# Study of the Process $e^+e^- \to \pi^+\pi^-\pi^0$ at 3.773 GeV

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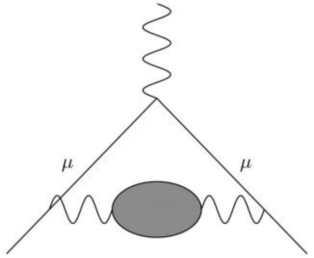
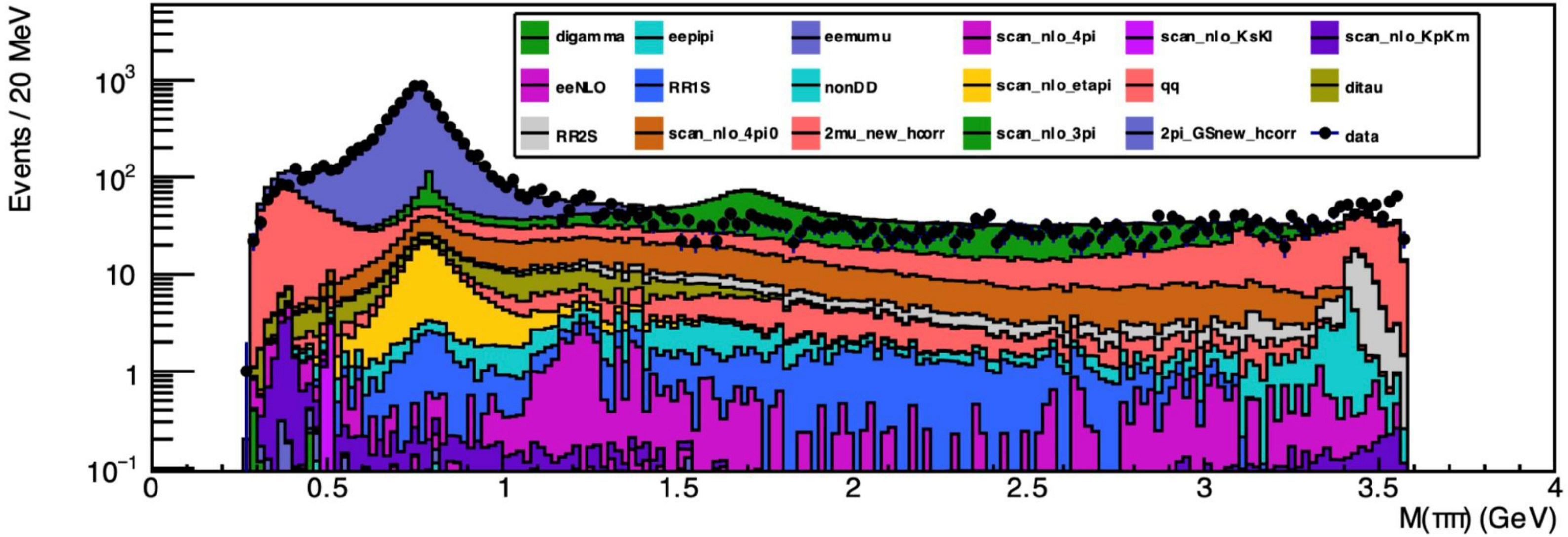
# MOTIVATION




- **Hadronic Vacuum Polarisation** is the main source of uncertainty in the Standard Model prediction of the **muon's anomalous magnetic moment**
- The **dominant channel** in data-driven approaches is: $e^+e^- \rightarrow \pi^+\pi^-$
  - Ongoing **2$\pi$ measurement** using ISR:




→ The **3π channel** acts as a large (≈ 10%) background to this measurement

→ This background **has not been measured yet** in this energy region

# DATA AND MC SIMULATIONS

**Data Sets**

- 2010 and 2011 data, total integrated luminosity of (2.932 ± 0.014) $fb^{-1}$
- 2021/2022 data, total integrated luminosty of (4.995 ± 0.019) $fb^{-1}$
- CM energy of $\sqrt{s} = 3.773$ GeV

**MC Simulations**

| Final State | Generator | $\sigma$ [pb] | LSF |
|---|---|---|---|
| $J/\psi\gamma_{\text{ISR}}$ (RR1S) | KKMC 4.15 | 1.1 | 0.100 |
| $\psi(2S)\gamma_{\text{ISR}}$ (RR2S) | KKMC 4.15 | 3.4 | 0.100 |
| $\tau^+\tau^-$ | KKMC 4.15 | 2.652 | 0.088 |
| $D^0\bar{D}^0$ | KKMC 4.15 | 3.66 | 0.101 |
| $D^+D^-$ | KKMC 4.15 | 2.88 | 0.102 |
| $\psi(3770) \to$ nonDD | KKMC 4.15 | 0.5 | 0.098 |
| $\gamma\gamma(+n\gamma)$ | KKMC 4.15 | 24.7 | 0.332 |
| $q\bar{q}$ | KKMC 4.15 | 15.463 | 0.091 |
| $\pi^+\pi^-\pi^0\pi^0(+n\gamma)$ | Phokhara 10.0 | 0.569 | 0.452 |
| $\pi^+\pi^-n\gamma$ | Phokhara 10.0 | 0.569 | 0.213 |
| $\mu^+\mu^-n\gamma$ | Phokhara 10.0 | 2.459 | 0.770 |
| $e^+e^-n\gamma$ ($e^+e^-$NLO) | BabaYaga@NLO | 270.860 | 10.58 |
| $\pi^+\pi^-\pi^0(+n\gamma)$ | Phokhara 10.0 | 0.243 | 0.193 |
| $\pi^+\pi^-\pi^0$phase space | EvtGen | | |

$$\text{LSF} = \frac{\mathcal{L}_{\text{exp}}\sigma_{\text{MC}}}{N_{\text{gen}}}$$

$\psi(3770) \to$ nonDD → exclude $\pi^+\pi^-\pi^0$

$q\bar{q}$ → exclude $\pi^+\pi^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\pi^0\pi^0$

$\pi^+\pi^-\pi^0(+n\gamma)$ →

$\pi^+\pi^-\pi^0$ :
$|M(\pi^+\pi^-\pi^0) - 3.773\text{GeV}| < 0.005$ GeV

$\pi^+\pi^-\pi^0\gamma$ :
$|M(\pi^+\pi^-\pi^0) - 3.773\text{GeV}| > 0.05$ GeV



JG|U

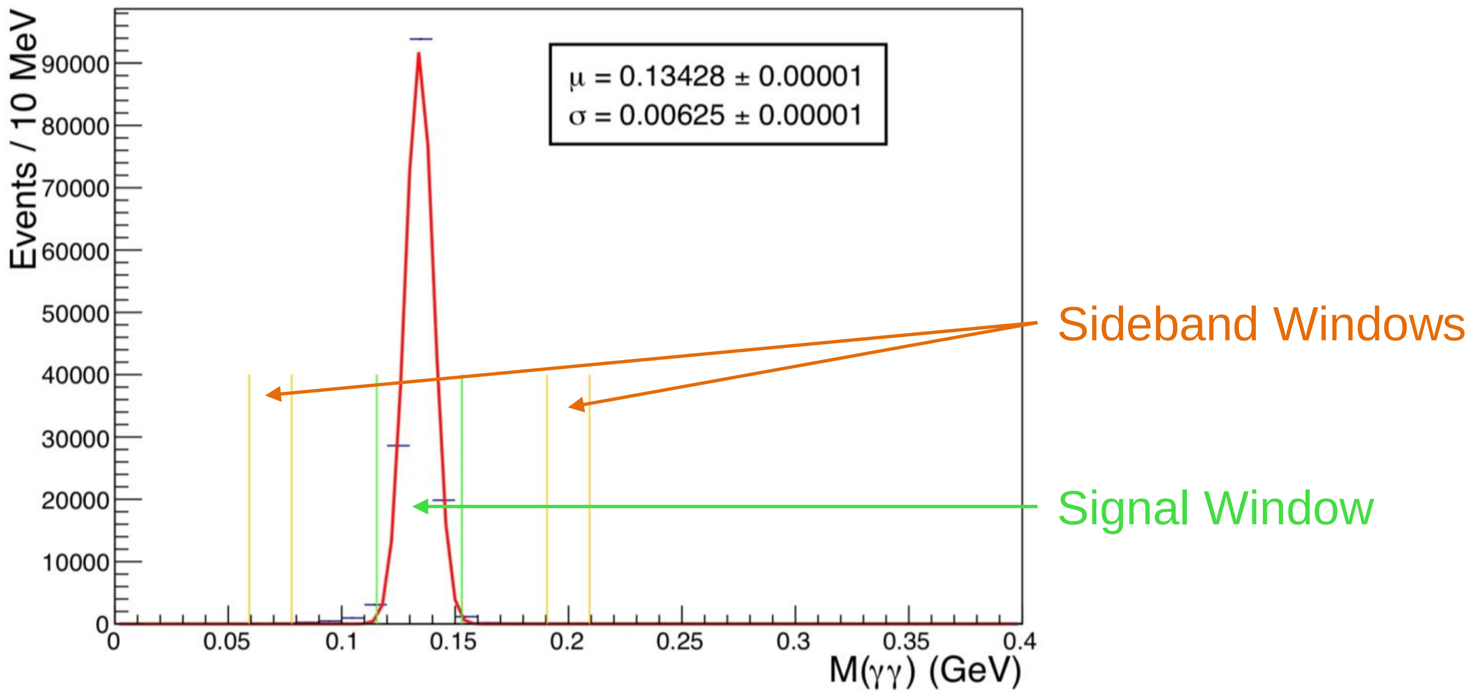
# PRE-SELECTION

Process of Interest: $e^+e^- \rightarrow \pi^+\pi^-\pi^0 \rightarrow \pi^+\pi^-\gamma\gamma$

| Criterion | Requirement |
|---|---|
| **Tracks** | |
| Production Vertex | $\mathrm{d}r < 1.0\mathrm{cm}$ |
| | $\mathrm{d}z < 10.0\mathrm{cm}$ |
| Number of Tracks | 2 |
| Track Charges | Oppositely Charged |
| Polar Angle | $\lvert\cos(\theta)\rvert < 0.93$ |
| Vertex Fit | Convergence |
| **Photons** | |
| Polar Angle | $\lvert\cos(\theta)\rvert < 0.8$ (Barrel) |
| | $0.86 < \lvert\cos(\theta)\rvert < 0.92$ (Endcaps) |
| Photon Energy | $E_\gamma > 25$ MeV (Barrel) |
| | $E_\gamma > 50$ MeV (Endcaps) |
| Time Difference | $0$ ns $< t < 700$ ns |
| Angle Shower-Track | $\beta > 10°$ |
| **4C-Kinematic Fit** | |
| Energy-Momentum Conservation, | |
| Convergence | |
| Photon pair with the lowest $\chi^2$ value is selected, | |

# EVENT SELECTION

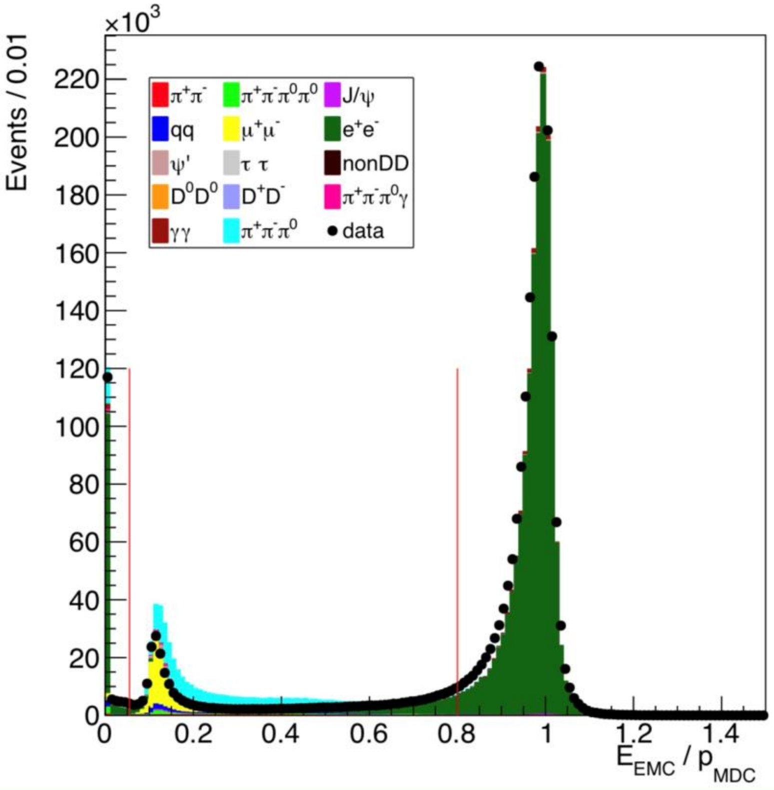
**Selection of $\pi^0$ Candidates**




$$|M(\gamma\gamma) - 0.13428\text{GeV}| < 3 \times 000625\text{GeV}$$

# EVENT SELECTION

**Energy to Mometum Ratio $E_{EMC}/p_{MDC}$**



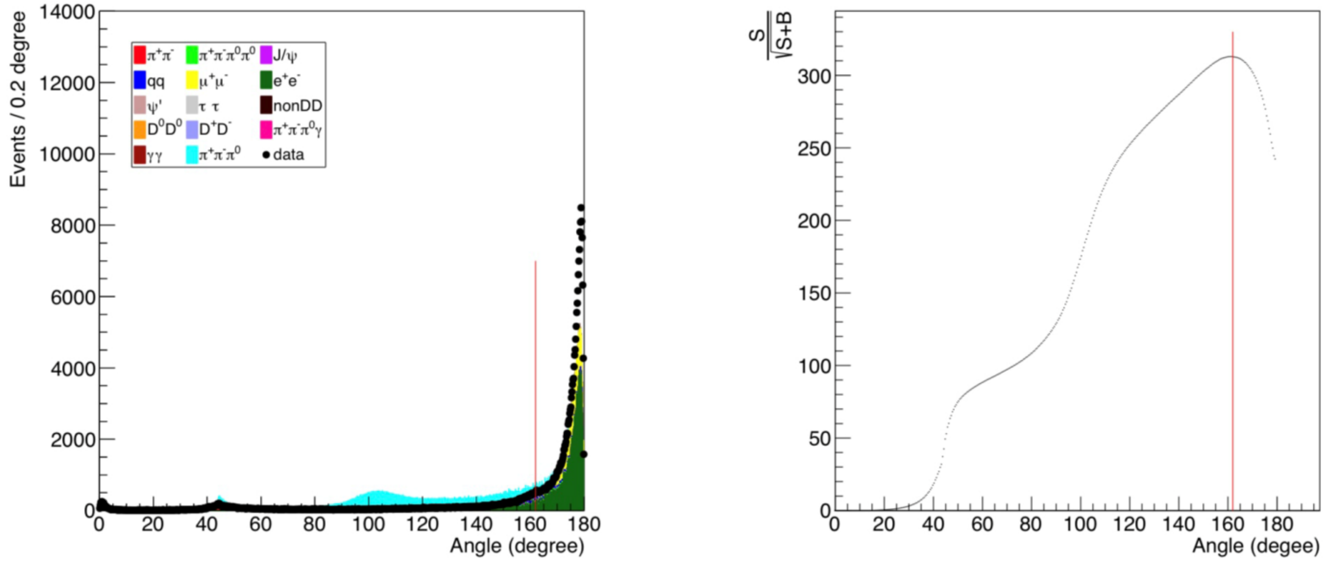

$$0.05 < E_{EMC}/p_{MDC} < 0.8, \quad \text{for} \geq 1 \text{ track}$$

# EVENT SELECTION

Optimization of Selection Cuts:

$$\frac{S}{\sqrt{S+B}}$$

**Opening Angle between the Charged Tracks in CM Frame**



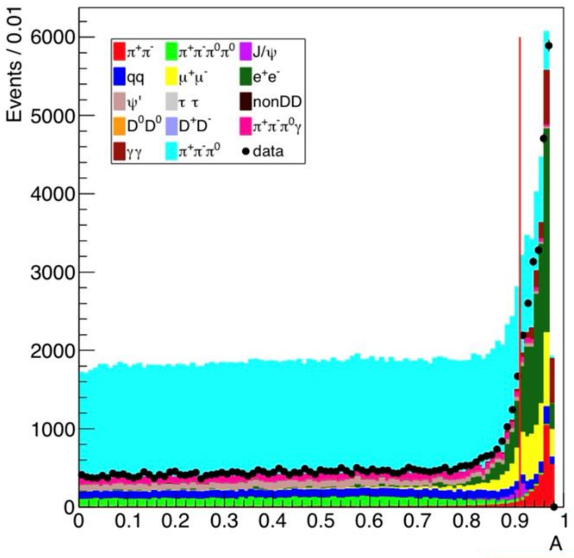

$\alpha < 162^\circ$

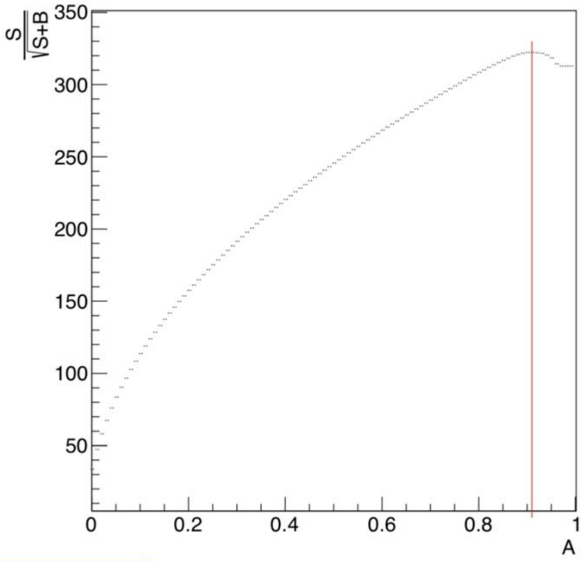
# EVENT SELECTION

Optimization of Selection Cuts:

$$\frac{S}{\sqrt{S+B}}$$

**Energy Asymmetry** $A = \frac{|E_{\gamma 1} - E_{\gamma 2}|}{E_{\gamma 1} + E_{\gamma 2}}$



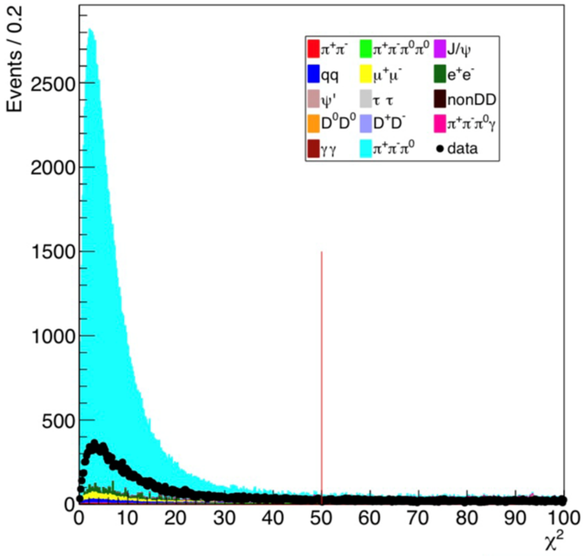

$A < 0.91$

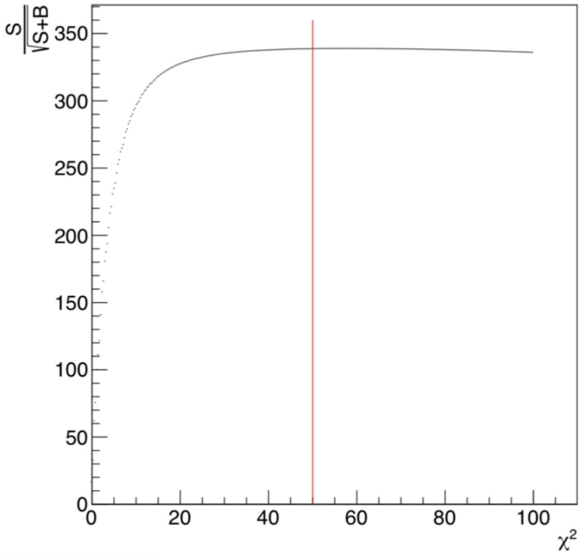
# EVENT SELECTION

Optimization of Selection Cuts:

$$\frac{S}{\sqrt{S+B}}$$

**Quality of the Kinematic Fit Result**

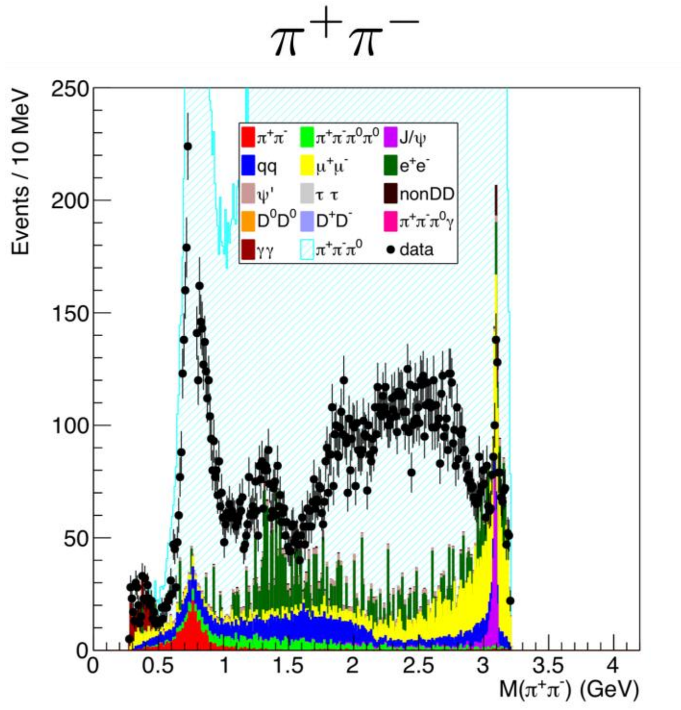


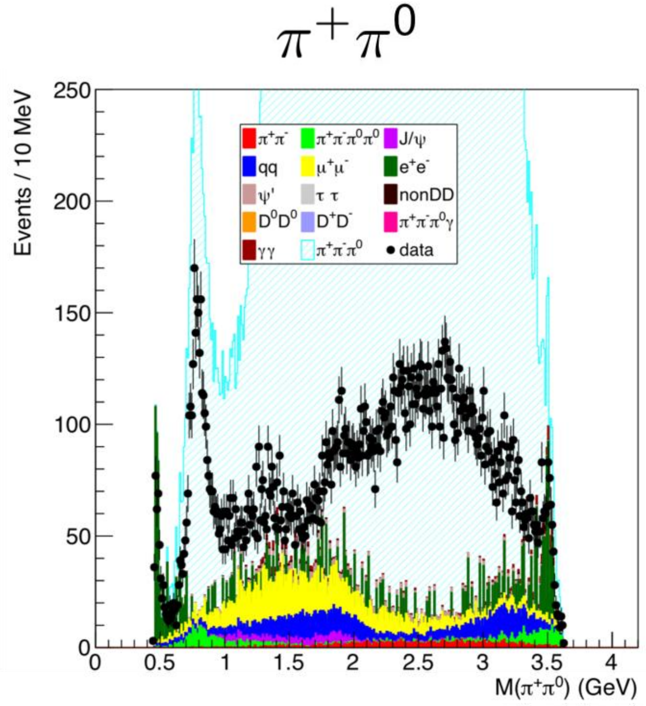




$$\chi^2 < 50$$

# BACKGROUND SUBSTRACTION

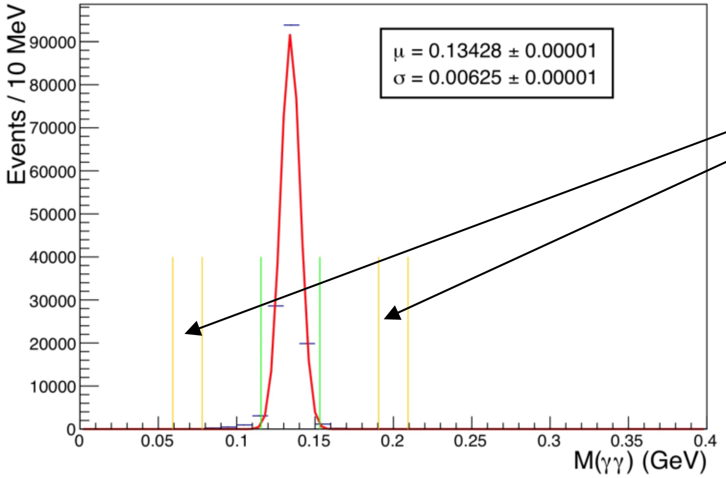
**$\pi\pi$ Invariant Mass Spectra**



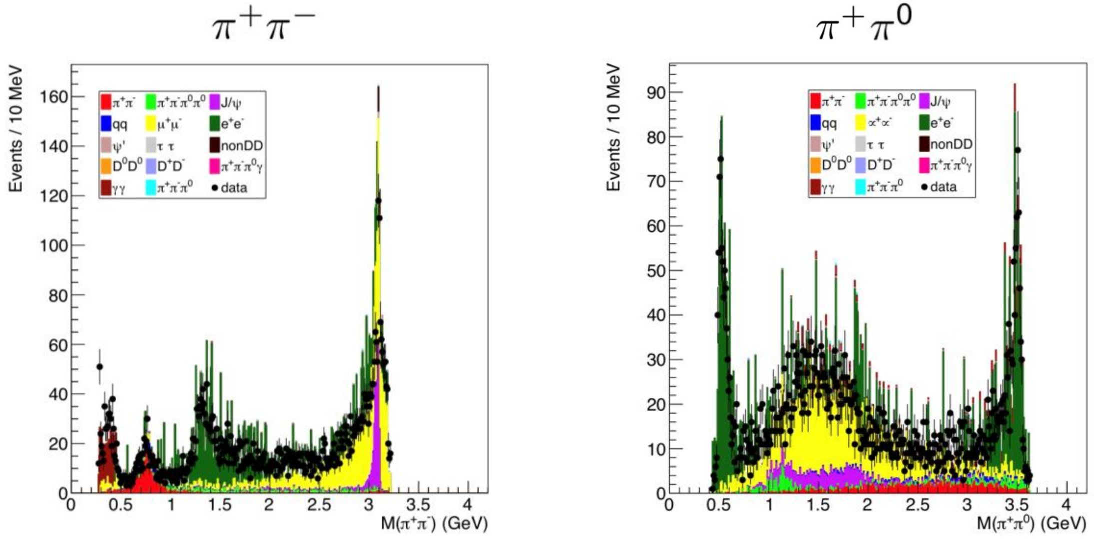

→ MC simulations indicate that background still remains in the selected data

# BACKGROUND SUBSTRACTION




$$9 \times 000625\text{GeV} < |M(\gamma\gamma) - 0.13428\text{GeV}| < 12 \times 000625\text{GeV}$$

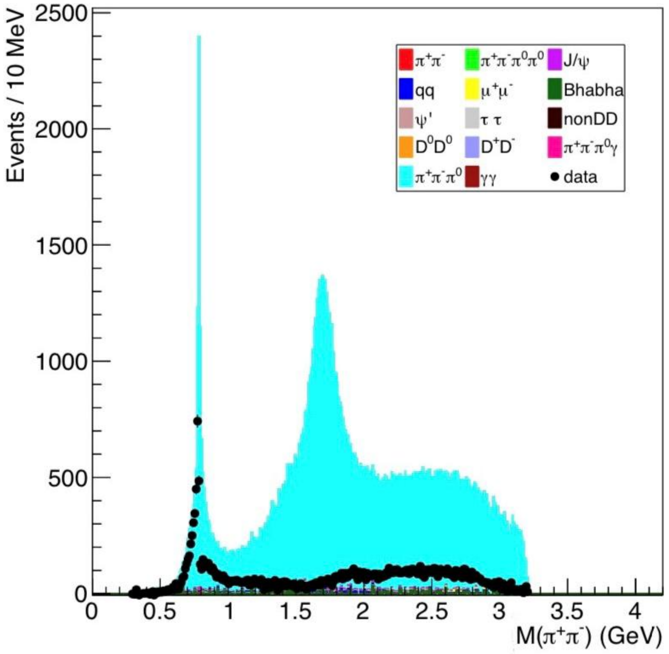
## $\pi\pi$ Mass Spectra in Sideband Regions



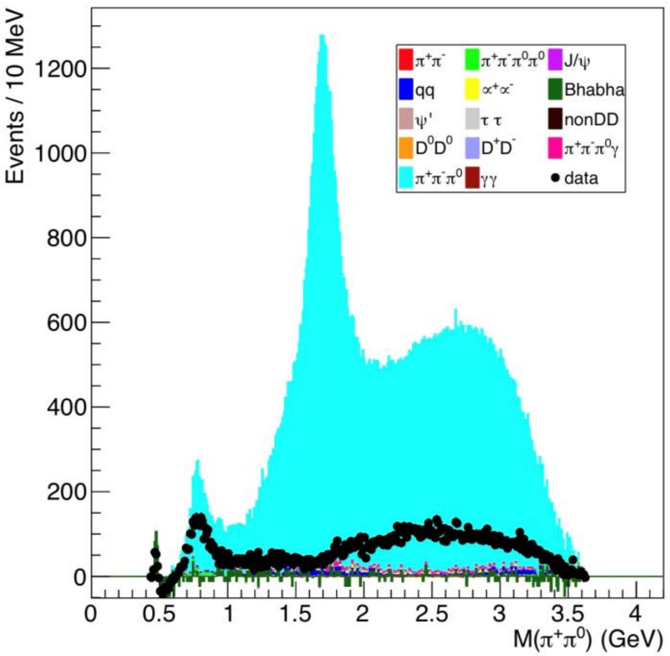
# BACKGROUND SUBSTRACTION

**Sideband Substracted $\pi\pi$ Mass Spectra**

$\pi^+\pi^-$

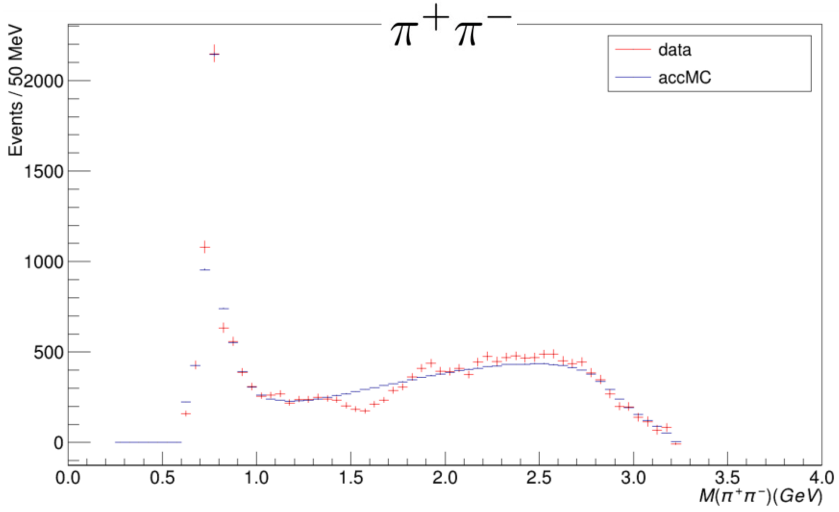


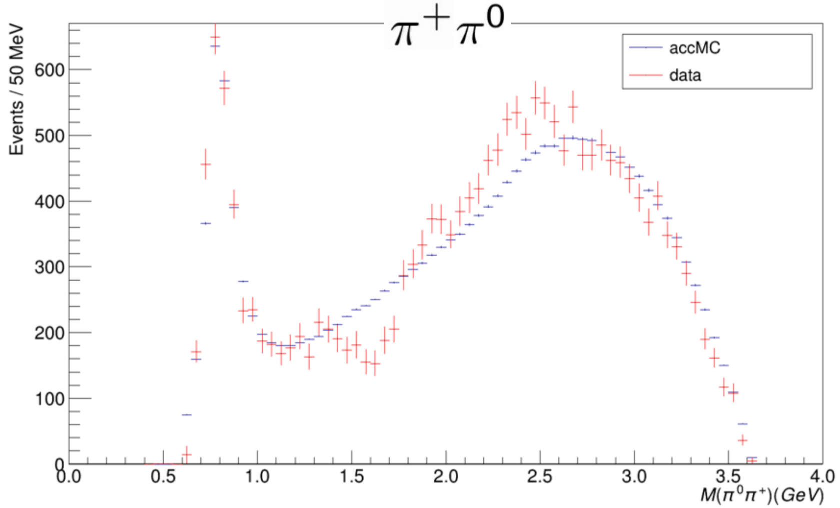

$\pi^+\pi^0$



# PARTIAL WAVE ANALYSIS

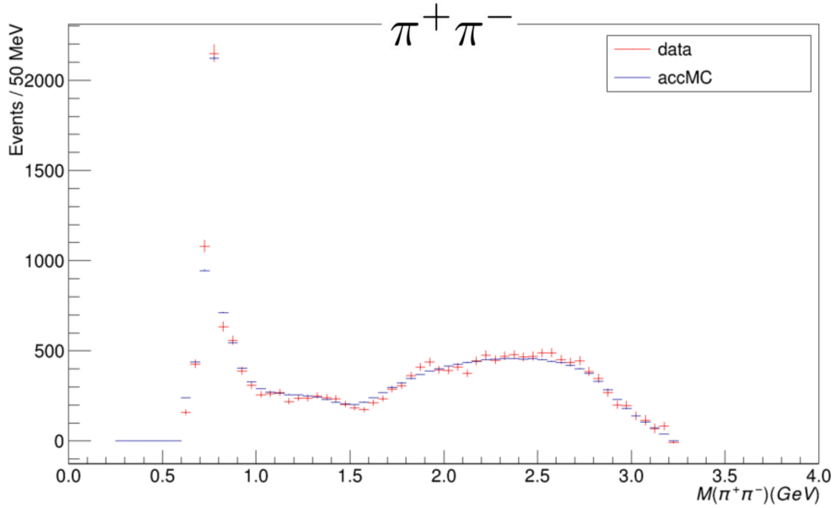
| | significance |
|---|---|
| $\omega(782)\rho(770)$ | null hypothesis |

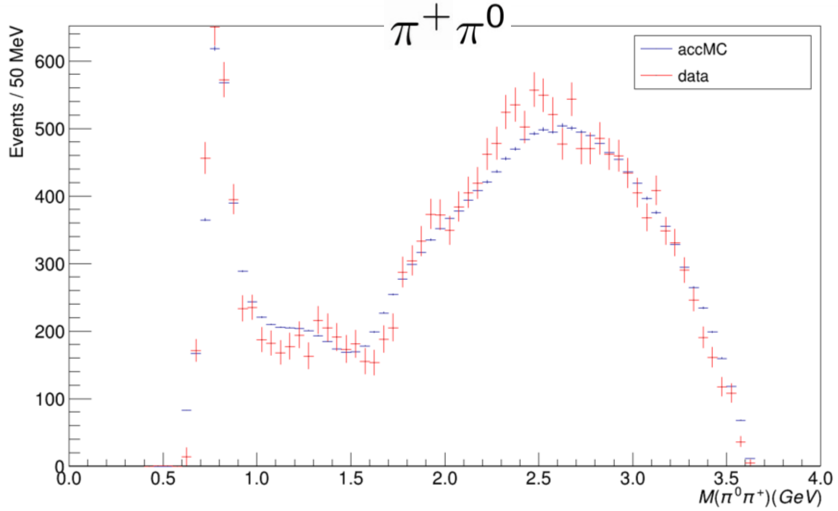


# PARTIAL WAVE ANALYSIS

| | significance |
|---|---|
| $\omega(782)\rho(770)$ | null hypothesis |
| $\omega(782)\rho(770)\rho(1450)$ | $22.2\sigma$ |



# PARTIAL WAVE ANALYSIS

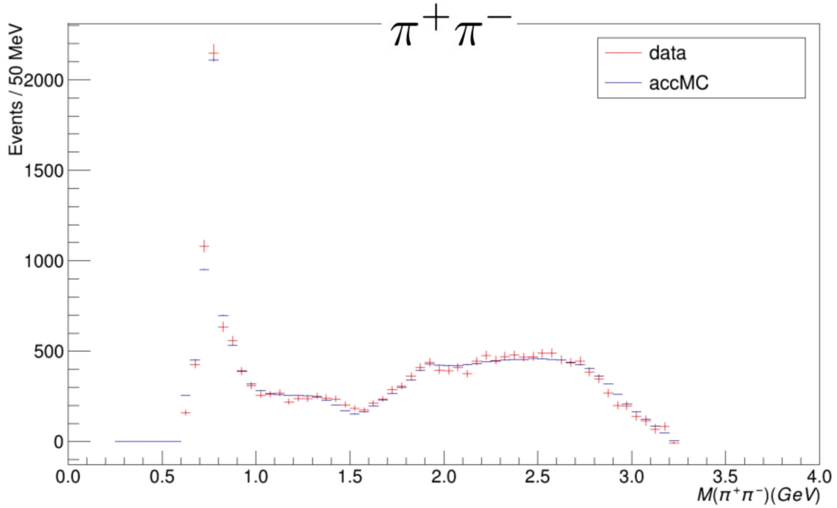
| | significance | |
|---|---|---|
| $\omega(782)\rho(770)$ | null hypothesis | |
| $\omega(782)\rho(770)\rho(1450)$ | $22.2\sigma$ | null hypothesis |
| $\omega(782)\rho(770)\rho(1450)\rho(1900)$ | $23.7\sigma$ | $8.6\sigma$ |

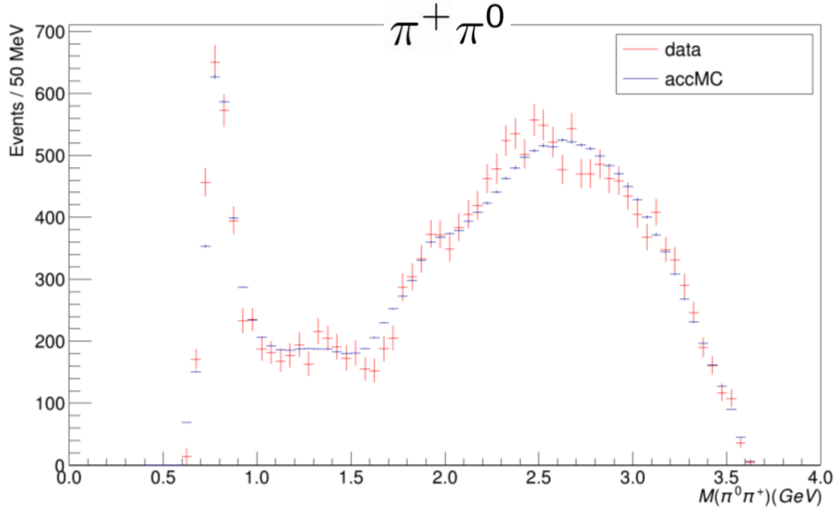
- Resonance parameters

$$\underbrace{\omega(782)}_{\text{neutral}},\ \underbrace{\rho(770),\ \rho(1450),\ \rho(1900)}_{\text{neutral and charged, amplitudes constrained in charged case}}$$

$$\underbrace{\omega(782),\ \rho(770)}_{\text{masses and widths held fixed during fit}},\ \underbrace{\rho(1450),\ \rho(1900)}_{\text{masses and widths determined with bounds}}$$

- Both spin orientations of $\Psi(3770)$ are considered



# DETERMINATION OF THE CROSS SECTION

number of signal events

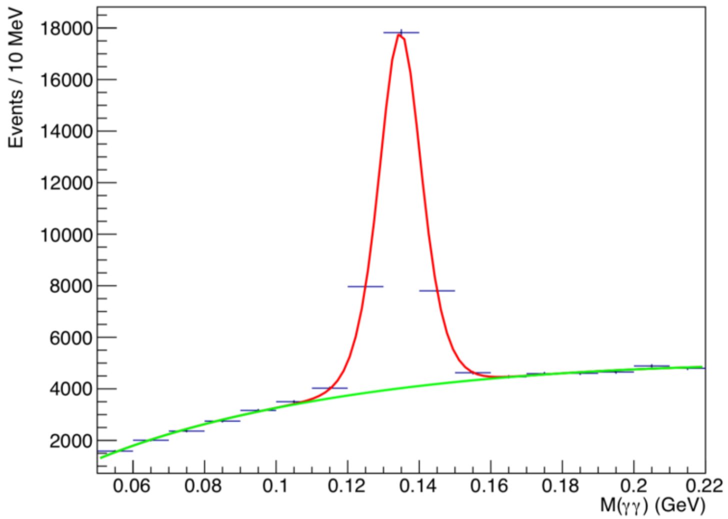
$$\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0) = \frac{N_{\pi^+\pi^-\pi^0}}{\mathcal{L} \cdot \epsilon}$$

total integrated luminosity

efficiency

# DETERMINATION OF THE CROSS SECTION

## Signal Yield Extraction $N_{\pi^+\pi^-\pi^0}$




$$\underbrace{P_0 \cdot e^{-0.5\left(\frac{x-P_1}{P_2}\right)^2} \cdot \left(1+\left(\frac{x-P_1}{P_2}\right)^2\right)^{-P_3}}_{\text{signal}} + \underbrace{P_4 \cdot x^{0.5} + P_5 \cdot x + P_6}_{\text{background}}$$

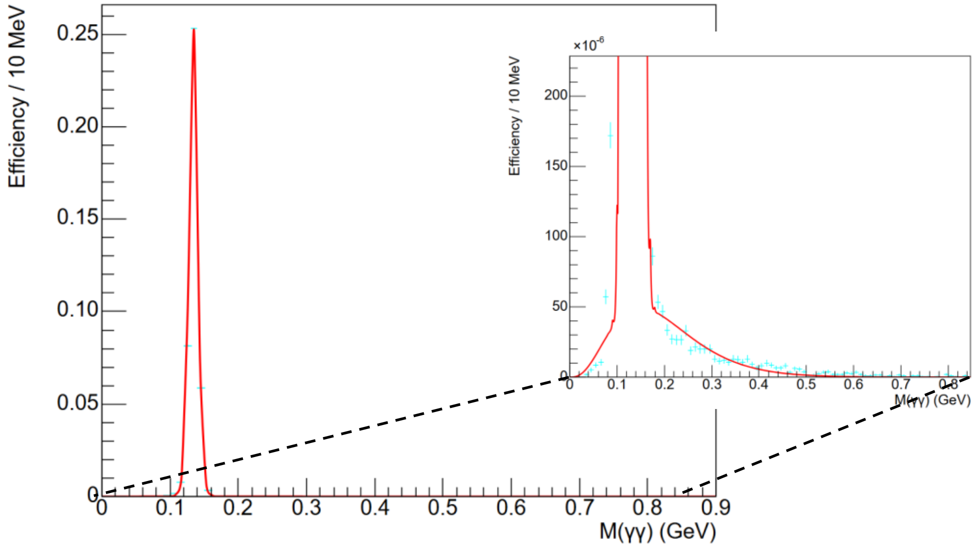
$\rightarrow N_{\pi^+\pi^-\pi^0} = 19038 \pm 176$

# DETERMINATION OF THE CROSS SECTION

## Efficiency

→ This calculation uses the MC events generated based on the PWA

$$\varepsilon = \frac{N_{acc}}{N_{gen}} = \frac{N_{acc}}{2000000}$$




$$P_0 \cdot e^{-0.5\left(\frac{x-P_1}{P_2}\right)^2} \cdot \left(1 + \left(\frac{x-P_1}{P_2}\right)^2\right)^{-P_3} + P_4 \cdot x^{P_5} \cdot e^{-P_6 x}$$

signal background

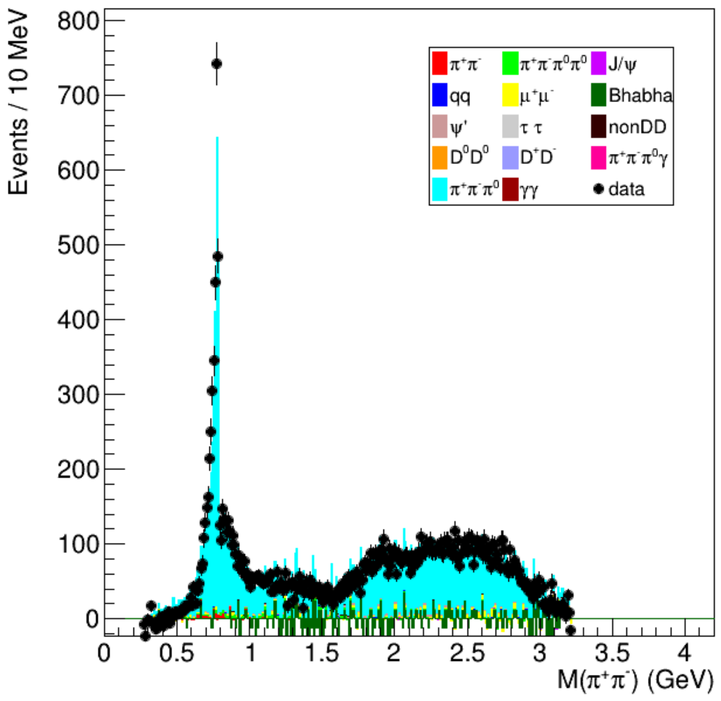
→ $\varepsilon = 0.401 \pm 0.001$

# DETERMINATION OF THE CROSS SECTION

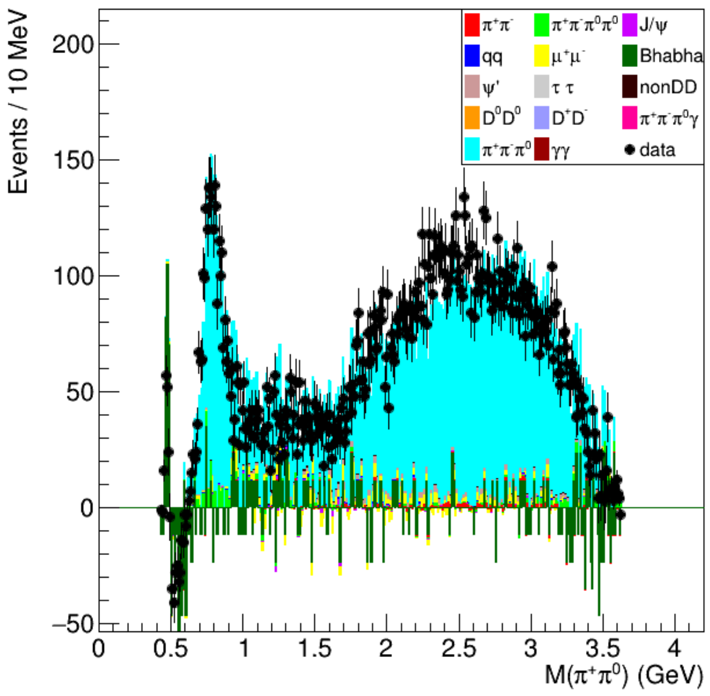
$$\Rightarrow \sigma(e^+e^- \to \pi^+\pi^-\pi^0) = (5.99 \pm 0.06_{\text{stat.}})\ \text{pb}$$

**Background Substracted ππ Invariant Mass**

$\pi^+\pi^-$

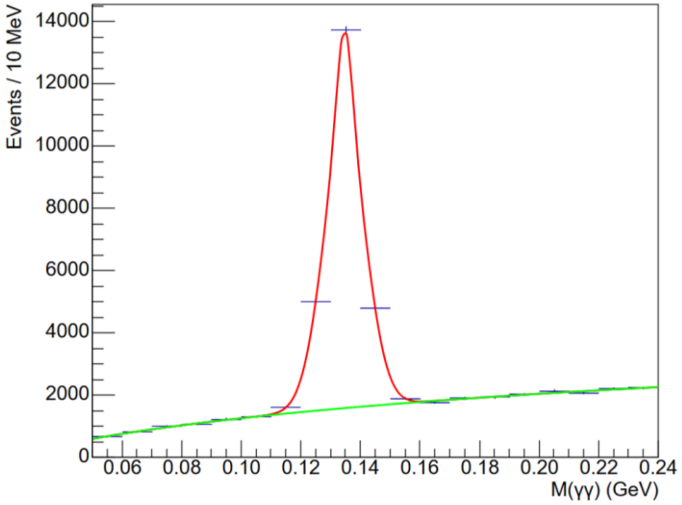



$\pi^+\pi^0$



# DETERMINATION OF THE CROSS SECTION

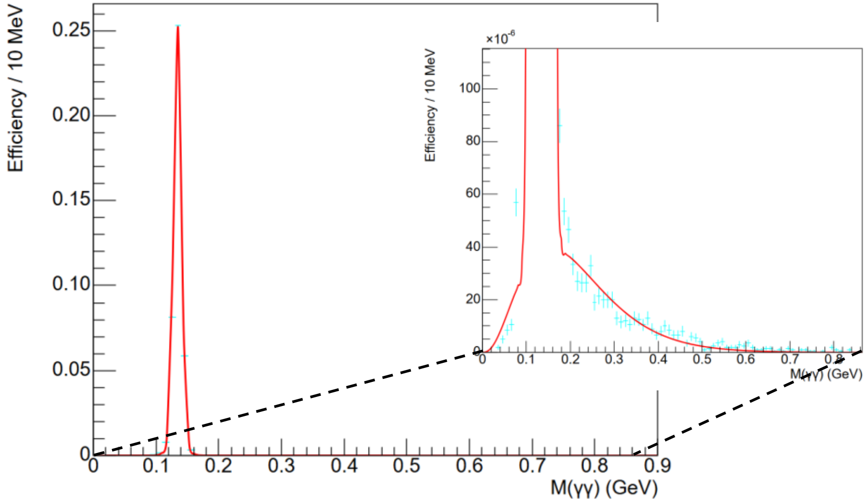
**Signal Yield Extraction $N_{\pi^+\pi^-\pi^0}$**




$$P_0 \cdot e^{-\frac{(x-P_2)^2}{2P_3^2}} + P_1 \cdot e^{-\frac{(x-P_2)^2}{2P_4^2}} + P_5 \cdot x^{0.5} + P_6 \cdot x + P_7$$

$\rightarrow N_{\pi^+\pi^-\pi^0} = 17929$

**Efficiency**




$$P_0 \cdot e^{-\frac{(x-P_2)^2}{2P_3^2}} + P_1 \cdot e^{-\frac{(x-P_2)^2}{2P_4^2}} + P_5 \cdot x^{P_6} \cdot e^{-P_7 x}$$

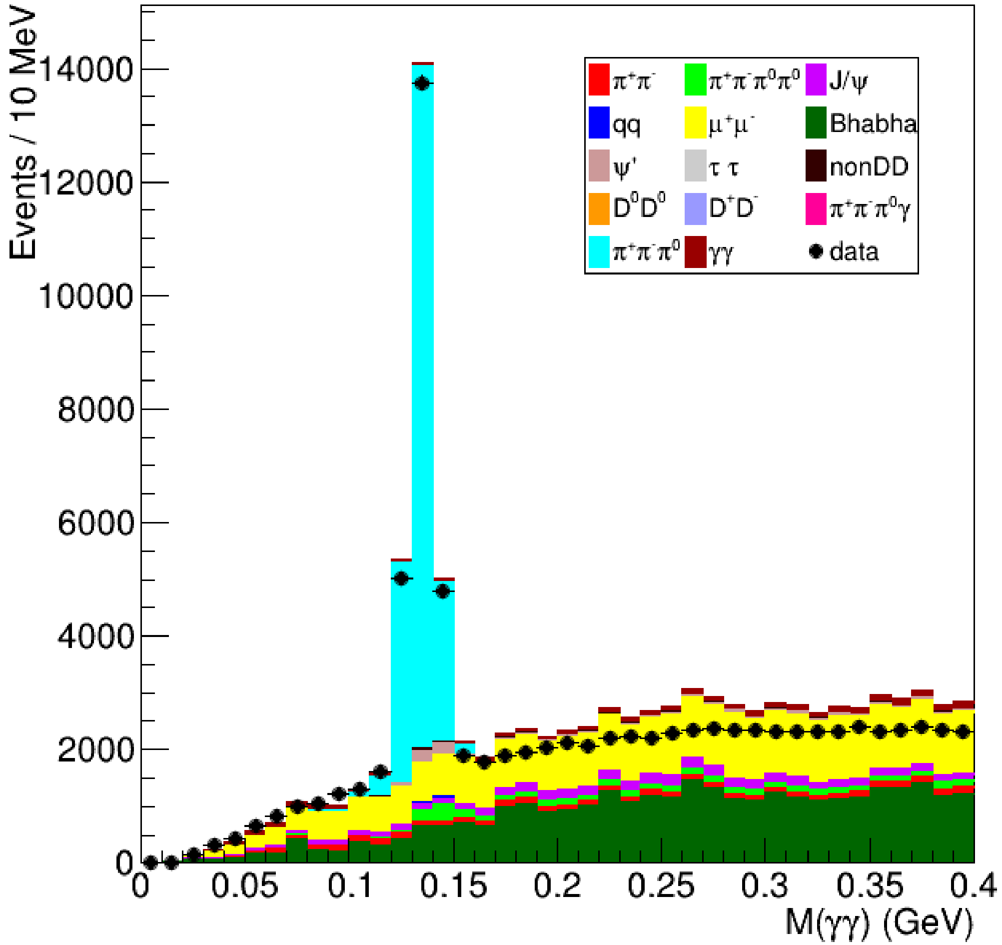
$\rightarrow \varepsilon = 0.404$

$$\Rightarrow \sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0) = (5.99 \pm 0.06_{\text{stat.}} \pm 0.55_{\text{sys.}})\ \text{pb}$$

→ Including photon & tracking efficiency

# BACK-UP

# YY INVARIANT MASS



# $\pi^-\pi^0$ INVARIANT MASS