中国科学技术大学
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Cross Section Measurement of $e^+e^- \to K_S^0 K_L^0$ via ISR Method

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Outline

1. Motivation
2. Data Set & MC Samples
3. Event Selection & Selection Results
4. Background analysis
5. Resolution and Unfolding
6. Cross Section
7. Systematic Uncertainty
8. Summary and Outlook

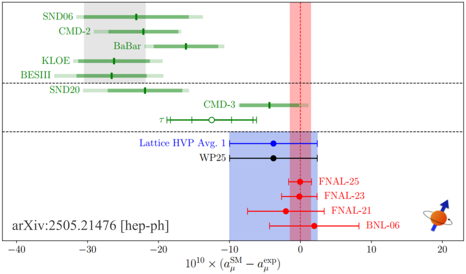
Motivation

Muon Anomalous Magnetic Moment:

- SM value (lattice QCD) of a_μ in WP25 agrees with the measured average.
- The data driven value in WP20 differs from the new result and CMD-3 value.
- It is important to understand the differences between these results.
- Uncertainty of the data driven value is dominated by HVP (hadron vacuum polarization) (83%) and HLbL (hadron light by light).

Contributions to Uncertainty of a_μ

Contribution	Value $\times 10^{11}$
Experiment (2023)	116 592 059(22)
Experiment (2025)	116 592 071.5(14.5)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value (2020)	116 591 810(43)
QED	116 584 718.8(2)
Electroweak	154.4(4)
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	7045(61)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)
Total SM Value (2025)	116 592 033(62)



Difference between Experiment(2025) and SM(2025)

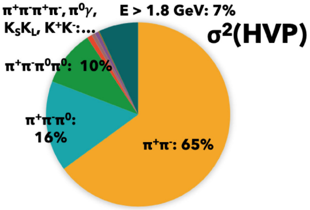
Motivation

Contributions to $a_\mu^{\mathrm{HVP,LO}}$ of Exclusive Channels:

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty} \frac{K(s)}{s} R(s)\, ds,\;\; R(s) = \frac{\sigma^0(e^+e^- \to hadrons(+\gamma))}{\sigma_{\mathrm{pt}}},\;\; \sigma_{\mathrm{pt}} = \frac{4\pi\alpha^2}{3s}$$

Contributions to $a_\mu^{\mathrm{HVP,LO}}$ of Exclusive Channels (2020)

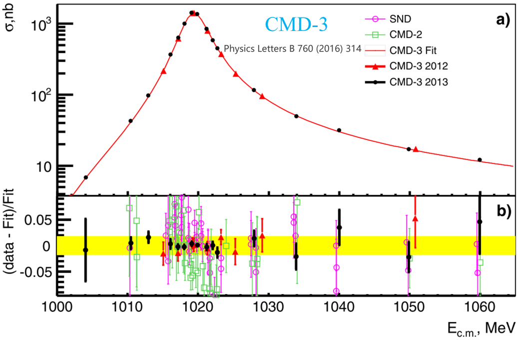
Channel	DHMZ19 ($\times 10^{10}$)	KNT19 ($\times 10^{10}$)
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(0.56)
J/ψ, $\psi(2S)$	7.76(12)	7.84(0.19)
[3.7, ∞) GeV	17.15(31)	16.95(0.19)
Total $a_\mu^{\mathrm{HVP,LO}}$	694.0(1.0)(3.5)(1.6)(0.1)$_\psi$(0.7)$_{\mathrm{DV+QCD}}$	692.8(2.4)

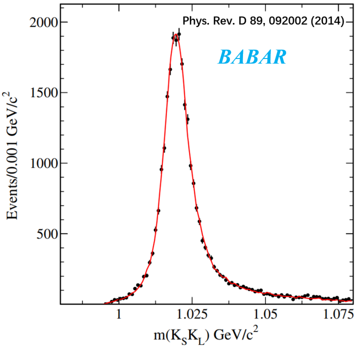


[Data from: Phys.Rep 887 (2020) 1-166]

Contributions to $a_\mu^{\mathrm{HVP,LO}}$ of Exclusive Channels (2020)

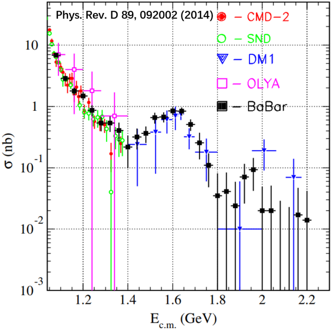
Motivation

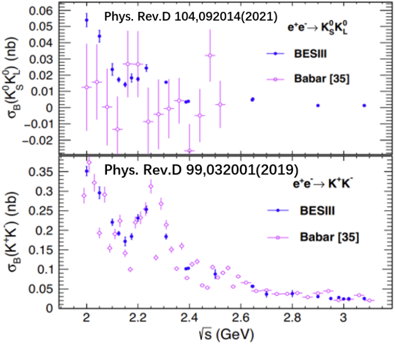




- These experiments are in good agreement for $K_S^0 K_L^0$.

Motivation





- Substantial structures are evident in the center-of-mass energy range 1.1 - 2.4 GeV.
- BESIII can also contributes to these problems using initial state radiation (ISR) method.

Data Set and MC Samples

Data sets

- $\sqrt{s}$ =4.178 GeV data for feasibility study at BOSS 7.0.3.
- Full set of $\psi(3770)$ data at BOSS 7.1.2 for the final result.

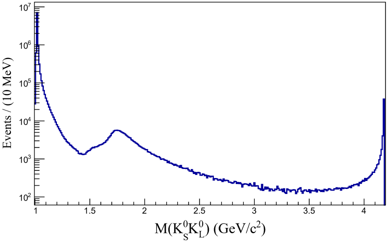
MC samples ($\sqrt{s}$ =4.178 GeV)

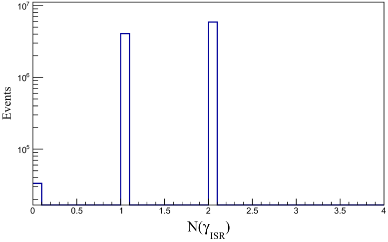
- $e^+e^- \to \gamma_{\rm ISR} K^0_S K^0_L$ PHOKHARA v9.1, 10M 59×data
- $e^+e^- \to \gamma_{\rm ISR} \pi^+\pi^-$ PHOKHARA v9.1, 10M 3×data
- $e^+e^- \to$ hadrons HYBRID, 80M 1.04×data
- $e^+e^- \to \gamma_{\rm ISR} e^+e^-$ BABAYAGA, 542M 0.4×data
- $e^+e^- \to \gamma_{\rm ISR} \mu^+\mu^-$ BABAYAGA, 88.94M 10.0×data
- $e^+e^- \to \gamma_{\rm ISR} \tau^+\tau^-$ KKMC, 11M 1×data
- $e^+e^- \to e^+e^- X$ exclusive MC:

$e^+e^- \to e^+e^- K^+K^-$ GALUGA, 5.4M 7.4×data
$e^+e^- \to e^+e^- \pi^+\pi^-$ GALUGA, 5.42M 0.64×data
$e^+e^- \to e^+e^- \eta$ EKHARA, 5.43M 5.15×data
$e^+e^- \to e^+e^- \eta'$ EKHARA, 5.43M 5.55×data
$e^+e^- \to e^+e^- e^+e^-$ DIAG36, 365M 4.0×data
$e^+e^- \to e^+e^- \mu^+\mu^-$ DIAG36, 108.94M 1.0×data

Signal MC Truth

$M(K_S^0 K_L^0)$ and $N(\gamma_{ISR})$





- The kaon form factor is obtained by Bruch, Khodjamirian and Kühn.
- Phokhara considers NLO ISR effect.

Preliminary Selection

Good showers and ISR photon selection

- $E_{\text{shower}} \geq 25$ MeV (Barrel, $|\cos\theta| \leq 0.83$)
- $E_{\text{shower}} \geq 50$ MeV (EndCap, $0.85 \leq |\cos\theta| \leq 0.93$)
- EMC time: 0 < t <700 ns
- $1 < N_{\text{good showers}} < 20$
- The most energetic shower should have an energy exceeding 0.4 GeV, which is regarded as the ISR photon.

Charged tracks selection

- $|V_z| < 20$ cm and $|\cos\theta| < 0.93$, no requirement for V_r
- PID: prob(e)<prob(π)
- $N_+ = N_- = 1$

Preliminary Selection

Vertex fit for K_S^0

- Decay point is reconstructed by a vertex fit of the π^+ and π^- tracks
- $|M(\pi^+\pi^-) - M_{K_S^0}^{\mathrm{PDG}}| \leq 0.05\ \mathrm{GeV}/c^2$
- A successful secondary vertex fit
- $L/\sigma_L \geq 2$

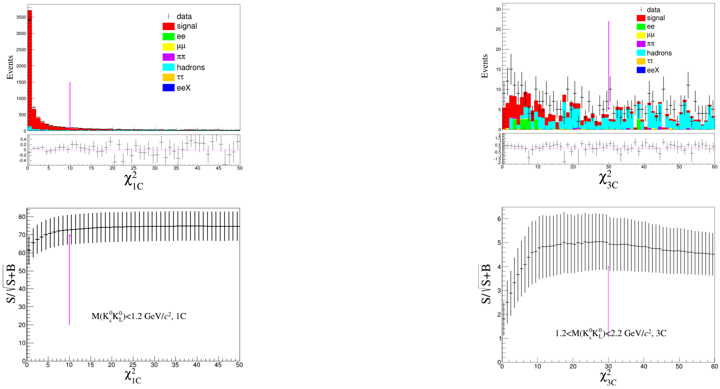
1C kinematic fit for $\boldsymbol{K_L^0}$ (low energy region, $\sqrt{s} < 1.2$ GeV)

- Include the ISR photon, K_S^0 and a missing K_L^0 track.
- Fix the mass of K_L^0 to the value provided by PDG.

3C kinematic fit for $\boldsymbol{K_L^0}$ (high energy region, $\sqrt{s} > 1.2$ GeV)

- Looping good showers other than ISR photon.
- Regarding the shower with minimum χ^2 as K_L^0 shower.
- $\theta(K_L^0\ \text{shower}, \text{fitted}\ K_L^0) < 0.5$ rad.

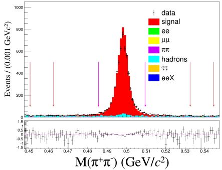
Further Selection

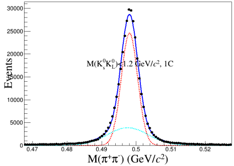


- S: scaled signal MC.
- S+B: data.
- χ^2_{1C} is required to be less than **10**.

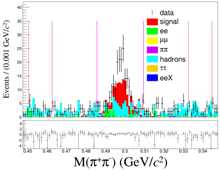
- S: scaled signal MC.
- S+B: data.
- χ^2_{3C} is required to be less than **30**.

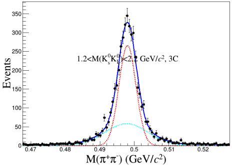
Further Selection





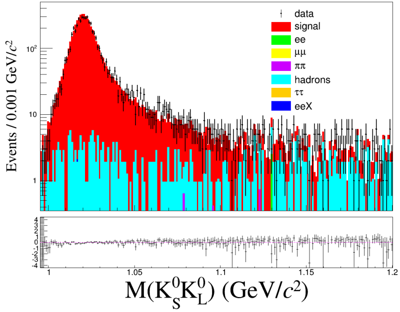
- $\sigma = 0.00374$ GeV/c^2
- Signal region:
  $|M(\pi^+\pi^-) - M_{K_S^0}^{PDG}| \leq 0.012$ GeV/c^2.
- Side band:
  0.035 GeV/$c^2 \leq |M(\pi^+\pi^-) - M_{K_S^0}^{PDG}| \leq 0.047$ GeV/c^2.

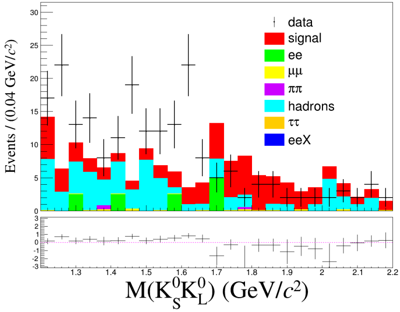




- $\sigma = 0.00428$ GeV/c^2
- Signal region:
  $|M(\pi^+\pi^-) - M_{K_S^0}^{PDG}| \leq 0.012$ GeV/c^2.
- Side band:
  0.035 GeV/$c^2 \leq |M(\pi^+\pi^-) - M_{K_S^0}^{PDG}| \leq 0.047$ GeV/c^2.

$M(K_S^0K_L^0)$



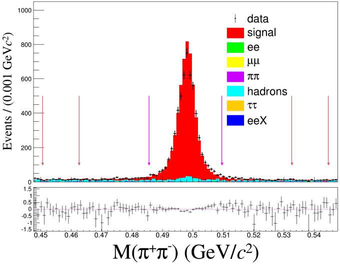


- $M(K_S^0K_L^0) < 1.2$ GeV/c^2
- 1C fit.

- $1.2 < M(K_S^0K_L^0) < 2.2$ GeV/c^2.
- 3C fit.

Hadron Topology (1C)

rowNo	decay tree	number
1	$x \to \pi^0\pi^+\pi^-\gamma^F\gamma^F$	63
2	$x \to \pi^0\pi^+\pi^-\gamma^F$	48
3	$x \to \pi^0 K_L^0 K_S^0 \gamma^f,\ K_S^0 \to \pi^+\pi^-$	39
4	$x \to \pi^0\pi^0\pi^+\pi^-\gamma^F\gamma^F$	28
5	$x \to \pi^0\pi^0\pi^+\pi^-\gamma^F$	25
6	$x \to \eta\phi,\ \eta \to \gamma\gamma,\ \phi \to K_L^0 K_S^0,\ K_S^0 \to \pi^+\pi^-$	19
7	$x \to vgam\gamma,\ vgam \to \eta\phi,$ $\eta \to \gamma\gamma,\ \phi \to K_L^0 K_S^0,\ K_S^0 \to \pi^+\pi^-$	13
8	$x \to K^+K^-\gamma^F\gamma^F$	13
9	$x \to K^+K^-\gamma^F$	11
10	$x \to \pi^0 a_0^0 \gamma^f,\ a_0^0 \to K_S^0 K_S^0,$ $K_S^0 \to \pi^0\pi^0,\ K_S^0 \to \pi^+\pi^-$	6

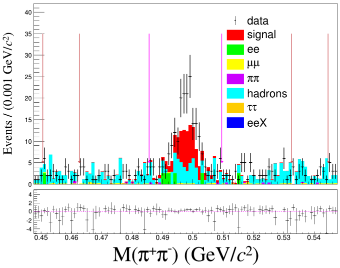


- The peaking backgrounds of $M(K_S^0)$ are the channels with K_S^0.
- Considering $\pi^0 K_S^0 K_L^0$ and $\phi\eta$.

Hadron Topology (3C)

rowNo	decay tree	number
1	$x \to \pi^0\pi^0\pi^+\pi^-\gamma^F$	16
2	$x \to \pi^0\pi^+\pi^-\gamma^F\gamma^F$	11
3	$x \to \pi^0\pi^+\pi^-\gamma^F$	10
4	$x \to \pi^0 K_L^0 K_S^0 \gamma^f,\ K_S^0 \to \pi^+\pi^-$	5
5	$x \to \pi^0\pi^0\pi^+\pi^-\gamma^F\gamma^F$	4
6	$x \to \pi^+\pi^-\gamma^F\gamma^F$	4
7	$x \to \eta\omega,\ \eta \to \gamma\gamma,\ \omega \to \pi^0\pi^+\pi^-$	3
8	$x \to K_S^0\bar{n}\Lambda\gamma^f,\ K_S^0 \to \pi^+\pi^-,\ \Lambda \to \pi^0 n$	2
9	$x \to K_S^0 n\bar{\Lambda}\gamma^f,\ K_S^0 \to \pi^+\pi^-,\ \bar{\Lambda} \to \pi^0\bar{n}$	2

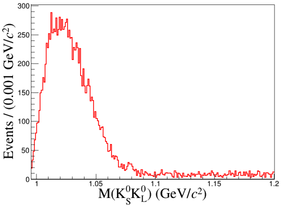
- Considering $\pi^0 K_S^0 K_L^0$ and $K_S^0\bar{n}\Lambda(K_S^0 n\bar{\Lambda})$.



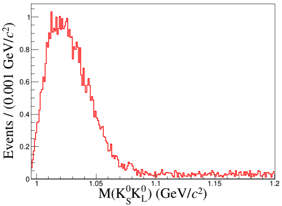
$\phi\eta$: MC Estimation

$\sqrt{s} = 4.178$ GeV, 1C fit method, $\eta \to \gamma\gamma$.

- Generating MC with reference to Phys. Rev. D 104, 032007 (2021) and Phys. Rev. D 108, 112011 (2023).
- $\sigma^{obs}(\phi\eta) = 12.126$ pb and $\mathcal{L}_{\text{int}} = 3149.5\ \text{pb}^{-1}$ at $\sqrt{s} = 4.178$ GeV
- Normalization constant: $\frac{12.126\times3149.5}{1000000} \times 0.3936 \times 0.339 \times 0.692 = 3.58 \times 10^{-3}$

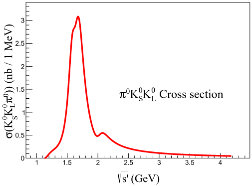


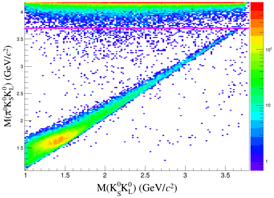
$\times$**3.58**$\times$ **10^{-3}** =

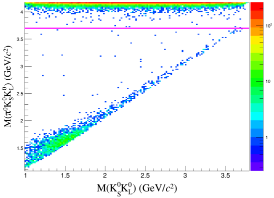


N = 46.69±0.82

$\pi^0 K_S^0 K_L^0$



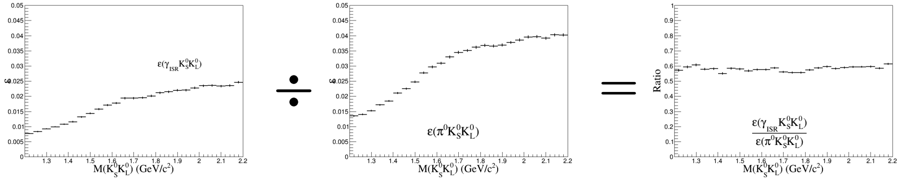




- $\pi^0 K_S^0 K_L^0$ cross section.
  - PHSP MC, 10M.
  - Born process: $M(\pi^0 K_S^0 K_L^0) > 3.7\ \text{GeV}/c^2$
  - ISR process: $M(\pi^0 K_S^0 K_L^0) < 3.7\ \text{GeV}/c^2$
- 1C kinematic fit.
- 3C kinematic fit.

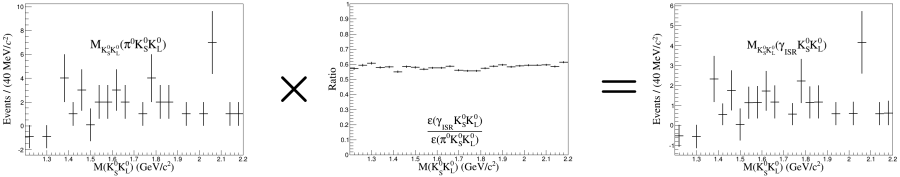
Born Process: Data Driven

$\sqrt{s} = 4.178$ GeV, 3C fit method.



- $\varepsilon(\gamma_{\mathrm{ISR}} K_S^0 K_L^0)$ is the efficiency of $\pi^0 K_S^0 K_L^0$ applied $\gamma_{\mathrm{ISR}} K_S^0 K_L^0$ selection criteria
- $\varepsilon(\pi^0 K_S^0 K_L^0)$ is the efficiency of $\pi^0 K_S^0 K_L^0$ applied $\pi^0 K_S^0 K_L^0$ selection criteria

Born Process: Data Driven

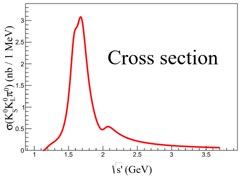


- $M_{K_S^0 K_L^0}(\pi^0 K_S^0 K_L^0)$ is data applied $\pi^0 K_S^0 K_L^0$ selection criteria.
- Result:

$$20.40 \pm 3.71$$

ISR Process: MC Estimation

$\sqrt{s} = 4.178$ GeV, 3C fit method.



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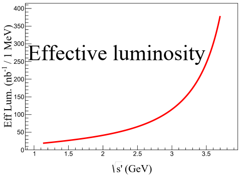

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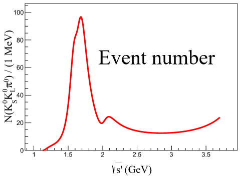
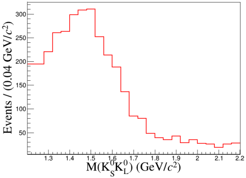


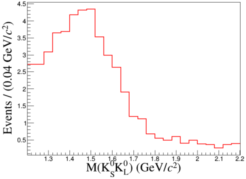
N$(e^+e^- \to \gamma_{\text{ISR}}\pi^0 K_S^0 K_L^0)$ is calculated as 59174.6

N$(e^+e^- \to \gamma_{\text{ISR}}\pi^0 K_S^0 K_L^0 \to \gamma_{\text{ISR}}\pi^0\pi^+\pi^- K_L^0) = 59174.6 \times 0.692 = 40948.82$

Normalization constant: $\frac{40948.82}{2925532} = 0.0140$



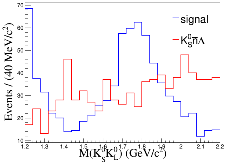
×0.0140 =



N = 46.98±0.82

$K_S^0\bar{n}\Lambda(K_S^0 n\bar{\Lambda})$: MVA

$\sqrt{s} = 4.178$ GeV, 3C fit method.



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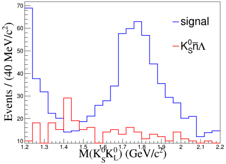
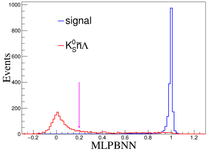
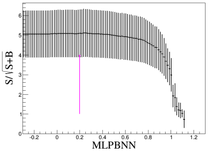




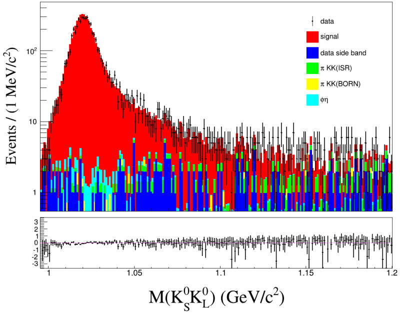


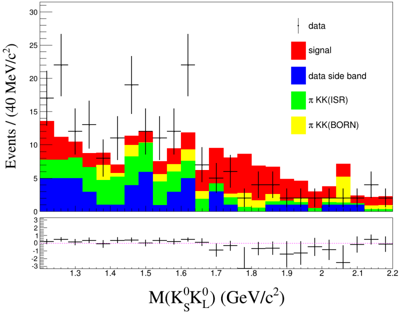
- Signal MC is scaled to the same size as $K_S^0\bar{n}\Lambda$ MC.
- Input variables:

  seed energy of $\gamma_{\rm ISR}$
  3×3 energy of $\gamma_{\rm ISR}$
  lateral moment of $\gamma_{\rm ISR}$
  total energy of $\gamma_{\rm ISR}$

- Requiring MLPBNN score>0.2.

$M(K_S^0 K_L^0)$

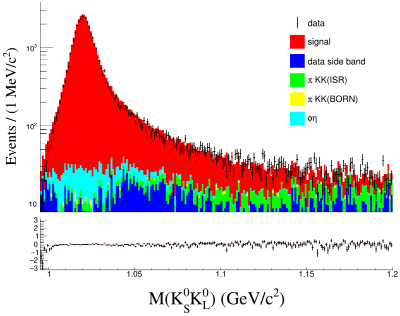


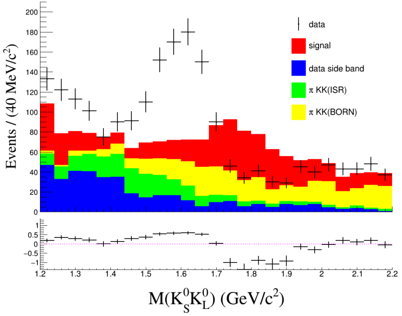


- $M(K_S^0 K_L^0) < 1.2\ \mathrm{GeV}/c^2$
- 1C kinematic fit.

- $1.2 < M(K_S^0 K_L^0) < 2.2\ \mathrm{GeV}/c^2$.
- 3C kinematic fit.

$M(K_S^0 K_L^0)$, $\psi(3770)$ Data





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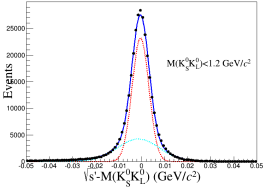
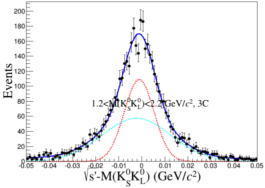
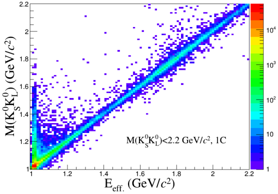
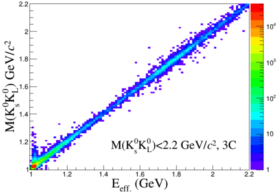
- 3C kinematic fit.

Resolution



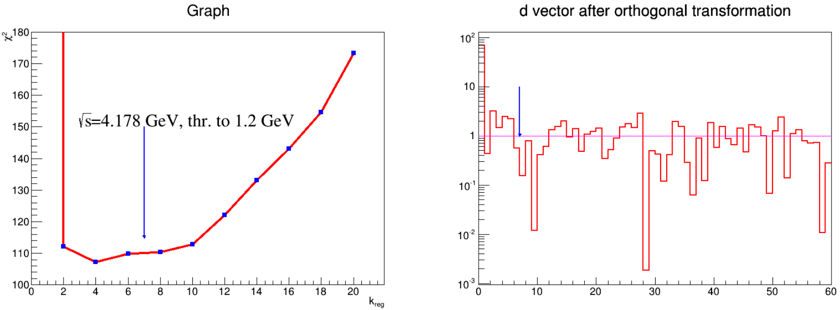






- $\sigma(M^{\text{truth}}_{K_S^0K_L^0} - M^{\text{rec.}}_{K_S^0K_L^0}) = 5.97\ \text{MeV}/c^2$
- $\sigma(M^{\text{truth}}_{K_S^0K_L^0} - M^{\text{rec.}}_{K_S^0K_L^0}) = 11.90\ \text{MeV}/c^2$
- An unfolding process is necessary for low energy region.

Unfolding



- $\chi^2 = \sum_{\text{bin}} \frac{(N_{\text{unfolded}} - N_{\text{truth}})^2}{\sigma^2_{\text{truth}}}$
- Unfolding region: from threshold to 1.2 GeV/c^2.
- Choosing $k_{\text{reg}} = 7$.

Cross Section

- Cross section calculation formula:

$$\sigma^0_{K^0_S K^0_L}(M_{K^0_S K^0_L}) = \frac{(\mathrm{d}N_{\mathrm{sig}}/\mathrm{d}M_{K^0_S K^0_L})_{\mathrm{unfolding}}}{\mathcal{B}(K^0_S \to \pi^+\pi^-)\varepsilon \cdot \mathrm{d}\mathcal{L}_{\mathrm{int}}/\mathrm{d}M_{K^0_S K^0_L}}$$

- $(\mathrm{d}N_{\mathrm{sig}}/\mathrm{d}M_{K^0_S K^0_L})_{\mathrm{unfolding}}$ is the ratio of the signal event number in each bin to the bin width.
- ε is the efficiency calculated by signal MC.
- $\mathcal{B}(K^0_S \to \pi^+\pi^-)$ is the branching fraction of $K^0_S \to \pi^+\pi^-$
- $\mathrm{d}\mathcal{L}_{\mathrm{int}}/\mathrm{d}M_{K^0_S K^0_L}$ is calculated by $\mathrm{d}\mathcal{L}_{\mathrm{int}}/\mathrm{d}M_{K^0_S K^0_L} = W(s,x) \cdot \mathcal{L}_{\mathrm{int}}$, and $W(s,x)$ is described by:

$$W(s,x) = kx^{k-1}[1 + \frac{\alpha}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) + \frac{3}{4}k + k^2(\frac{37}{96} - \frac{\pi^2}{12} - \frac{1}{72}\ln\frac{s}{m_e^2})]$$
$$-k(1 - \frac{1}{2}x) + \frac{1}{8}k^2[4(2-x)\ln\frac{1}{x} - \frac{1+3(1-x)^2}{x}\ln(1-x) - 6 + x],$$
$$k = \frac{2\alpha}{\pi}(\ln\frac{s}{m_e^2} - 1),\ x = 1 - \frac{q^2}{s}.$$

VP Correction

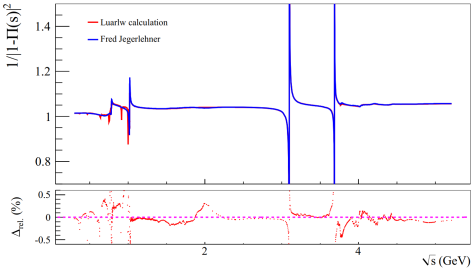
- In perturbative theory, the vacuum polarization (VP) changes the form of the photon propagator in following way:

$$\frac{-ig_{\mu\nu}}{q^2} \rightarrow \frac{-ig_{\mu\nu}}{q^2(1-\Pi(q^2))}$$

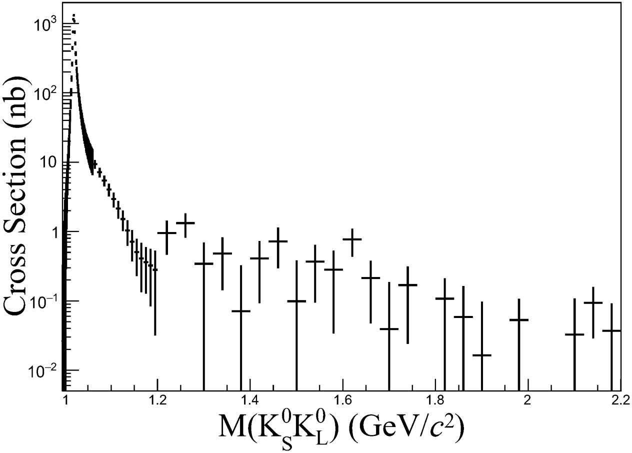
- q^2 is the square of the momentum of the propagator.
- Due to vacuum polarization (VP), the Born cross section will be modified to:

$$\sigma^0(q^2) \rightarrow \frac{\sigma^0(q^2)}{|1-\Pi(q^2)|^2}$$

- Adopting the VP factors of the Fred Jegerlehner group.

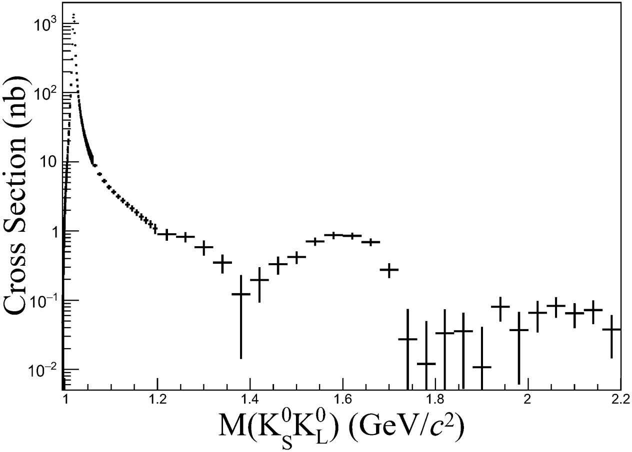


Results $\sqrt{s} = 4.178$ GeV



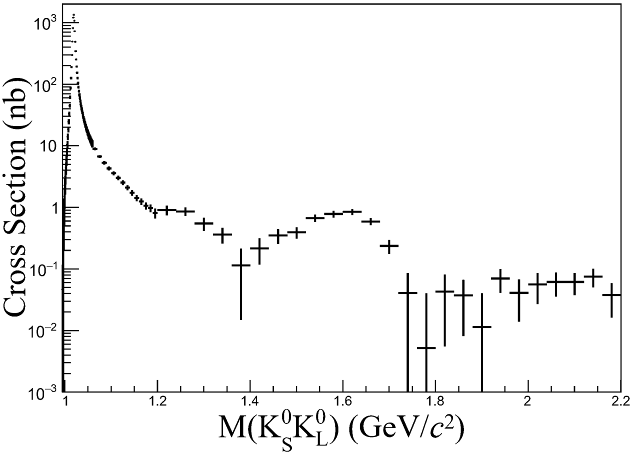
- Uncertainties are statistical only.
- A gap around $M(K^0_S K^0_L) = 1.2$ GeV$/c^2$.

Results $\sqrt{s} = \mathbf{3.773}$ GeV



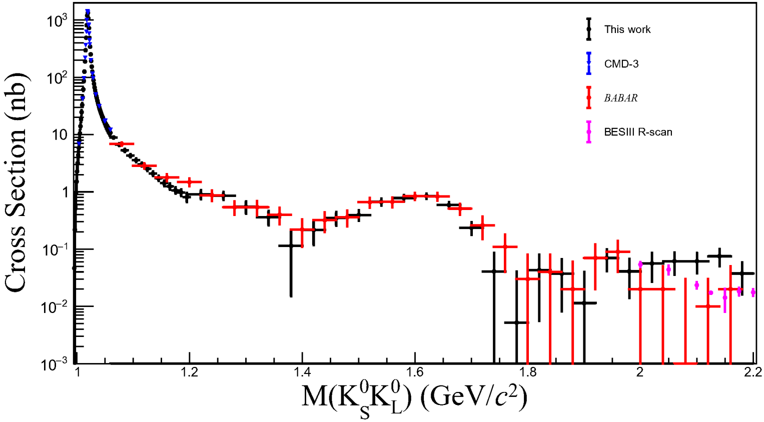
- Uncertainties are statistical only.

Combined Results



- Uncertainties are statistical only.

Comparisons between BESIII and other Experiments



- Uncertainties are statistical only.

Summary and Outlook

Summary

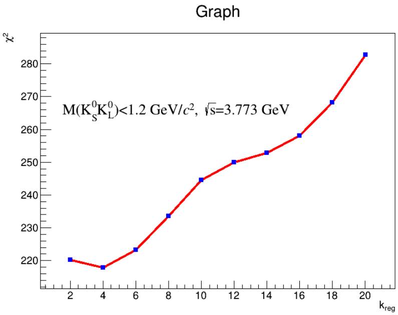
- The event selection criteria have been optimized.
- Preliminary cross sections have been obtained.
- The results are broadly consistent with those of other experiments.

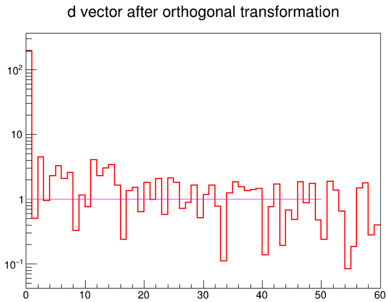
Outlook

- More check about MVA.
- Finish systematic uncertainties analyzing.

Thank You!

Unfolding





- $\sqrt{s} = 3.773$ GeV.
- $k_{reg} = 4$.

Systematic Uncertainty (remain ongoing)

Uncertainty Source(1C)	Method	Uncertainty Value(%)
Effective luminosity	-	0.71($\sqrt{s} = 3.773$ GeV) 1.2($\sqrt{s} = 4.178$ GeV)
ISR photon detection	-	1.0
K_S^0 branching fraction	-	0.08
π PID		
$N_+ = N_- = 1$		
K_S^0 reconstruction	Control sample	1.1($p_{K_S^0} < 1.0$ GeV$/c$) 1.0($p_{K_S^0} > 1.4$ GeV$/c$)
$\chi^2_{1C} < 10$		
MC model		
$\pi 0 K_S^0 K_L^0$		
$\phi\eta$		
other peaking backgrounds		
Side band		
Unfolding		
VP factors		