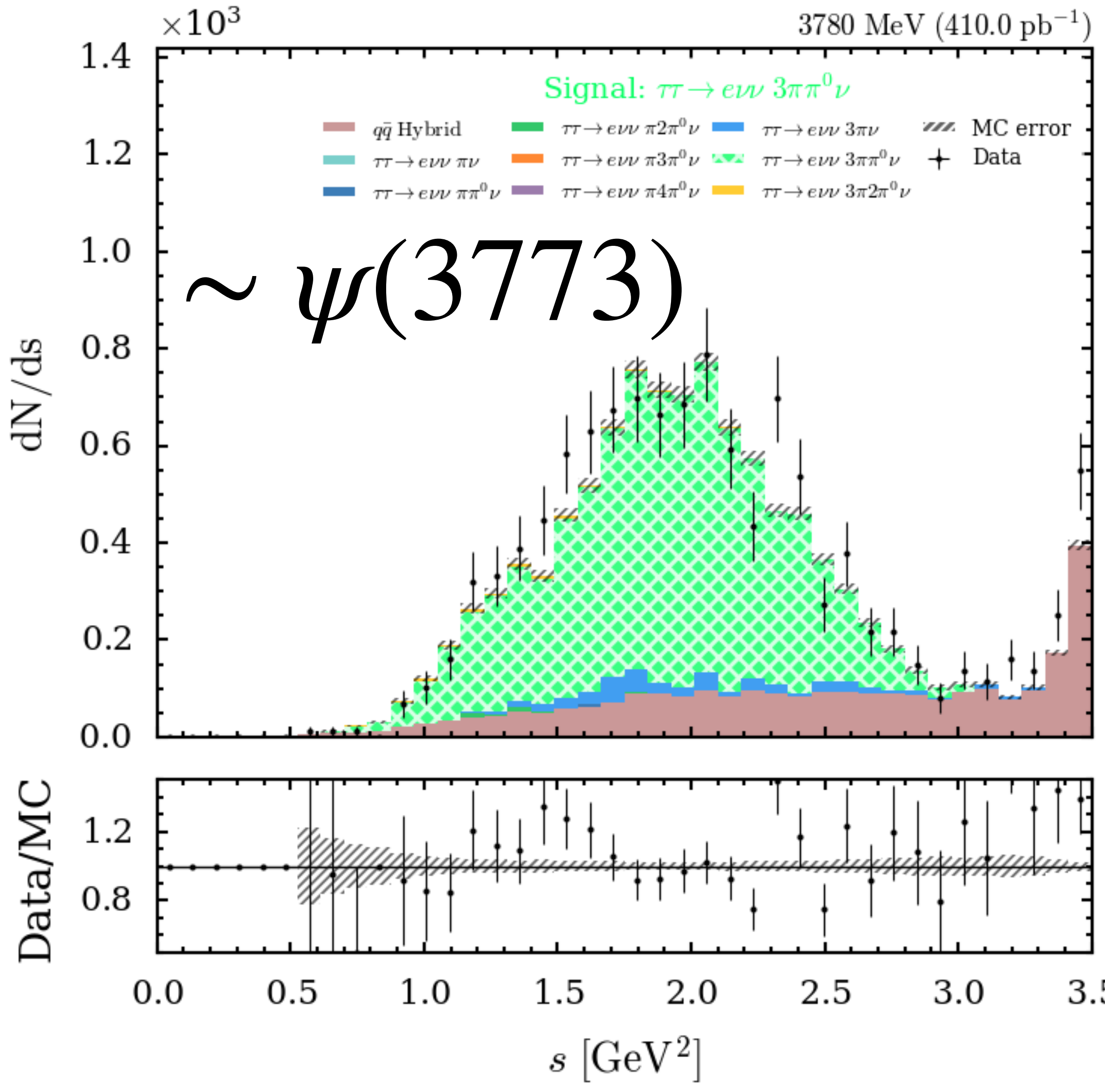
**Higher than the open charm threshold and lower than excited charm threshold**, the data at $E_{cm} = 3780\,\mathrm{MeV}$ shows:

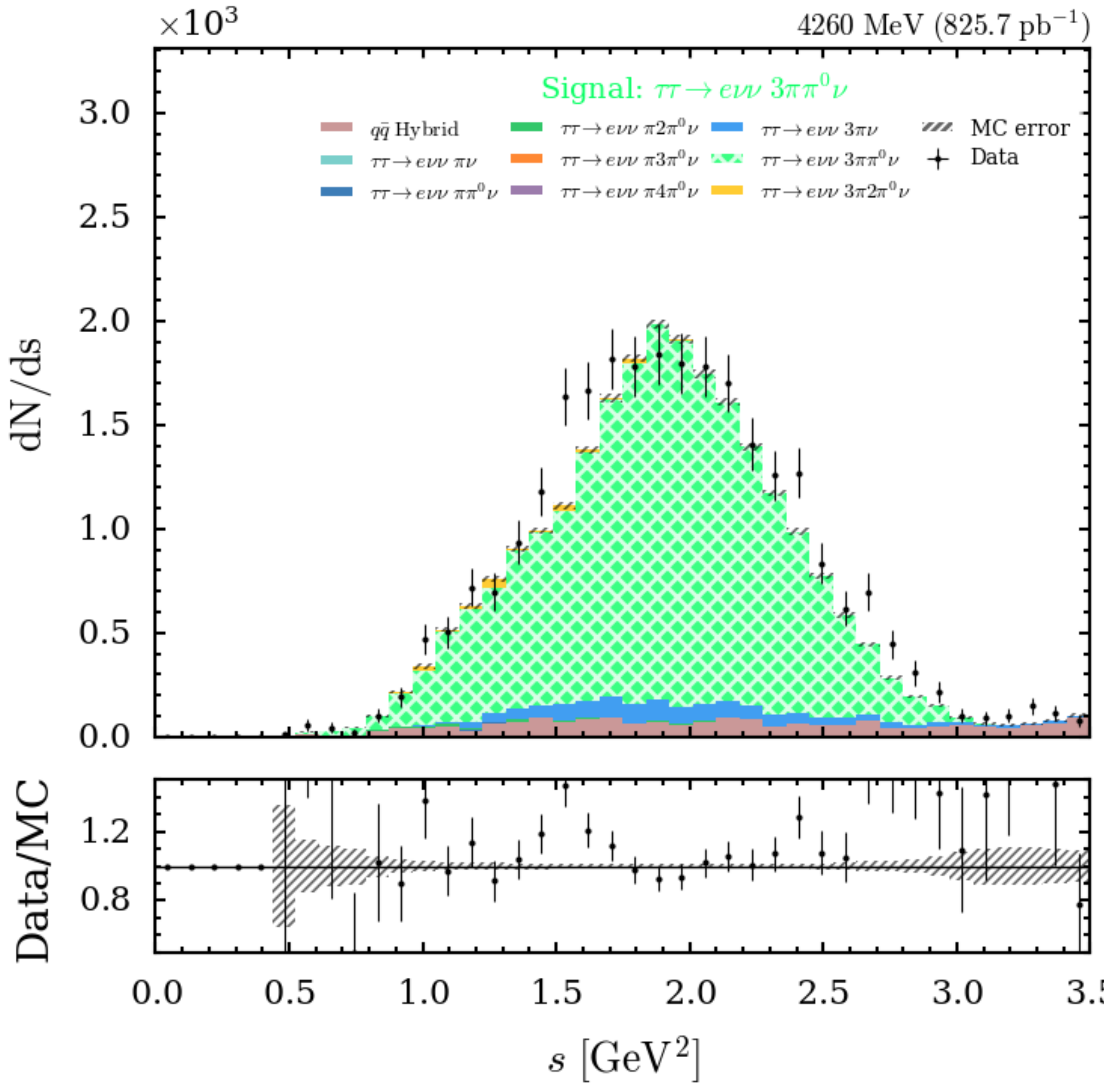
- **Good MC-data consistency.**
- another peak higher than $m_\tau^2$, explained by inclusive MC.

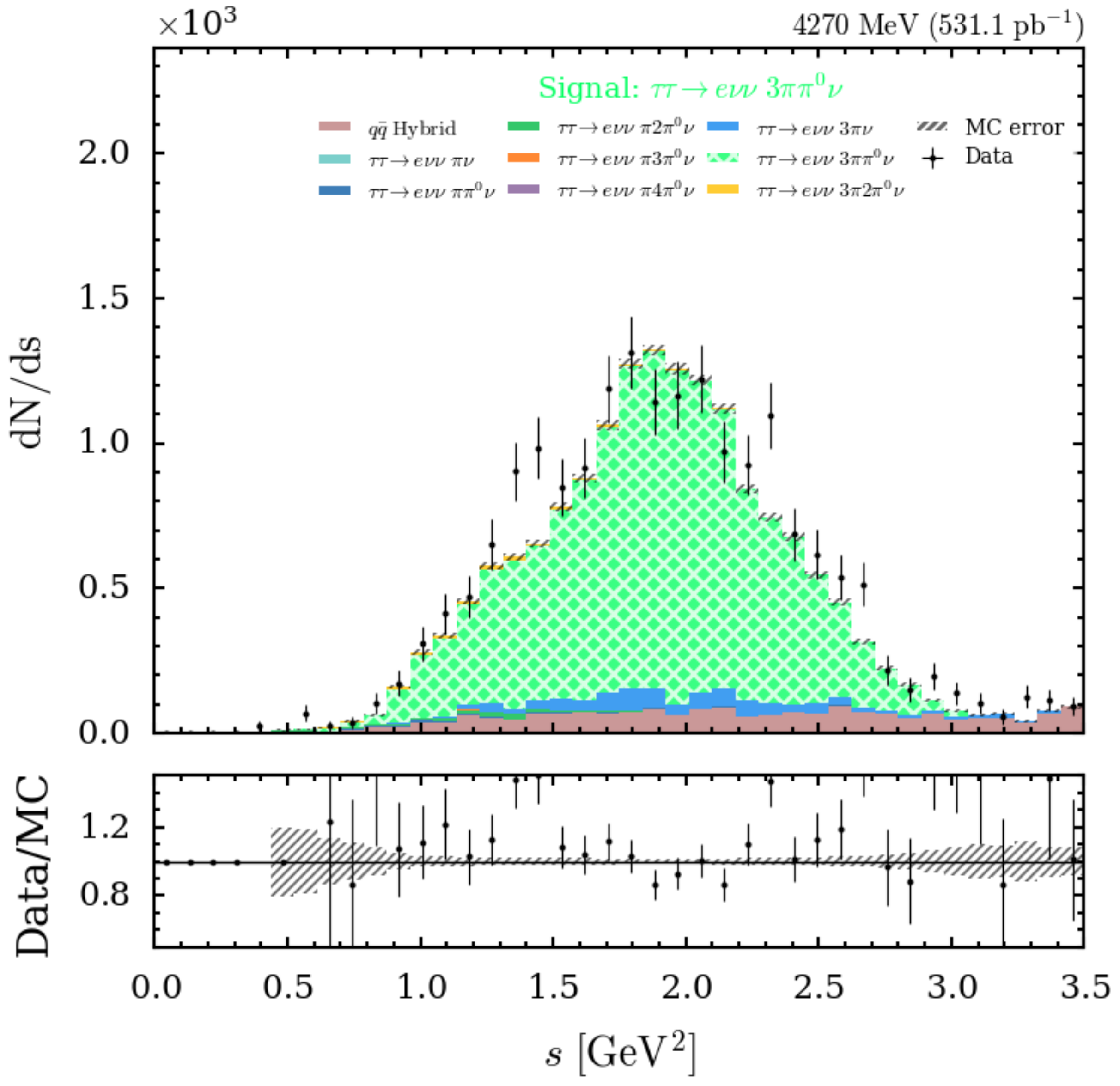


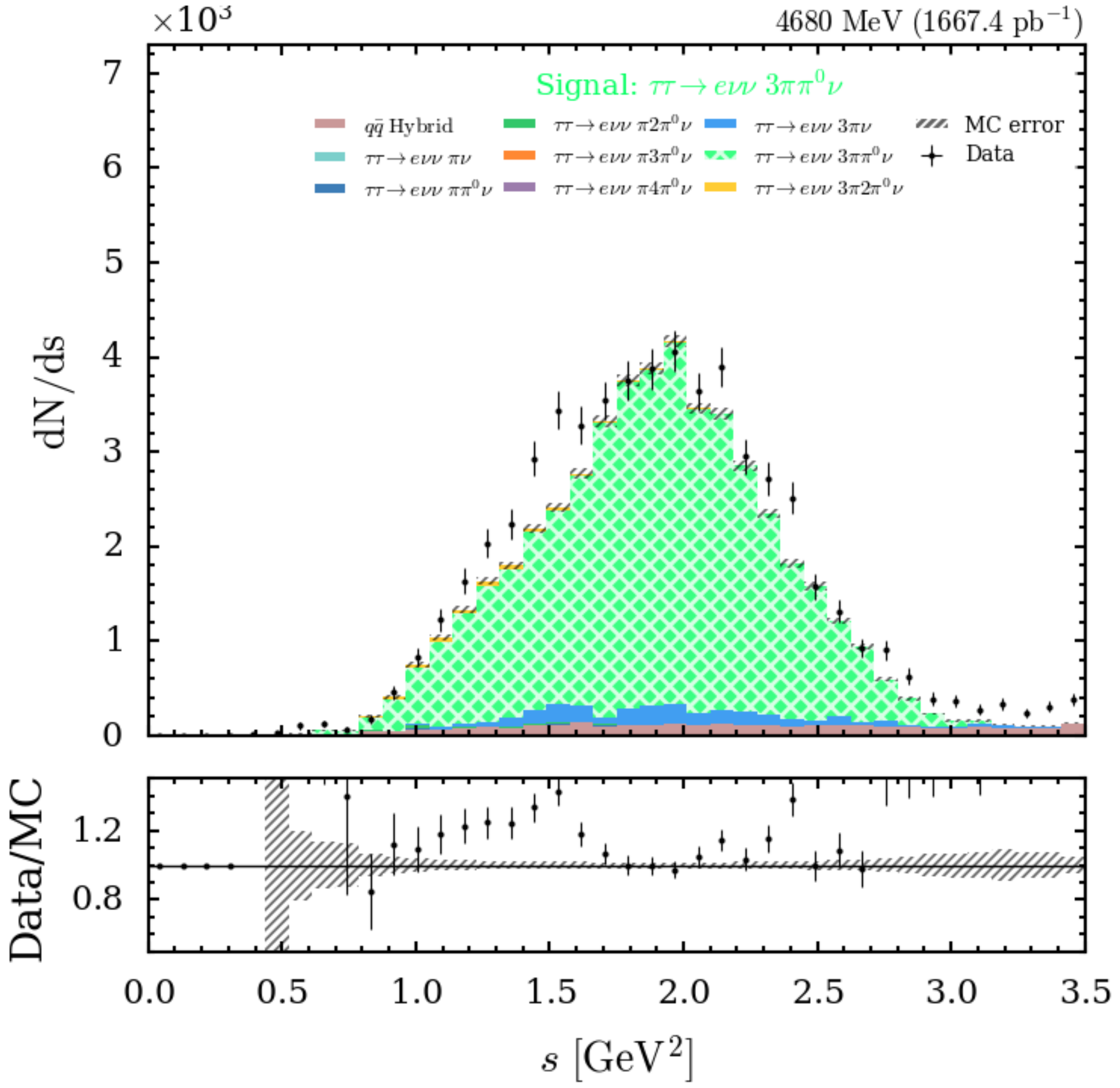



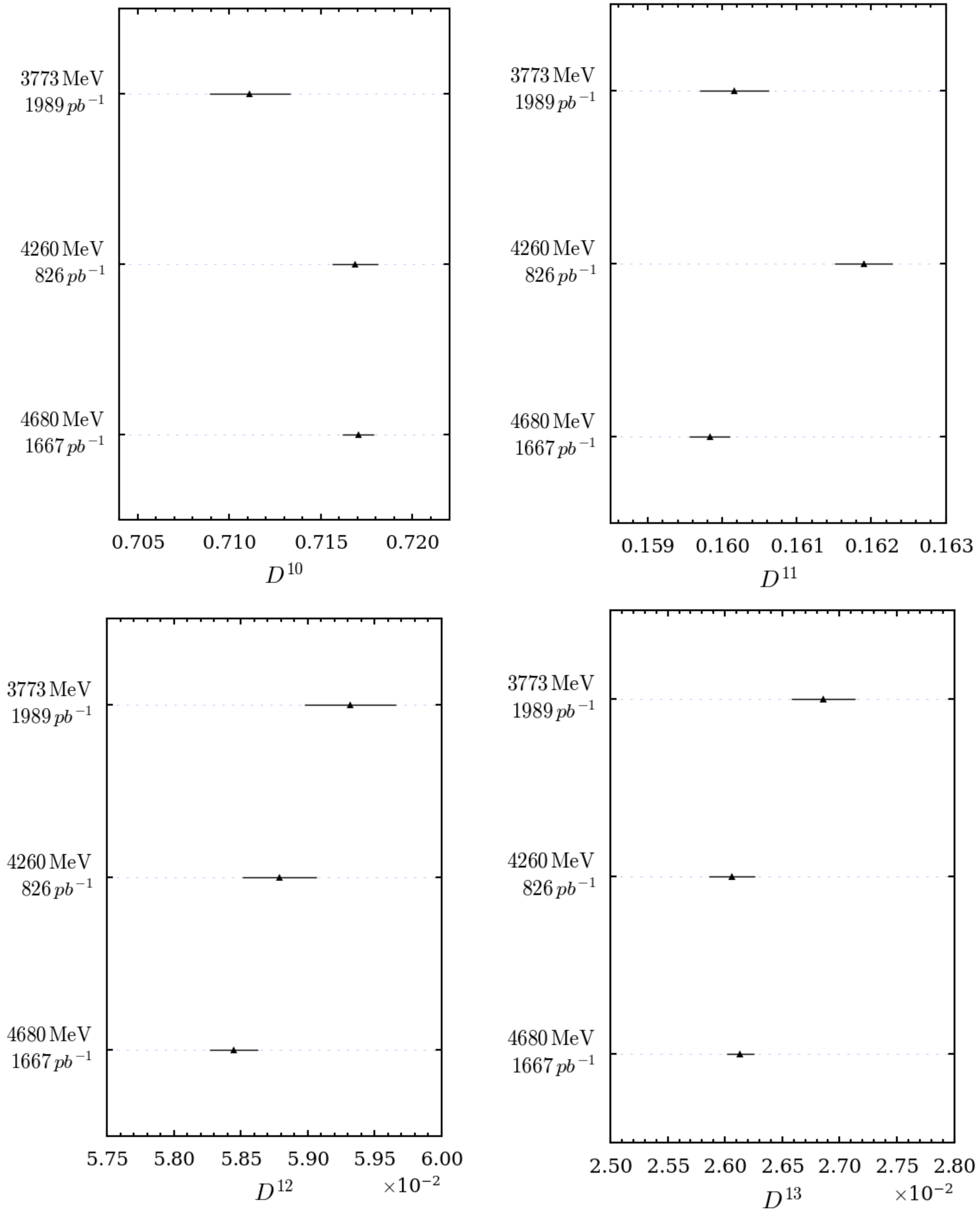



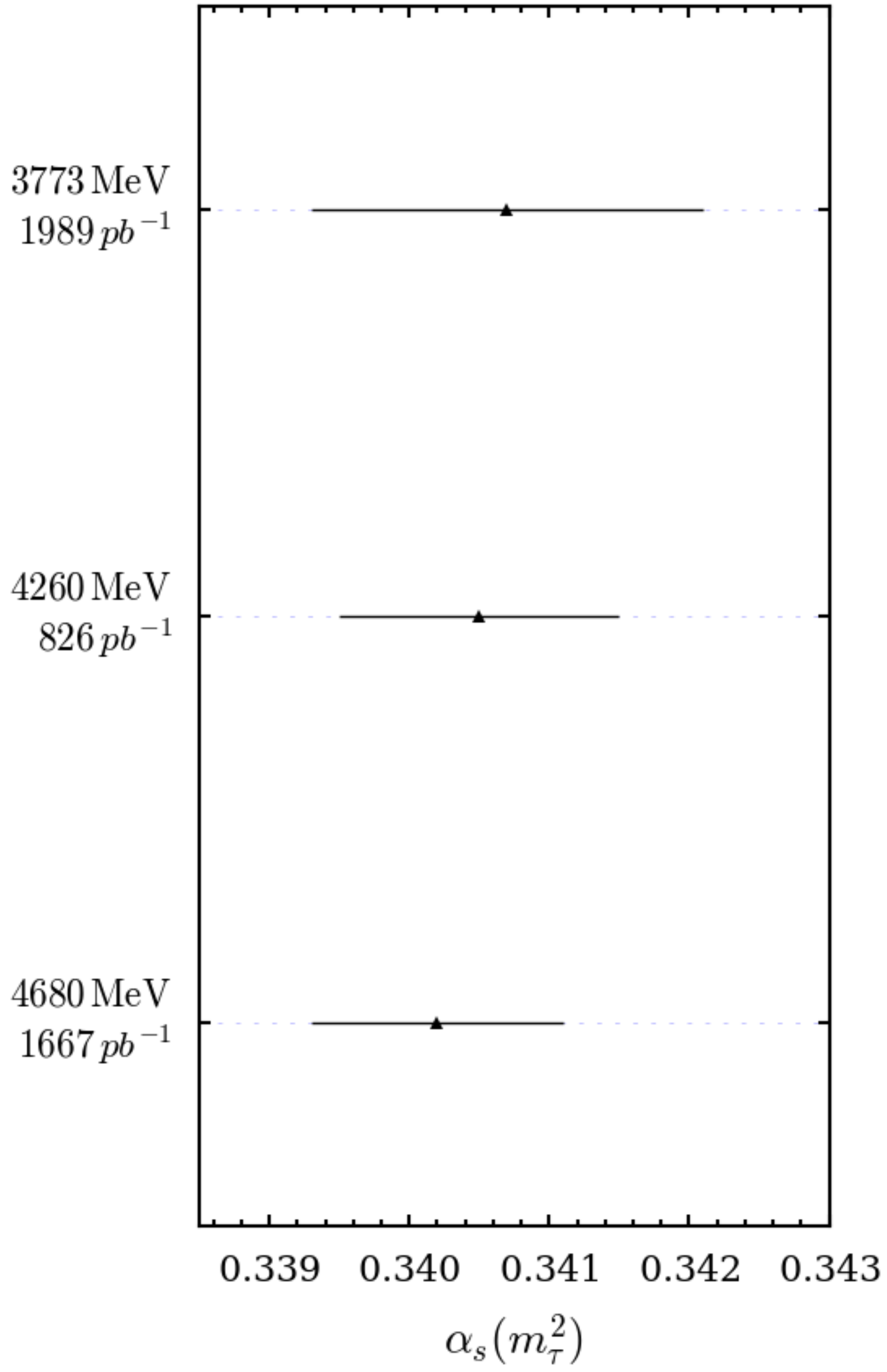




$q\bar{q}$ samples for 3773 MeV data also includes $D\bar{D}$ components.

# Efficiency for high multiplicity channel at low $E_{cm}$ is lower

- Comparing the results derived from data at three $E_{cm}$ points,
  - fluctuation larger than $1\sigma$ statistic uncertainty on $D^{kl}$ is observed.
  - however, $\alpha_s(m_\tau^2)$ seems to be rather stable, due to the constrain of $R_\tau$.




The error bars only include statistic uncertainty.

# Summary of Systematic Uncertainties

- The systematic uncertainties on the spectral moments are dominated by:
    - $e^+e^- \to \mathrm{had}$ generator
    - Cut on iso-photons
    - Unexpected background (fluctuation versus $E_{cm}$)
- The $\alpha_s(m_\tau^2)$ accuracy is dominated by the external input value of $R_\tau$.

Table 9: The nominal value, statistical and systematic uncertainties on the spectral moments.

| BESIII 4260 MeV | $D^{10}$ | $D^{11}$ | $D^{12}$ | $D^{13}$ | $\alpha_s(m_\tau^2)$ |
|---|---|---|---|---|---|
| Nominal | 0.71686 | 0.16190 | 0.05879 | 0.02606 | 0.3390 |
| Statistic | 0.00121 | 0.00037 | 0.00028 | 0.00020 | 0.0014 |
| Systematic | 0.00393 | 0.00149 | 0.00071 | 0.00049 | 0.0052 |
| Cross-sections of the backgrounds | 0.00007 | 0.00001 | 0.00001 | 0.00001 | 0.00003 |
| Modeling of the $e^+e^- \to$ hadrons | 0.00153 | 0.00010 | 0.00020 | 0.00018 | 0.0006 |
| Unfolding parameter | 0.00052 | 0.00030 | 0.00011 | 0.00005 | 0.0007 |
| Photon simulation | 0.00199 | 0.00101 | 0.00050 | 0.00020 | 0.0016 |
| Unexpected background | 0.00296 | 0.00104 | 0.00044 | 0.00040 | 0.0002 |
| $\pi^0$ Reconstruction | 0.00023 | 0.00011 | 0.00004 | 0.00003 | 0.0002 |
| $\pi^\pm$ Tracking efficiency | 0.00021 | 0.00011 | 0.00005 | 0.00002 | 0.0002 |
| $R_\tau = 0.475 \pm 0.011$ | | | | | 0.0049 |
| Total uncertainty | 0.00411 | 0.00154 | 0.00076 | 0.00053 | 0.0054 |

# Comparison of Spectral moments

Although the BESIII accuracy of spectral moments is similar with OPAL and CLEO-II, worse than ALEPH, the uncertainty on $\alpha_s$ is comparable with ALEPH and better than OPAL and CLEO-II, since the experimental uncertainty is controlled by the $R_\tau$ and theoretical uncertainty dominates the result.

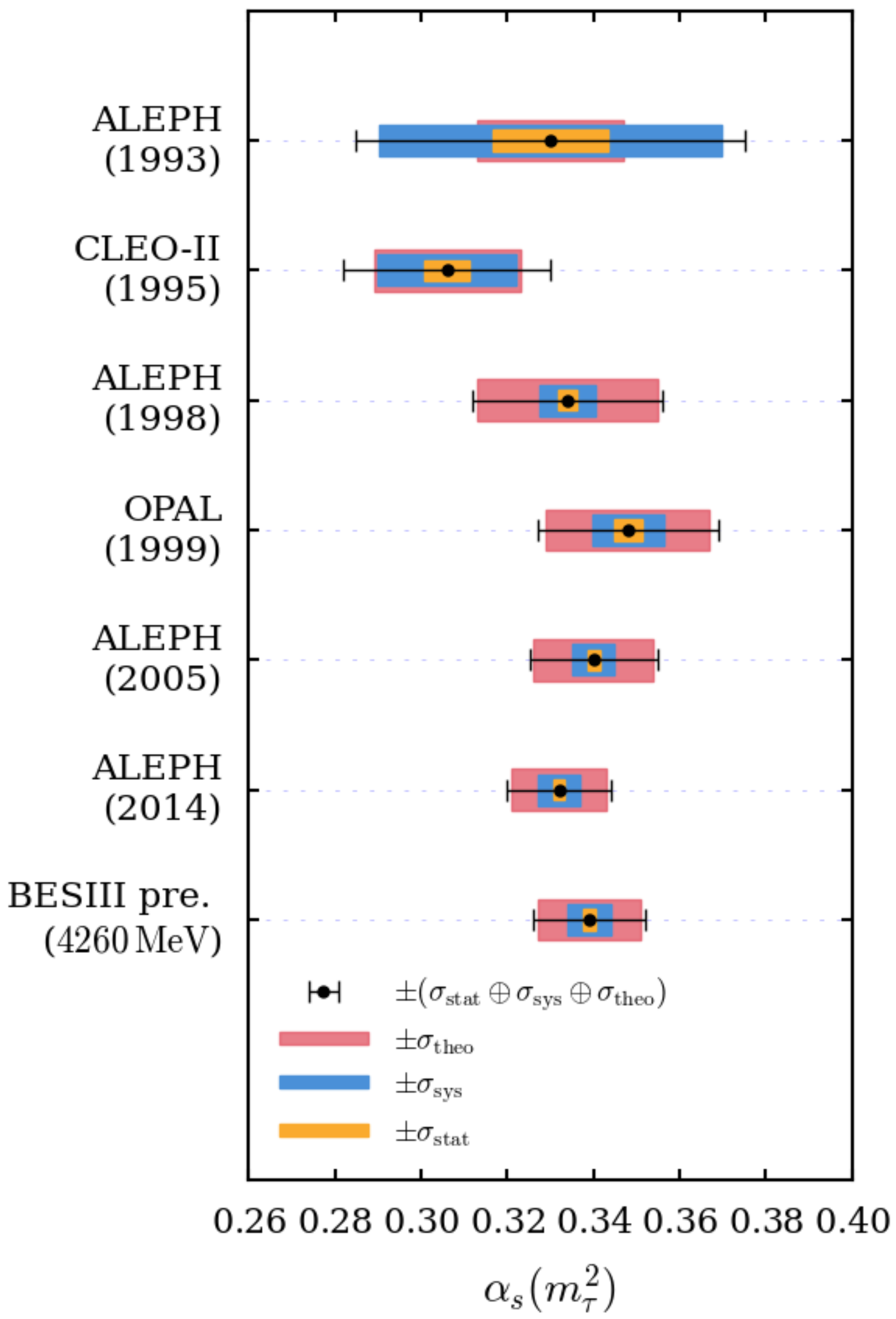

The statistical uncertainties for ALEPH's results after 2005 are estimated from the results in 1998 based on statistical improvements.

# Summary

- **Using BESIII data at center-of-mass energy of 4.26 GeV with integral luminosity of $825.7\ \mathrm{pb}^{-1}$ ,**
  - measured invariant mass spectrum of $\tau^- \to \pi^-\pi^0\nu_\tau$, $\pi^-2\pi^0\nu_\tau$, $\pi^-3\pi^0\nu_\tau$, $2\pi^-\pi^+\nu_\tau$, $2\pi^-\pi^+\pi^0\nu_\tau$, $2\pi^-\pi^+2\pi^0\nu_\tau$ decays.
  - Determined $\alpha_s(m_\tau^2)$ based on CIPT theory.
  - Preliminary result：

$$\alpha_s(m_\tau^2) = 0.3342 \pm 0.0010_{\mathrm{stat}} \pm 0.0052_{\mathrm{sys}} \pm 0.011_{\mathrm{theo}}$$

  - The uncertainty is dominated by the theoretical uncertainty, which depends on the theoretical method.
  - The statistical uncertainty is close to ALEPH, when data at only one $E_{cm}$ point is utilized.
  - The systematic uncertainty is dominated by the $e^+e^- \to \mathrm{had}$ background modeling and MC simulation for photons, which will be further studied in the future.
- The memo is ready.



# Thanks for Attention

# Back Up

Mar. 26th, 2025, Beijing

# Systematic: MC background simulation

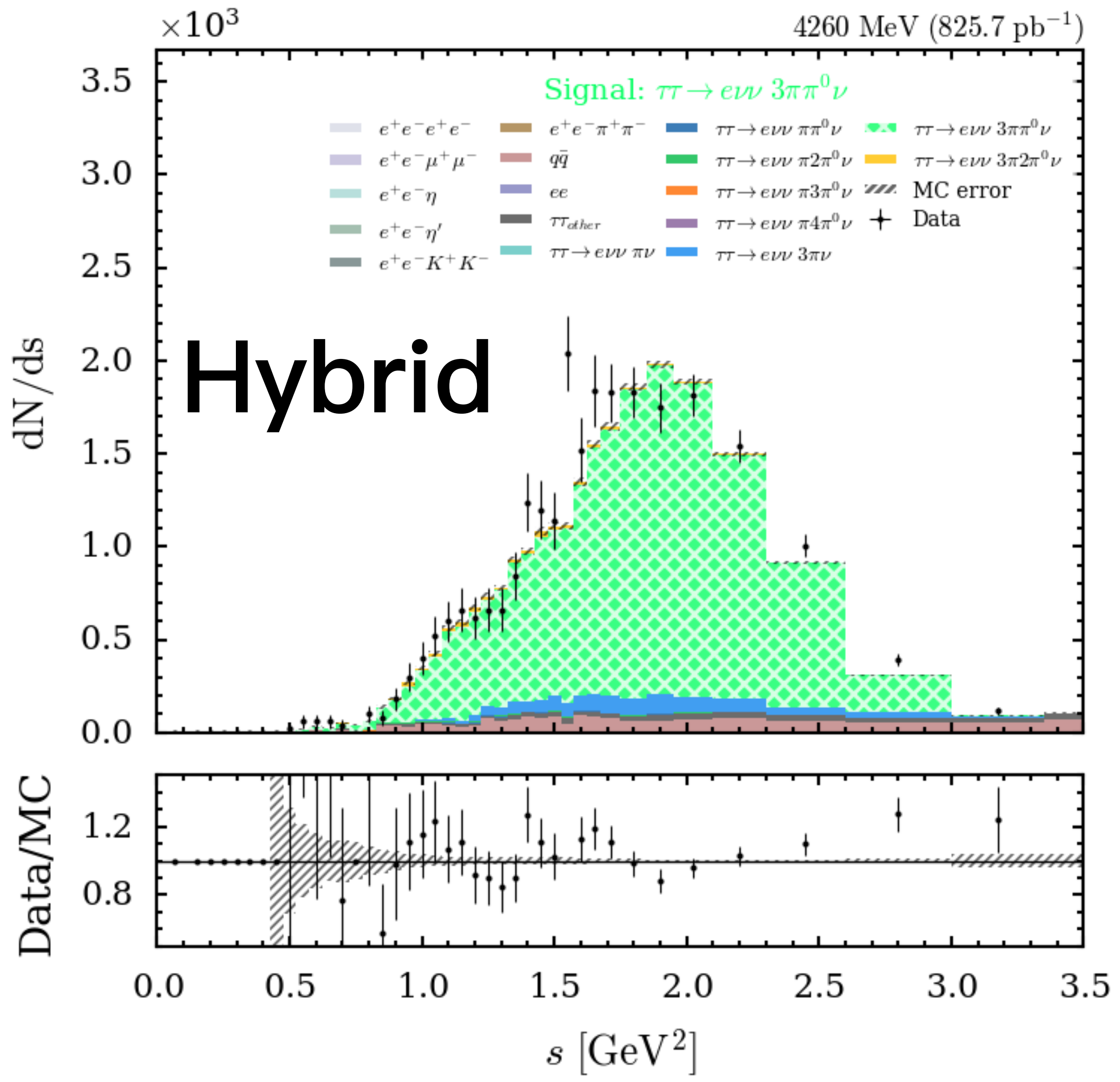
- Hybrid vs. LAUNDA
  - Hybrid sample: $\sim 4.9 \times 10^7$ (3.3x of data)
  - LUNDA sample: $\sim 1.5 \times 10^7$ (1x of data)
- The systematic uncertainty is estimated at half of the difference in results obtained from these two generators (Hybrid: 0.3390 —> LUND:0.3378), which is $\pm 0.0006$.
- Hybrid and LUNDA exhibit large difference on spectral moments measurement, which will be further investigated.

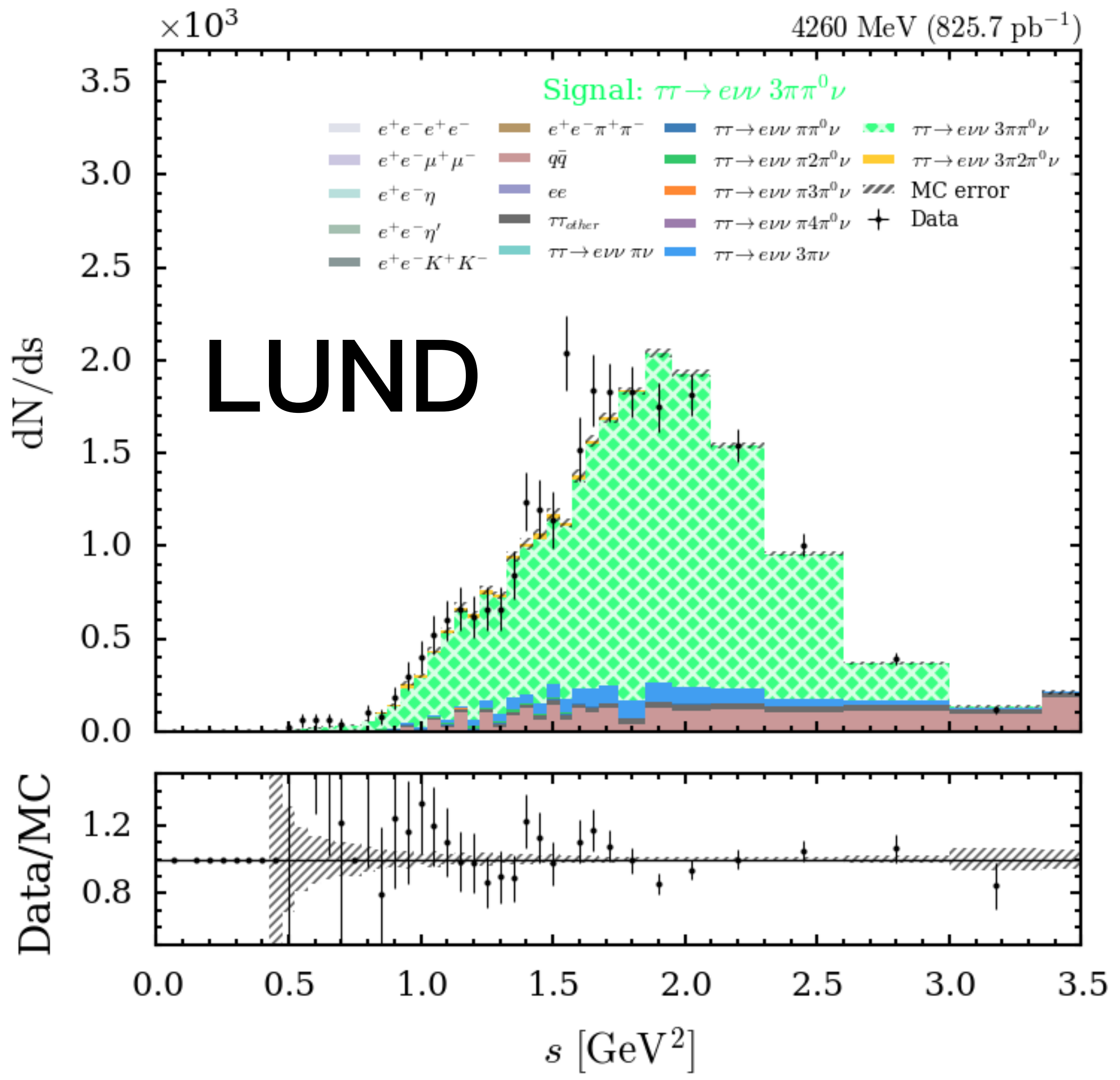
**Hybrid**

| | $R_{\tau}$ | $D^{10}$ | $D^{11}$ | $D^{12}$ | $D^{13}$ |
|---|---|---|---|---|---|
| Measurement | 3.47500 | 0.71686 | 0.16190 | 0.05879 | 0.02606 |
| Exp. Error | 0.01100 | 0.00127 | 0.00039 | 0.00028 | 0.00020 |
| Theory Prediction | 3.47665 | 0.71485 | 0.16104 | 0.05886 | 0.02614 |
| Theory Error | 0.03300 | 0.00350 | 0.00280 | 0.00040 | 0.00020 |

**LUND**

| | $R_{\tau}$ | $D^{10}$ | $D^{11}$ | $D^{12}$ | $D^{13}$ |
|---|---|---|---|---|---|
| Measurement | 3.47500 | 0.71991 | 0.16171 | 0.05840 | 0.02570 |
| Exp. Error | 0.01100 | 0.00126 | 0.00037 | 0.00028 | 0.00020 |
| Theory Prediction | 3.47797 | 0.71636 | 0.16018 | 0.05853 | 0.02585 |
| Theory Error | 0.03300 | 0.00350 | 0.00280 | 0.00040 | 0.00020 |



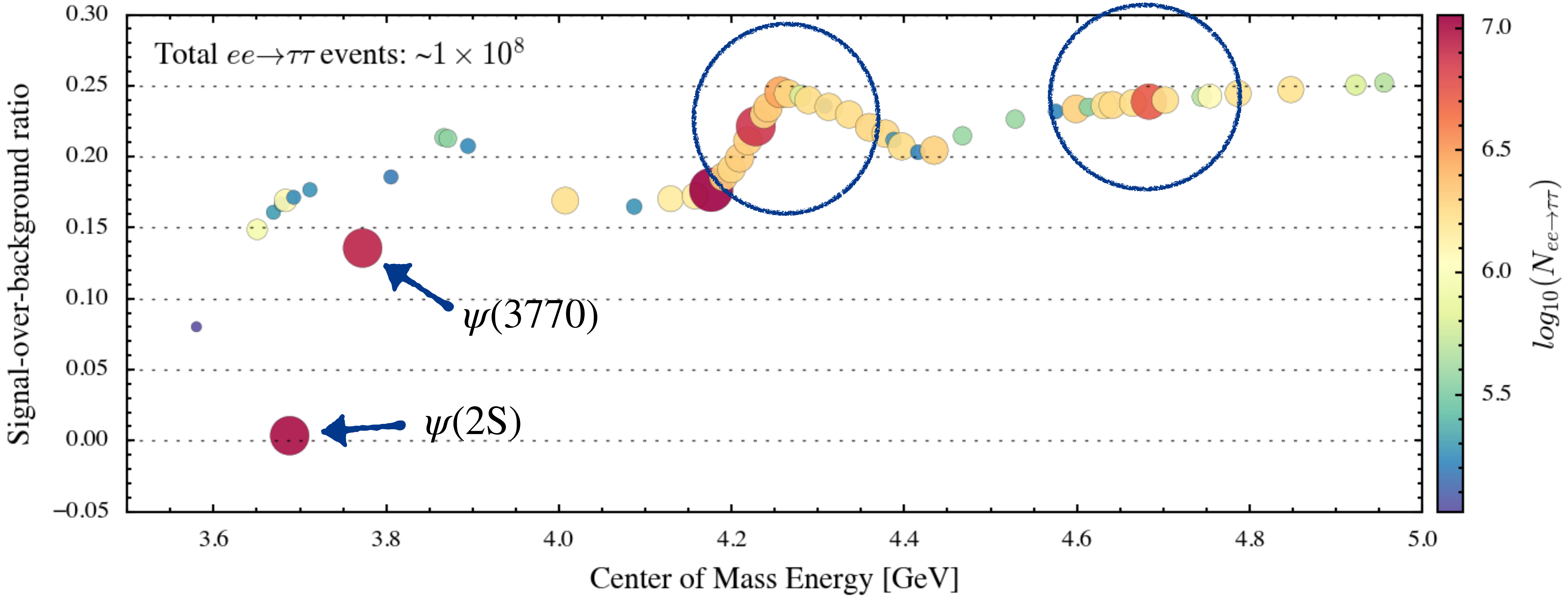



# BESIII Data set, $\tau$–pair yield and signal–background ratio

Cross section ratios of $e^+e^- \to \tau^+\tau^-$ and $e^+e^- \to \text{hadrons}$:

$$S/B = \frac{\sigma^{\text{QED}}(e^+e^- \to \tau^+\tau^-)}{\sigma(e^+e^- \to \text{hadron})}$$

The data sets in center-of-mass energy region of $4.2\ \text{GeV} \sim 4.7\ \text{GeV}$ have higher signal-over-background ratio.

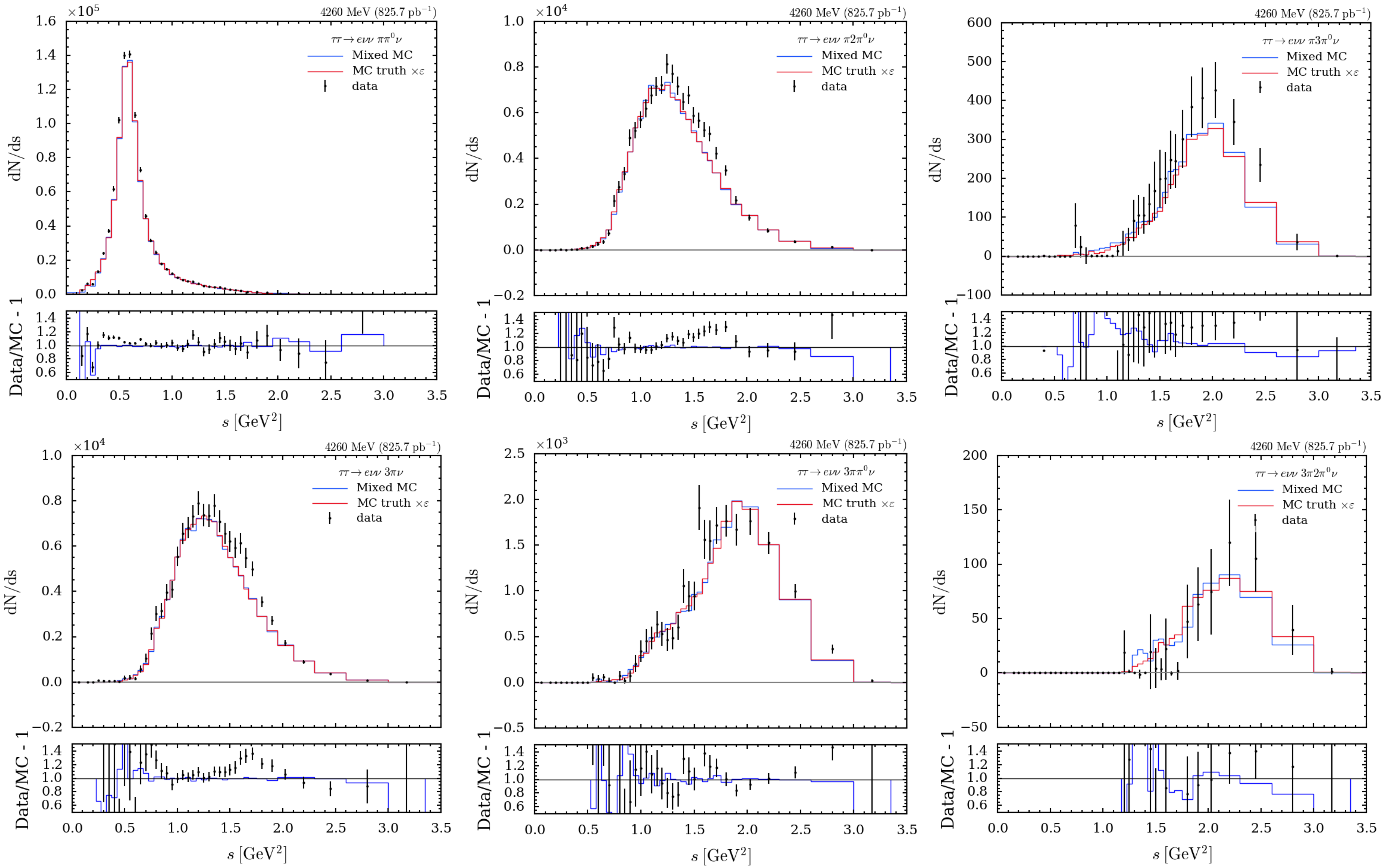



The size and color of the scatter points denote the estimated $\tau$-pair yields, and the height denotes the signal-over-background ratio.

# Invariant mass distribution after unfolding

**Intermediate results after unfolding**

- Mixed MC is used to validate the unfolding procedure.
- The unfolded Mixed MC was corrected back to the truth distribution.



# Systematic: $\pi^0$ reconstruction

Temporarily estimated using Achim Denig's systematic study result.

This study was performed on the $\psi(3770) \to \omega\pi^0$ data (2010-2011) and $\psi(3686) \to \pi^0\pi^0 J/\psi(\gamma^{ISR})$ data (2009), with BOSS version 6.6.4.p01, which does not perfectly suitable to our analysis.

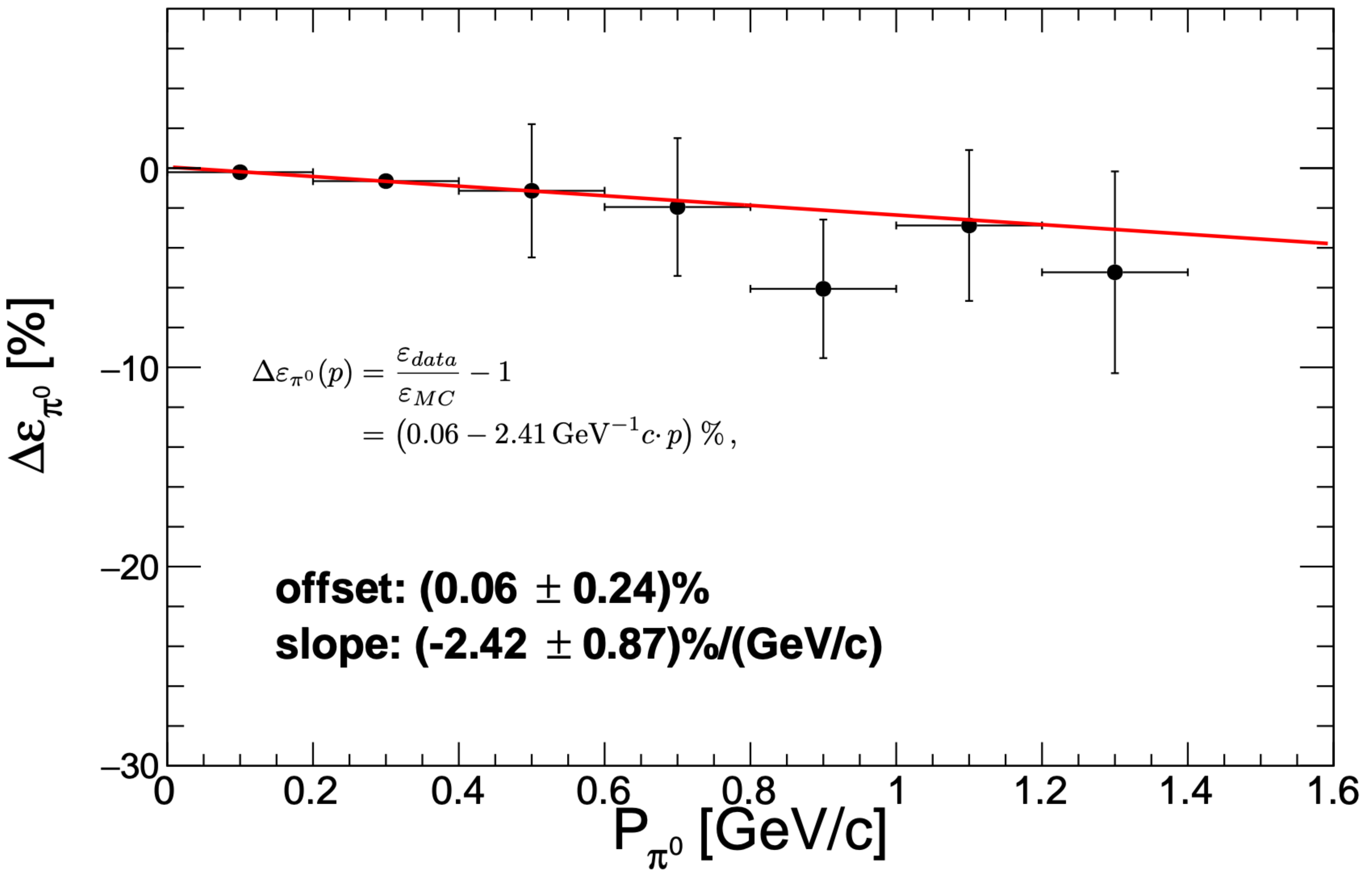
**Method**:

1. Apply a weight for each event, calculated by momenta of $\pi^0$'s.

$$w\left(P_{\pi_1^0}, P_{\pi_2^0}, \dots\right) = \left(1 + \Delta\varepsilon_{\pi^0}\left(P_{\pi_1^0}\right)\right) \cdot \left(1 + \Delta\varepsilon_{\pi^0}\left(P_{\pi_2^0}\right)\right) \cdot \dots$$

   Redo the data correction (background subtraction + unfolding) using the re-weighted MC samples.

2. Compare the resulting spectral moments and $\alpha_s(m_\tau^2)$ obtained w/i (w/o) re-weighting.




**For spectral moments: only 0.19% and 0.3% for $D^{12}$ and $D^{13}$.**

**For $\alpha_s(m_\tau^2)$: only 0.0005 (0.15%)**

# Systematic: $\pi^{\pm}$ reconstruction

Estimated using K/π Tracking Efficiency @4178 in BOSS7.0.3, which is quoted in Observation of $D_s^+ \to f_0(980)\mu^+\nu_\mu$ [DocDB: https://docbes3.ihep.ac.cn/cgi-bin/DocDB/ShowDocument?docid=1446]

This study was performed using continuum process $e^+e^- \to K^+K^-\pi^+\pi^-$ , with BOSS version 7.0.3.

**Method**:

1. Apply a weight for each event, calculated by momenta of $\pi^0$'s.
$$w\left(P_{\pi_1^{\pm}}, P_{\pi_2^{\pm}}, \dots\right) = \left(1 + \Delta\varepsilon_{\pi^{\pm}}\left(P_{\pi_1^{\pm}}\right)\right) \cdot \left(1 + \Delta\varepsilon_{\pi^{\pm}}\left(P_{\pi_2^{\pm}}\right)\right) \cdot \dots$$
Redo the data correction (background subtraction + unfolding) using the re-weighted MC samples.

2. Compare the resulting spectral moments and $\alpha_s(m_\tau^2)$ obtained w/i (w/o) re-weighting.

**The systematic uncertainty associated to charged pions seems marginal.**

## Tracking Efficiency of Pion

Table 1: Pion track efficiency and difference of data and MC.

| $p_T$ (GeV/c) | $\pi^+$ | | | $\pi^-$ | | |
|---|---|---|---|---|---|---|
| | $\epsilon_{data}(\%)$ | $\epsilon_{MC}(\%)$ | $\epsilon_{data}/\epsilon_{MC} - 1(\%)$ | $\epsilon_{data}(\%)$ | $\epsilon_{MC}(\%)$ | $\epsilon_{data}/\epsilon_{MC} - 1(\%)$ |
| 0.1-0.2 | 87.75±0.72 | 87.93±0.65 | -0.20±1.04 | 86.56±0.80 | 88.92±0.63 | -2.65±1.09 |
| 0.2-0.3 | 94.51±0.57 | 94.58±0.35 | -0.07±0.72 | 94.75±0.56 | 94.09±0.37 | 0.70±0.70 |
| 0.3-0.4 | 97.13±0.48 | 97.00±0.23 | 0.13±0.55 | 96.49±0.50 | 97.03±0.24 | -0.56±0.57 |
| 0.4-0.5 | 98.32±0.59 | 98.09±0.17 | 0.23±0.63 | 98.29±0.46 | 97.89±0.19 | 0.41±0.51 |
| 0.5-0.6 | 98.91±0.76 | 98.60±0.15 | 0.31±0.79 | 99.11±0.50 | 98.80±0.14 | 0.31±0.53 |
| 0.6-0.7 | 99.75±0.74 | 99.05±0.12 | 0.71±0.76 | 99.15±0.46 | 99.00±0.13 | 0.15±0.48 |
| 0.7-0.8 | 99.72±0.15 | 99.39±0.11 | 0.33±0.19 | 99.50±0.31 | 99.32±0.11 | 0.18±0.33 |
| 0.8-0.9 | 99.03±0.20 | 99.35±0.11 | -0.32±0.23 | 99.63±0.40 | 99.37±0.11 | 0.26±0.42 |
| 0.9-1.0 | 99.97±0.22 | 99.51±0.10 | 0.46±0.24 | 99.43±0.27 | 99.37±0.12 | 0.06±0.30 |
| 1.0-1.1 | 99.82±0.29 | 99.67±0.09 | 0.15±0.30 | 99.81±0.34 | 99.44±0.12 | 0.37±0.36 |
| 1.1-1.2 | 99.85±0.56 | 99.64±0.10 | 0.21±0.57 | 99.46±0.42 | 99.55±0.12 | -0.09±0.44 |

# Comparison of Spectral moments

Although the BESIII accuracy of spectral moments is similar with OPAL and CLEO-II, worse than ALEPH, the uncertainty of $\alpha_s$ is comparable with ALEPH and better than OPAL and CLEO-II, since the experimental uncertainty is controlled by the $R_\tau$ and theoretical uncertainty dominates the result.


The statistical uncertainties for ALEPH's results after 2005 are estimated from the results in 1998 based on statistical improvements.