JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

Measurement of the $\gamma^*\gamma^* \to \pi^0\pi^0$ Cross Section

Max Lellmann

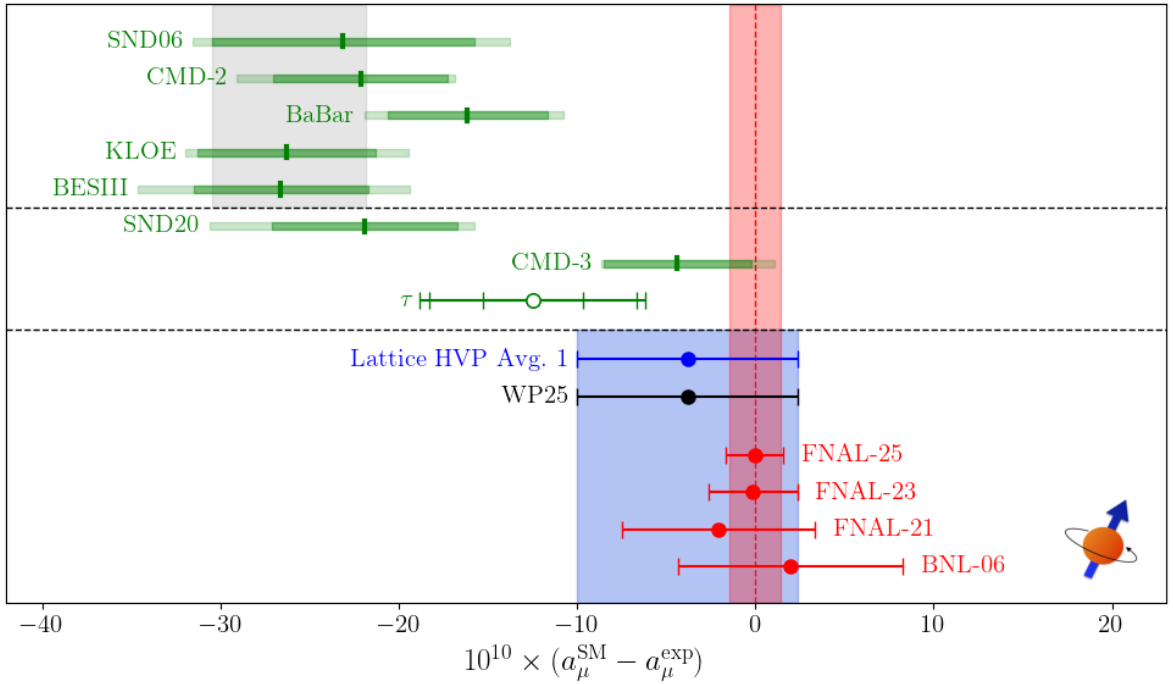
τQCD Meeting

15.10.2025

Motivation – The Muon Anomaly

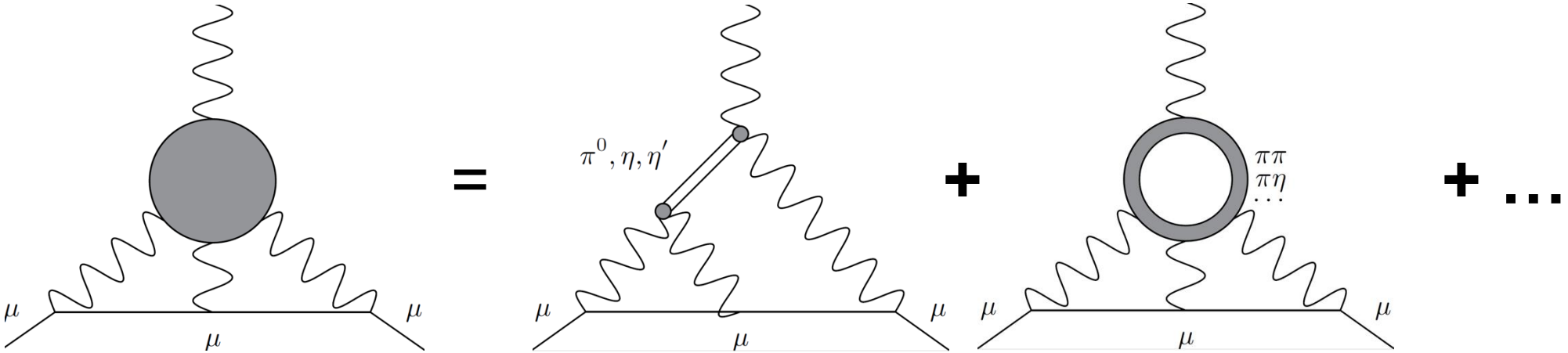
- Muon anomaly $a_\mu = \frac{g_\mu - 2}{2}$
- Possible discrepancy between SM and direct measurtement
- Need to increase accuracy SM prediction
- Next-to-leading-order hadronic interactions from hadronic Light-by-Light scattering

[Aliberti et. al. (2025)]



The Hadronic Light by Light Contribution

[Aliberti et. al. (2025)]



Four-Point Function Calculated by Dispersion Relations:

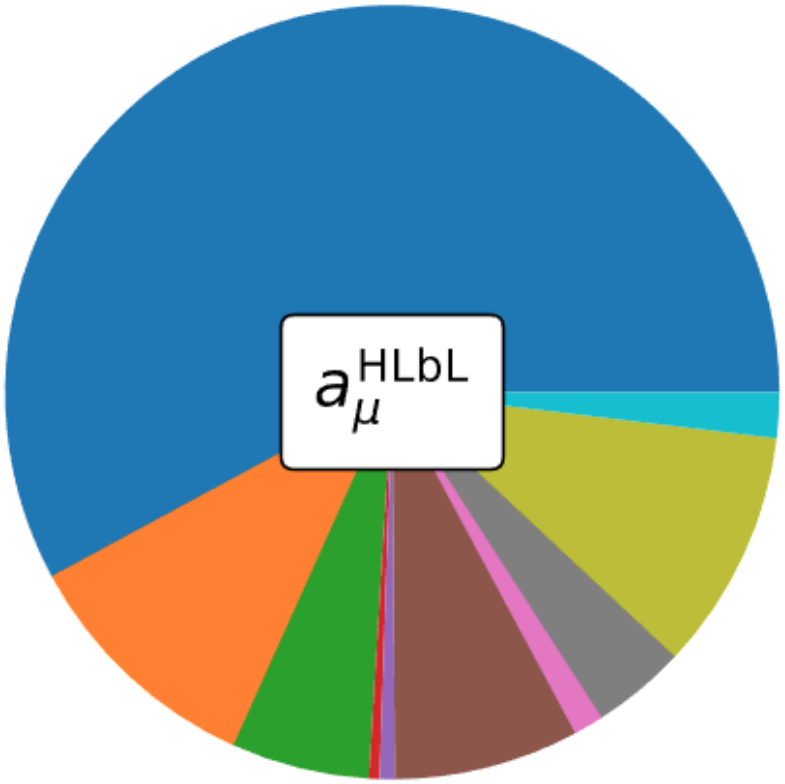
Two-Photon Couplings

$$a_\mu^{\mathrm{HLbL}} = \frac{\alpha^3}{432\pi^2} \int_0^\infty d\Sigma\, \Sigma^3 \int_0^1 dr\, r\sqrt{1-r^2} \int_0^{2\pi} d\phi \sum_{i=1}^{12} T_i(\Sigma, r, \phi)\, \bar{\Pi}_i(Q_1^2, Q_2^2, Q_3^2)$$

[Colangelo, Hoferichter, Procura, Stoffer (2017)]

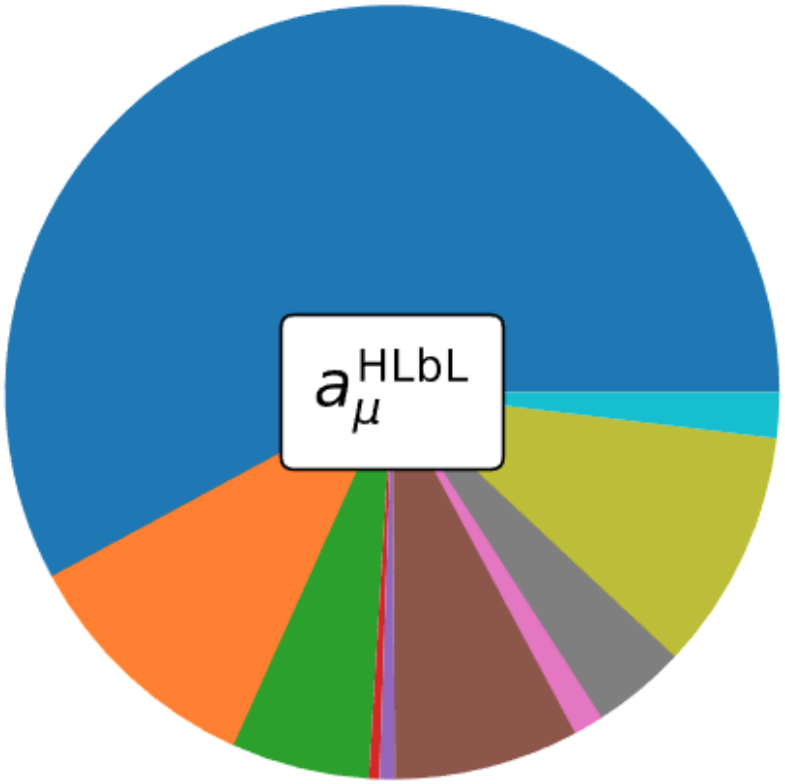
The Hadronic Light by Light Contribution

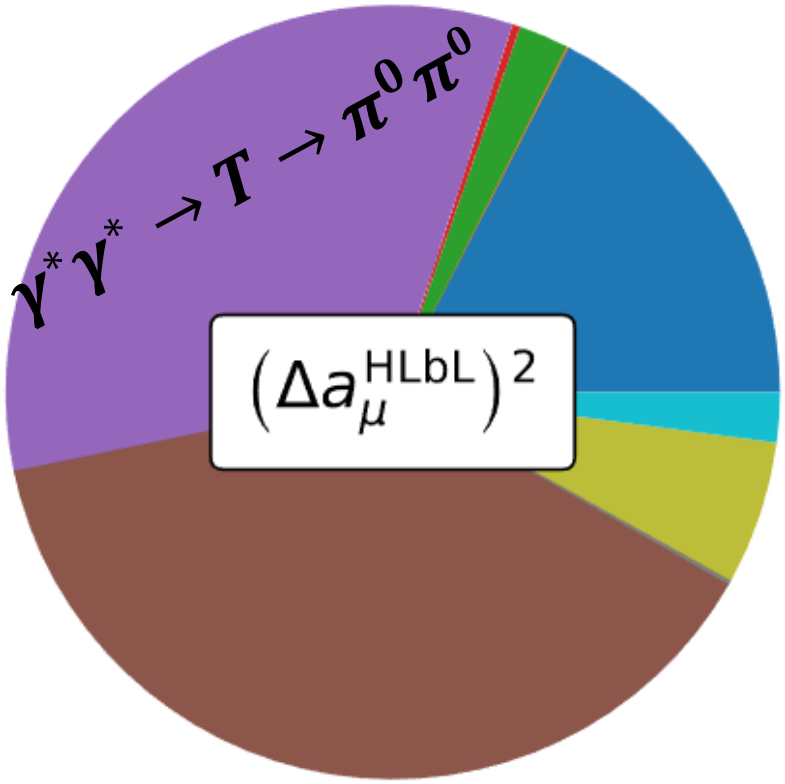
[Aliberti et. al. (2025)]



The Hadronic Light by Light Contribution

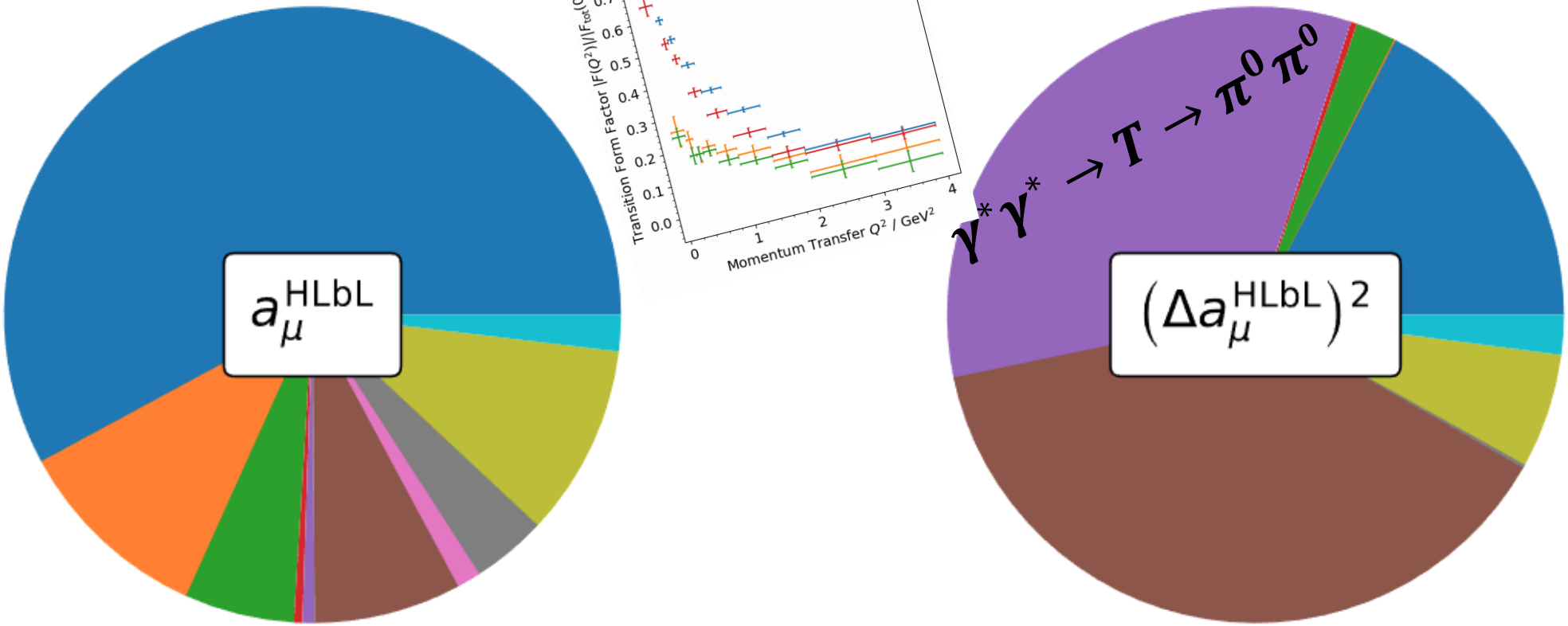
[Aliberti et. al. (2025)]



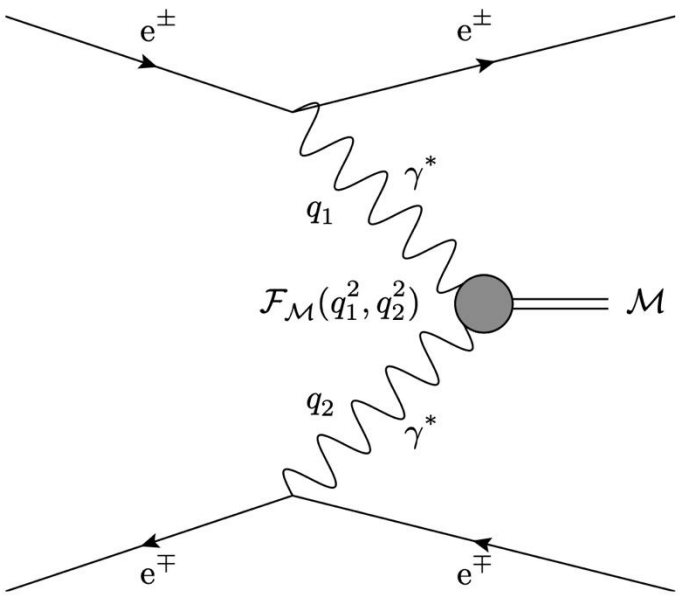


The Hadronic Light by Light Contribution

[Aliberti et. al. (2025)]



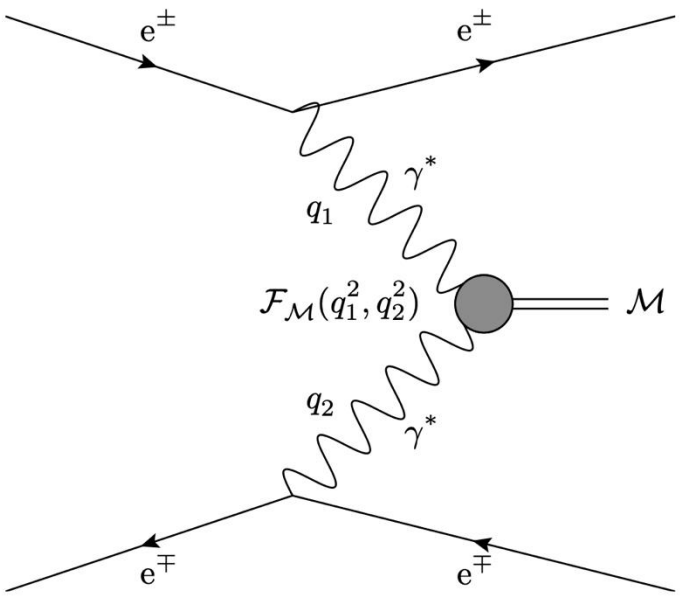
Two-Photon Physics at e^+e^- Machines



$$
\begin{aligned}
d\sigma \;=\; & \frac{\alpha^2}{16\,\pi^4\,Q_1^2\,Q_2^2}\,\frac{2\sqrt{X}}{s_{ee}\,(1-4\,m_e^2/s_{ee})^{1/2}}\cdot\frac{d^3\vec{p}\,'_1}{E'_1}\cdot\frac{d^3\vec{p}\,'_2}{E'_2} \\
\times\; & \Big\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT}+\rho_1^{00}\,\rho_2^{00}\,\sigma_{LL}+2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL}+2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT} \\
& +2\,\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\cos(2\tilde{\phi})\tau_{TT} \\
& +8\left[\frac{\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)}{\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)}\right]^{1/2}\cos\tilde{\phi}\,\tilde{\tau}_{TL} \\
& +h_1h_2\,4\,\left[\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)\right]^{1/2}\,\tilde{\tau}_{TT} \\
& +\,h_1h_2\,8\,\left[\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\right]^{1/2}\cos\tilde{\phi}\,\tau_{TL}\Big\}\,,
\end{aligned}
$$

Two-Photon Physics at e^+e^- Machines

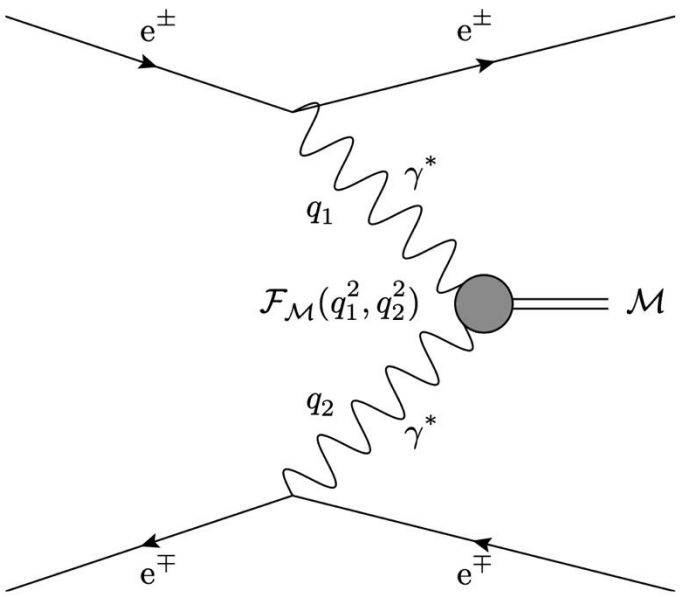
$e^+e^- \to e^+e^-\pi^0\pi^0$



$$
\begin{aligned}
d\sigma = \; & \frac{\alpha^2}{16\,\pi^4\,Q_1^2\,Q_2^2}\,\frac{2\sqrt{X}}{s_{ee}\,(1-4\,m_e^2/s_{ee})^{1/2}}\cdot\frac{d^3\vec{p}\,'_1}{E'_1}\cdot\frac{d^3\vec{p}\,'_2}{E'_2} \\
\times \; & \Big\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT} \\
& +2\,\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\cos(2\tilde{\phi})\,\tau_{TT} \\
& +8\left[\frac{\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)}{\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)}\right]^{1/2}\cos\tilde{\phi}\,\tilde{\tau}_{TL} \qquad \gamma\gamma \to \pi^0\pi^0 \\
& +h_1h_2\,4\,\left[\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)\right]^{1/2}\tilde{\tau}_{TT} \\
& +\,h_1h_2\,8\,\left[\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\right]^{1/2}\cos\tilde{\phi}\,\tau_{TL}\Big\}\,,
\end{aligned}
$$

Two-Photon Physics at e^+e^- Machines

$\mathbf{e^+e^- \to e^+e^-\pi^0\pi^0}$



$$d\sigma = \frac{\alpha^2}{16\,\pi^4\,Q_1^2\,Q_2^2}\,\frac{2\sqrt{X}}{s_{ee}\,(1-4\,m_e^2/s_{ee})^{1/2}}\cdot\frac{d^3\vec{p}_1'}{E_1'}\cdot\frac{d^3\vec{p}_2'}{E_2'}$$

$$\times\ \Big\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT}$$

$$+ 2\,(\rho_1^{++}-1)\,(\rho_2^{++}-1)\cos(2\tilde{\phi})\,\tau_{TT}$$

$$+ 8\left[\frac{(\rho_1^{00}+1)\,(\rho_2^{00}+1)}{(\rho_1^{++}-1)\,(\rho_2^{++}-1)}\right]^{1/2}\cos\tilde{\phi}\,\tilde{\tau}_{TL}$$

$\boldsymbol{\gamma\gamma \to \pi^0\pi^0}$

$$+ h_1 h_2\,4\,[(\rho_1^{00}+1)\,(\rho_2^{00}+1)]^{1/2}\,\tilde{\tau}_{TT}$$

$$+ h_1 h_2\,8\,[(\rho_1^{++}-1)\,(\rho_2^{++}-1)]^{1/2}\cos\tilde{\phi}\,\tau_{TL}\Big\},$$

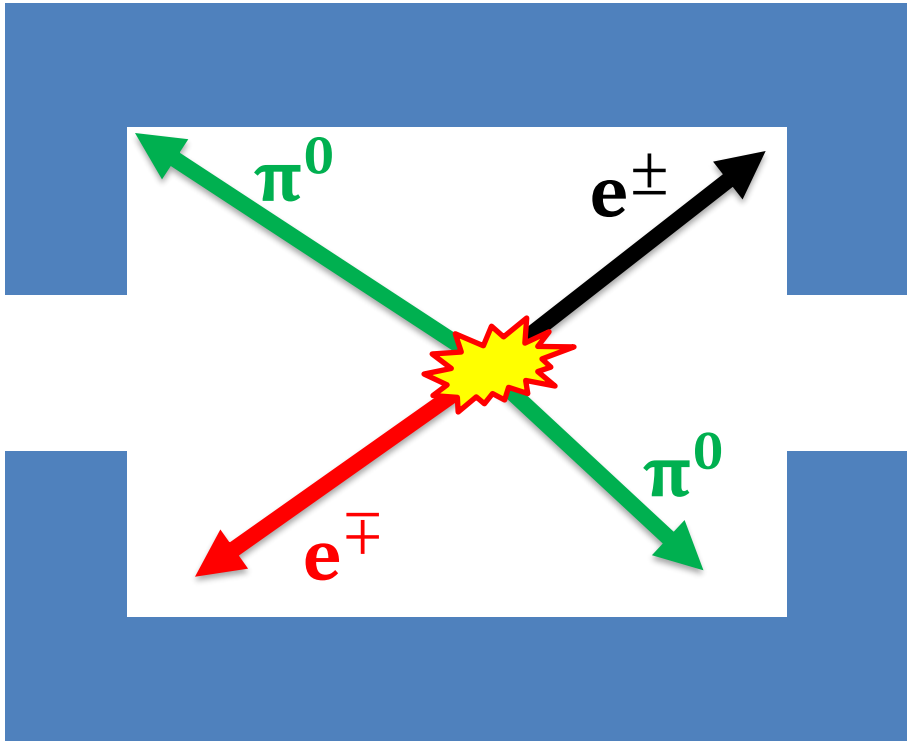
$$\frac{d^3\sigma}{dW\,dQ_1^2\,dQ_2^2} = \frac{d^3L_{TT}}{dW\,dQ_1^2 dQ_2^2}\,\sigma_{TT} + \frac{d^3L_{LT}}{dW\,dQ_1^2 dQ_2^2}\,\sigma_{LT} + \frac{d^3L_{TL}}{dW\,dQ_1^2 dQ_2^2}\,\sigma_{TL} + \frac{d^3L_{LL}}{dW\,dQ_1^2 dQ_2^2}\,\sigma_{LL}$$

The Double Tag Approach

- Photon virtuality equals lepton momentum transfers

$$Q_i^2 \approx 4E_i E_i' \sin^2 \frac{\theta_i}{2}$$

- Very strong drop of cross-sections with virtuality
- Actual double tagging most likely unfeasible
- Solution: Extension of the Single Tagged approach (BAM-1046)

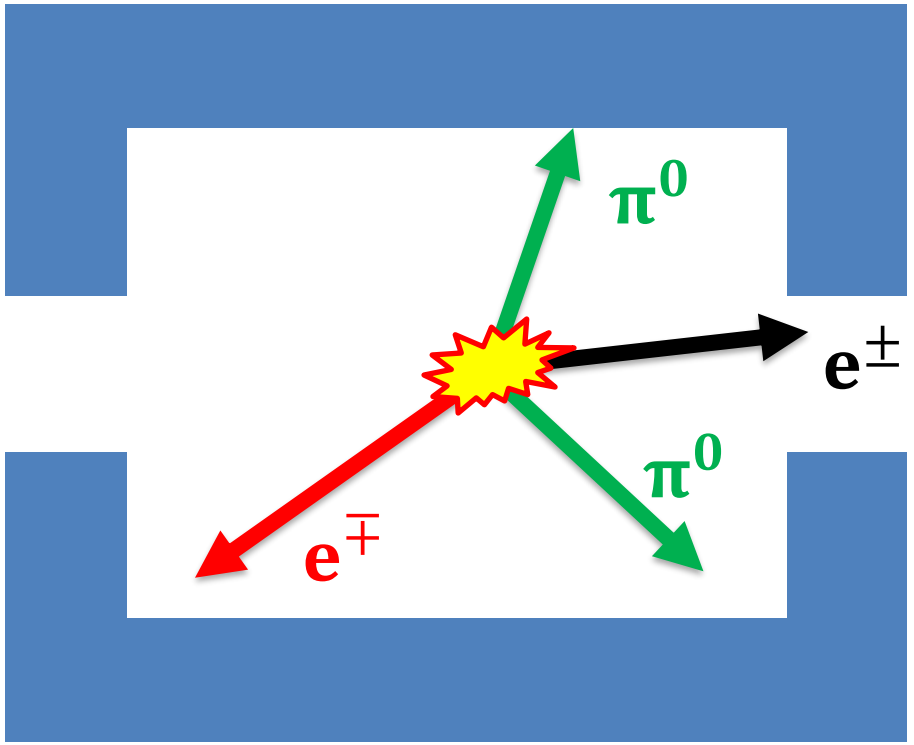


The Double Tag Approach

- Photon virtuality equals lepton momentum transfers

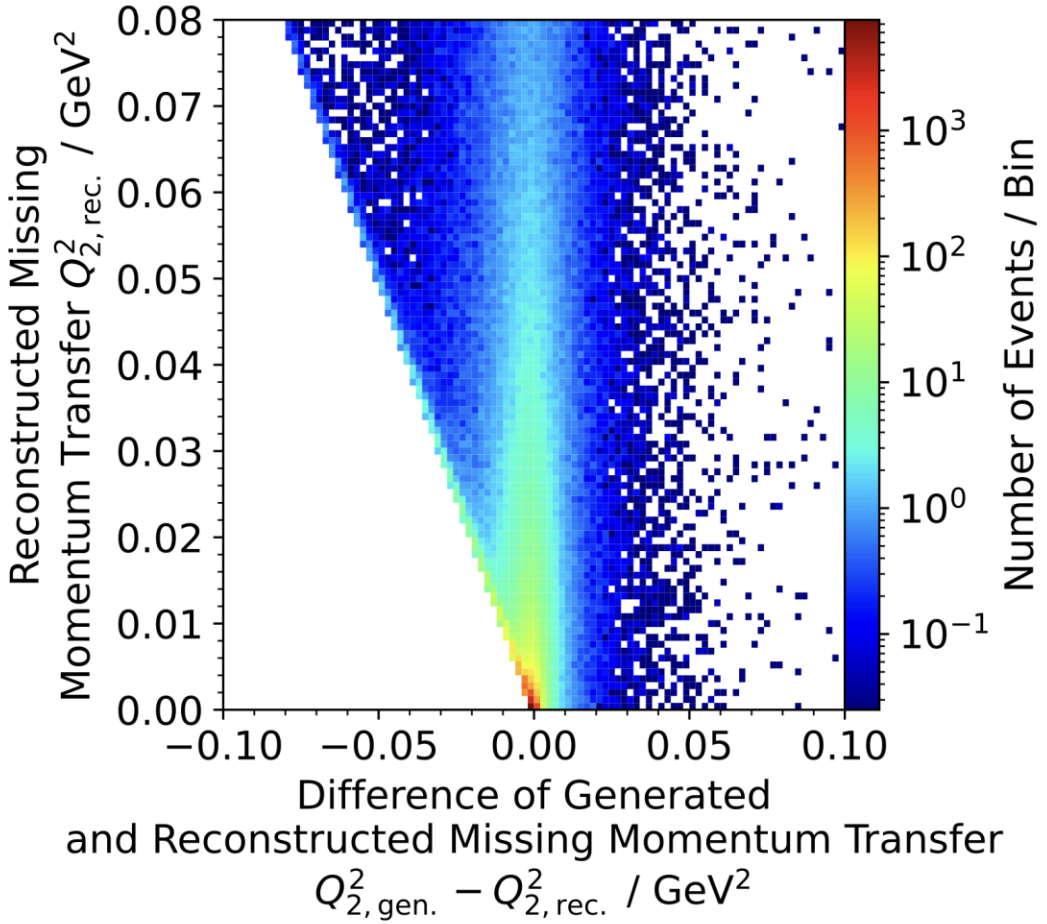
$$Q_i^2 \approx 4E_i E_i' \sin^2 \frac{\theta_i}{2}$$

- Very strong drop of cross-sections with virtuality
- Actual double tagging unfeasible with current statistics
- Solution: Extension of the Single Tagged approach (BAM-1046)



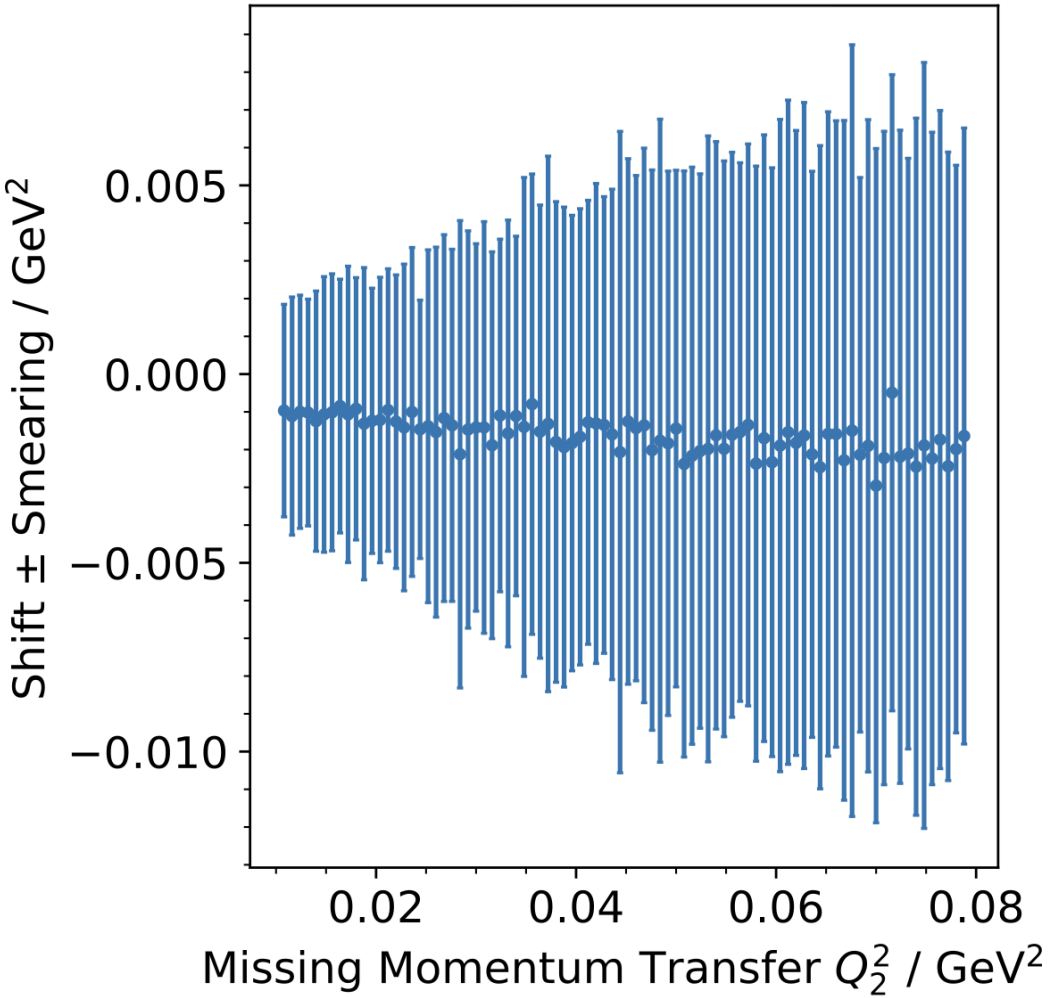
Quick Reminder on BAM-1046

- Event Selection
  - One good charged track
  - Identified as electron/Positron by EoP criterion
  - At least four good photon candidates
  - 1C Kin. Fit (Missing Lepton, $\chi^2 < 200$)
  - 3C Kin. Fit (Two Pions, no convergence required)
  - Use all of the 3.773 GeV data
- Constraint to singly virtual event using cut on the missing momentum transfer
  - Dramatically increased resolution due to kinematic fit
- Use same selection in this analysis!



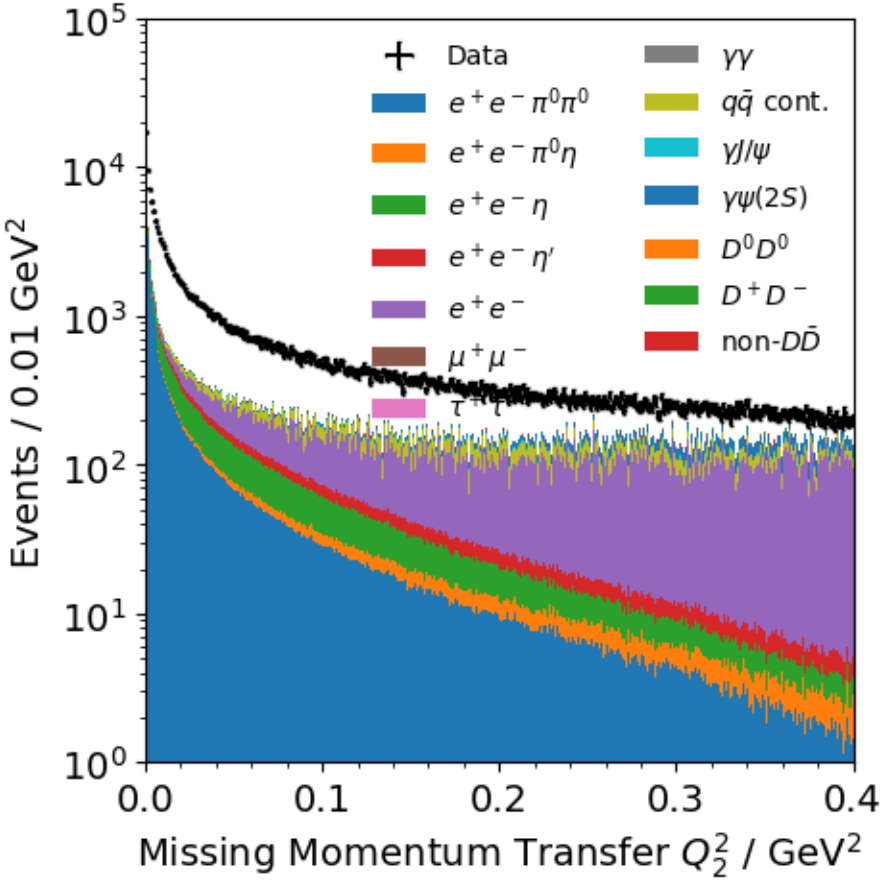
Quick Reminder on BAM-1046

- Event Selection
  - One good charged track
  - Identified as electron/Positron by EoP criterion
  - At least four good photon candidates
  - 1C Kin. Fit (Missing Lepton, $\chi^2 < 200$)
  - 3C Kin. Fit (Two Pions, no convergence required)
  - Use all of the 3.773 GeV data
- Constraint to singly virtual event using cut on the missing momentum transfer
  - Dramatically increased resolution due to kinematic fit
- Use same selection in this analysis!



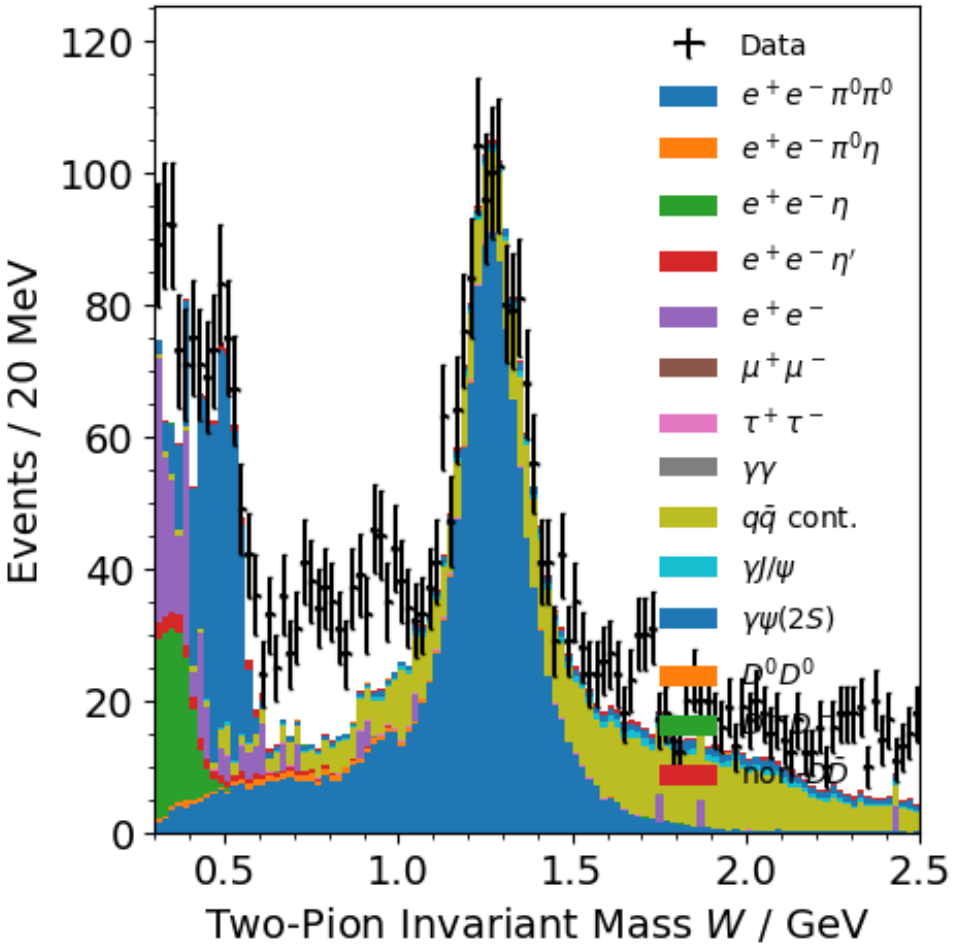
Event Selection – Missing Q²

- High level of background contamination at larger Momentum transfer
- Going much above Q²~0.3 GeV² probably not possible due to large background and small signal
- At very small Q² contamination from singly virtual production
- Q² range ~ 0.08 GeV² - 0.3 GeV²



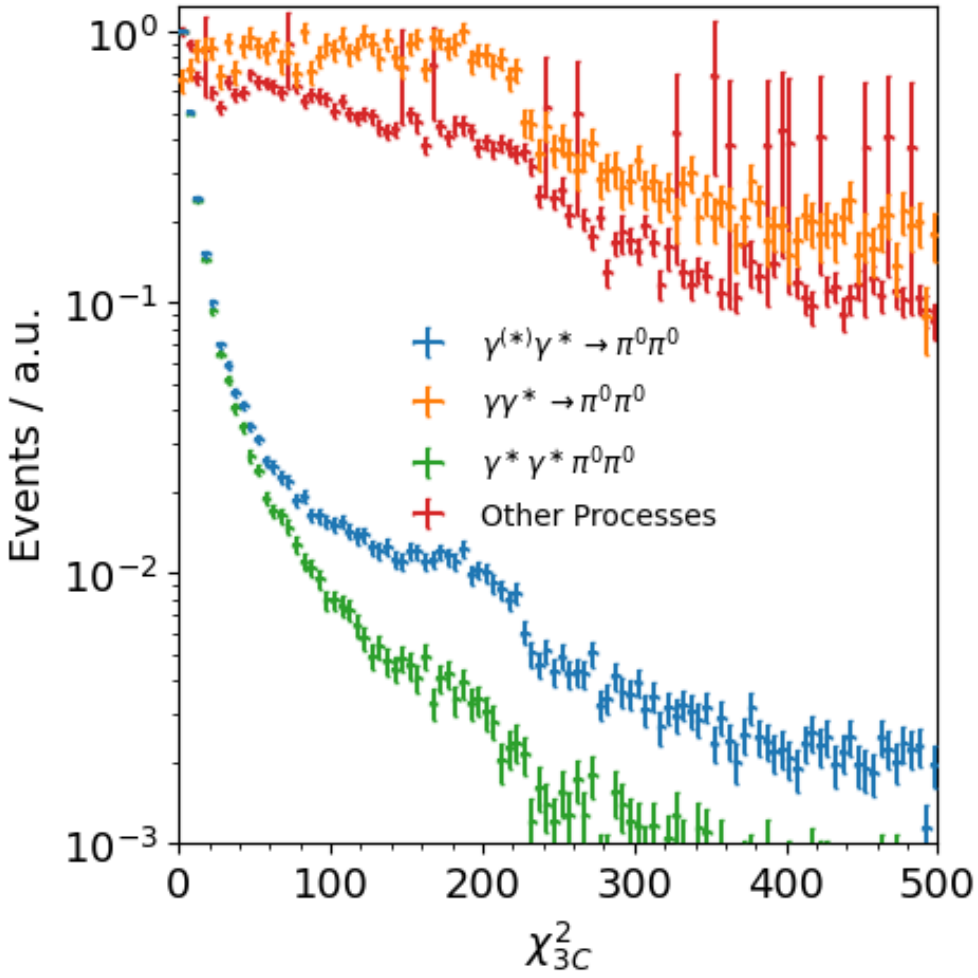
Event Selection – Converged 3C Kin. Fit

- After convergence requirement of clear indication of $f_2(1270)$
- Large amount of backgrounds at smaller and larger masses
- Generally small statistics
- Restrict cross section measurement to $f_2(1270)$ mass range 1-1.5 GeV



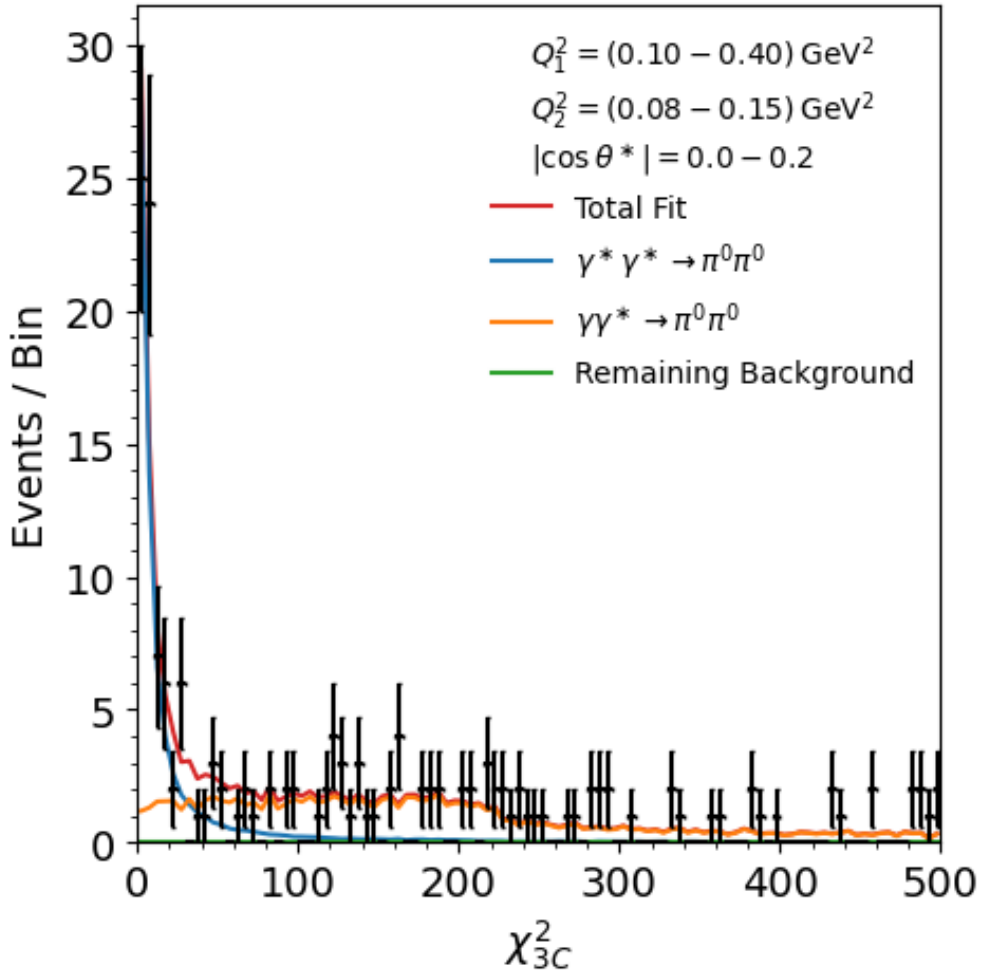
Background Subtraction

- Fit χ^2_{3C} spectra
  - Similar to BAM-1046
- Additional complication: Contamination from Singly virtual events
  - Significantly different shapes of χ^2 distributions
- Fit different MC distributions and polynomials to the data to extract signal



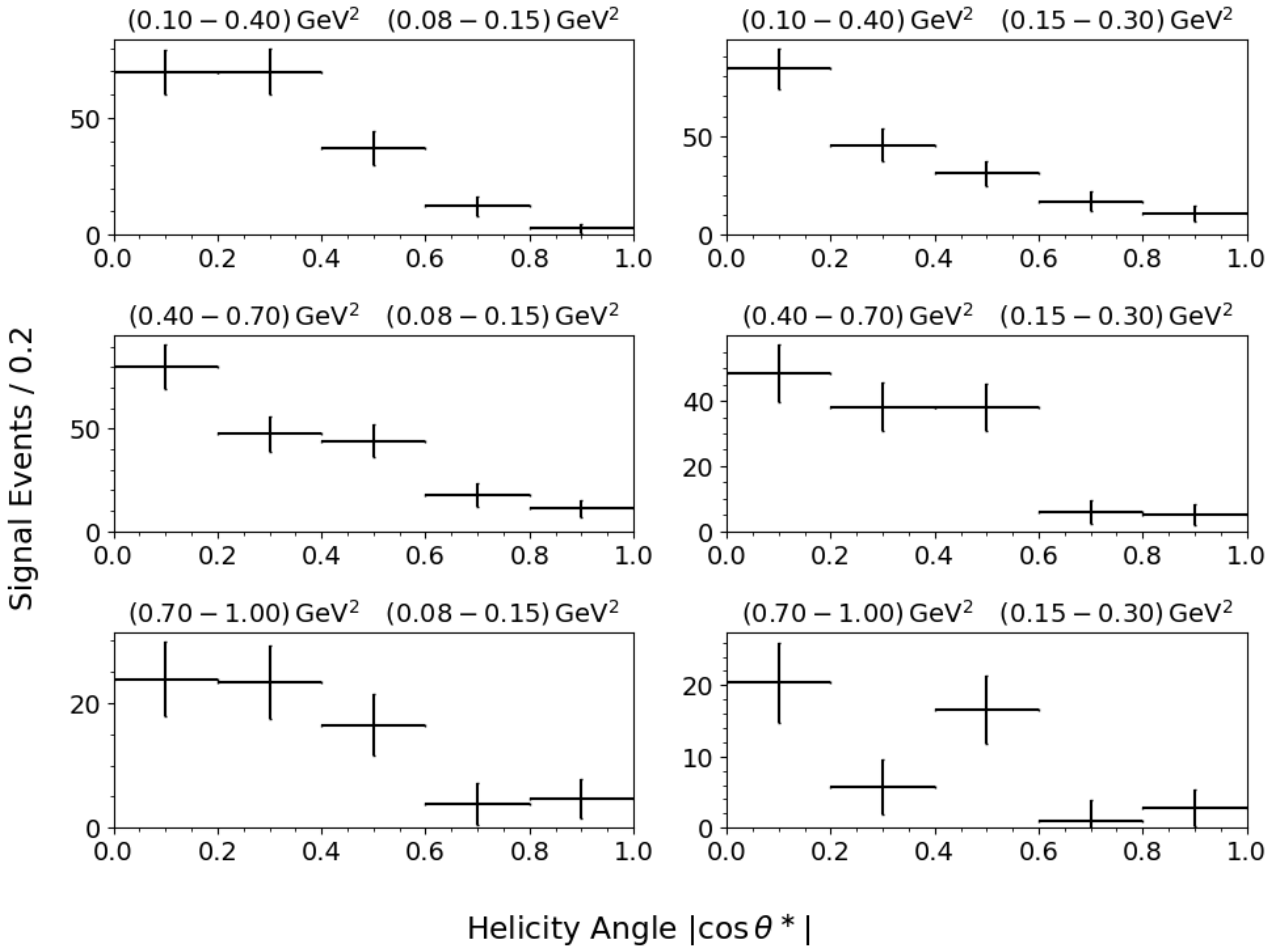
Background Subtraction

- Fit χ^2_{3C} spectra
  - Similar to BAM-1046
- Additional complication: Contamination from Singly virtual events
  - Significantly different shapes of χ^2 distributions
- Fit different MC distributions and polynomials to the data to extract signal

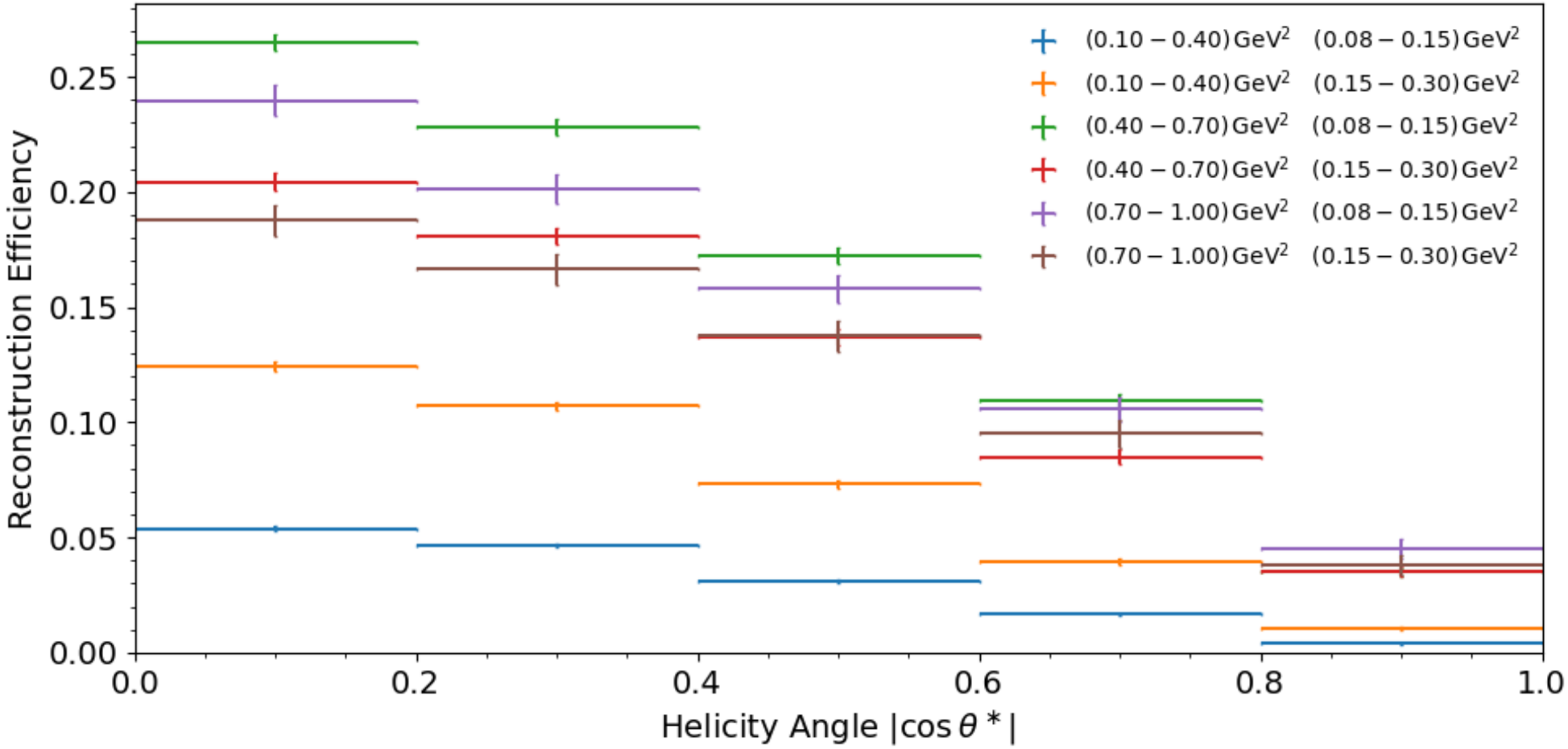


Background Subtraction

- Fit χ^2_{3C} spectra
  - Similar to BAM-1046
- Additional complication: Contamination from Singly virtual events
  - Significantly different shapes of χ^2 distributions
- Fit different MC distributions and polynomials to the data to extract signal

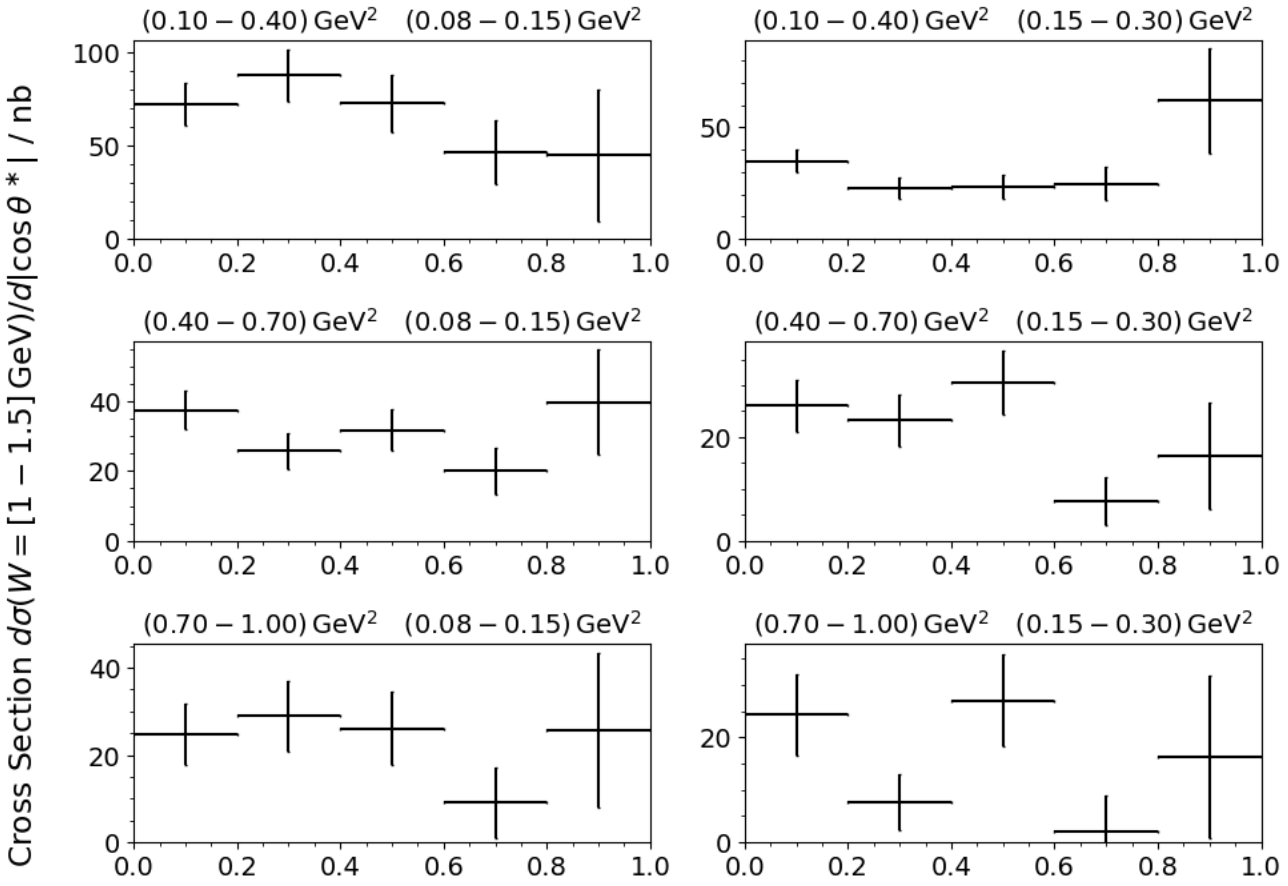


Reconstruction Efficiency



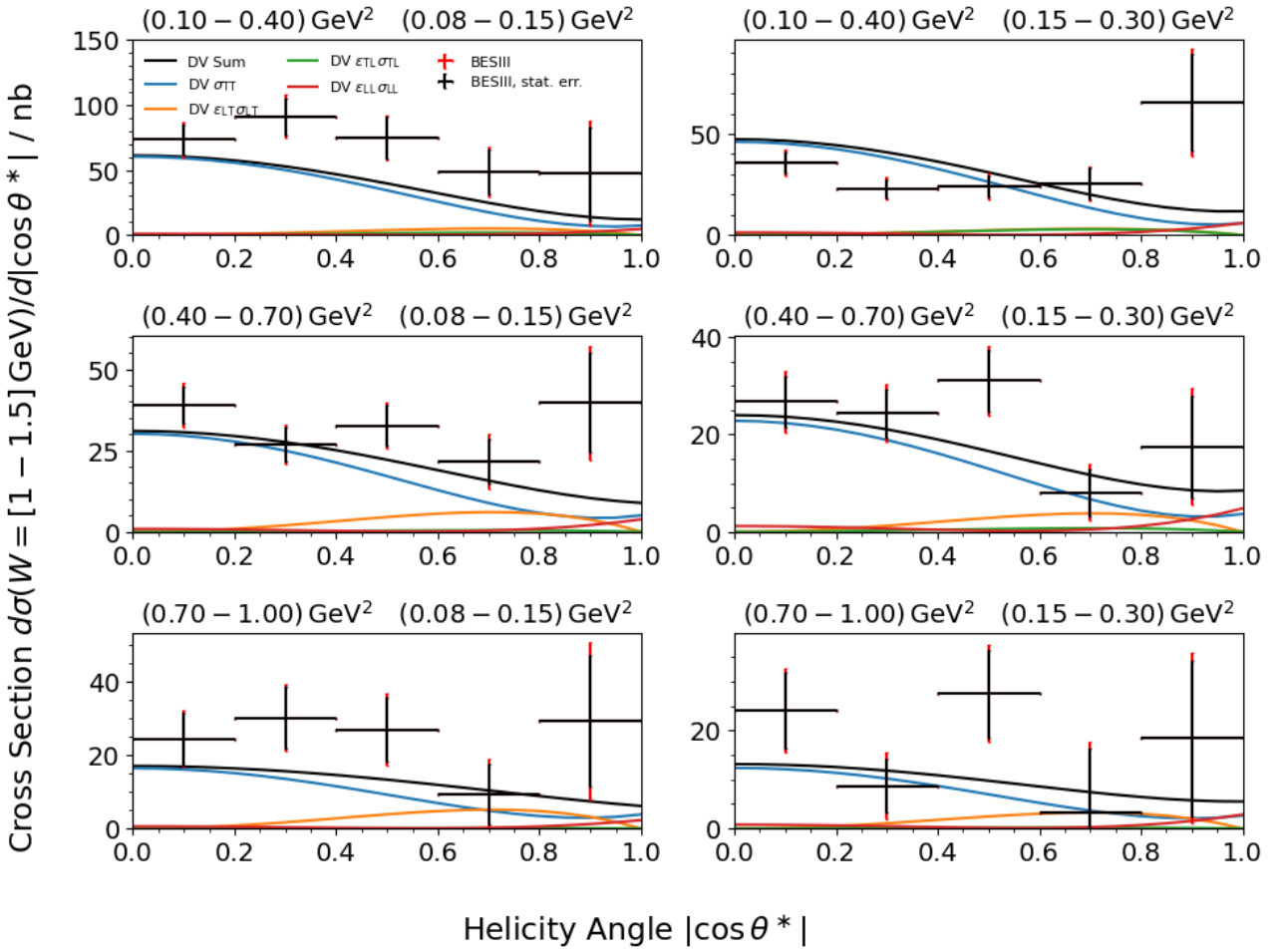
Determination of Cross Sections

$$d\sigma/d|\cos\theta^*| = d(\sigma_{TT} + \varepsilon_{LT}\sigma_{LT} + \varepsilon_{TL}\sigma_{TL} + \varepsilon_{LL}\sigma_{LL})/|\cos\theta^*| = \frac{N}{\Delta_{Q_1^2}\Delta_{Q_2^2}\Delta_W\Delta_{|\cos\theta^*|}\,\varepsilon\, L_{int}\,\frac{d^3L_{TT}}{dQ_1^2 dQ_2^2\, dW}}$$

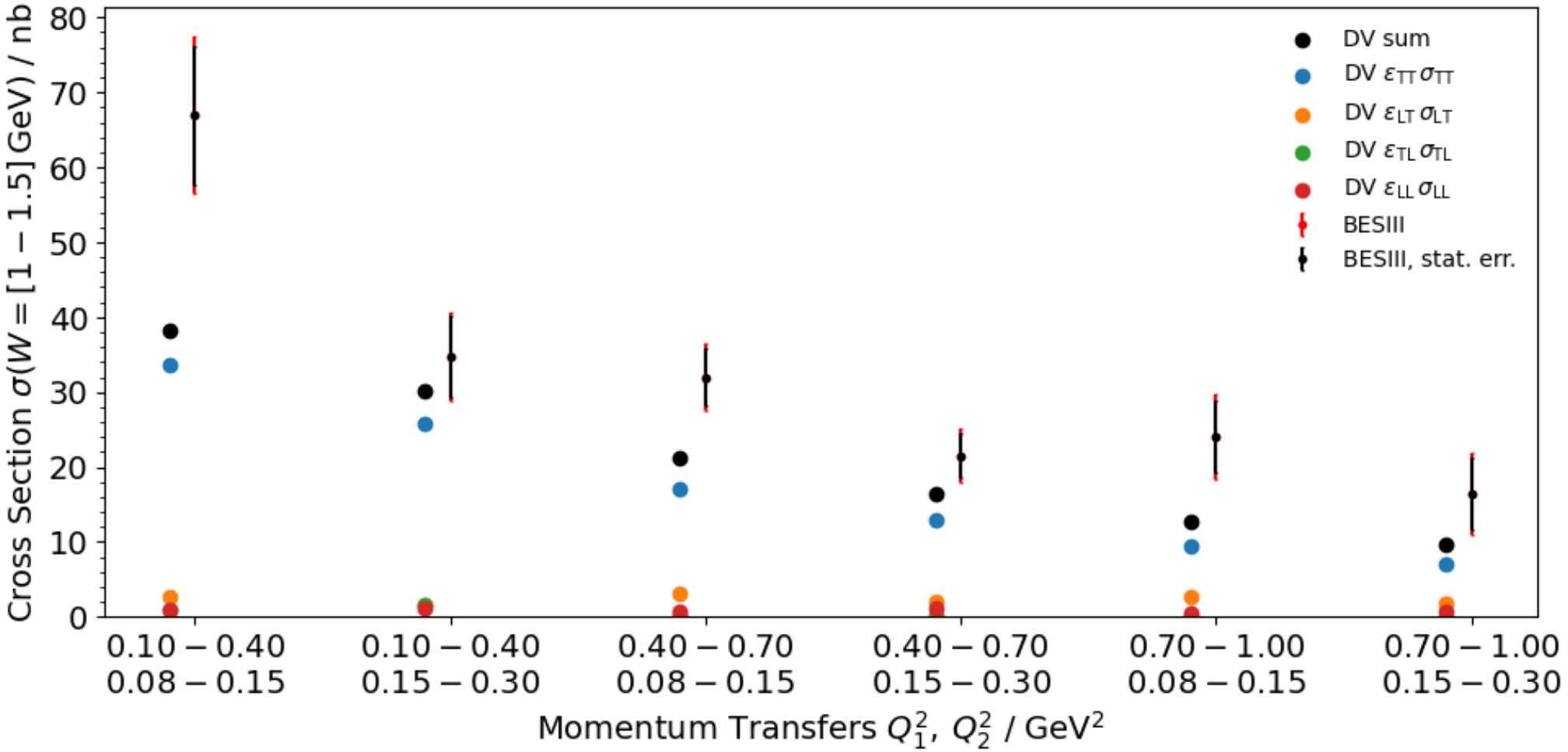


Systematic Uncertainties

- Determination equivalent to BAM-1046
  - Electron/Pion Tagging
  - Luminosity
  - Analysis Constraints
  - ~~Luminosity Function~~
  - Efficiency Correction
  - Background Subtraction
  - ~~Other Two-Photon Processes~~
- Rather conservative but small compared to statistical uncertainty
- Comparison to Dispersive Theory by Danilkin and Vanderhaeghen



Helicity Angle Integrated Cross Section



Extraction of Transition Form Factors

- In BAM-1046 the extraction of transition form factor was possible using PWA
- Here are 5 different helicity contributions
- Only 5 data points in each Q^2 bin: Determination not possible
- Also: Two pairs of two form factors have same helicity angle behavior
  - Dispersive constraints required for determination of Form Factors
  - Model dependent
- S-wave contribution?

Publication of the Cross Sections probably better

Summary and Outlook

- First double virtual measurement of the two photon producton cross section of a tensor state
- Completely new ansatz
- Second doubly virtual measurement of any hadronic final state (after BaBar $\gamma^*\gamma^* \rightarrow \eta'$)
- Muon g-2 relevant momentum transfer range
- Measurement statistically limited
- Memo in preparation